Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control

Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

Lecture 3

Accelerations in planar mechanisms.

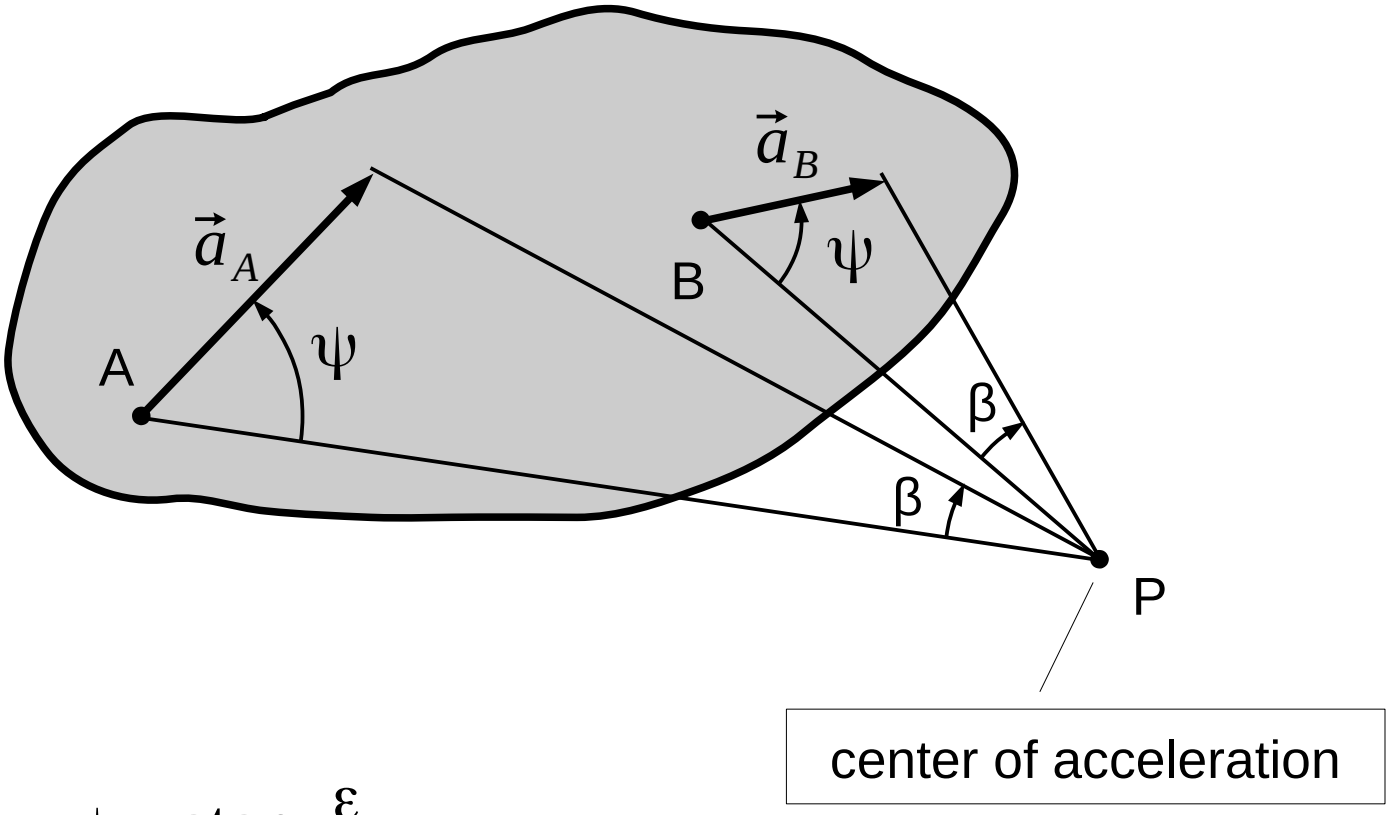
Methods of velocities and accelerations determination

Graphical methods

- velocity projection method,
- instantaneous center of rotation method,
- instantaneous center of acceleration method,
- method of rotated velocities,
- velocity decomposition method,
- acceleration decomposition method,
- velocity scheme (diagram) method,
- accelerations scheme (diagram) method.

Analytical method

Instantaneous center of acceleration



$$\psi = \operatorname{atan} \frac{\varepsilon}{\omega^2}$$

ε - angular acceleration

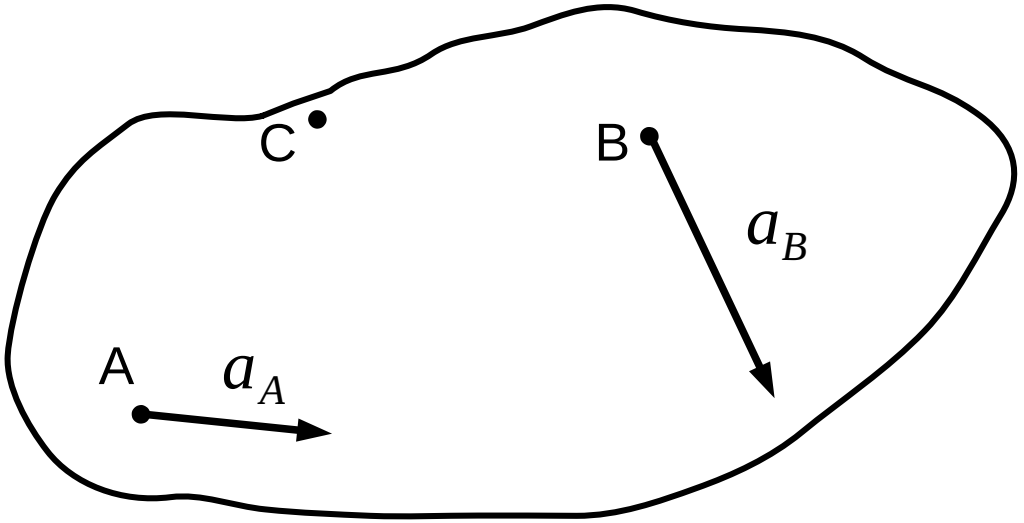
ω - angular velocity

Instantaneous center of acceleration method

Example

Given: $\bar{a}_A$ and $\bar{a}_B$

Searched: $\bar{a}_C$



Instantaneous center of acceleration method

Example

Given: a_A and a_B

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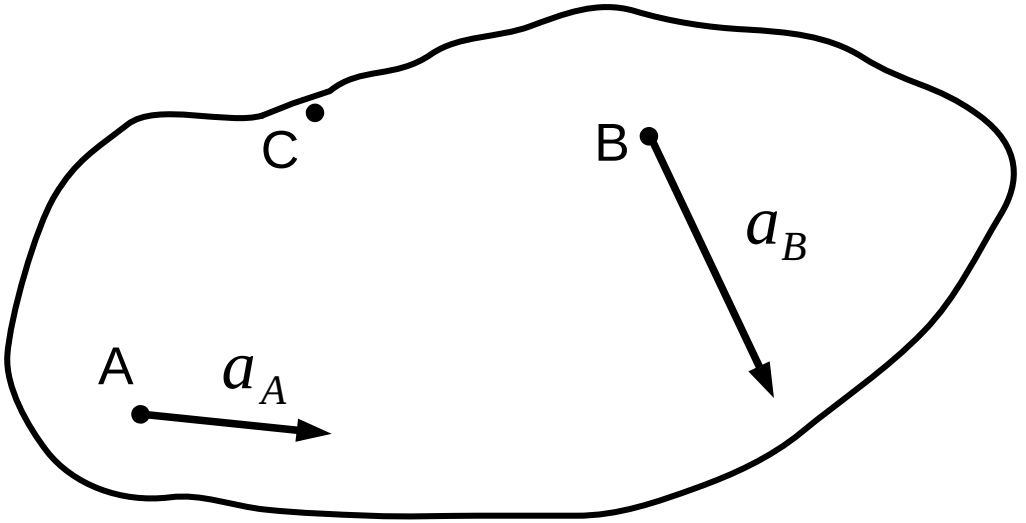
1. STEP:
construction of ψ



Instantaneous center of acceleration method

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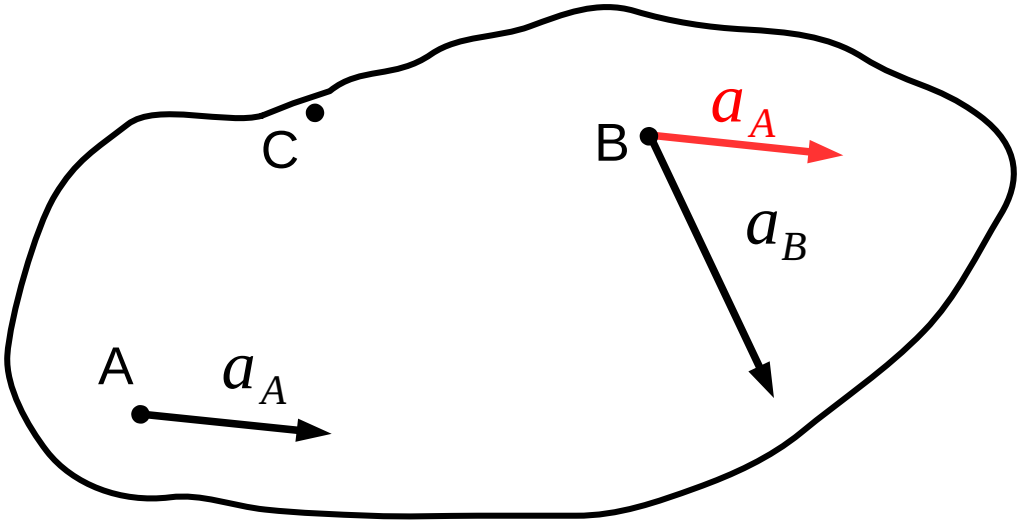

Instantaneous center of acceleration method

Example

$$a_B = a_A + a_{BA}$$

Given: a_A and a_B

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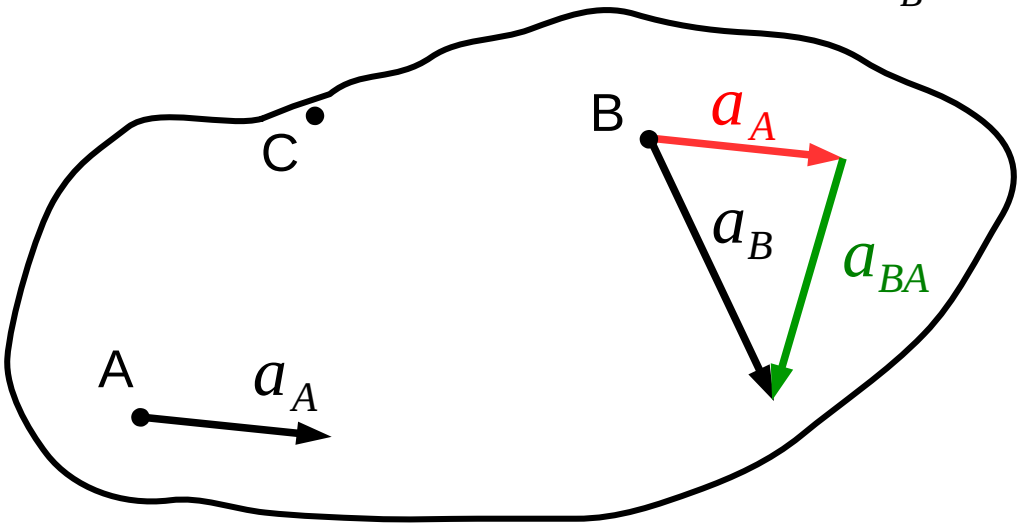

