



Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

Lecture 5 cont.

Dynamics of planar mechanisms.

Dynamics of planar mechanisms

Inverse dynamics problem

Calculation of forces and torques that cause given motion of a mechanism (kinetostatics)

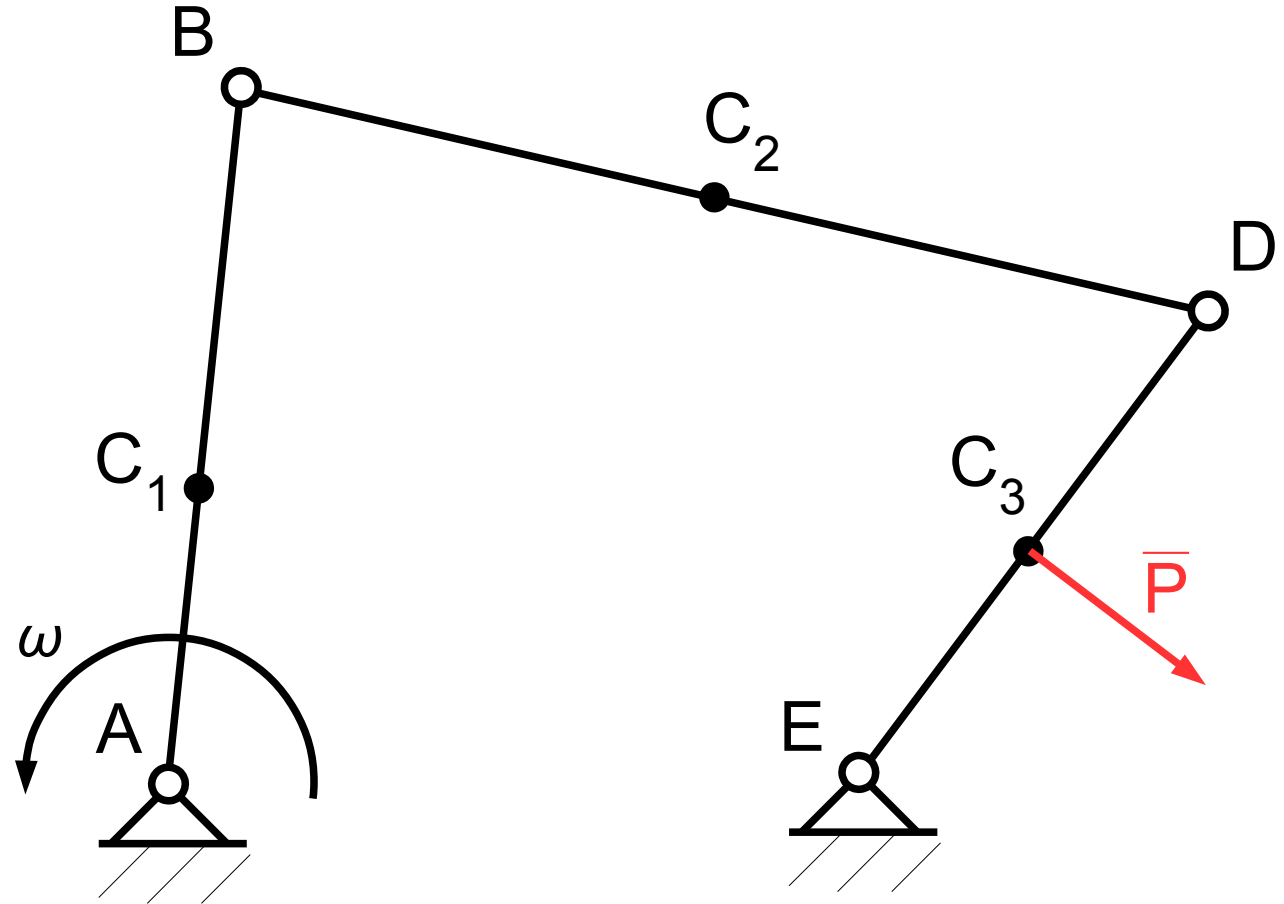
0. Mechanism and its geometry, driving and operating forces/torques, displacement, velocity and acceleration functions are given.
1. Calculation of inertia forces and torques acting moving members of the mechanism.
2. Decomposition of the mechanism with reaction disclosure.
3. Write down vector sums of external forces, reactions and inertia forces (d'Alembert equations).
4. Solve the equations with graphical and/or analytical method.

Dynamics of planar mechanisms

Inverse dynamic problem – example

Given:

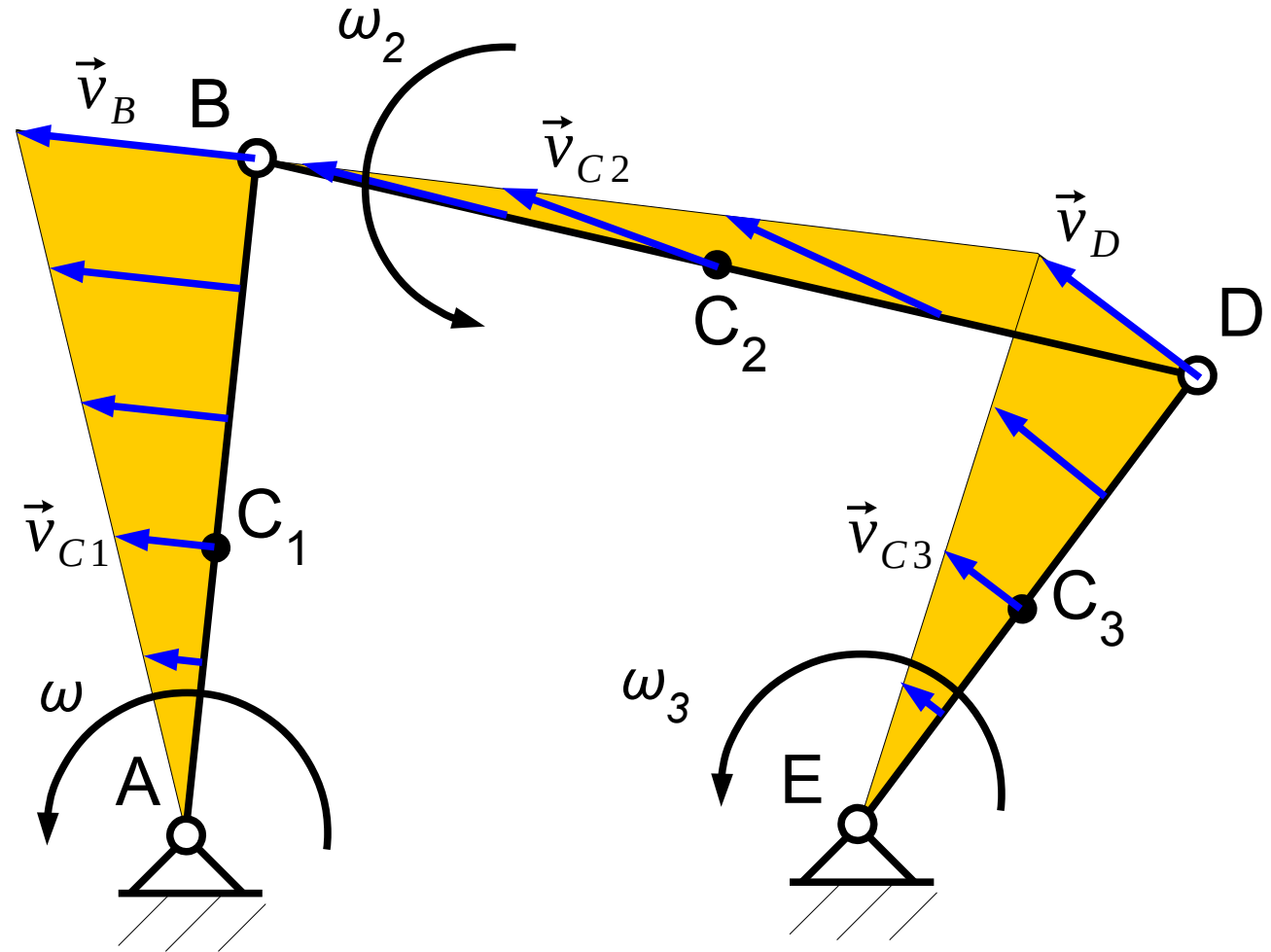
geometry, mass, center of a mass locations, mass moments of inertia for all mechanism members, constant angular velocity ω of a driven element, operating force P .



Dynamics of planar mechanisms

Inverse dynamic problem – example

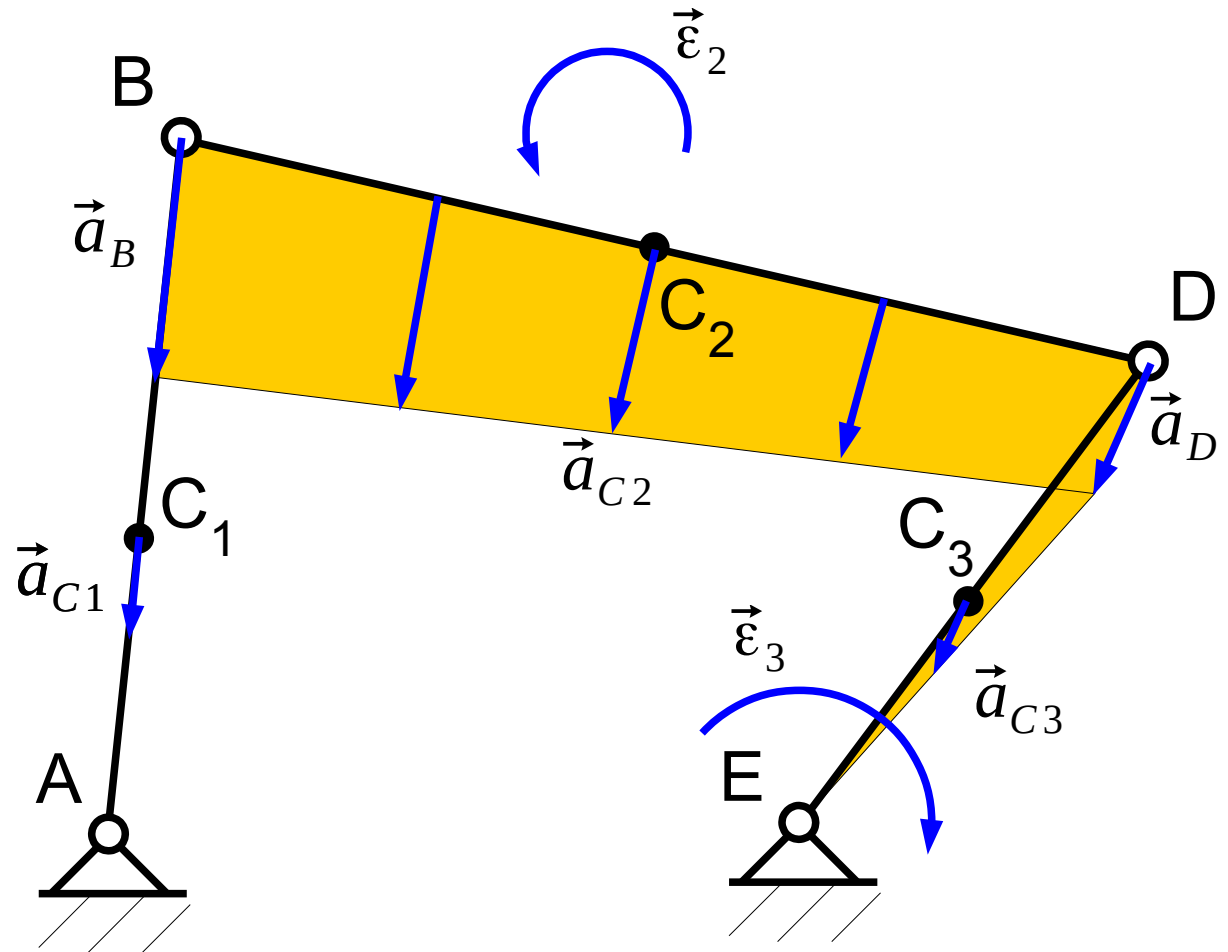
velocity scheme



Dynamics of planar mechanisms

Inverse dynamic problem – example

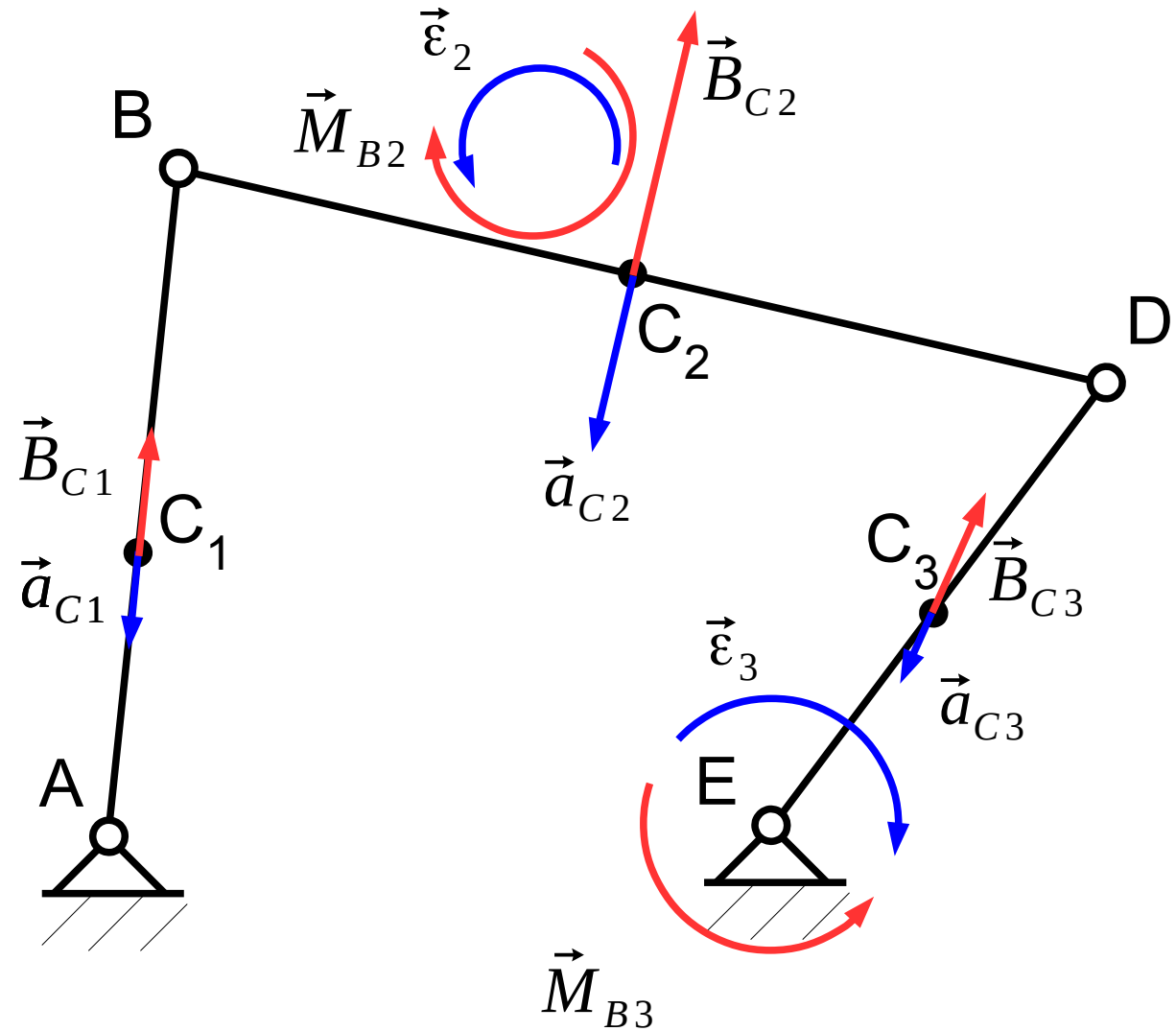
acceleration scheme



Dynamics of planar mechanisms

Inverse dynamic problem – example

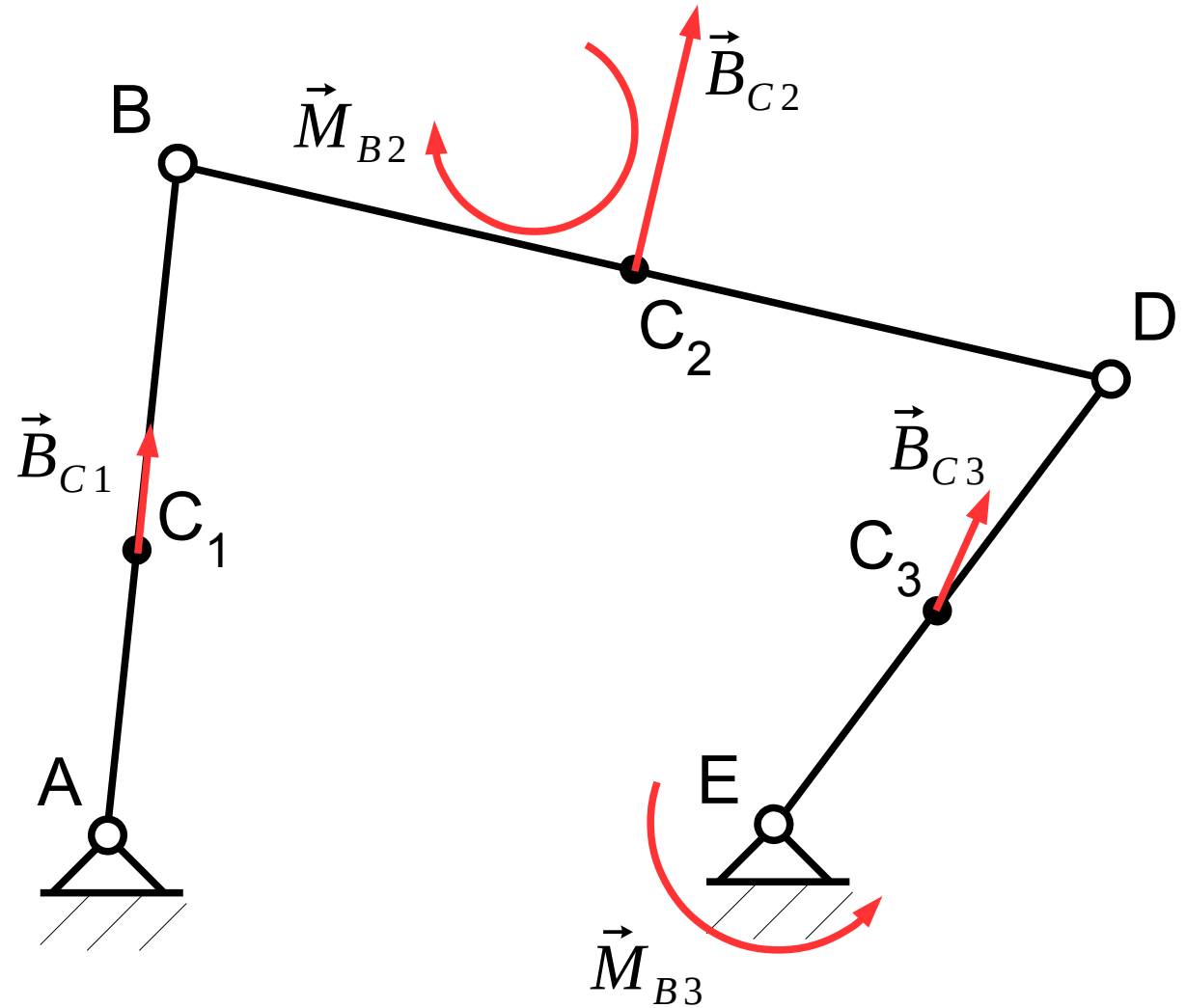
inertial forces and torques



Dynamics of planar mechanisms

Inverse dynamic problem – example

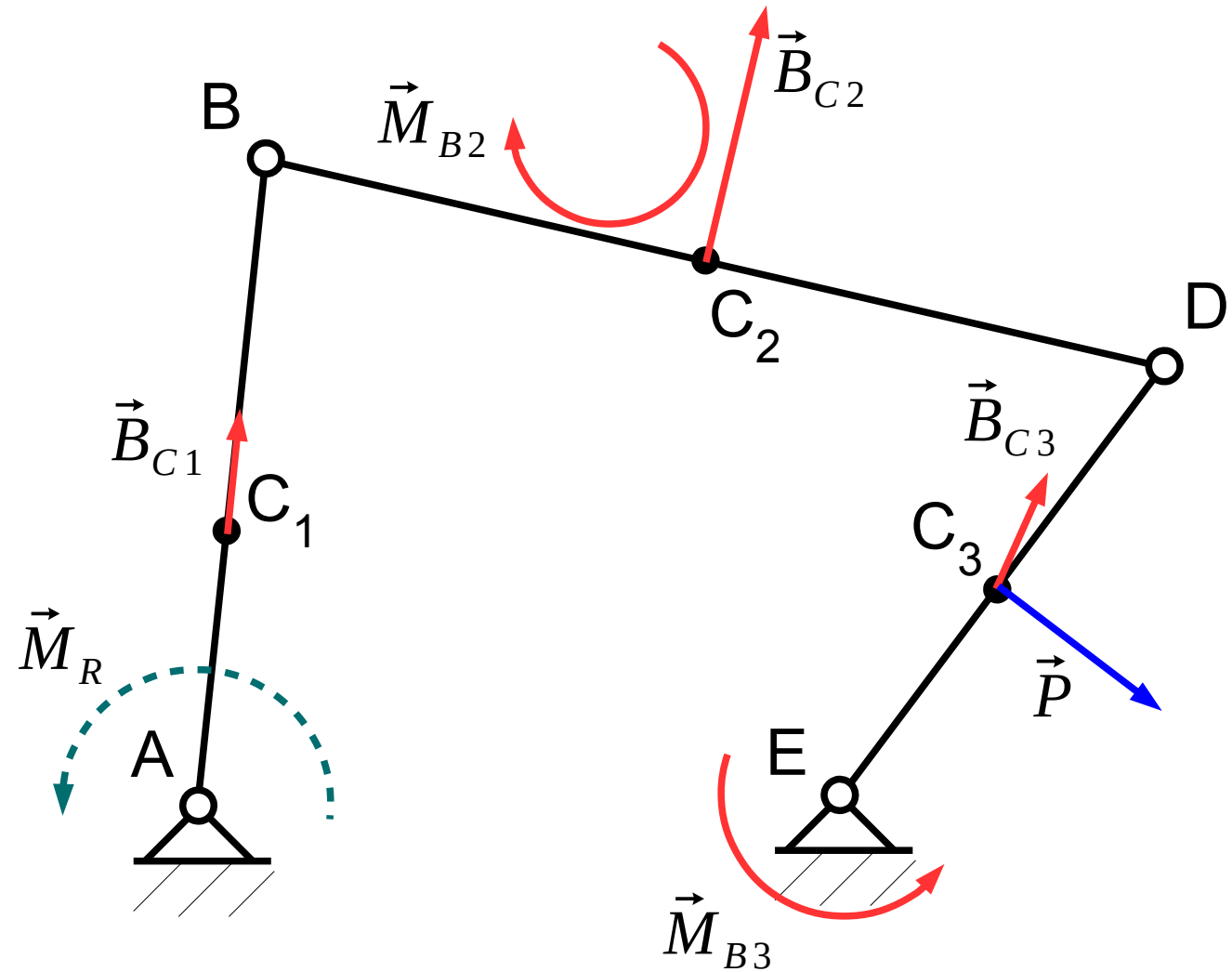
inertial forces and torques



Dynamics of planar mechanisms

Inverse dynamic problem – example

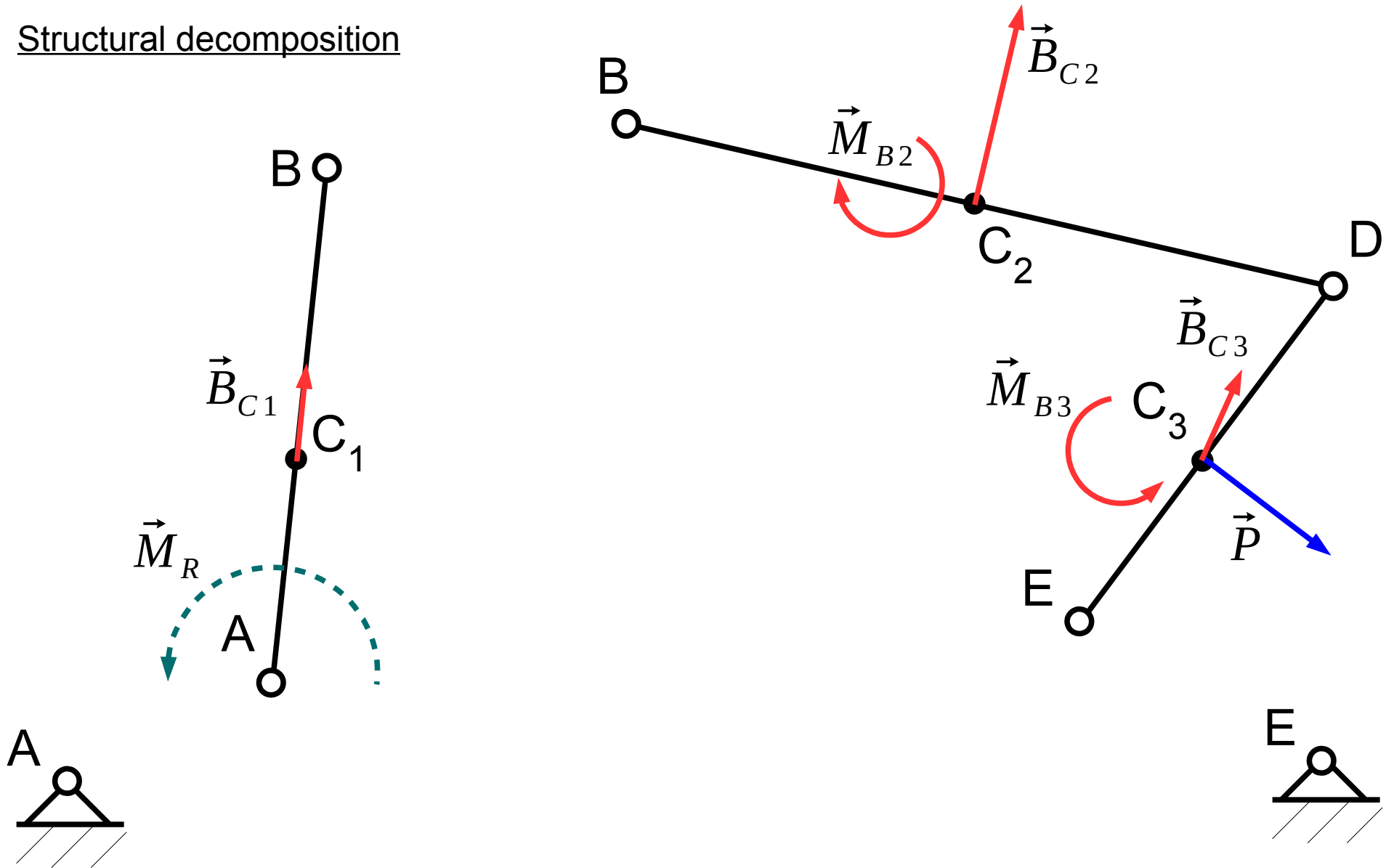
inertial forces and torques
+
external and operating forces and torques



