Faculty of Automotive and Construction Machinery Engineering
WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control

Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

Lecture 10

Classification of basic automatic systems with examples.

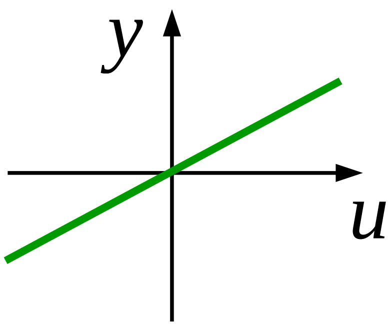
Classification of basic automatic systems

Element name	Transfer function
proportional	k
first order (inertial)	$\frac{k}{Ts+1}$
integrator	$\frac{k}{s}$
differentiator	ks
differentiator with inertia	$\frac{ks}{Ts+1}$
delay	$e^{-\tau s}$
second order (oscillator)	$\frac{k}{T_1^2 s^2 + T_2 s + 1}$

Proportional element

1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

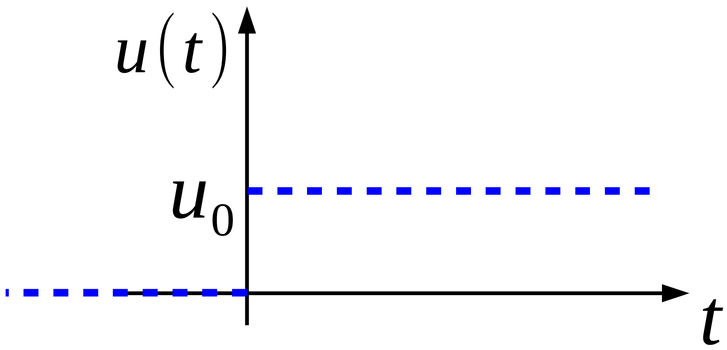
2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$

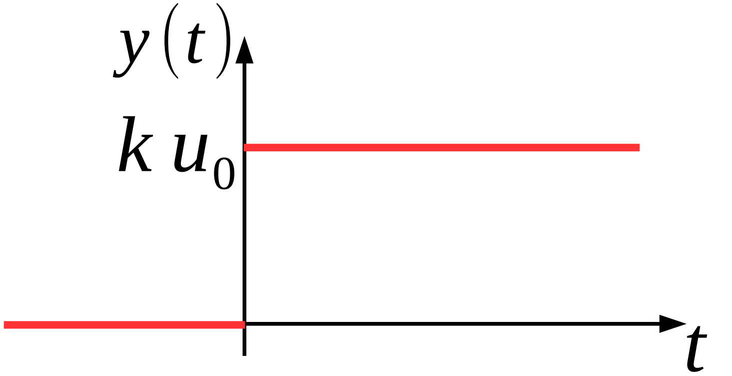


for $k > 0$

3. Transfer function: $H(s) = k$

4. Step response: $y(t) = k\, u_0 1(t)$ for $u(t) = u_0 1(t)$

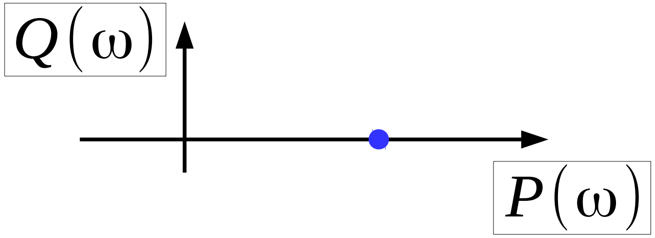




Proportional element

5. Frequency response: $H(j\omega)=k$ $\quad P(\omega)=k,\ Q(\omega)=0$

6. Nyquist plot:

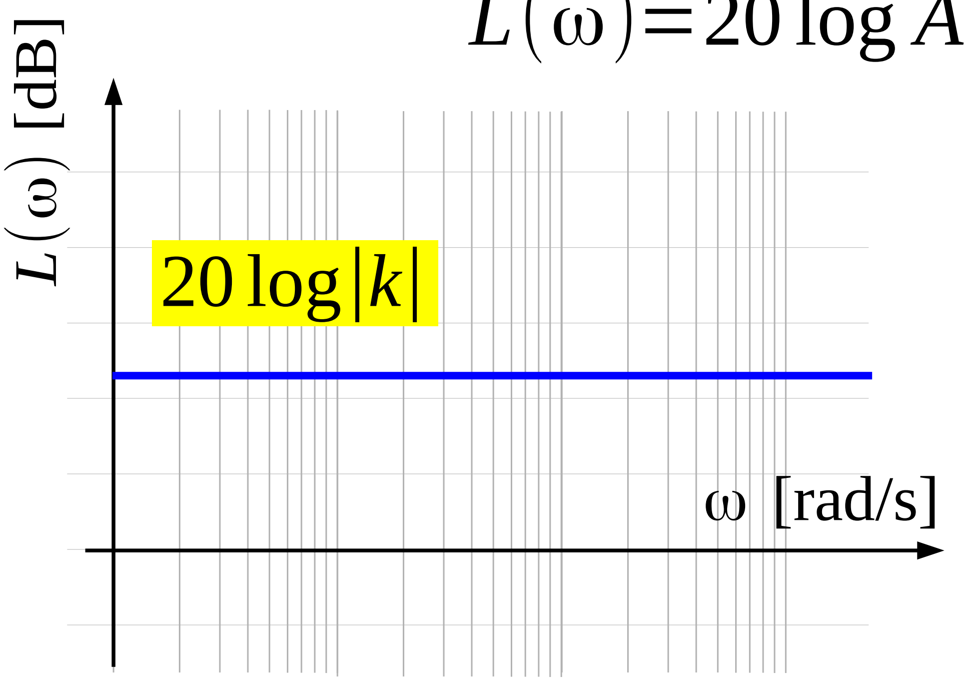


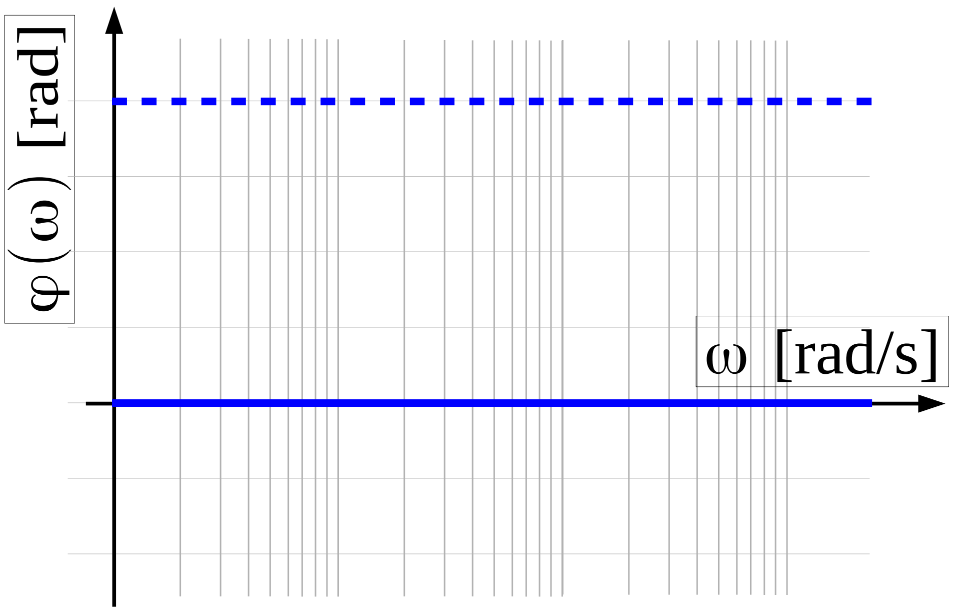
for $k>0$

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k|$

$$\varphi(\omega)=\arctan\frac{Q}{P}=\begin{cases}0, \text{ dla } k\geq 0\\ \pi, \text{ dla } k<0\end{cases}$$