Instantaneous center of acceleration method

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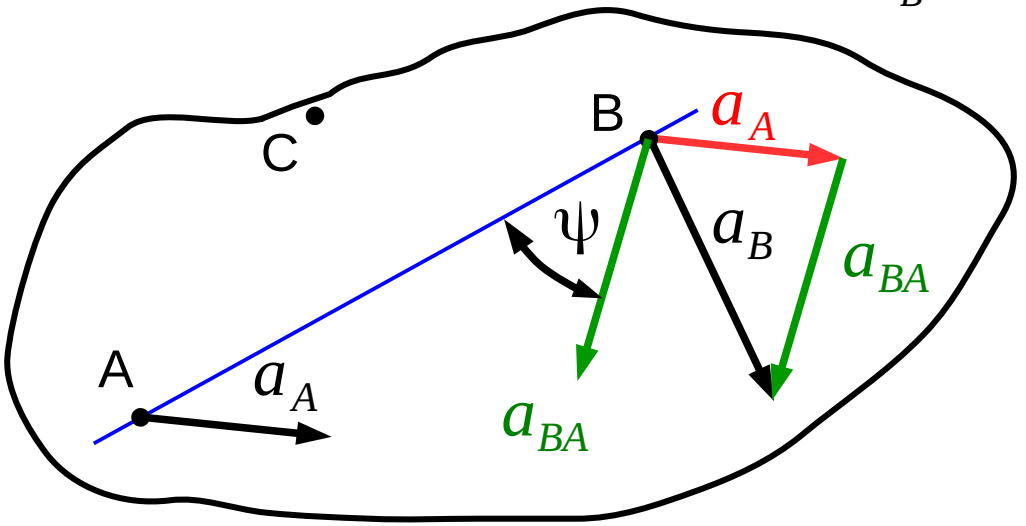

Instantaneous center of acceleration method

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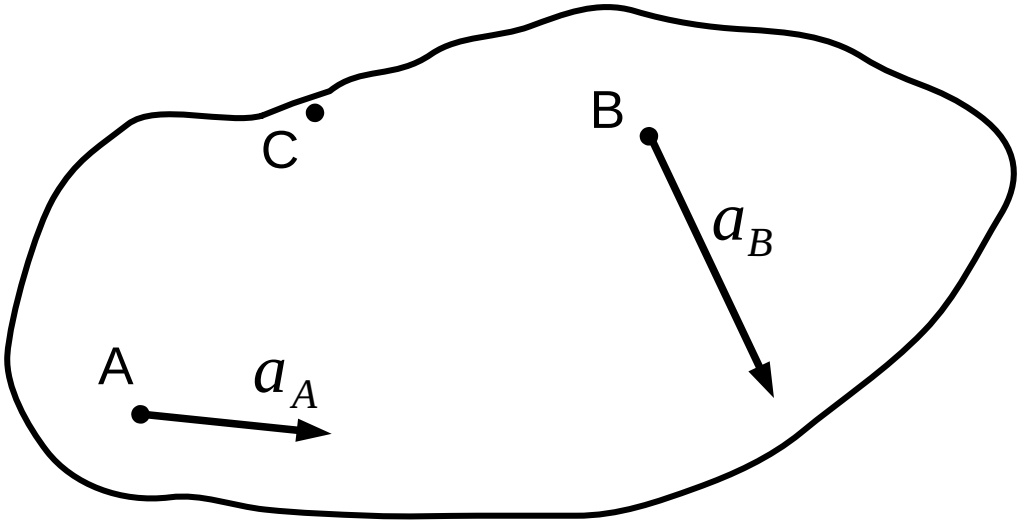

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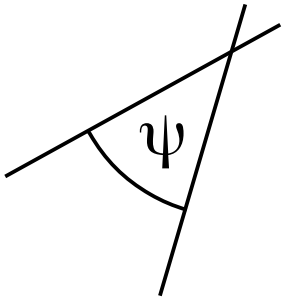

Instantaneous center of acceleration method

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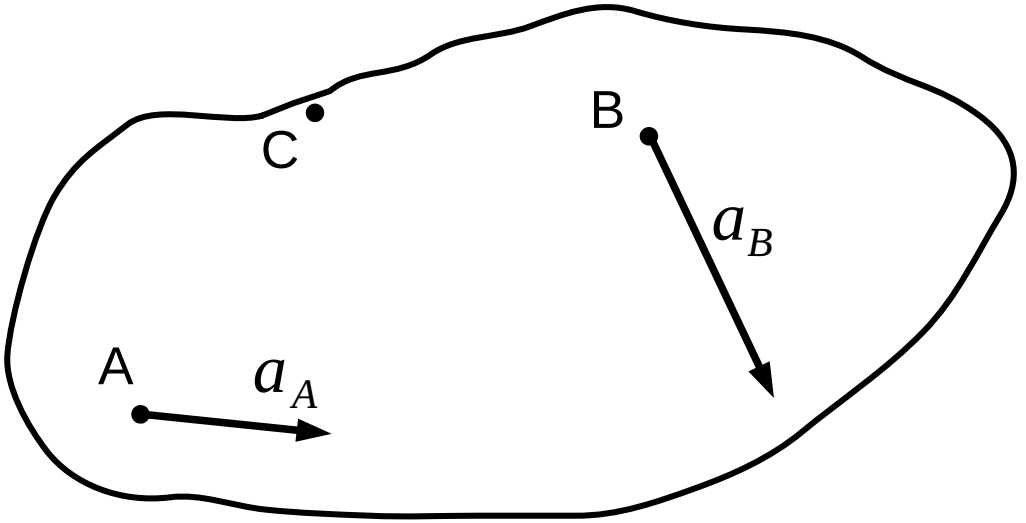

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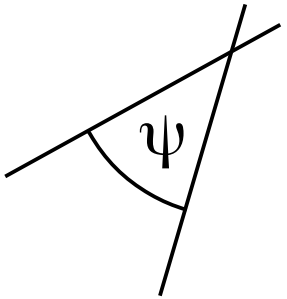
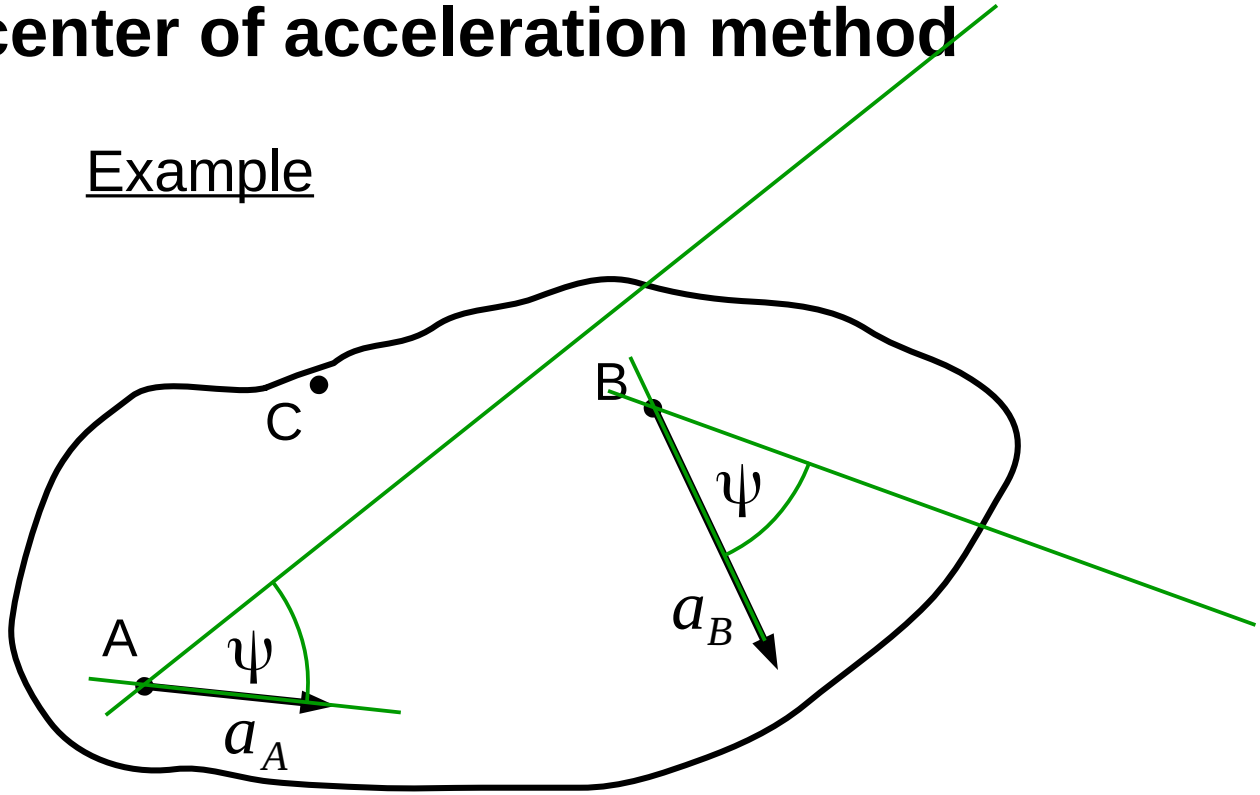


2. STEP: find out the center of acceleration

Instantaneous center of acceleration method

Example



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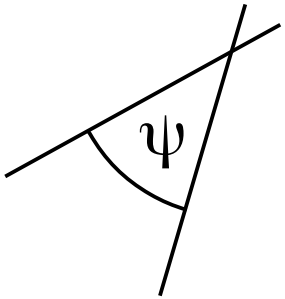

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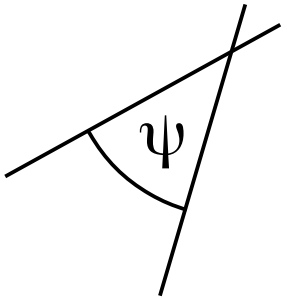