Dynamics of planar mechanisms

Inverse dynamic problem – example

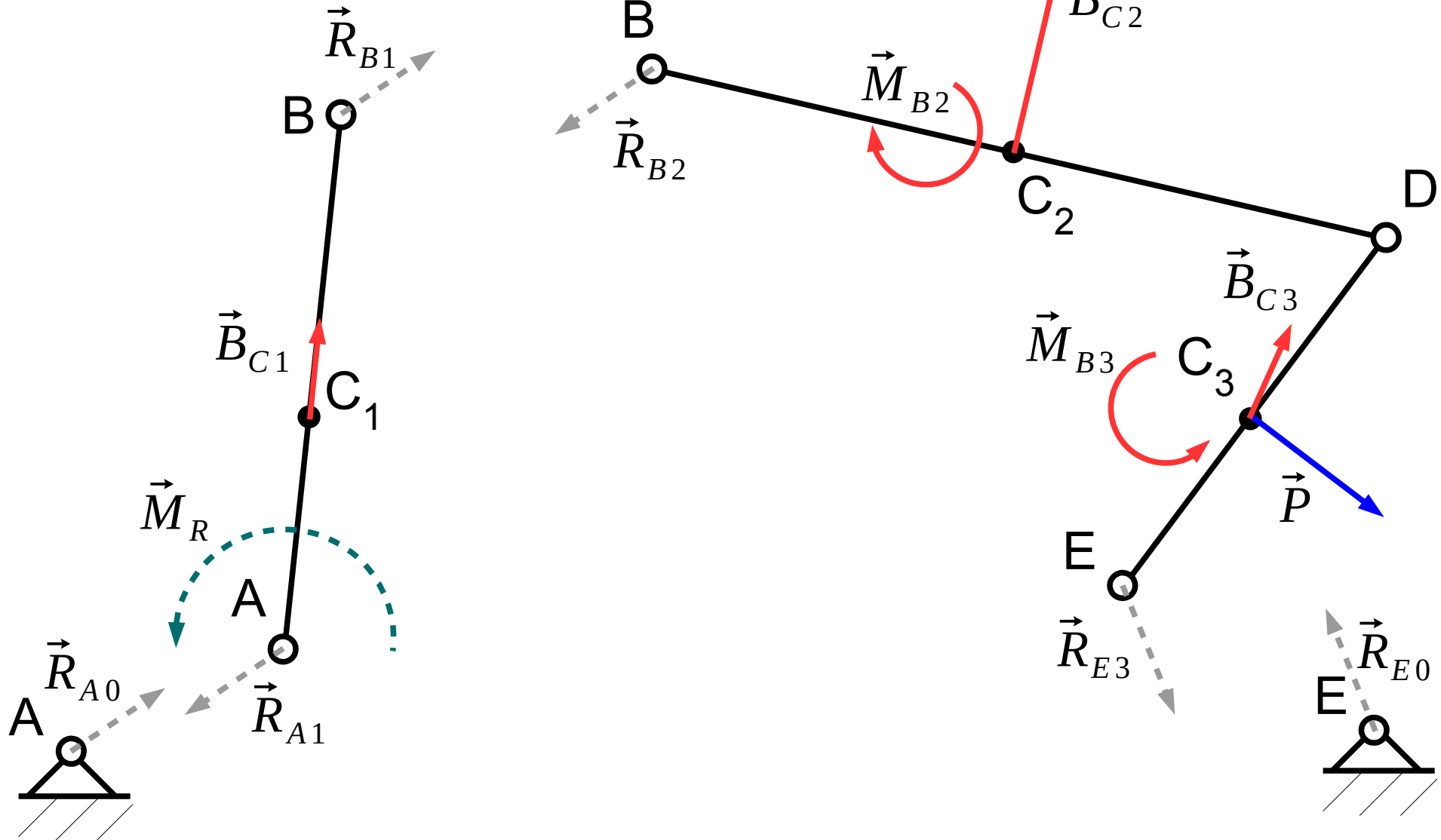
Structural decomposition



Dynamics of planar mechanisms

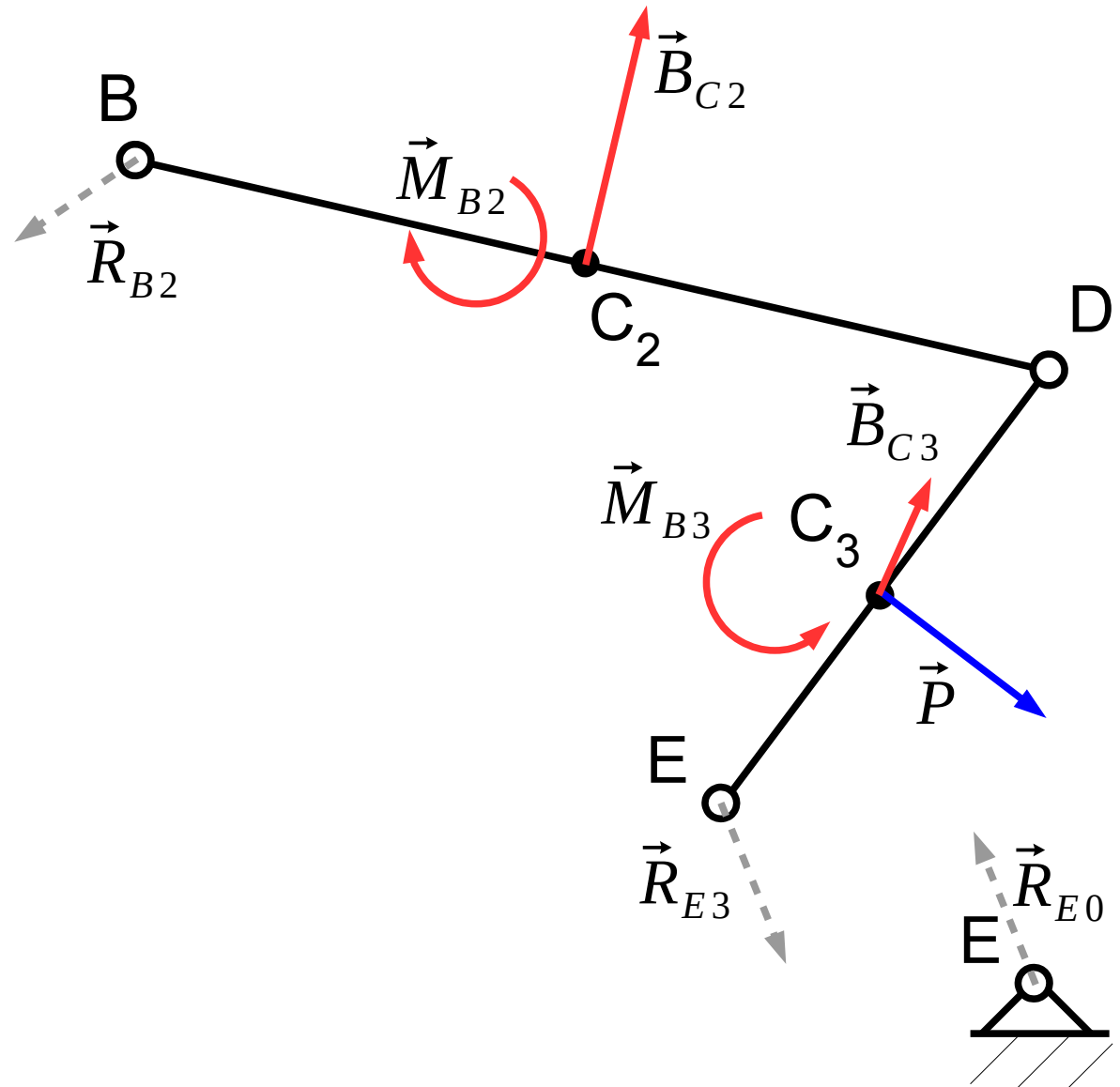
Inverse dynamic problem – example

Unknown
reaction forces



Dynamics of planar mechanisms

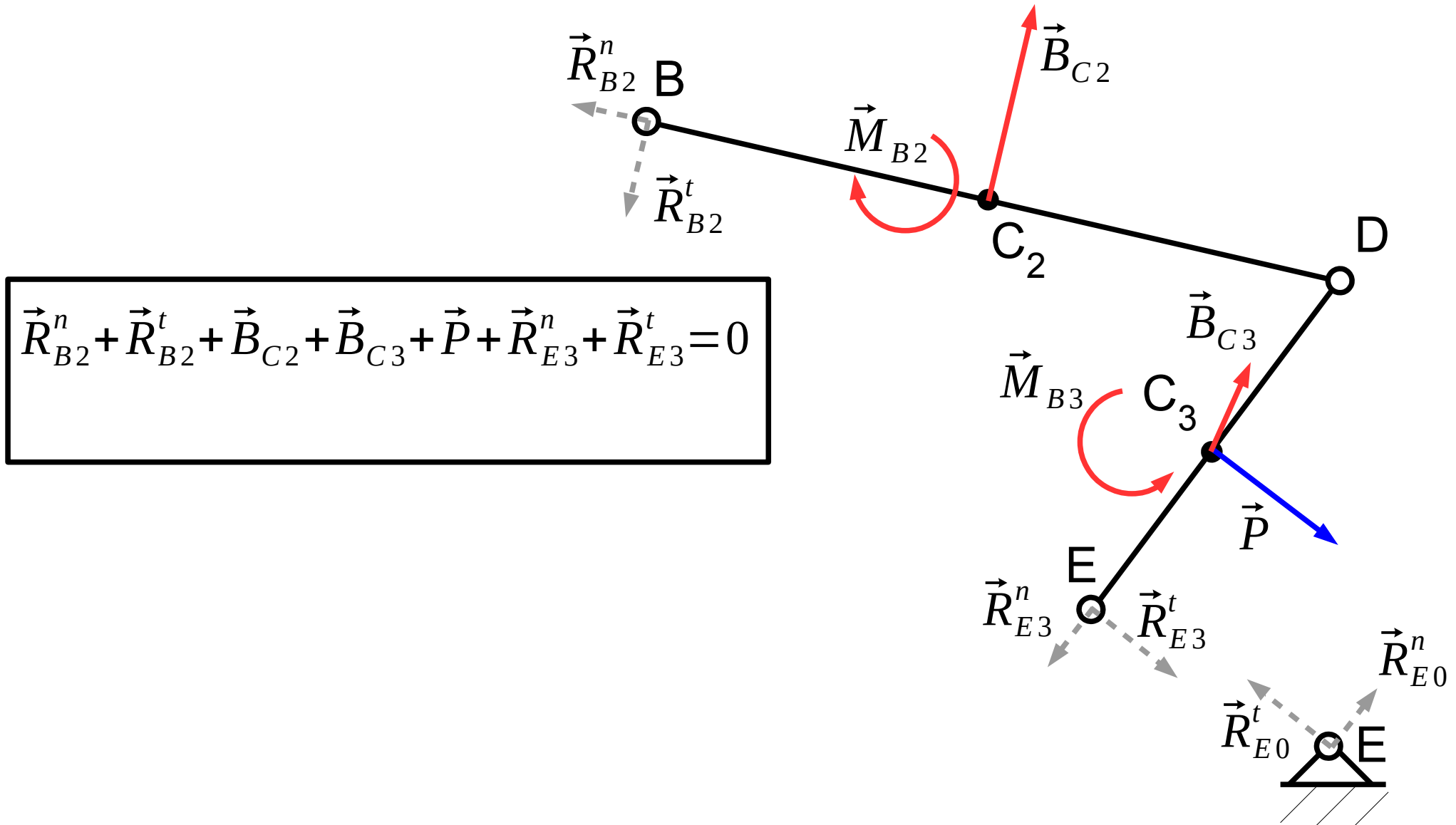
Inverse dynamic problem – example



$$\vec{R}_{B2} + \vec{B}_{C2} + \vec{B}_{C3} + \vec{P} + \vec{R}_{E3} = 0$$

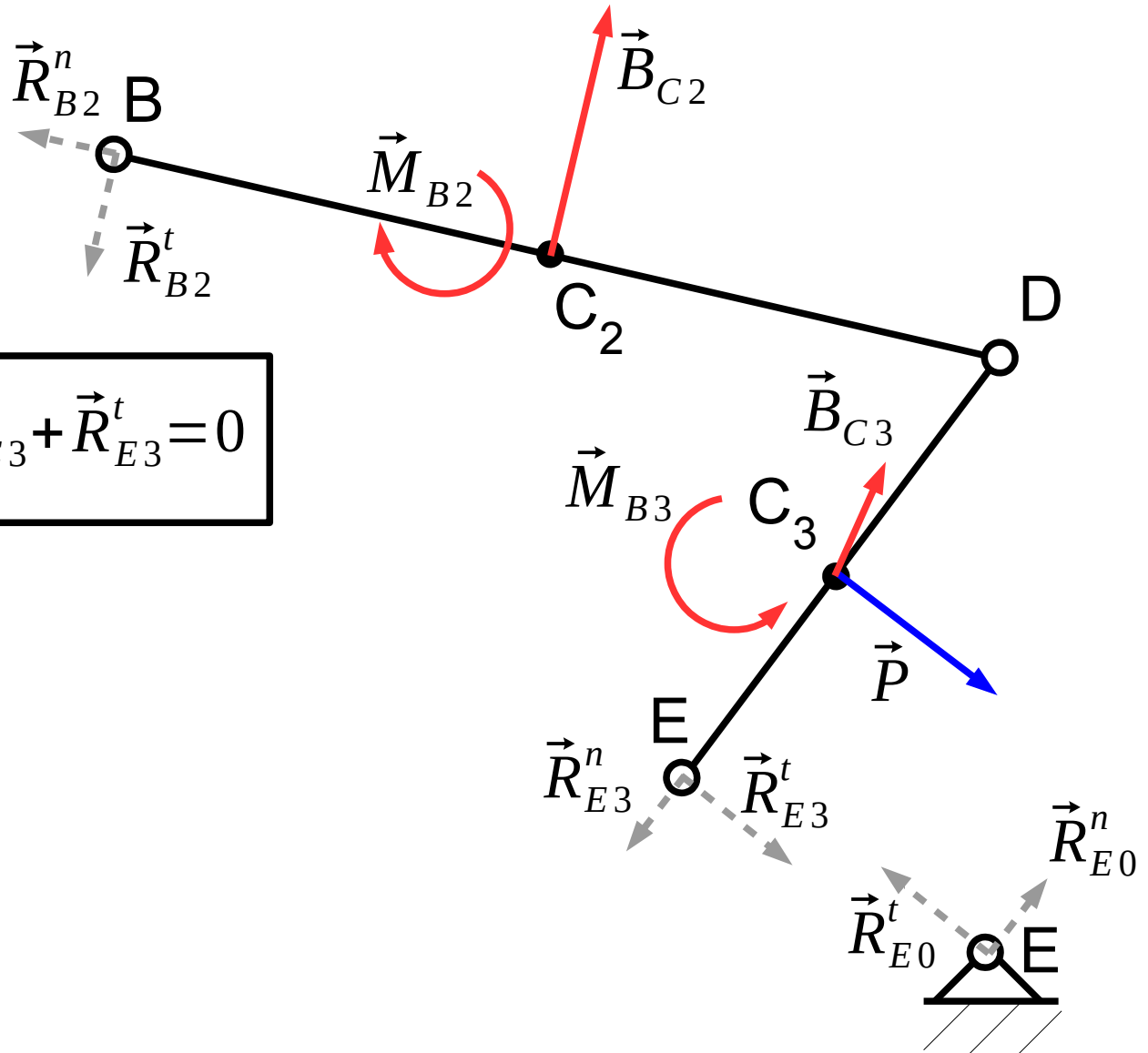
Dynamics of planar mechanisms

Inverse dynamic problem – example



Dynamics of planar mechanisms

Inverse dynamic problem – example



$$\vec{R}_{B2}^n + \vec{R}_{B2}^t + \vec{B}_{C2} + \vec{B}_{C3} + \vec{P} + \vec{R}_{E3}^n + \vec{R}_{E3}^t = 0$$

part BD: $\sum M_D : \dots$

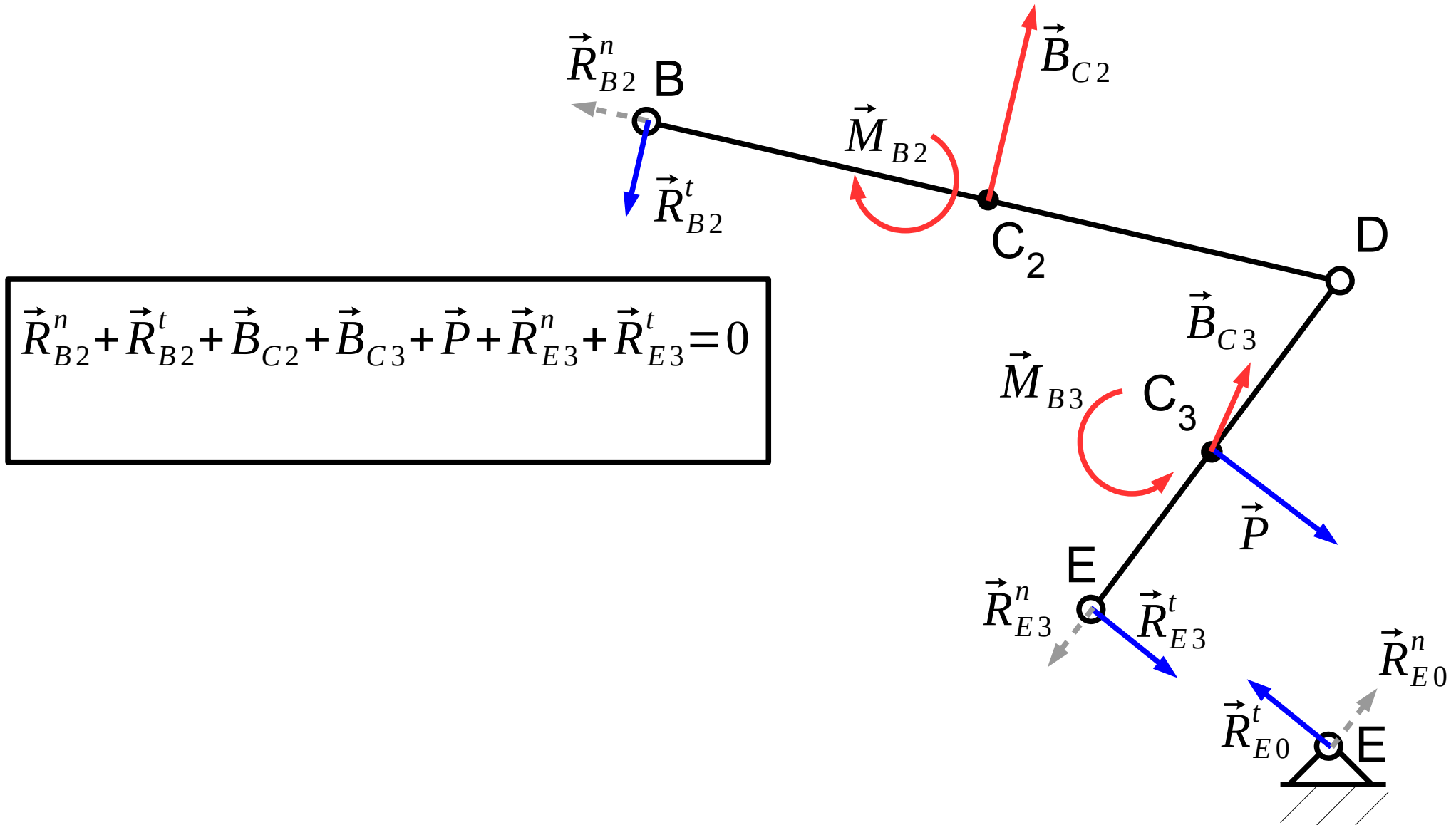
$$\vec{R}_{B2}^t = \dots$$

part ED: $\sum M_D : \dots$

$$\vec{R}_{E3}^t = \dots$$

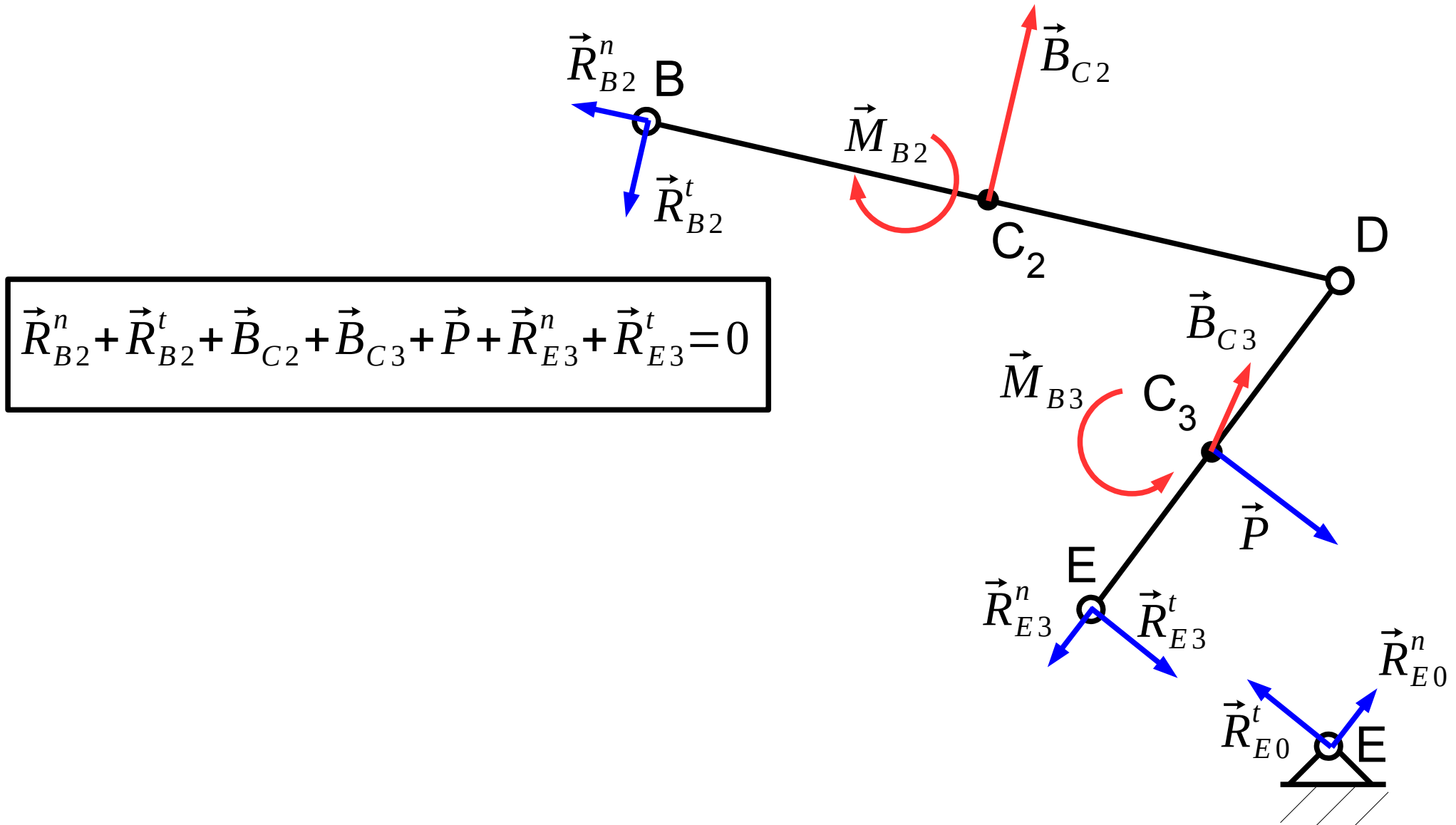
Dynamics of planar mechanisms

Inverse dynamic problem – example



Dynamics of planar mechanisms

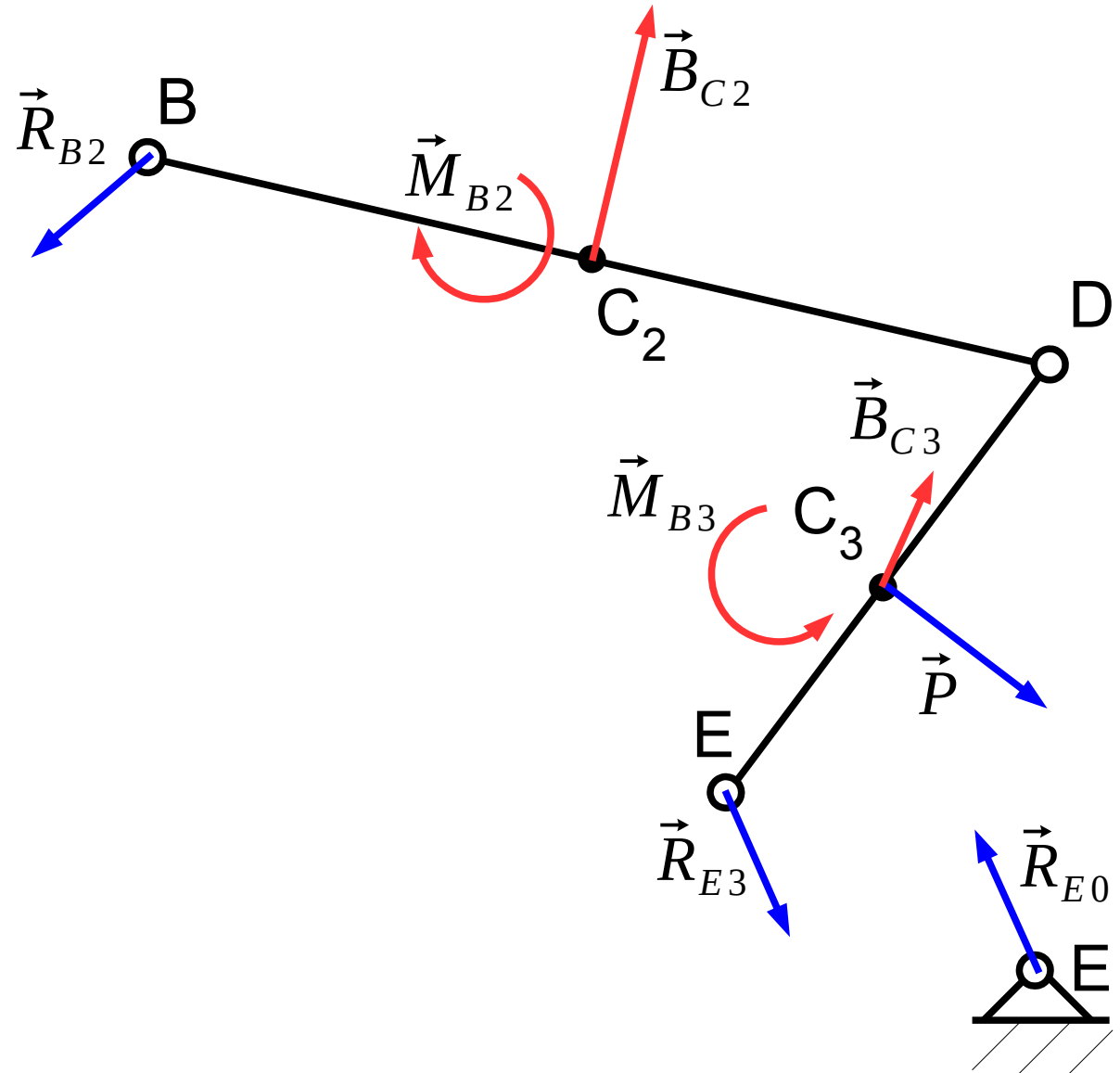
Inverse dynamic problem – example



$$\vec{R}_{B2}^n + \vec{R}_{B2}^t + \vec{B}_{C2} + \vec{B}_{C3} + \vec{P} + \vec{R}_{E3}^n + \vec{R}_{E3}^t = 0$$

