Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control

Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

Theory of Machines and Automatic Control - project class

The Faculty of Automotive and Construction Machinery Engineering

Winter 2019/2020

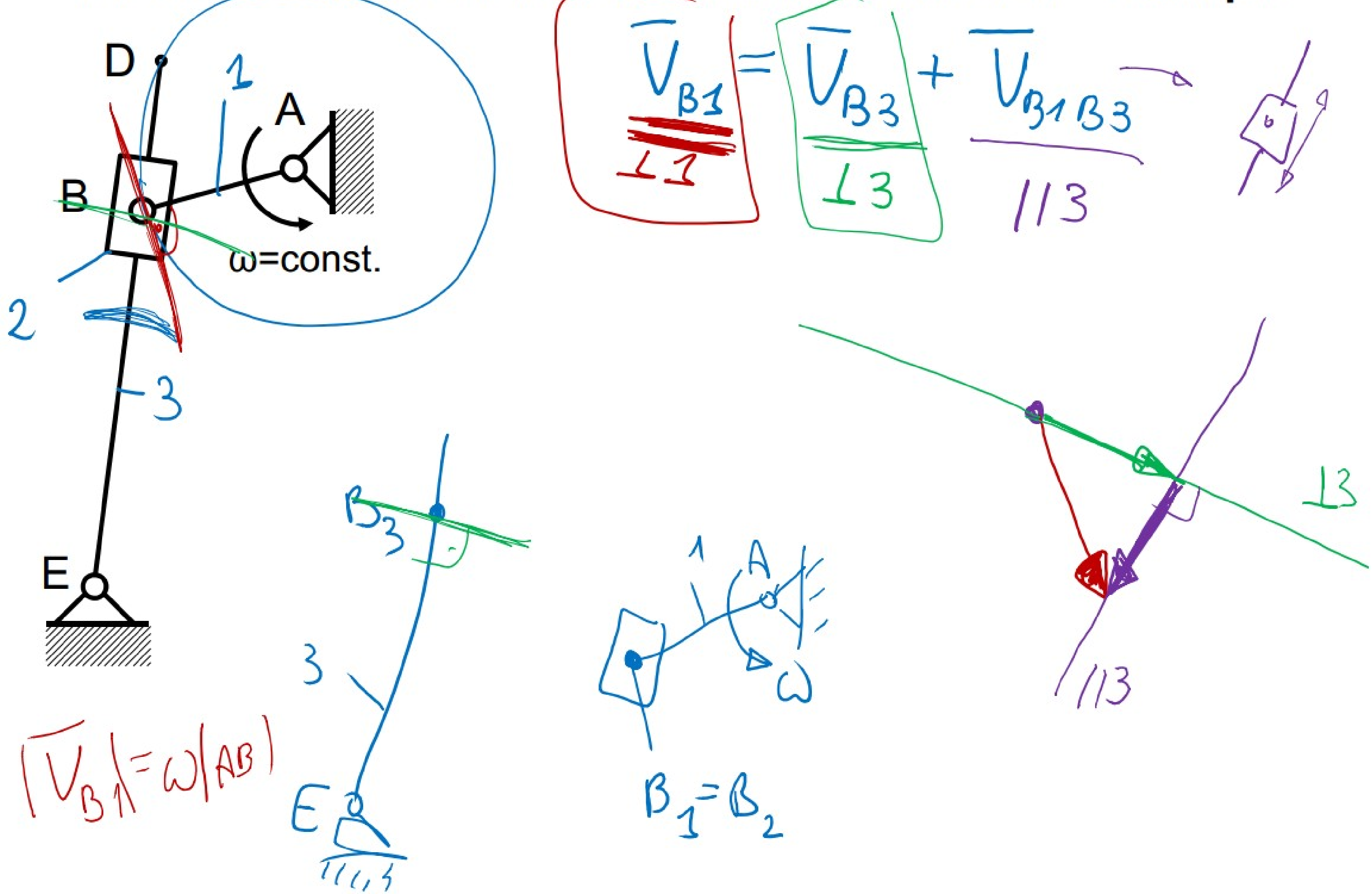
2.1 EHVE – Wednesday, 8:15-10:00, room 3.14, S. Korczak, P. Wawrzyniak
2.2 EHVE – Wednesday, 10:15-12:00, room 3.11, S. Korczak, P. Wawrzyniak
2.1 MTR – Friday, 8:15-10:00, room 0.3, M. Parafiniak

Group / date		**topics**	**assessment**
2.1 EHVE 2.2 EHVE	**2.1 MTR**		
23.10.2019	~~25.10.2019~~ 8.11.2019	Introduction. 1st project topics distribution. Graphical methods.	---
30.10.2019	---	---	---
6.11.2019	~~8.11.2019~~ 15.11.2019	1st project consultations. Analytical method.	---
13.11.2019	~~15.11.2019~~	---	---
20.11.2019	22.11.2019	---	---
27.11.2019	29.11.2019	1st project commitment. 2nd project topics distribution.	1st project evaluation.
4.12.2019	6.12.2019		
11.12.2019	13.12.2019	2nd project consultations.	---
18.12.2019	20.12.2019	2nd project commitment. 3rd project topics distribution. Characteristics of basic automatic control elements. Block diagram algebra.	2nd project evaluation.
Winter break (23.12.2019 – 6.02.2020)			
8.01.2020	10.01.2020	---	---
15.01.2020	17.01.2020	3rd project consultations. PID control.	---
22.01.2020	24.01.2020	3rd project consultations & commitment.	3rd project evaluation.
23.01.2020	31.01.2020	Final class evaluation.	
1.01.2020 – 14.02.2020: exam session			

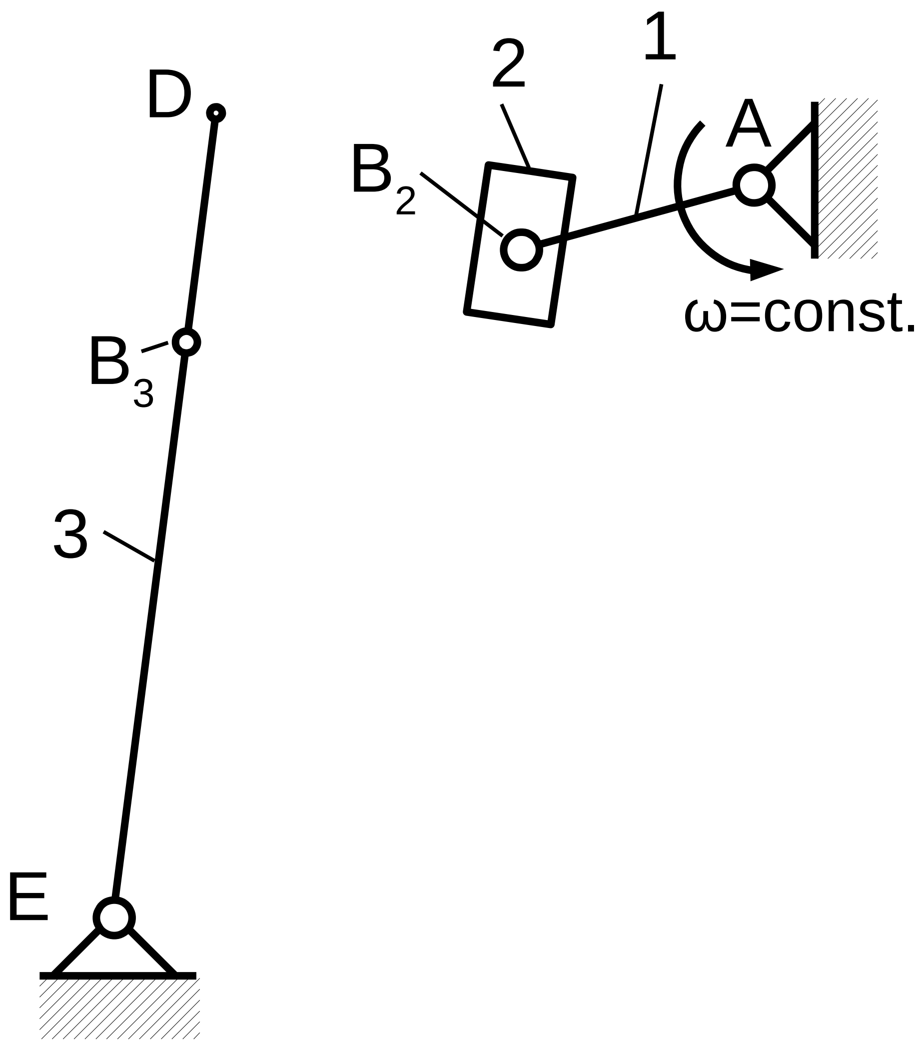
Lecture 3 cont.

Relative motion – example.

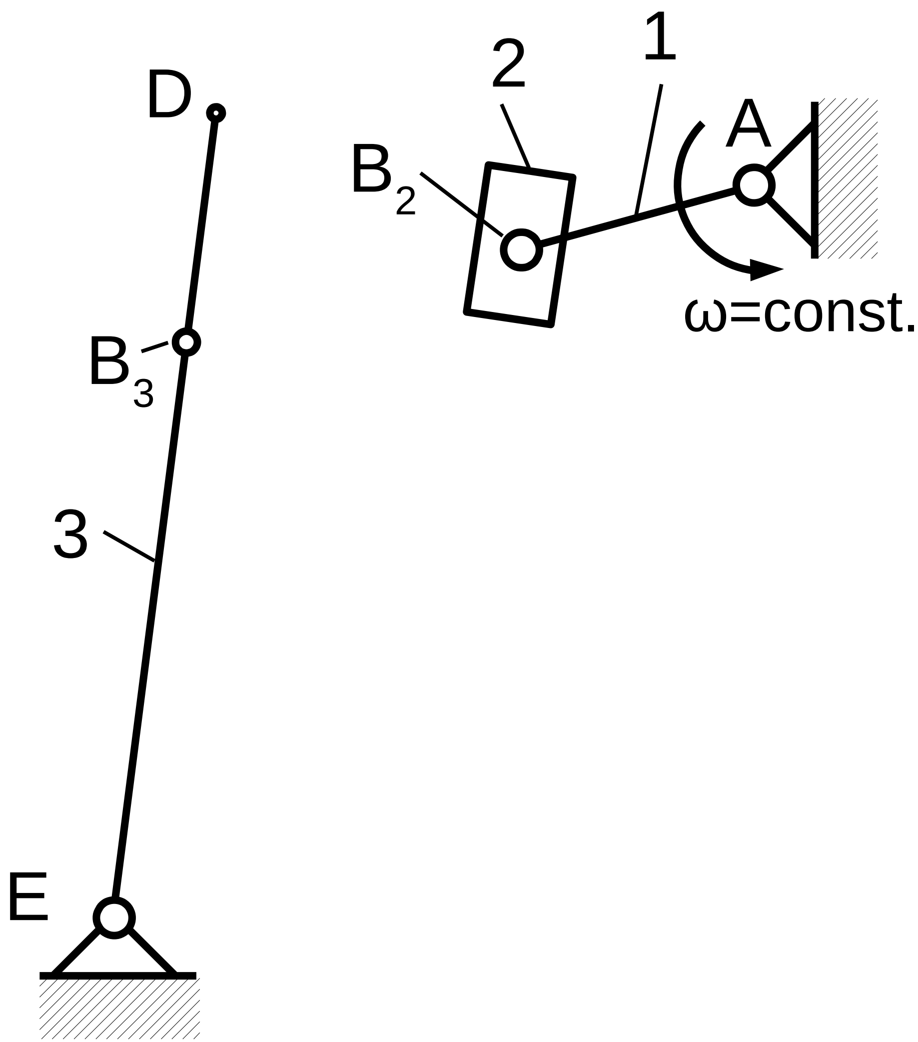
Velocities and accelerations in relative motion – example



Velocities and accelerations in relative motion – example



Velocities and accelerations in relative motion – example



Lecture 4

Analytical method. Cam mechanisms.

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

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4. Define “directed angles” for all vectors defined in the same manner. Assume that this angles are created by the positive x axis counter-clockwise rotation.

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4. Define “directed angles” for all vectors defined in the same manner. Assume that this angles are created by the positive x axis counter-clockwise rotation.

5. Fore each of polygon write down sum of vectors, e.g.:

$$\sum_{i=1}^{i=n} \vec{l}_i = 0$$

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

6a. Write down projections of each polygon onto coordinate system's axes:

$$\text{x:}\quad \sum_{i=1}^{i=n} l_i \cos\varphi_i = 0 \qquad\qquad \text{y:}\quad \sum_{i=1}^{i=n} l_i \sin\varphi_i = 0$$

(we do not need to analyze signs because of „directed angles” setup procedure)

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(we do not need to analyze signs because of „directed angles” setup procedure)

6b. Define which vectors' lengths and angles are given and/or constant (related to geometry), and which are variable in time and unknown.

(for a proper defined system number of unknown variables is equal to the number of equations)

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

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(we do not need to analyze signs because of „directed angles” setup procedure)

6b. Define which vectors' lengths and angles are given and/or constant (related to geometry), and which are variable in time and unknown.