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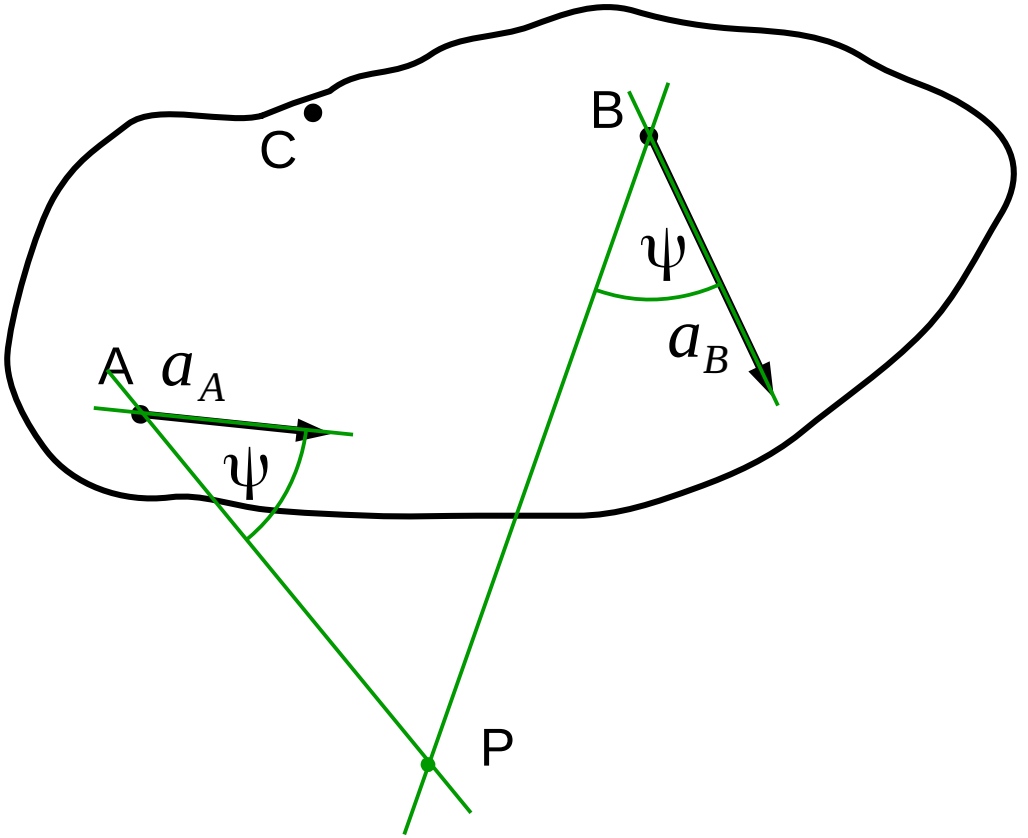

Instantaneous center of acceleration method

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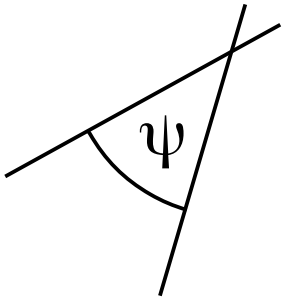

2. STEP: find out the center of acceleration

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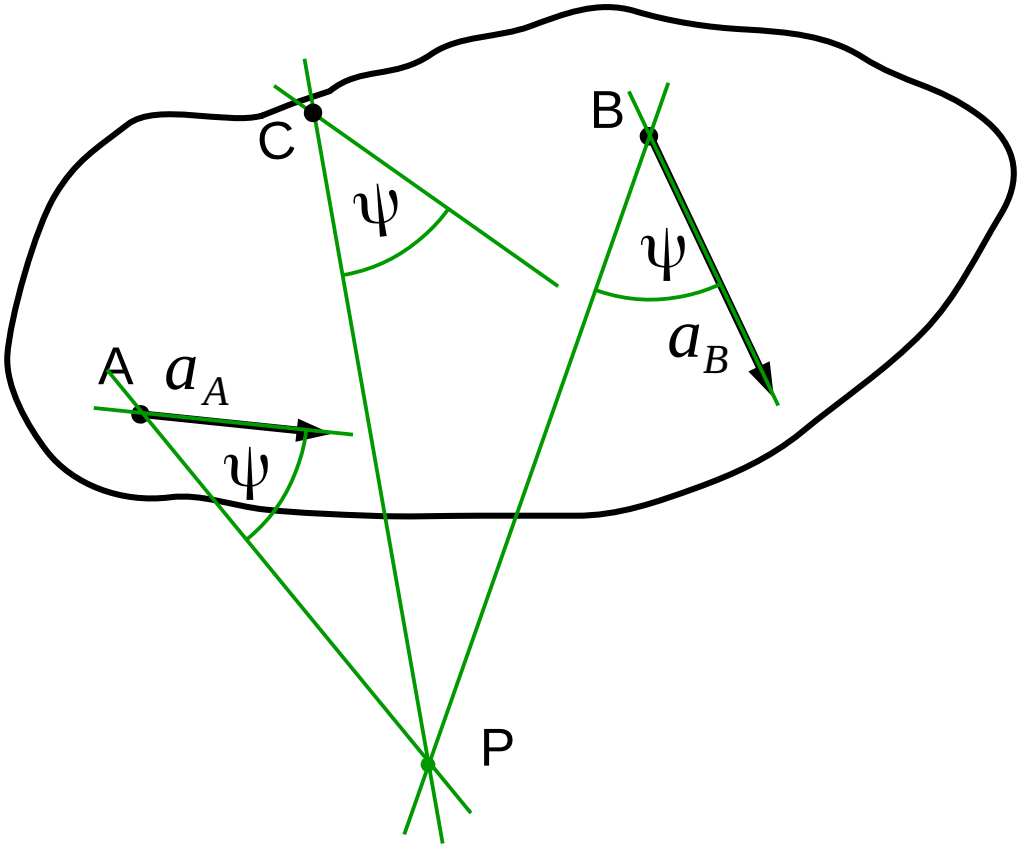

Instantaneous center of acceleration method

Example

Given: $\bar{a}_A$ and $\bar{a}_B$

Searched: $\bar{a}_C$

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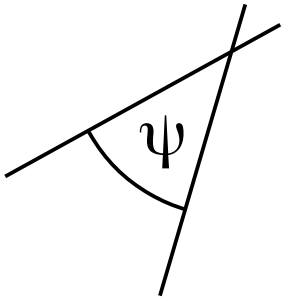

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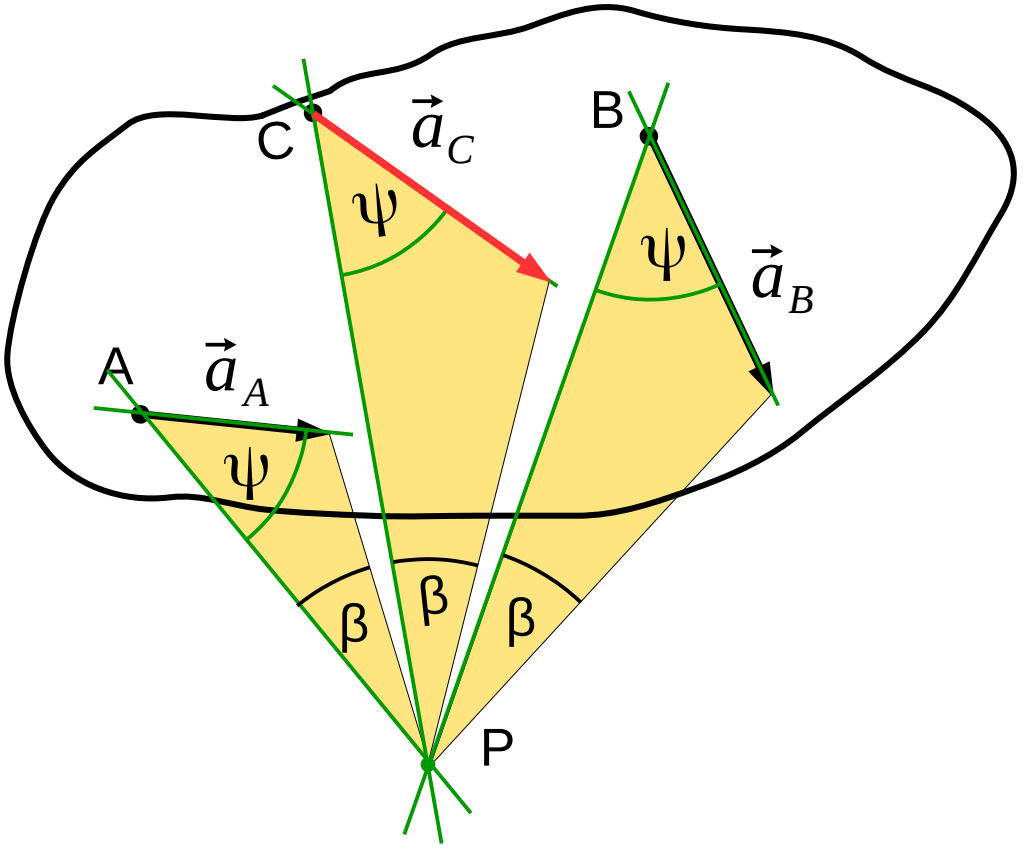
3. STEP: find out $\bar{a}_C$

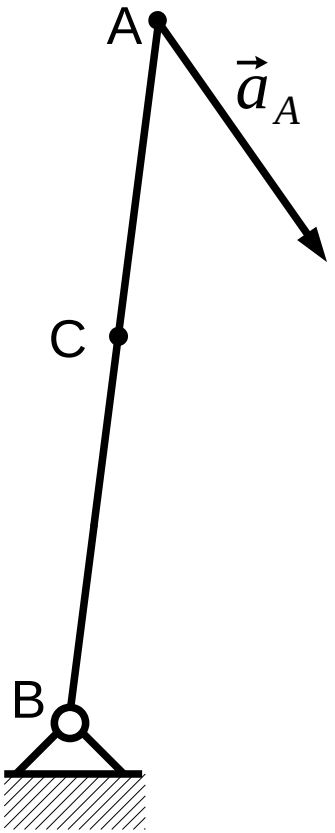


Instantaneous center of acceleration method

Example 2

Given: $\bar{a}_A$

Searched: $\bar{a}_C$



Instantaneous center of acceleration method

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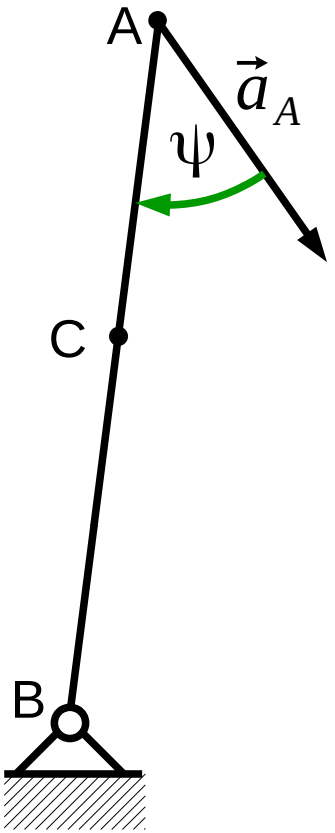

Instantaneous center of acceleration method

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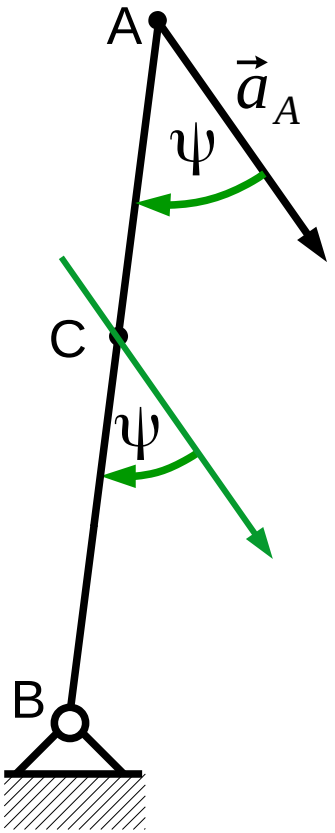

