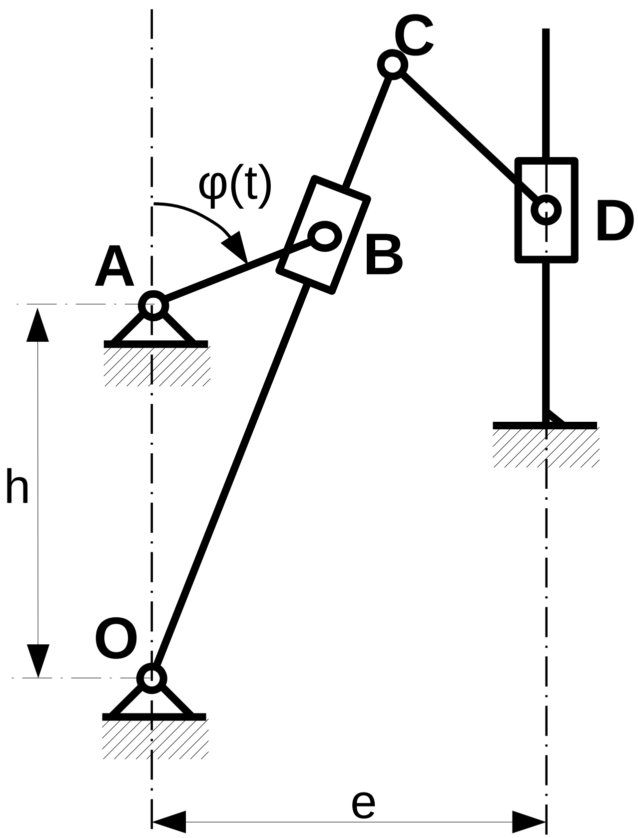
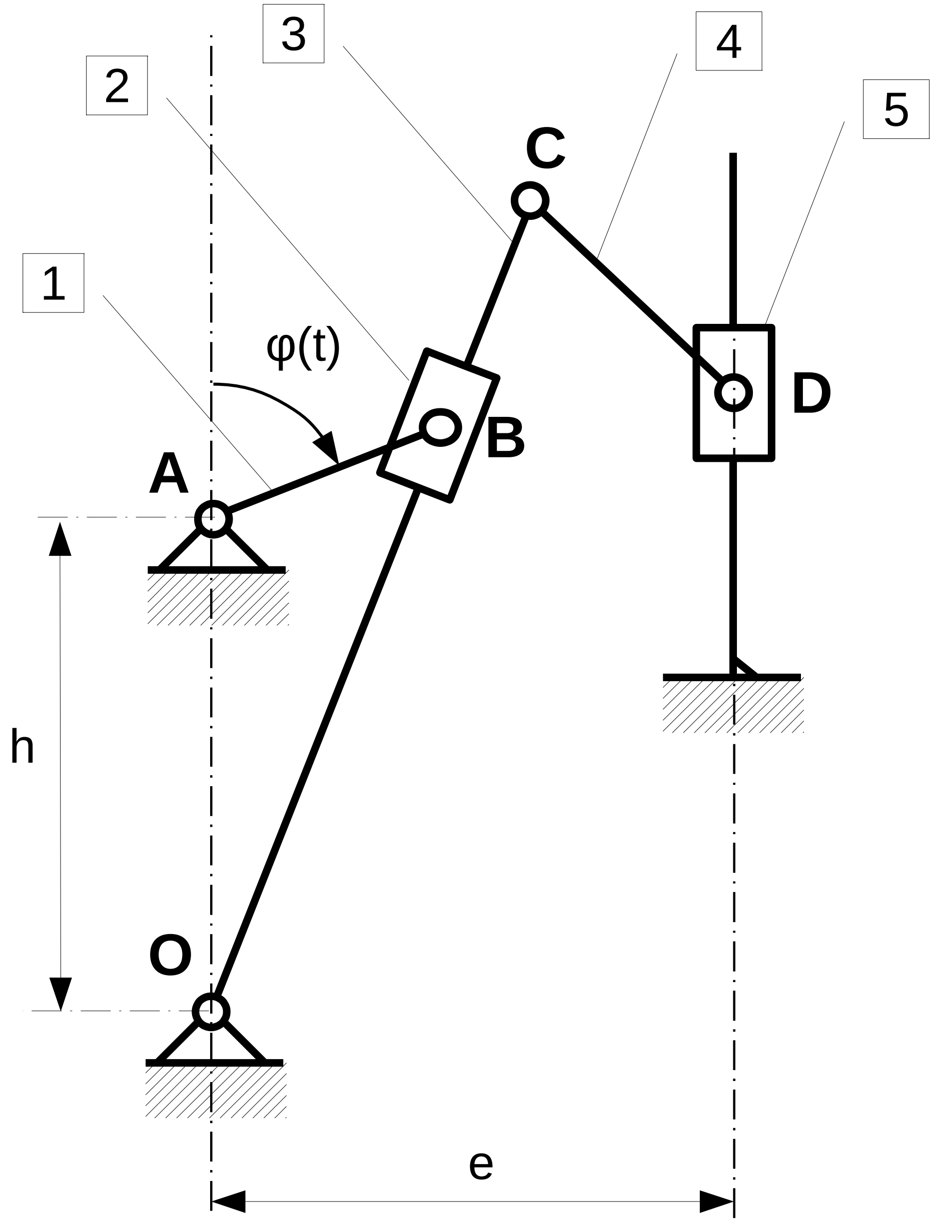
TM&AC - Winter 2019/2020

velocities and accelerations in planar mechanisms
EXAMPLE

Given: mechanism geometry and
angular displacement φ(t) of driven element.



Warsaw University of Technology, The Faculty of Automotive and Construction Machinery Engineering
Institute of Machine Design Fundamentals, Department of Mechanics, http://www.ipbm.simr.pw.edu.pl/,
Sebastian Korczak, PhD Eng.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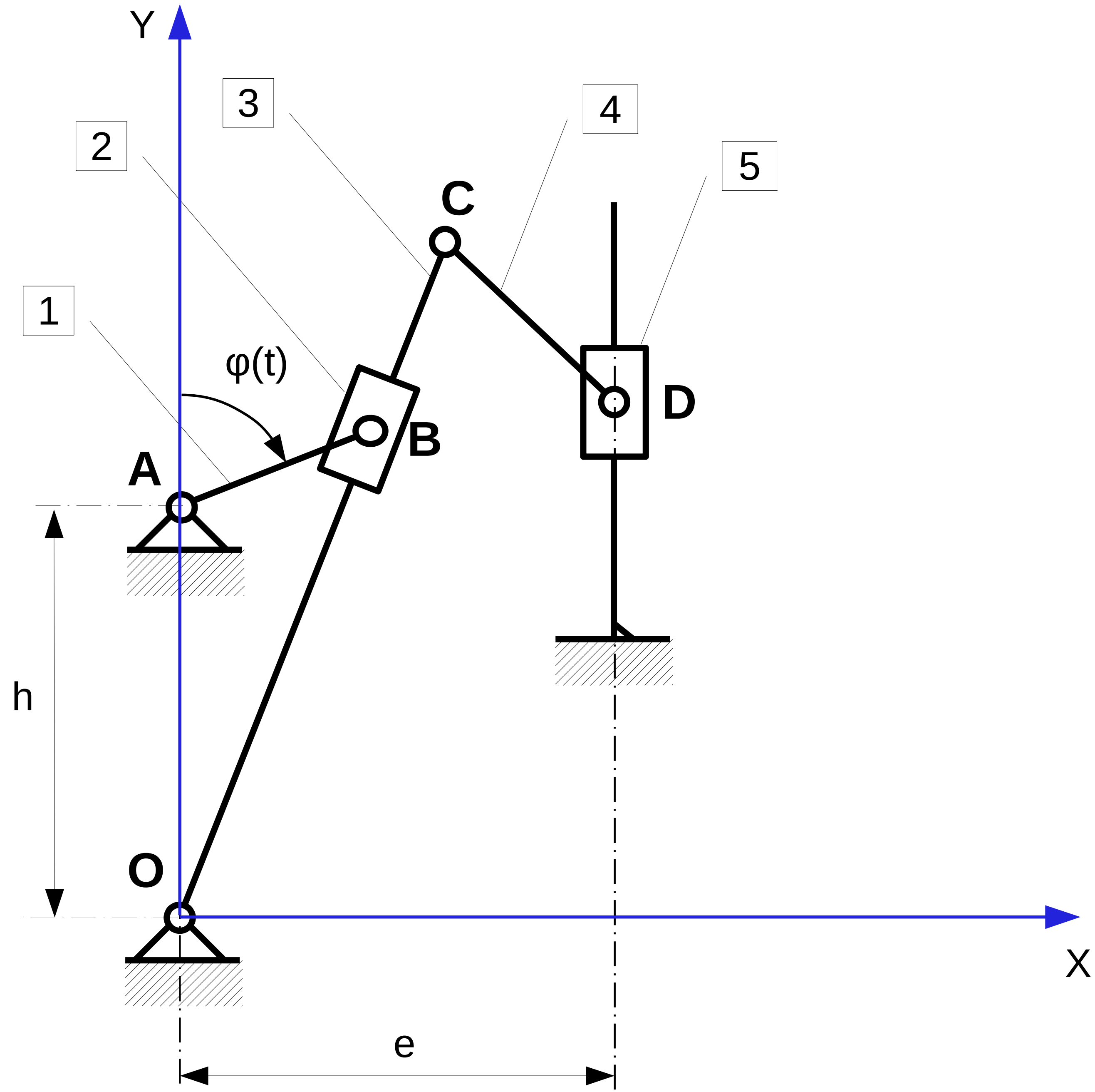
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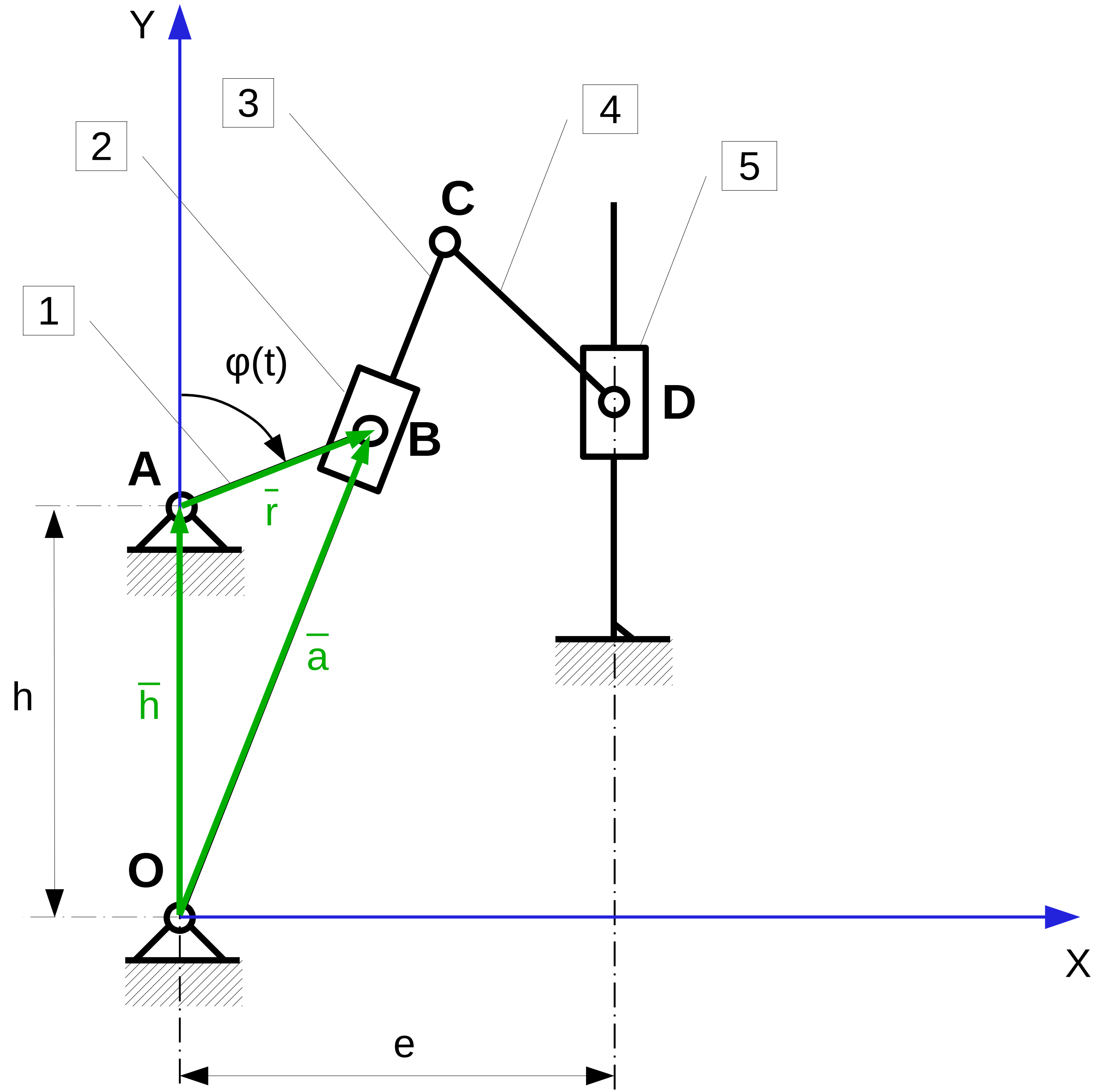
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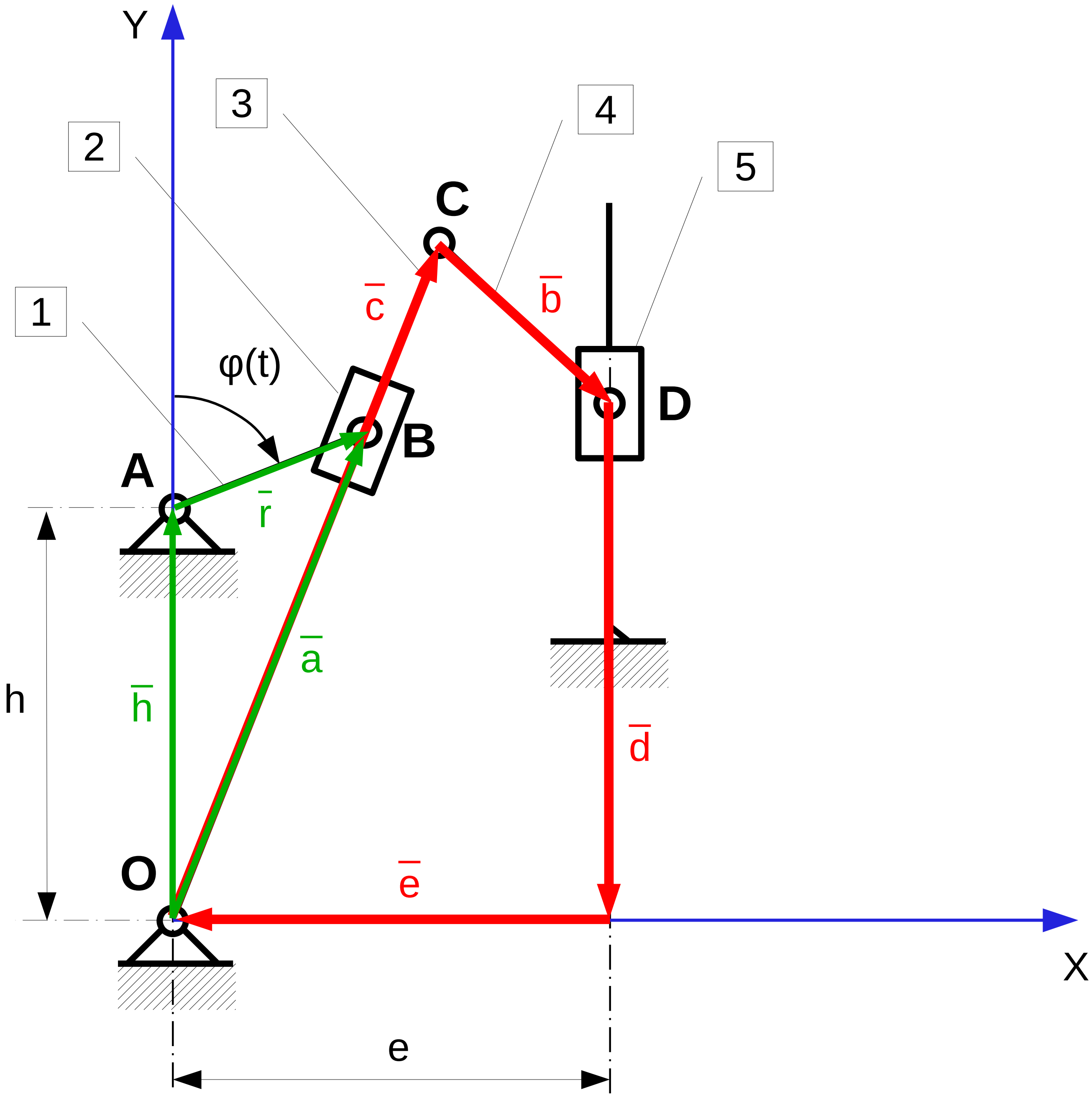