(for a proper defined system number of unknown variables is equal to the number of equations)

7. Solve the equations. The resulting functions describe motion of the mechanism.

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

8. Differentiate functions achieved in p.7 to obtain velocities. Differentiate once again to obtain accelerations.

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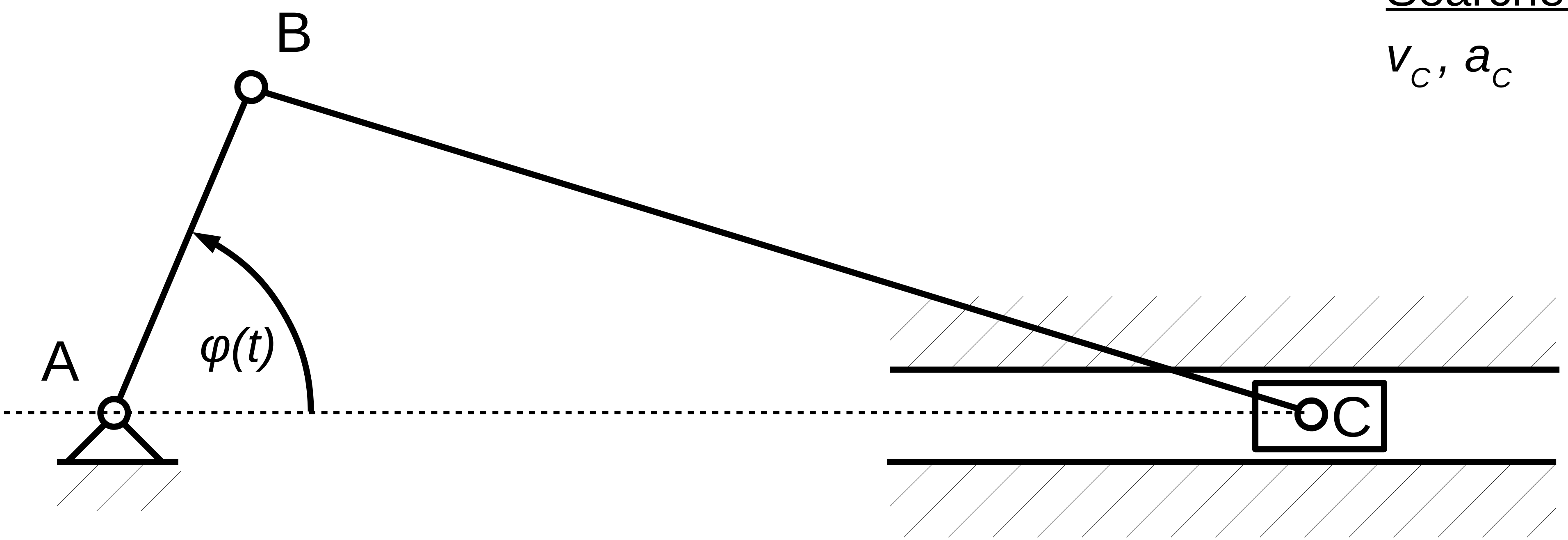
9. If desired information was not obtained in p.8, differentiate equations from p.6. Sometimes rotation of the coordinate system is useful here.

Analytical method – example:
crank-slider mechanism



Given:

$|AB| = r$

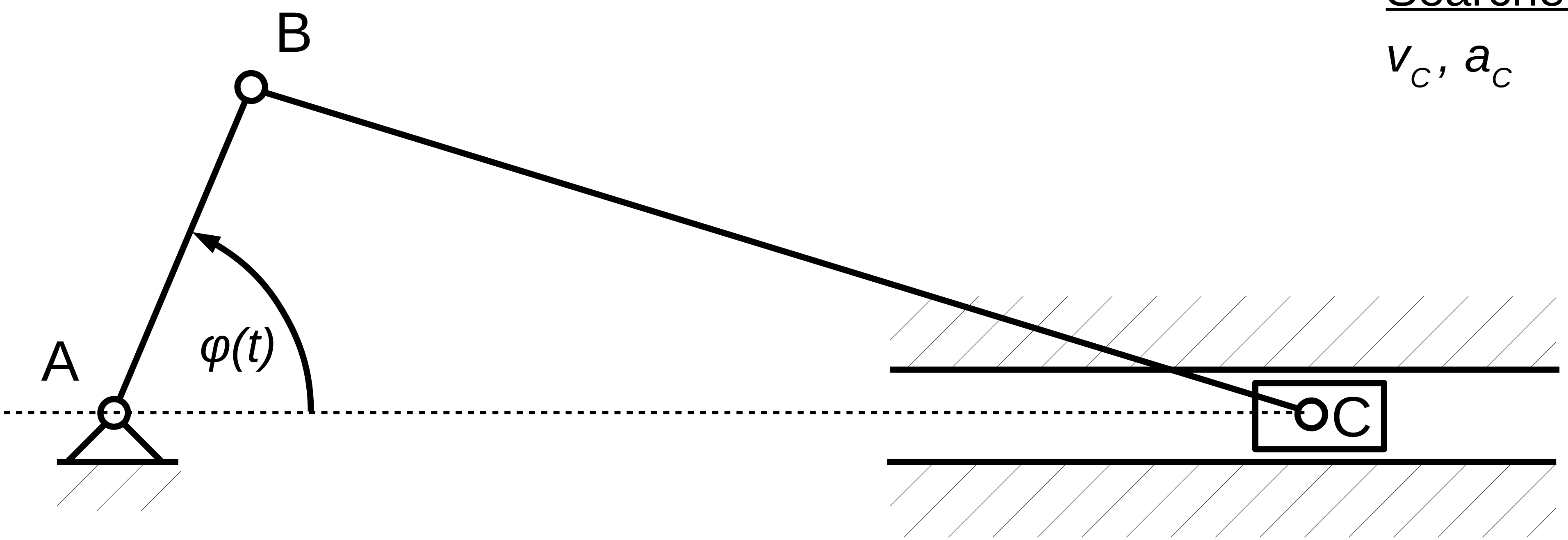
$|BC| = l$

$\varphi(t)$

Searched:

v_C, a_C

Analytical method – example:
crank-slider mechanism



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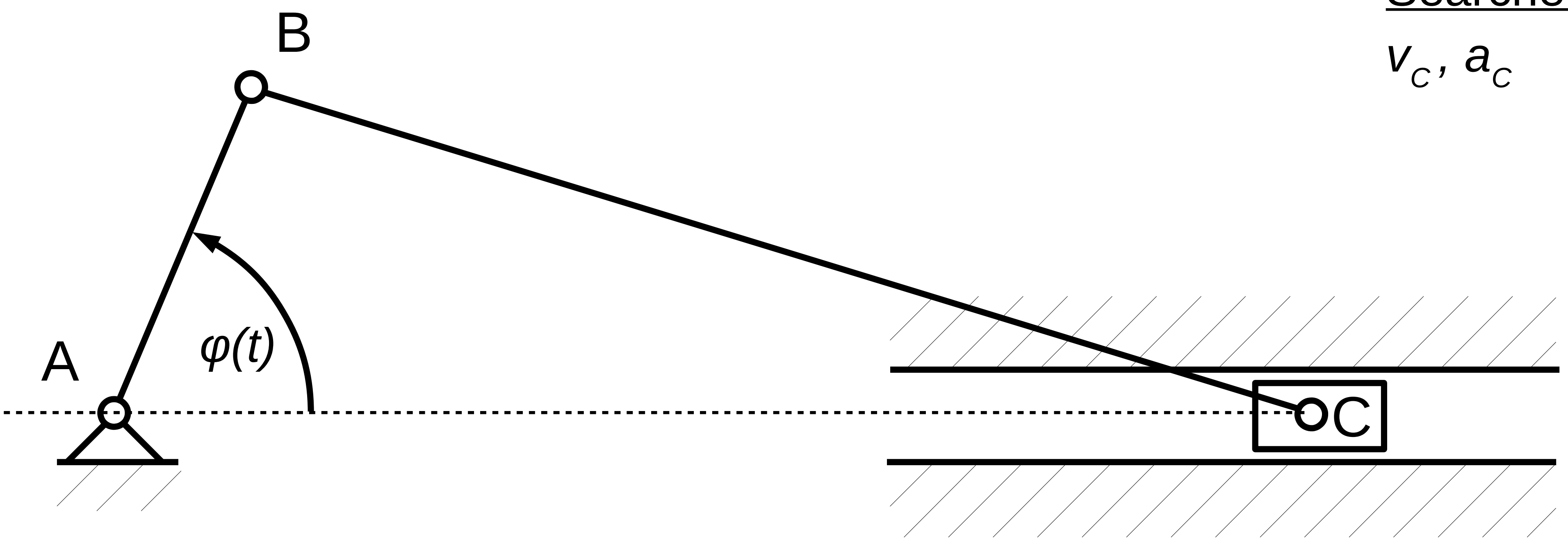

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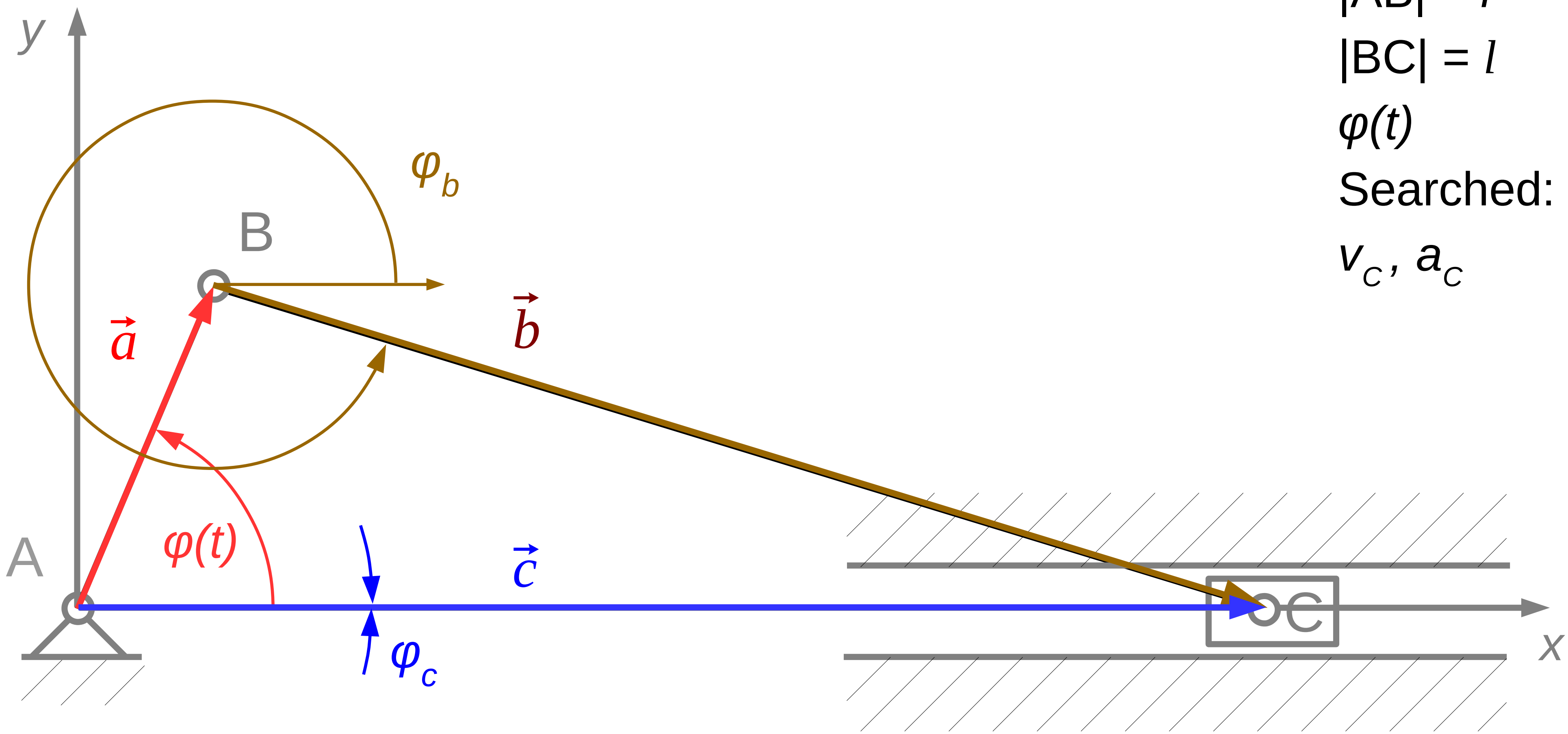

Given:

|AB| = *r*

|BC| = *l*

φ(t)

Searched:

v_C, a_C

$$|\vec{a}| = r \qquad \varphi(t)$$

$$|\vec{b}| = l \qquad \varphi_b(t)$$

$$|\vec{c}| = c(t) \qquad \varphi_c = 0$$

$$\vec{a} + \vec{b} = \vec{c}$$

$$\text{x:} \quad r\cos\varphi(t) + l\cos\varphi_b(t) = c(t)\cos 0$$

$$\text{y:} \quad r\sin\varphi(t) + l\sin\varphi_b(t) = c(t)\sin 0$$

Analytical method – example:
crank-slider mechanism

$$r\cos\varphi(t)+l\cos\varphi_b(t)=c(t)\cos 0$$
$$r\sin\varphi(t)+l\sin\varphi_b(t)=c(t)\sin 0$$

2 unknowns

Analytical method – example:
crank-slider mechanism

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Analytical method – example:
crank-slider mechanism

$$r\cos\varphi(t)+l\cos\varphi_b(t)=c(t)\cos 0$$
$$r\sin\varphi(t)+l\sin\varphi_b(t)=c(t)\sin 0$$