Instantaneous center of acceleration method

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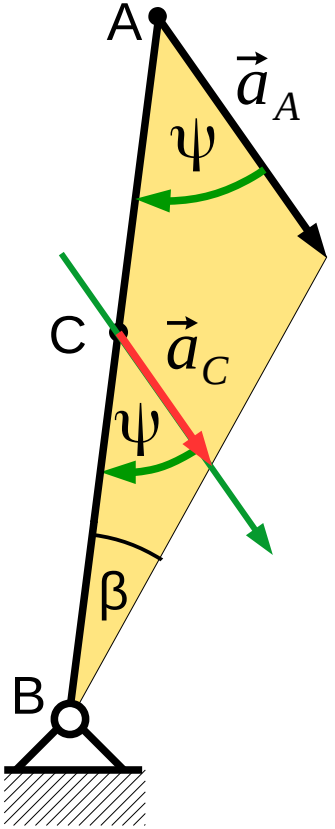
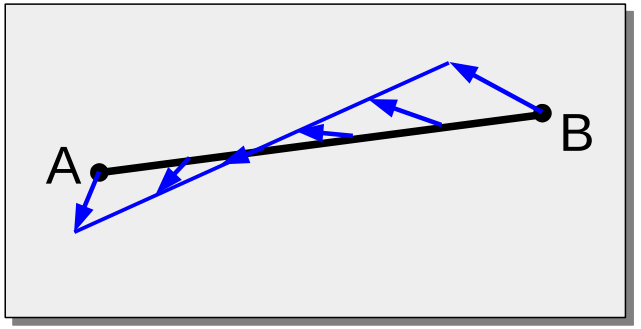


Acceleration decomposition method

Example



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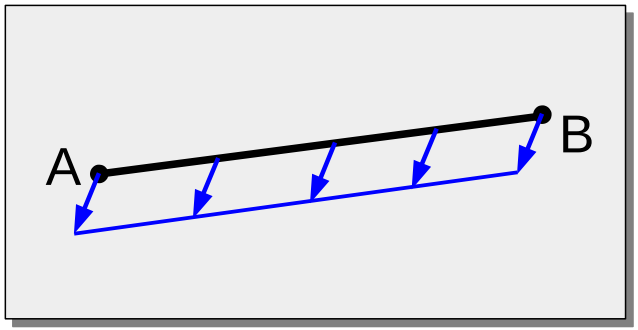

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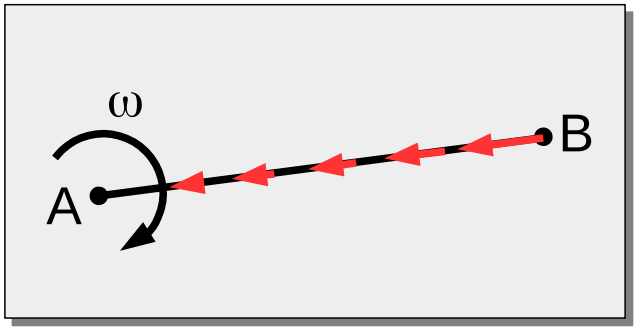

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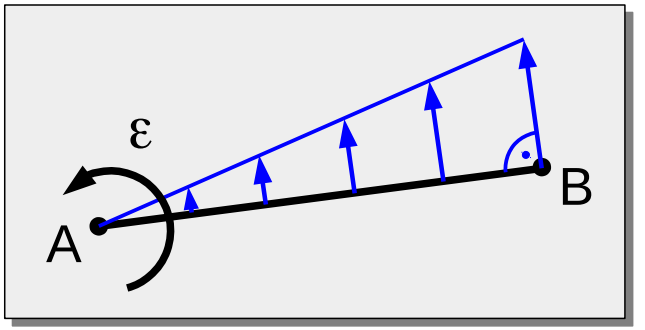

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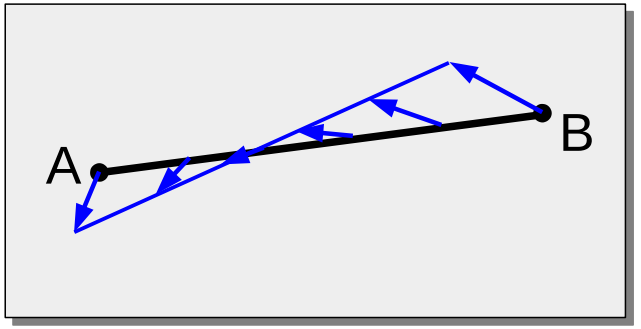

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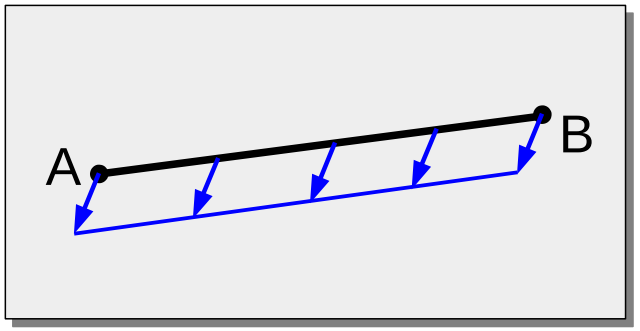

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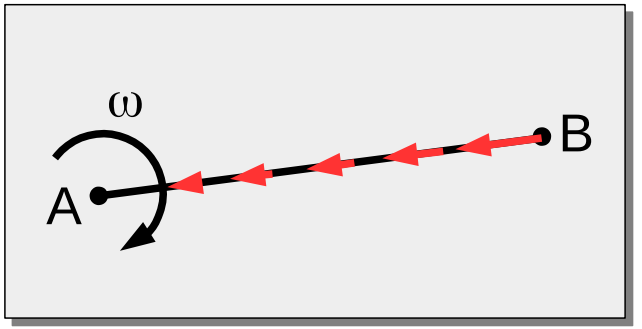

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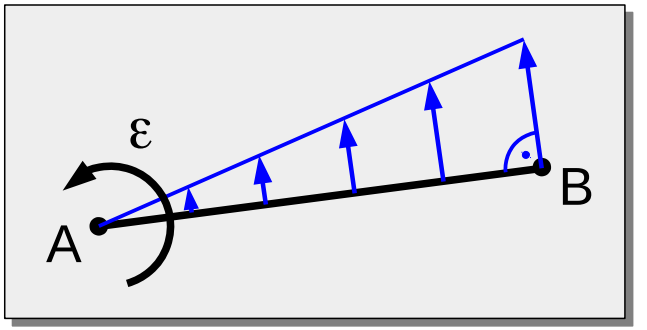


$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}^{\,n}_{BA} + \vec{a}^{\,t}_{BA}$$

absolute acceleration of point B

absolute acceleration of point A

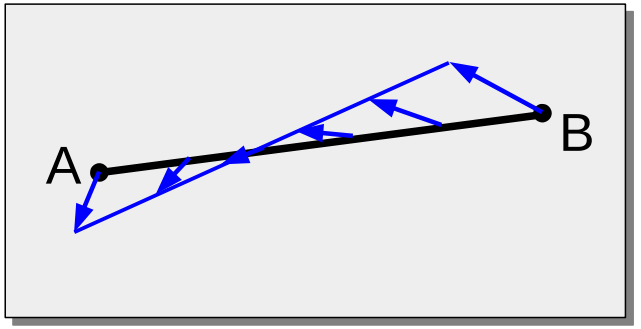
Angular acceleration of point B in rotation around point A.

Centripetal acceleration (normal)

Rotary acceleration (tangential)

Acceleration decomposition method

Example



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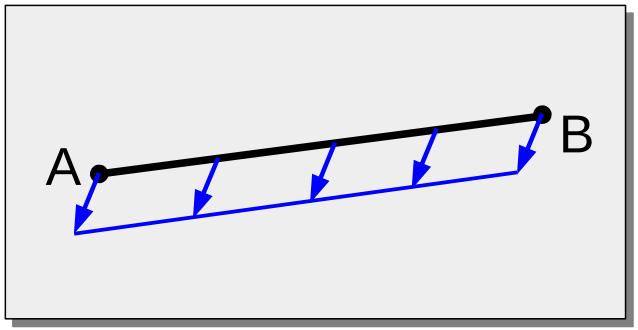

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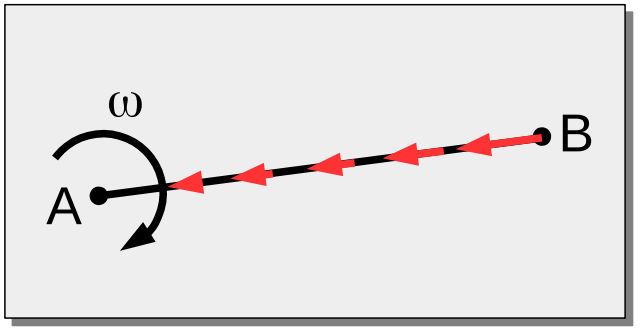

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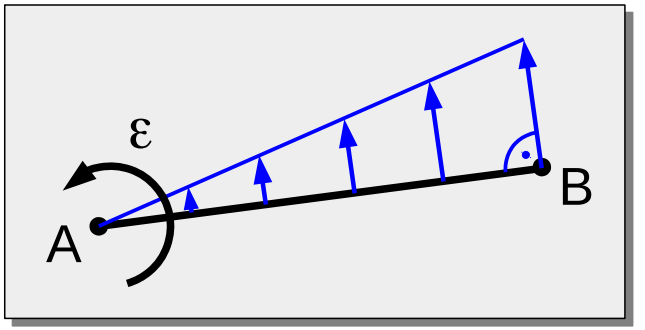


$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}^{\,n}_{BA} + \vec{a}^{\,t}_{BA}$$

Centripetal acceleration (normal)

Rotary acceleration (tangential)

$$\vec{a}^{\,n}_{BA} = \vec{\omega} \times \left(\vec{\omega} \times \overrightarrow{AB} \right) = -\omega^2 \overrightarrow{AB}$$

$$\vec{a}^{\,t}_{BA} = \vec{\varepsilon} \times \overrightarrow{AB}$$

Acceleration scheme (diagram)

Acceleration scheme of a rigid body – geometry created by the ends of it's acceleration vectors moved to the common starting point (acceleration scheme's pole).