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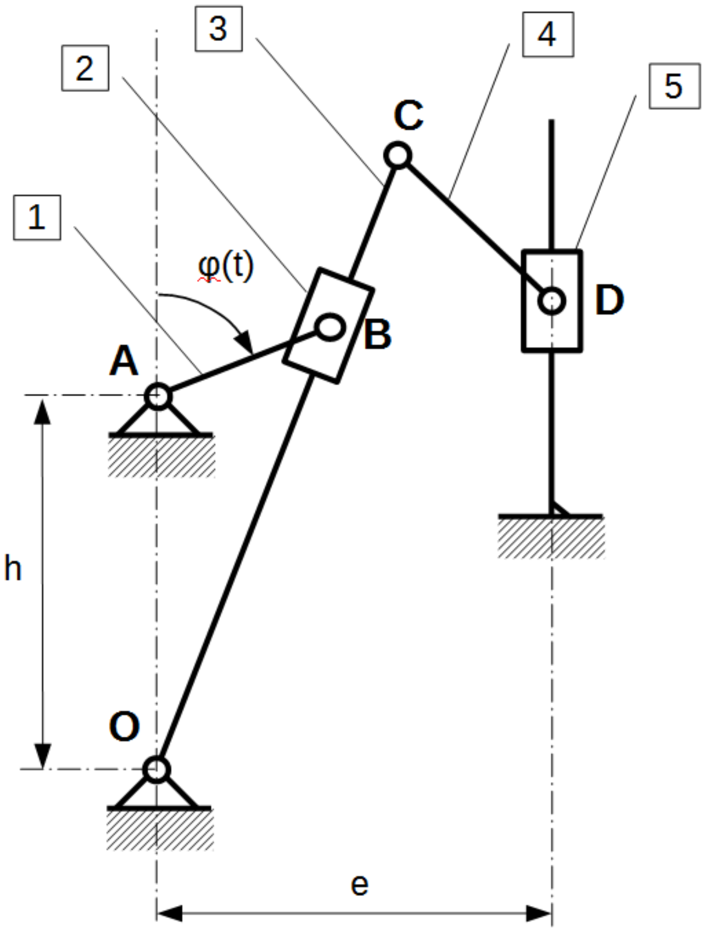
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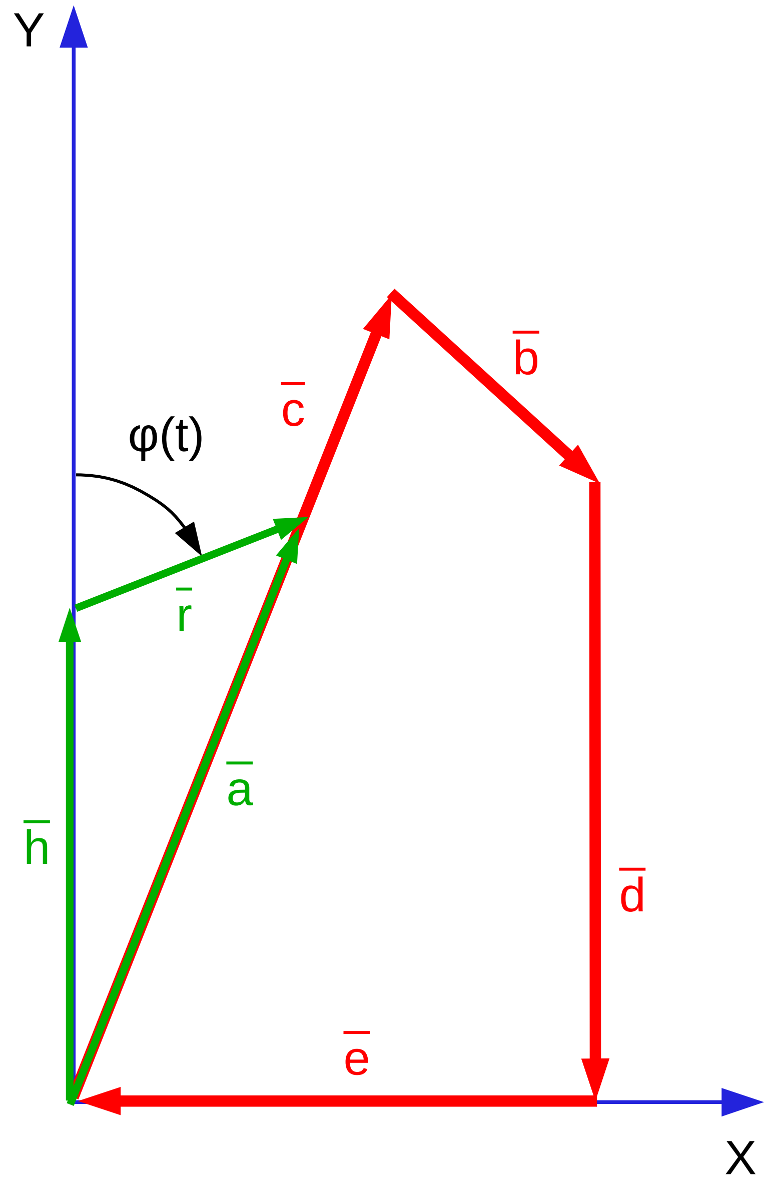
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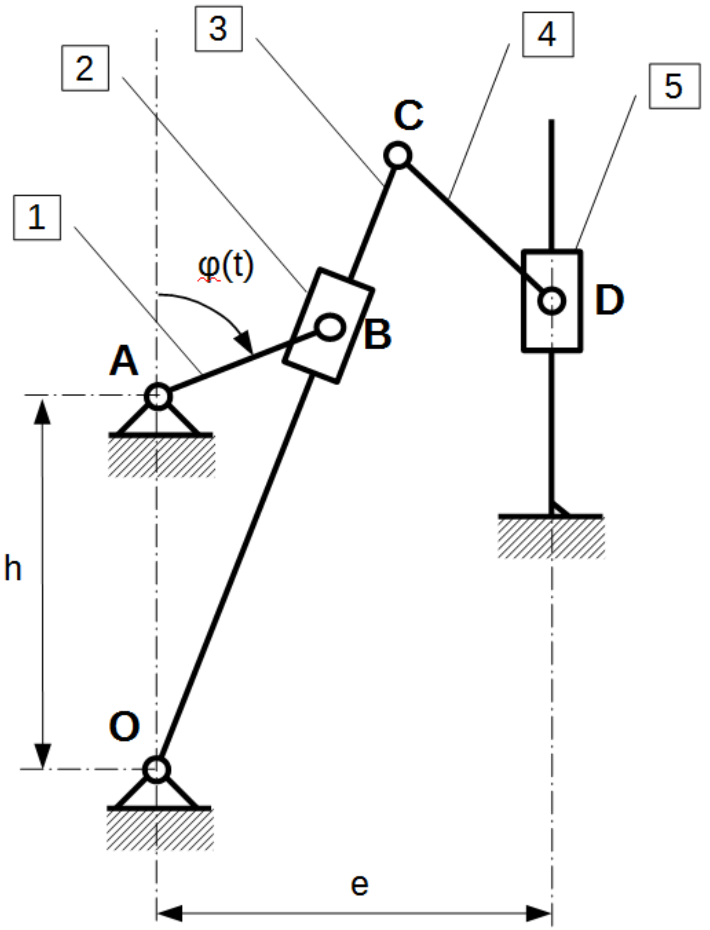
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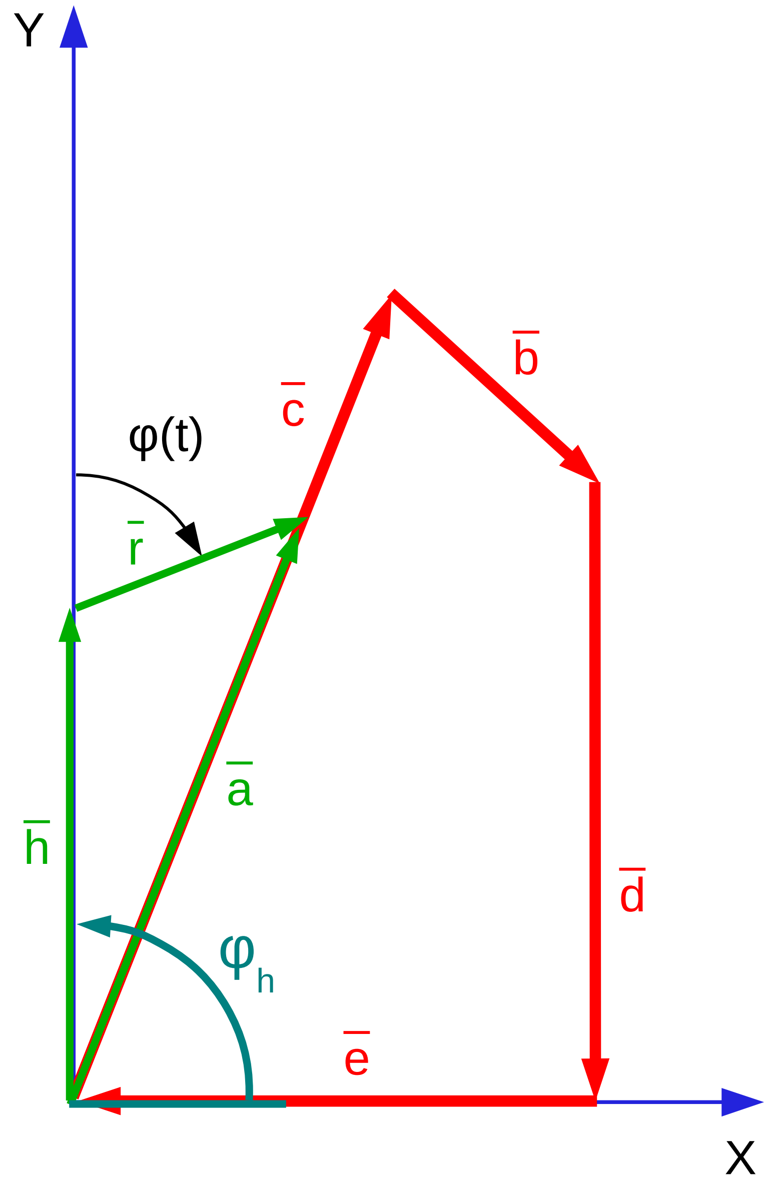
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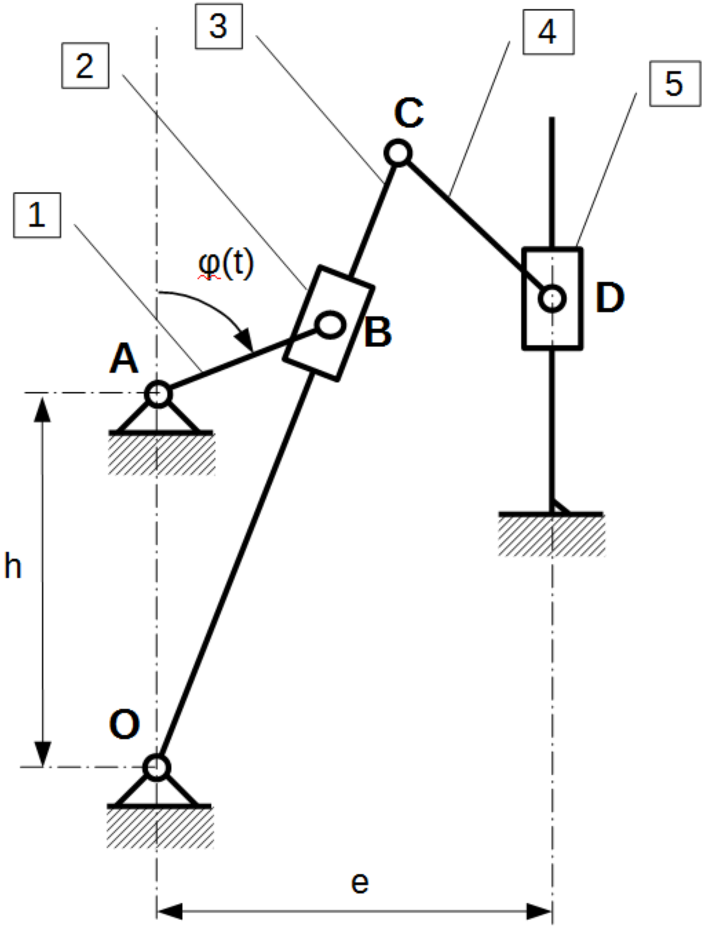
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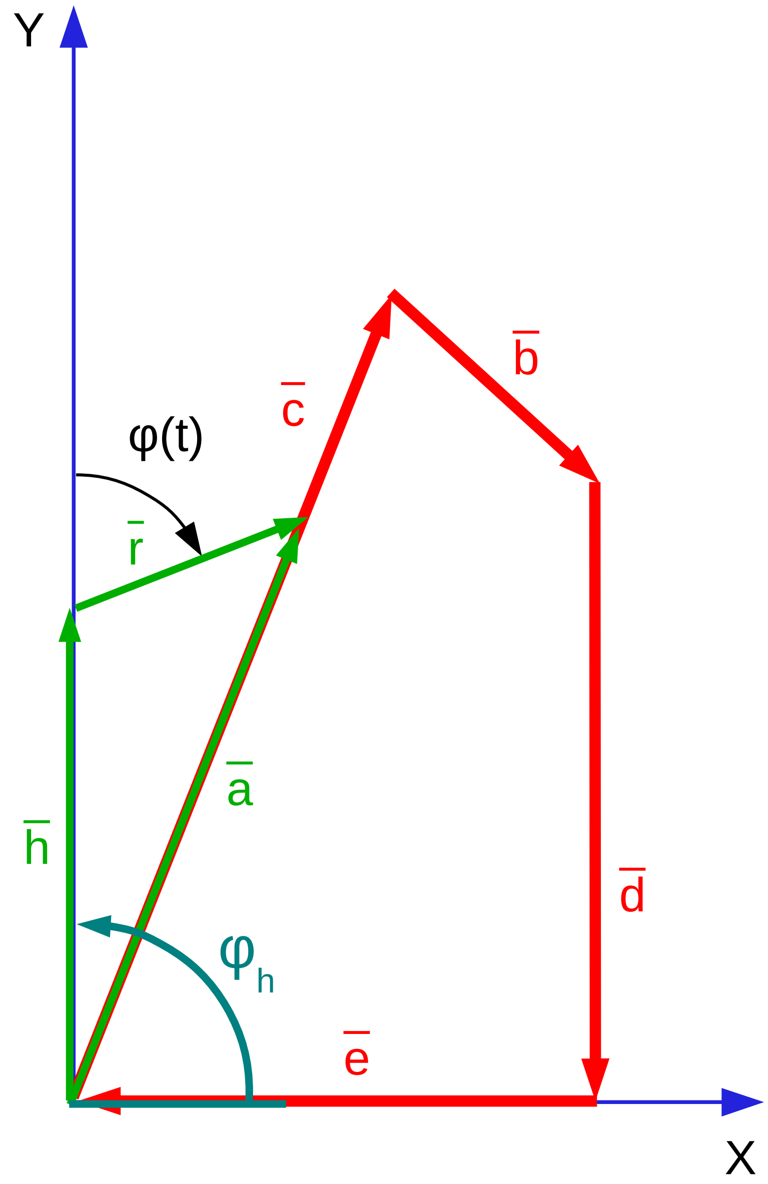
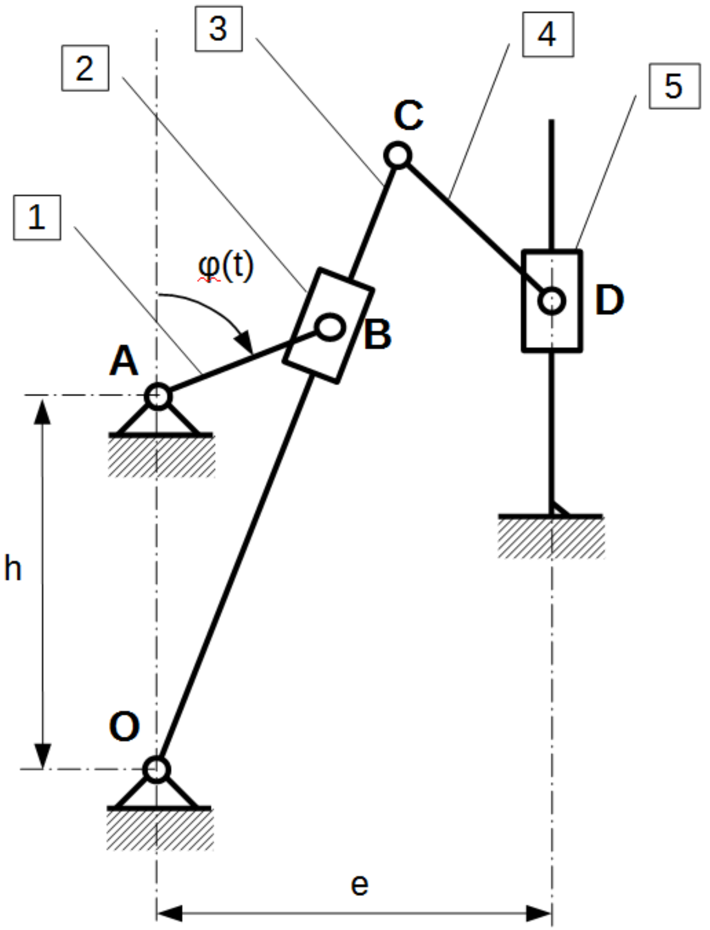

Y
$\bar{b}$
$\bar{c}$
φ(t)
$\bar{r}$
$\bar{a}$
$\bar{h}$
$\bar{d}$
φ_h
$\bar{e}$
X

$\varphi_h = 90^o$



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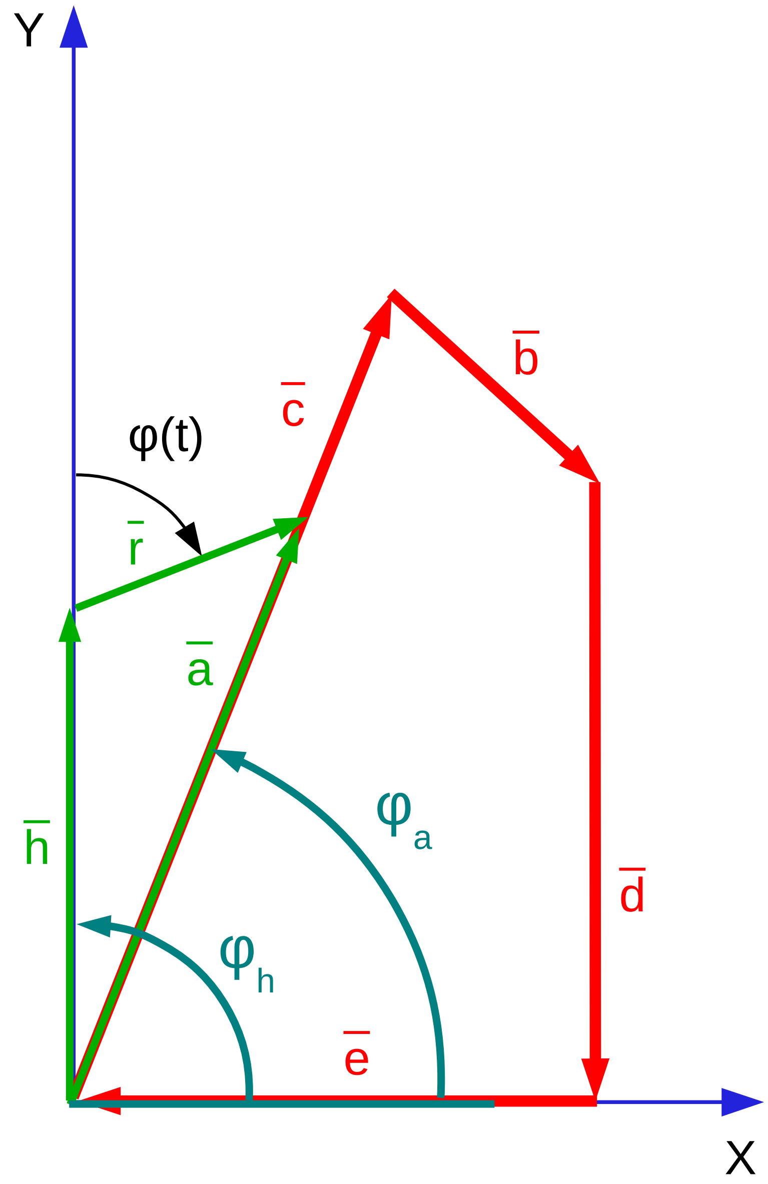
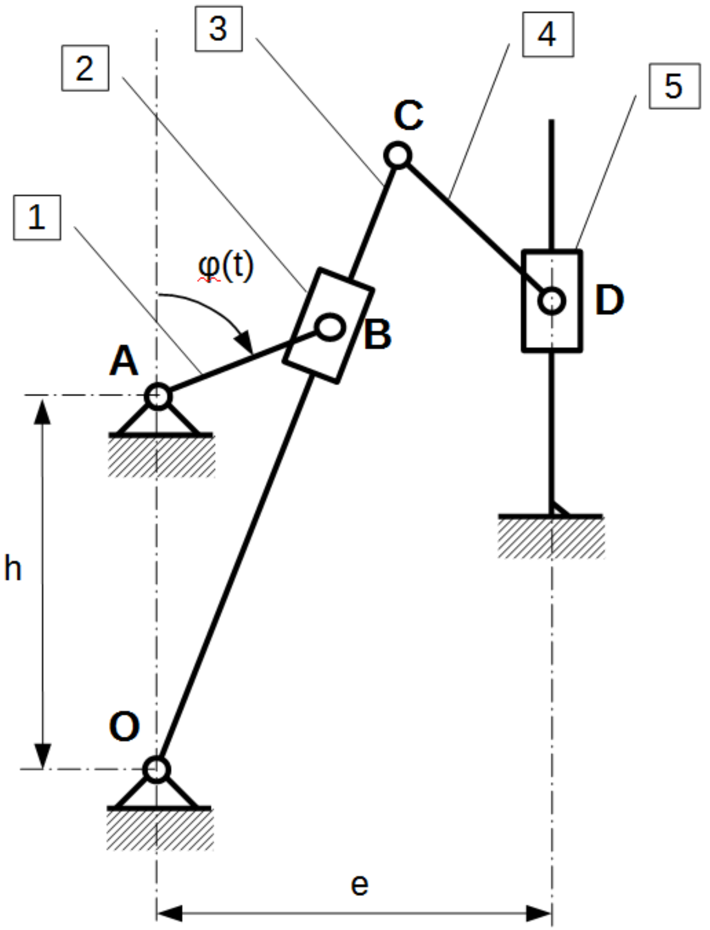
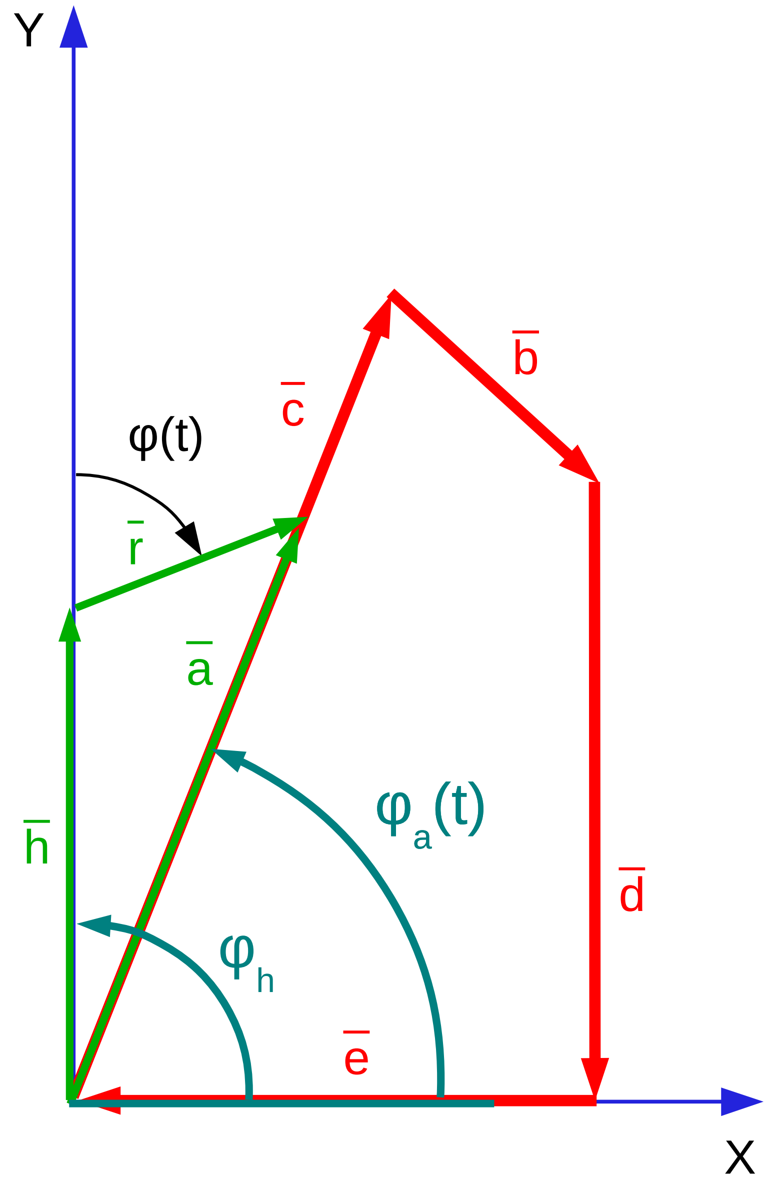
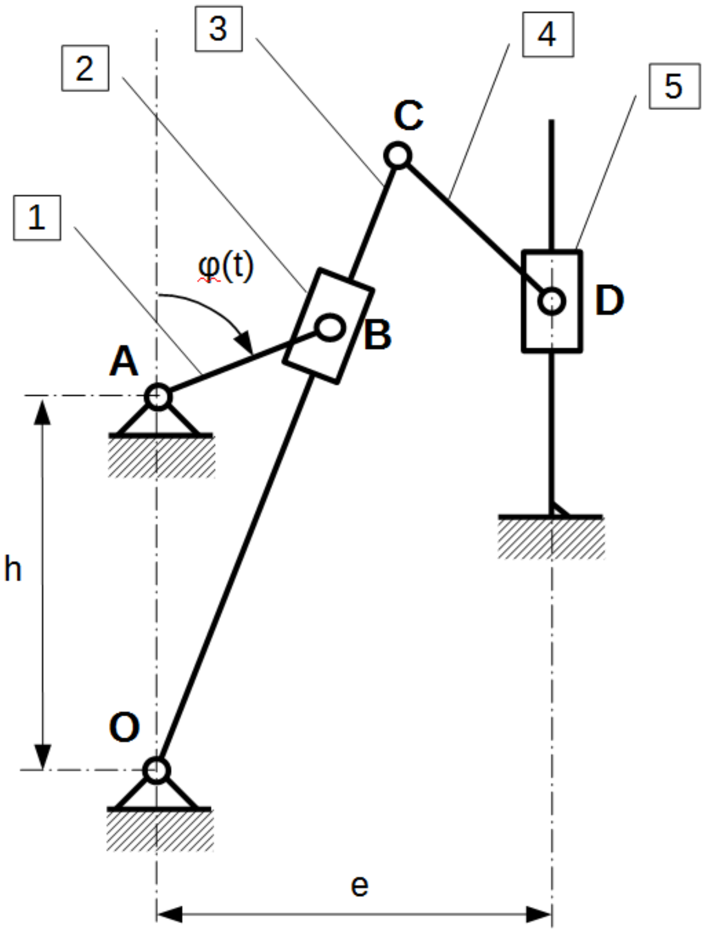