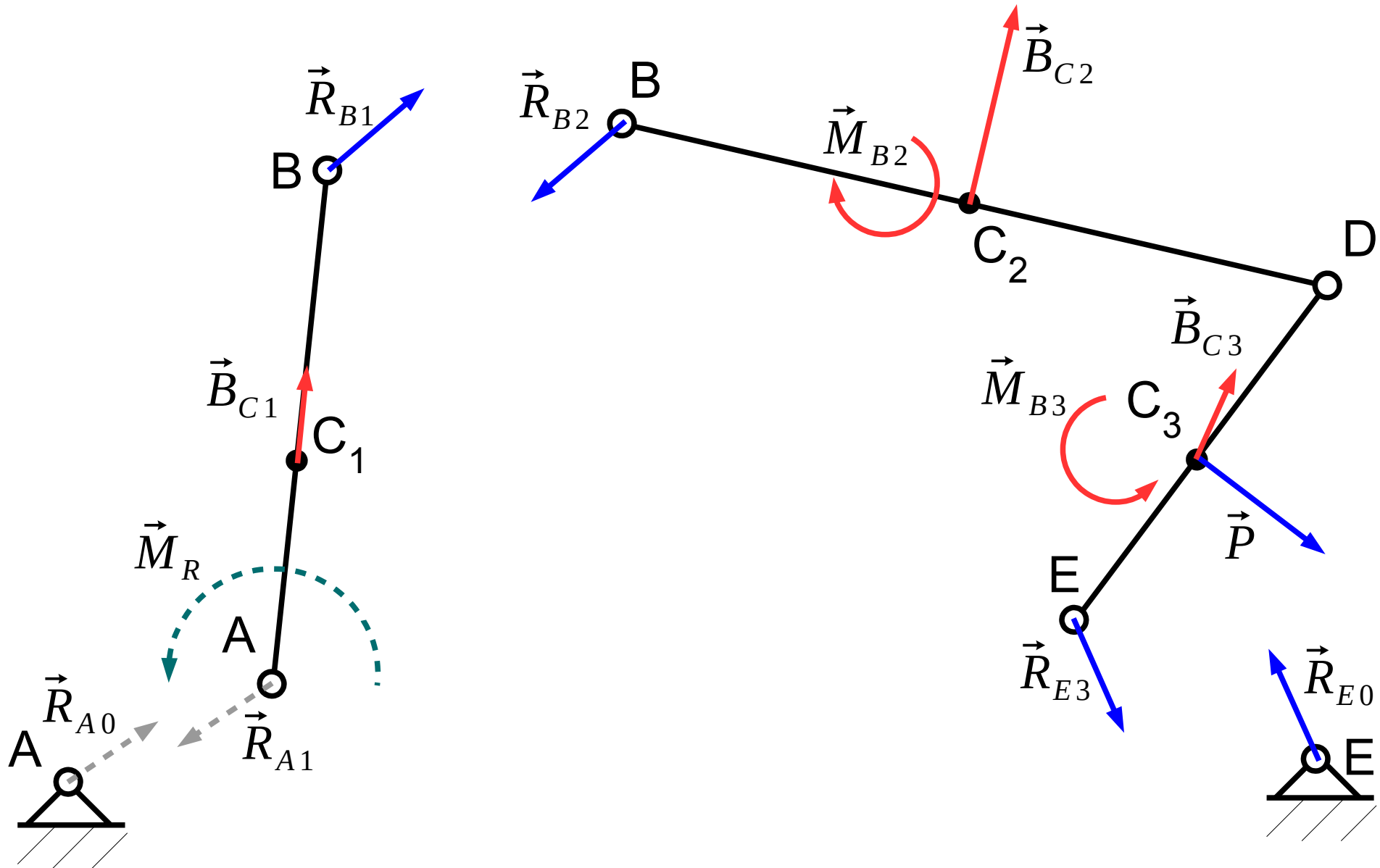
Dynamics of planar mechanisms

Inverse dynamic problem – example



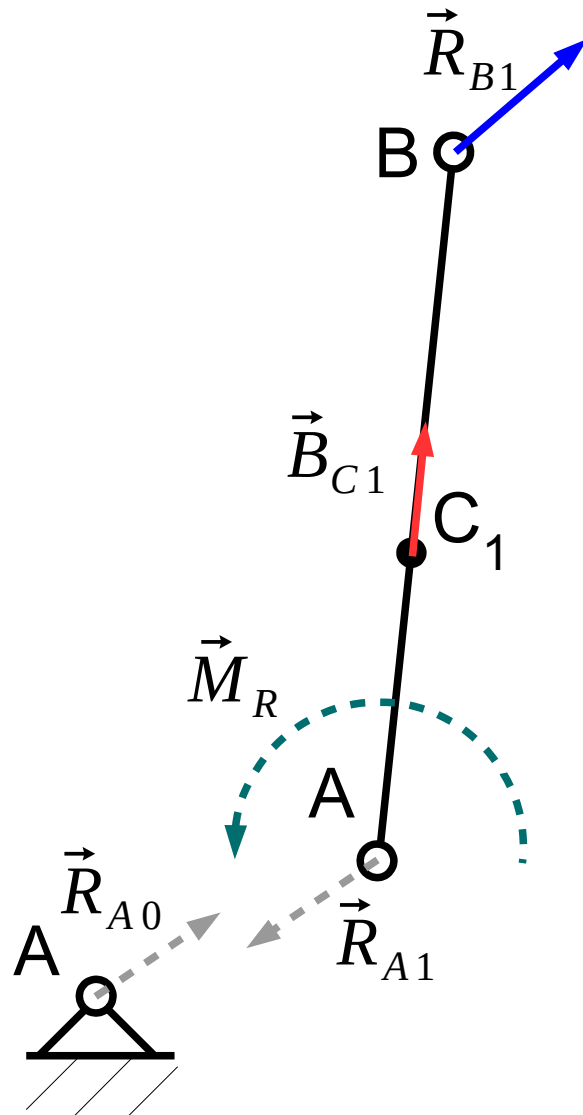
Dynamics of planar mechanisms

Inverse dynamic problem – example



Dynamics of planar mechanisms

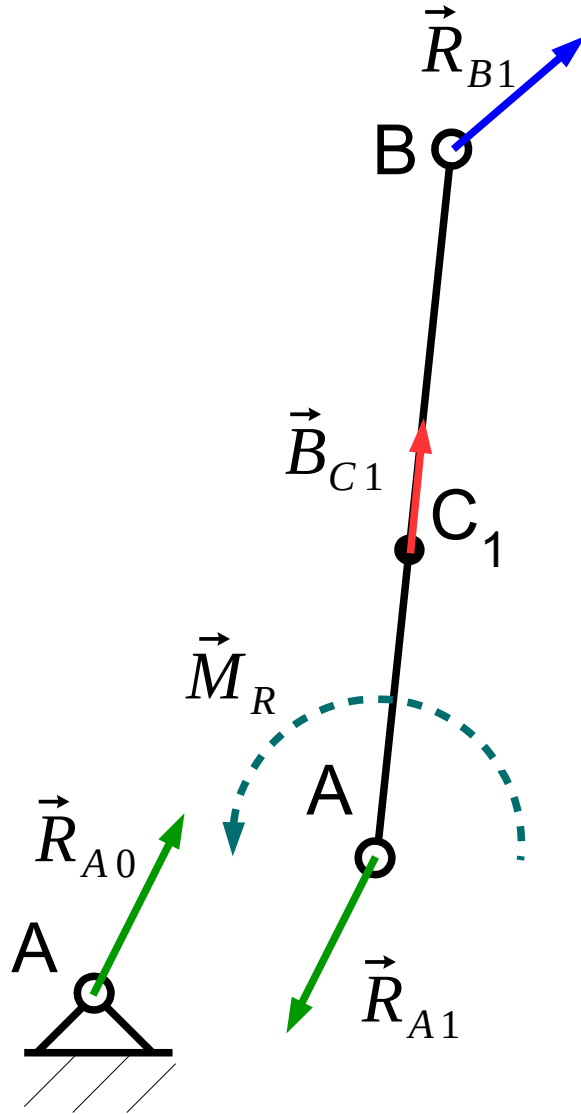
Inverse dynamic problem – example



$$\text{part } AB: \vec{R}_{A1} + \vec{B}_{C1} + \vec{R}_{B1} = 0$$

Dynamics of planar mechanisms

Inverse dynamic problem – example



$$\sum M_A : \dots$$

$$M_R = \dots$$

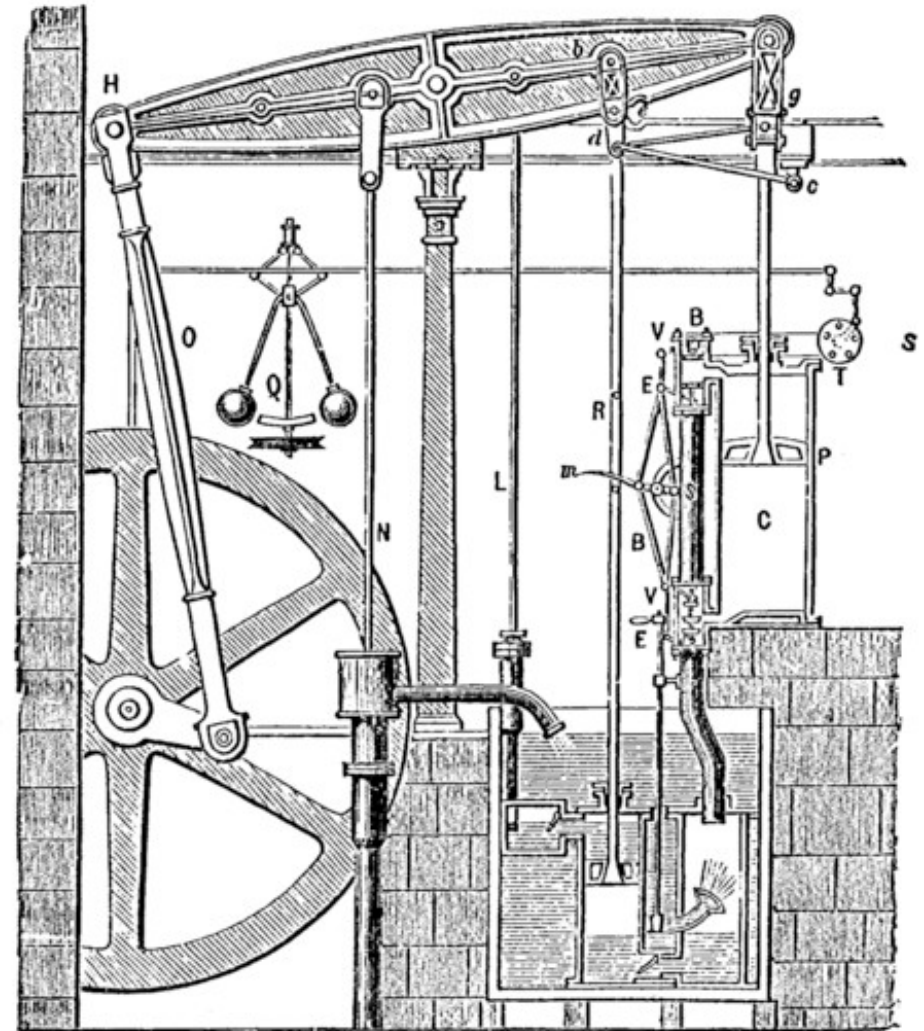
Lecture 6

Machine dynamics.
Reduction of masses and forces.
Machine equation of motion.

Machine dynamics

Overview

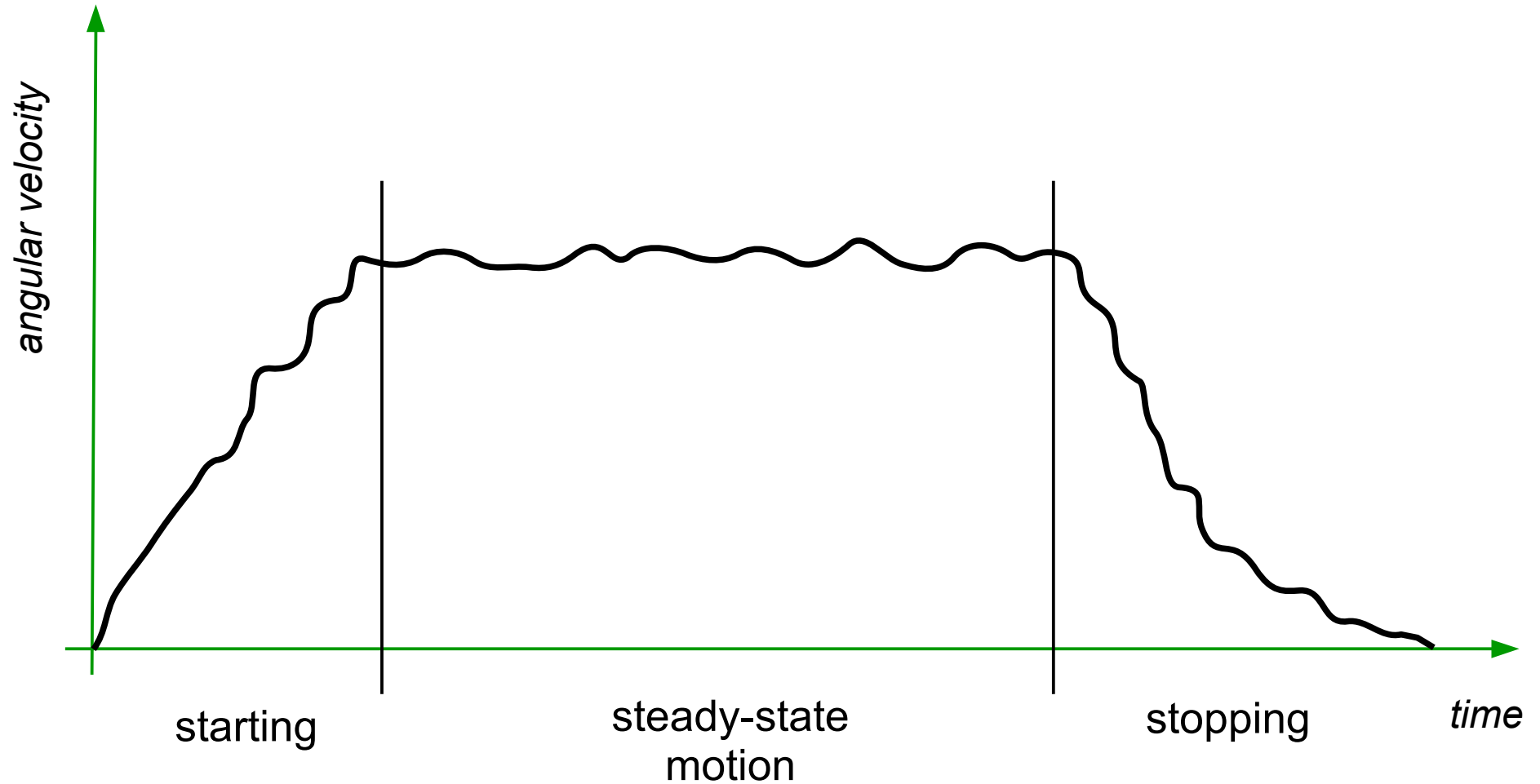
Machine – a tool containing one or more parts that uses energy to perform an intended action. Machines are assembled from components.



source: wikipedia.org, *The Boulton & Watt Steam Engine, 1784*

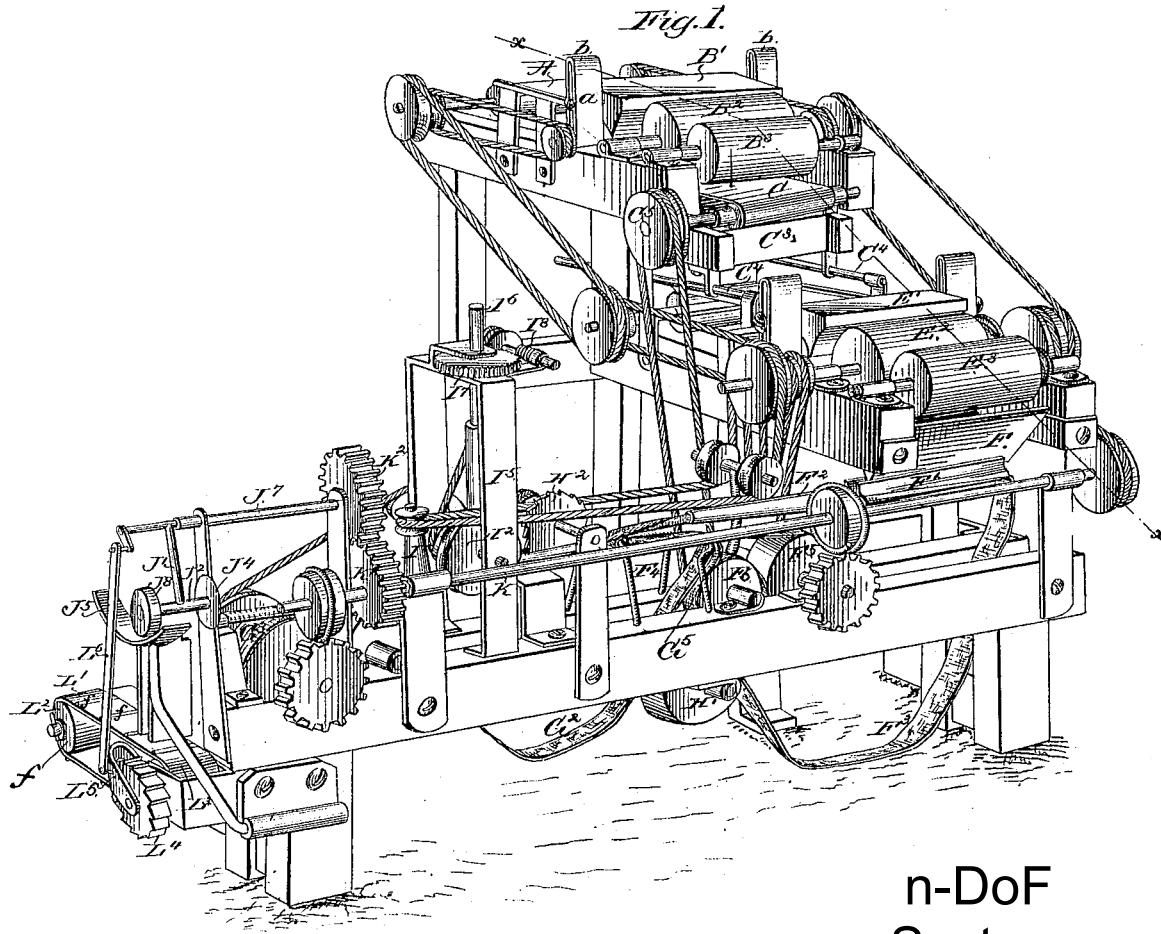
Machine dynamics

Overview

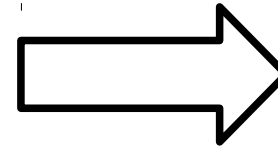


Reduction of masses and forces

Idea of reduction



complicated



$$\ddot{x}_1(t) = F_1(x_1, x_2, \dots, t)$$

$$\ddot{x}_2(t) = F_2(x_1, x_2, \dots, t)$$

$$\dots$$
$$\ddot{x}_n(t) = F_n(x_1, x_2, \dots, t)$$

+ constraints

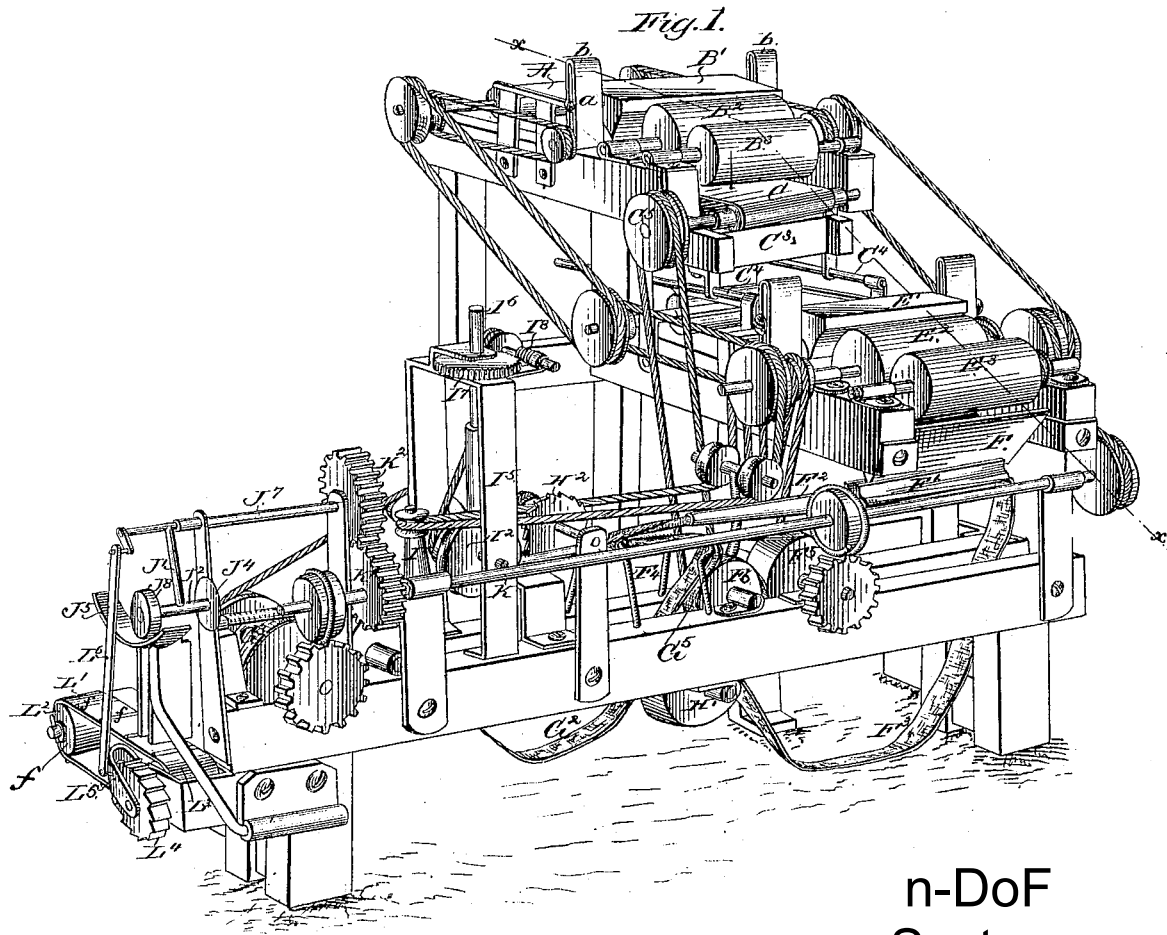
+ limitations

n-DoF
System

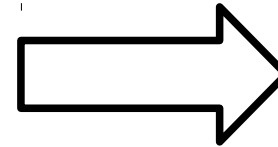
Source: James Albert Bonsack (1859 – 1924) - U.S. patent 238,640
cigarette rolling machine, invented in 1880 and patented in 1881