2 unknowns

$$r\cos\varphi(t)+l\cos\varphi_b(t)=c(t)$$
$$r\sin\varphi(t)+l\sin\varphi_b(t)=0$$

$$\sin\varphi_b(t)=-\frac{r}{l}\sin\varphi(t)=-\lambda\sin\varphi(t)$$

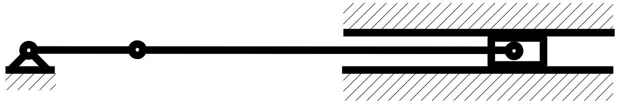
$$\varphi_b(t)=-\arcsin(\lambda\sin\varphi(t))$$

$$\sin^2\varphi_b(t)+\cos^2\varphi_b(t)=1$$

$$\cos\varphi_b(t)=\pm\sqrt{1-\sin^2\varphi_b(t)}$$

$$\cos\varphi_b(t)=\pm\sqrt{1-\lambda^2\sin^2\varphi(t)}$$

$$c(t)=r\cos\varphi(t)\pm l\sqrt{1-\lambda^2\sin^2\varphi(t)}$$



for $\varphi(t)=0$

$$c(t)=r+l$$

$$\boxed{c(t)=r\cos\varphi(t)+l\sqrt{1-\lambda^2\sin^2\varphi(t)}}$$

Analytical method – example:
crank-slider mechanism

slider movement

$$c(t) = r\cos\varphi(t) + l\sqrt{1-\lambda^2\sin^2\varphi(t)}$$

$$v_C(t) = \frac{dc(t)}{dt} = -r\frac{d\varphi(t)}{dt}\sin\varphi(t) - \frac{-2l\lambda^2\frac{d\varphi(t)}{dt}\sin\varphi(t)\cos\varphi(t)}{2\sqrt{1-\lambda^2\sin^2\varphi(t)}}$$

$$a_C(t) = \frac{dv_C(t)}{dt} = \ldots$$

Analytical method – example:
crank-slider mechanism

calculations with wxmaxima

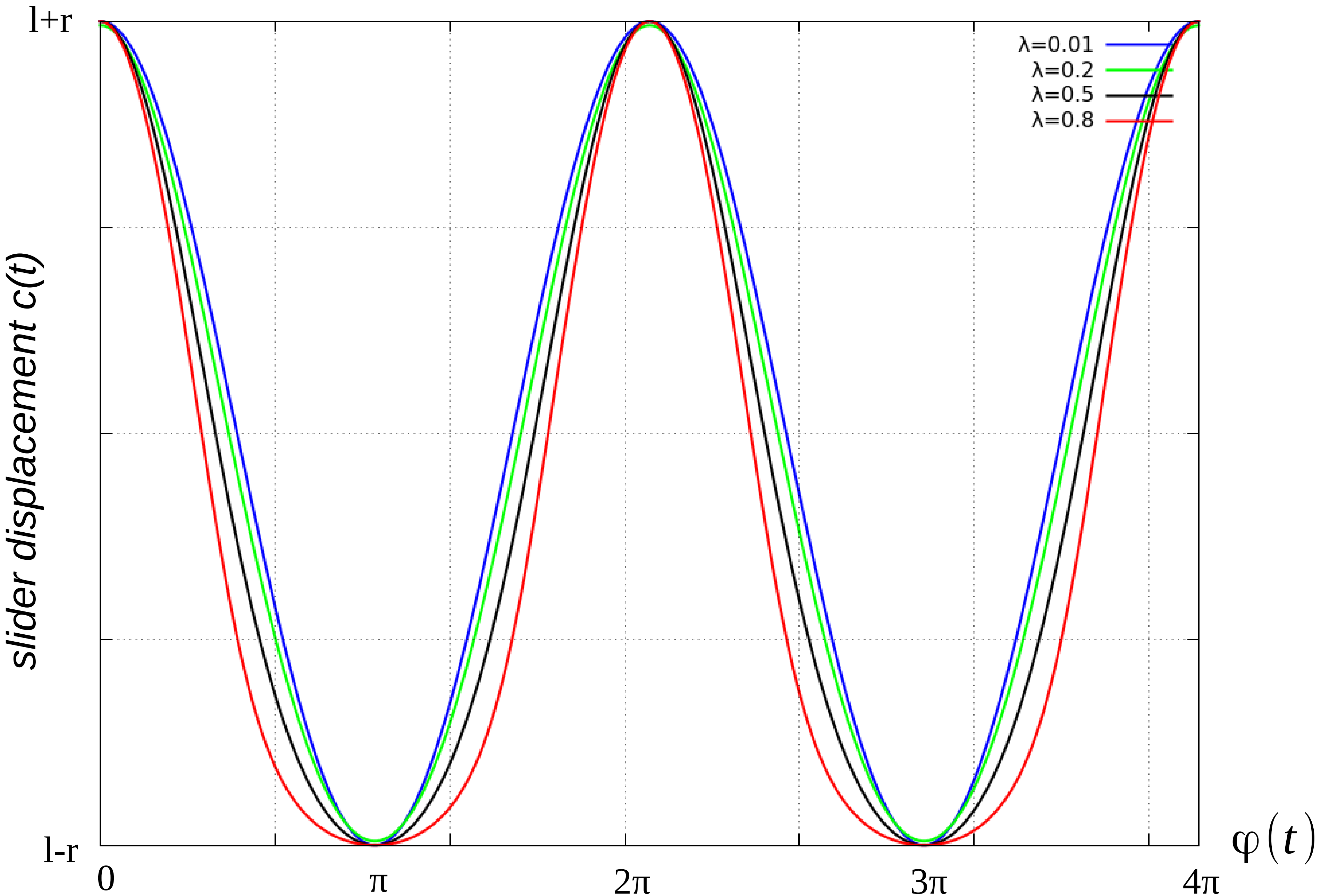
```
(%i14)  c: r*cos(%phi(t))+l*sqrt(1-%lambda^2*(sin(%phi(t)))^2);
        v: diff(c,t,1);
        a: diff(v,t,1);
```

(c) $$l\sqrt{1-\lambda^2\sin(\varphi(t))^2}+r\cos(\varphi(t))$$

(v) $$-\frac{\lambda^2 l\cos(\varphi(t))\sin(\varphi(t))\left(\frac{d}{dt}\varphi(t)\right)}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}}-r\sin(\varphi(t))\left(\frac{d}{dt}\varphi(t)\right)$$

(a) $$-\frac{\lambda^2 l\cos(\varphi(t))\sin(\varphi(t))\left(\frac{d^2}{dt^2}\varphi(t)\right)}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}}-r\sin(\varphi(t))\left(\frac{d^2}{dt^2}\varphi(t)\right)+\frac{\lambda^2 l\sin(\varphi(t))^2\left(\frac{d}{dt}\varphi(t)\right)^2}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}}$$
$$\frac{\lambda^2 l\cos(\varphi(t))^2\left(\frac{d}{dt}\varphi(t)\right)^2}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}}-\frac{\lambda^4 l\cos(\varphi(t))^2\sin(\varphi(t))^2\left(\frac{d}{dt}\varphi(t)\right)^2}{\left(1-\lambda^2\sin(\varphi(t))^2\right)^{3/2}}-r\cos(\varphi(t))\left(\frac{d}{dt}\varphi(t)\right)^2$$

Analytical method – example:
crank-slider mechanism



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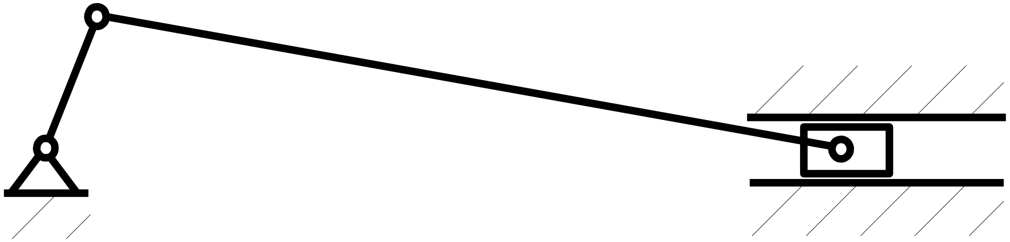
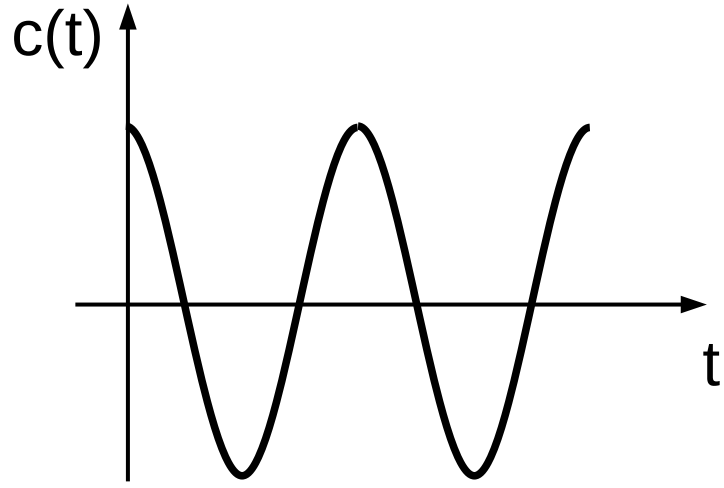
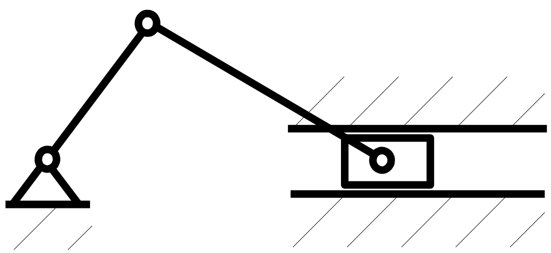
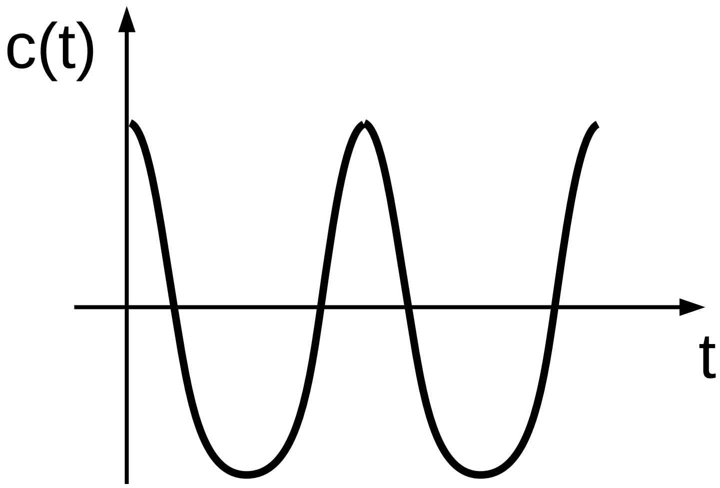








Interesting reeding: http://www.enginebuildermag.com/2016/08/understanding-rod-ratios/

Analytical method – example:
crank-slider mechanism

connecting rod motion

$$\varphi_b(t) = -\arcsin(\lambda \sin\varphi(t))$$

$$\omega_b(t) = \frac{d\varphi_b(t)}{dt} = \frac{-\lambda\dot{\varphi}(t)\cos\varphi(t)}{\sqrt{1-\lambda^2\sin^2\varphi(t)}}$$

$$\varepsilon_b(t) = \frac{d\omega_b(t)}{dt} = -\frac{\lambda\cos(\varphi(t))\left(\frac{d^2}{dt^2}\varphi(t)\right)}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}} + \frac{\lambda\sin(\varphi(t))\left(\frac{d}{dt}\varphi(t)\right)^2}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}} - \frac{\lambda^3\cos(\varphi(t))^2\sin(\varphi(t))\left(\frac{d}{dt}\varphi(t)\right)^2}{\left(1-\lambda^2\sin(\varphi(t))^2\right)^{3/2}}$$

Analytical method – example:
slider-yoke

Given:

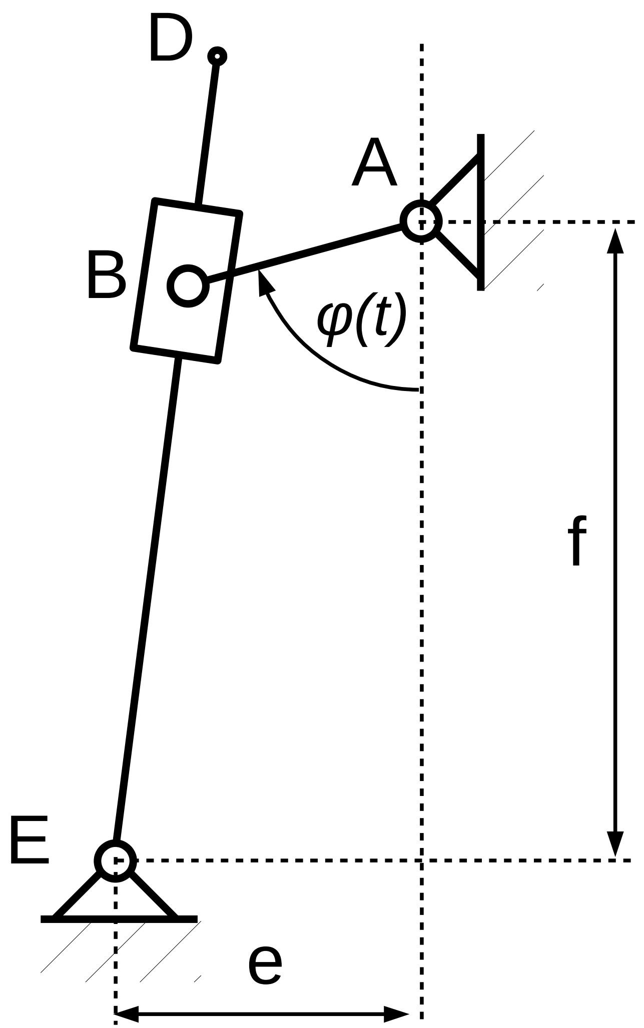
|AB| = *r*

e, f, $\varphi(t)$

Searched:

angular velocity ω_2 and acceleration ε_2

of ED element



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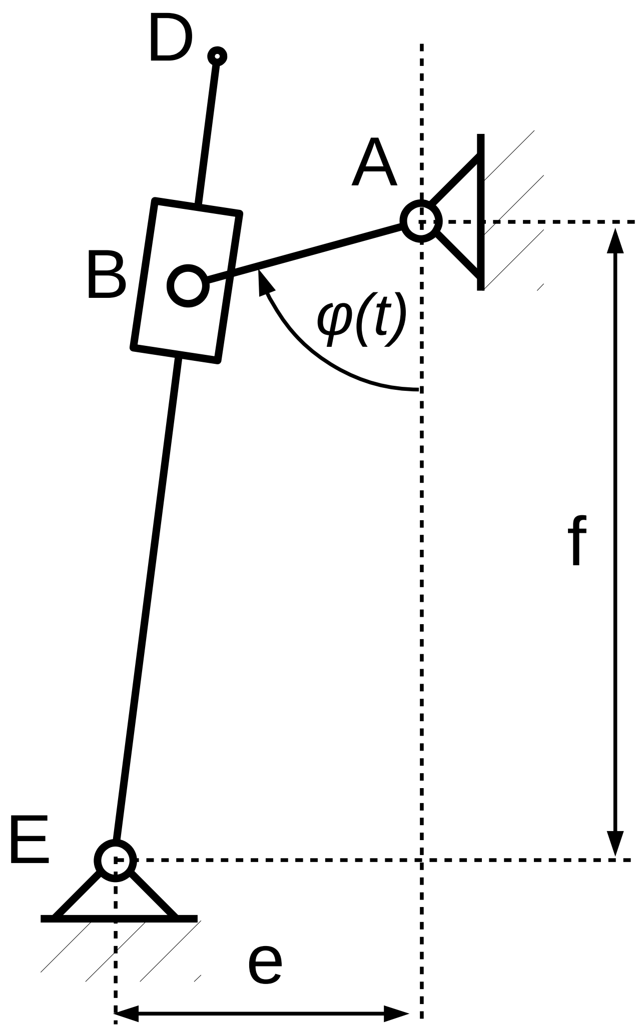