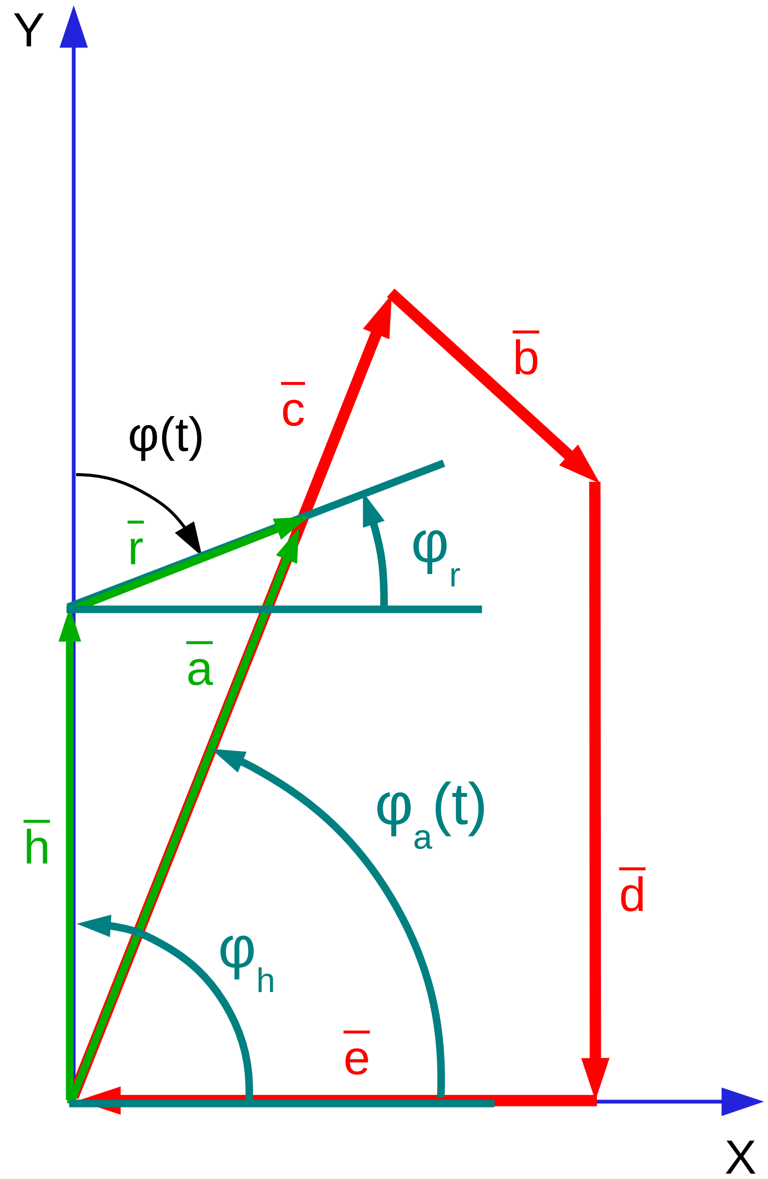




$\varphi_h = 90^o$

$\varphi_a(t) \neq$ const.





$\varphi_h = 90^o$

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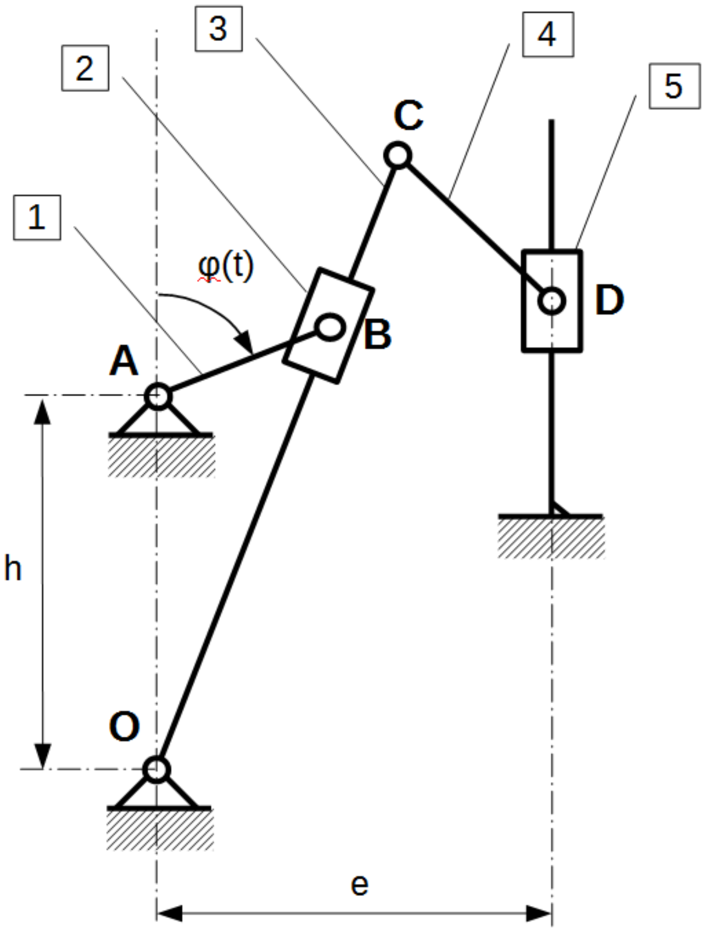

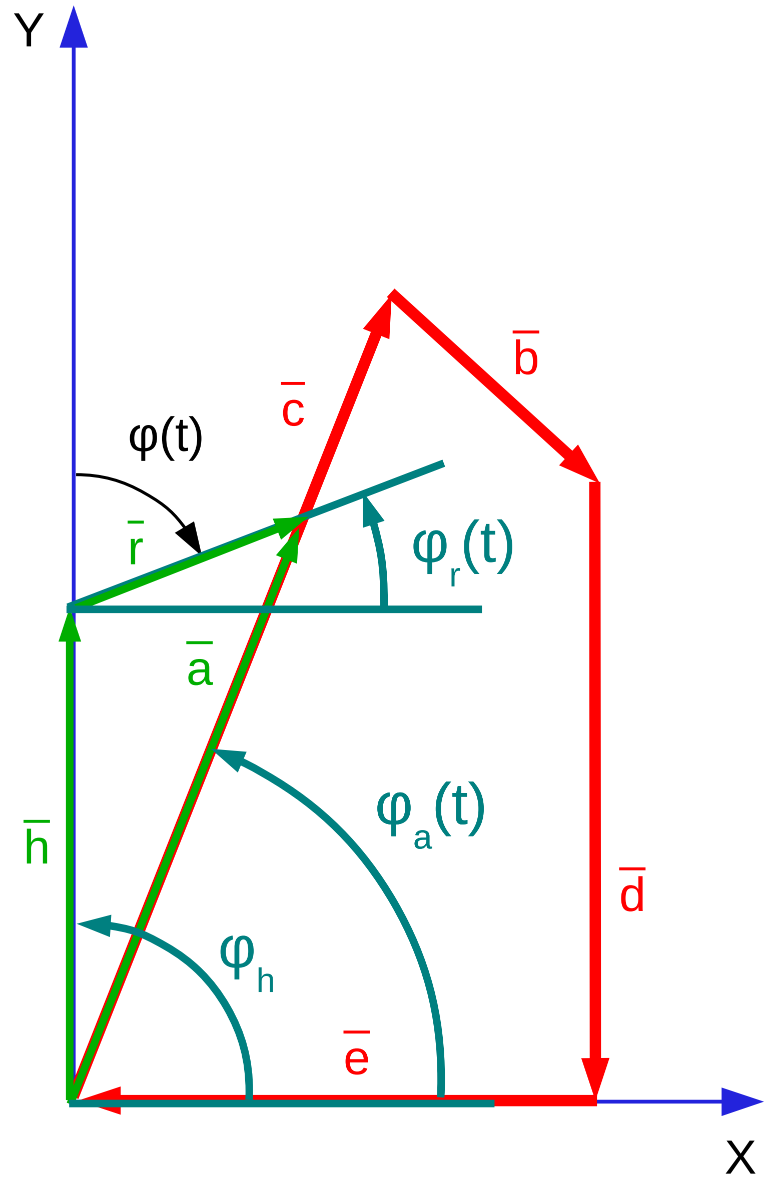

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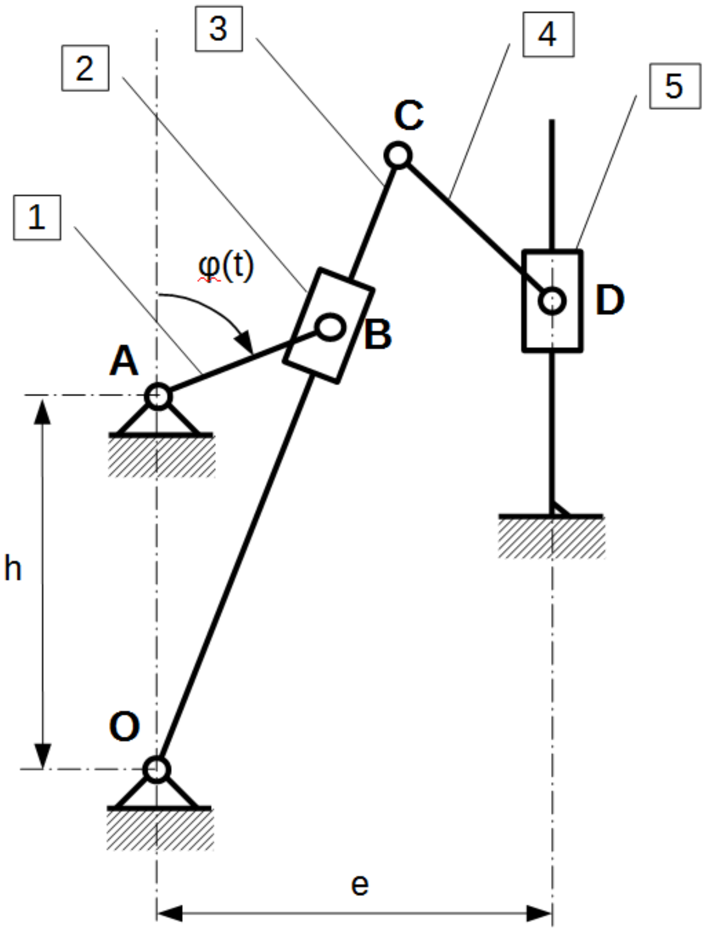
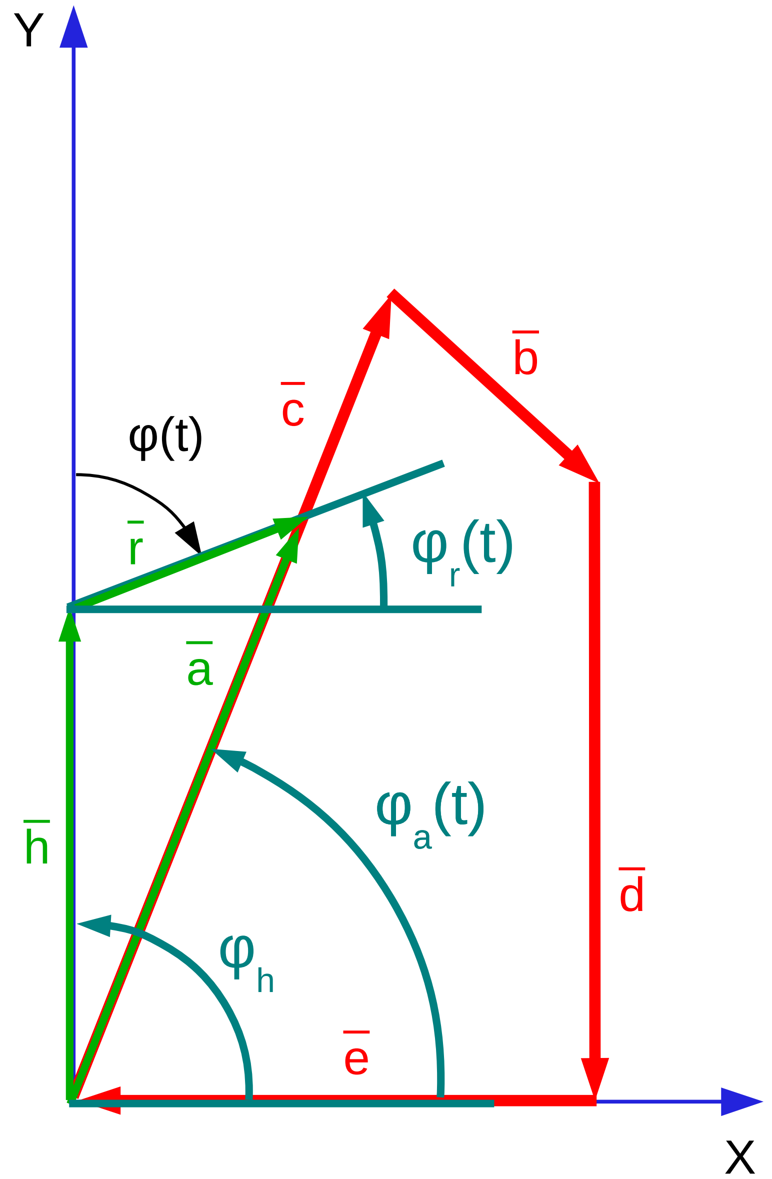
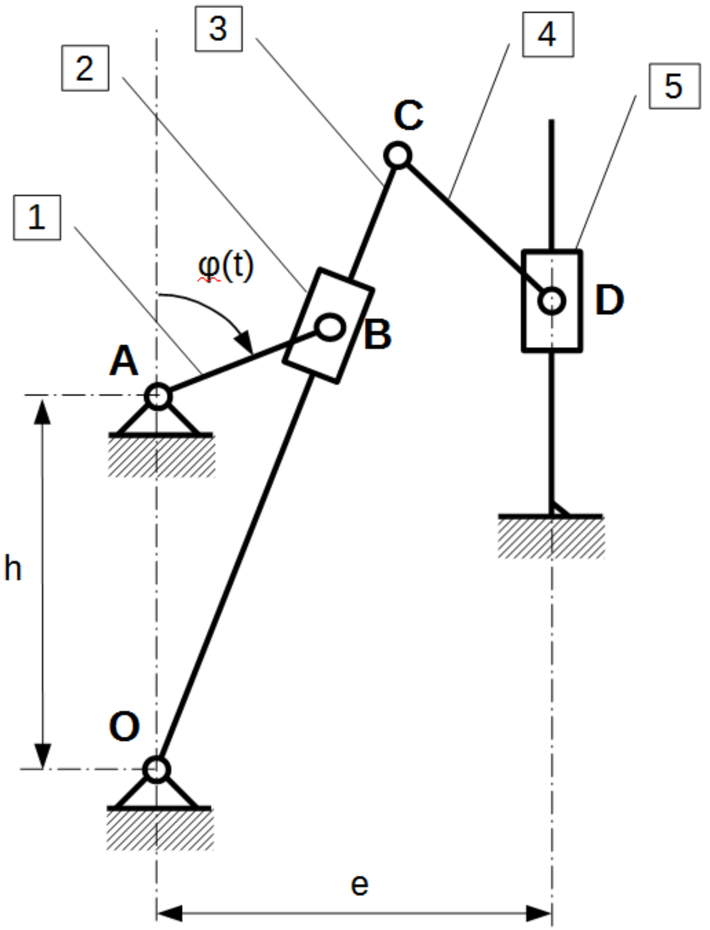