Reduction of masses and forces

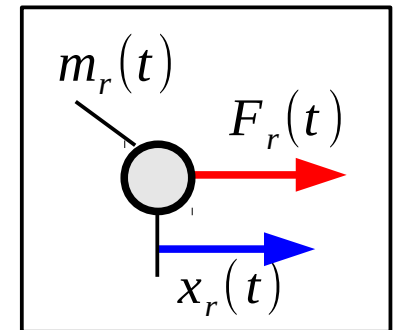
Idea of reduction



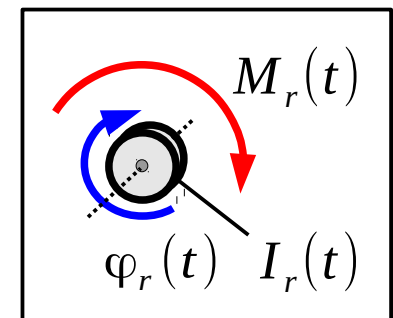
simpler, but
not always
possible



1-DoF
System



or

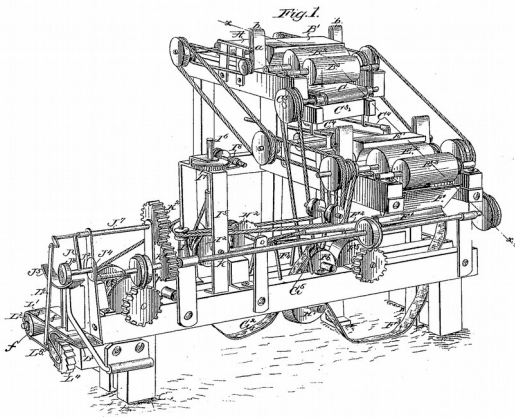


Source: James Albert Bonsack (1859 – 1924) - U.S. patent 238,640
cigarette rolling machine, invented in 1880 and patented in 1881

n-DoF
System

Reduction of masses

Kinetic energy

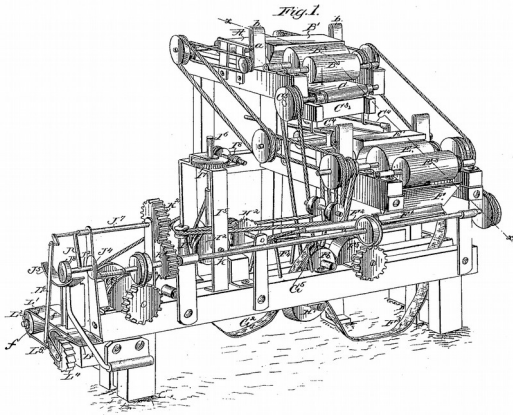


Total kinetic
energy

$$T = \sum_{i=1}^n \left(\frac{1}{2} m_i v_i^2 + \frac{1}{2} I_i \omega_i^2 \right)$$

Reduction of masses

Kinetic energy

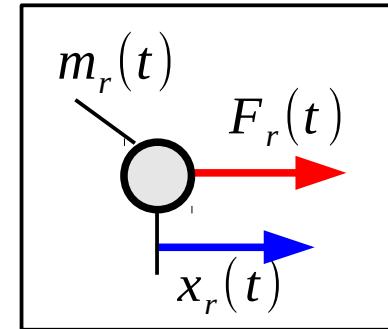


Total kinetic energy

$$T = \sum_{i=1}^n \left(\frac{1}{2} m_i v_i^2 + \frac{1}{2} I_i \omega_i^2 \right)$$

$$T = \frac{1}{2} m_r v_r^2$$

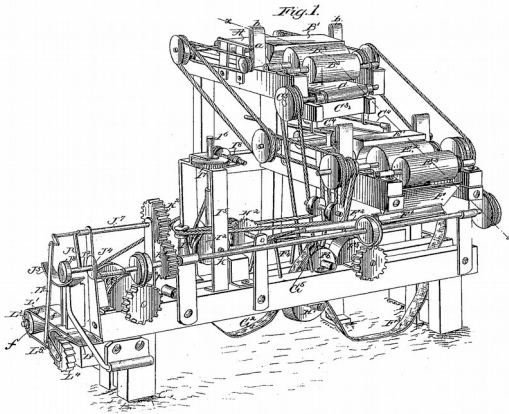
reduced mass



$$v_r = \frac{dx_r(t)}{dt}$$

Reduction of masses

Kinetic energy



Total kinetic energy

or

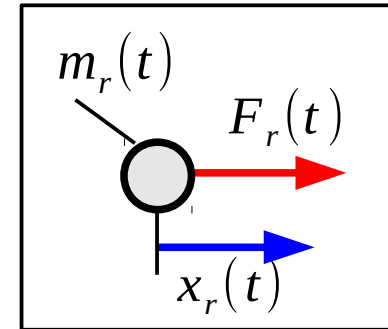
$$T = \sum_{i=1}^n \left(\frac{1}{2} m_i v_i^2 + \frac{1}{2} I_i \omega_i^2 \right)$$

$$T = \frac{1}{2} m_r v_r^2$$

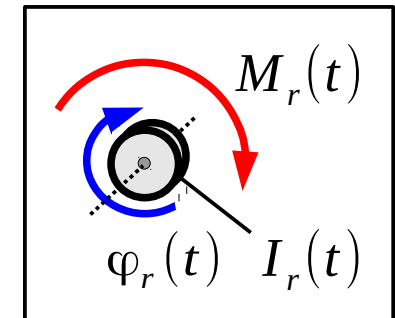
reduced mass

$$T = \frac{1}{2} I_r \omega_r^2$$

reduced moment of inertia



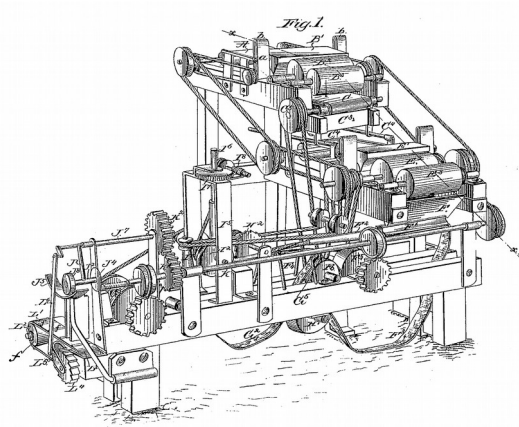
$$v_r = \frac{dx_r(t)}{dt}$$



$$\omega_r = \frac{d\varphi_r(t)}{dt}$$

Reduction of forces

System power

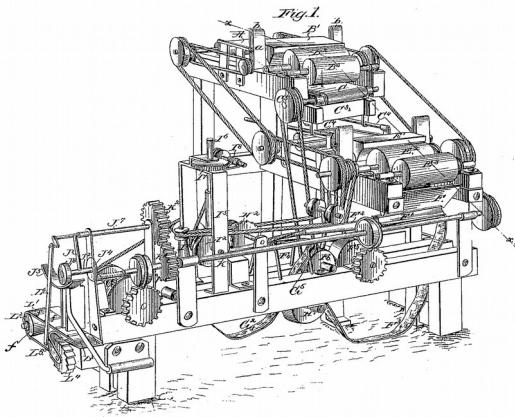


Total system's
power

$$P(F_i, M_i, \omega_i, v_i, \dots)$$

Reduction of forces

System power

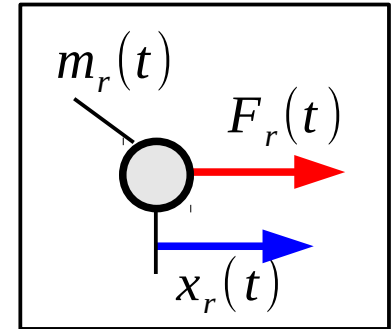


Total system's
power

$$P(F_i, M_i, \omega_i, v_i, \dots)$$

$$P = F_r v_r$$

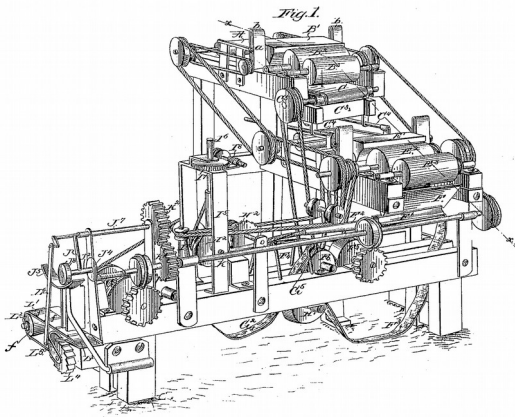
reduced
force



$$v_r = \frac{dx_r(t)}{dt}$$

Reduction of forces

System power



Total system's
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$$P(F_i, M_i, \omega_i, v_i, \dots)$$

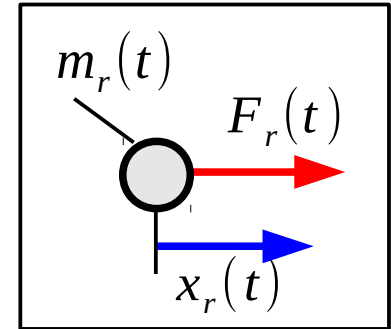
or

$$P = F_r v_r$$

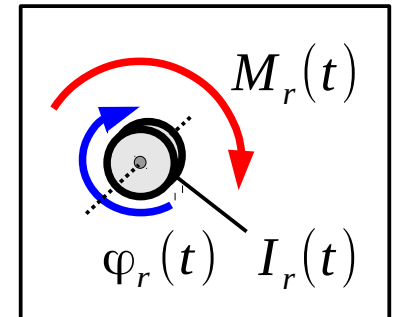
reduced
force

$$P = M_r \omega_r$$

reduced torque



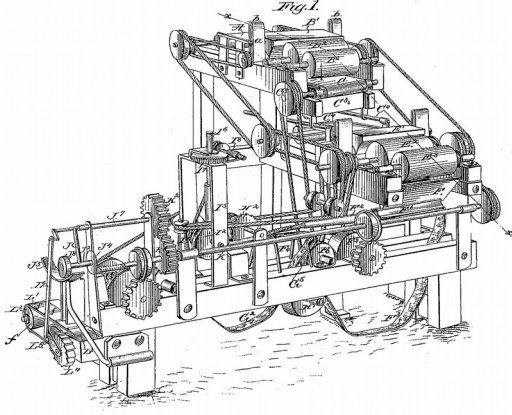
$$v_r = \frac{dx_r(t)}{dt}$$



$$\omega_r = \frac{d\varphi_r(t)}{dt}$$

Reduction of masses – details

Kinetic energy

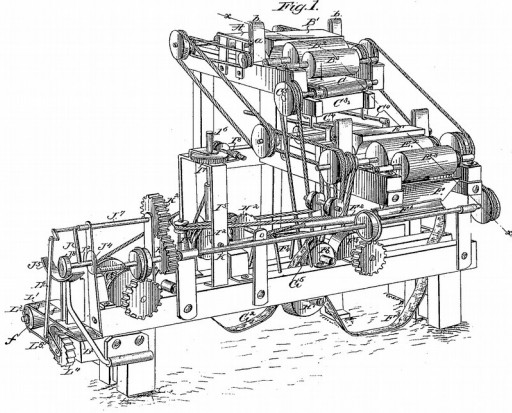


$$T = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

n – translating
elements
k – rotating
elements

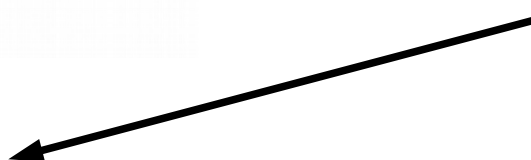
Reduction of masses – details

Kinetic energy



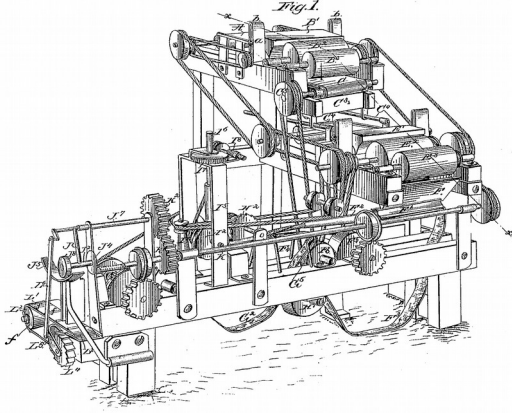
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n – translating elements
k – rotating elements


$$\frac{1}{2} m_r v_r^2 = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

Reduction of masses – details

Kinetic energy



$$T = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

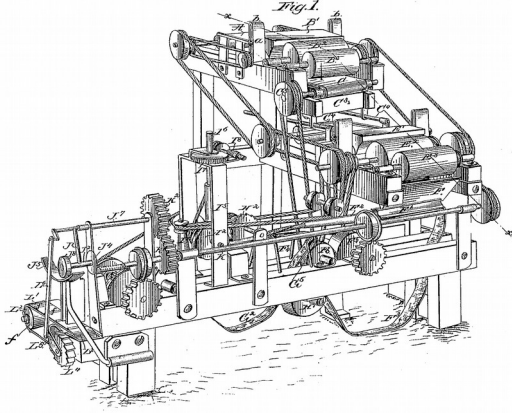
n – translating
elements
k – rotating
elements

$$\frac{1}{2} m_r v_r^2 = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

$$m_r = \sum_{i=1}^n m_i \frac{v_i^2}{v_r^2} + \sum_{j=1}^k I_j \frac{\omega_j^2}{v_r^2}$$

Reduction of masses – details

Kinetic energy



$$T = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

n – translating elements
k – rotating elements

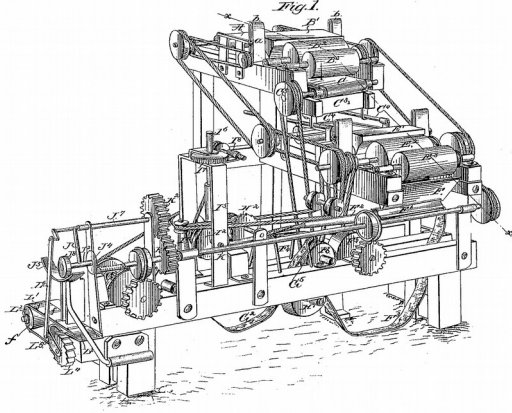
$$\frac{1}{2} m_r v_r^2 = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

$$\frac{1}{2} I_r \omega_r^2 = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

$$m_r = \sum_{i=1}^n m_i \frac{v_i^2}{v_r^2} + \sum_{j=1}^k I_j \frac{\omega_j^2}{v_r^2}$$

Reduction of masses – details

Kinetic energy



$$T = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

n – translating elements
k – rotating elements

$$\frac{1}{2} m_r v_r^2 = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

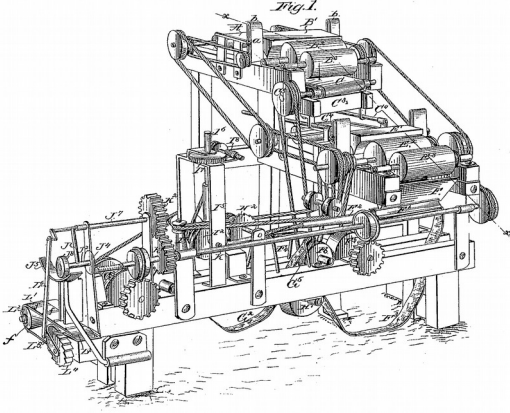
$$\frac{1}{2} I_r \omega_r^2 = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

$$m_r = \sum_{i=1}^n m_i \frac{v_i^2}{v_r^2} + \sum_{j=1}^k I_j \frac{\omega_j^2}{v_r^2}$$

$$I_r = \sum_{i=1}^n m_i \frac{v_i^2}{\omega_r^2} + \sum_{j=1}^k I_j \frac{\omega_j^2}{\omega_r^2}$$

Reduction of masses – details

Kinetic energy



$$T = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

n – translating elements
k – rotating elements

$$\frac{1}{2} m_r v_r^2 = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

$$\frac{1}{2} I_r \omega_r^2 = \sum_{i=1}^n \frac{1}{2} m_i v_i^2 + \sum_{j=1}^k \frac{1}{2} I_j \omega_j^2$$

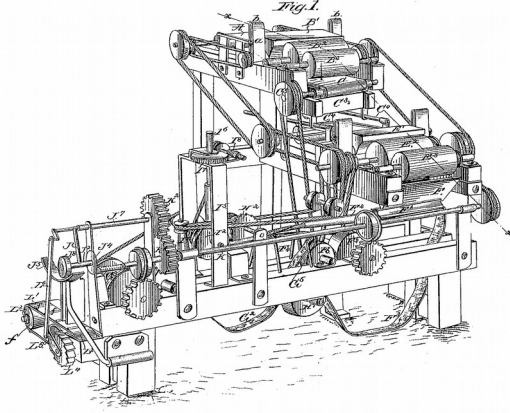
$$m_r = \sum_{i=1}^n m_i \frac{v_i^2}{v_r^2} + \sum_{j=1}^k I_j \frac{\omega_j^2}{v_r^2}$$

$$I_r = \sum_{i=1}^n m_i \frac{v_i^2}{\omega_r^2} + \sum_{j=1}^k I_j \frac{\omega_j^2}{\omega_r^2}$$

v_r, ω_r – arbitrary chosen velocities

Reduction of forces – details

Work



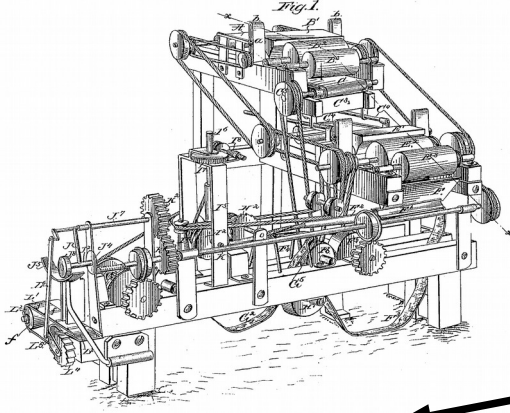
$$dW = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

$$\alpha_i = \sphericalangle(P_i, ds_i)$$

n – translating
elements
k – rotating
elements

Reduction of forces – details

Work



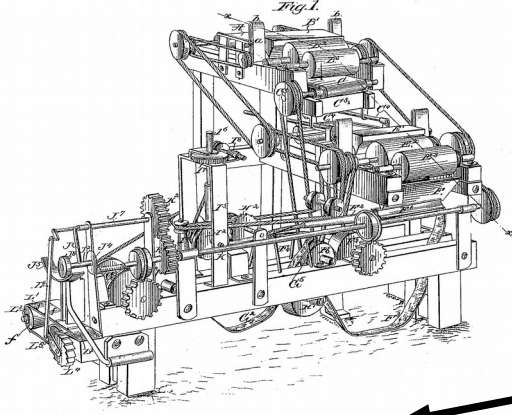
$$dW = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

n – translating elements
k – rotating elements

$$P_r ds_r = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

Reduction of forces – details

Work



$$dW = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

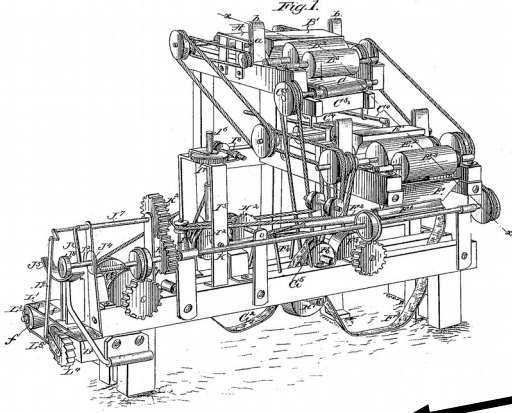
n – translating elements
k – rotating elements

$$P_r ds_r = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

$$P_r = \sum_{i=1}^n P_i \frac{ds_i}{ds_r} \cos \alpha_i + \sum_{j=1}^k M_j \frac{d\varphi_j}{ds_r}$$

Reduction of forces – details

Work



$$dW = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

n – translating elements
k – rotating elements

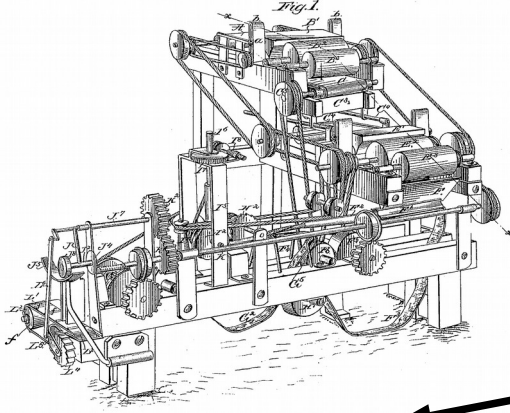
$$P_r ds_r = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

$$P_r = \sum_{i=1}^n P_i \frac{ds_i}{ds_r} \cos \alpha_i + \sum_{j=1}^k M_j \frac{d\varphi_j}{ds_r}$$

$$P_r = \sum_{i=1}^n P_i \frac{v_i dt}{v_r dt} \cos \alpha_i + \sum_{j=1}^k M_j \frac{\omega_j dt}{v_r dt}$$

Reduction of forces – details

Work



$$dW = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

n – translating elements
k – rotating elements

$$P_r ds_r = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

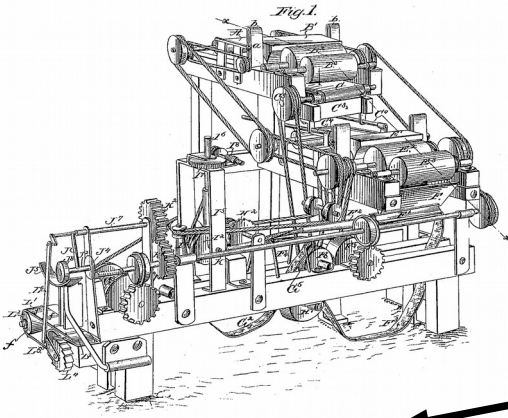
$$P_r = \sum_{i=1}^n P_i \frac{ds_i}{ds_r} \cos \alpha_i + \sum_{j=1}^k M_j \frac{d\varphi_j}{ds_r}$$

$$P_r = \sum_{i=1}^n P_i \frac{v_i dt}{v_r dt} \cos \alpha_i + \sum_{j=1}^k M_j \frac{\omega_j dt}{v_r dt}$$

$$P_r = \sum_{i=1}^n P_i \frac{v_i}{v_r} \cos \alpha_i + \sum_{j=1}^k M_j \frac{\omega_j}{v_r}$$

Reduction of forces – details

Work



$$dW = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

n – translating elements
k – rotating elements

$$P_r ds_r = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

$$M_r d\varphi_r = \sum_{i=1}^n P_i ds_i \cos \alpha_i + \sum_{j=1}^k M_j d\varphi_j$$

$$P_r = \sum_{i=1}^n P_i \frac{ds_i}{ds_r} \cos \alpha_i + \sum_{j=1}^k M_j \frac{d\varphi_j}{ds_r}$$

$$M_r = \sum_{i=1}^n P_i \frac{ds_i}{d\varphi_r} \cos \alpha_i + \sum_{j=1}^k M_j \frac{d\varphi_j}{d\varphi_r}$$

$$P_r = \sum_{i=1}^n P_i \frac{v_i dt}{v_r dt} \cos \alpha_i + \sum_{j=1}^k M_j \frac{\omega_j dt}{v_r dt}$$

$$M_r = \sum_{i=1}^n P_i \frac{v_i dt}{\omega_r dt} \cos \alpha_i + \sum_{j=1}^k M_j \frac{\omega_j dt}{\omega_r dt}$$

$$P_r = \sum_{i=1}^n P_i \frac{v_i}{v_r} \cos \alpha_i + \sum_{j=1}^k M_j \frac{\omega_j}{v_r}$$

$$M_r = \sum_{i=1}^n P_i \frac{v_i}{\omega_r} \cos \alpha_i + \sum_{j=1}^k M_j \frac{\omega_j}{\omega_r}$$

Reduction of masses/moments of inertia

$$m_r = \sum_{i=1}^n m_i \frac{v_i^2}{v_r^2} + \sum_{j=1}^k I_j \frac{\omega_j^2}{v_r^2}$$

$$I_r = \sum_{i=1}^n m_i \frac{v_i^2}{\omega_r^2} + \sum_{j=1}^k I_j \frac{\omega_j^2}{\omega_r^2}$$

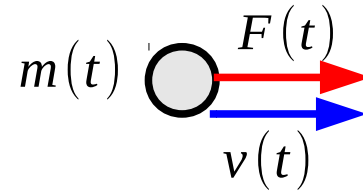
Reduction of forces/torques

$$P_r = \sum_{i=1}^n P_i \frac{v_i}{v_r} \cos \alpha_i + \sum_{j=1}^k M_j \frac{\omega_j}{v_r}$$

$$M_r = \sum_{i=1}^n P_i \frac{v_i}{\omega_r} \cos \alpha_i + \sum_{j=1}^k M_j \frac{\omega_j}{\omega_r}$$

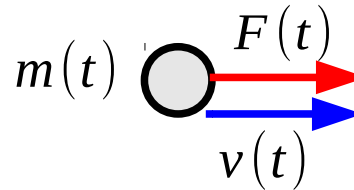
Machine equation of motion

Linear motion



Machine equation of motion

Linear motion



elementary
change of kinetic
energy

elementary
work

$$dT = dW$$

complete
differential
of kinetic
energy

$$d\left(\frac{1}{2} m(t) v(t)^2\right) = F(t) dx$$

$$\frac{1}{2} dm(t) v(t)^2 + m(t) v(t) dv(t) = F(t) dx$$

$$\frac{1}{2} dm(t) v(t)^2 + m(t) \frac{dx(t)}{dt} dv(t) = F(t) dx$$

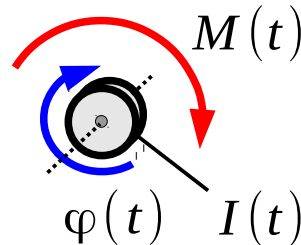
$$\frac{dm(t)}{dx} \frac{v(t)^2}{2} + m \frac{dv(t)}{dt} = F(t)$$

$$\boxed{\frac{dm(t)}{dt} \frac{v(t)}{2} + m \frac{dv(t)}{dt} = F(t)}$$

$$\text{if } m = \text{const.} \Rightarrow m \frac{dv(t)}{dt} = P(t) \text{ or } m \ddot{x}(t) = F(t)$$

Machine equation of motion

Angular motion



$$dT = dW$$

$$d\left(I \frac{\omega(t)^2}{2}\right) = M(t) d\varphi$$

...