Acceleration scheme is similar to the corresponding rigid body:

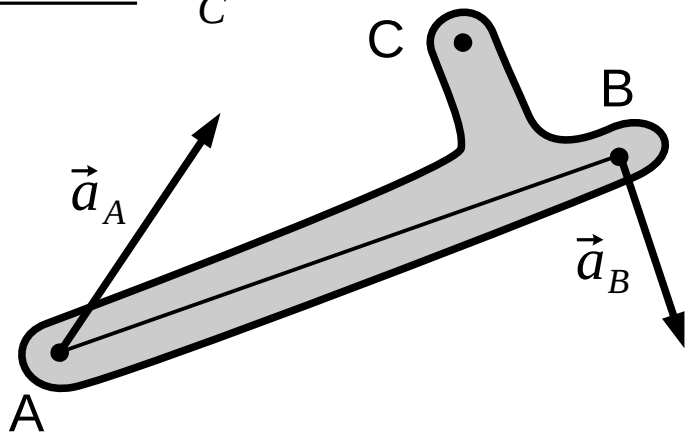
it is scaled and rotated by $(180^{o}-\psi)$ *angle in the direction of body's angular velocity if* $\mathrm{sgn}\omega=\mathrm{sgn}\varepsilon$ *(or opposite direction if* $\mathrm{sgn}\omega\neq\mathrm{sgn}\varepsilon$*).*

Acceleration scheme method

Example

Given: $\bar{a}_A$ and $\bar{a}_B$ + geometry

Searched: $\bar{a}_C$



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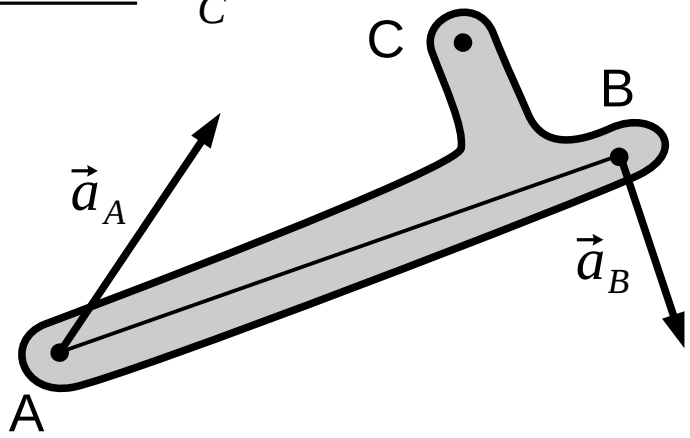

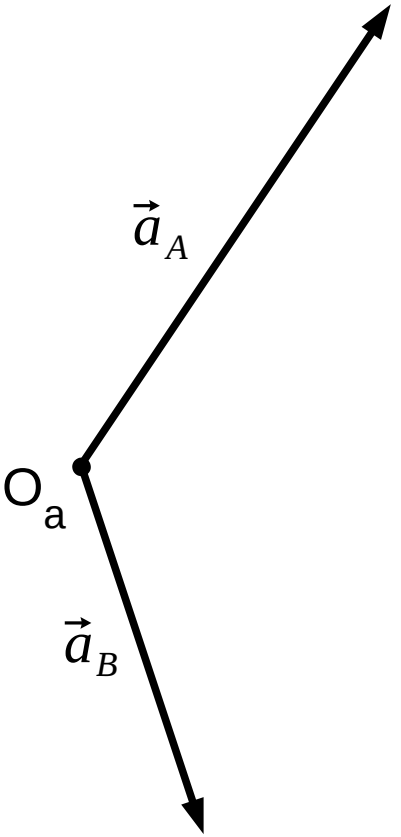

acceleration scale, e.g.: 1cm → 1m/s

Acceleration scheme method

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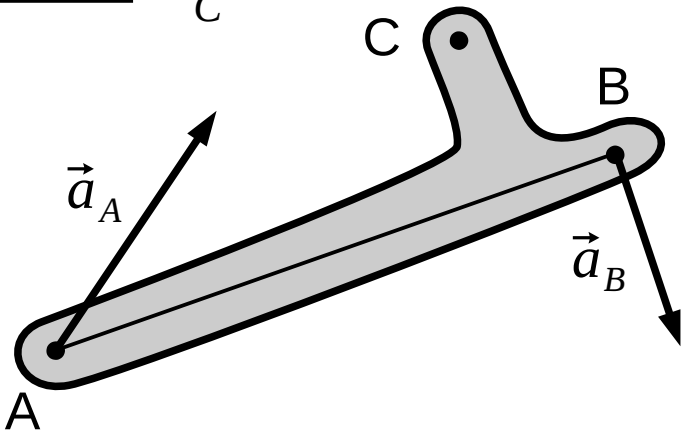

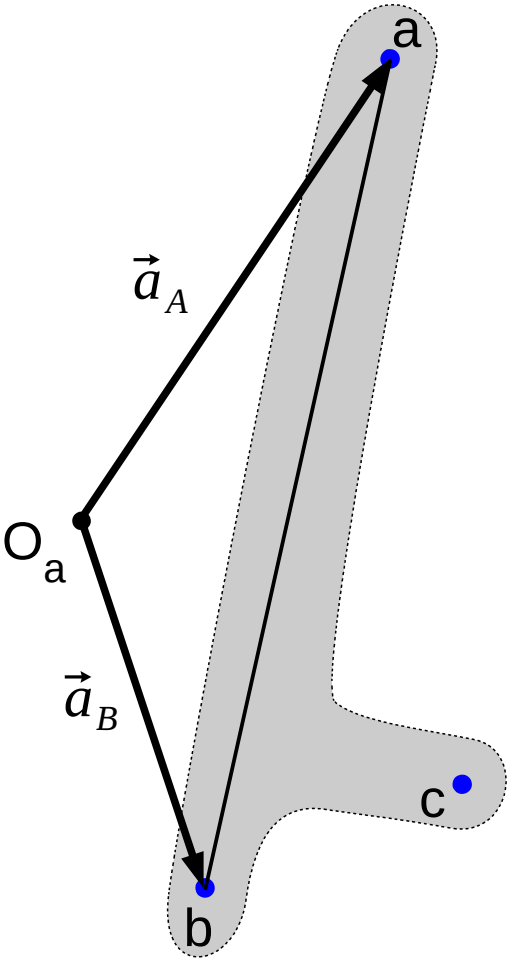

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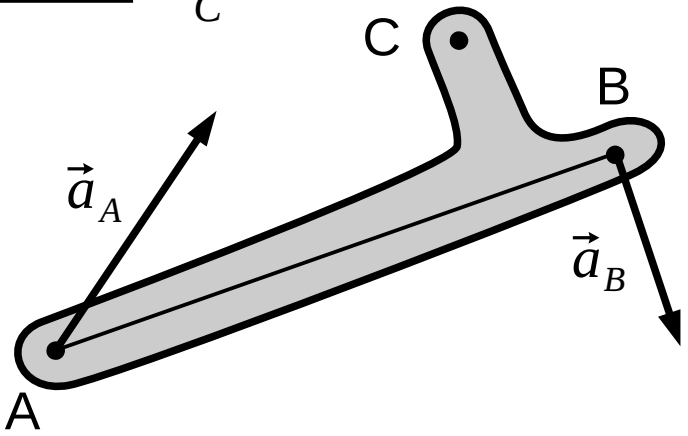