$\varphi_h = 90^o$

$\varphi_a(t) \neq const.$

$\varphi_r(t) = 90^o - \varphi(t)$



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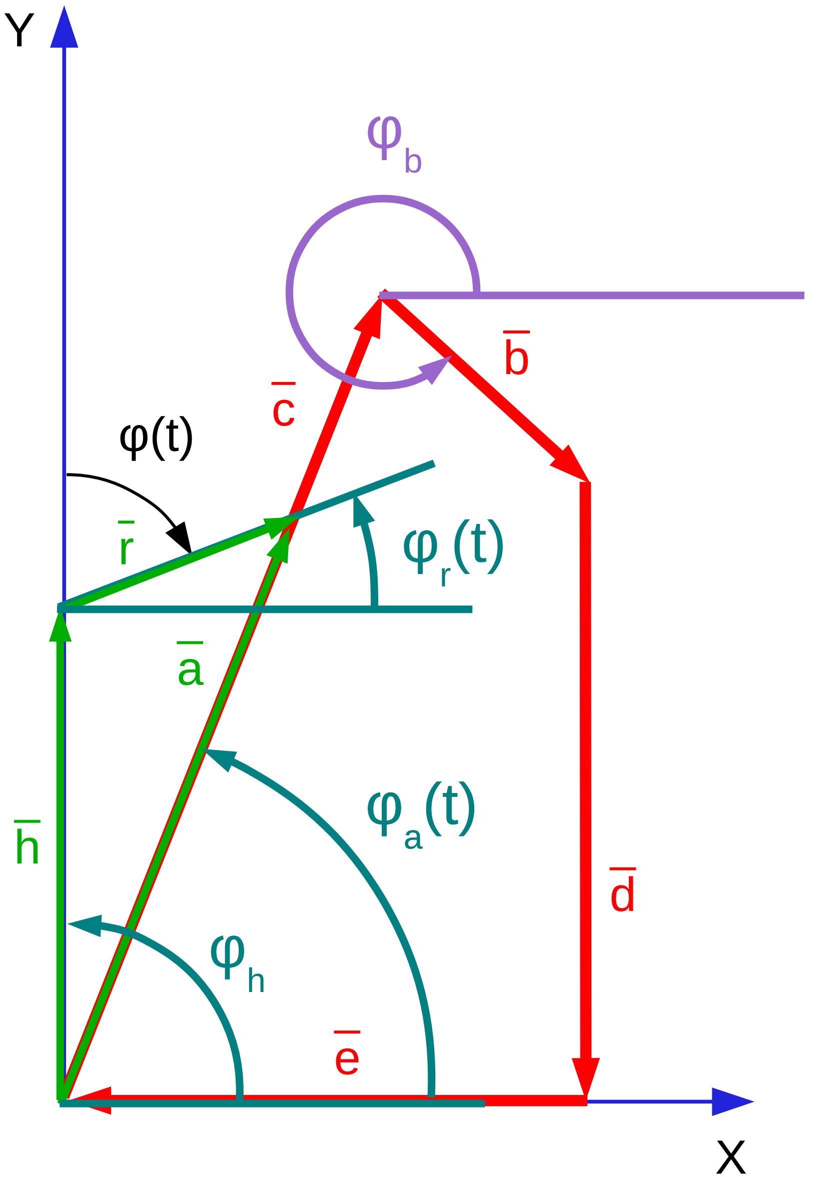
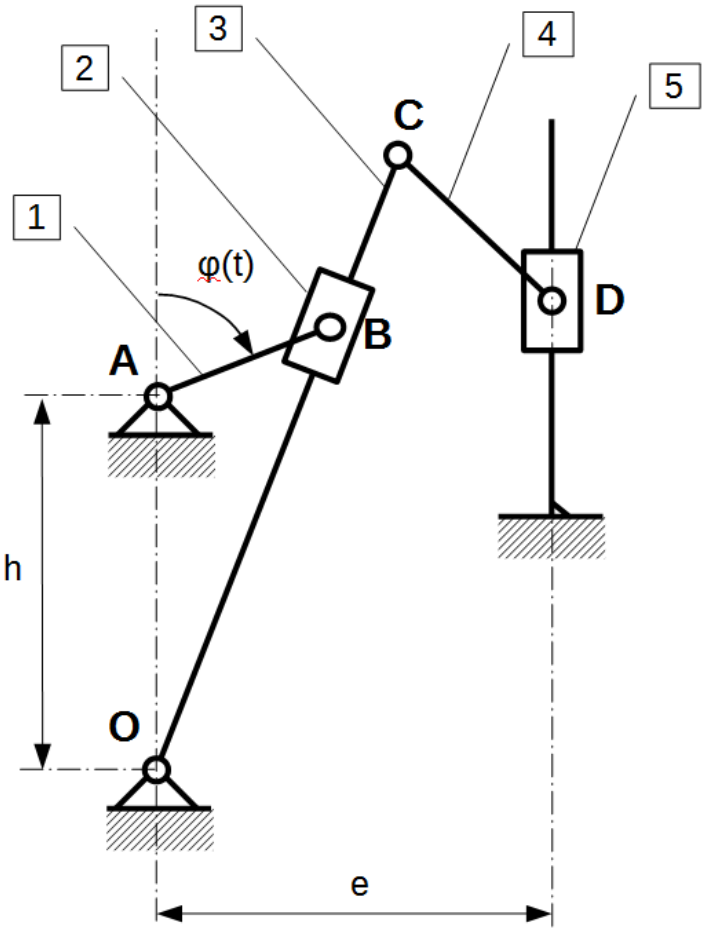
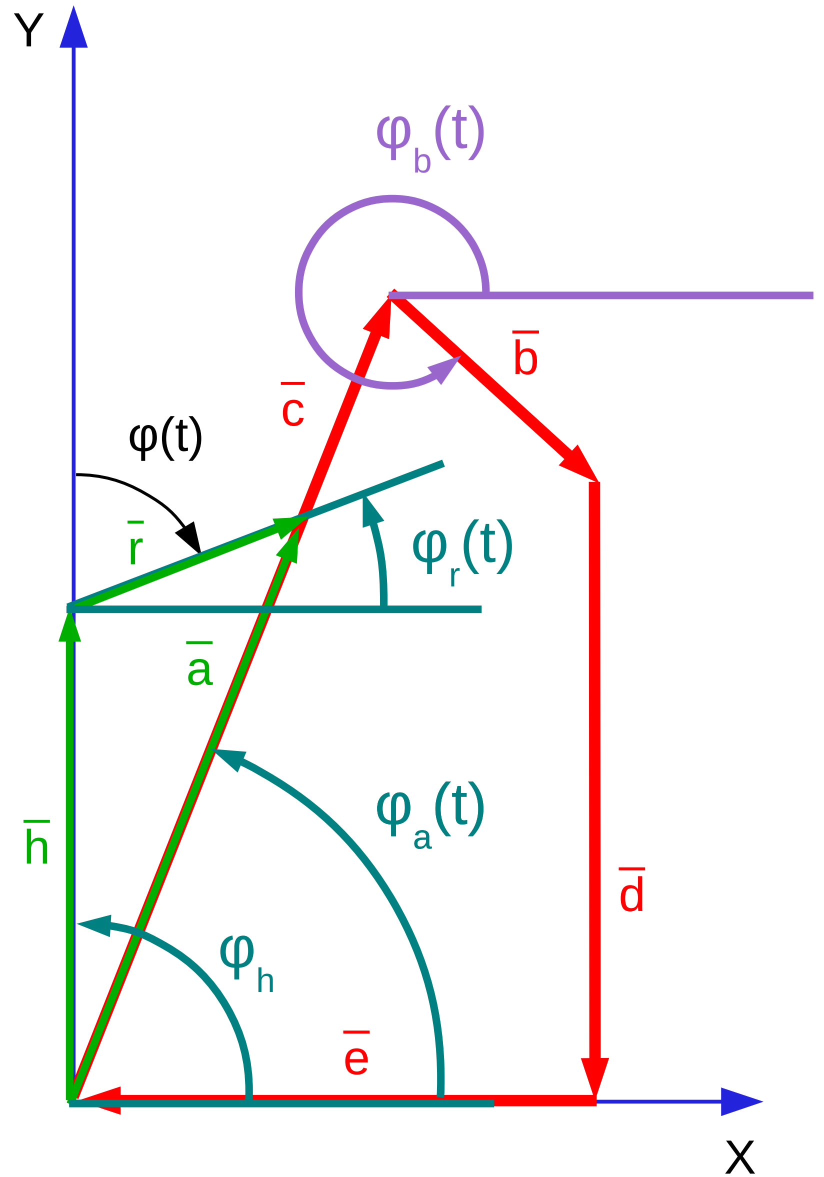




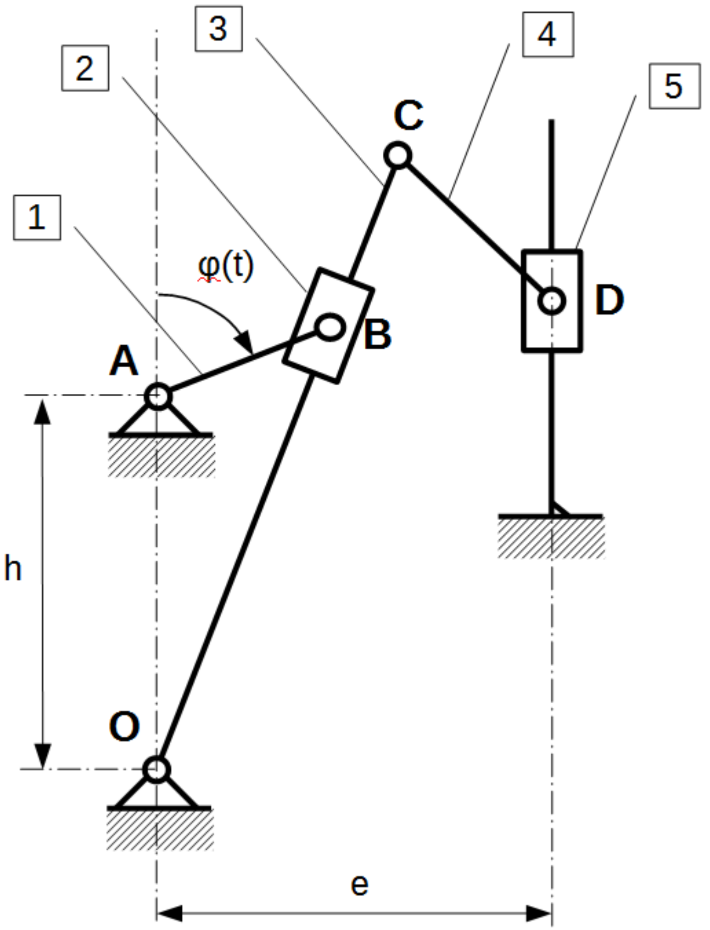


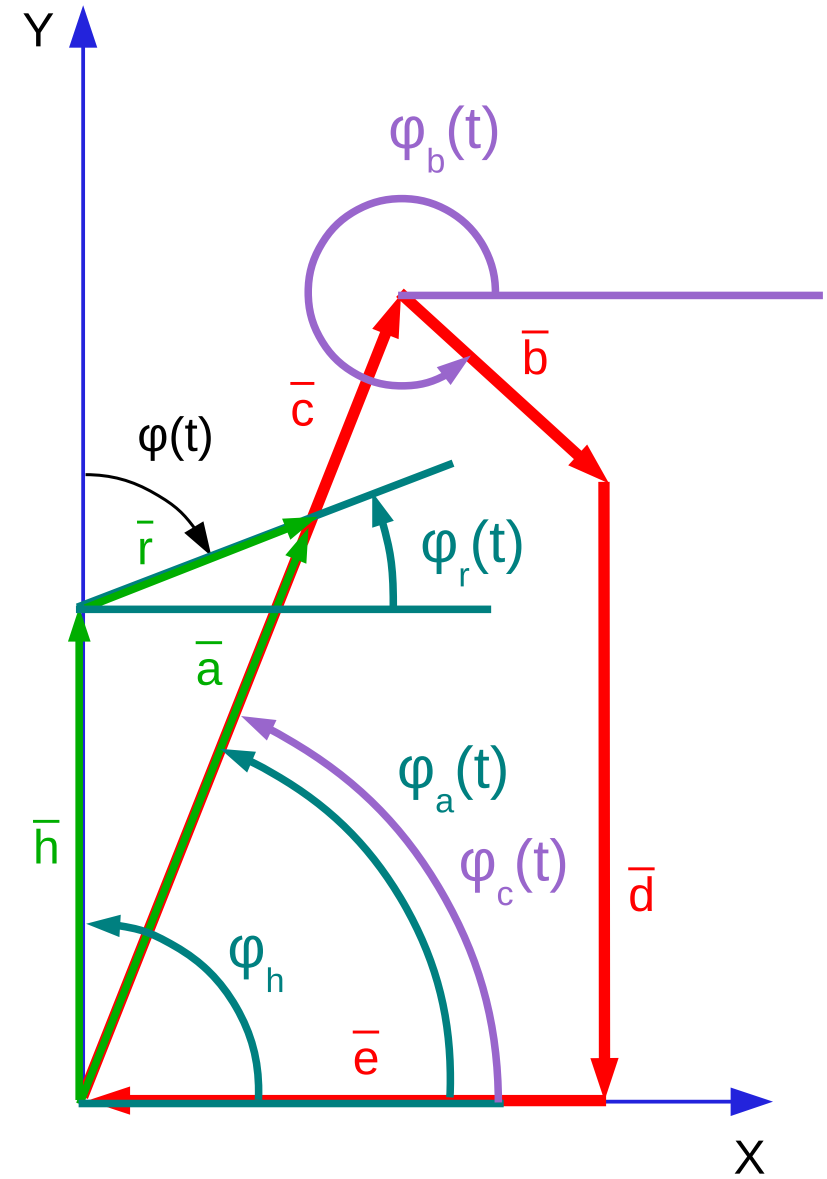
$\varphi_h = 90^o$

$\varphi_a(t) \neq \text{const.}$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_b(t) \neq \text{const.}$





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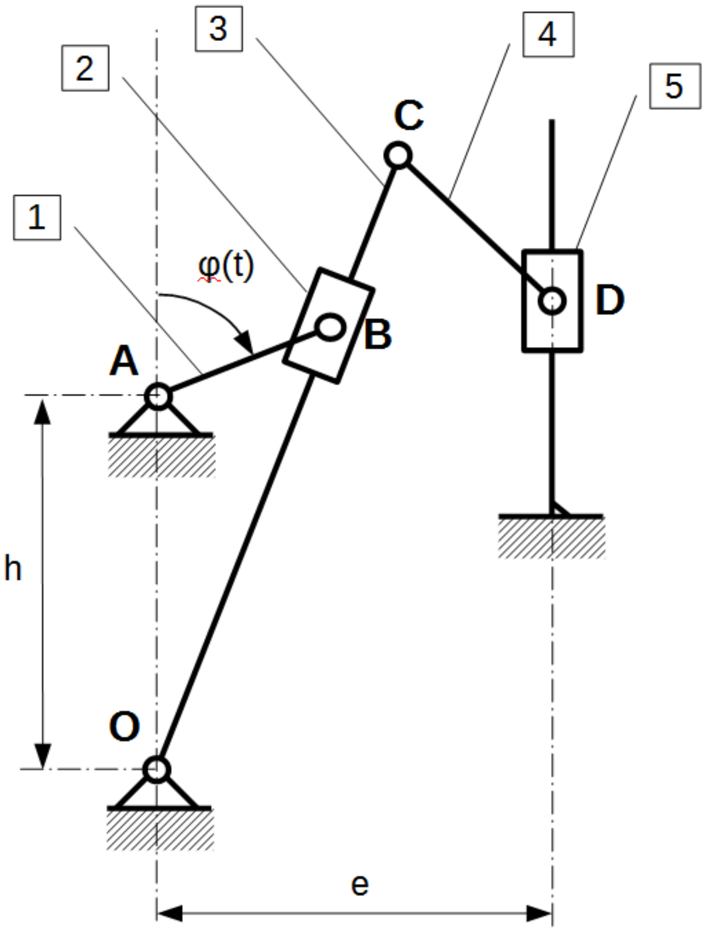
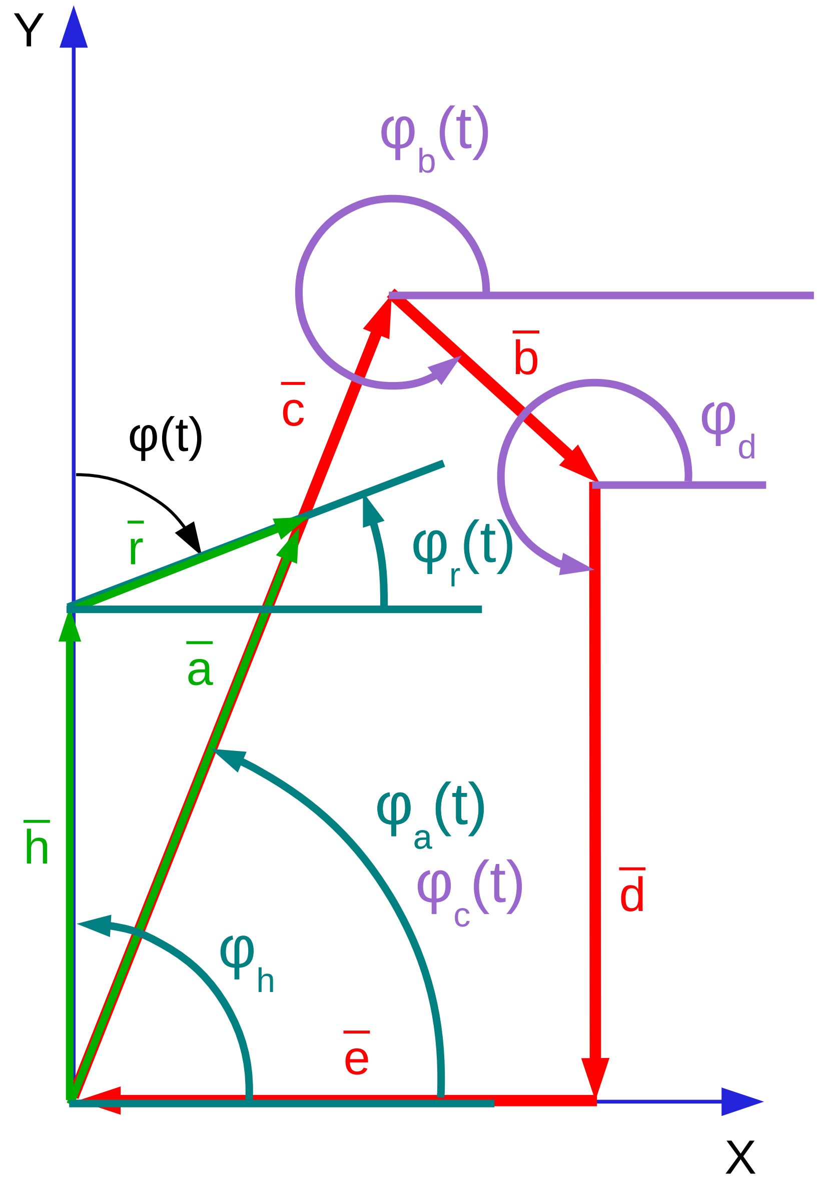
$\varphi_b(t) \neq \text{const.}$

$\varphi_c(t) = \varphi_a(t)$





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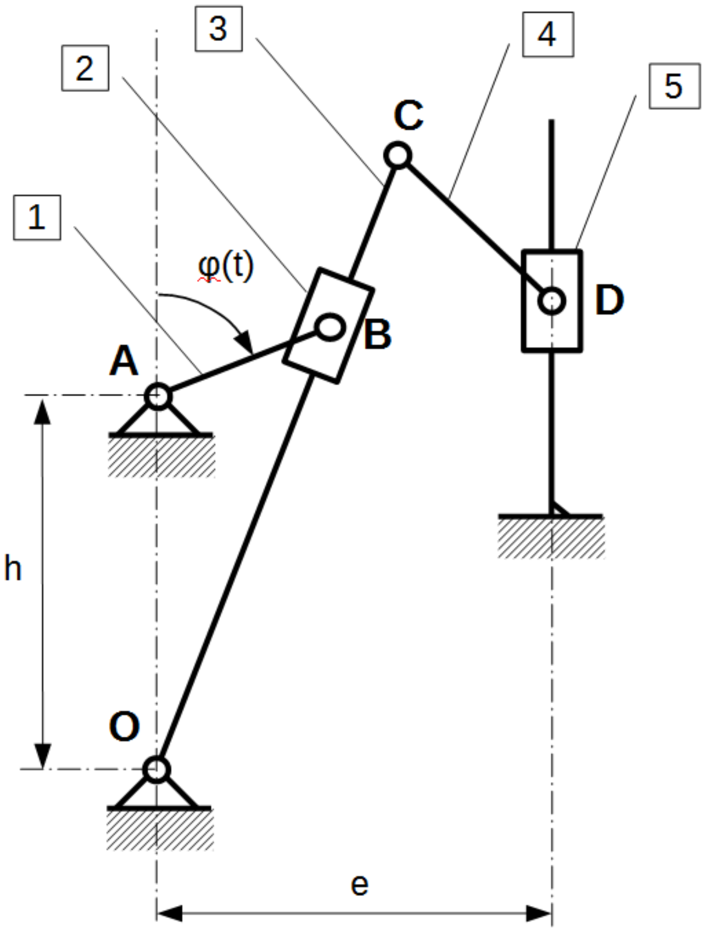
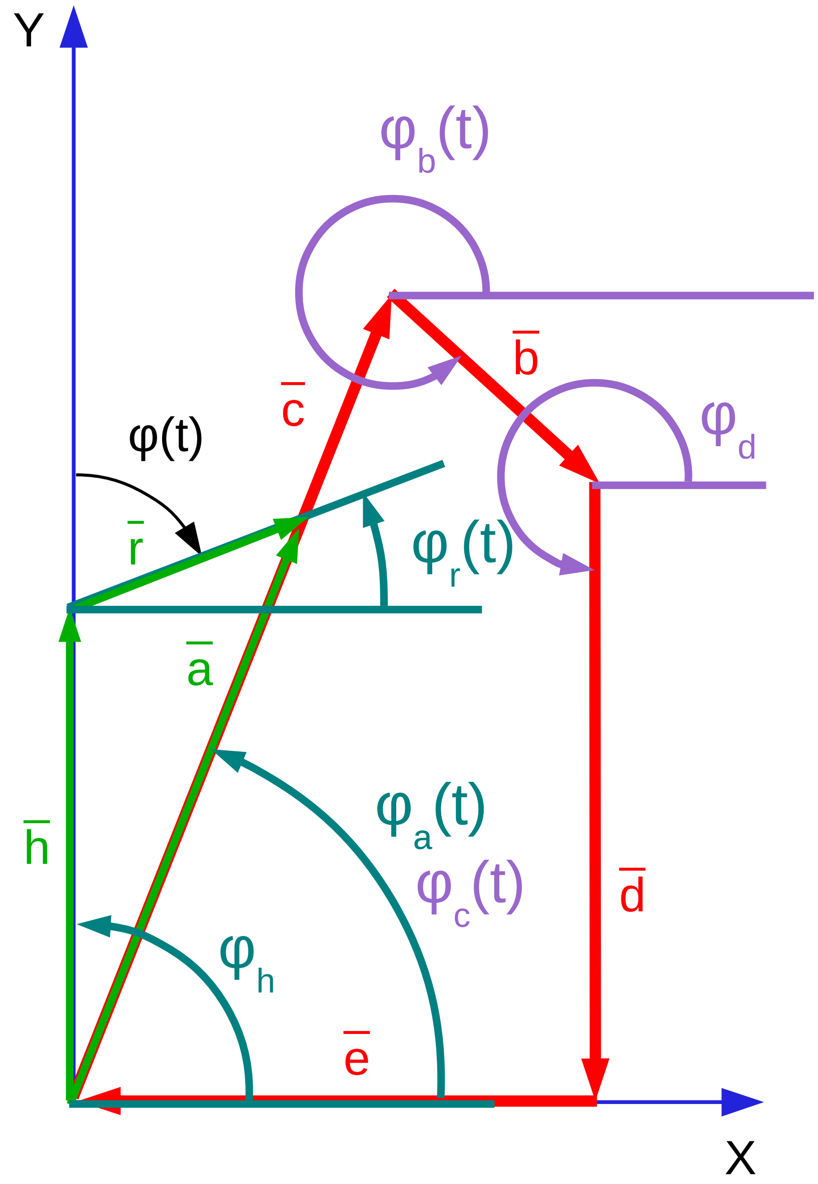
$\varphi_c(t) = \varphi_a(t)$