...

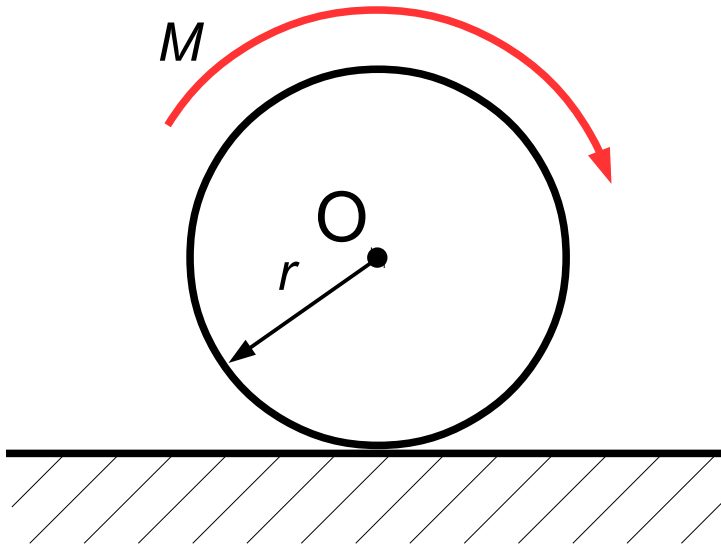
$$\frac{dI(t)}{d\varphi} \frac{\omega(t)^2}{2} + I(t) \frac{d\omega(t)}{dt} = M(t)$$

$$\boxed{\frac{dI(t)}{dt} \frac{\omega(t)}{2} + I(t) \frac{d\omega(t)}{dt} = M(t)}$$

$$\text{if } I = \text{const.} \Rightarrow I \frac{d\omega(t)}{dt} = M(t) \text{ or } I \ddot{\varphi}(t) = M(t)$$

Reduction of masses and forces

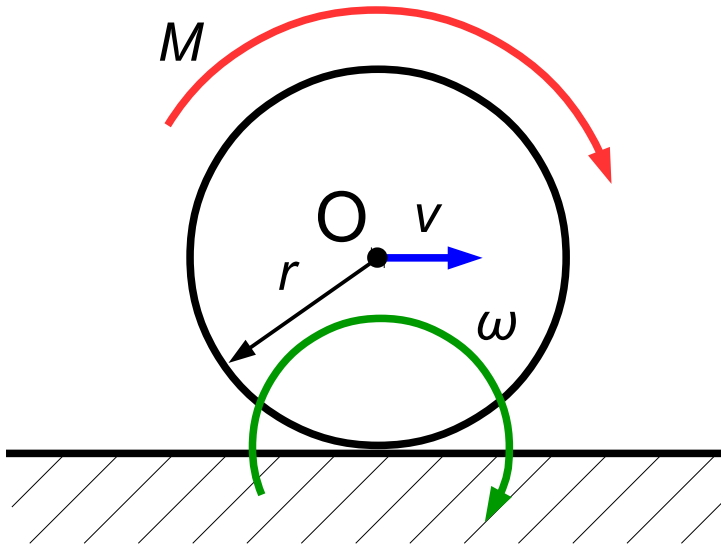
Rolling wheel (without a slip)



Given: m – wheel's mass,
 I_O – wheel's mass moment of inertia in point O ,
 r – wheel's radius,
 M – torque.

Reduction of masses and forces

Rolling wheel (without a slip)



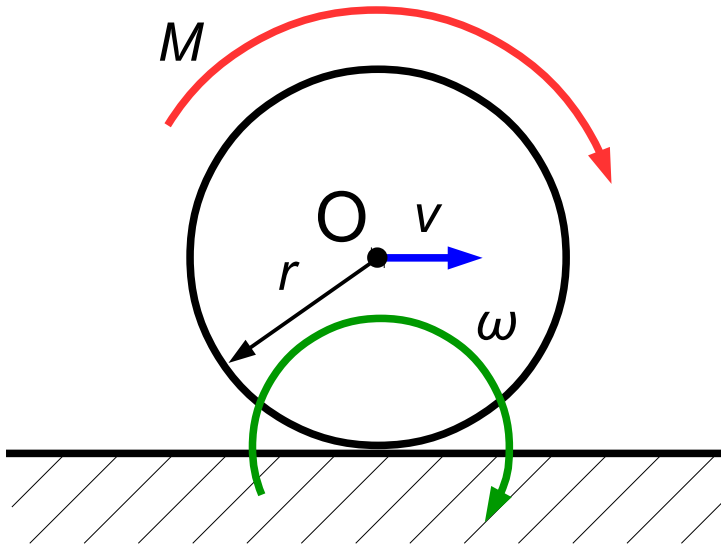
Given: m – wheel's mass,
 I_O – wheel's mass moment of inertia in point O,
 r – wheel's radius,
 M – torque.

Assume:

v – velocity of the wheel's center,
 ω – angular velocity of the wheel.

Reduction of masses and forces

Rolling wheel (without a slip)



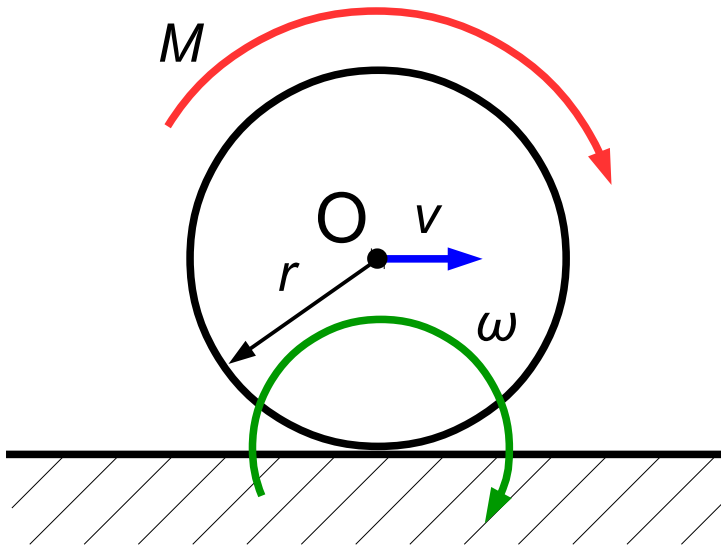
Given: m – wheel's mass,
 I_O – wheel's mass moment of inertia in point O,
 r – wheel's radius,
 M – torque.

Assume:

v – velocity of the wheel's center,
 ω – angular velocity of the wheel.

Reduction of masses and forces

Rolling wheel (without a slip)



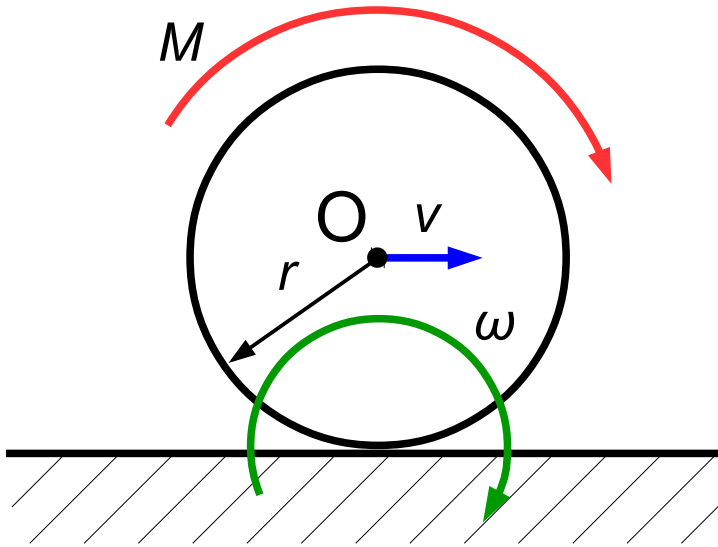
Given: m – wheel's mass,
 I_O – wheel's mass moment of inertia in point O,
 r – wheel's radius,
 M – torque.

Assume:

v – velocity of the wheel's center,
 ω – angular velocity of the wheel.

Reduction of masses and forces

Rolling wheel (without a slip)



Given: m – wheel's mass,
 I_o – wheel's mass moment of inertia in point O,
 r – wheel's radius,
 M – torque.

Assume:
 v – velocity of the wheel's center,
 ω – angular velocity of the wheel.

$$T = \frac{1}{2} m v^2 + \frac{1}{2} I_o \omega^2 \quad \text{but } v = \omega r$$

$$T = \frac{1}{2} m v^2 + \frac{1}{2} I_o \frac{v^2}{r^2} = \frac{1}{2} \left(m + \frac{I_o}{r^2} \right) v^2 = \frac{1}{2} m_r v^2$$

$$m_r = m + \frac{I_o}{r^2} = \text{const.}$$

$$P = M \omega$$

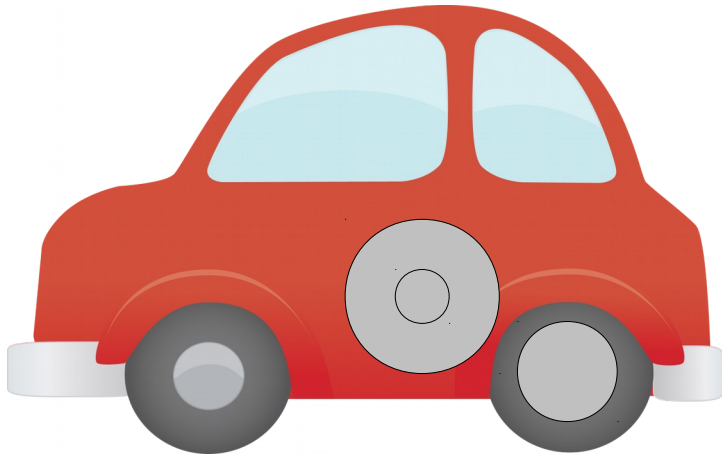
$$P = M \frac{v}{r} = \frac{M}{r} v = F_r v$$

$$F_r = \frac{M}{r}$$

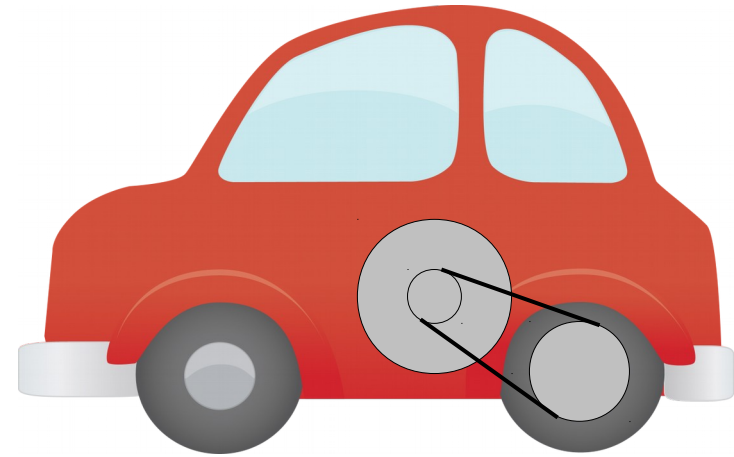
$$m_r \frac{dv}{dt} = F_r$$

$$\boxed{\left(m + \frac{I_o}{r^2} \right) \frac{dv}{dt} = \frac{M}{r}}$$

Reduction of masses and forces – quiz



m_1 – total mass
 m_{r1} – reduced mass



m_2 – total mass
 m_{r2} – reduced mass

$$m_1 = m_2$$

$$m_{r1} ? m_{r2}$$

Reduction of masses and forces

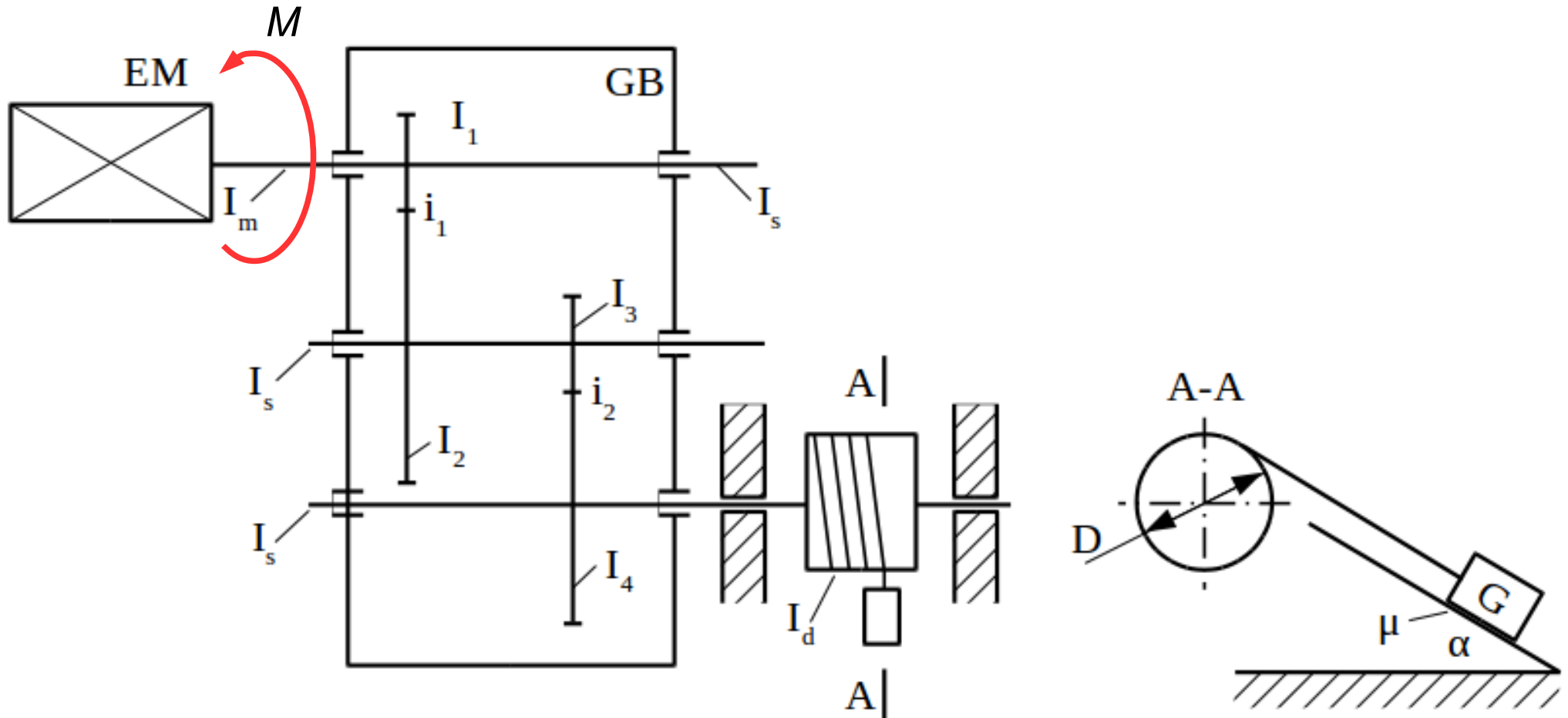
Example 1

In this example a drum winch is analyzed. It consist of:

- electric motor EM, which generates torque as a function of angular velocity ω : $M=A-B\omega$ where A and B are given constants; rotor moment of inertia is equal to I_m ;
- two stage gearbox GB (reducer) with given gears moments of inertia I_1, I_2, I_3, I_4 ; shafts moments of inertia are equal to I_s ; gears ratio are given as $i_1=\omega_2/\omega_1$ and $i_2=\omega_4/\omega_3$
- winch's drum has diameter D and moment of inertia I_d ; drum is set on two ball bearing that generates resistance M_f assumed to be constant;
- inclined plane with angle α related to the horizon line;
- box of weight G pulled up with winch. Friction between object and inclined plane is represented as dry friction with μ coefficient.

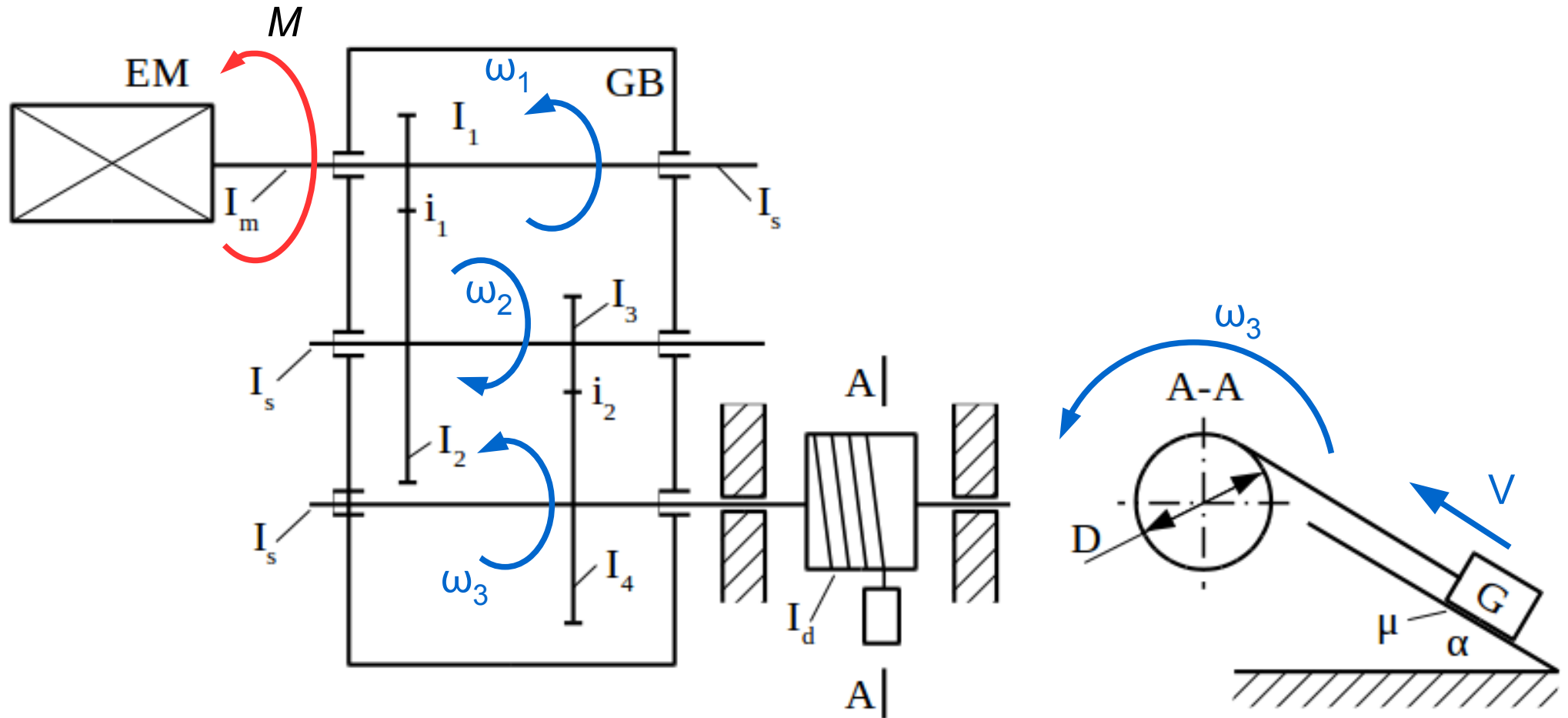
Reduction of masses and forces

Example 1



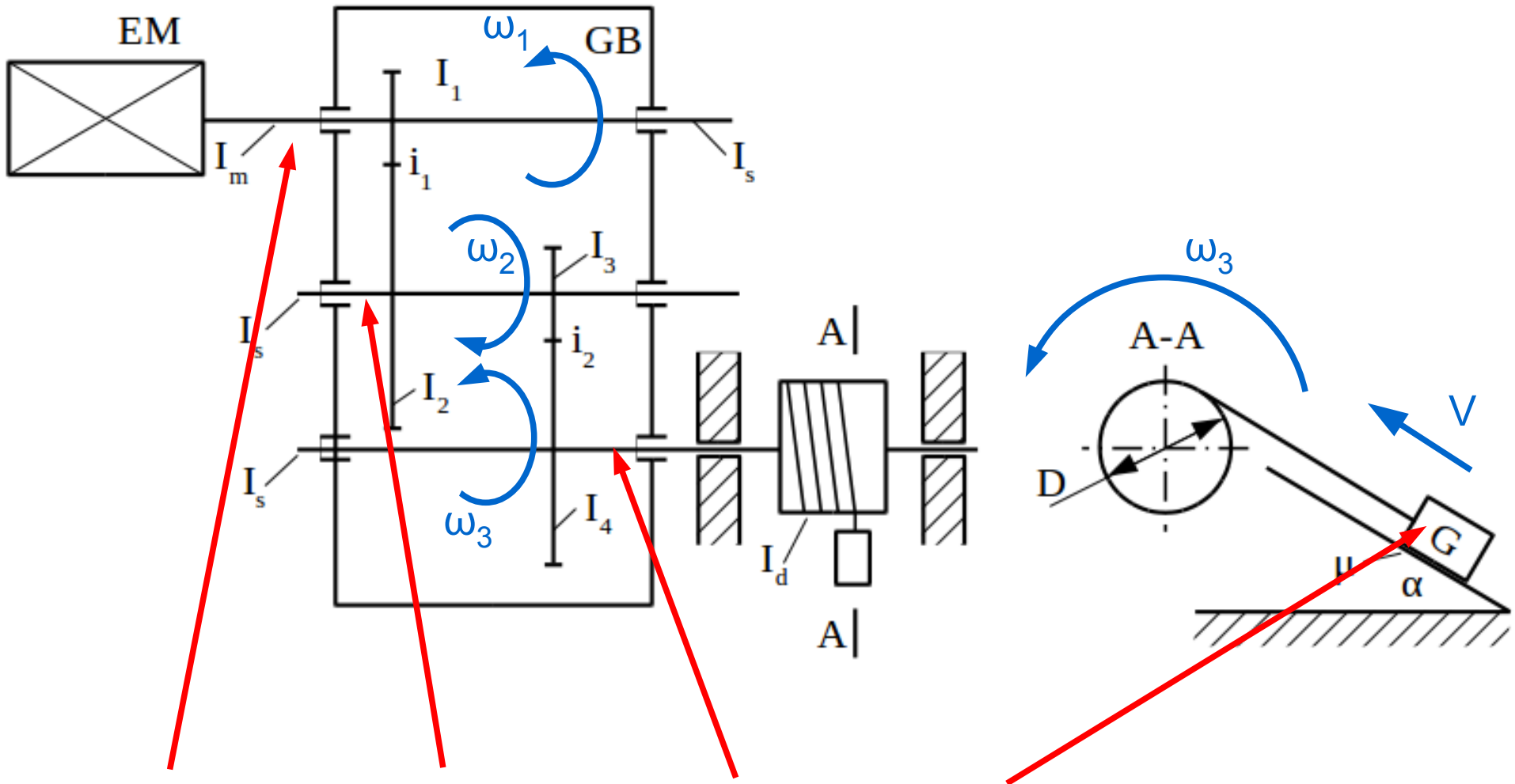
Reduction of masses and forces

Example 1



Reduction of masses and forces

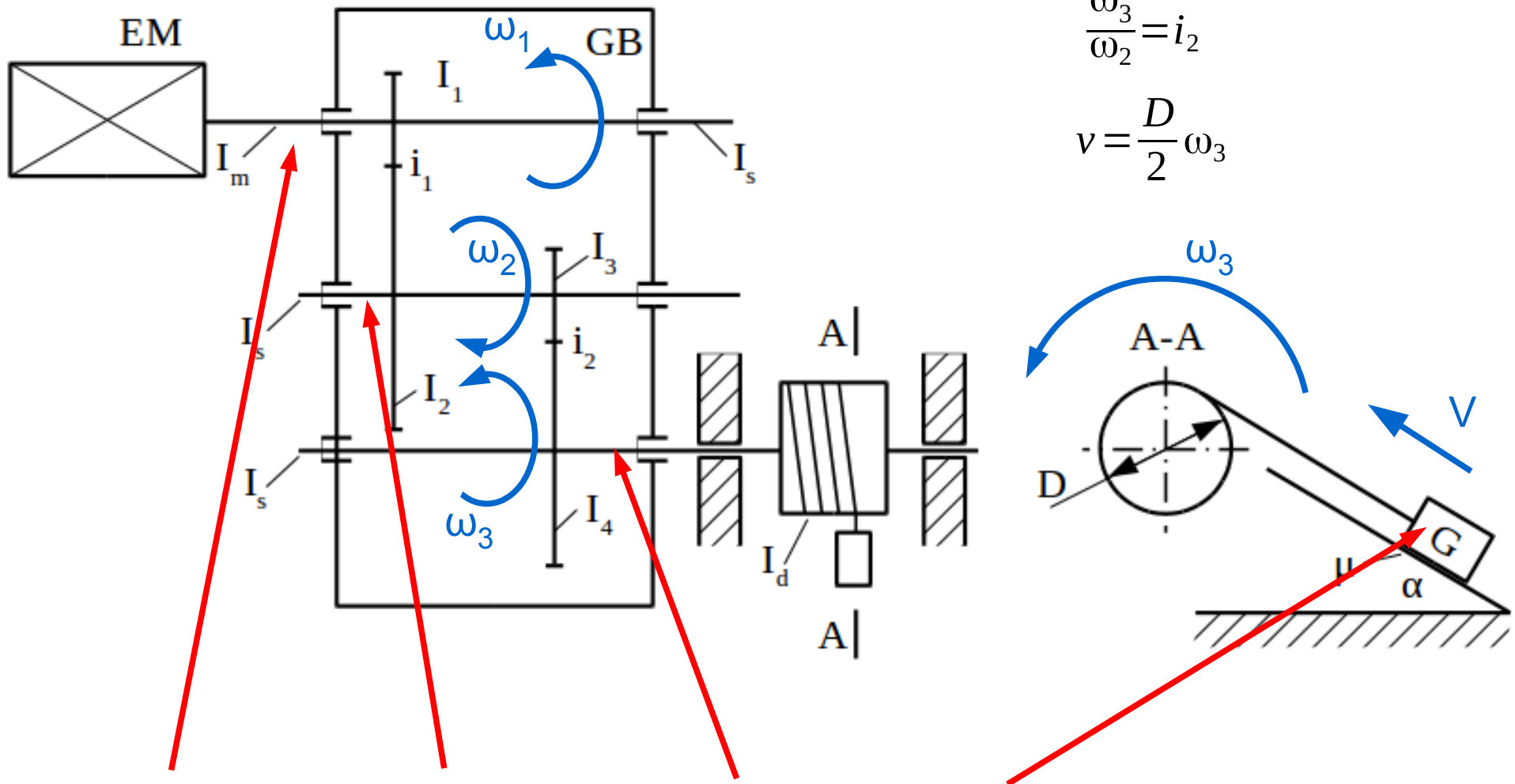
Example 1



$$T = \frac{1}{2}(I_m + I_1 + I_s)\omega_1^2 + \frac{1}{2}(I_2 + I_3 + I_s)\omega_2^2 + \frac{1}{2}(I_4 + I_d + I_s)\omega_3^2 + \frac{1}{2}\frac{G}{g}v^2$$