$L(\omega)=20\log A(\omega)$

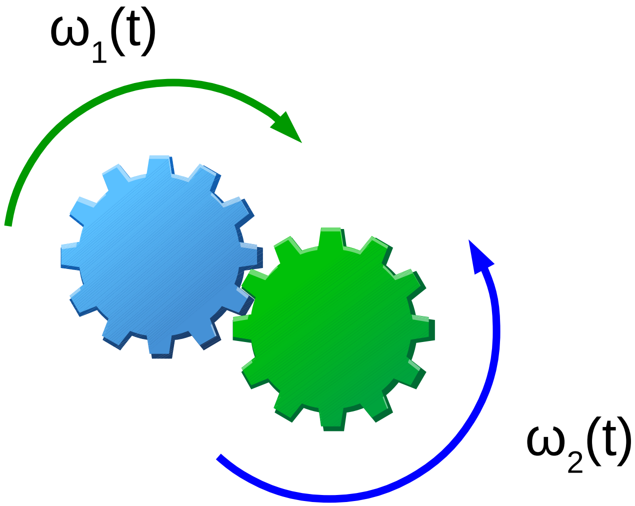




Proportional element

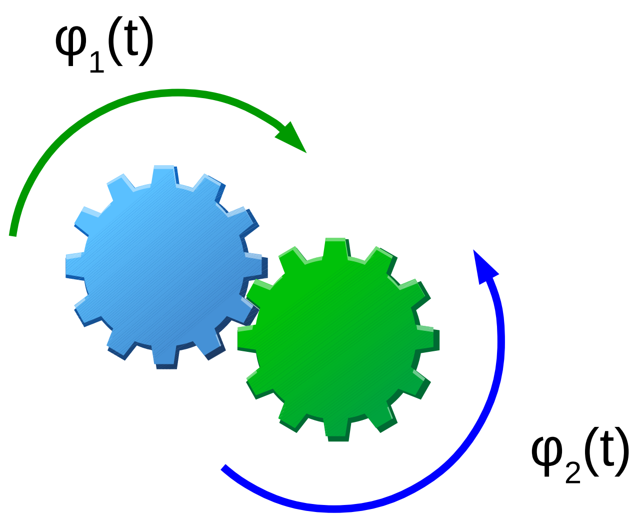
Examples

1



GEARBOX:
input – angular velocity $\omega_1(t)$
output – angular velocity $\omega_2(t)$

2

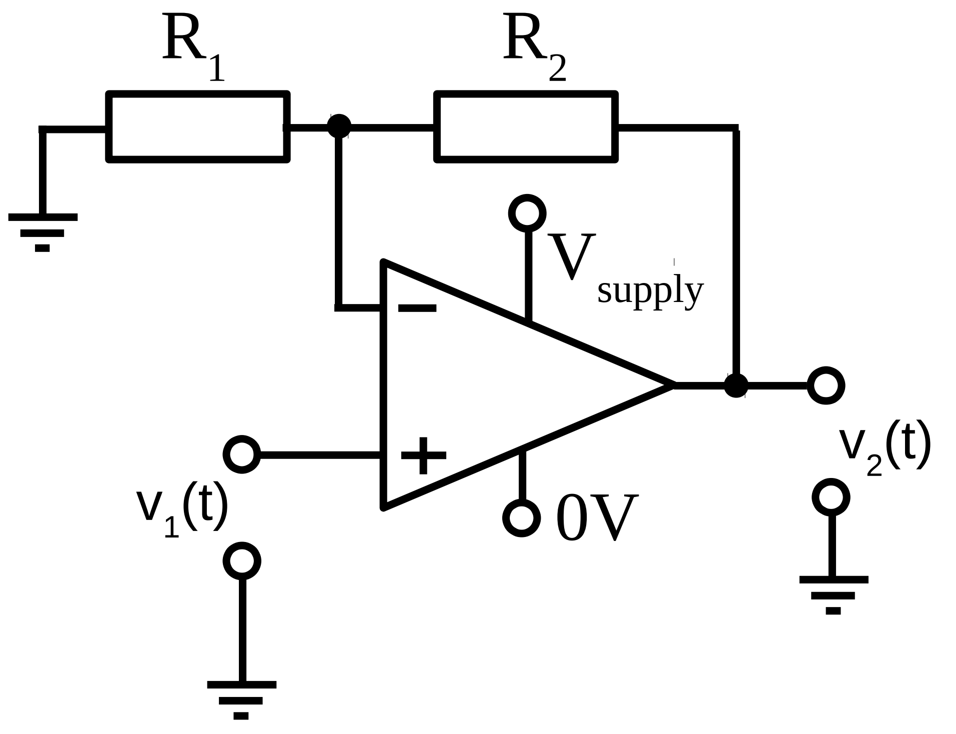


GEARBOX:
input – rotation angle $\varphi_1(t)$
output – rotation angle $\varphi_2(t)$

Proportional element

<u>Examples</u>

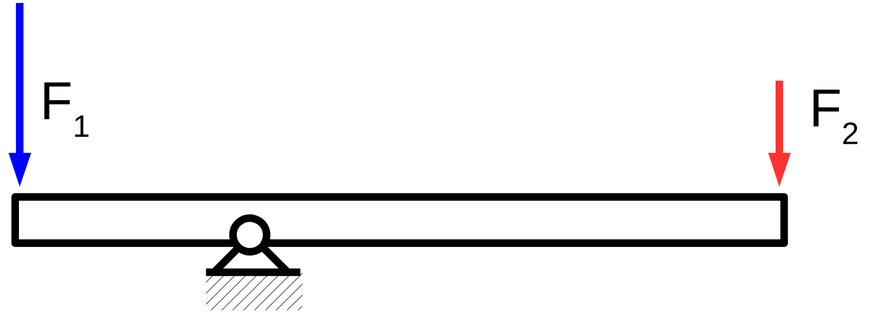
3



OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = v_1(t)\left(1 + \frac{R_2}{R_1}\right)$$

4

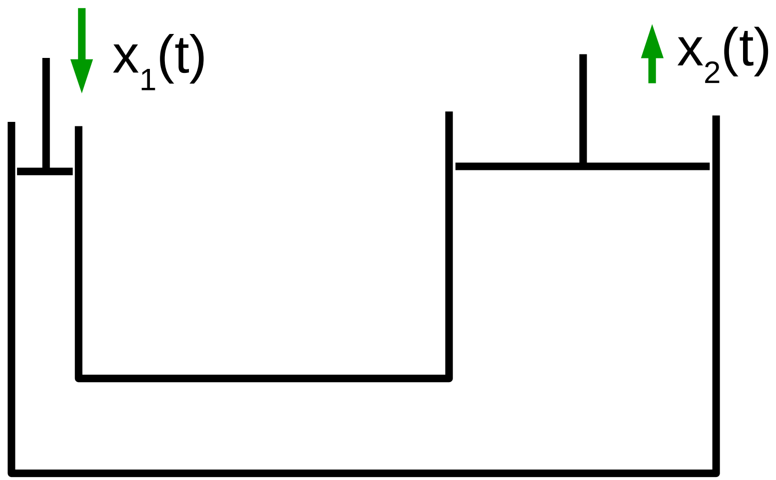


BEAM in steady state:
input – force F_1
output – force F_2

Proportional element

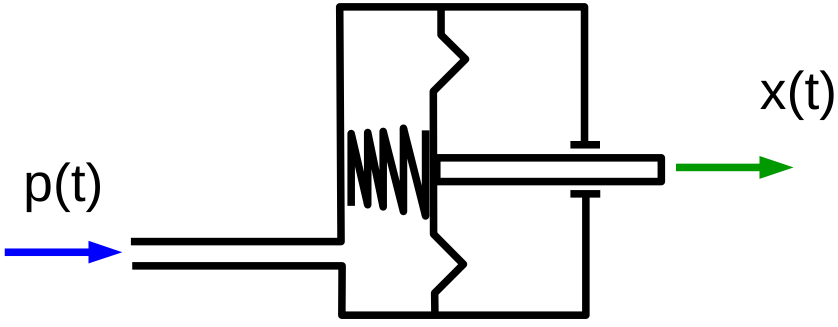
Examples

5



HYDRAULIC LEVER:
input – displacement $x_1(t)$
output – displacement $x_2(t)$

6



PRESSURE ACTUATOR:
input – pressure $p_1(t)$
output – displacement x(t)

First-order inertial element

1. Element equation: $$T\frac{dy(t)}{dt}+y(t)=ku(t)$$

$u(t)$ - input

$y(t)$ - output

First-order inertial element

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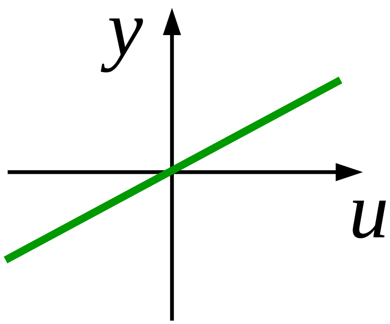
2. Static characteristic (steady state):