$\varphi_h = 90^o$

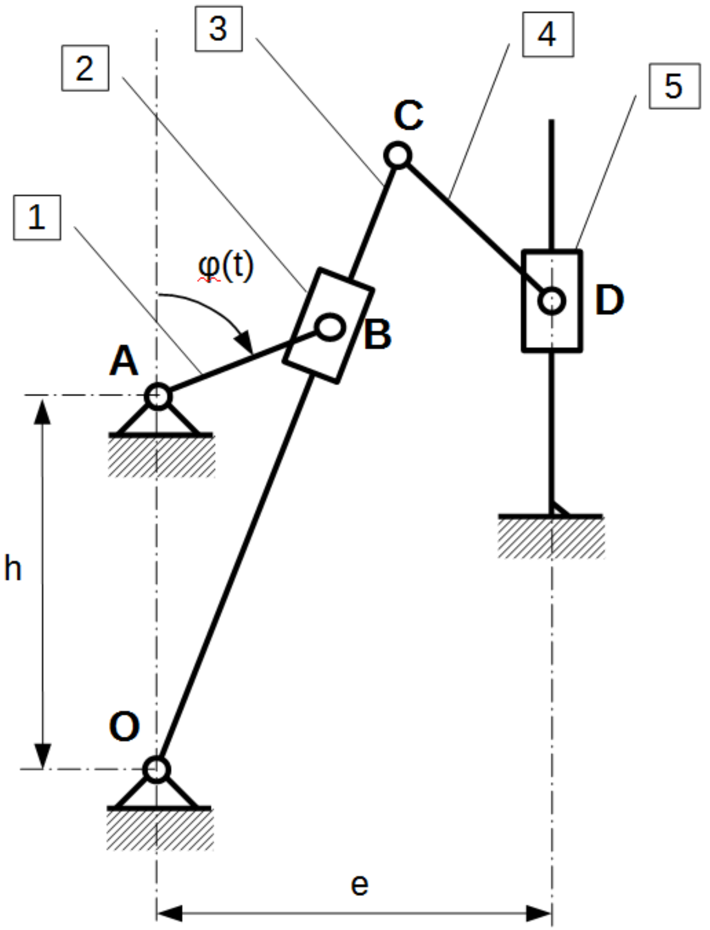
$\varphi_d = 270^o$

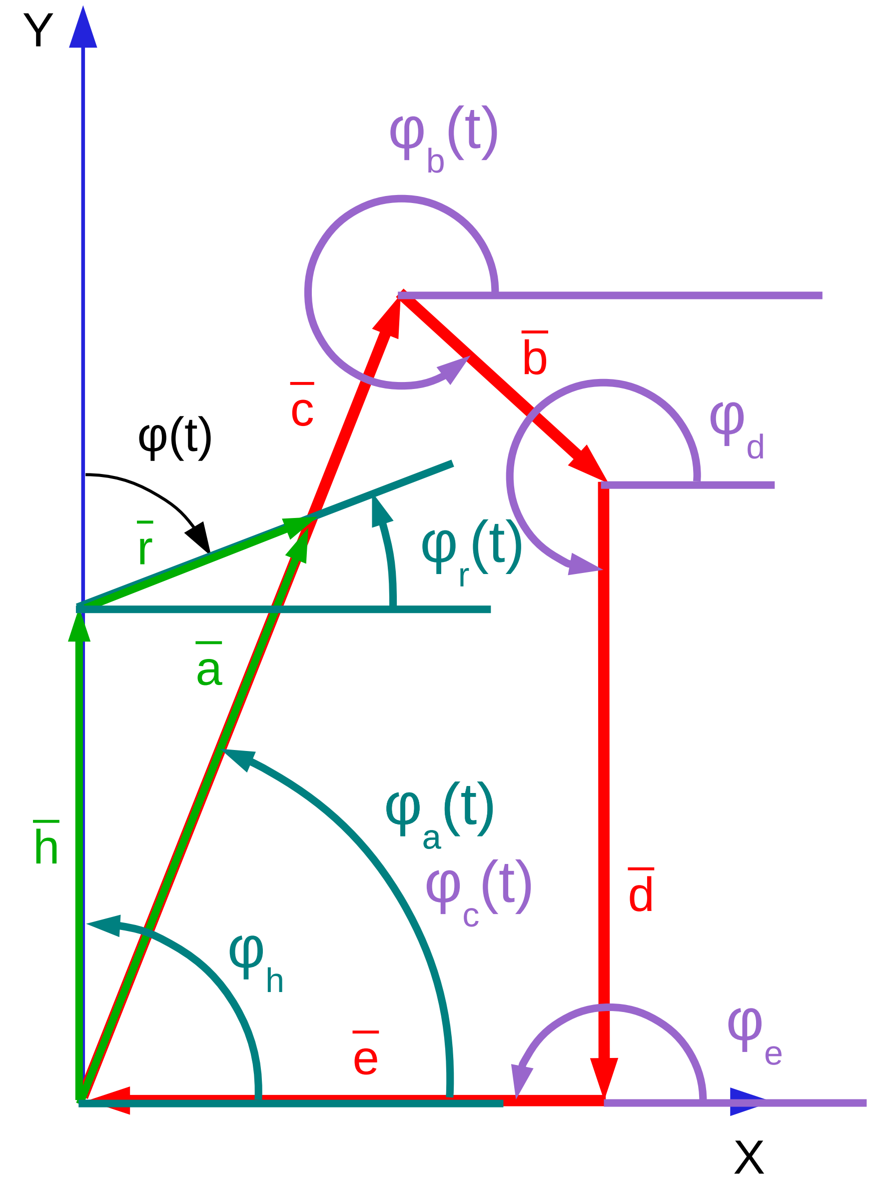
$\varphi_a(t) \neq \text{const.}$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_b(t) \neq \text{const.}$

$\varphi_c(t) = \varphi_a(t)$





$\varphi_h = 90^o$

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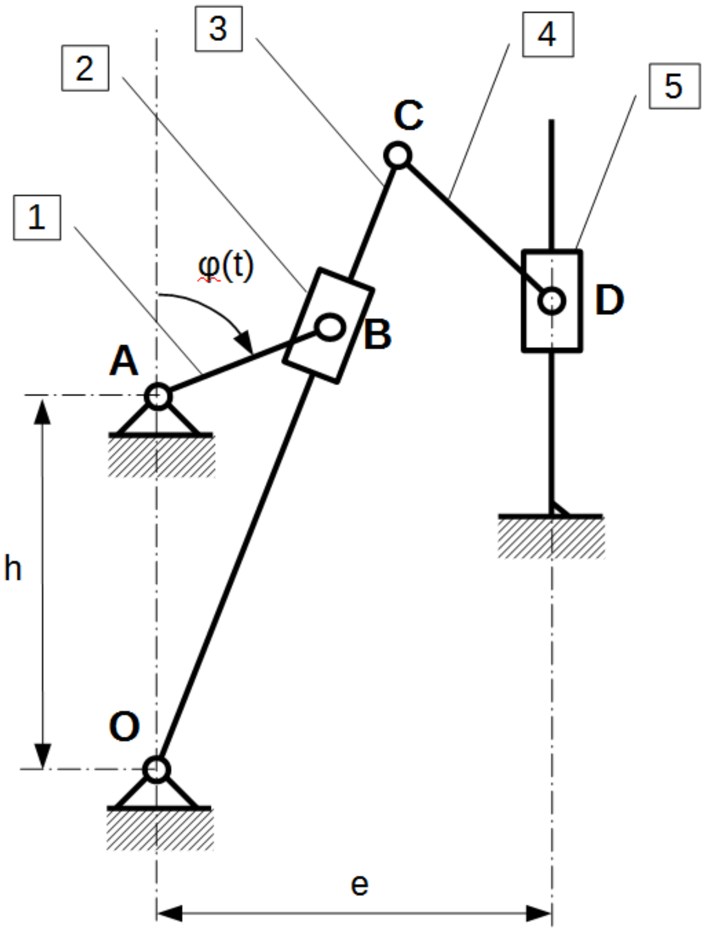
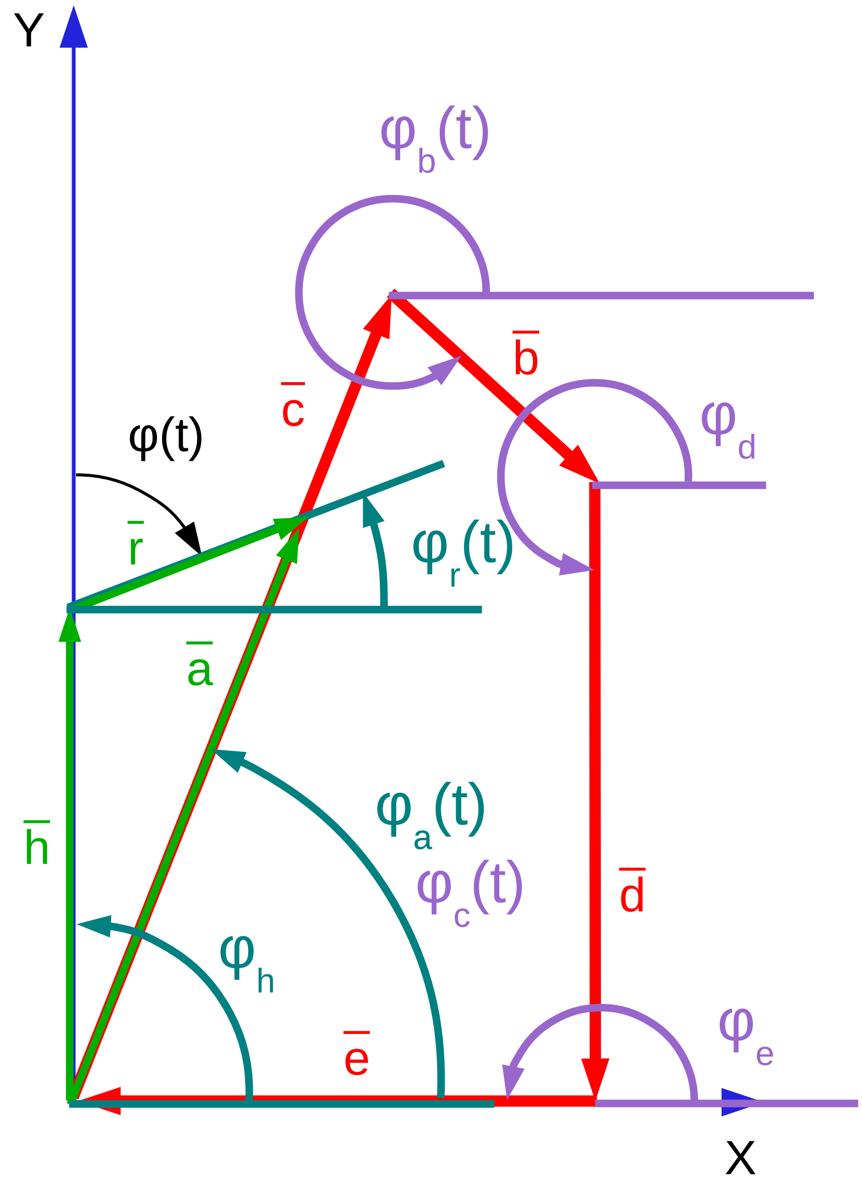




$\varphi_h = 90^o$

$\varphi_a(t) \neq \text{const.}$

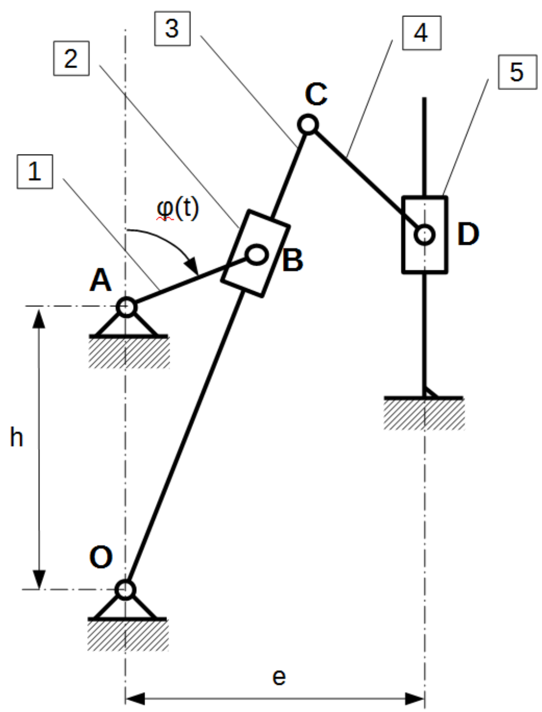
$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_b(t) \neq \text{const.}$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_d = 270^o$

$\varphi_e = 180^o$



$\varphi_h = 90^o$

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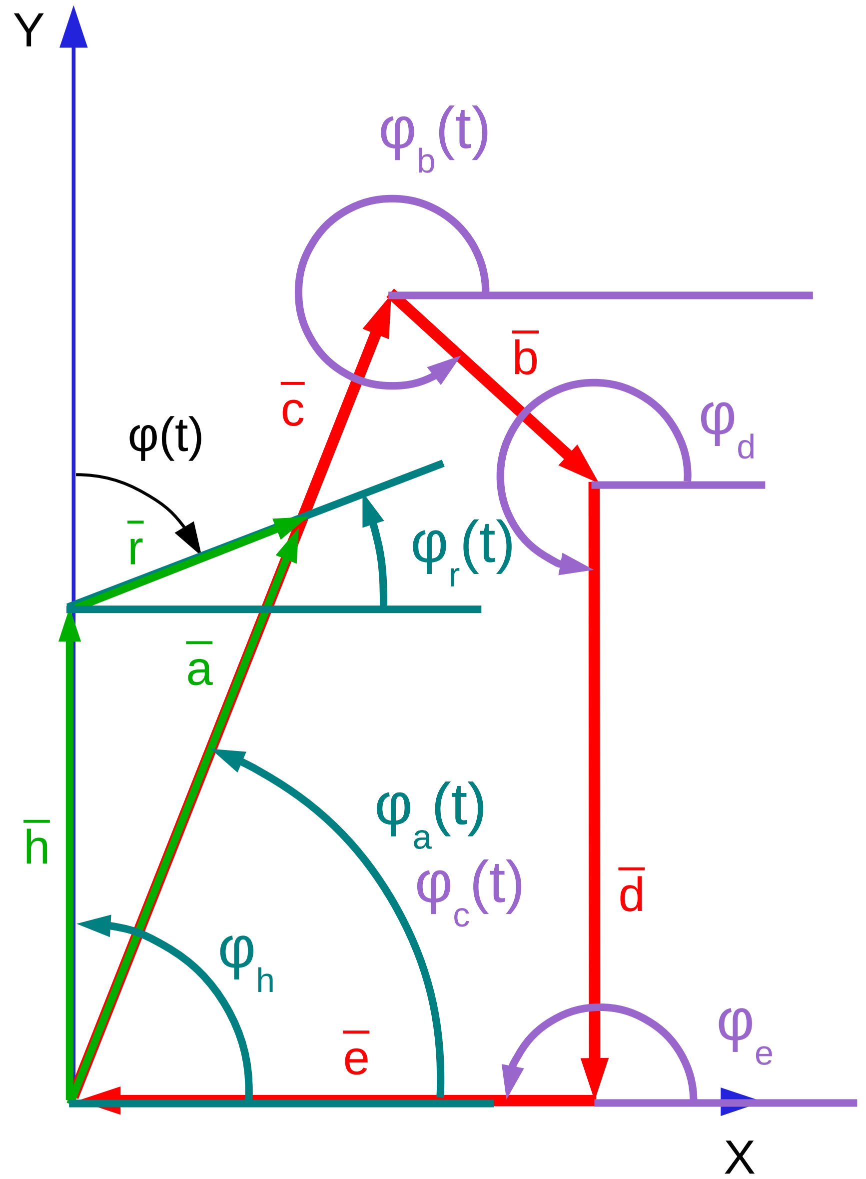
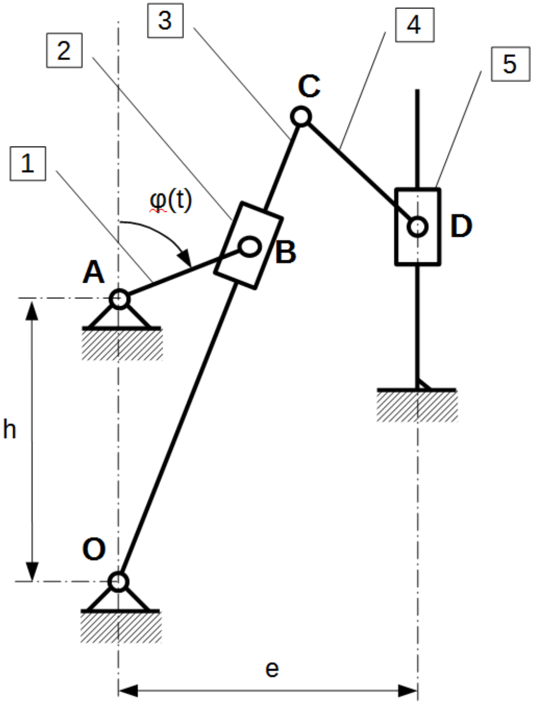
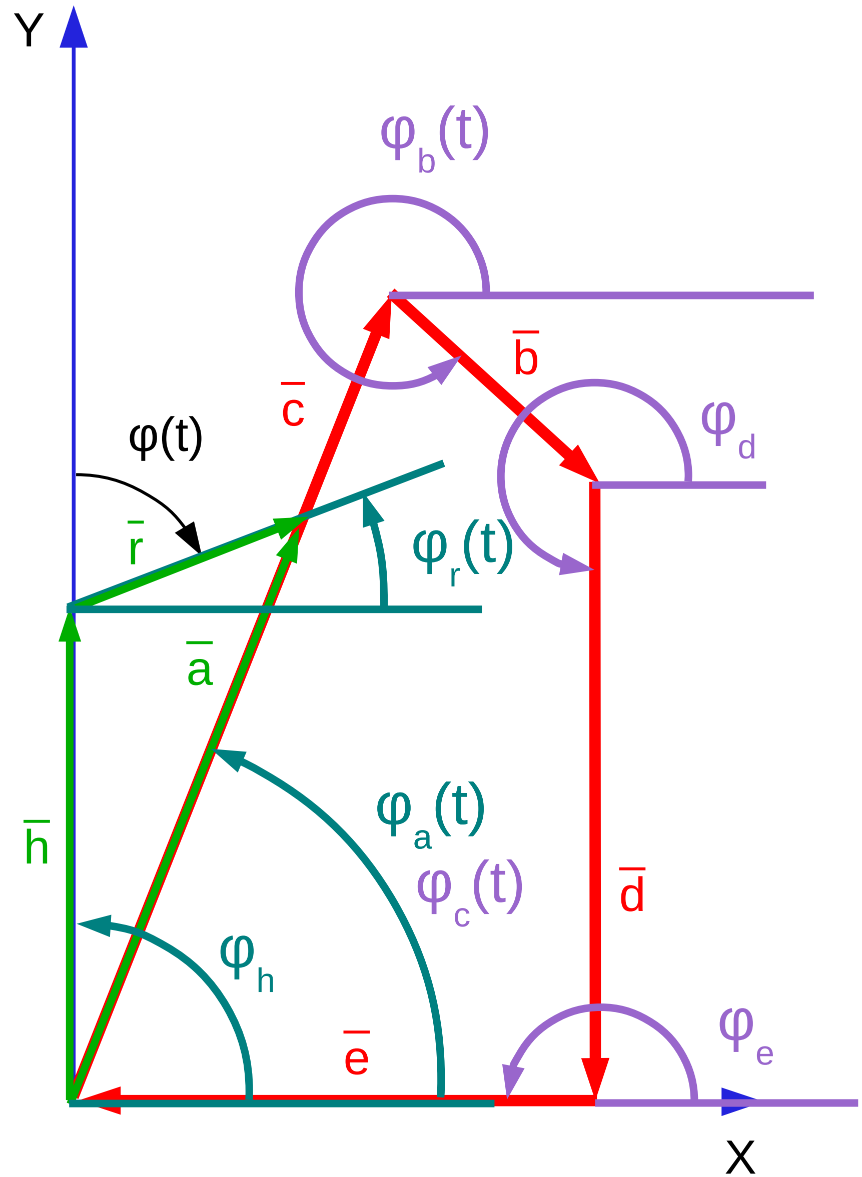
$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_d = 270^o$