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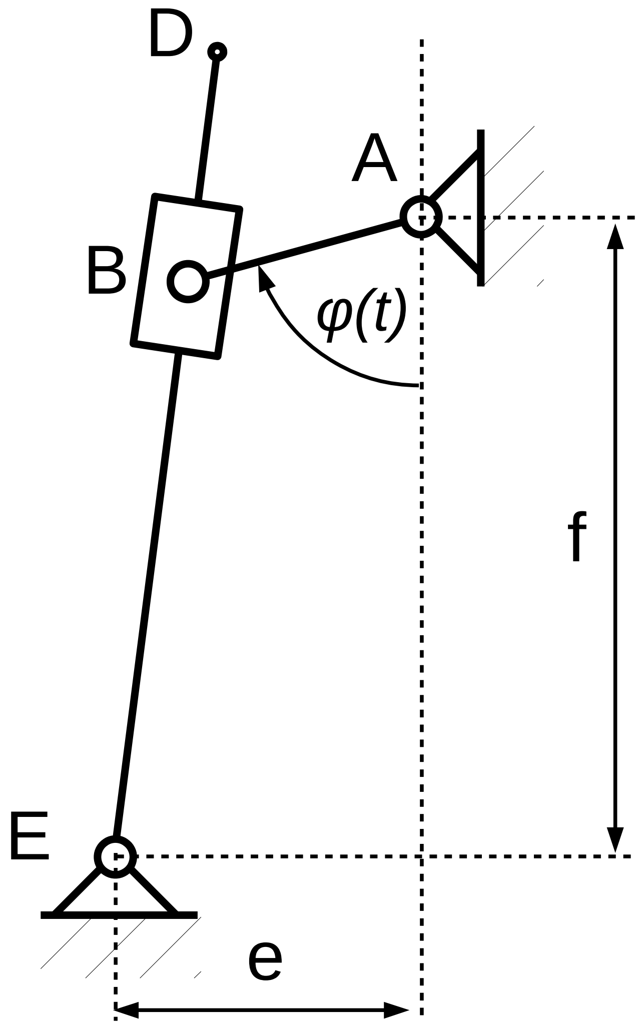

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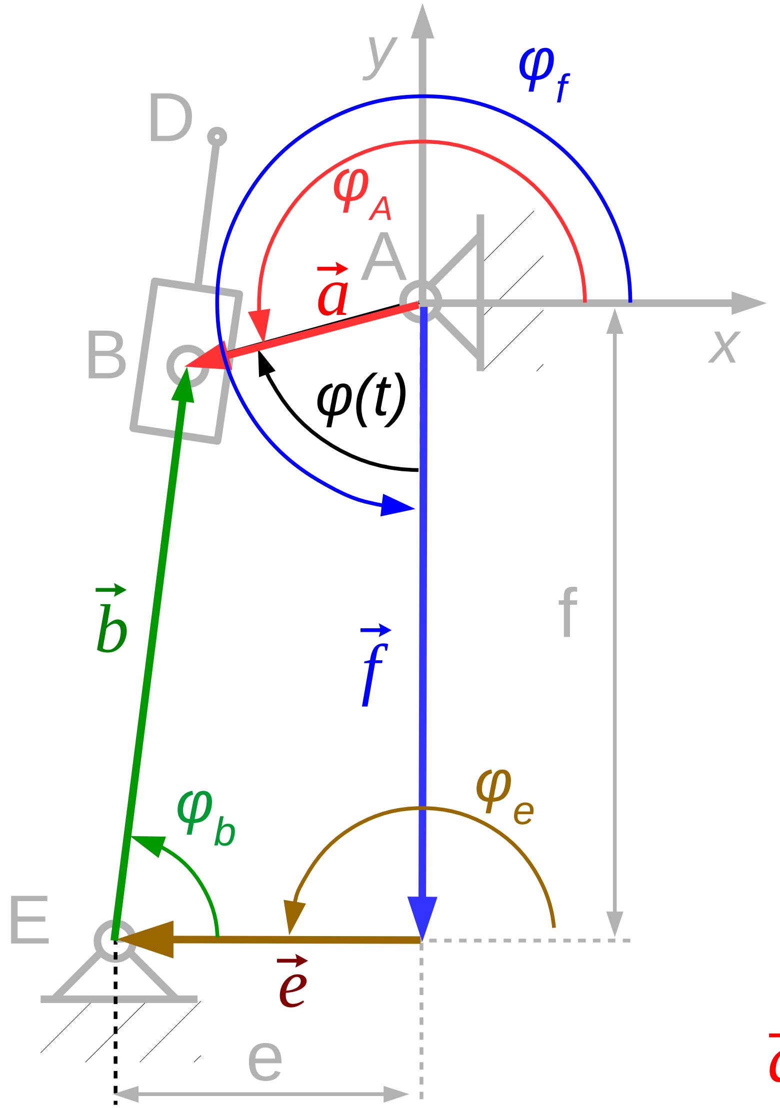


$$|\vec{a}| = r$$
$$\varphi_a(t) = 270^o - \varphi(t)$$

$$|\vec{b}| = b(t)$$
$$\varphi_b(t)$$

$$|\vec{e}| = e$$
$$\varphi_e = 180^o$$

$$|\vec{f}| = f$$
$$\varphi_f = 270^o$$

$$\vec{a} = \vec{b} + \vec{e} + \vec{f}$$

Analytical method – example:
slider-yoke

<u>Given</u>:

|AB| = *r*

e, f, *φ(t)*

<u>Searched</u>:

angular velocity ω_2 and acceleration ε_2

of ED element

$$|\vec{a}|=r$$
$$\varphi_a(t)=270^o-\varphi(t)$$
$$|\vec{b}|=b(t)$$
$$\varphi_b(t)$$

$$|\vec{e}|=e$$
$$\varphi_e=180^o$$
$$|\vec{f}|=f$$
$$\varphi_f=270^o$$

$$\vec{a}=\vec{b}+\vec{e}+\vec{f}$$

$$\text{x:}\quad r\cos(270^o-\varphi(t))=b(t)\cos\varphi_b(t)+e\cos 180^o+f\cos 270^o$$
$$\text{y:}\quad r\sin(270^o-\varphi(t))=b(t)\sin\varphi_b(t)+e\sin 180^o+f\sin 270^o$$

$$\text{x:}\quad -r\sin\varphi(t)=b(t)\cos\varphi_b(t)-e$$
$$\text{y:}\quad -r\cos\varphi(t)=b(t)\sin\varphi_b(t)-f$$

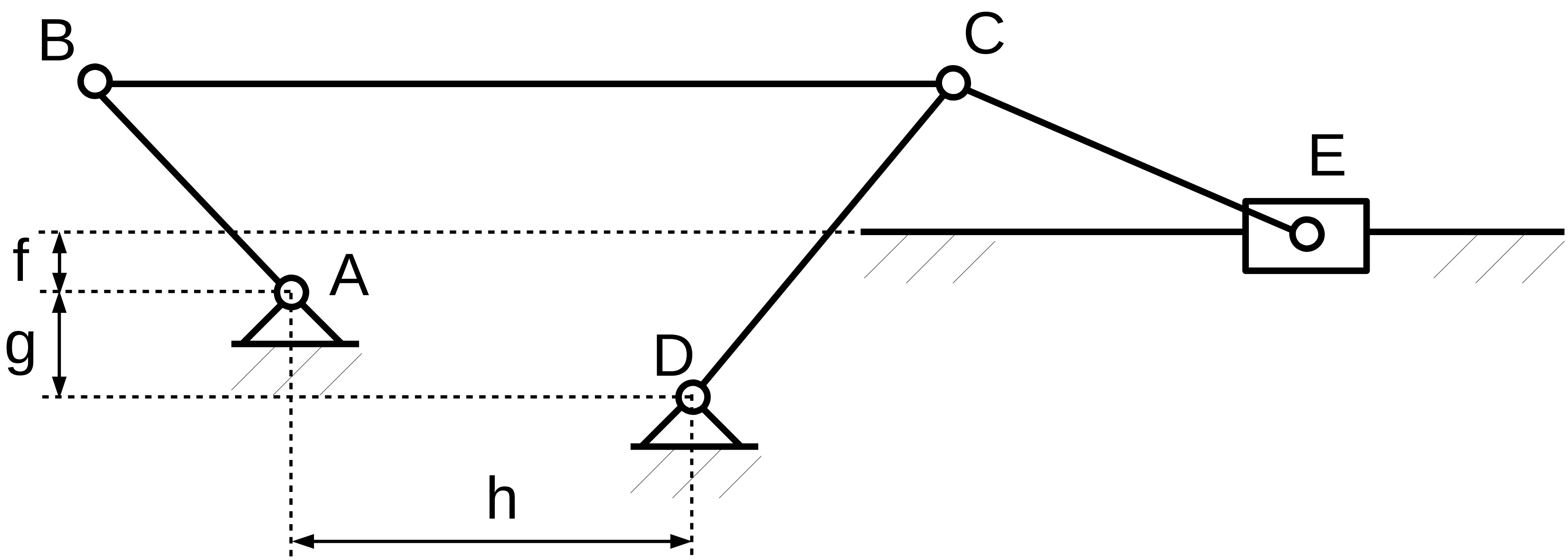
Analytical method – example:
slider-yoke

$$e - r\sin\varphi(t) = b(t)\cos\varphi_b(t)$$
$$f - r\cos\varphi(t) = b(t)\sin\varphi_b(t)$$

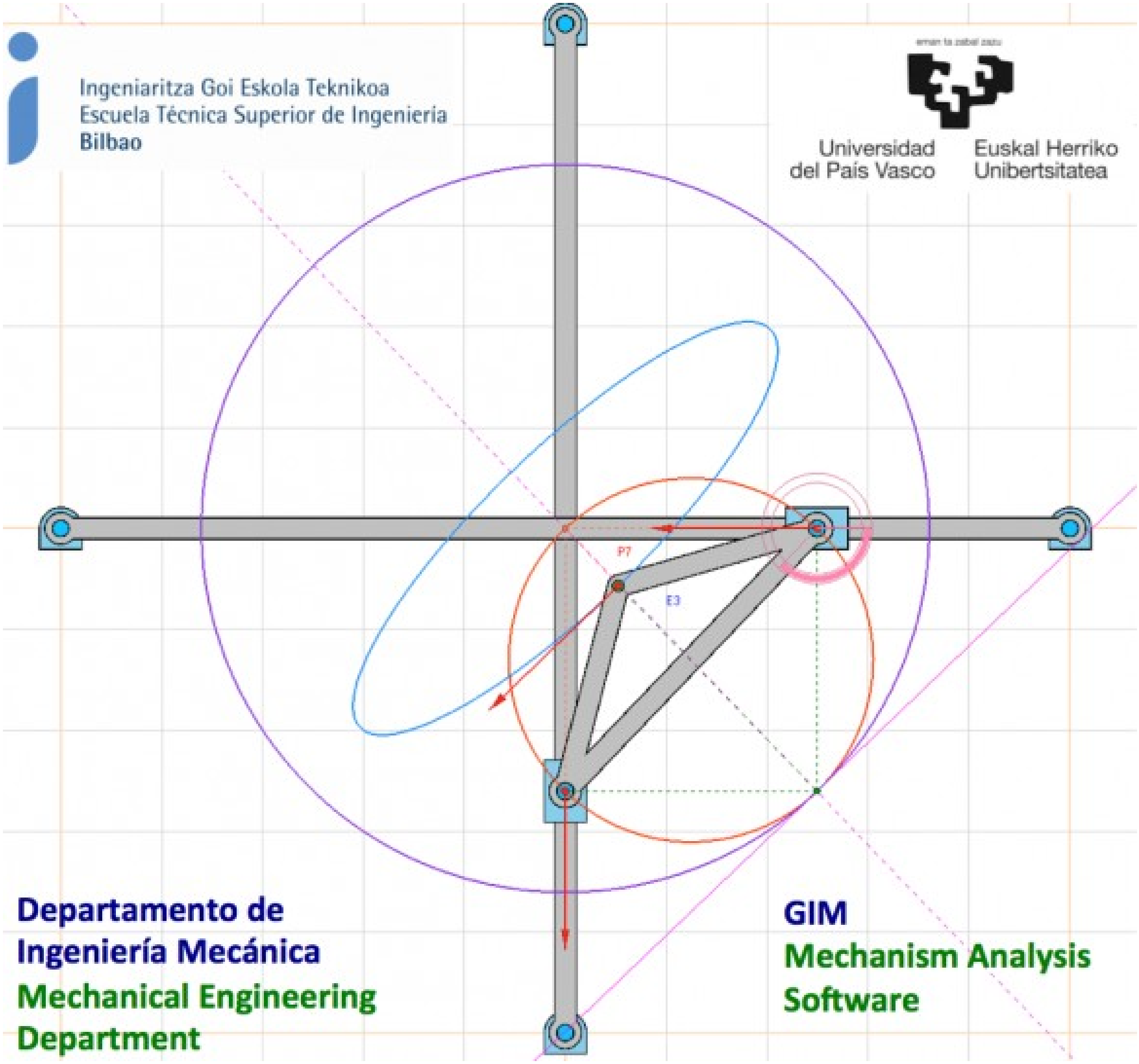
Analytical method – example:
slider-yoke

$$e - r\sin\varphi(t) = b(t)\cos\varphi_b(t)$$
$$f - r\cos\varphi(t) = b(t)\sin\varphi_b(t)$$