First-order inertial element

1. Element equation: $T\dfrac{dy(t)}{dt}+y(t)=ku(t)$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $y=ku$ for $\dfrac{dy}{dt}=0\wedge\dfrac{du}{dt}=0$



for $k>0$

3. Transfer function:

First-order inertial element

1. Element equation: $T\frac{dy(t)}{dt}+y(t)=ku(t)$

$u(t)$ - input
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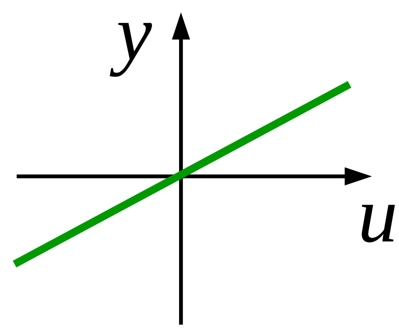


for $k>0$

3. Transfer function: $H(s)=\frac{k}{Ts+1}$

First-order inertial element

4. Step response:

First-order inertial element

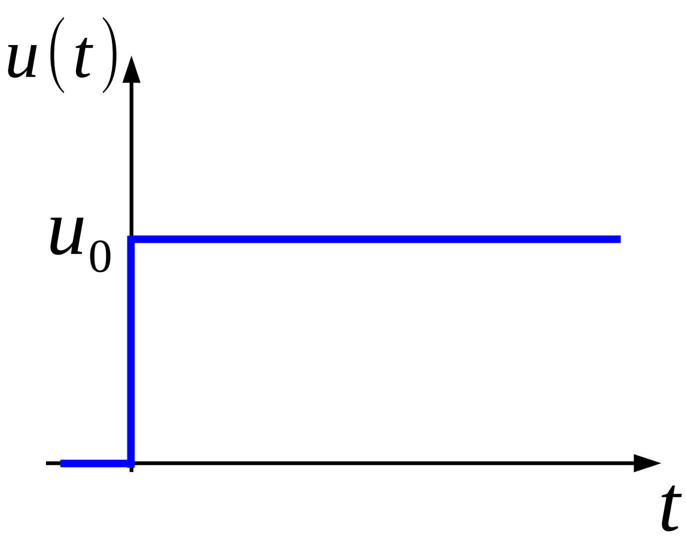
4. Step response:

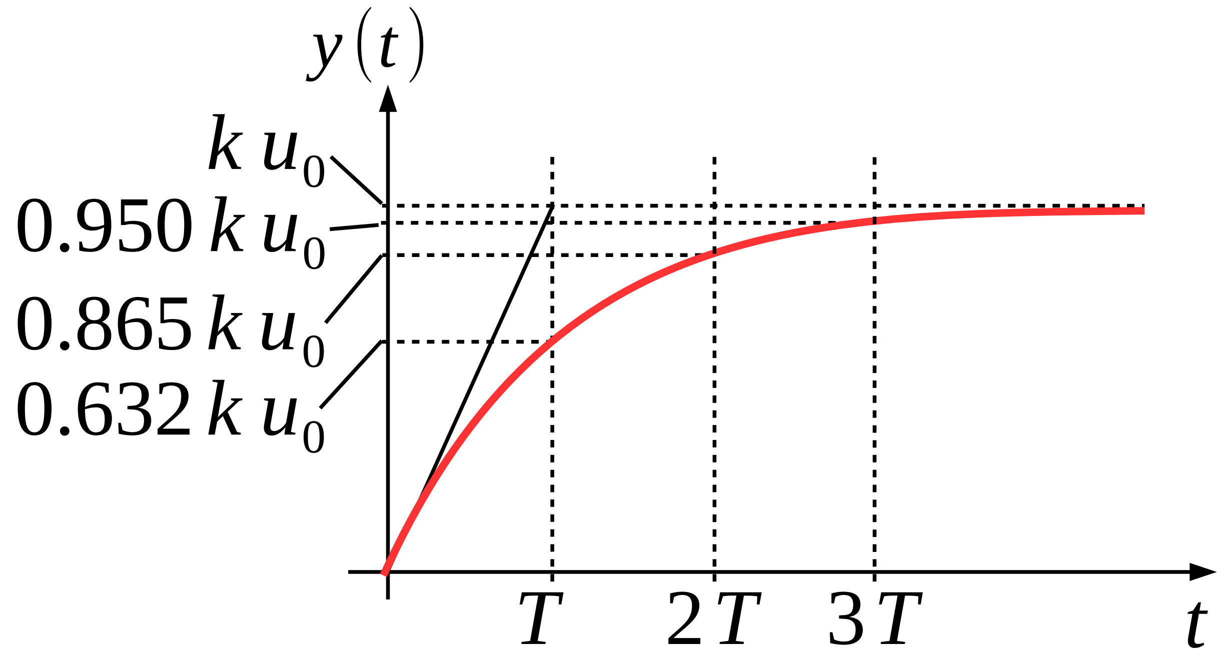
input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0\frac{1}{s}$

Laplace of output: $Y(s)=H(s)U(s)=\frac{k u_0}{s(Ts+1)}$

output: $y(t)=L^{-1}\{Y(s)\}=k u_0(1-e^{-t/T})$





First-order inertial element

5. Frequency response:

First-order inertial element

5. Frequency response: $H(j\omega)=\dfrac{k}{Tj\omega+1}$

$$P(\omega)=\frac{k}{T^2\omega^2+1},\quad Q(\omega)=\frac{-kT\omega}{T^2\omega^2+1}$$

6. Nyquist plot:

First-order inertial element

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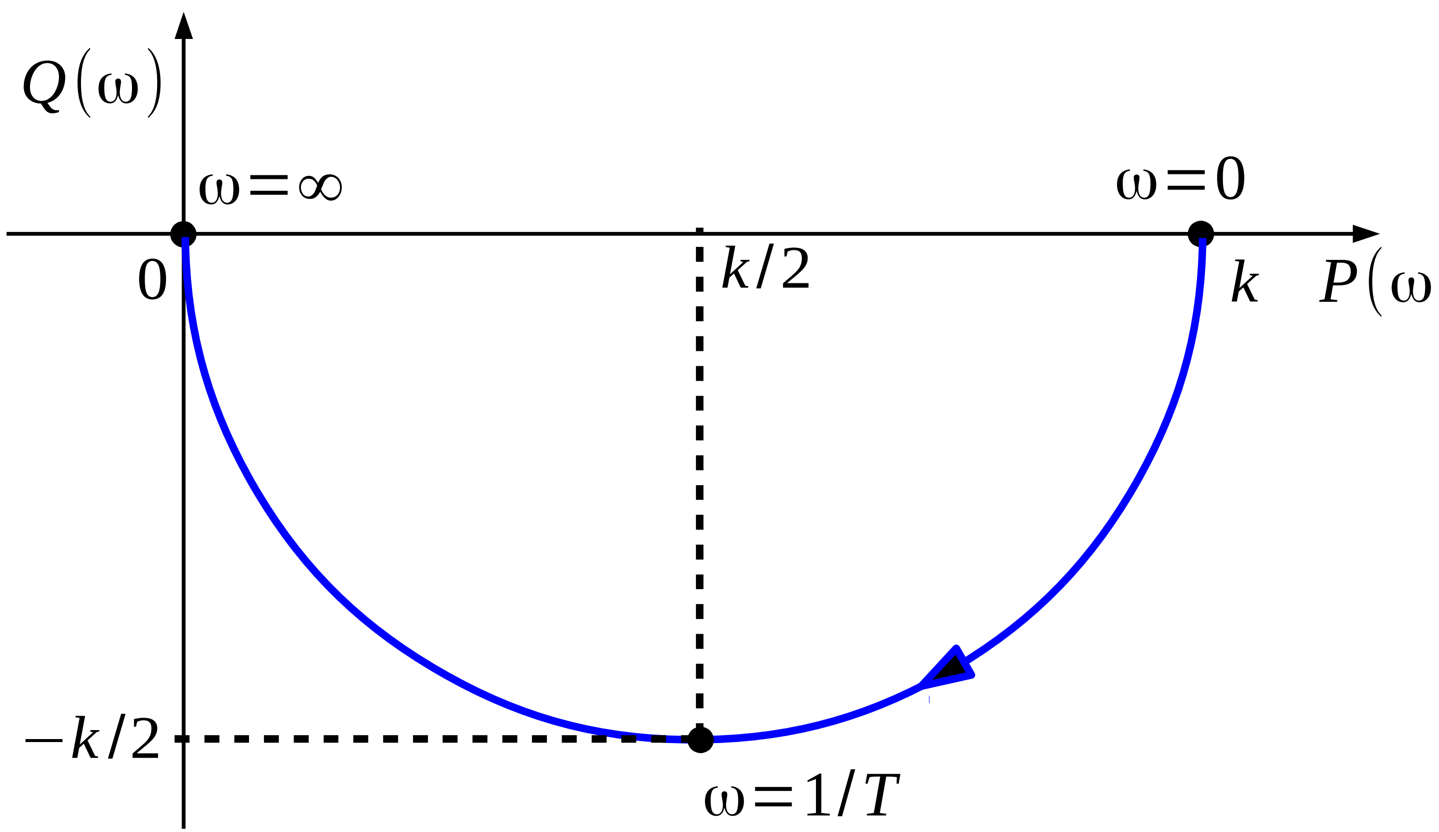
6. Nyquist plot:

for $k>0$



First-order inertial element

7. Bode plot:

$$P(\omega) = \frac{k}{T^2 \omega^2 + 1}, \quad Q(\omega) = \frac{-k T \omega}{T^2 \omega^2 + 1}$$