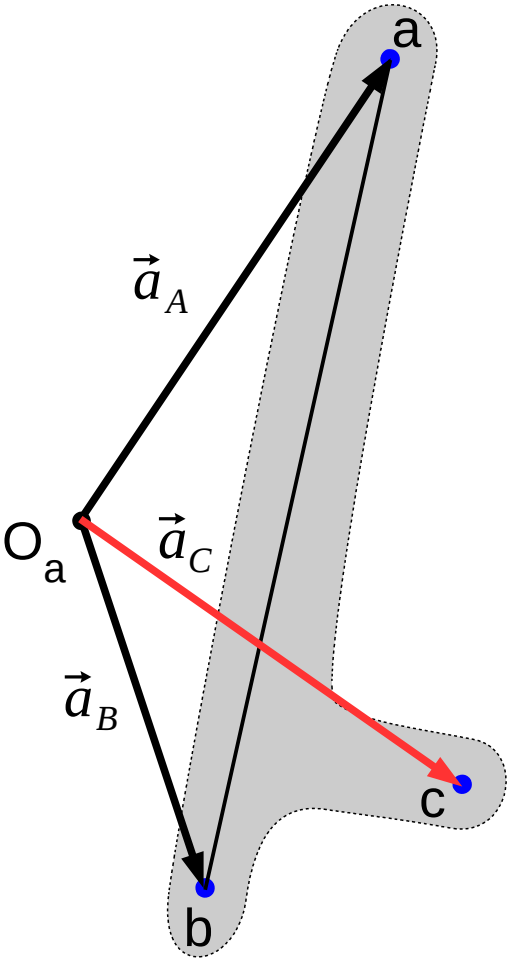

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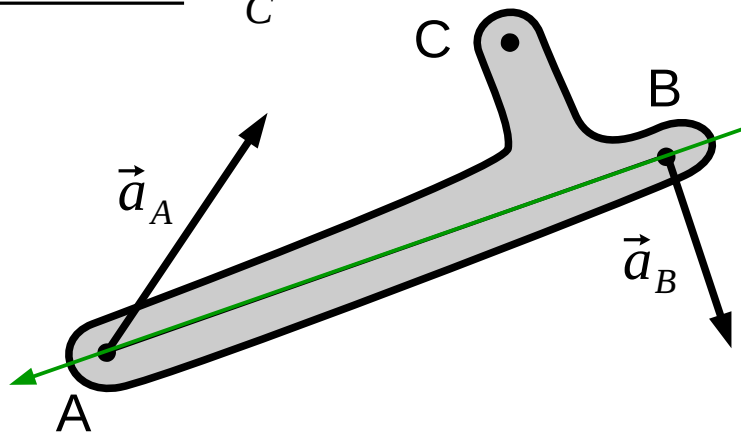
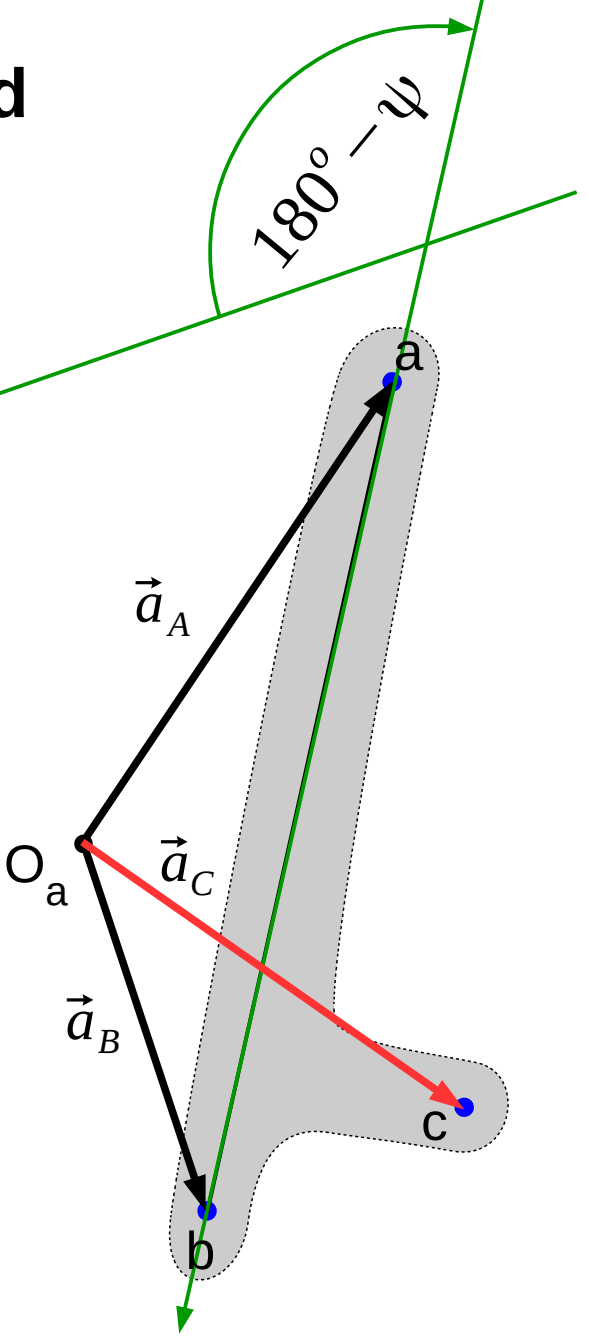
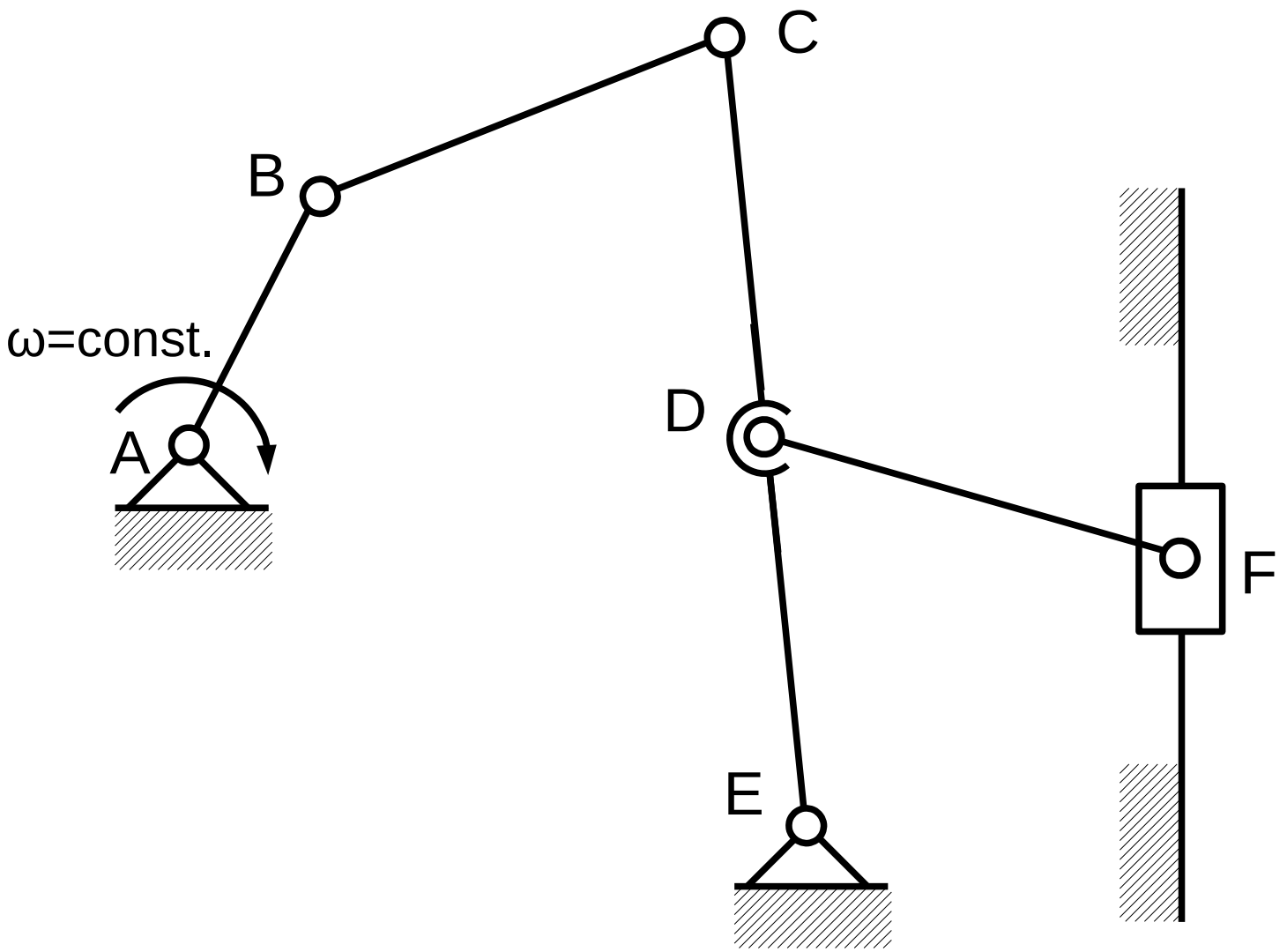




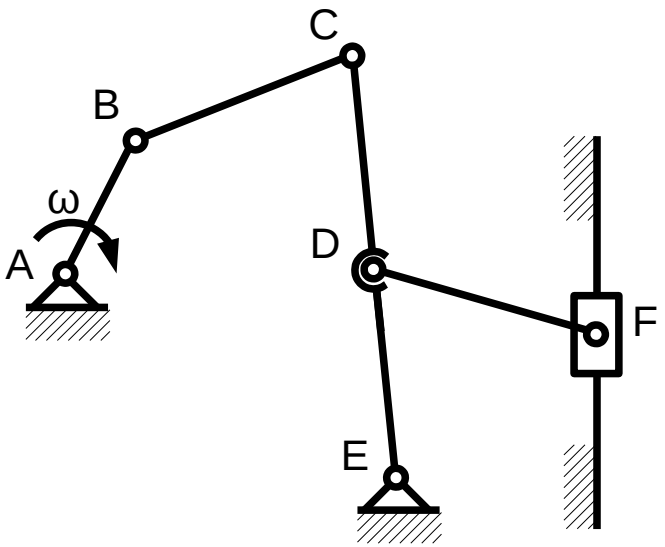
acceleration scale, e.g.: 1cm → 1m/s

Velocities and accelerations – example

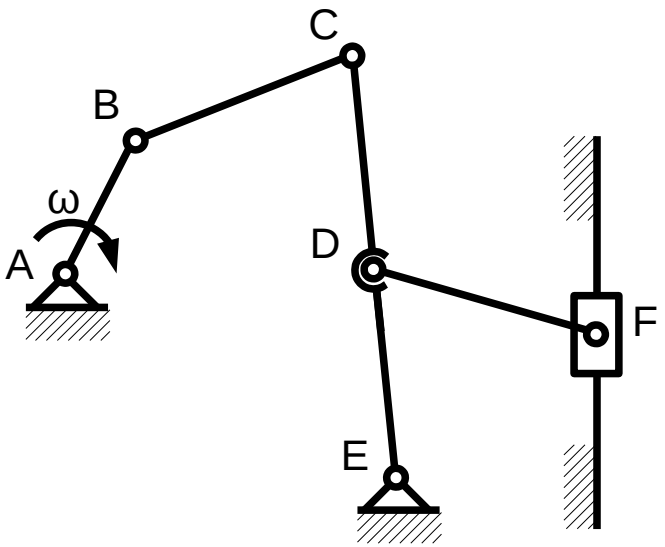
Find velocity and acceleration of the point F using decompositiom methods



Velocities and accelerations – example



Velocities and accelerations – example



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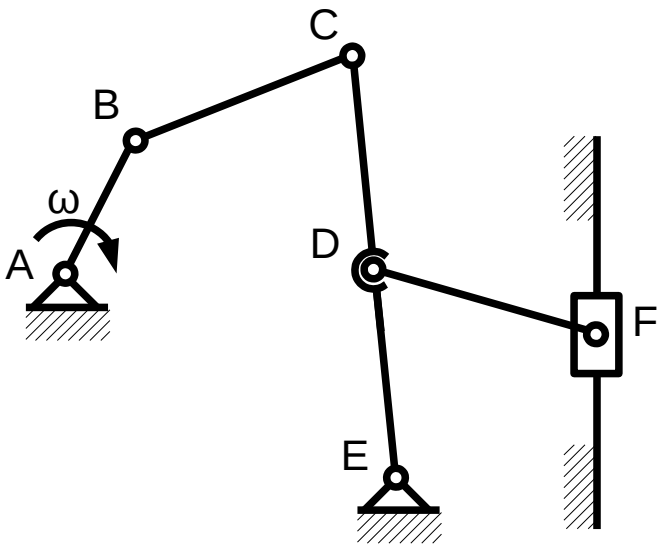

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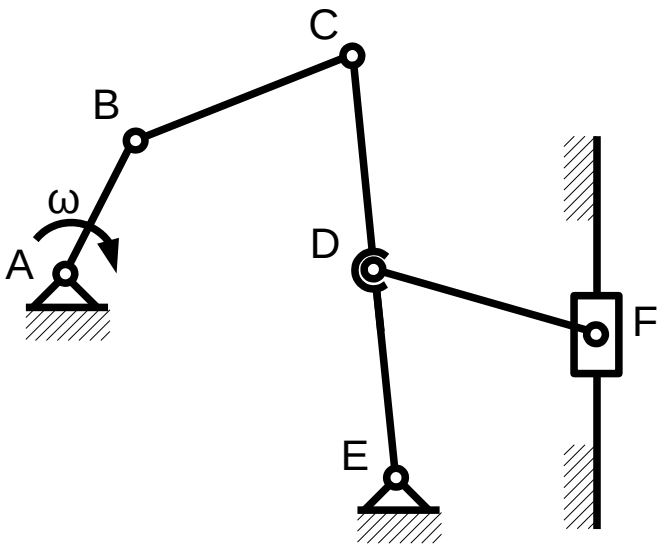