Analytical method – example



Software

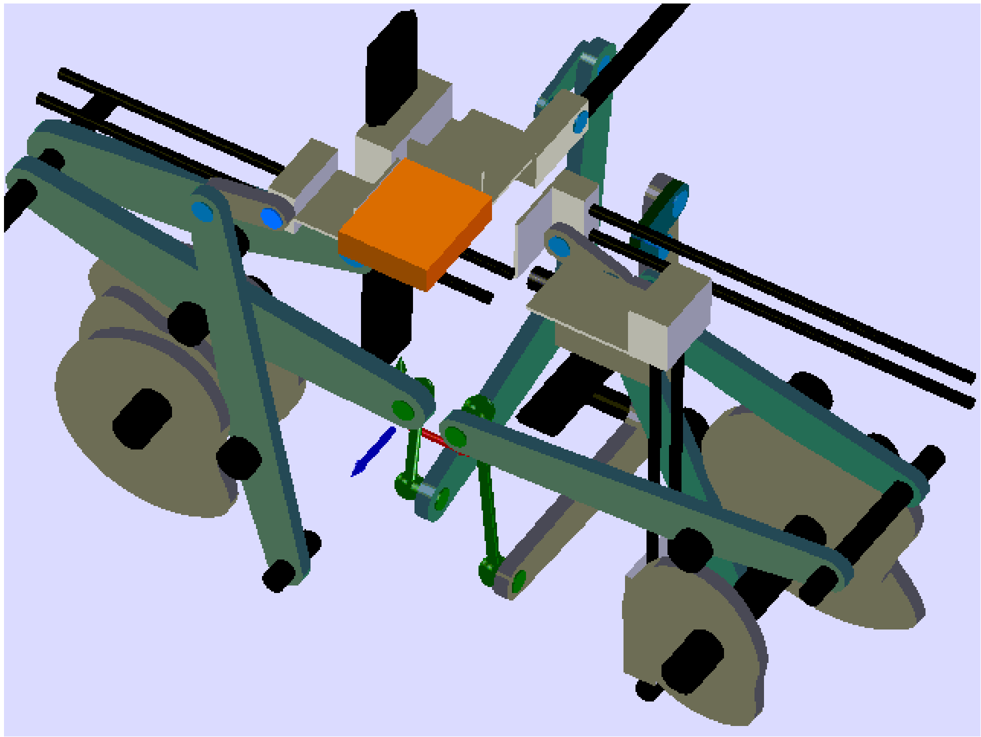


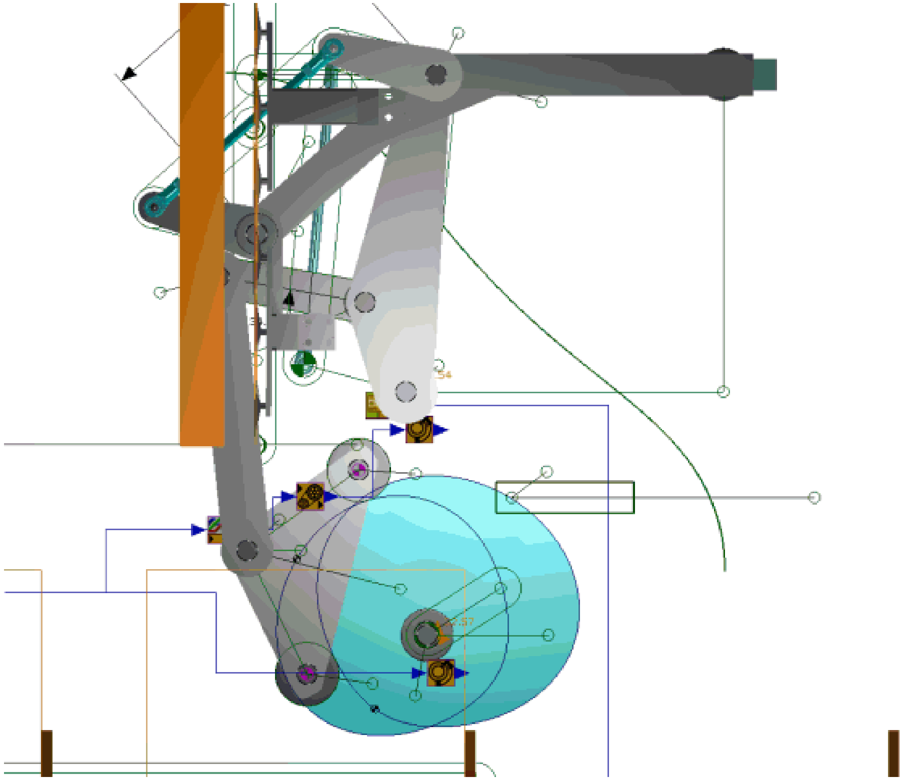
http://www.ehu.eus/compmech/software/

Cam-follower mechanisms

Cam-follower

Inspirations



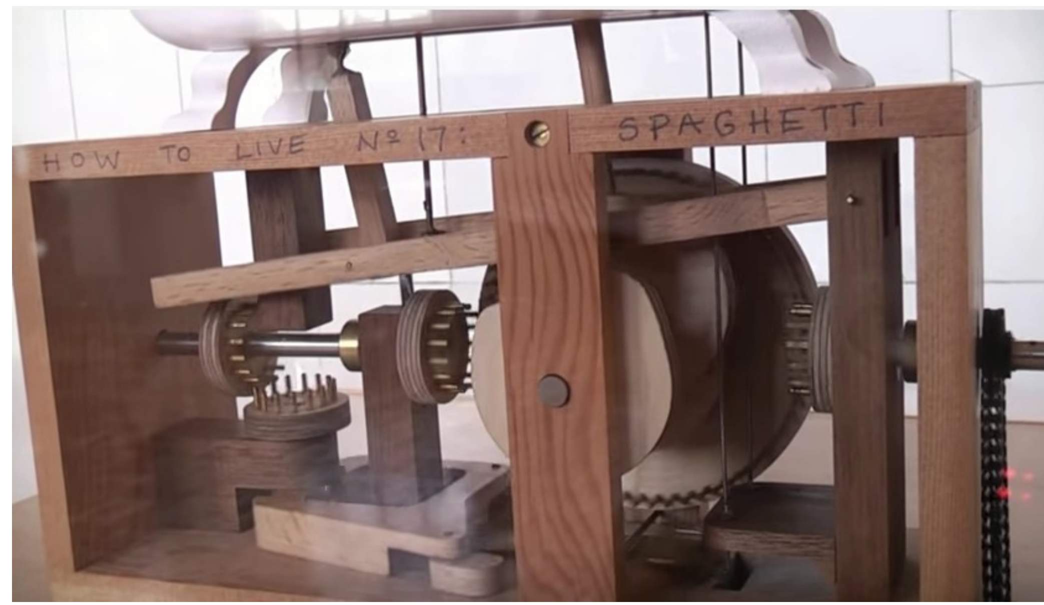


source: psmotion.com

Cam-follower

art inspiration

The "Draughtsman-Writer" automaton by Henri Maillardet
https://en.wikipedia.org/wiki/File:Maillardet
%27s_automaton_at_the_Franklin_Institute.webm



Mechanics Alive! Cabaret Mechanical
Theatre Automata Exhibition
https://www.youtube.com/watch?v=kv1CpJi60xQ

Cam-follower

Cam-follower mechanism – mechanism build of a cam and a follower (tappet) connected as a IV class kinematic pair.

Cam is rotating (sometimes is translating)

follower is reciprocating (sometimes is swinging/oscillating)

advantages

- simple to construction,
- simple to create,
- any dimensions,
- simple to create advanced motions.

disadvantages

- no adaptation possible.