First-order inertial element

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k|/\sqrt{T^2\omega^2+1}$

$$L(\omega)=20\log A(\omega)=20\log|k|-20\log\sqrt{T^2\omega^2+1}$$

$$\varphi(\omega)=\arctan\frac{Q}{P}=\arctan(-T\omega)$$

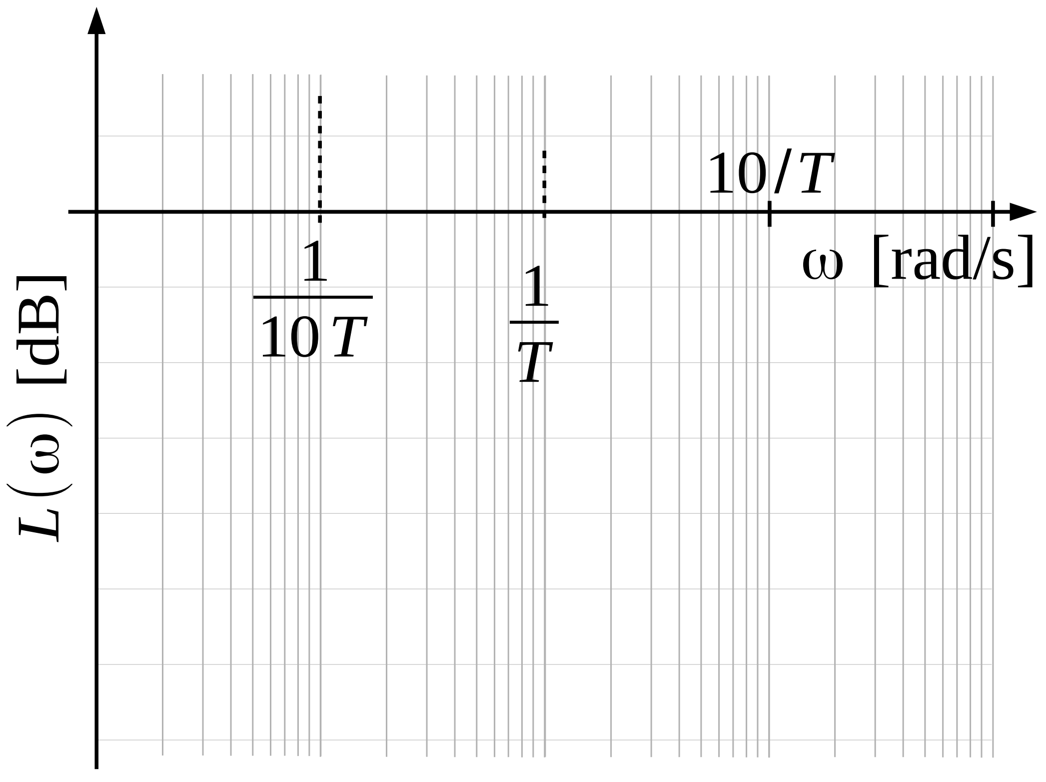
First-order inertial element

7. Bode plot: $A(\omega) = \sqrt{P^2 + Q^2} = |k| / \sqrt{T^2 \omega^2 + 1}$

$$L(\omega) = 20 \log A(\omega) = 20 \log |k| - 20 \log \sqrt{T^2 \omega^2 + 1}$$

$$\varphi(\omega) = \arctan \frac{Q}{P} = \arctan(-T\omega)$$

for $k > 0$



First-order inertial element

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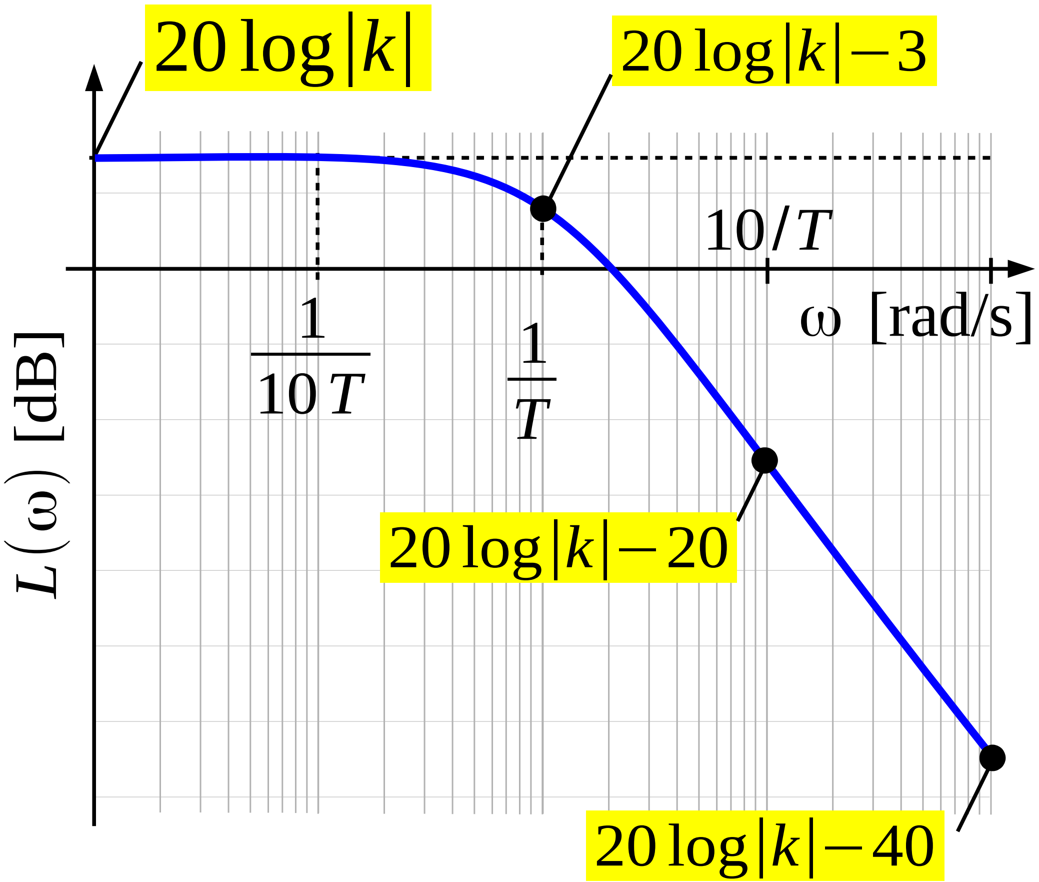

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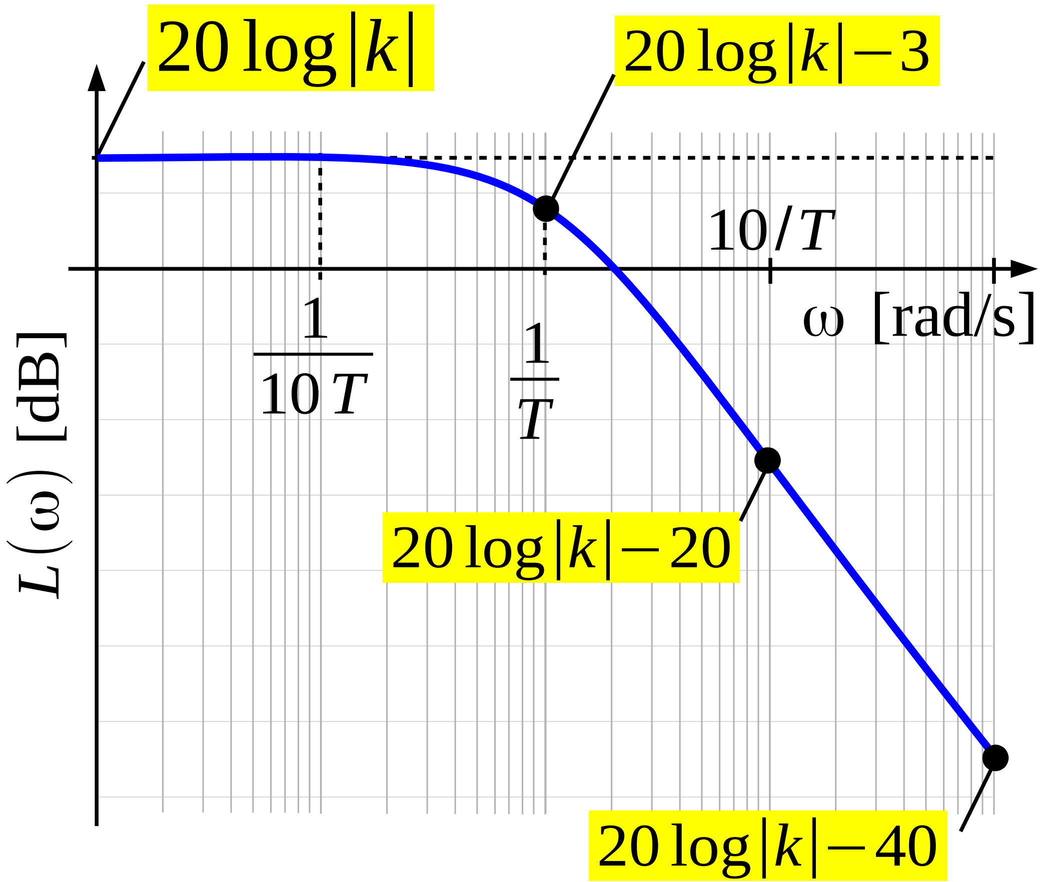
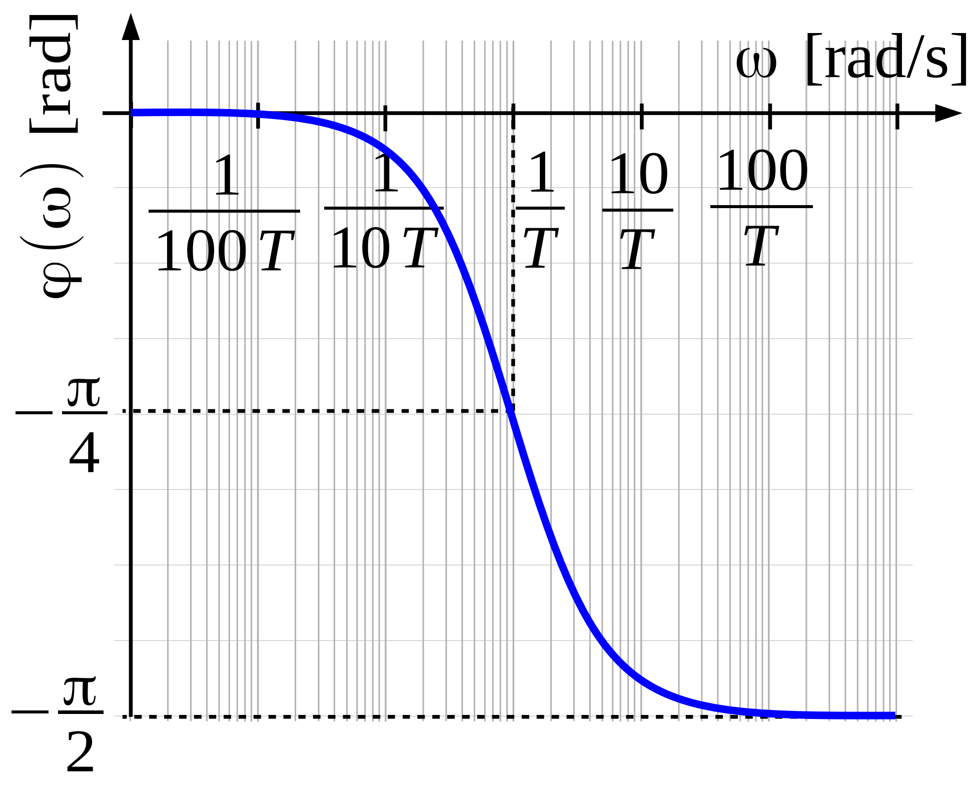
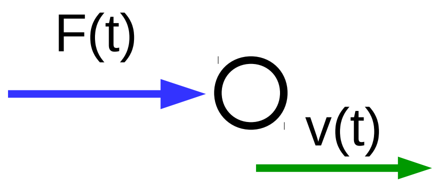