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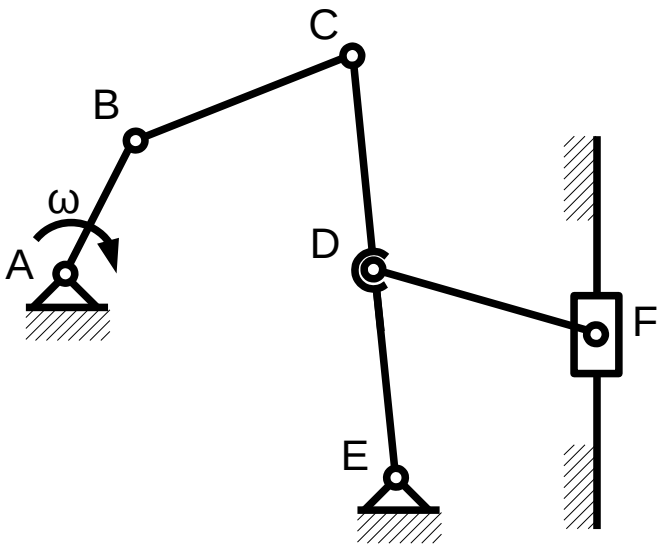
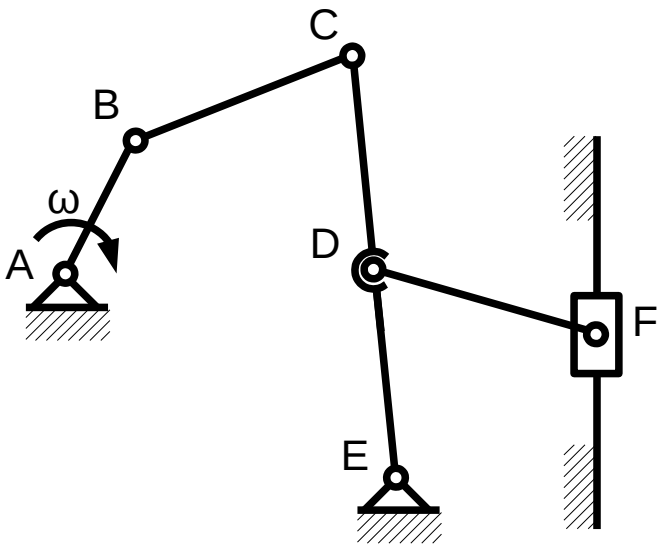
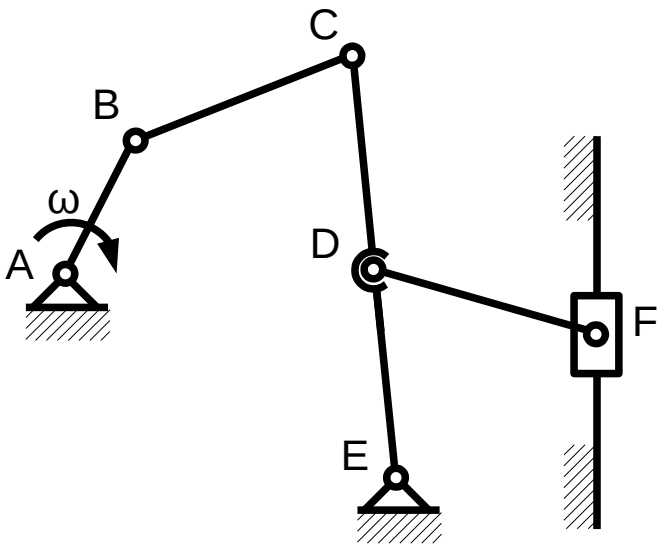


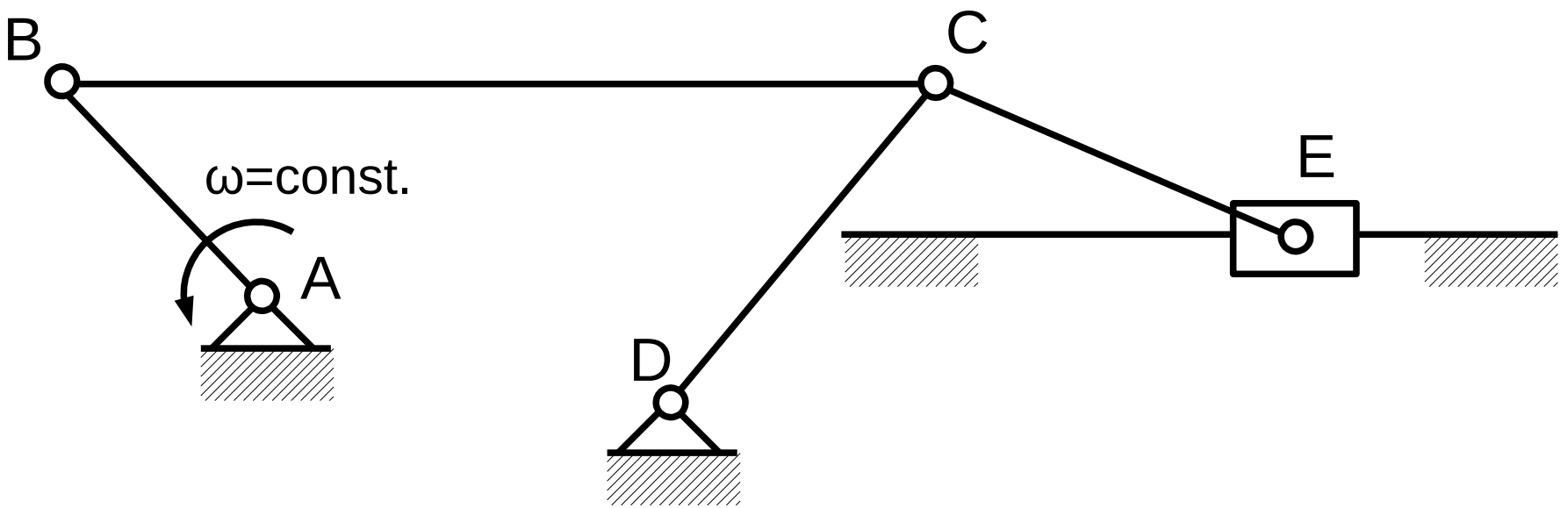
Velocities and accelerations – example



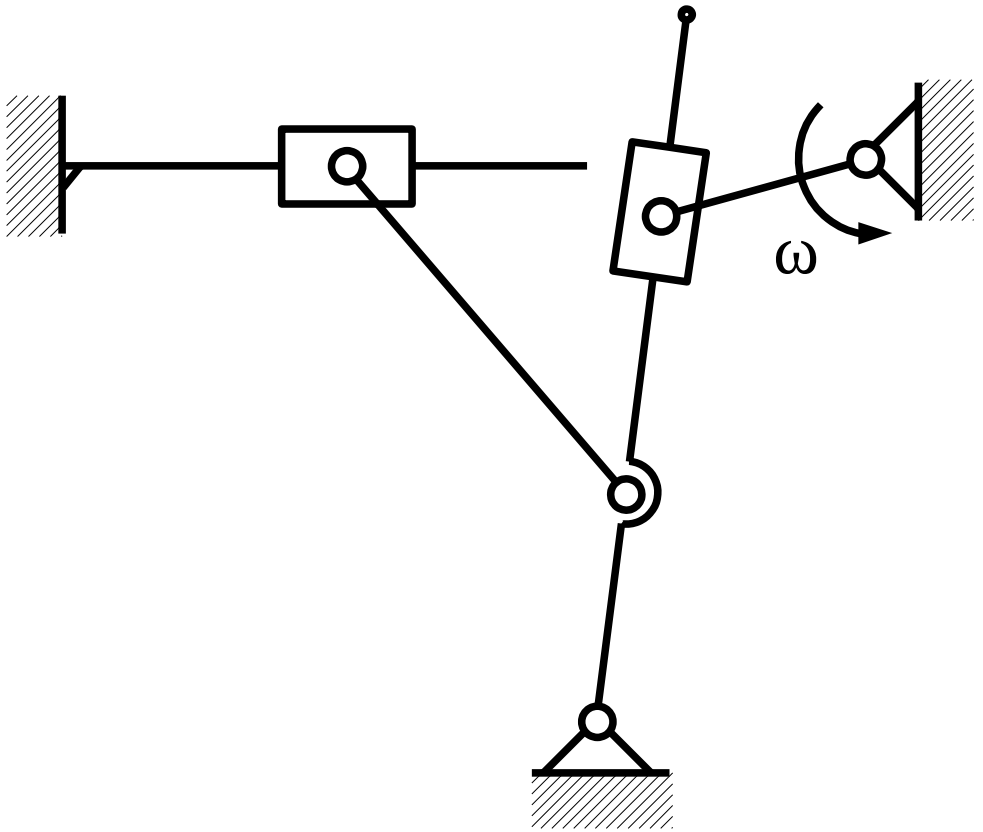
Velocities and accelerations – example



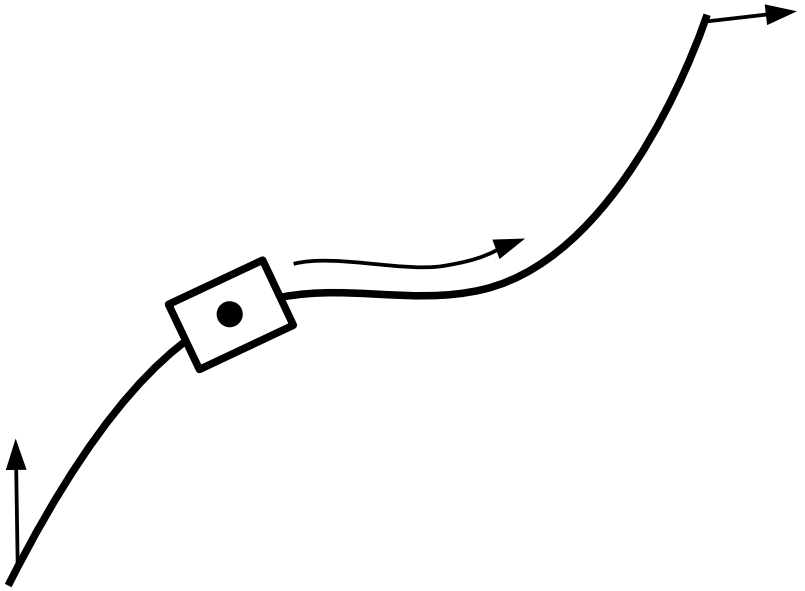
Velocities and accelerations – example 2



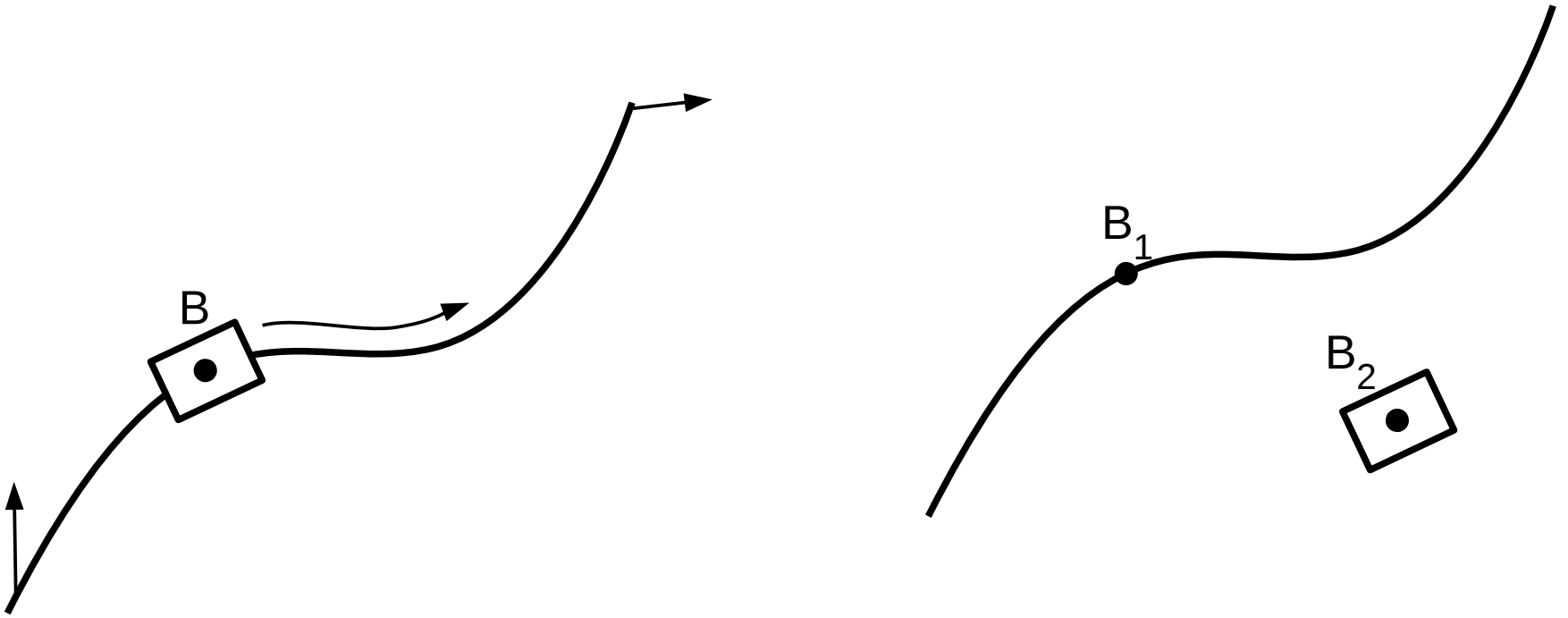
Velocities and accelerations – example 3 (appendix)