Cam-follower

Classification

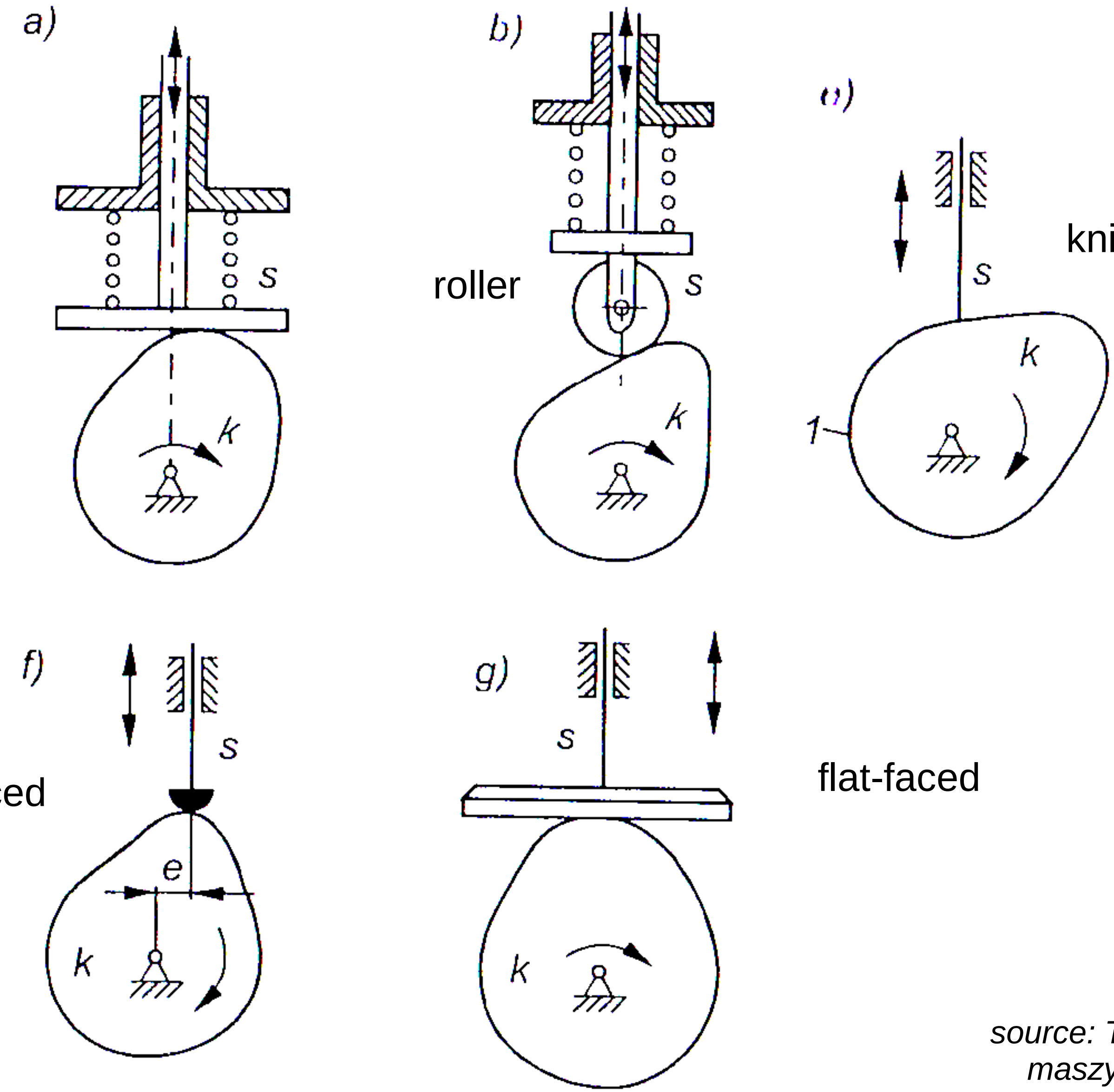
flat / spatial

with in-line (central) follower / with offset (eccentric) follower

closed with geometry / closed with force

Cam-follower

Followers

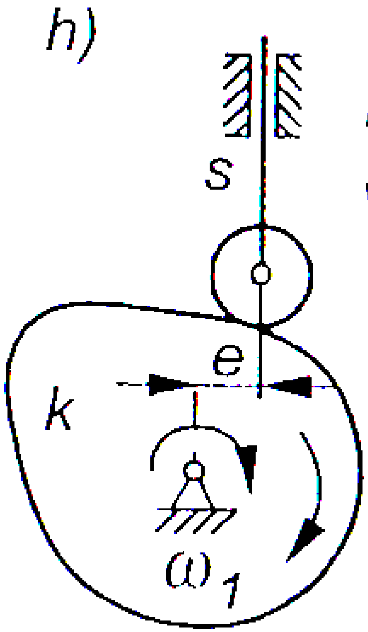


source: T. Kołacin, „Podstawy teorii maszyn i automatyki", OW PW

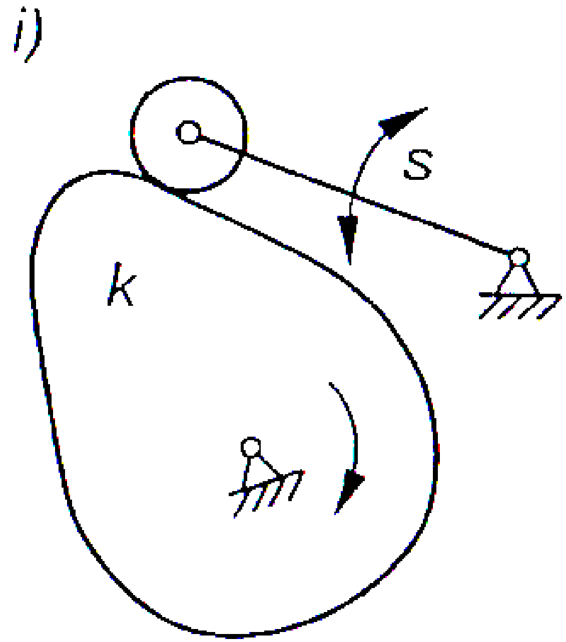
Cam-follower

Followers

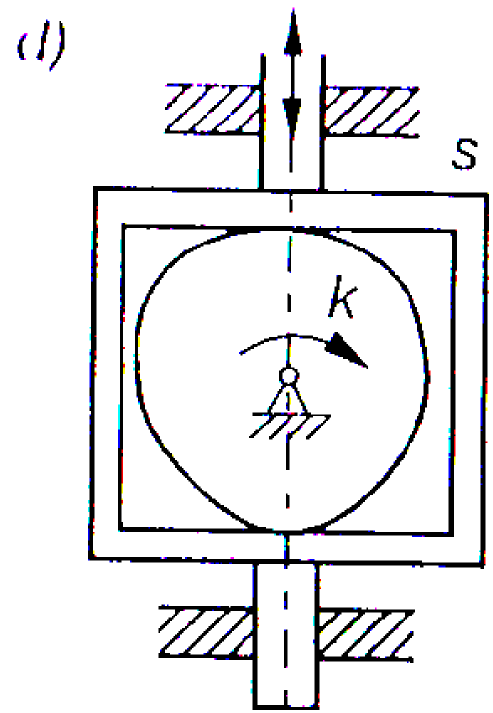
eccentric follower



swinging



flat frame

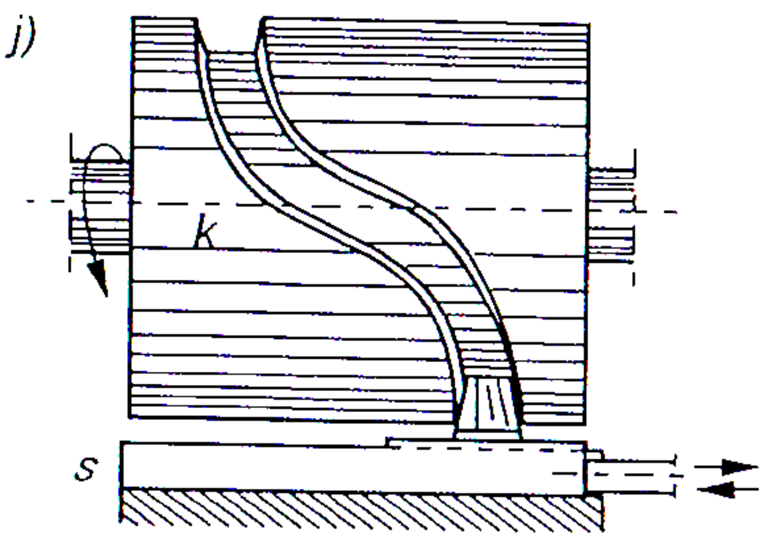


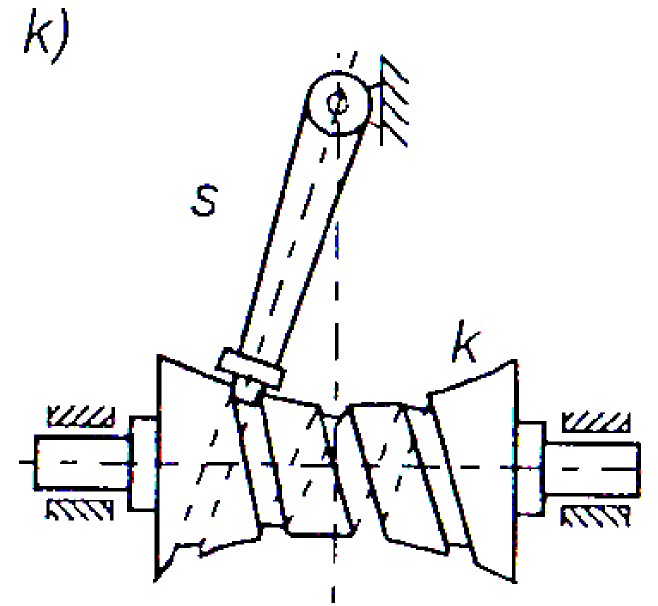
source: T. Kołacin, „Podstawy teorii maszyn i automatyki", OW PW

Cam-follower

examples

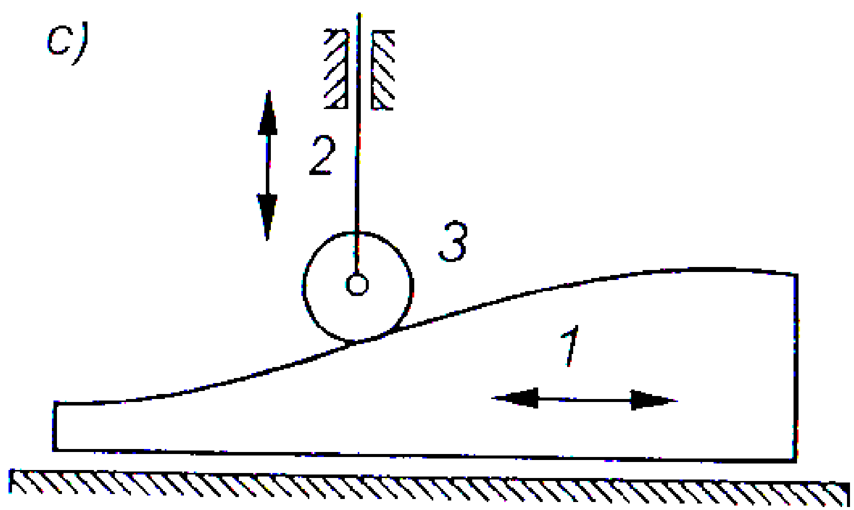
cylindrical cam





globoidal

translating cam



source: T. Kołacin, „Podstawy teorii maszyn i automatyki", OW PW

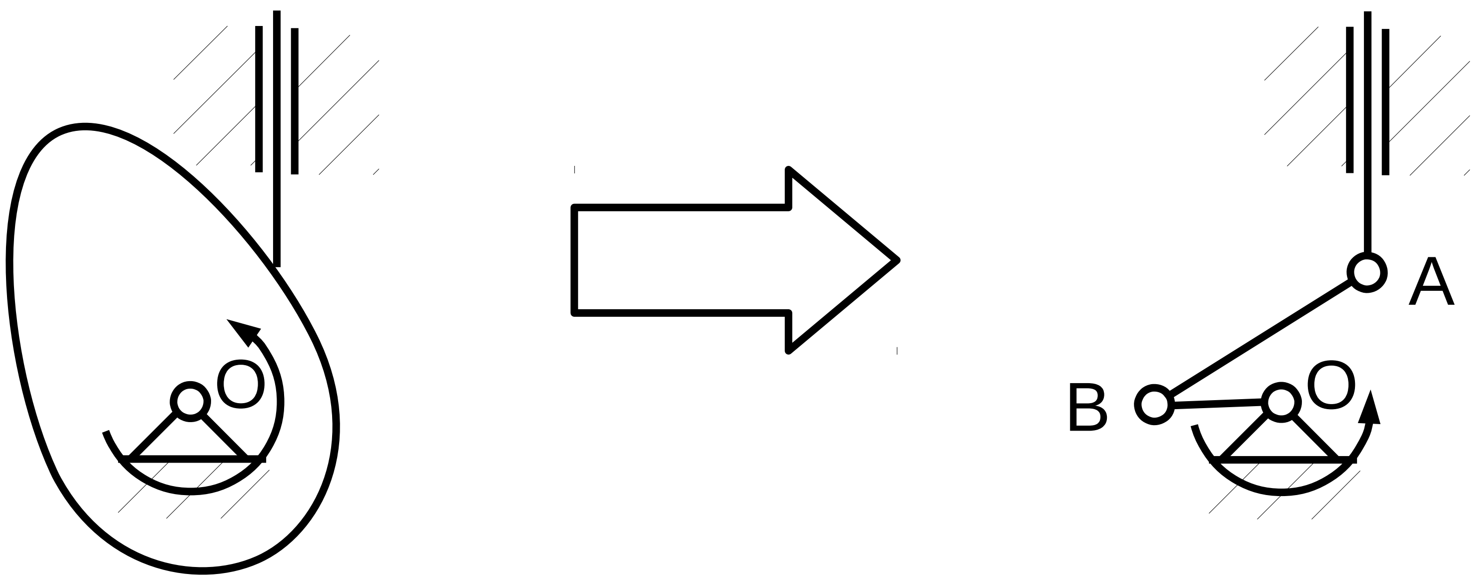
Analysis and synthesis of cam-follower mechanism

Analysis – calculation of displacement, velocity and acceleration functions for a follower motion with respect to a cam's rotations angle for arbitrary given geometry.

Synthesis – calculation of a cam geometry needed to obtain given displacement/velocity/acceleration functions. Limitations must be included, i.e. some maximum values, geometry limitations and jerk values (third derivative).

Analysis of a cam-follower mechanism

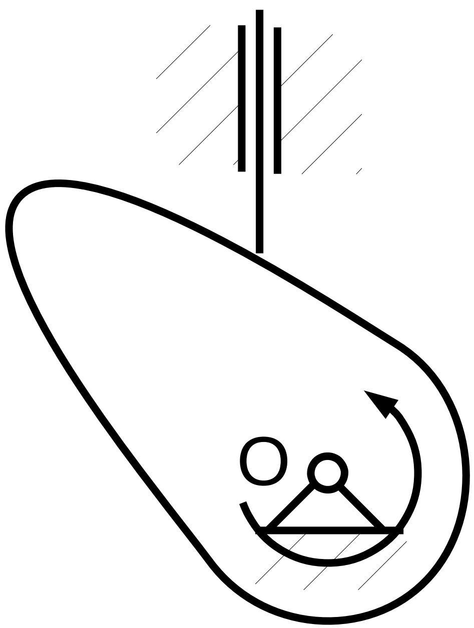
<u>Graphical method with IV. class kinematic pair substitution with V. class kinematic pairs.</u>

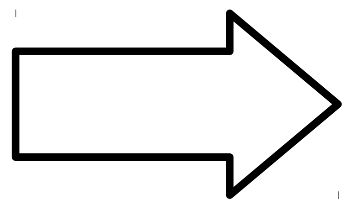


AB – radius of curvature in the contact point

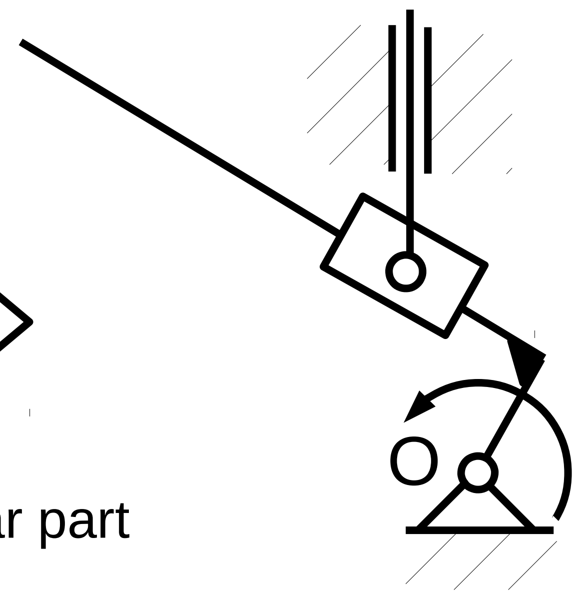
Analysis of a cam-follower mechanism

<u>Graphical method with IV. class kinematic pair substitution with V. class kinematic pairs.</u>