First-order inertial element

Examples

1



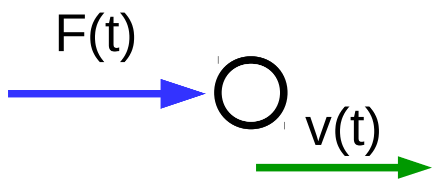
LINEAR MOTION OF A MATERIAL POINT WITH LINEAR DAMPING:
input – force F(t)
output – velocity v(t)

example: car is driving on a flat surface with air resistance proportional To its velocity, described using machine equation of motion, with assumption of constant reduced mass.

First-order inertial element

Examples

1



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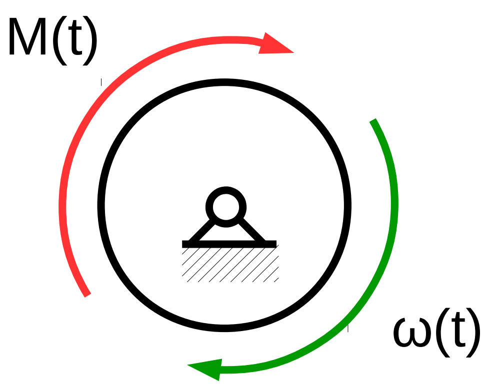
2

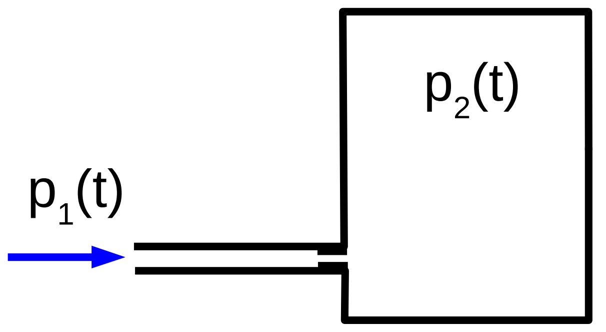


ANGULAR MOTION OF A RIGID BODY WITH LINEAR DAMPING:
input – torque M(t)
output – angular velocity ω(t)

First-order inertial element

Examples

3

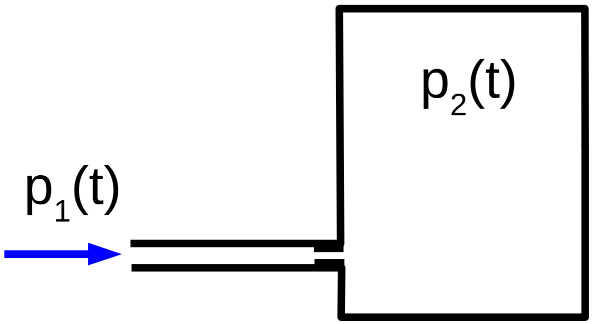


AIR CONTAINER:
input – pressure $p_1(t)$
output – pressure $p_2(t)$

First-order inertial element

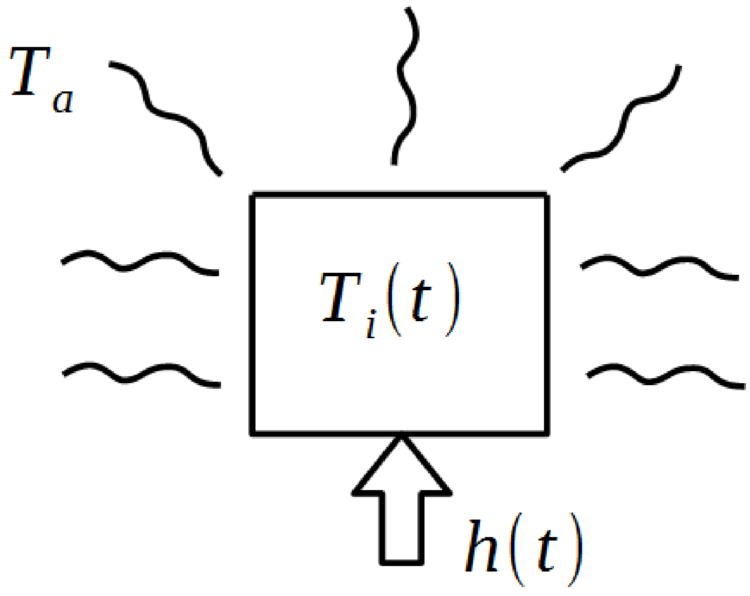
Examples

3



AIR CONTAINER:
input – pressure $p_1(t)$
output – pressure $p_2(t)$

4



HEATED OBJECT WITH SMALL INERTIA:
input – heater power h(t)
output – object temperature $T_i(t)$

Integrator

1. Element equation: $$\frac{dy(t)}{dt} = k\,u(t)$$

$u(t)$ - input

$y(t)$ - output

Integrator

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$u(t)$ - input
$y(t)$ - output

---

2. Static characteristic (steady state):

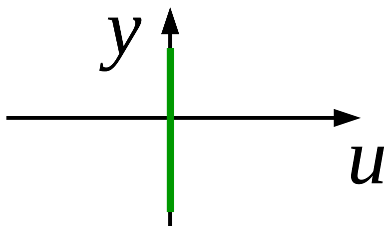
Integrator

1. Element equation: $\frac{dy(t)}{dt} = k\,u(t)$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $u = 0$

for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



3. Transfer function:

Integrator

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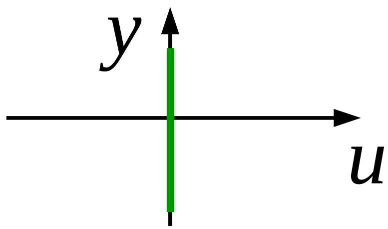