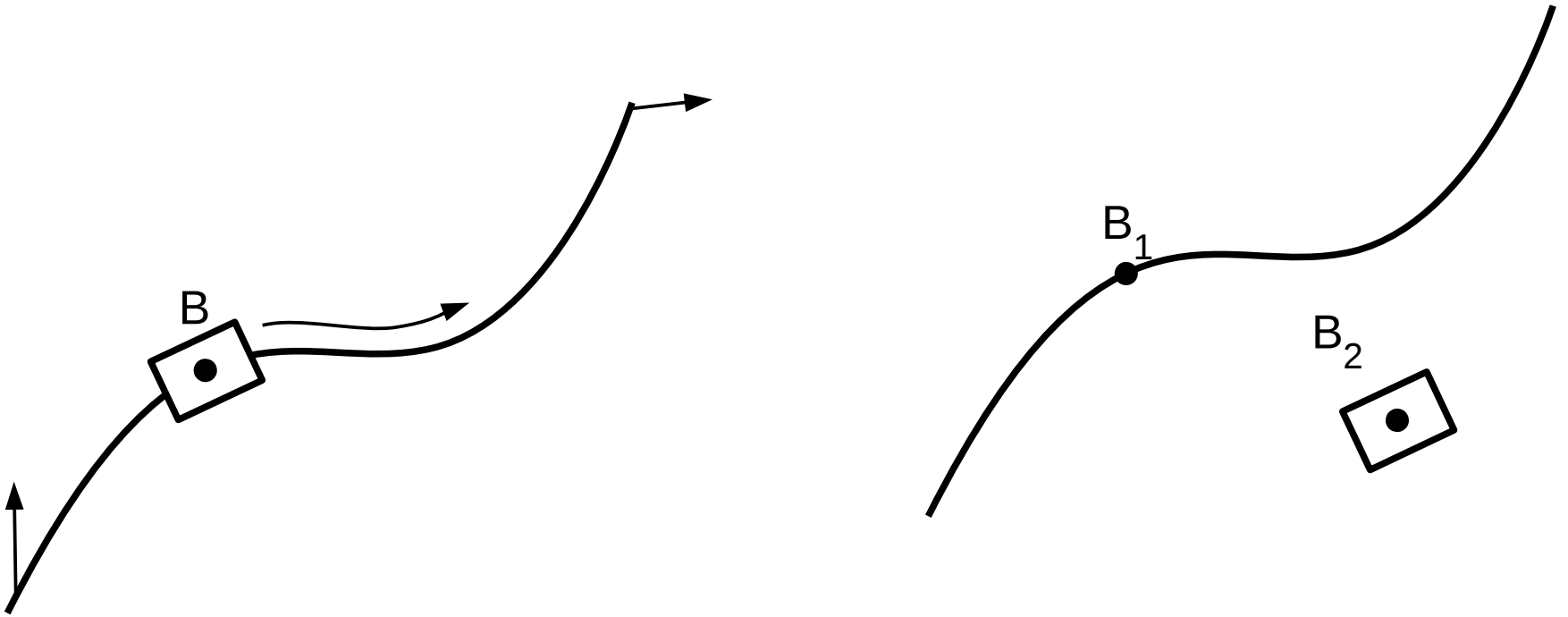
Velocities in relative motion



Velocities in relative motion



Velocities in relative motion



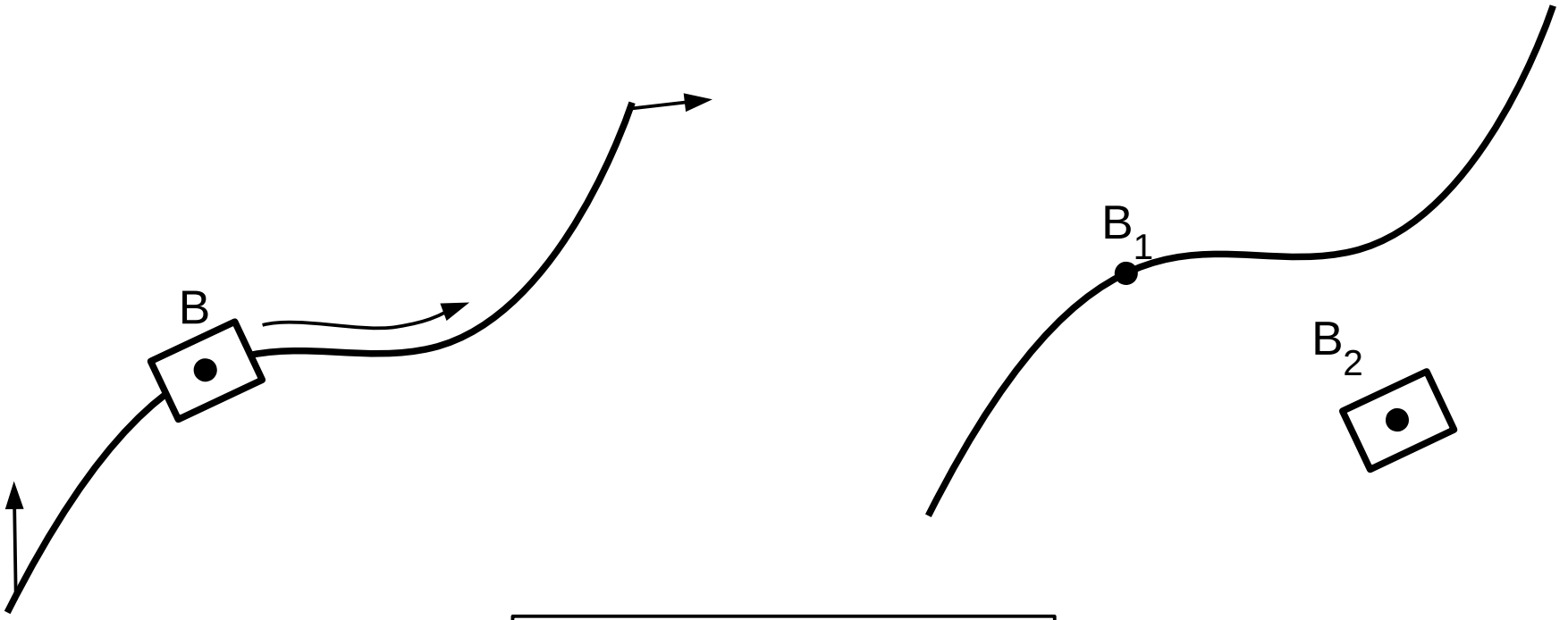
$$\vec{v}_{B2} = \vec{v}_{B1} + \vec{v}_{B2B1}$$

absolute velocity of point B_2

transportation velocity

relative velocity

Accelerations in relative motion



$$\vec{a}_{B2} = \vec{a}^{u}_{B1} + \vec{a}^{rel}_{B2B1} + \vec{a}^{c}$$

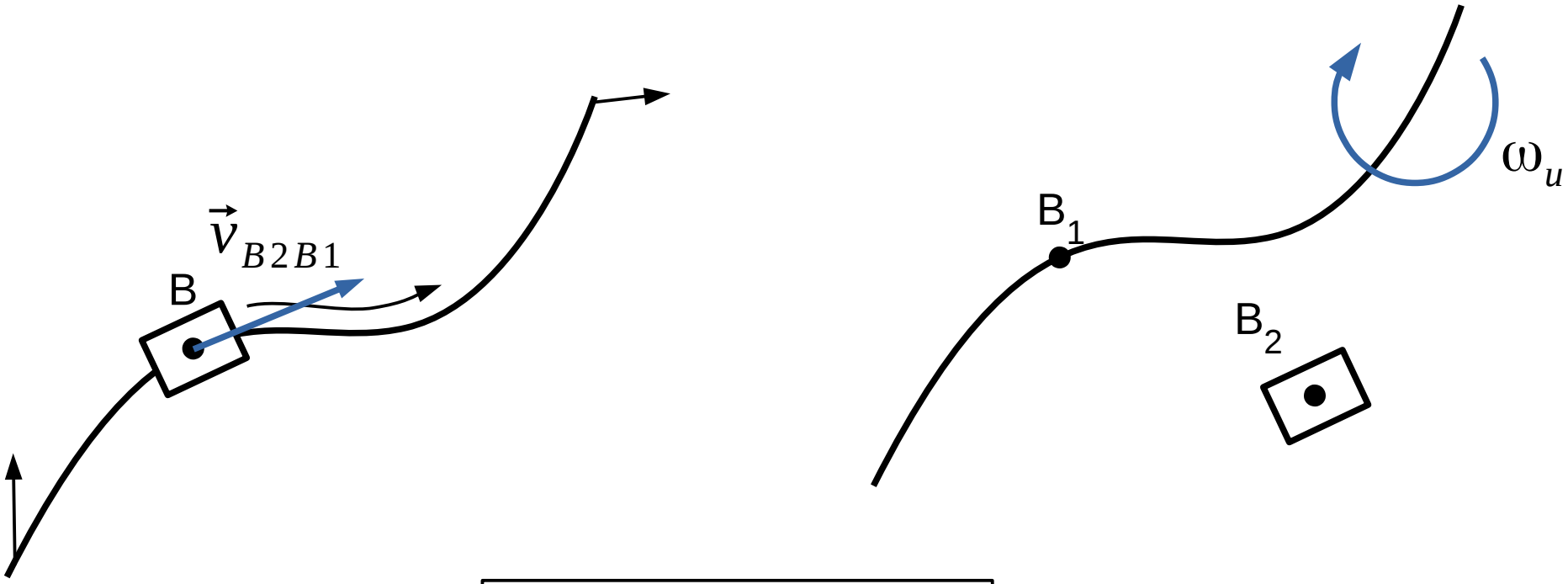
absolute acceleration of point B2

Transportation acceleration (absolute acceleration of point B1)

Relative acceleration

Coriolis acceleration

Accelerations in relative motion



$$\vec{a}_{B2} = \vec{a}^{u}_{B1} + \vec{a}^{rel}_{B2B1} + \vec{a}^{c}$$

absolute acceleration of point B2

Transportation acceleration (absolute acceleration of point B1)

Relative acceleration

Coriolis acceleration

$$\vec{a}^{c} = 2\,\vec{\omega}_u \times \vec{v}_{B2B1}$$

Velocities and accelerations in relative motion – example