Reduction of masses and forces

Example 1



$$\frac{\omega_2}{\omega_1} = i_1$$

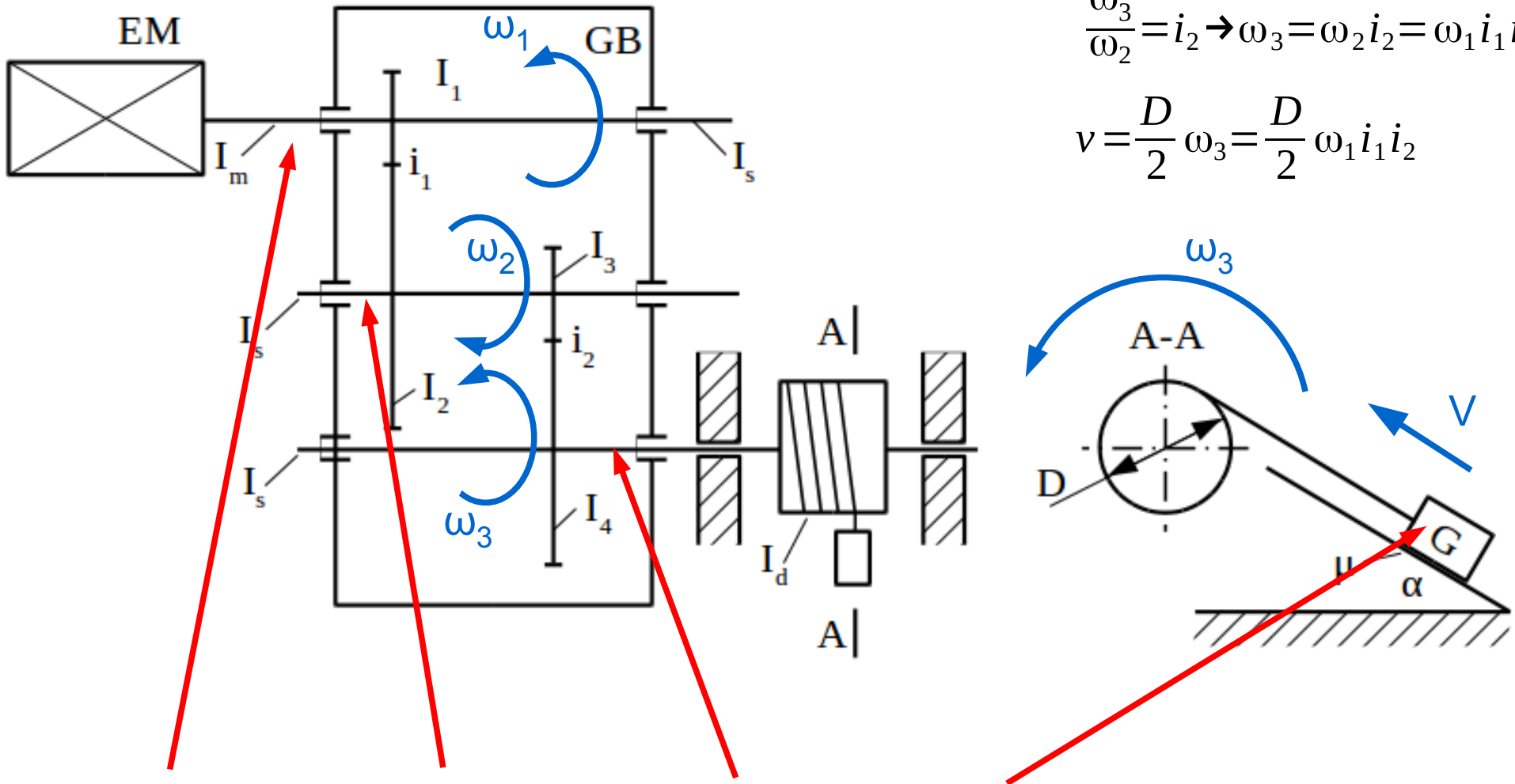
$$\frac{\omega_3}{\omega_2} = i_2$$

$$v = \frac{D}{2} \omega_3$$

$$T = \frac{1}{2} (I_m + I_1 + I_s) \omega_1^2 + \frac{1}{2} (I_2 + I_3 + I_s) \omega_2^2 + \frac{1}{2} (I_4 + I_d + I_s) \omega_3^2 + \frac{1}{2} \frac{G}{g} v^2$$

Reduction of masses and forces

Example 1



$$\frac{\omega_2}{\omega_1} = i_1 \rightarrow \omega_2 = \omega_1 i_1$$

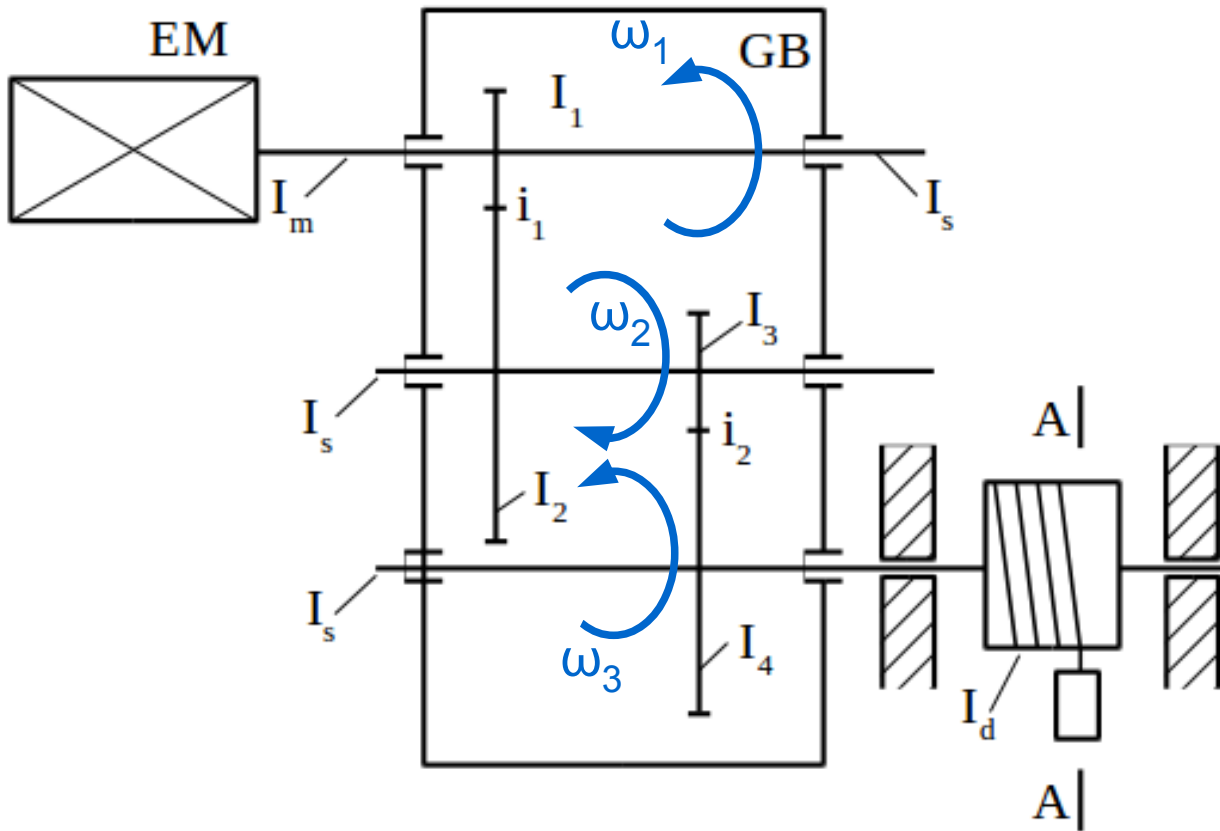
$$\frac{\omega_3}{\omega_2} = i_2 \rightarrow \omega_3 = \omega_2 i_2 = \omega_1 i_1 i_2$$

$$v = \frac{D}{2} \omega_3 = \frac{D}{2} \omega_1 i_1 i_2$$

$$T = \frac{1}{2} (I_m + I_1 + I_s) \omega_1^2 + \frac{1}{2} (I_2 + I_3 + I_s) \omega_2^2 + \frac{1}{2} (I_4 + I_d + I_s) \omega_3^2 + \frac{1}{2} \frac{G}{g} v^2$$

Reduction of masses and forces

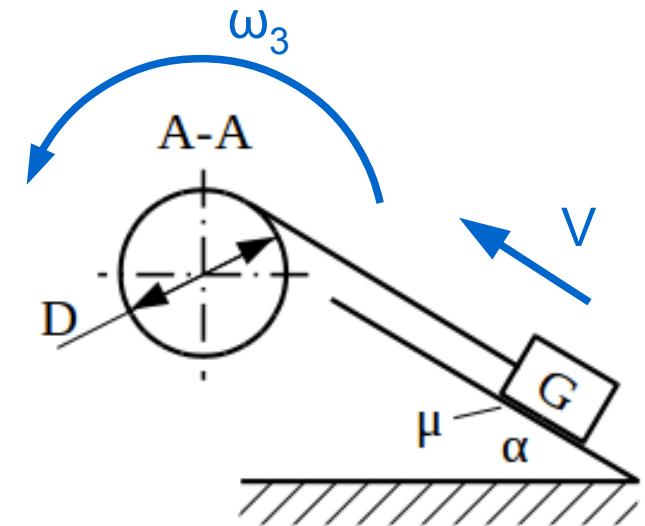
Example 1



$$\frac{\omega_2}{\omega_1} = i_1 \rightarrow \omega_2 = \omega_1 i_1$$

$$\frac{\omega_3}{\omega_2} = i_2 \rightarrow \omega_3 = \omega_2 i_2 = \omega_1 i_1 i_2$$

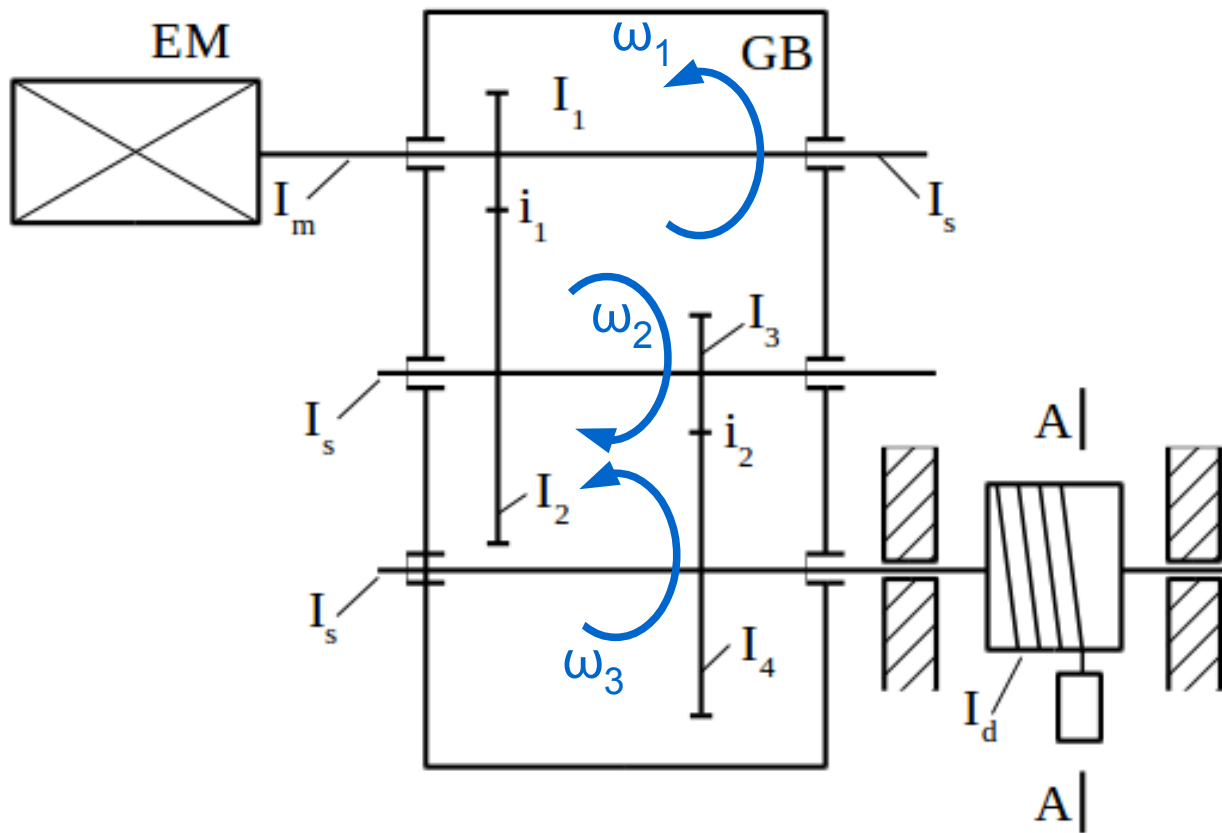
$$v = \frac{D}{2} \omega_3 = \frac{D}{2} \omega_1 i_1 i_2$$



$$T = \frac{1}{2} (I_m + I_1 + I_s) \omega_1^2 + \frac{1}{2} (I_2 + I_3 + I_s) \omega_1^2 i_1^2 + \frac{1}{2} (I_4 + I_d + I_s) \omega_1^2 i_1^2 i_2^2 + \frac{1}{2} \frac{G}{g} \frac{D^2}{4} \omega_1^2 i_1^2 i_2^2$$

Reduction of masses and forces

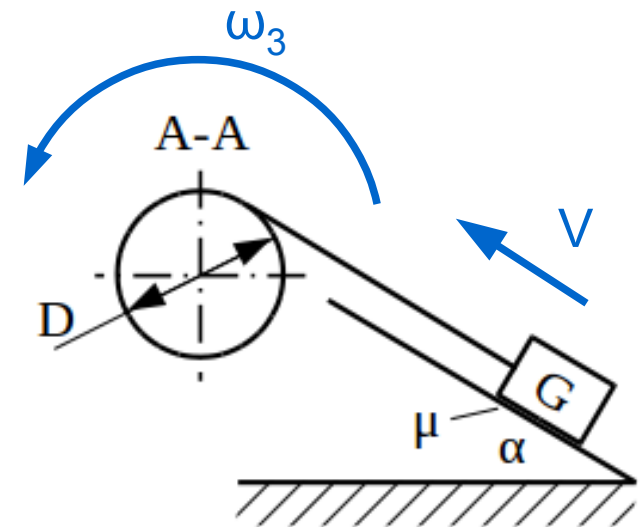
Example 1



$$\frac{\omega_2}{\omega_1} = i_1 \rightarrow \omega_2 = \omega_1 i_1$$

$$\frac{\omega_3}{\omega_2} = i_2 \rightarrow \omega_3 = \omega_2 i_2 = \omega_1 i_1 i_2$$

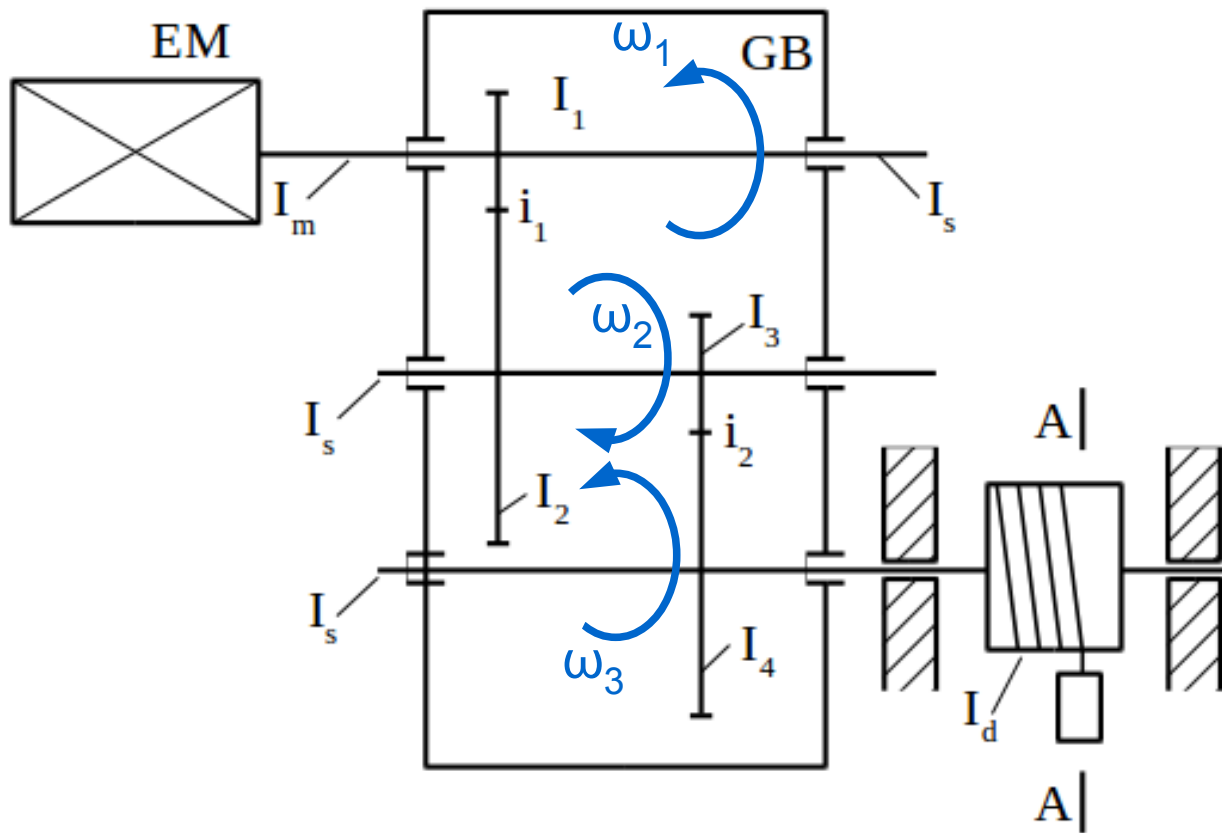
$$v = \frac{D}{2} \omega_3 = \frac{D}{2} \omega_1 i_1 i_2$$



$$T = \frac{1}{2} \left[(I_m + I_1 + I_s) + (I_2 + I_3 + I_s) i_1^2 + (I_4 + I_d + I_s) i_1^2 i_2^2 + \frac{G}{g} \frac{D^2}{4} i_1^2 i_2^2 \right] \omega_1^2$$

Reduction of masses and forces

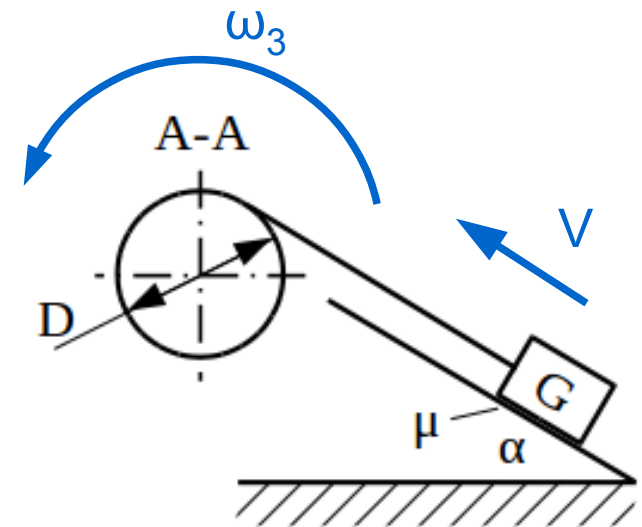
Example 1



$$\frac{\omega_2}{\omega_1} = i_1 \rightarrow \omega_2 = \omega_1 i_1$$

$$\frac{\omega_3}{\omega_2} = i_2 \rightarrow \omega_3 = \omega_2 i_2 = \omega_1 i_1 i_2$$

$$v = \frac{D}{2} \omega_3 = \frac{D}{2} \omega_1 i_1 i_2$$

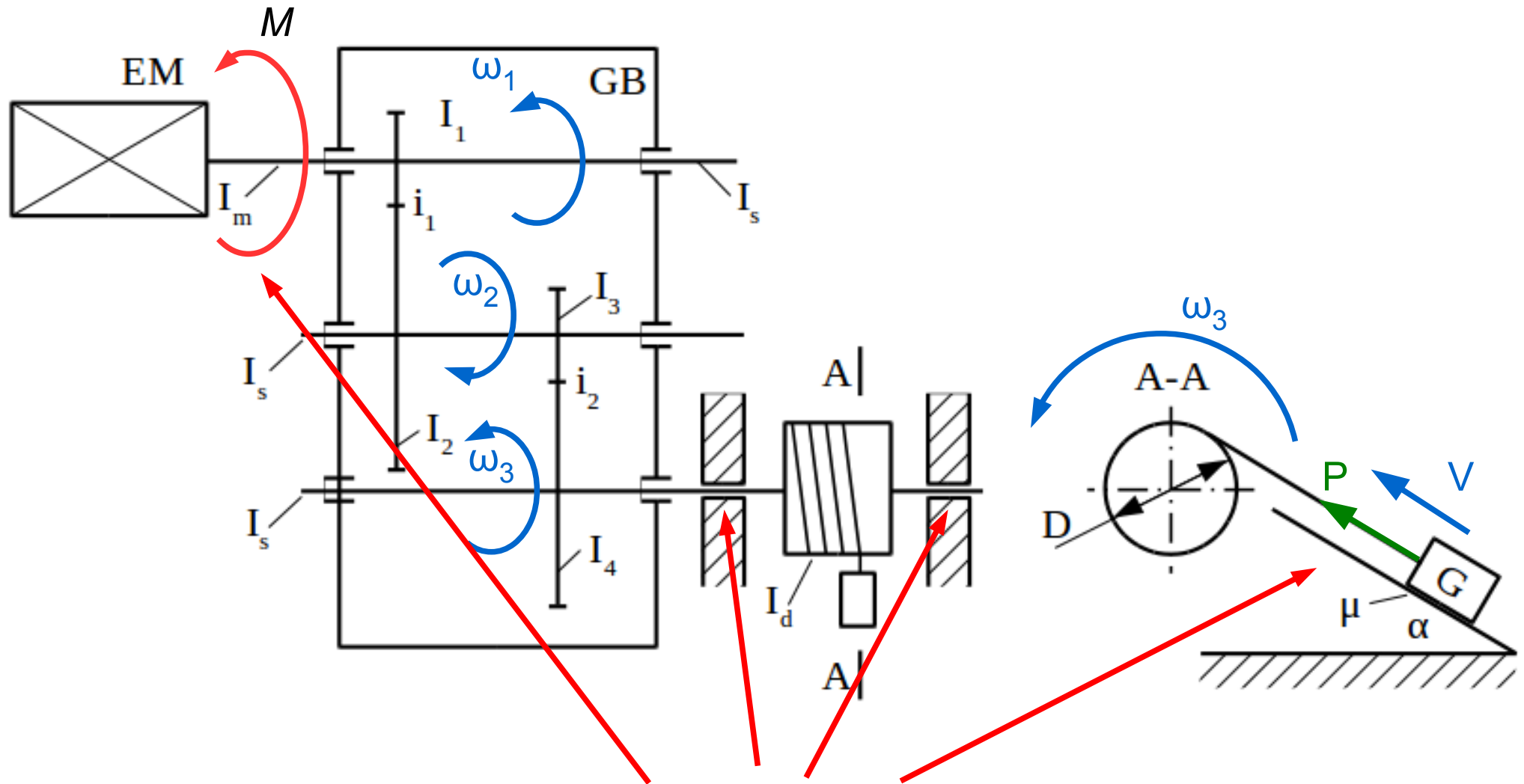


reduced moment of inertia

$$I_R = I_m + I_1 + I_s + (I_2 + I_3 + I_s) i_1^2 + (I_4 + I_d + I_s) i_1^2 i_2^2 + \frac{G}{g} \frac{D^2}{4} i_1^2 i_2^2$$