3. Transfer function: $H(s) = \frac{k}{s}$

Integrator

4. Step response:

input: $u(t) = u_0 1(t)$

Laplace of input: $U(s) = u_0 \frac{1}{s}$

Integrator

4. Step response:

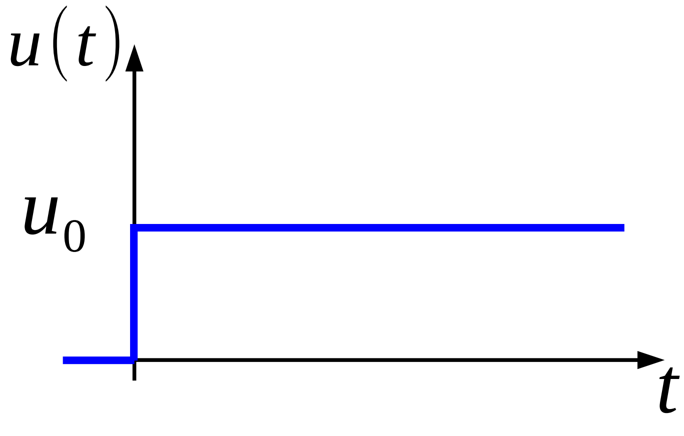
input: $u(t) = u_0 1(t)$

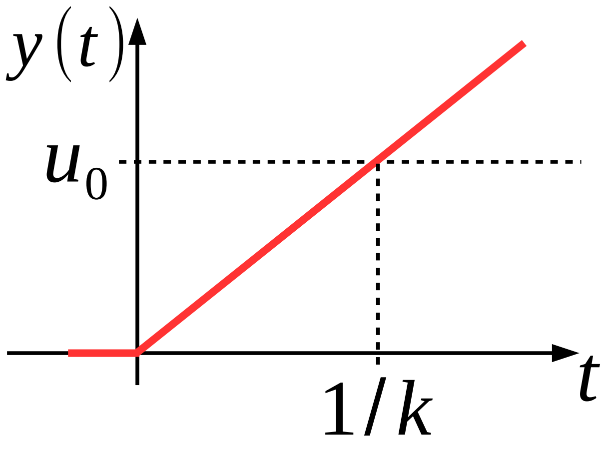
Laplace of input: $U(s) = u_0 \frac{1}{s}$

Laplace of output: $Y(s) = H(s)U(s) = \frac{k u_0}{s^2}$

output: $y(t) = L^{-1}\{Y(s)\} = k u_0 t$

for $k > 0$





Integrator

5. Frequency response:

Integrator

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$$P(\omega)=0,\quad Q(\omega)=-\frac{k}{\omega}$$

6. Nyquist plot:

Integrator

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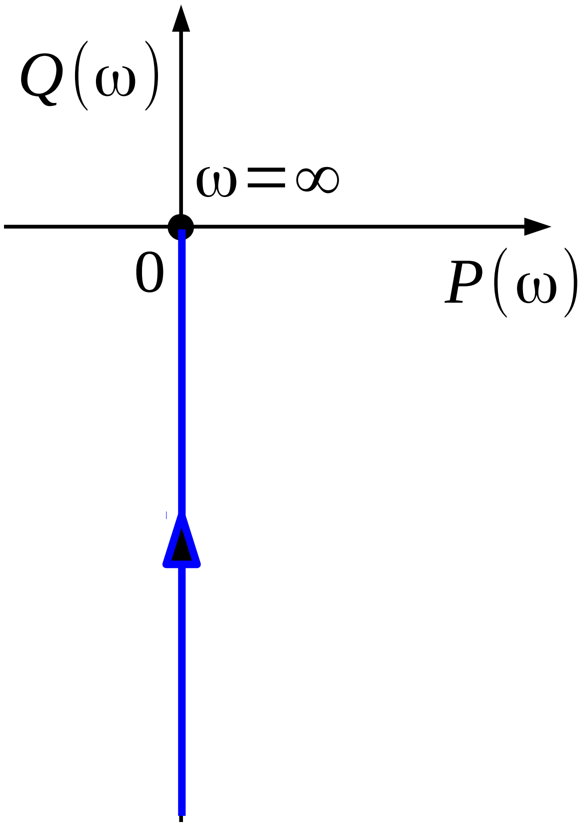
6. Nyquist plot:

for $k>0$



Integrator

7. Bode plot:

$$P(\omega)=0,\quad Q(\omega)=-\frac{k}{\omega}$$

Integrator

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|\frac{k}{\omega}|$

$$L(\omega)=20\log A(\omega)=20\log|\frac{k}{\omega}| \qquad \varphi(\omega)=\arctan\frac{Q}{P}=\arctan(-\infty)$$

Integrator

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|\frac{k}{\omega}|$ for $k>0$

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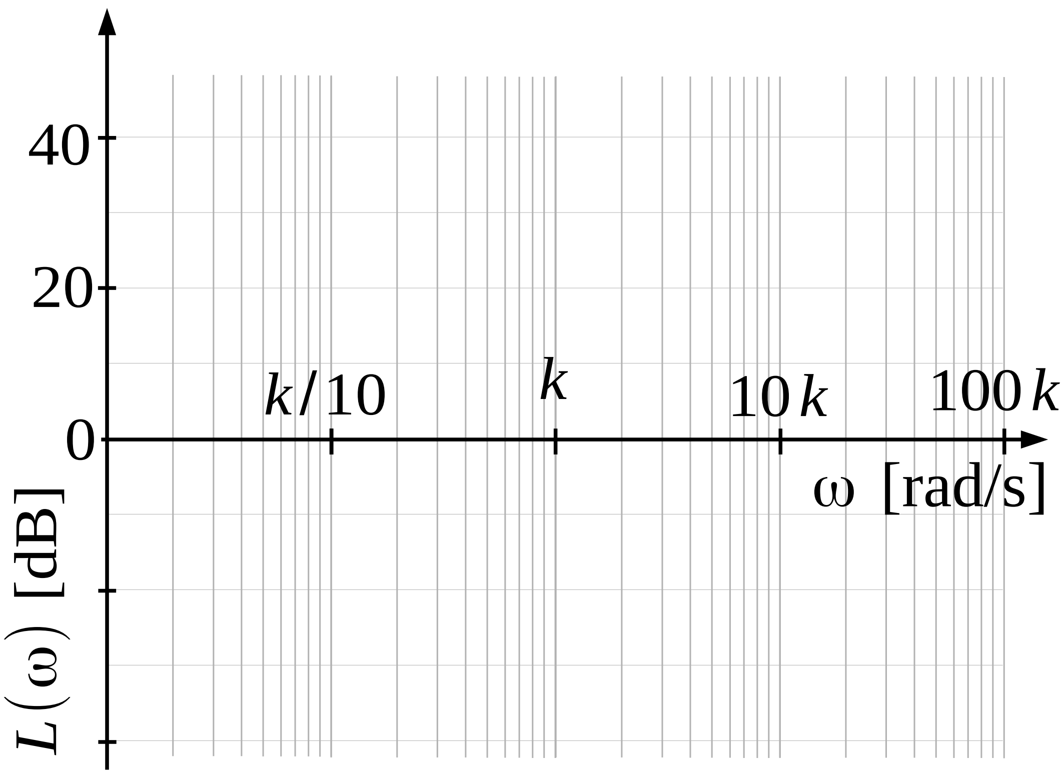

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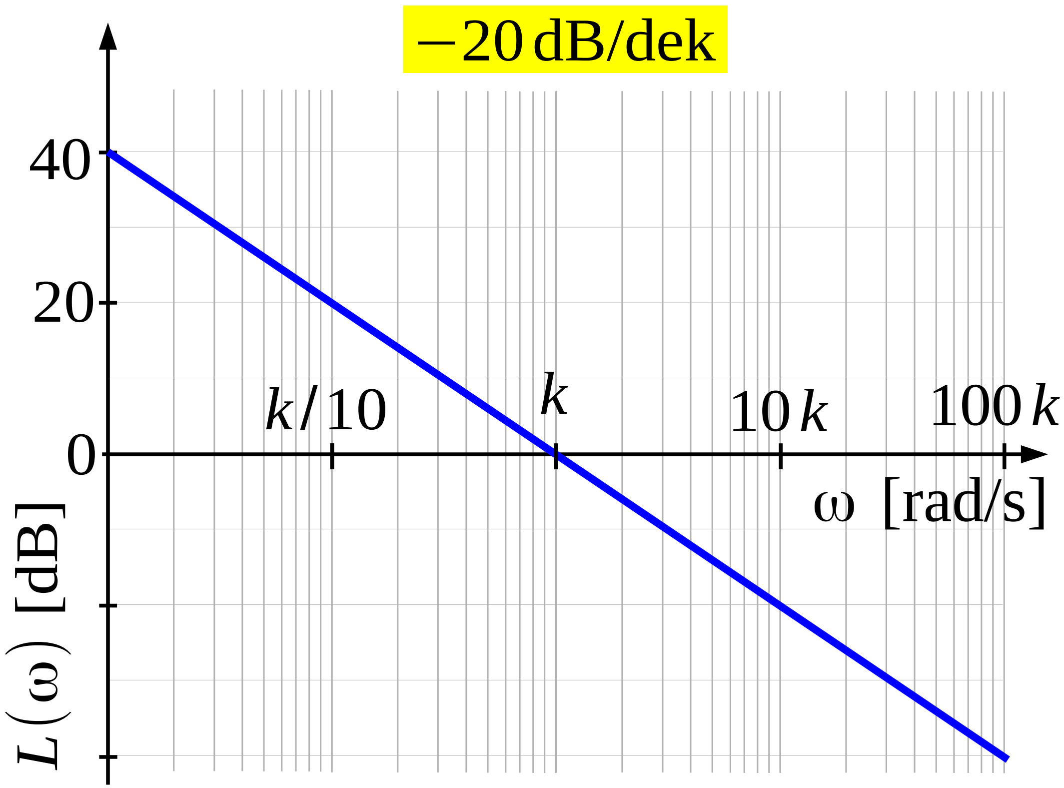