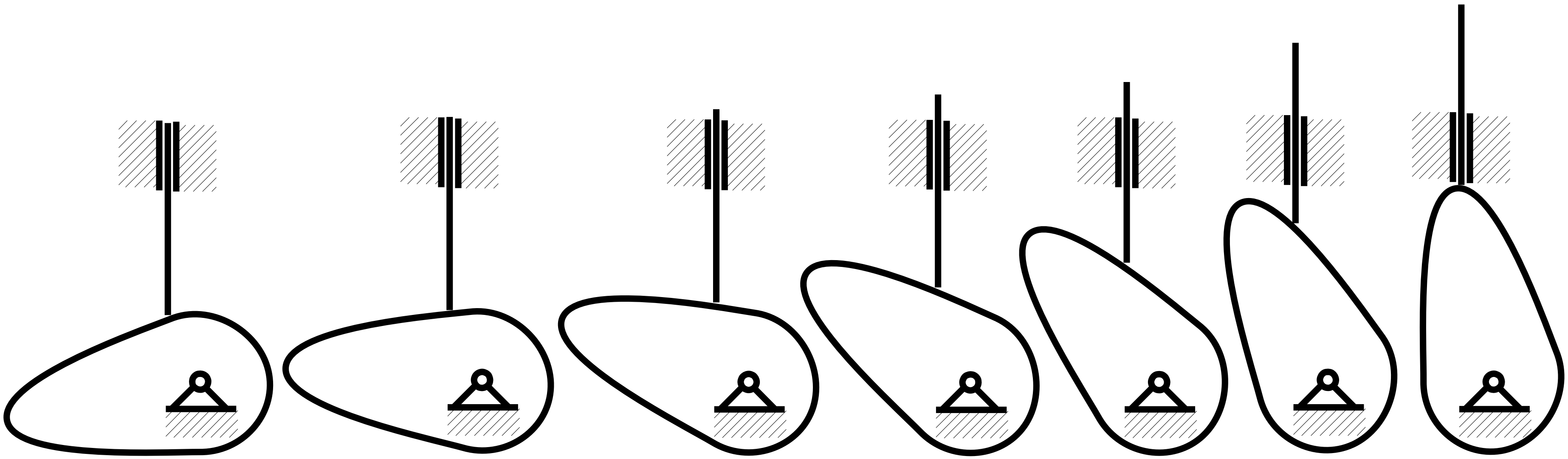


for a rectilinear part
of a cam



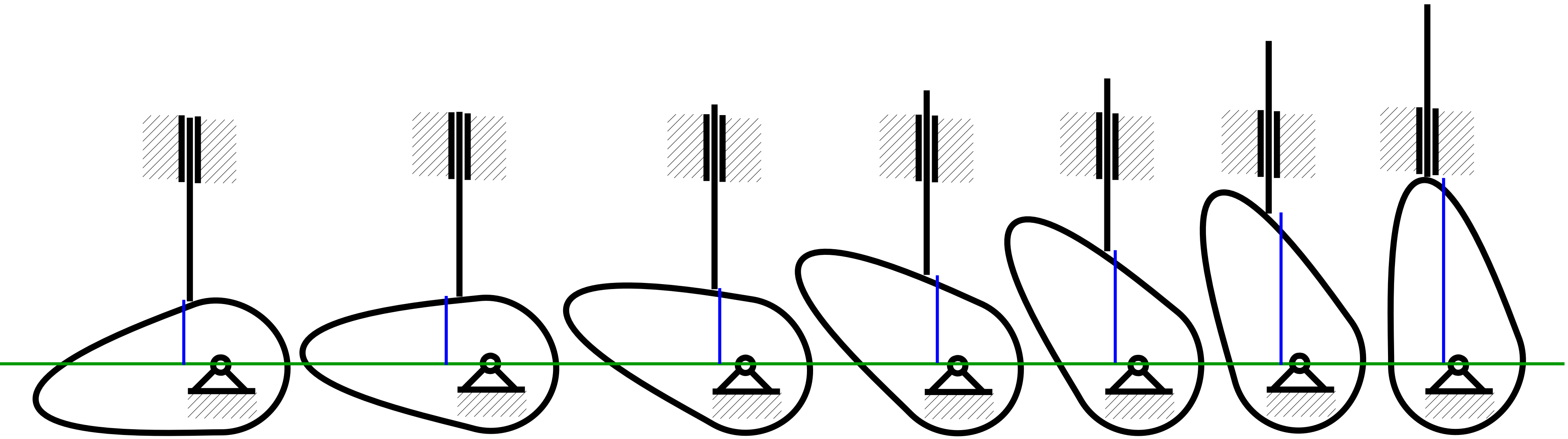
Analysis of a cam-follower mechanism

Graphical determination of a follower movement



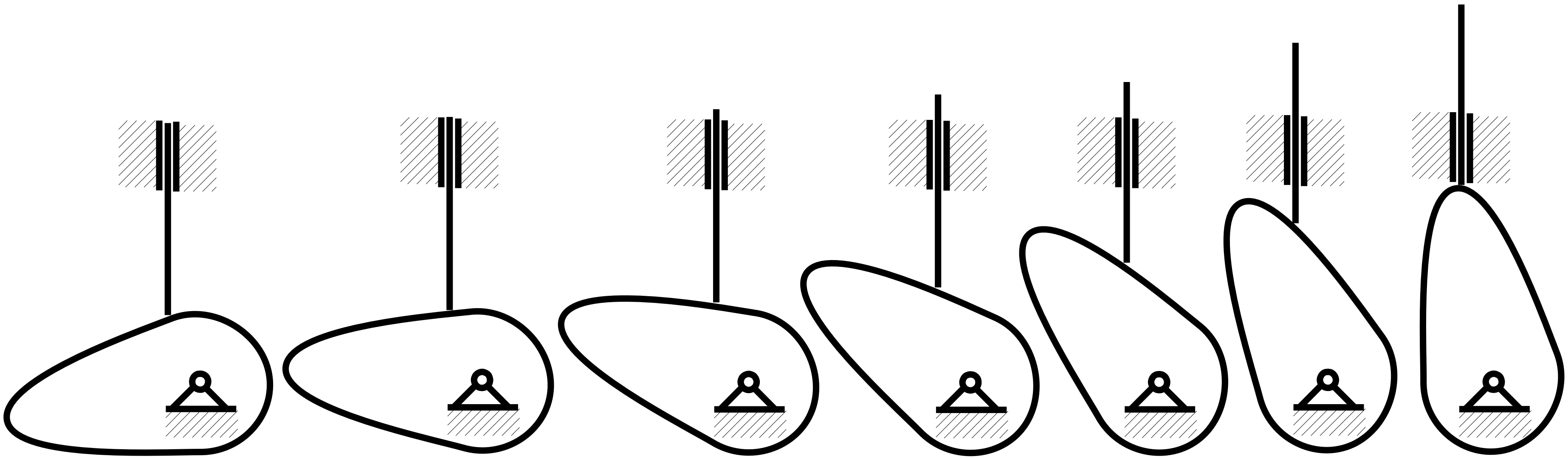
Analysis of a cam-follower mechanism

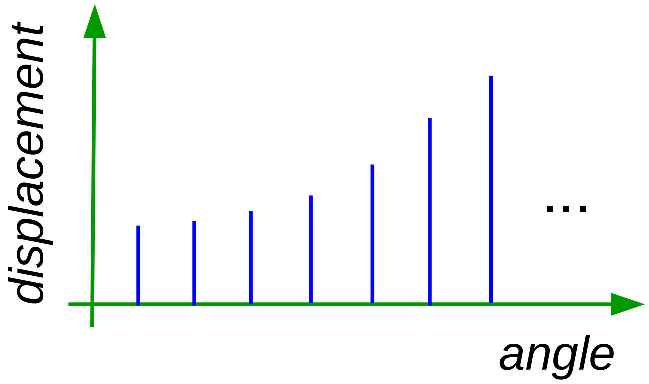
Graphical determination of a follower movement



Analysis of a cam-follower mechanism

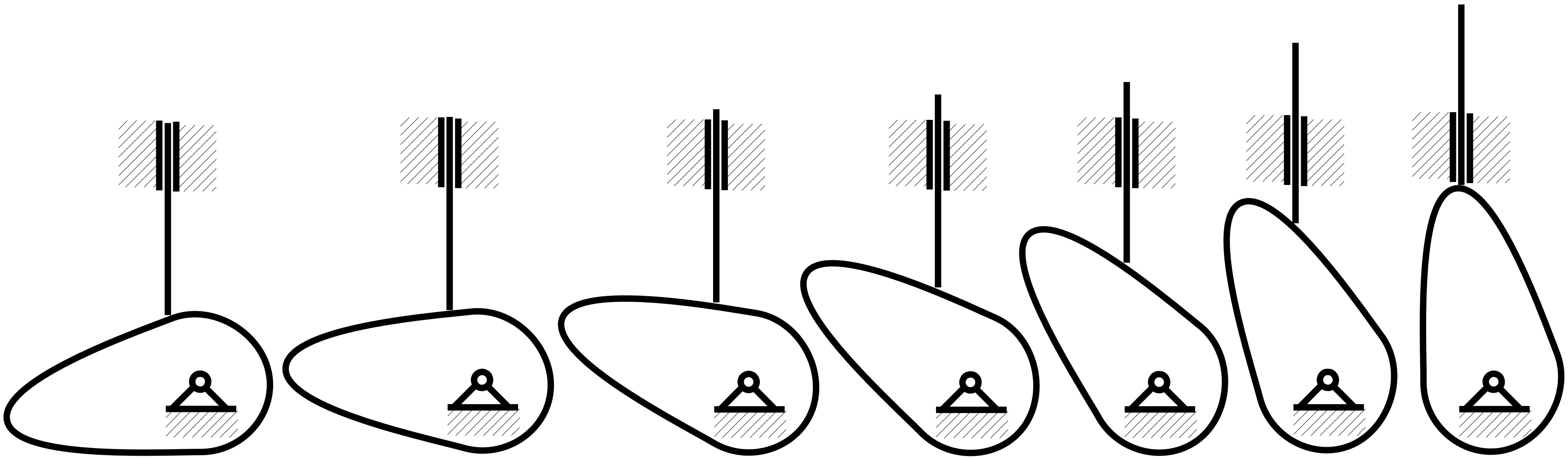
Graphical determination of a follower movement

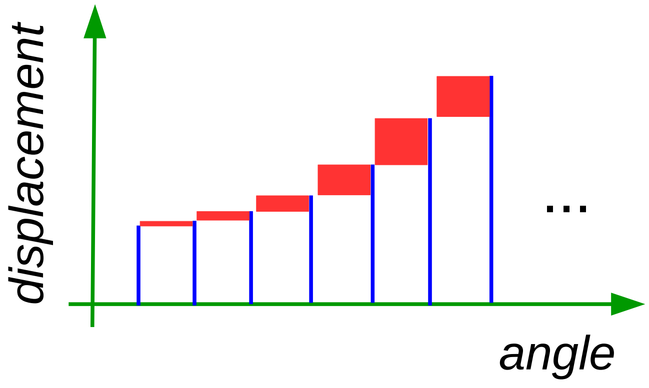




Analysis of a cam-follower mechanism

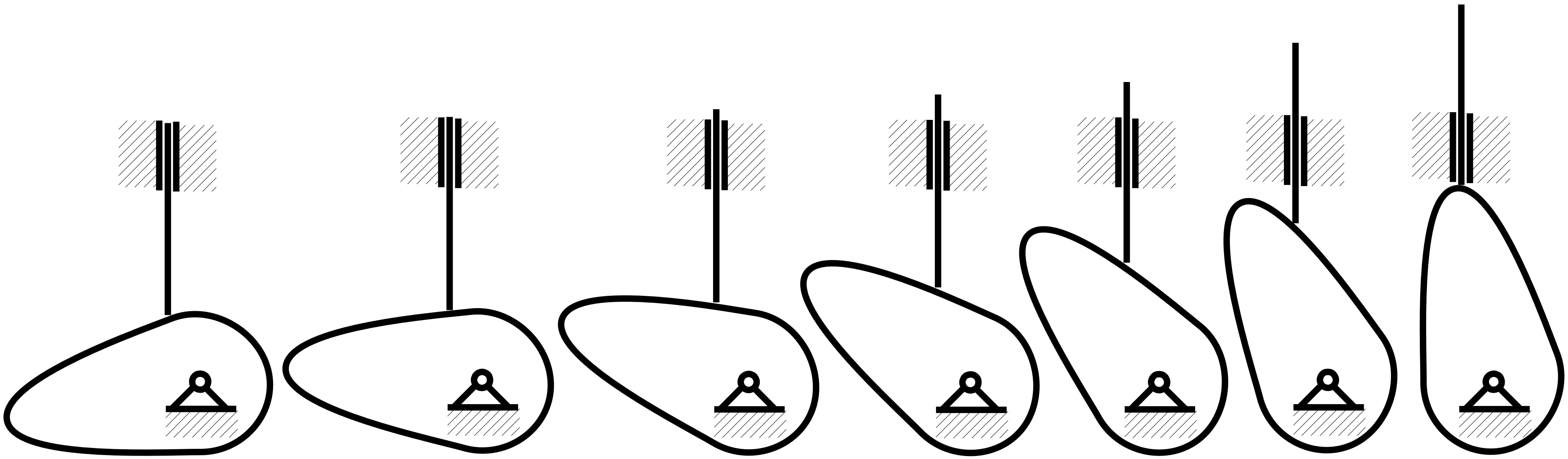
Graphical determination of a follower movement