$\varphi_e = 180^o$







$\varphi_h = 90^o$

$|\bar{r}| = |AB| = r = \text{const.}$

$\varphi_a(t) \neq \text{const.}$

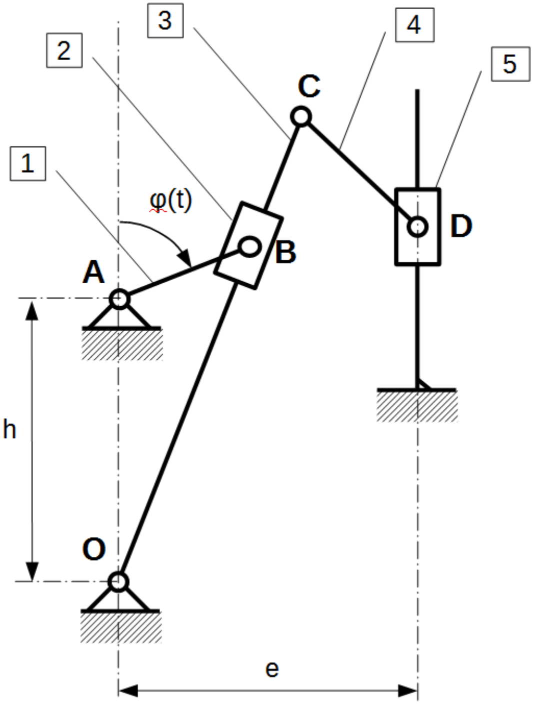
$\varphi_b(t) \neq \text{const.}$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_d = 270^o$

$\varphi_e = 180^o$



$\varphi_h = 90^o$

$\varphi_a(t) \neq const.$

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$\varphi_c(t) = \varphi_a(t)$

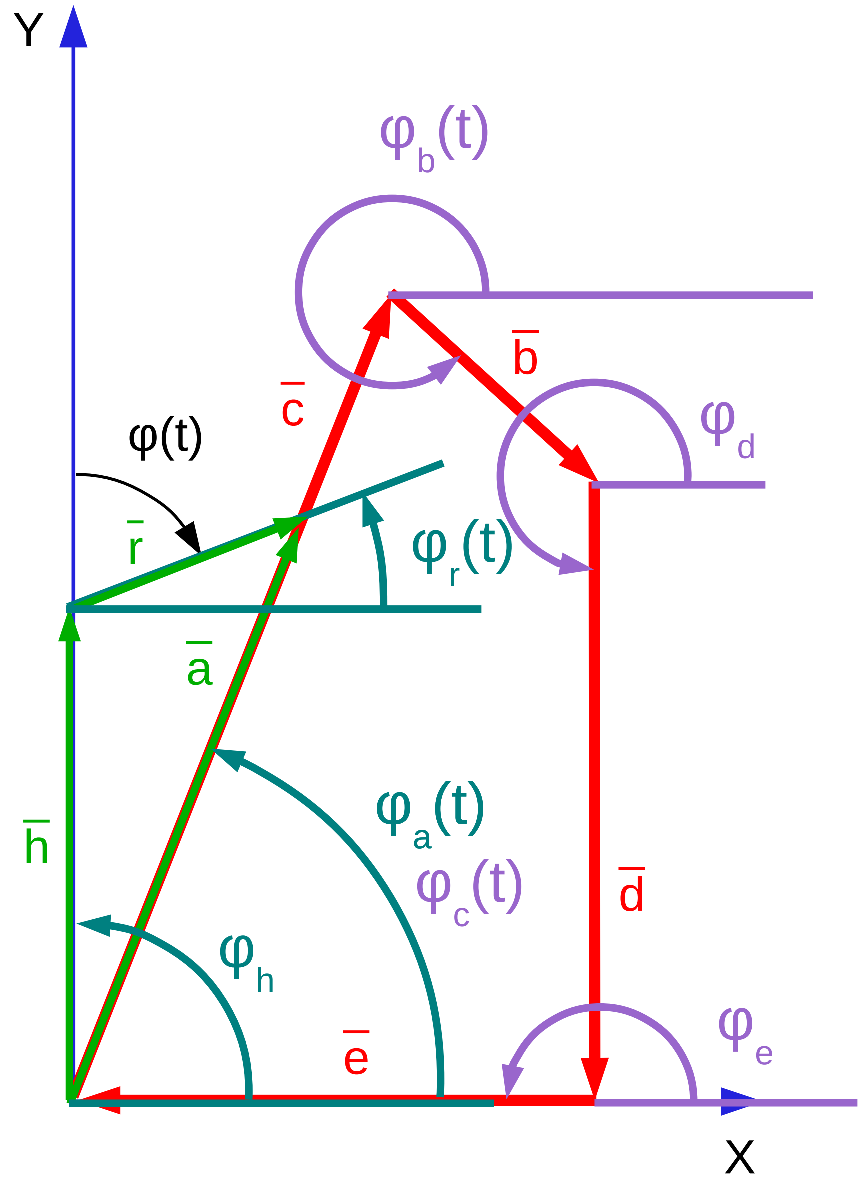
$\varphi_r(t) = 90^o - \varphi(t)$

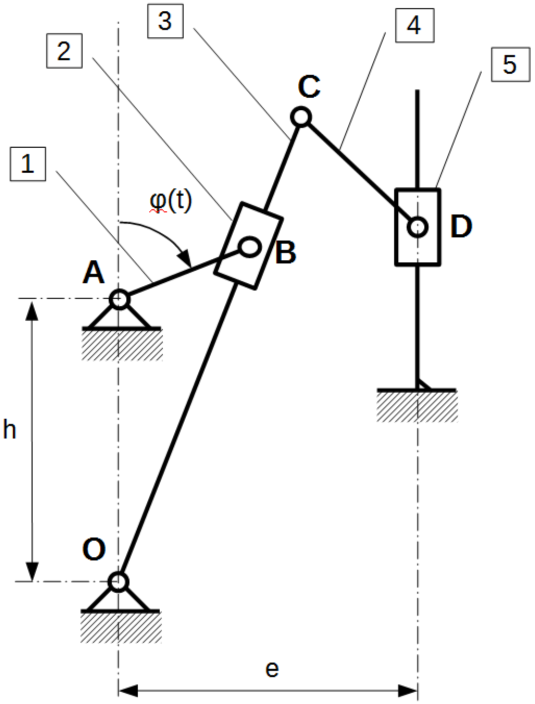
$\varphi_d = 270^o$

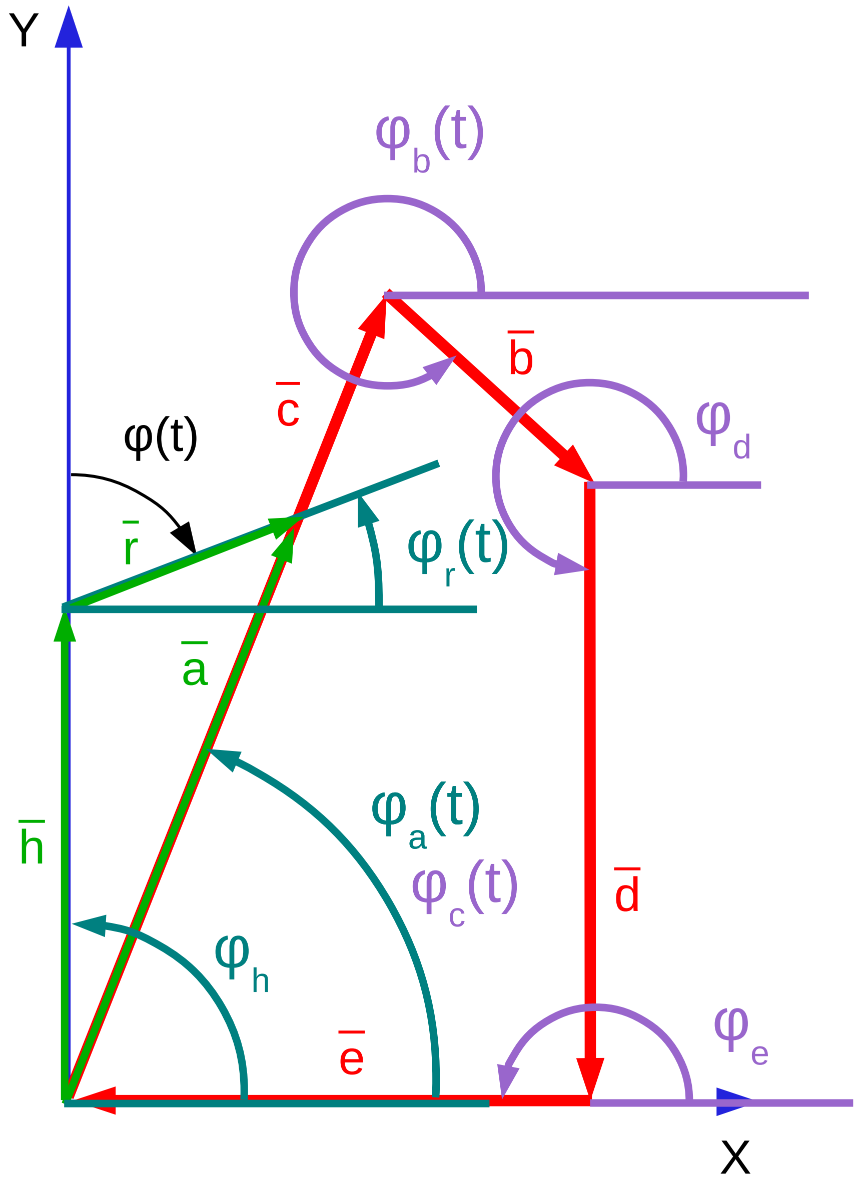
$\varphi_e = 180^o$

$|\bar{r}| = |AB| = r = const.$

$|\bar{h}| = |OA| = h = const.$







$|\bar{r}| = |AB| = r = \text{const.}$

$|\bar{h}| = |OA| = h = \text{const.}$

$|\bar{a}| = |OB| = a(t)$

$\varphi_h = 90^o$

$\varphi_a(t) \neq \text{const.}$

$\varphi_b(t) \neq \text{const.}$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_d = 270^o$

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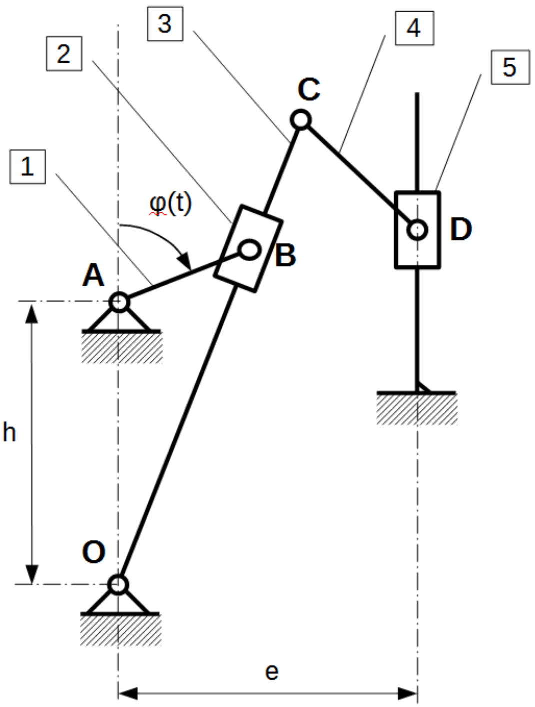

$\varphi_h = 90^o$

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$\varphi_c(t) = \varphi_a(t)$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_d = 270^o$