Integrator

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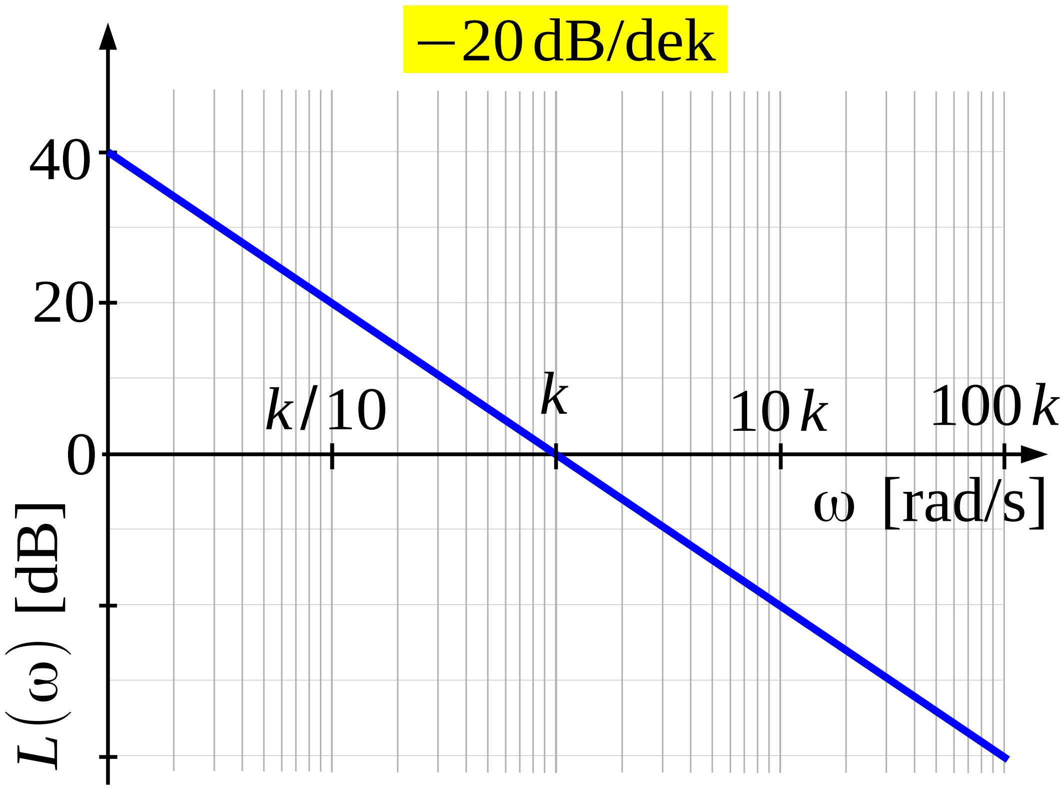
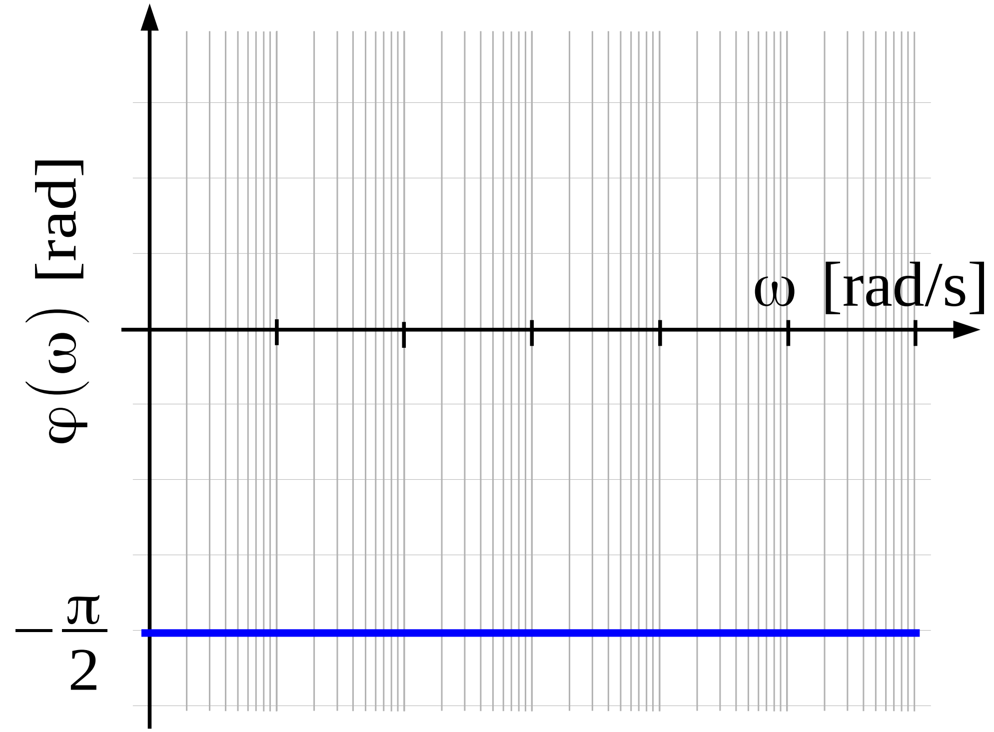
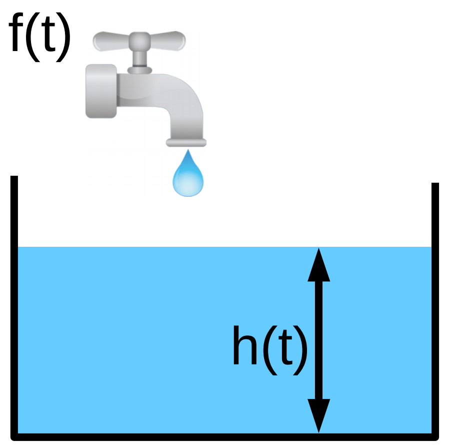




Integrator

Examples

1

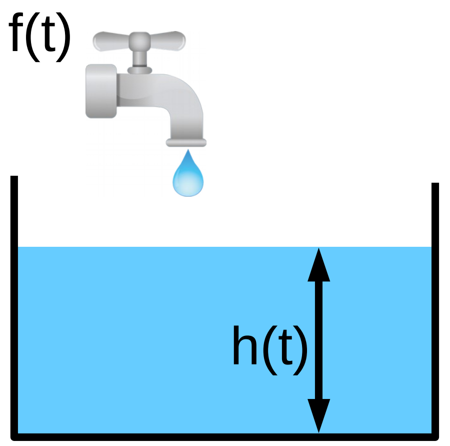


PRISM LIQUID TANK:
input – liquid inflow f(t)
output – liquid level h(t)

Integrator

Examples

1



PRISM LIQUID TANK:
input – liquid inflow f(t)
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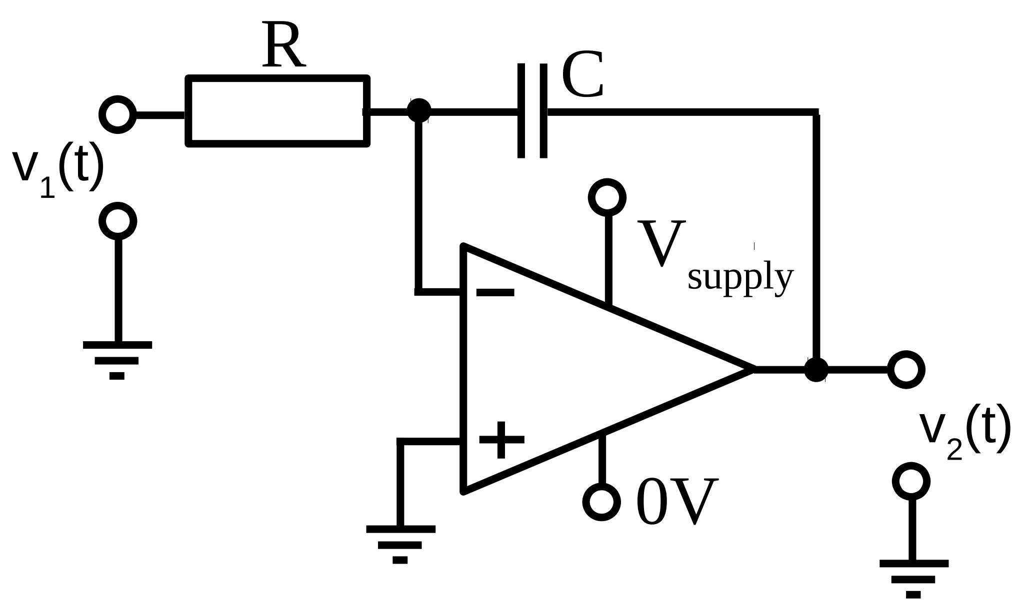