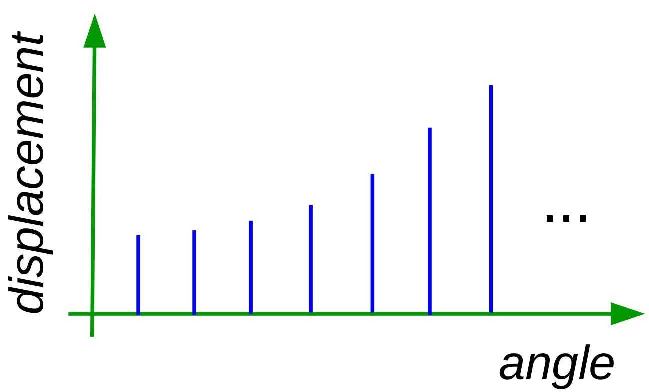


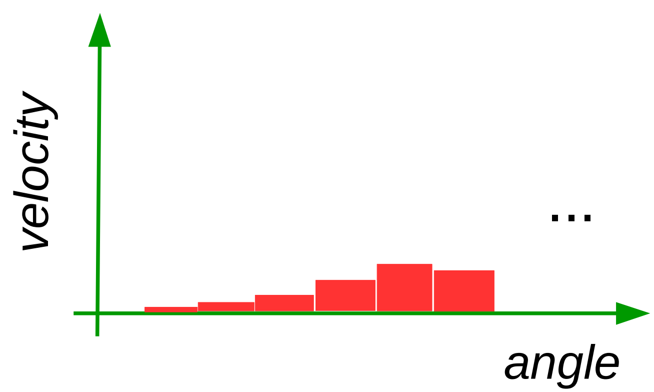


Analysis of a cam-follower mechanism

Graphical determination of a follower movement

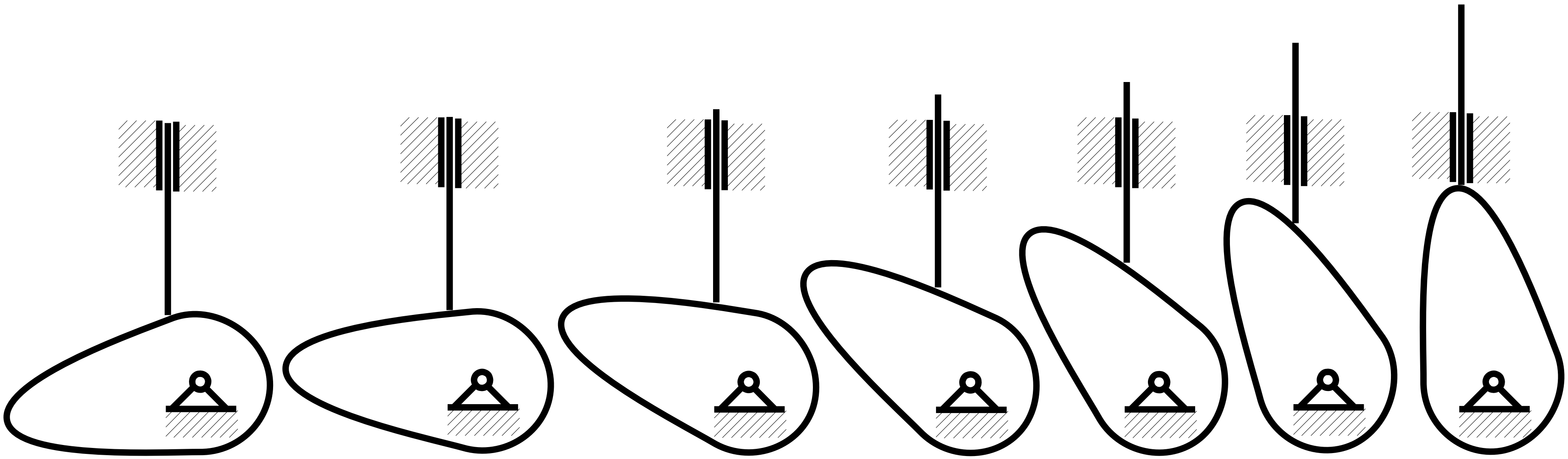


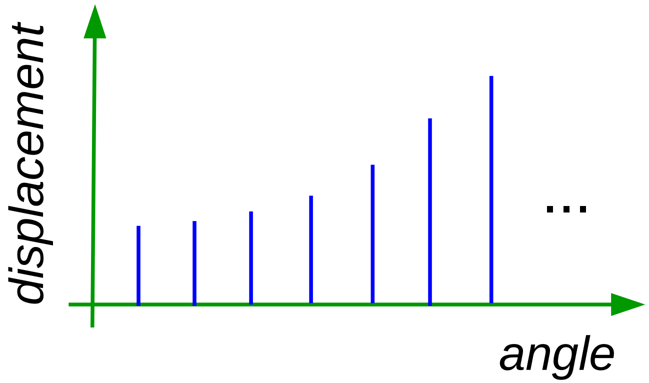


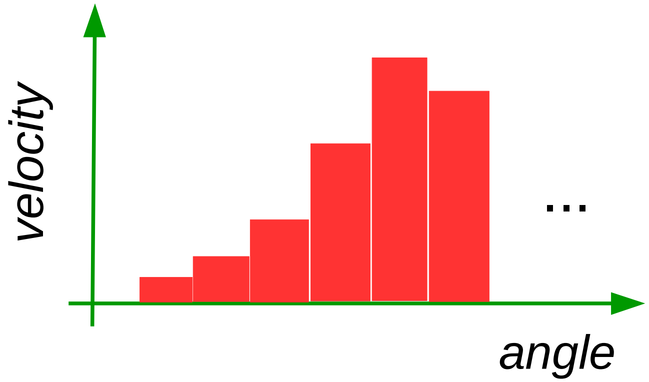


Analysis of a cam-follower mechanism

Graphical determination of a follower movement

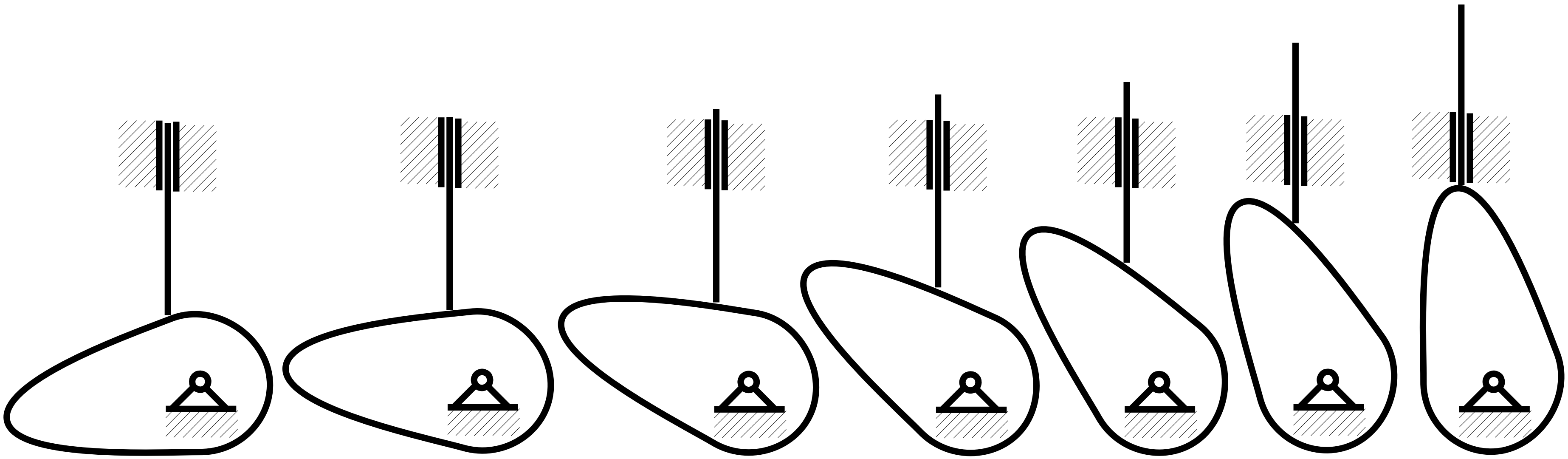


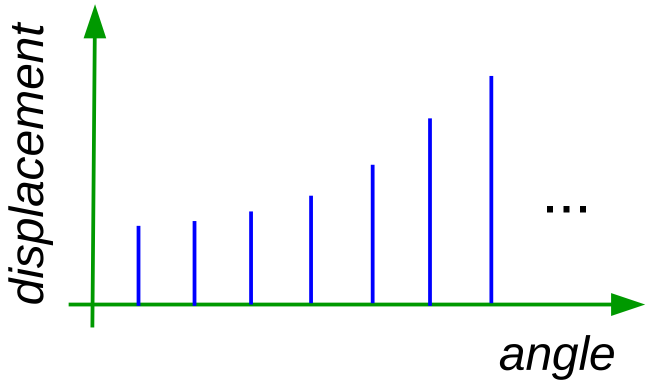


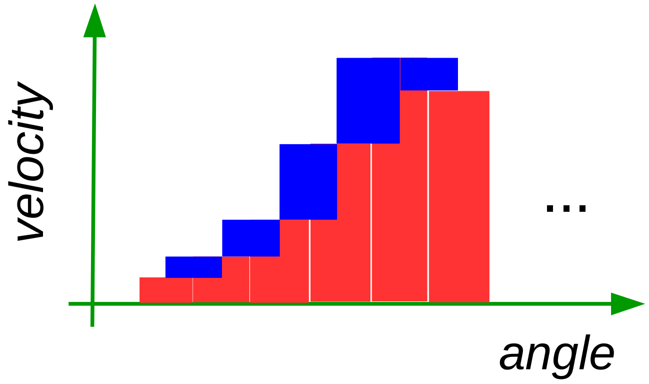


Analysis of a cam-follower mechanism

Graphical determination of a follower movement

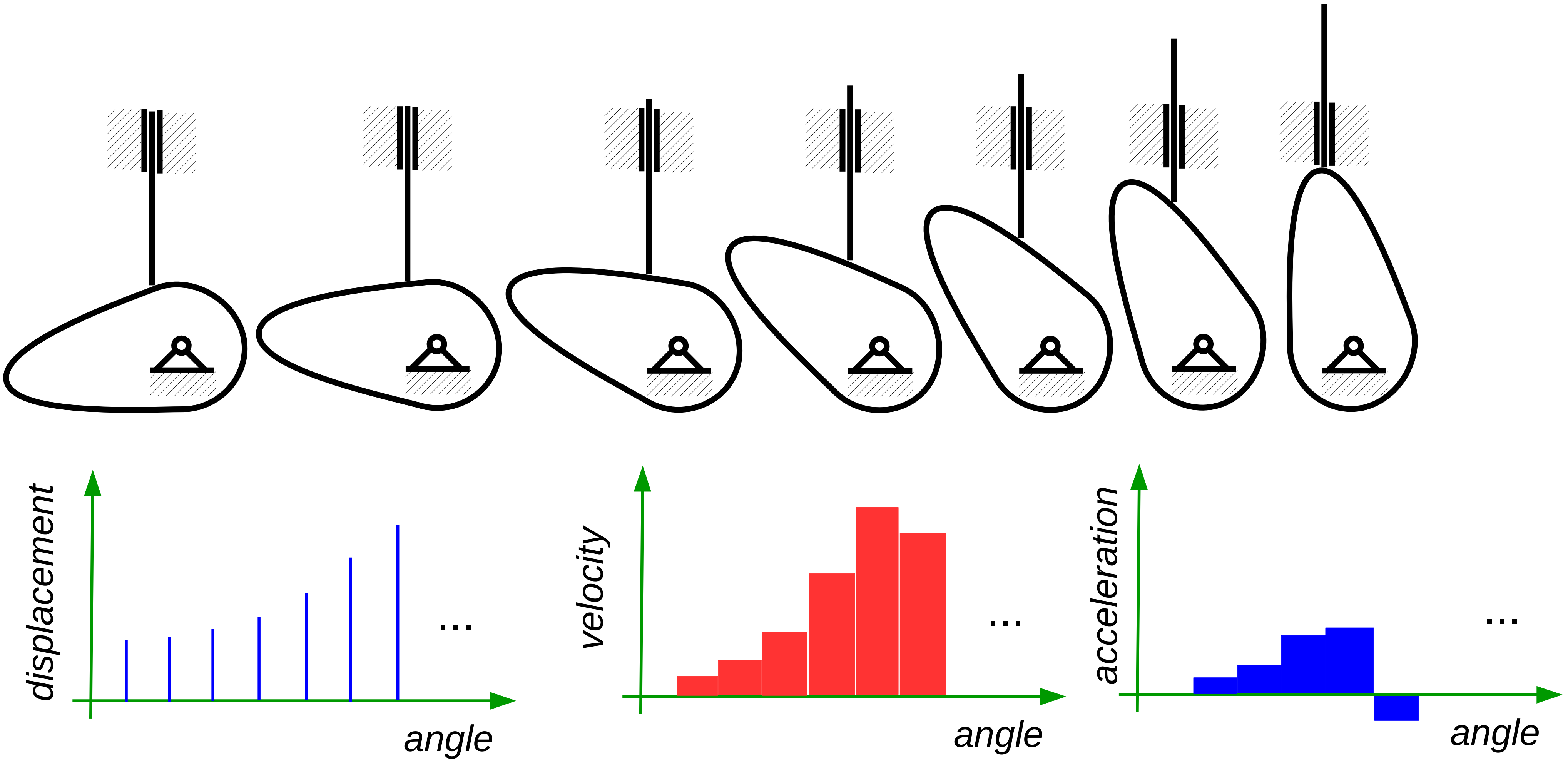






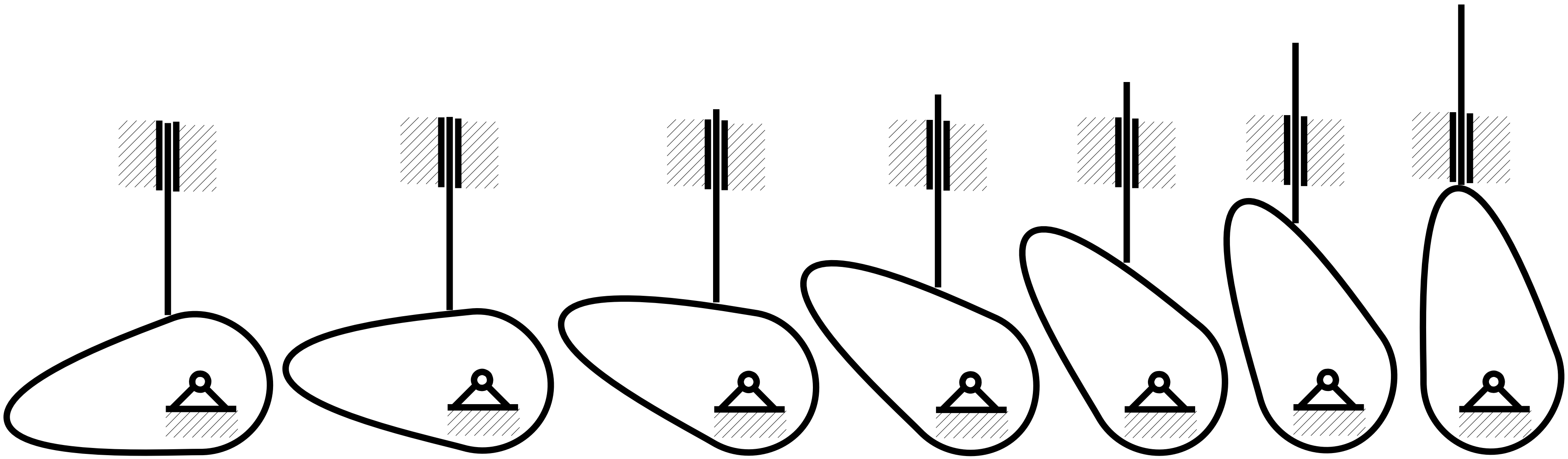
Analysis of a cam-follower mechanism

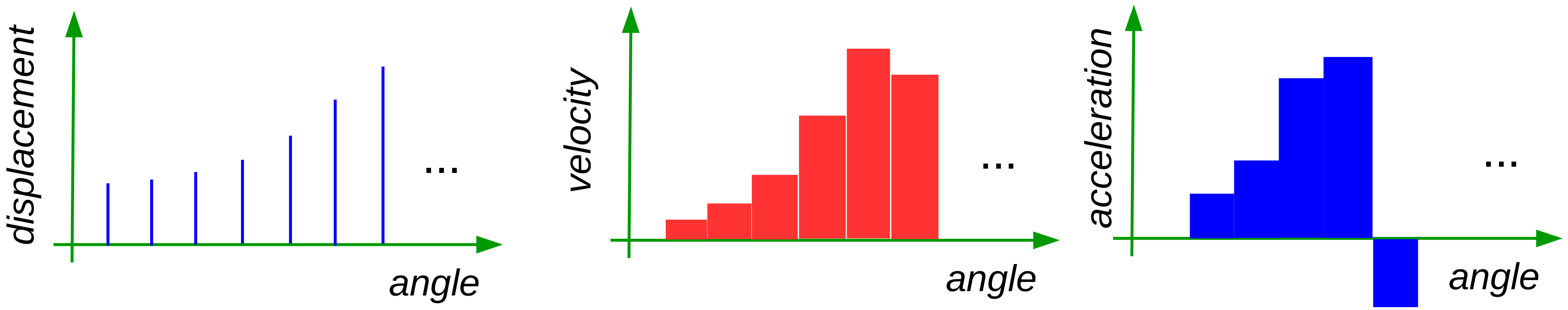
Graphical determination of a follower movement



Analysis of a cam-follower mechanism

Graphical determination of a follower movement





Analysis of a cam-follower mechanism

Analytical method - example