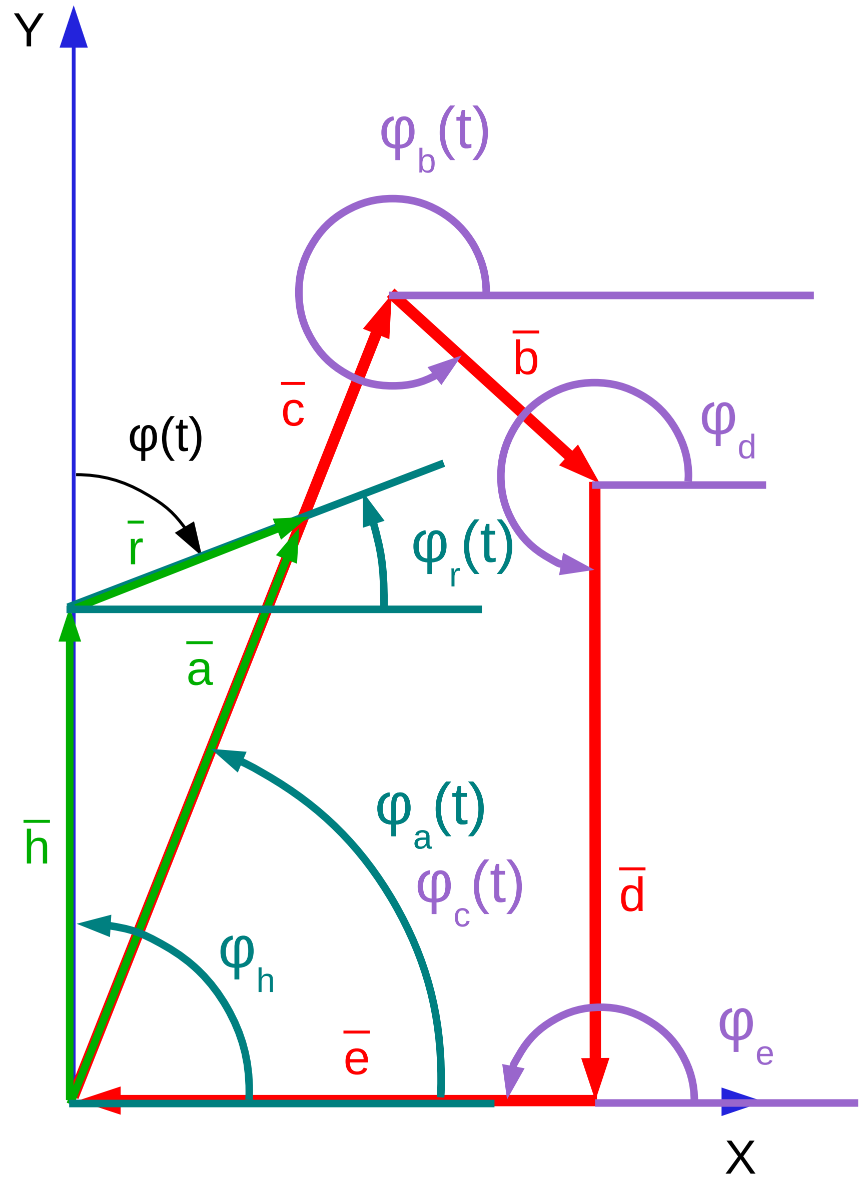
$\varphi_e = 180^o$

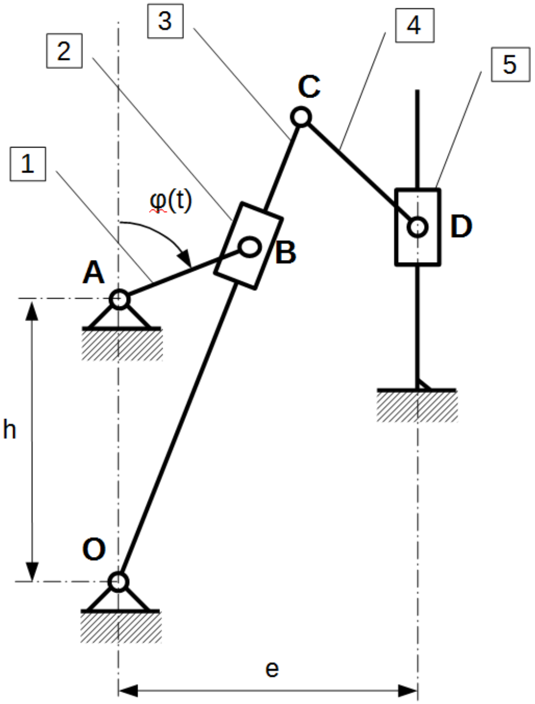
$|\bar{r}| = |AB| = r = \text{const.}$

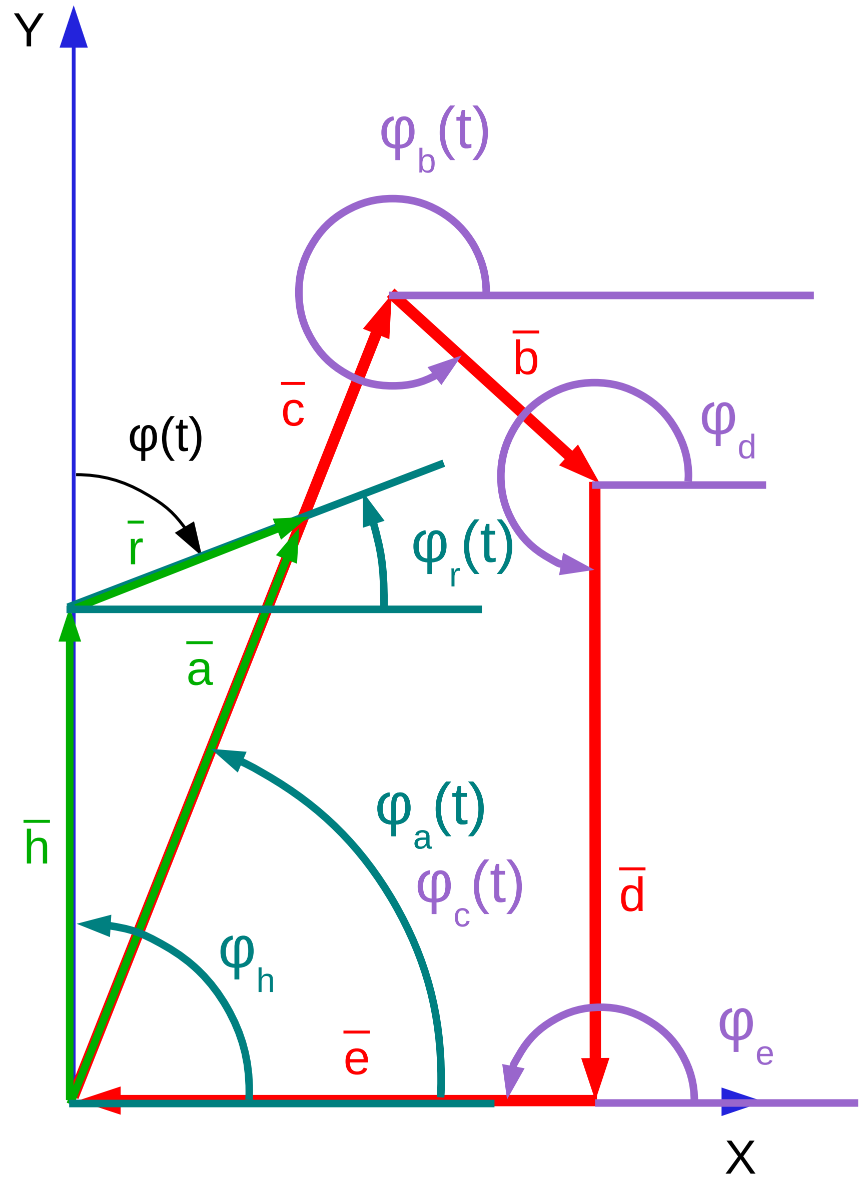
$|\bar{h}| = |OA| = h = \text{const.}$

$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = \text{const.}$







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$\varphi_d = 270^o$

$\varphi_e = 180^o$

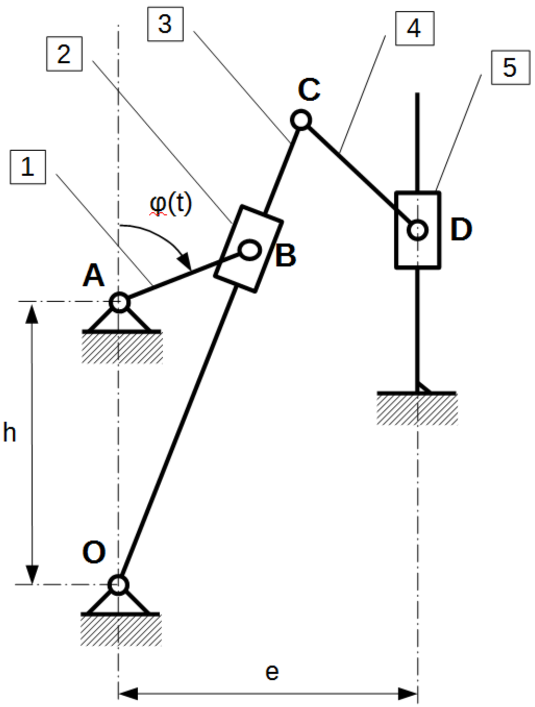
$|\bar{r}| = |AB| = r = \text{const.}$

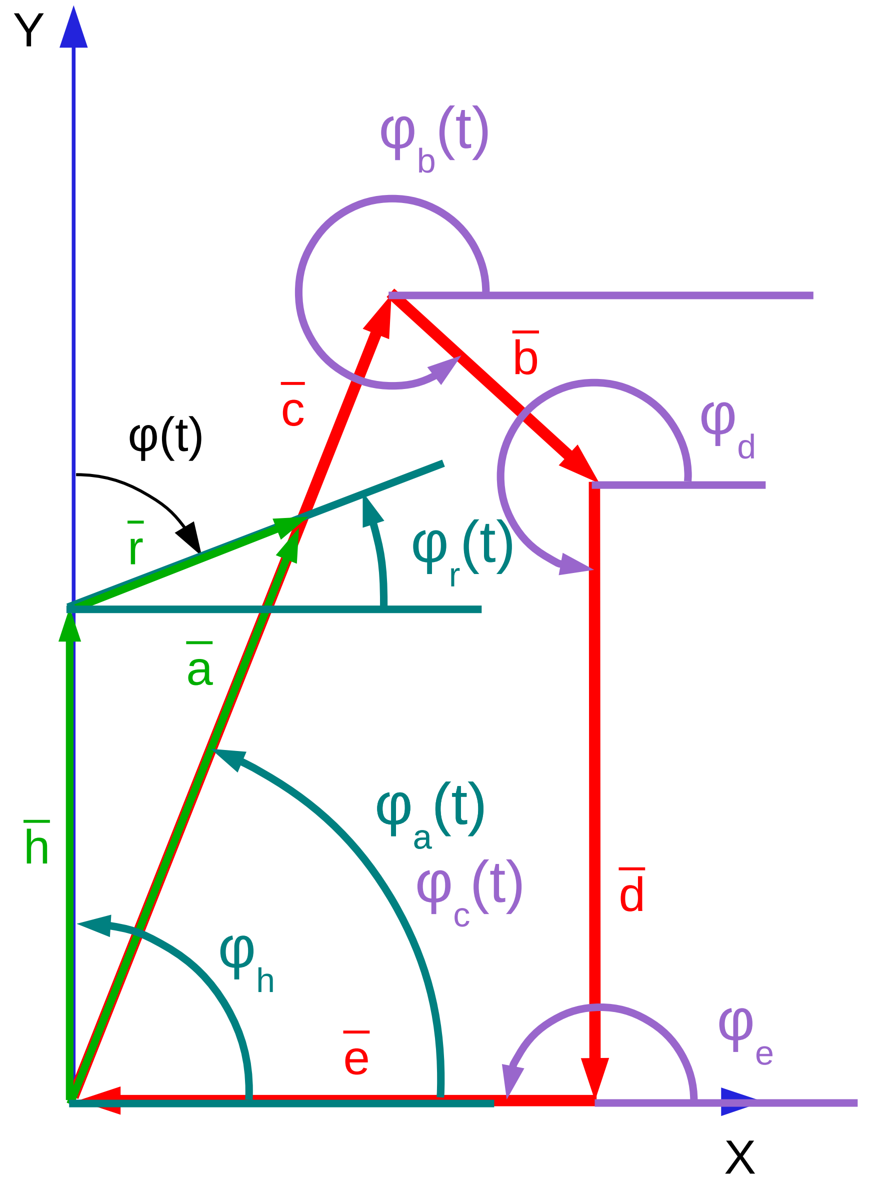
$|\bar{h}| = |OA| = h = \text{const.}$

$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = \text{const.}$

$|b| = |CD| = b = \text{const.}$





$\varphi_h = 90^o$

$\varphi_a(t) \neq \text{const.}$

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$\varphi_c(t) = \varphi_a(t)$

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$|\bar{r}| = |AB| = r = \text{const.}$

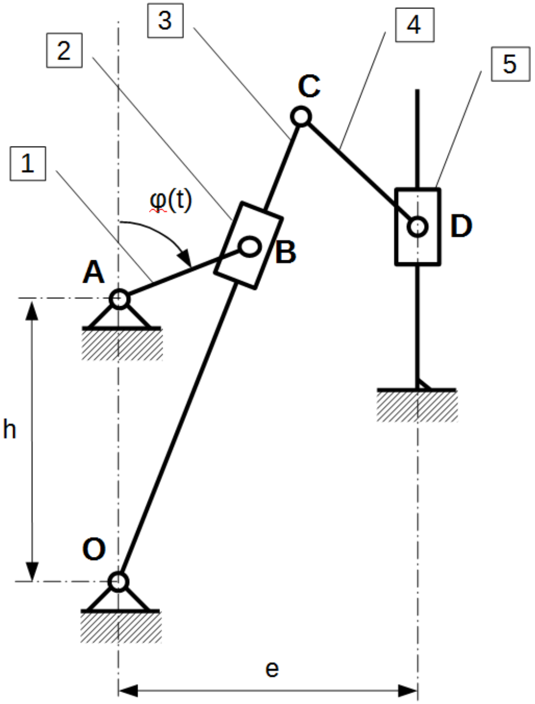
$|\bar{h}| = |OA| = h = \text{const.}$

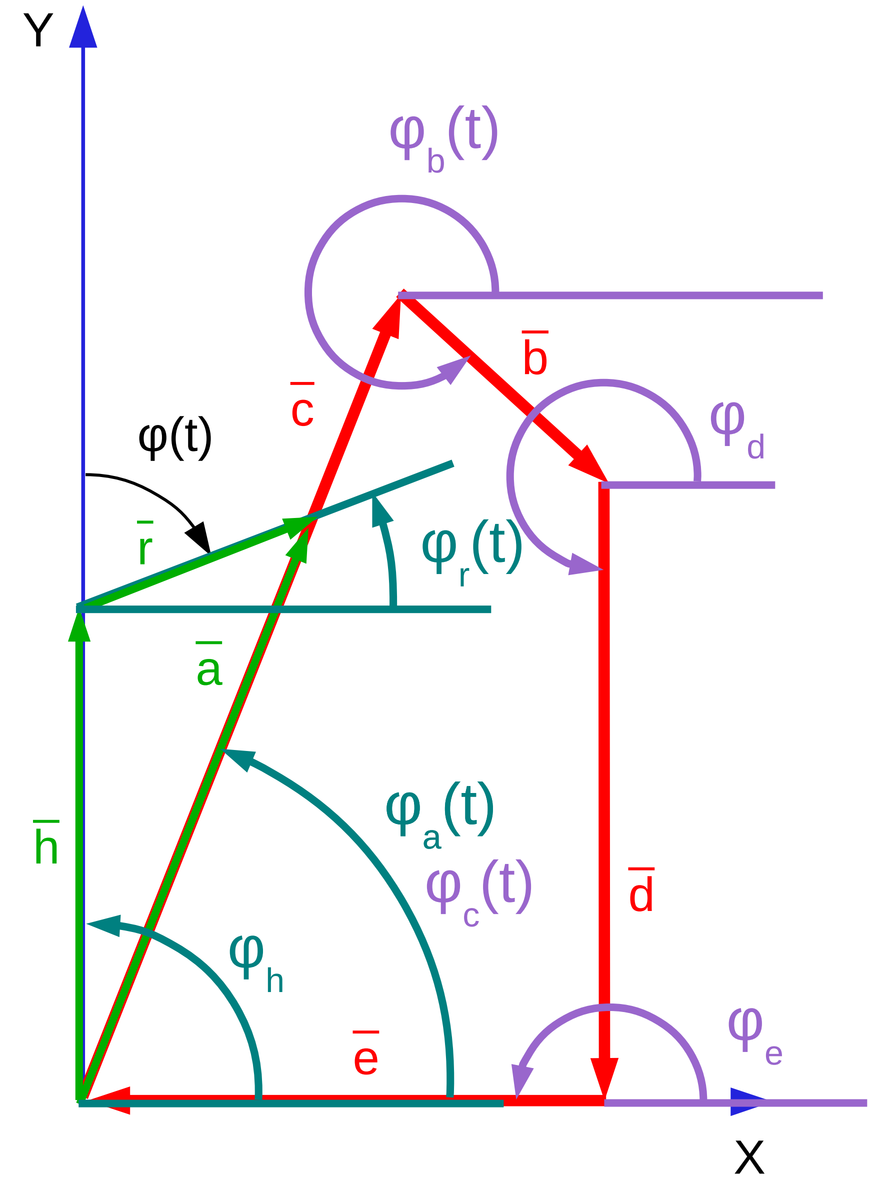
$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = \text{const.}$

$|b| = |CD| = b = \text{const.}$

$|d| = d(t)$





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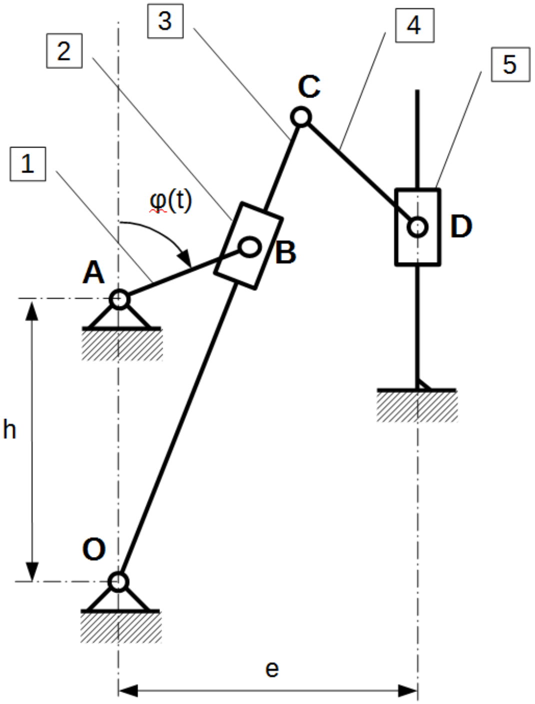
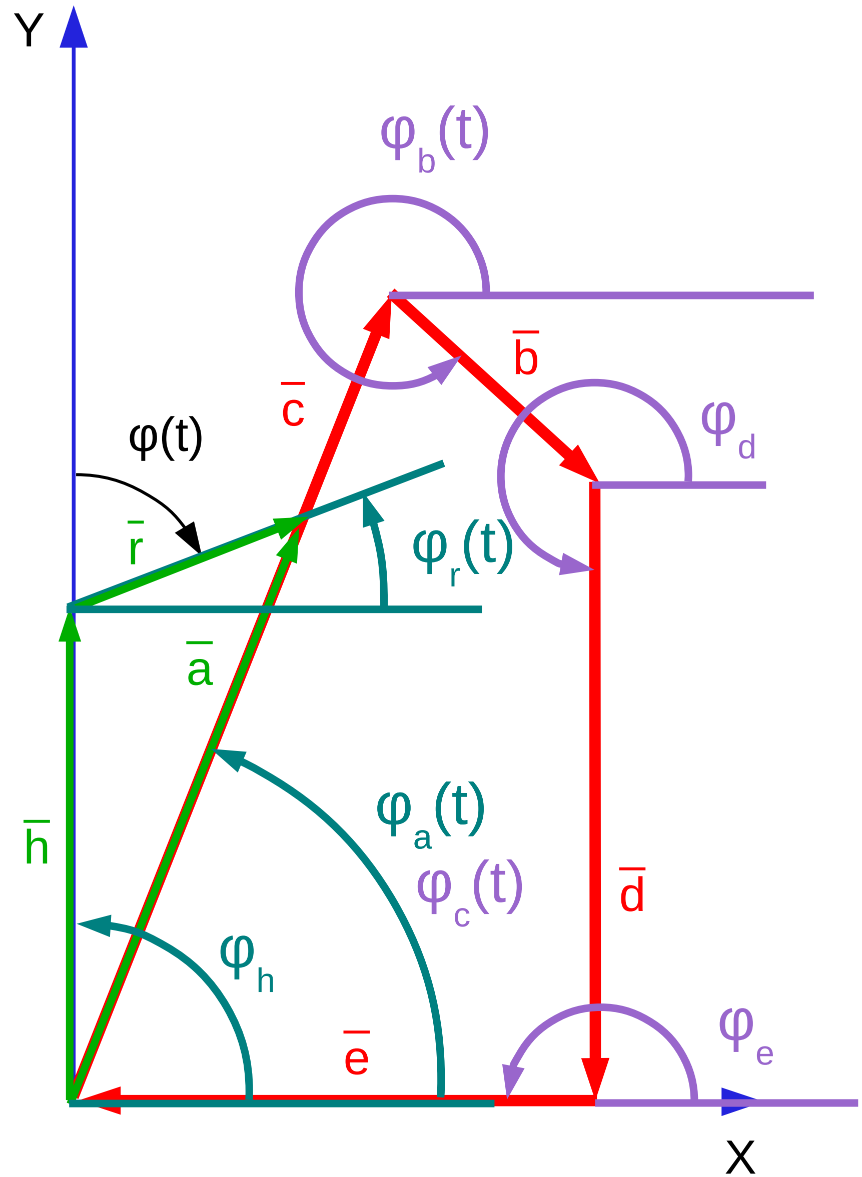
$|b| = |CD| = b = const.$

$|d| = d(t)$

$|\bar{e}| = e = const.$





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$\varphi_d = 270^o$

$\varphi_e = 180^o$

$|\bar{r}| = |AB| = r = \text{const.}$

$|\bar{h}| = |OA| = h = \text{const.}$

$|\bar{a}| = |OB| = a(t)$

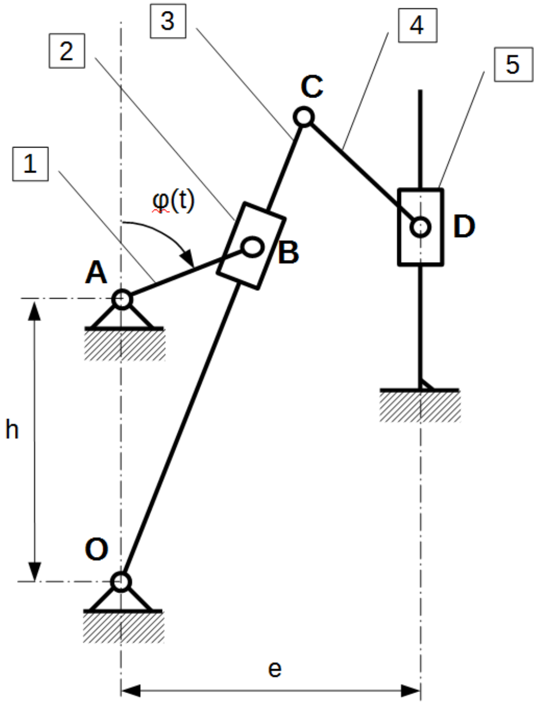
$|\bar{c}| = |OC| = c = \text{const.}$

$|b| = |CD| = b = \text{const.}$

$|d| = d(t)$

$|\bar{e}| = e = \text{const.}$

$$\bar{h} + \bar{r} = \bar{a}$$
$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$



$\varphi_h = 90^o$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_a(t) \neq const.$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_b(t) \neq const.$

$\varphi_d = 270^o$

$\varphi_e = 180^o$

$|\bar{h}| = |OA| = h = const.$

$|\bar{r}| = |AB| = r = const.$

$|\bar{a}| = |OB| = a(t)$

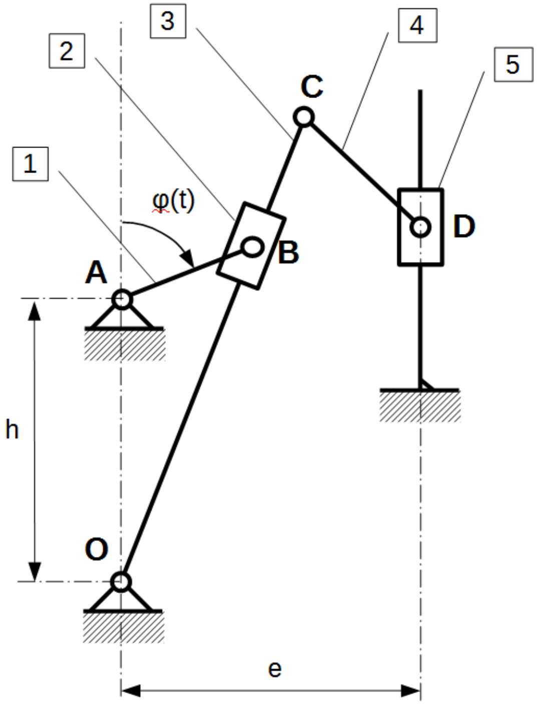
$|\bar{c}| = |OC| = c = const.$

$|b| = |CD| = b = const.$

$|d| = d(t)$

$|\bar{e}| = e = const.$

$$\bar{h} + \bar{r} = \bar{a}$$
$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$



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$|\bar{c}| = |OC| = c = const.$

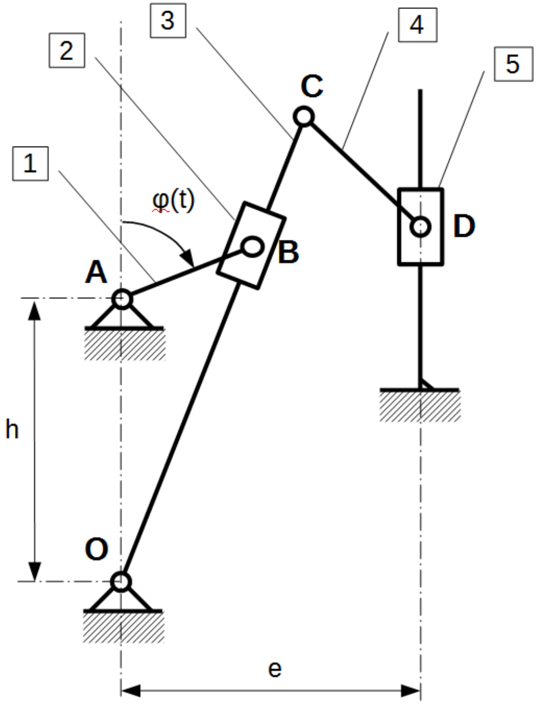
$|b| = |CD| = b = const.$

$|d| = d(t)$

$|\bar{e}| = e = const.$

$$\bar{h} + \bar{r} = \bar{a}$$
$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

$$x: \quad h\cos\phi_h + r\cos\phi_r(t) = a(t)\cos\phi_a(t)$$
$$y: \quad h\sin\phi_h + r\sin\phi_r(t) = a(t)\sin\phi_a(t)$$



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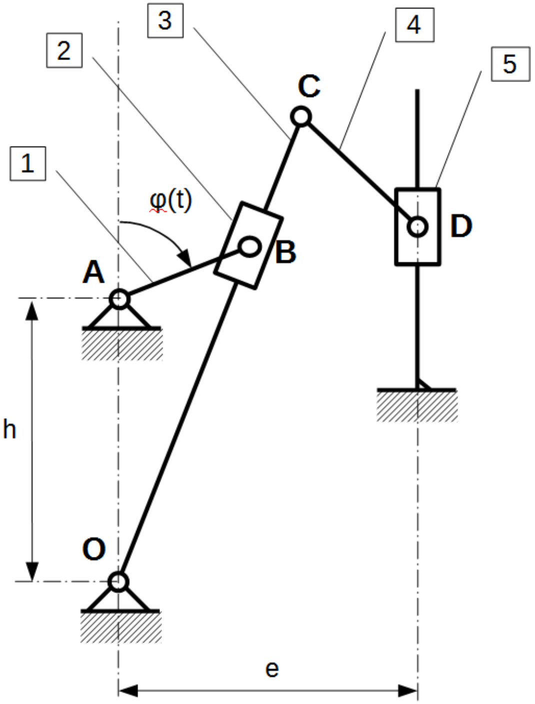
$|b| = |CD| = b = const.$

$|d| = d(t)$

$|\bar{e}| = e = const.$

$$\bar{h} + \bar{r} = \bar{a}$$
$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

$$x:\quad h\cos\phi_h + r\cos\phi_r(t) = a(t)\cos\phi_a(t)$$
$$y:\quad h\sin\phi_h + r\sin\phi_r(t) = a(t)\sin\phi_a(t)$$
$$x:\quad c\cos\phi_c(t) + b\cos\phi_b(t) + d(t)\cos\phi_d + e\cos\phi_e = 0$$
$$y:\quad c\sin\phi_c(t) + b\sin\phi_b(t) + d(t)\sin\phi_d + e\sin\phi_e = 0$$



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$|\bar{h}| = |OA| = h = const.$