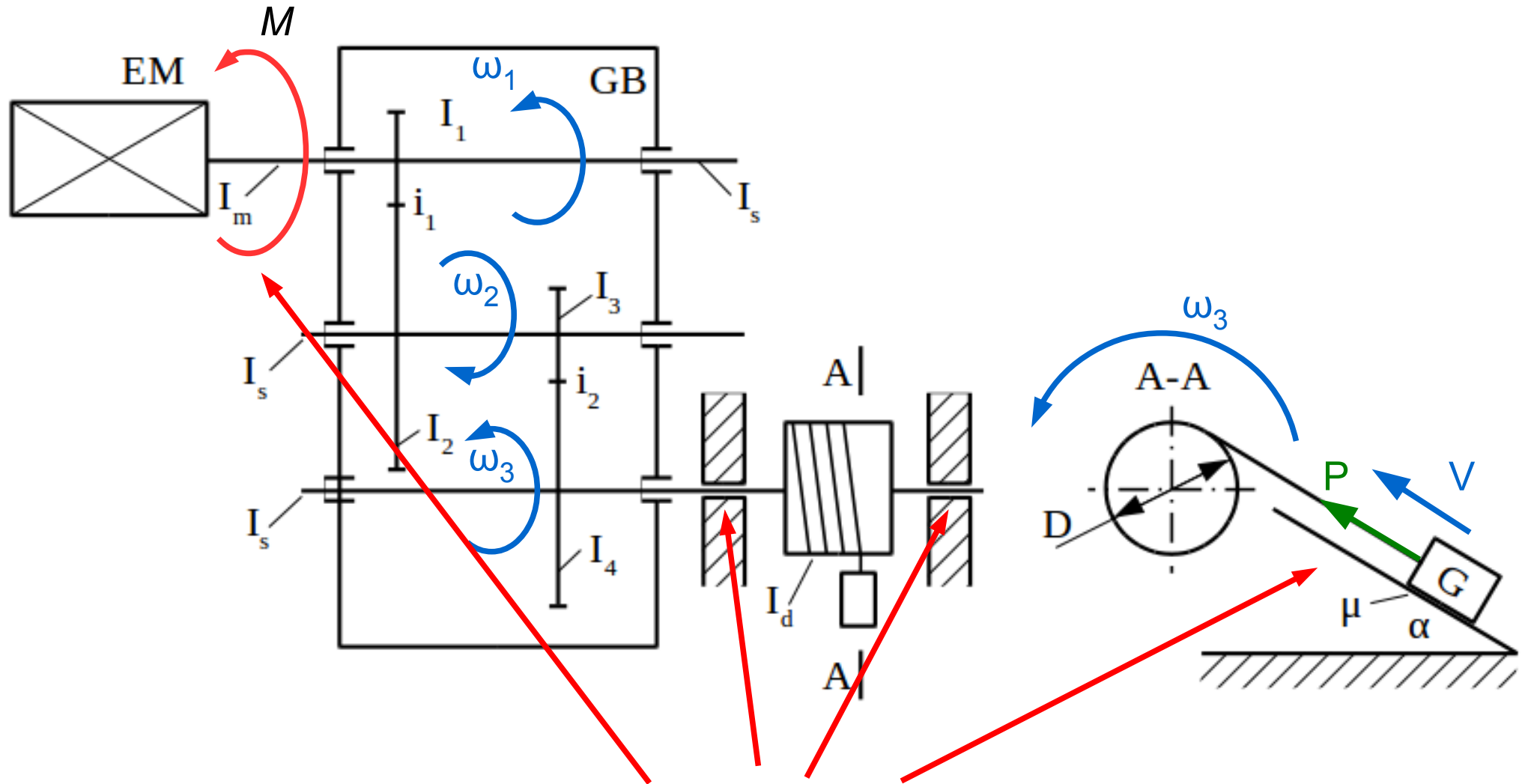
Reduction of masses and forces

Example 1



Reduction of masses and forces

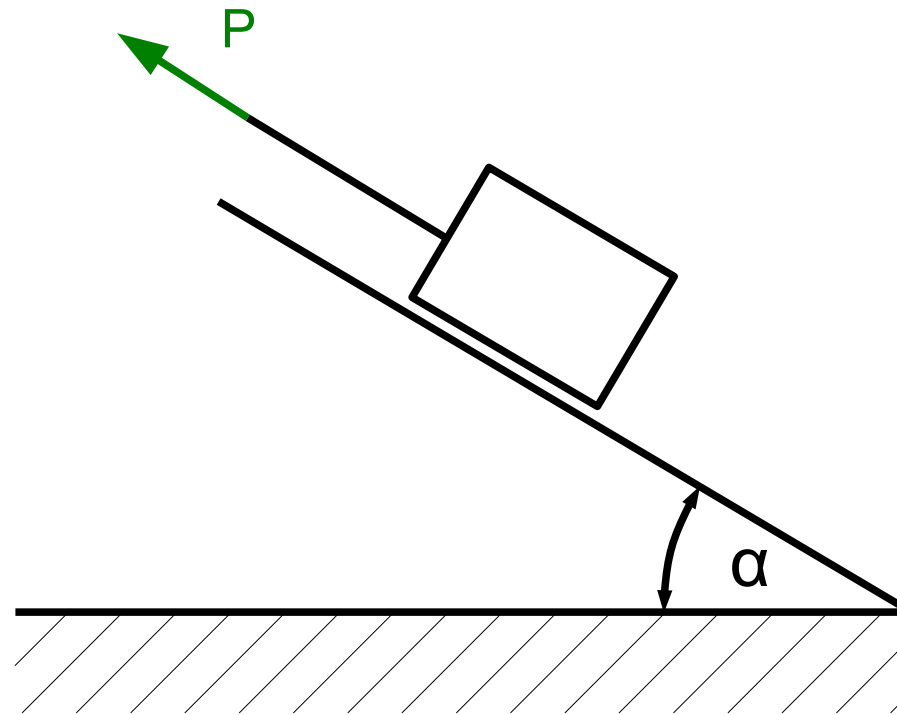
Example 1



$$N = M_s \omega_1 - M_f \omega_3 - P v$$

Reduction of masses and forces

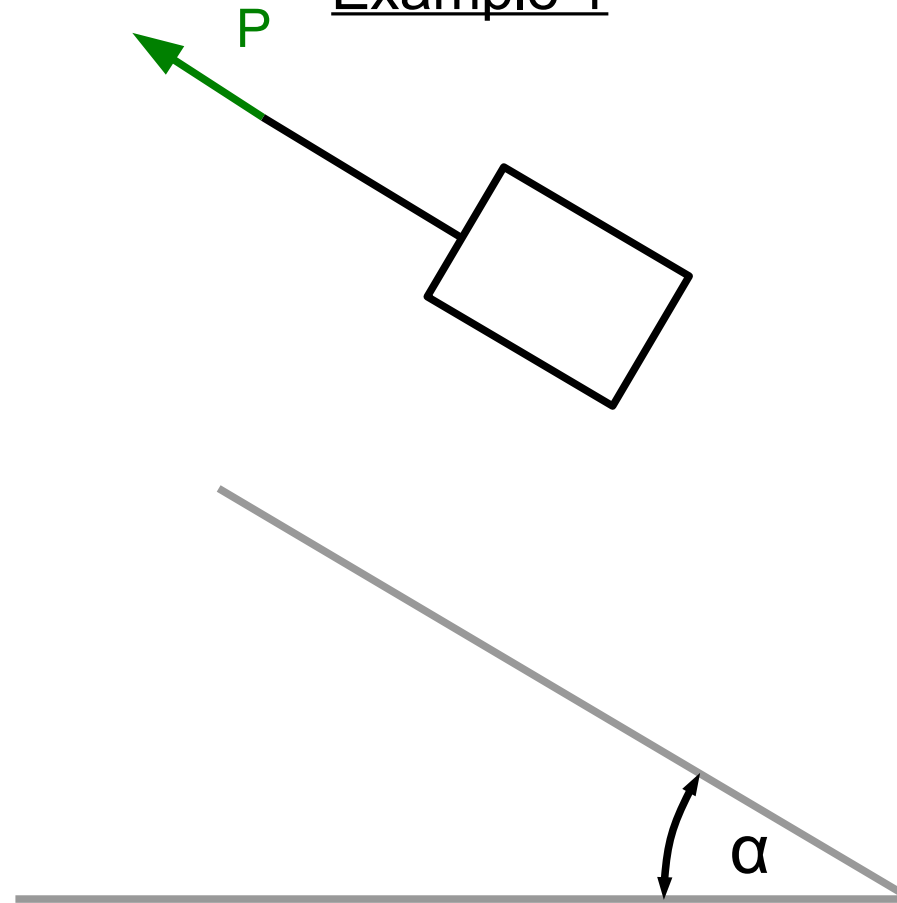
Example 1



$P = \dots$

Reduction of masses and forces

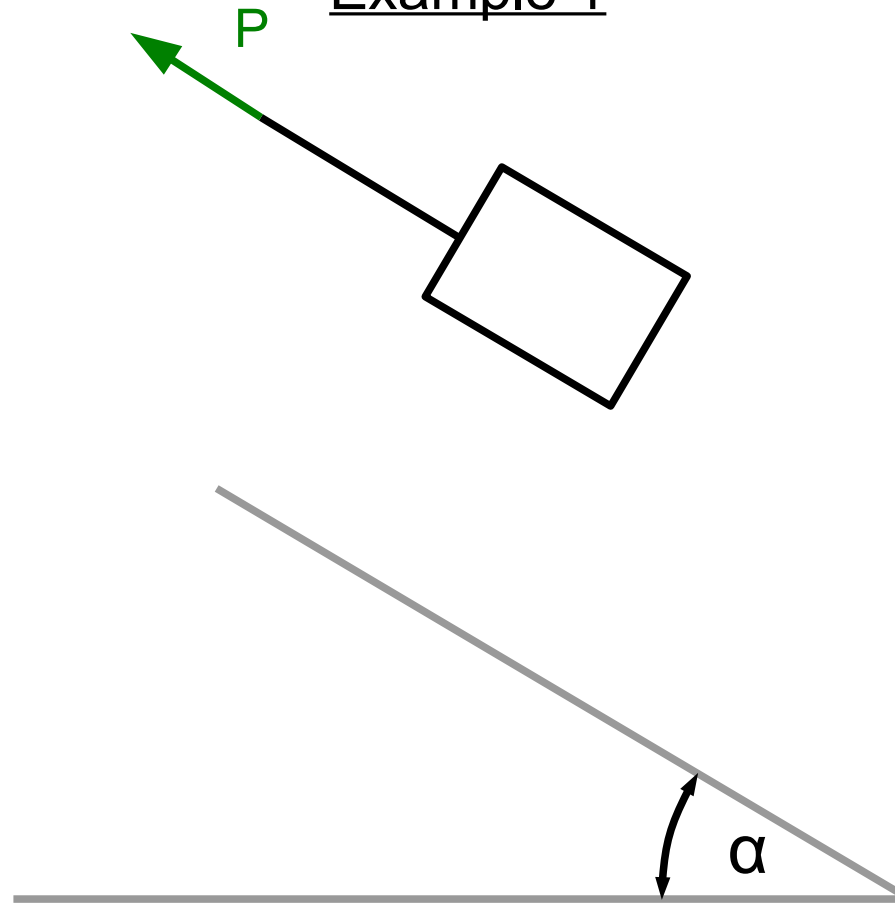
Example 1



$P = \dots$

Reduction of masses and forces

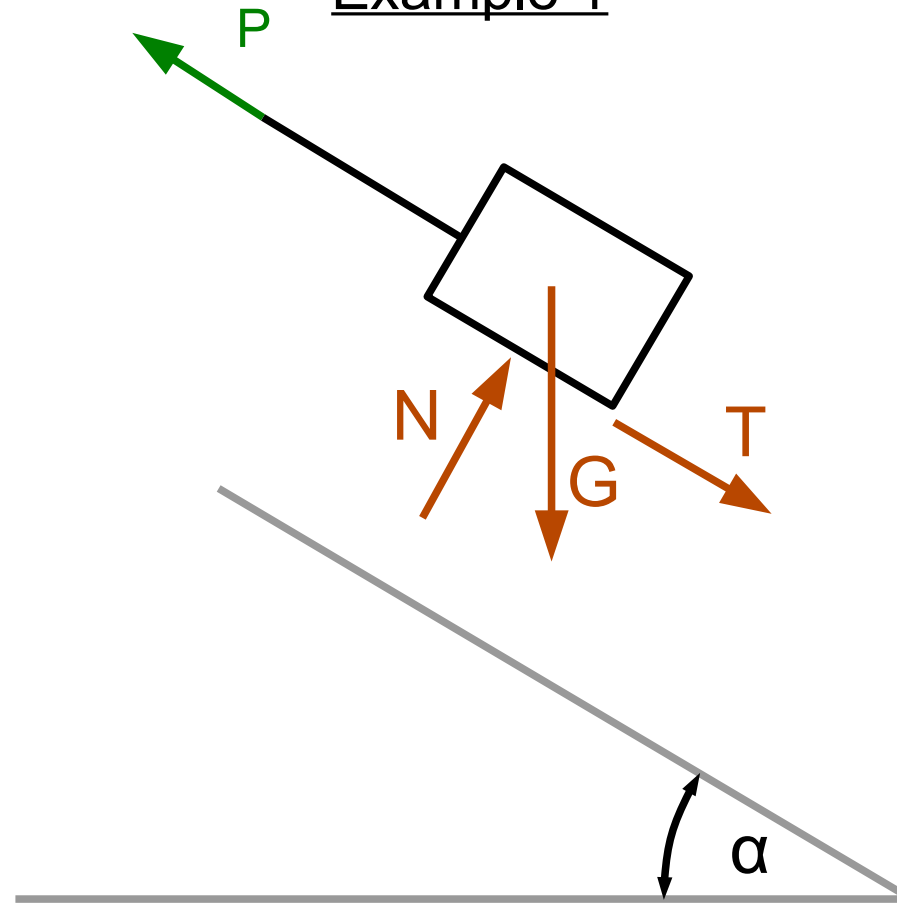
Example 1



$P = \dots$

Reduction of masses and forces

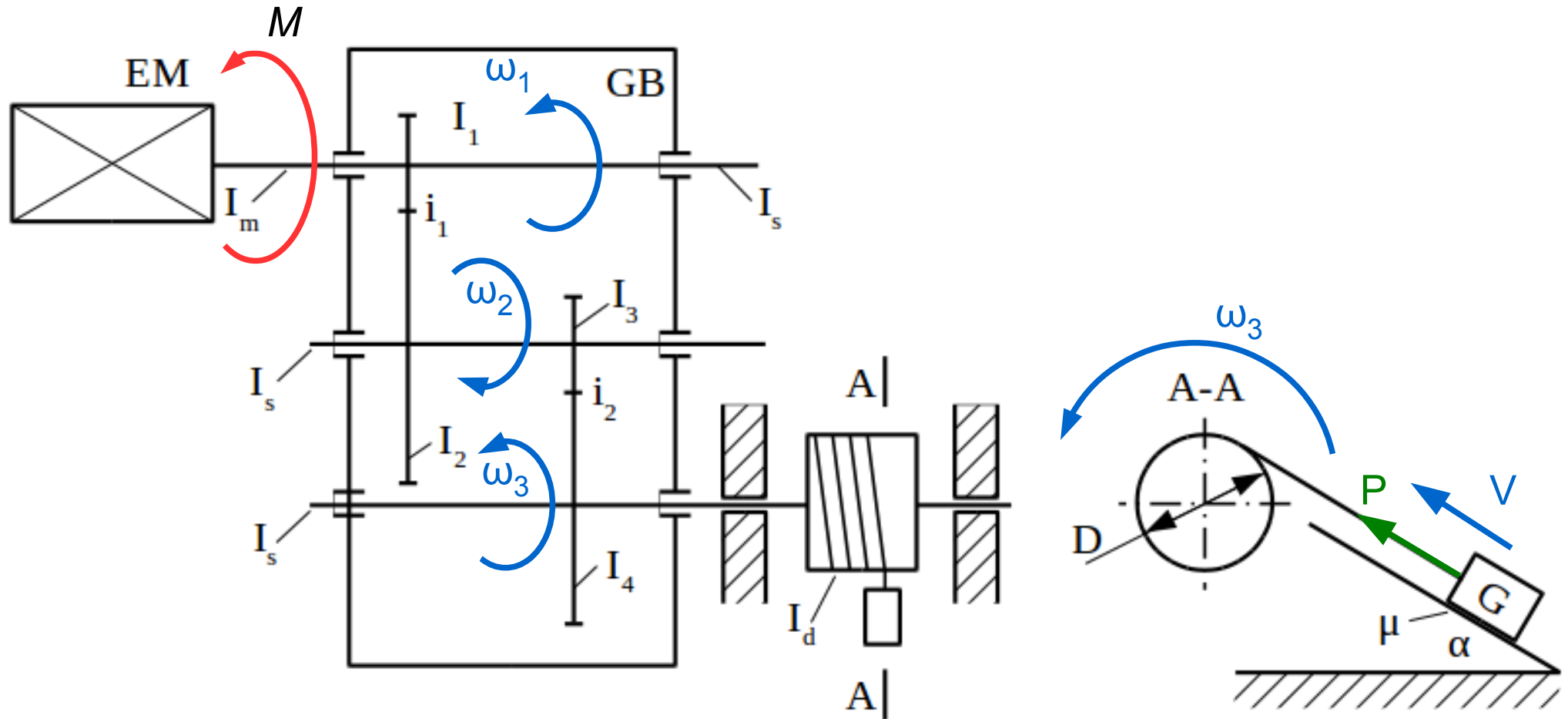
Example 1



$$P = T + G \sin \alpha = \mu N + G \sin \alpha = \mu G \cos \alpha + G \sin \alpha$$

Reduction of masses and forces

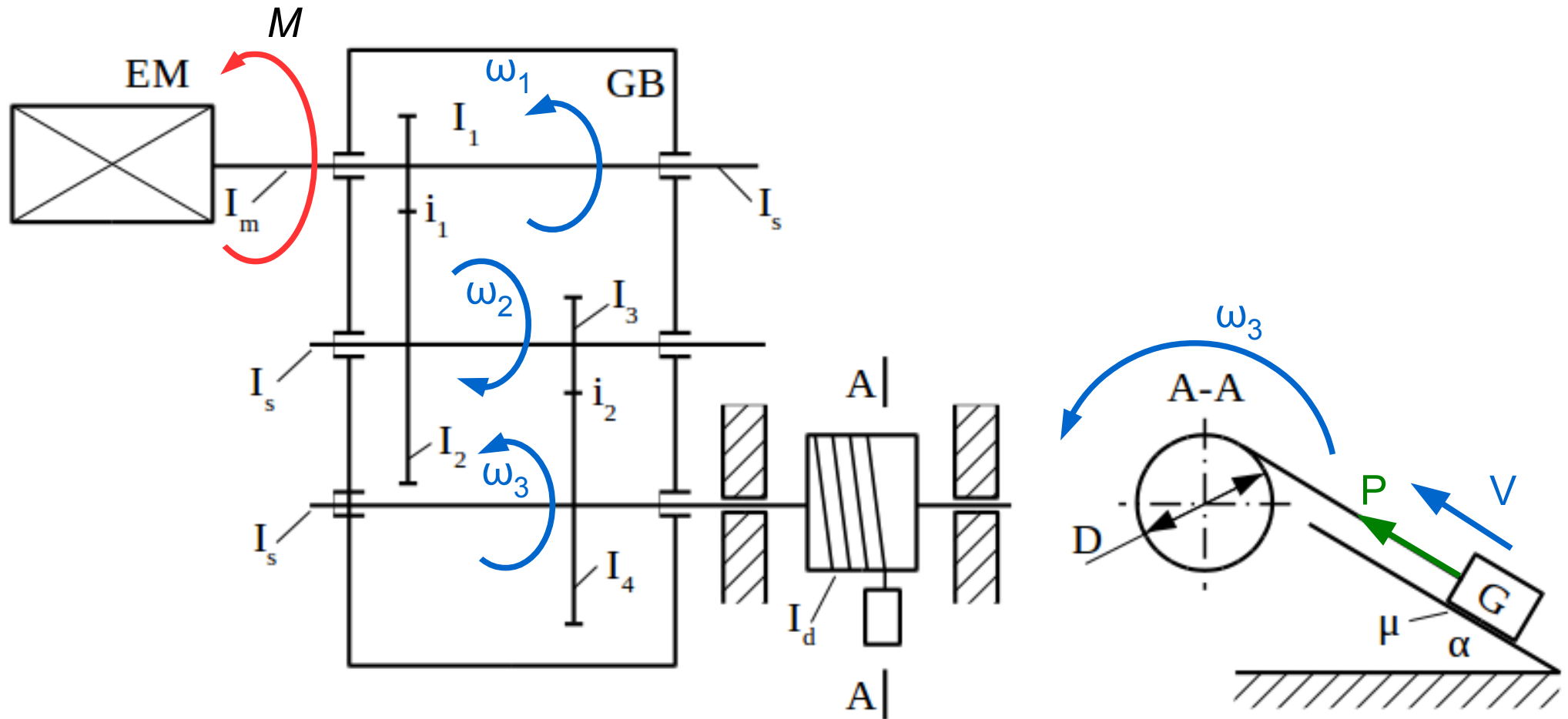
Example 1



$$N = M_s \omega_1 - M_f \omega_3 - (\mu G \cos \alpha + G \sin \alpha) v$$

Reduction of masses and forces

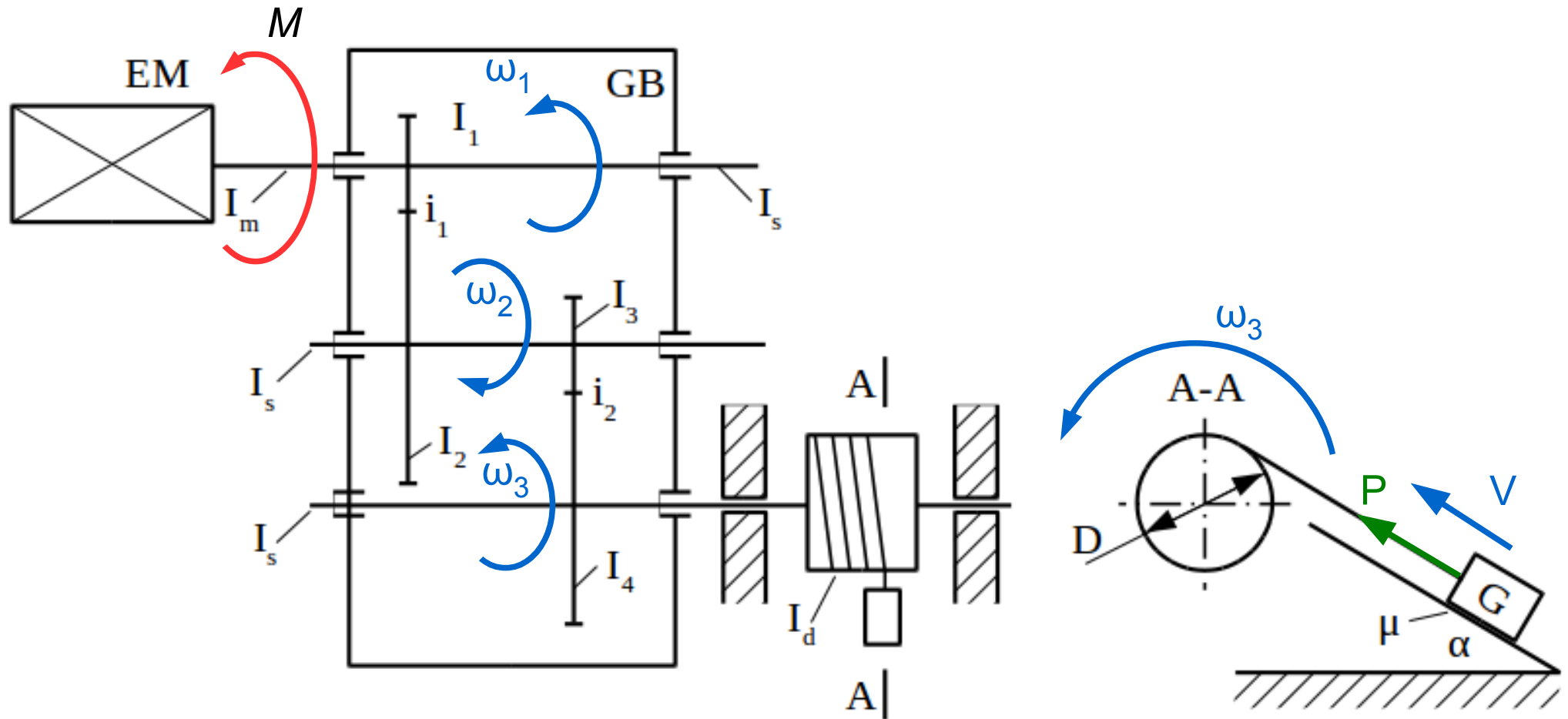
Example 1



$$N = M_s \omega_1 - M_f \omega_1 i_1 i_2 - (\mu G \cos \alpha + G \sin \alpha) \frac{D}{2} \omega_1 i_1 i_2$$