$|\bar{r}| = |AB| = r = const.$

$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = const.$

$|b| = |CD| = b = const.$

$|d| = d(t)$

$|\bar{e}| = e = const.$

$$\begin{aligned} \bar{h} + \bar{r} &= \bar{a} \\ \bar{c} + \bar{b} + \bar{d} + \bar{e} &= \bar{0} \end{aligned}$$

$$x: \quad h\cos\phi_h + r\cos\phi_r(t) = a(t)\cos\phi_a(t)$$

$$y: \quad h\sin\phi_h + r\sin\phi_r(t) = a(t)\sin\phi_a(t)$$

$$x: \quad c\cos\phi_c(t) + b\cos\phi_b(t) + d(t)\cos\phi_d + e\cos\phi_e = 0$$

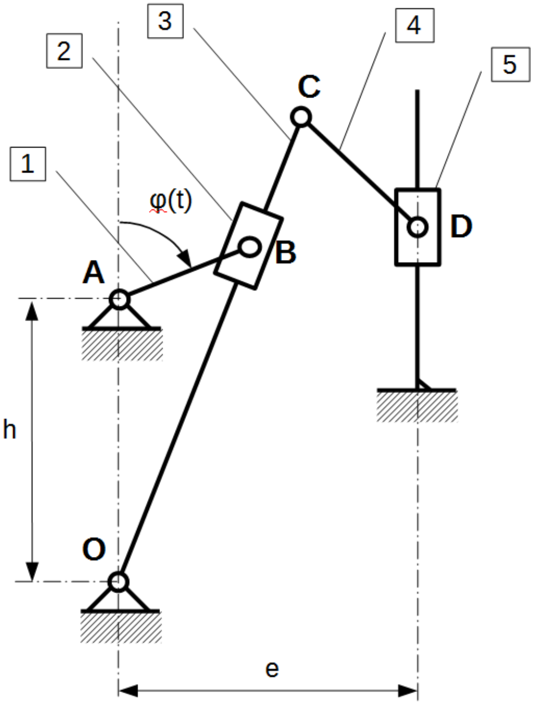
$$y: \quad c\sin\phi_c(t) + b\sin\phi_b(t) + d(t)\sin\phi_d + e\sin\phi_e = 0$$

$$x: \quad h\cos 90^o + r\cos(90^o - \phi(t)) = a(t)\cos\phi_a(t)$$

$$y: \quad h\sin 90^o + r\sin(90^o - \phi(t)) = a(t)\sin\phi_a(t)$$

$$x: \quad c\cos\phi_a(t) + b\cos\phi_b(t) + d(t)\cos 270^o + e\cos 180^o = 0$$

$$y: \quad c\sin\phi_a(t) + b\sin\phi_b(t) + d(t)\sin 270^o + e\sin 180^o = 0$$



$\varphi_h = 90^o$

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$\varphi_a(t) \neq const.$

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$|\bar{h}| = |OA| = h = const.$

$|\bar{r}| = |AB| = r = const.$

$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = const.$

$|b| = |CD| = b = const.$

$|d| = d(t)$

$|\bar{e}| = e = const.$

$$\bar{h} + \bar{r} = \bar{a}$$
$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

$$x: \; h\cos\phi_h + r\cos\phi_r(t) = a(t)\cos\phi_a(t)$$
$$y: \; h\sin\phi_h + r\sin\phi_r(t) = a(t)\sin\phi_a(t)$$
$$x: \; c\cos\phi_c(t) + b\cos\phi_b(t) + d(t)\cos\phi_d + e\cos\phi_e = 0$$
$$y: \; c\sin\phi_c(t) + b\sin\phi_b(t) + d(t)\sin\phi_d + e\sin\phi_e = 0$$

$$x: \; h\cos 90^o + r\cos(90^o - \phi(t)) = a(t)\cos\phi_a(t)$$
$$y: \; h\sin 90^o + r\sin(90^o - \phi(t)) = a(t)\sin\phi_a(t)$$
$$x: \; c\cos\phi_a(t) + b\cos\phi_b(t) + d(t)\cos 270^o + e\cos 180^o = 0$$
$$y: \; c\sin\phi_a(t) + b\sin\phi_b(t) + d(t)\sin 270^o + e\sin 180^o = 0$$

$$x: \; r\sin\phi(t) = a(t)\cos\phi_a(t)$$
$$y: \; h + r\cos\phi(t) = a(t)\sin\phi_a(t)$$
$$x: \; c\cos\phi_a(t) + b\cos\phi_b(t) - e = 0$$
$$y: \; c\sin\phi_a(t) + b\sin\phi_b(t) - d(t) = 0$$

$$① \quad x: \; r\sin\phi(t) = a(t)\cos\phi_a(t)$$
$$② \quad y: \; h + r\cos\phi(t) = a(t)\sin\phi_a(t)$$
$$③ \quad x: \; c\cos\phi_a(t) + b\cos\phi_b(t) - e = 0$$
$$④ \quad y: \; c\sin\phi_a(t) + b\sin\phi_b(t) - d(t) = 0$$

1 $x:\quad r\sin\phi(t)=a(t)\cos\phi_a(t)$

2 $y:\quad h+r\cos\phi(t)=a(t)\sin\phi_a(t)$

3 $x:\quad c\cos\phi_a(t)+b\cos\phi_b(t)-e=0$

4 $y:\quad c\sin\phi_a(t)+b\sin\phi_b(t)-d(t)=0$

$$a(t)=\sqrt{r^2\sin^2(t)+(h+r\cos\phi(t))^2}$$

$$\phi_a(t)=atan\left(\frac{h+r\cos\phi(t)}{r\sin\phi(t)}\right)$$

$$\phi_b(t)=\arccos\frac{e-c\cos\phi_a(t)}{b}$$

$$d(t)=c\sin\phi_a(t)+b\sqrt{1-\left(\frac{e-c\cos\phi_a(t)}{b}\right)^2}$$

1 $x: \; r\sin\phi(t) = a(t)\cos\phi_a(t)$

2 $y: \; h + r\cos\phi(t) = a(t)\sin\phi_a(t)$

3 $x: \; c\cos\phi_a(t) + b\cos\phi_b(t) - e = 0$

4 $y: \; c\sin\phi_a(t) + b\sin\phi_b(t) - d(t) = 0$

$$a(t) = \sqrt{r^2\sin^2(t) + (h + r\cos\phi(t))^2}$$

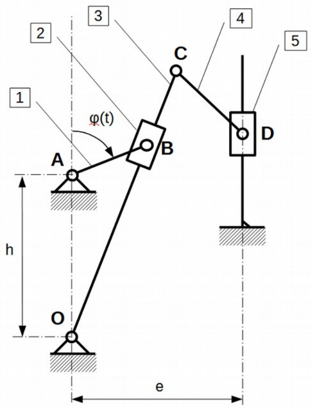
$$\phi_a(t) = atan\left(\frac{h + r\cos\phi(t)}{r\sin\phi(t)}\right)$$

$$\phi_b(t) = \arccos\frac{e - c\cos\phi_a(t)}{b}$$

$$d(t) = c\sin\phi_a(t) + b\sqrt{1 - \left(\frac{e - c\cos\phi_a(t)}{b}\right)^2}$$

$$v_D(t) = \dot{d}(t)$$

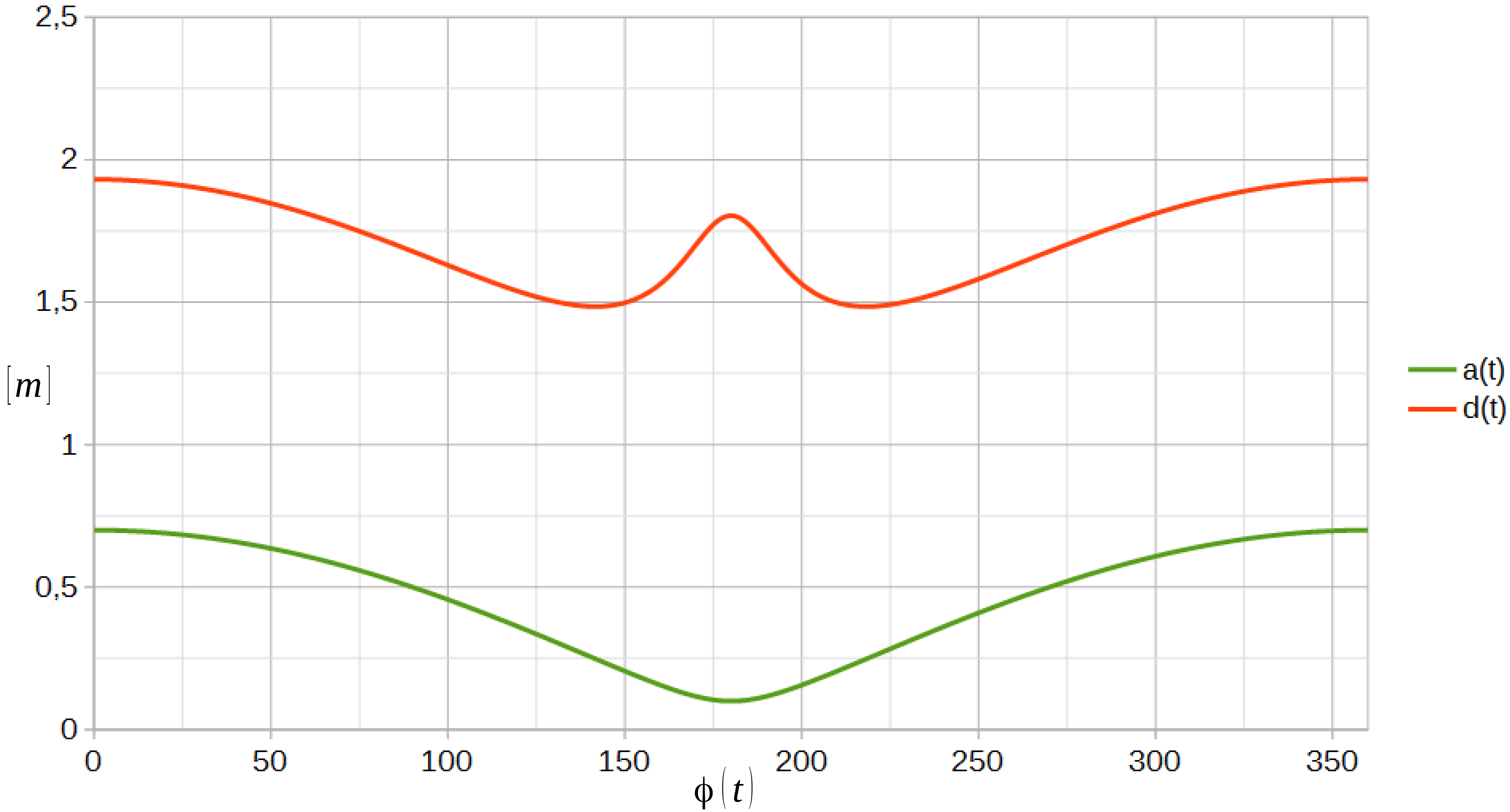
$$a_D(t) = \ddot{d}(t)$$

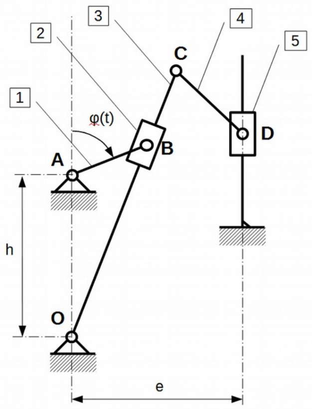


$$a(t)=\sqrt{r^2\sin^2(t)+(h+r\cos\phi(t))^2}$$

$$d(t)=c\sin\phi_a(t)+b\sqrt{1-\left(\frac{e-c\cos\phi_a(t)}{b}\right)^2}$$

r=	0,3	[m]
h=	0,4	[m]
c=	1	[m]
e=	0,4	[m]
b=	1	[m]
ω=	1	[rad/s]

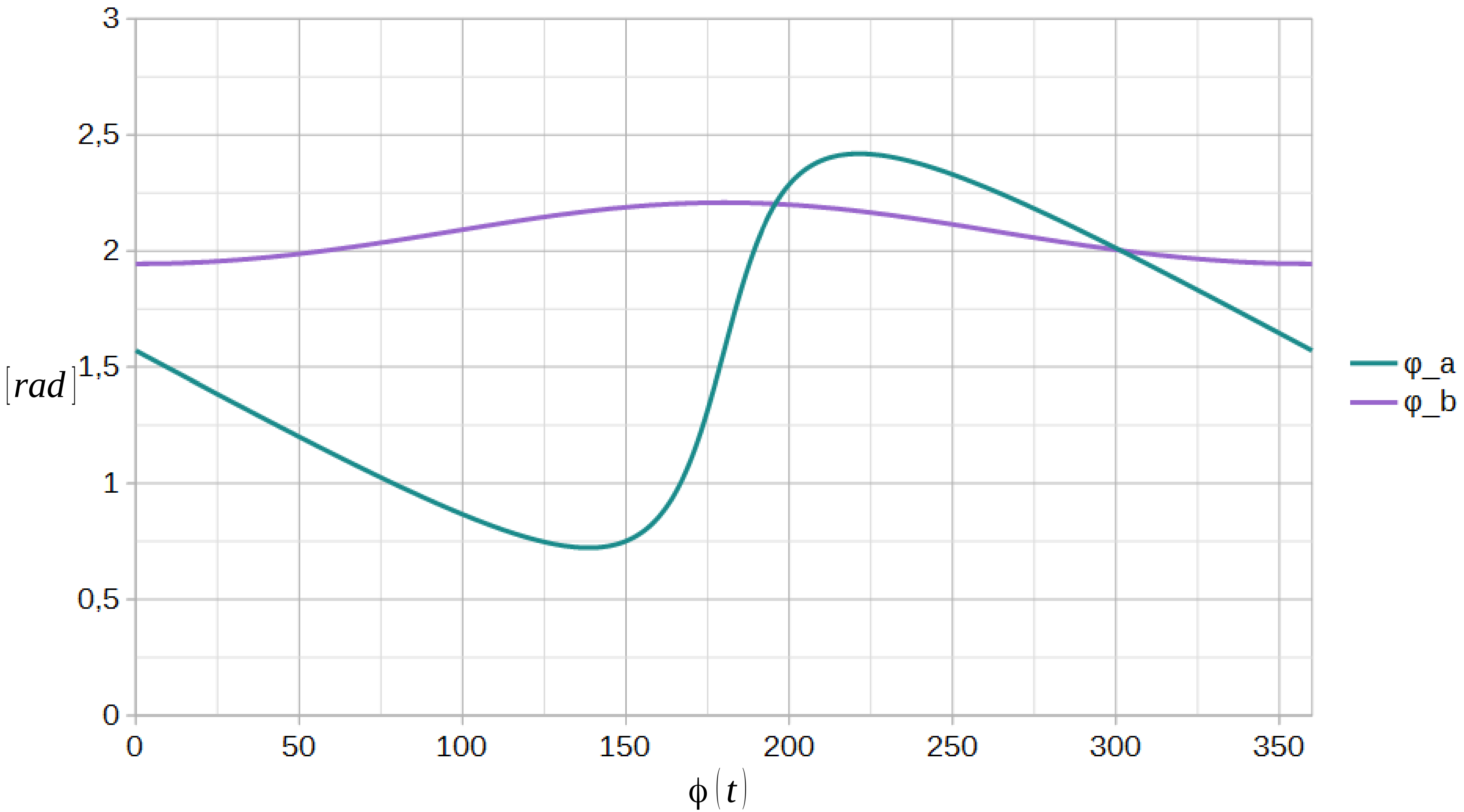


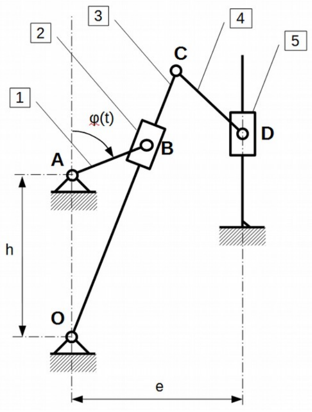


$$\phi_a(t) = atan\left(\frac{h + r\cos\phi(t)}{r\sin\phi(t)}\right)$$

$$\phi_b(t) = \arccos\frac{e - c\cos\phi_a(t)}{b}$$

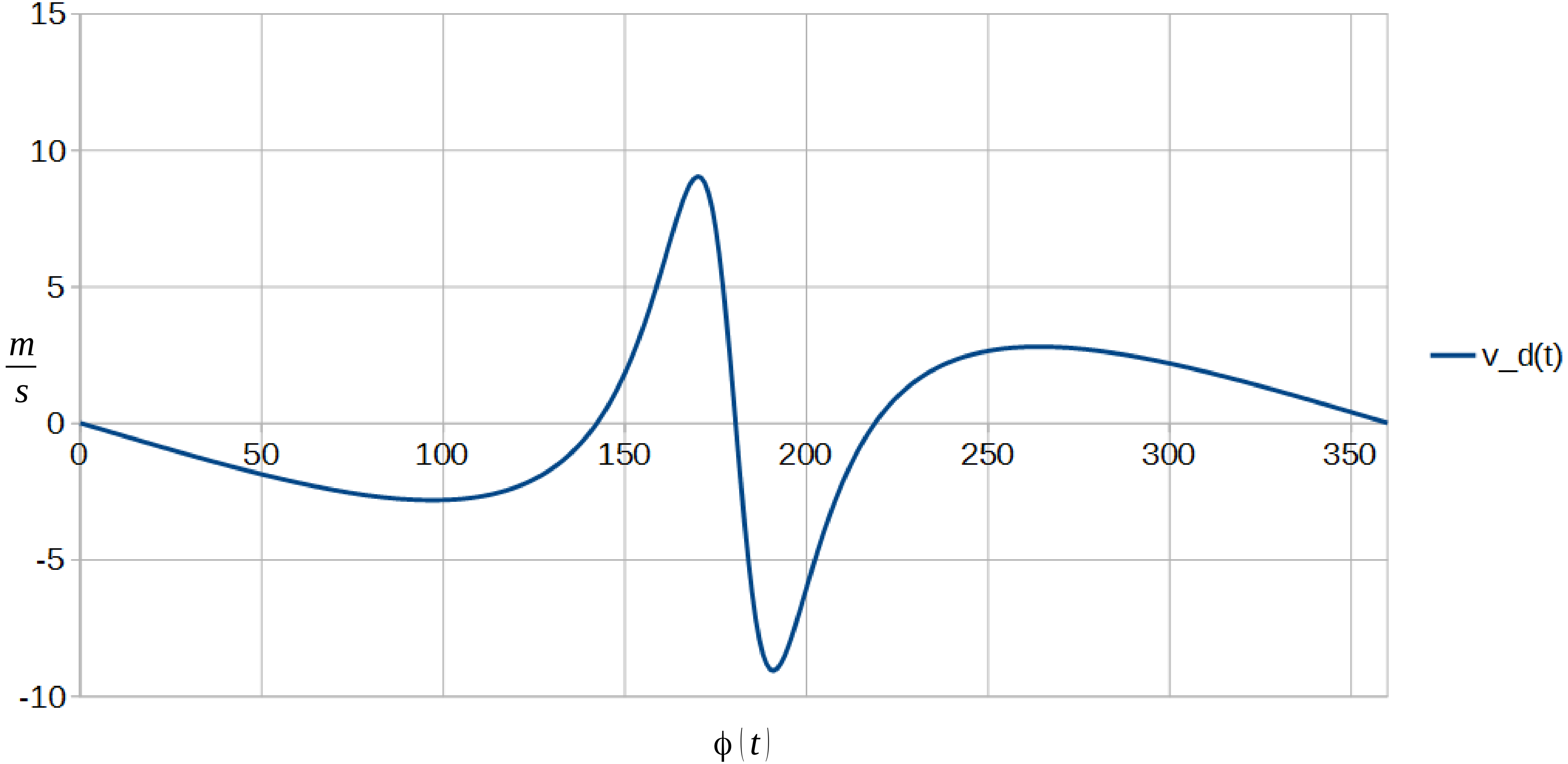
r=	0,3	[m]
h=	0,4	[m]
c=	1	[m]
e=	0,4	[m]
b=	1	[m]
ω=	1	[rad/s]

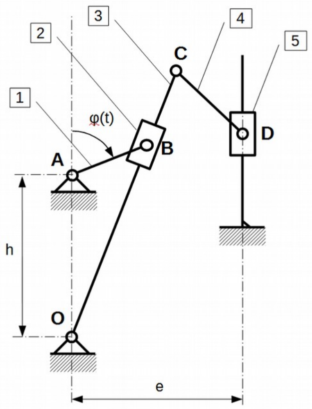




r=	0,3	[m]
h=	0,4	[m]
c=	1	[m]
e=	0,4	[m]
b=	1	[m]
ω=	1	[rad/s]

$$v_D(t) = \dot{d}(t)$$





r=	0,3	[m]
h=	0,4	[m]
c=	1	[m]
e=	0,4	[m]
b=	1	[m]
ω=	1	[rad/s]

$$a_D(t) = \ddot{d}(t)$$