Reduction of masses and forces

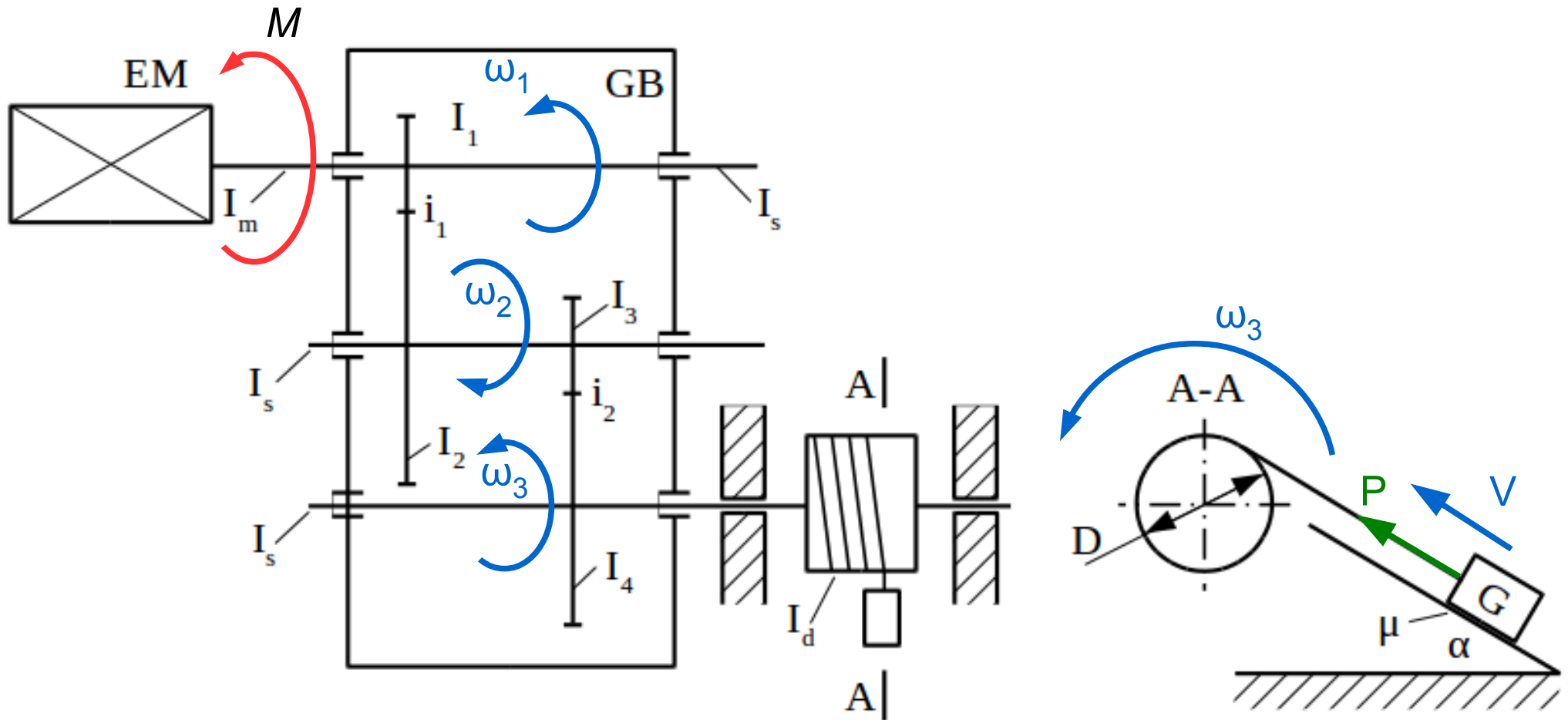
Example 1



$$N = \left[M_s - M_f i_1 i_2 - (\mu G \cos \alpha + G \sin \alpha) \frac{D}{2} i_1 i_2 \right] \omega_1$$

Reduction of masses and forces

Example 1



reduced torque

$$M_R = M_s - M_f i_1 i_2 - (\mu G \cos \alpha + G \sin \alpha) \frac{D}{2} i_1 i_2$$

Reduction of masses and forces

Example 1

Reduced torque

$$M_R = M_s - \left(M_f i_1 i_2 + (\mu G \cos \alpha + G \sin \alpha) \frac{D}{2} i_1 i_2 \right)$$

Reduction of masses and forces

Example 1

Reduced torque

$$M_R = M_s - \left(M_f i_1 i_2 + (\mu G \cos \alpha + G \sin \alpha) \frac{D}{2} i_1 i_2 \right)$$

driving torque M_D
(electric motor torque)

passive torque M_P

Reduction of masses and forces

Example 1

Reduced torque

$$M_R = M_s - \left(M_f i_1 i_2 + (\mu G \cos \alpha + G \sin \alpha) \frac{D}{2} i_1 i_2 \right)$$

driving torque M_D
(electric motor torque)

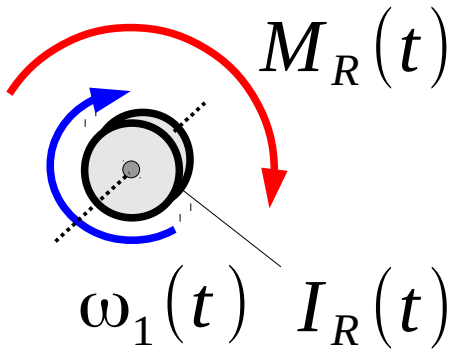
passive torque M_P

$$M_R = M_D - M_P = (A - B \omega_1) - M_P$$

Reduction of masses and forces

Example 1

Start-up process



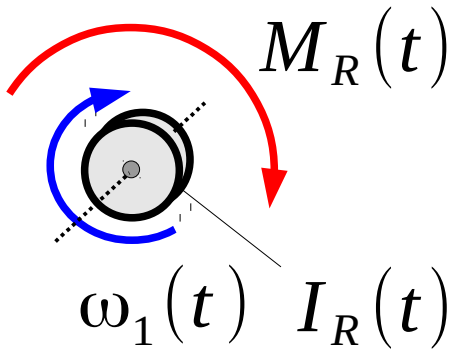
$$I_R \frac{d\omega_1}{dt} = M_R$$

$$M_R = A - B\omega_1 - M_P$$

Reduction of masses and forces

Example 1

Start-up process



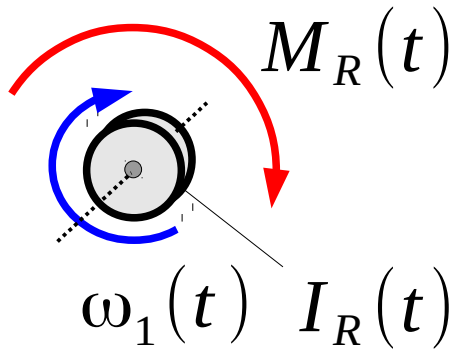
$$I_R \frac{d\omega_1}{dt} = M_R$$

$$M_R = A - B\omega_1 - M_P$$

Reduction of masses and forces

Example 1

Start-up process



$$I_R \frac{d\omega_1}{dt} = M_R$$

$$M_R = A - B\omega_1 - M_P$$

Reduction of masses and forces

Example 1

Start-up process

$$\frac{d\omega_1}{dt} + \frac{B}{I_R}\omega_1 = \frac{A - M_P}{I_R} \quad A, M_P, B, I_R - \text{constants}$$

non-homogeneous
1st order ODE
with const. coef.

general solution

particular solution

initial condition

Reduction of masses and forces

Example 1

Start-up process

$$\frac{d\omega_1}{dt} + \frac{B}{I_R}\omega_1 = \frac{A - M_P}{I_R} \quad A, M_P, B, I_R - \text{constants}$$

non-homogeneous
1st order ODE
with const. coef.

general solution

particular solution

initial condition

Reduction of masses and forces

Example 1

Start-up process

$$\frac{d\omega_1}{dt} + \frac{B}{I_R}\omega_1 = \frac{A - M_P}{I_R} \quad A, M_P, B, I_R - \text{constants}$$

non-homogeneous
1st order ODE
with const. coef.

general solution

$$\omega_{1g}(t) = E e^{-\frac{B}{I_R}t}$$

particular solution

$$\omega_{1p}(t) = F$$

initial condition

$$\omega_1(t=0) = 0$$

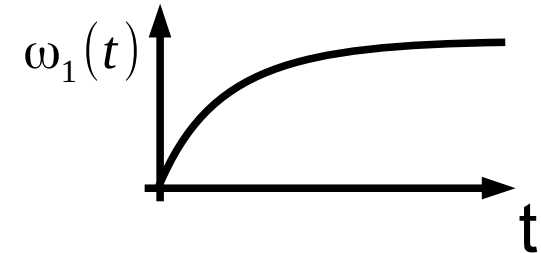
$$\omega_1(t) = \frac{A - M_P}{B} \left(1 - e^{-\frac{B}{I_R}t} \right)$$

Reduction of masses and forces

Example 1

Start-up process

$$\omega_1(t) = \frac{A - M_P}{B} \left(1 - e^{-\frac{B}{I_R} t} \right)$$

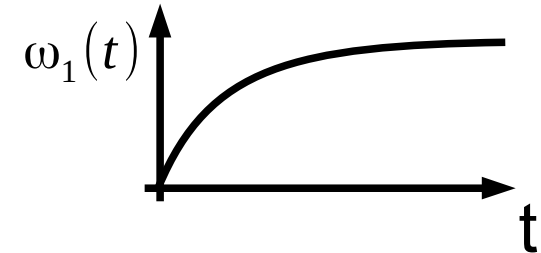


Reduction of masses and forces

Example 1

Start-up process

$$\omega_1(t) = \frac{A - M_P}{B} \left(1 - e^{-\frac{B}{I_R} t} \right)$$



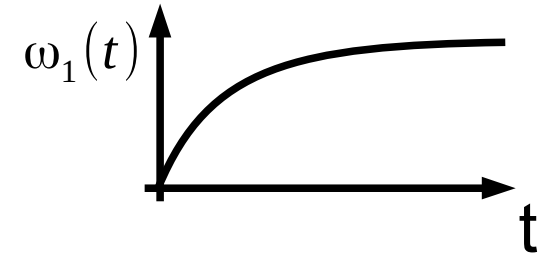
steady state value (maximum) $\omega_{max} = \frac{A - M_P}{B}$

Reduction of masses and forces

Example 1

Start-up process

$$\omega_1(t) = \frac{A - M_P}{B} \left(1 - e^{-\frac{B}{I_R} t} \right)$$



steady state value (maximum) $\omega_{max} = \frac{A - M_P}{B}$

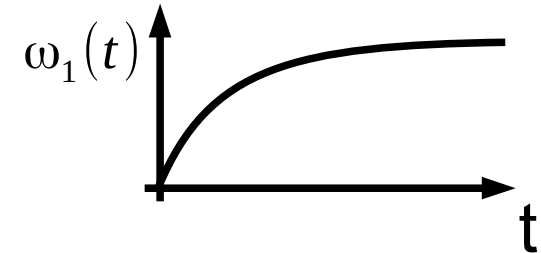
START-UP TIME
(95% of maximum value) $0,95 \frac{A - M_P}{B} = \frac{A - M_P}{B} \left(1 - e^{-\frac{B}{I_R} t_{95}} \right)$

Reduction of masses and forces

Example 1

Start-up process

$$\omega_1(t) = \frac{A - M_P}{B} \left(1 - e^{-\frac{B}{I_R} t} \right)$$



steady state value (maximum) $\omega_{max} = \frac{A - M_P}{B}$

START-UP TIME
(95% of maximum value) $0,95 \frac{A - M_P}{B} = \frac{A - M_P}{B} \left(1 - e^{-\frac{B}{I_R} t_{95}} \right)$

$$t_{95} \approx 3 \frac{I_R}{B}$$

Reduction of masses and forces

Example 1

Start-up process

$$\omega_1(t) = \frac{A - M_P}{B} \left(1 - e^{-\frac{B}{I_R} t} \right)$$

electric motor
angular velocity

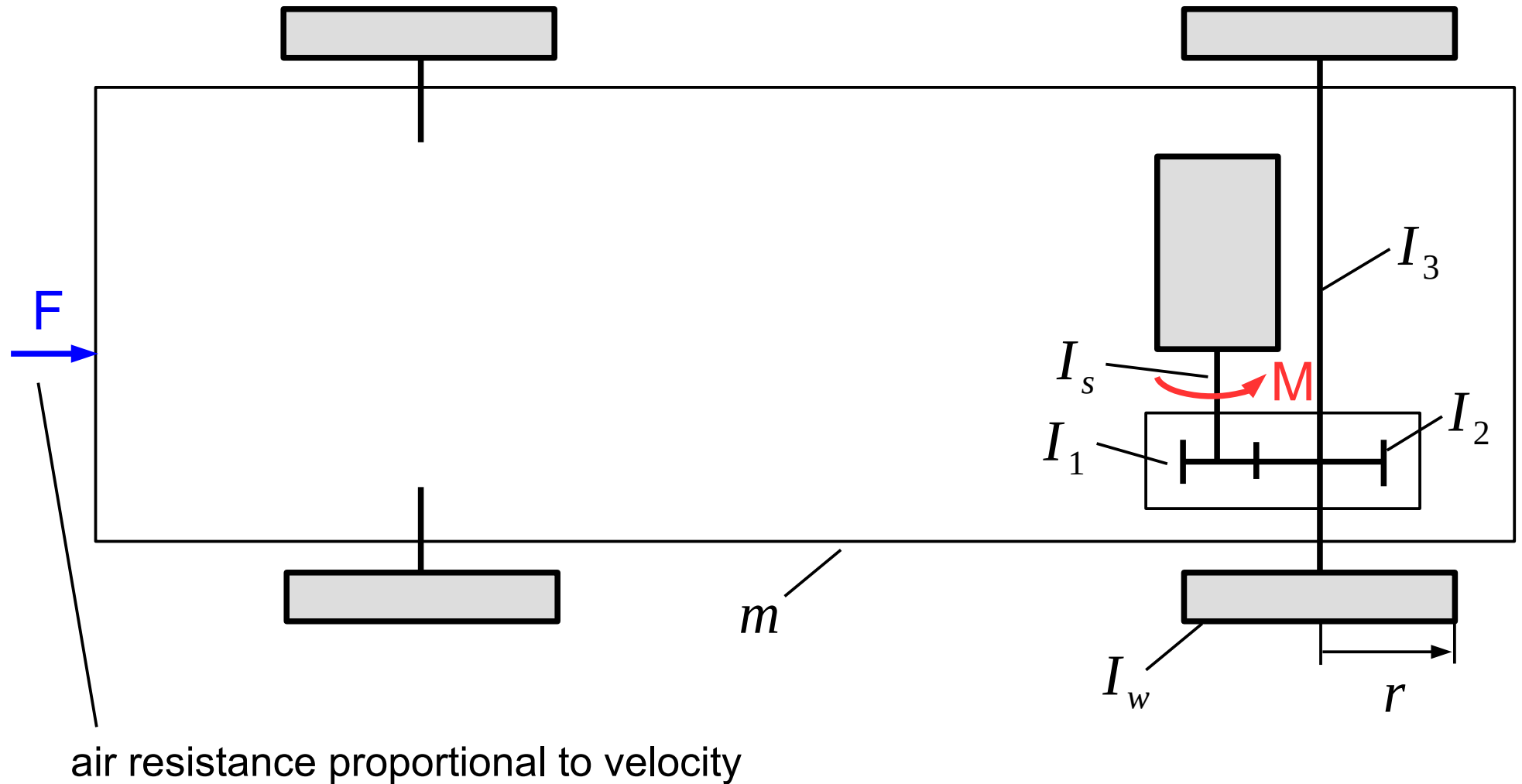


$$v(t) = \frac{D}{2} \omega_1(t) i_1 i_2$$

box linear velocity

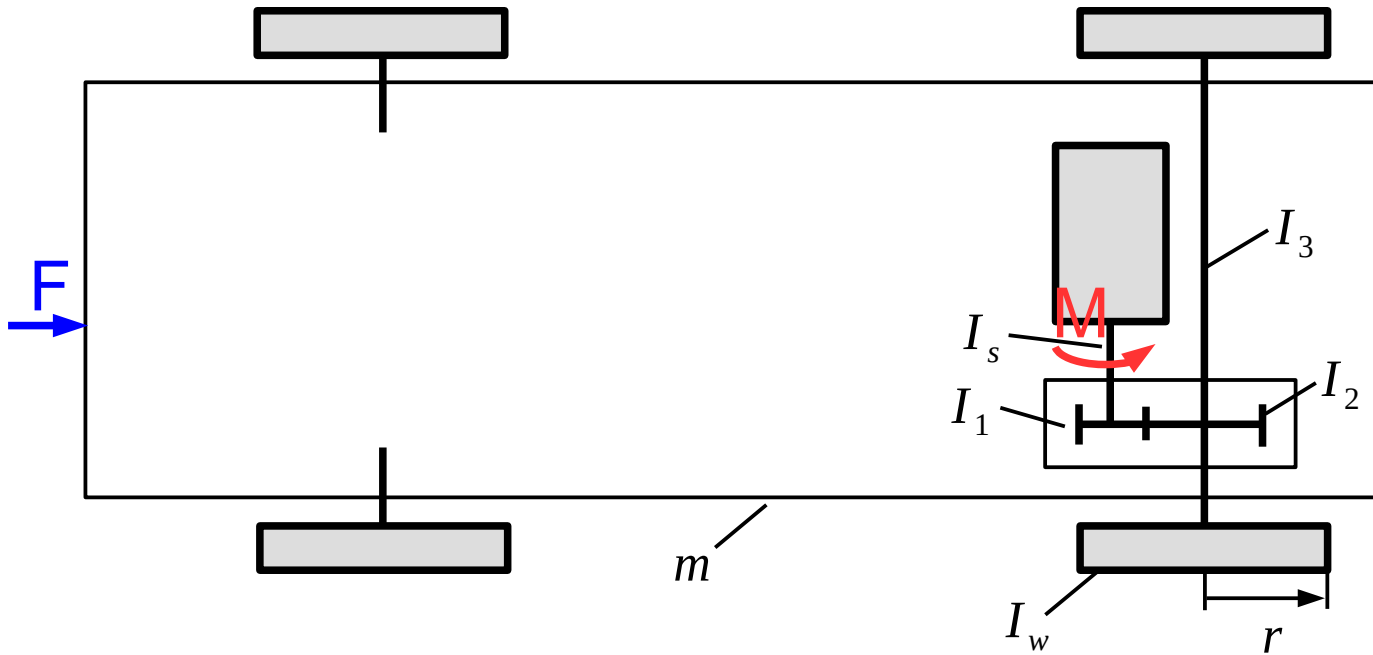
Reduction of masses and forces

Example 2



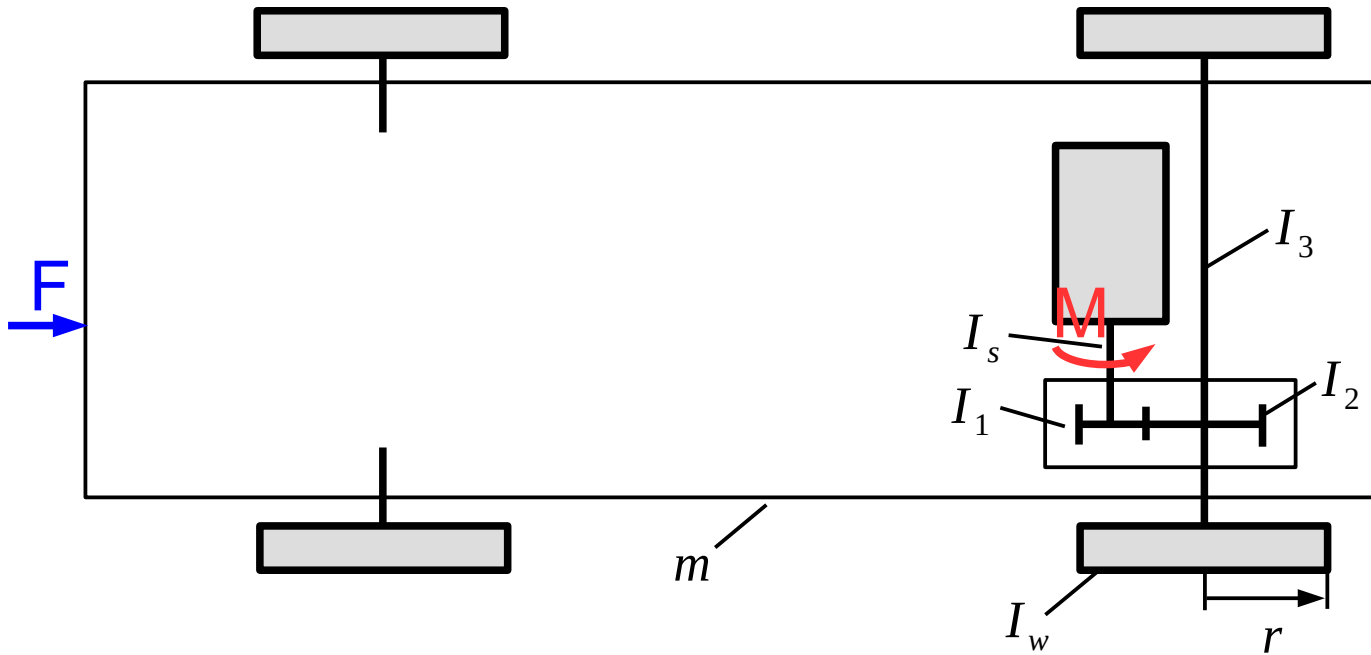
Reduction of masses and forces

Example 2



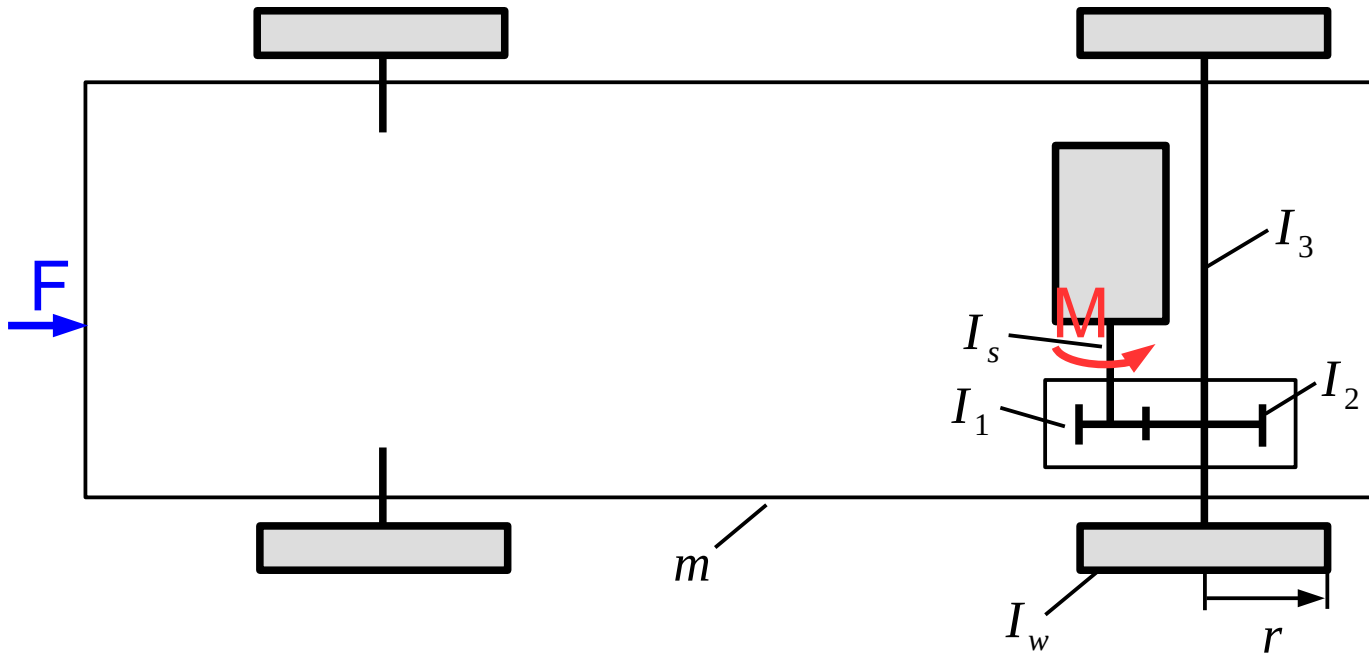
Reduction of masses and forces

Example 2



Reduction of masses and forces

Example 2



Reduction of masses and forces

Example 2