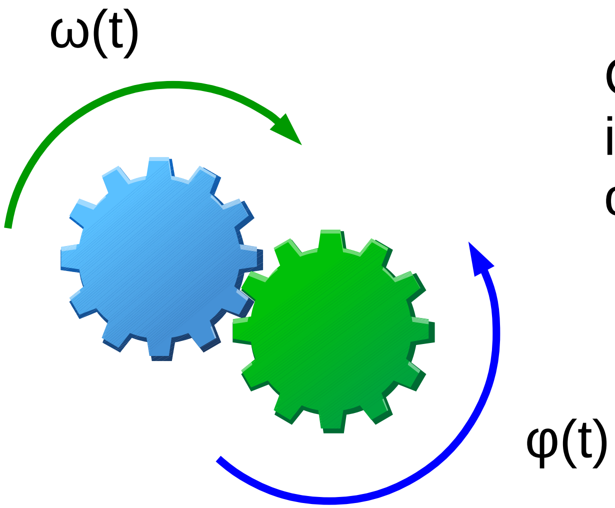
OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = \frac{1}{RC} \int_0^t v_1(t)\,dt$$

Integrator

Examples

(3)

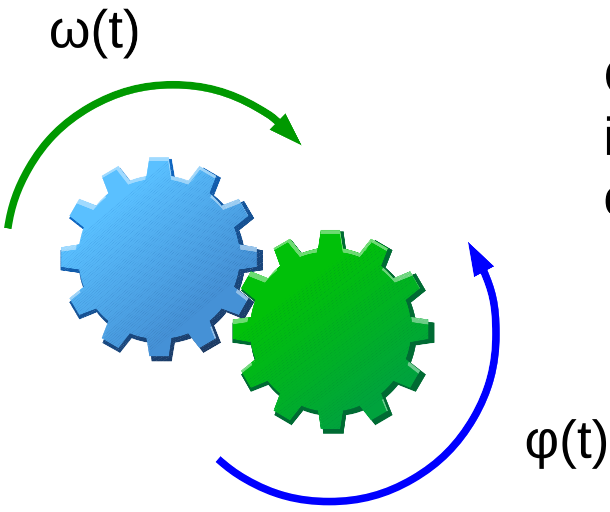


GEARBOX:
input – angular velocity ω(t)
output – rotation angle φ(t)

Integrator

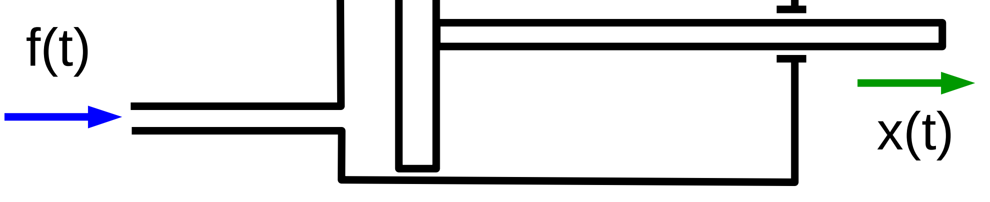
Examples

3



GEARBOX:
input – angular velocity ω(t)
output – rotation angle φ(t)

4



HYDRAULIC CYLINDER:
input – volume inflow f(t)
output – displacement x(t)

Differentiator

1. Element equation: $y(t) = k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

Differentiator

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2. Static characteristic (steady state):

Differentiator

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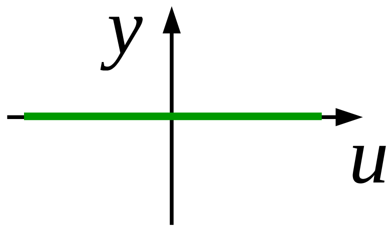


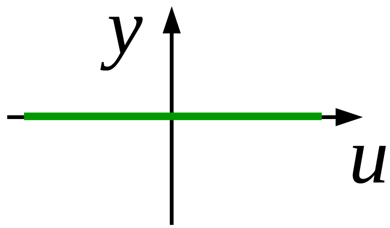
3. Transfer function:

Differentiator

1. Element equation: $y(t) = k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $y = 0$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



3. Transfer function: $H(s) = k\,s$

Differentiator

4. Step response:

input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0 \frac{1}{s}$

Differentiator

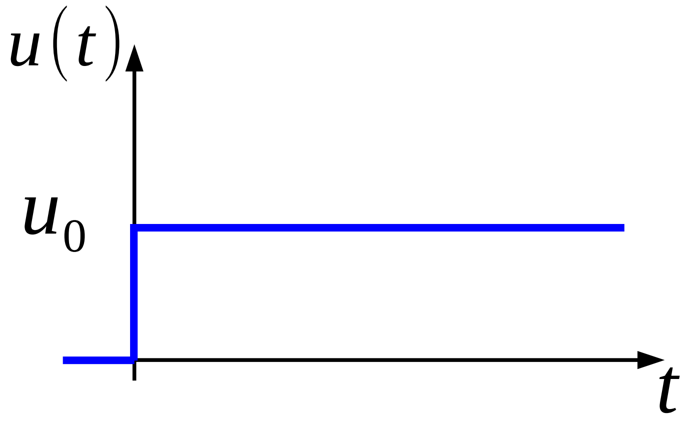
4. Step response:

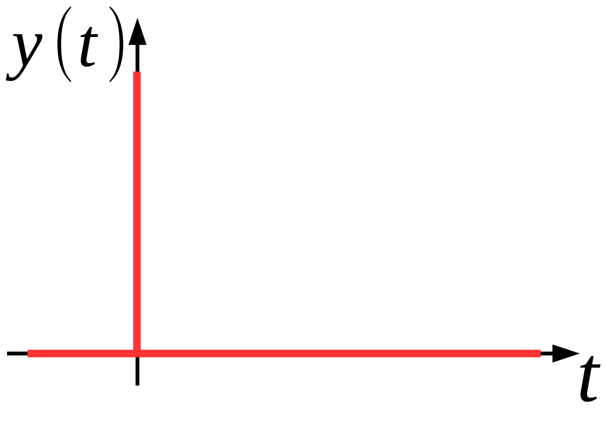
input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0 \frac{1}{s}$

Laplace of output: $Y(s)=H(s)U(s)=k\,u_0$

output: $y(t)=L^{-1}\{Y(s)\}=k\,u_0\,\delta(t)$





Differentiator

5. Frequency response:

Differentiator

5. Frequency response: $H\left(j\omega\right)=jk\omega$

$$P\left(\omega\right)=0,\ \ Q\left(\omega\right)=k\omega$$

6. Nyquist plot:

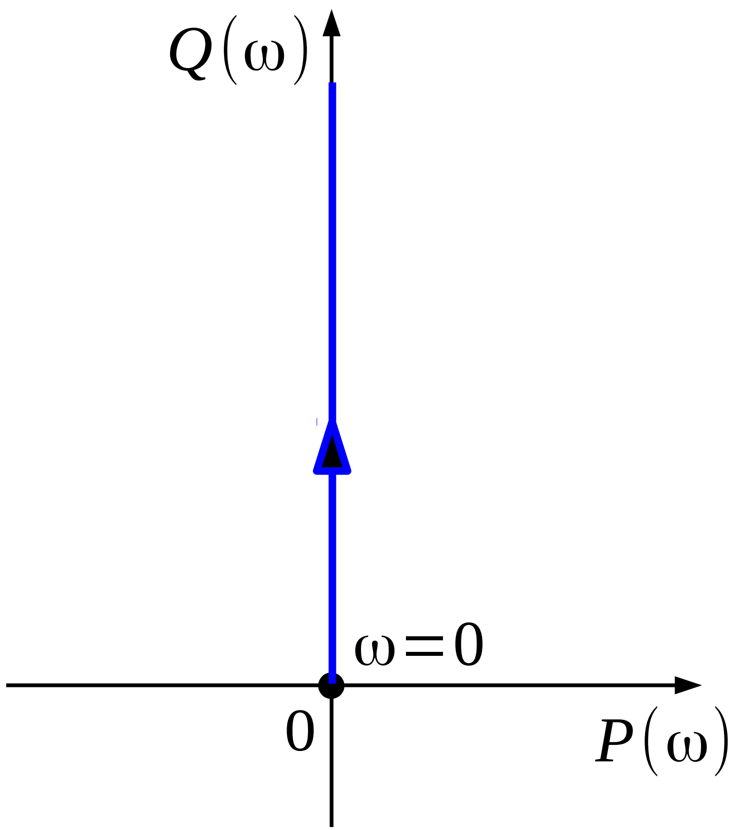
Differentiator

5. Frequency response: $H(j\omega) = jk\omega$

$$P(\omega) = 0, \quad Q(\omega) = k\omega$$

6. Nyquist plot:

for $k > 0$



Differentiator

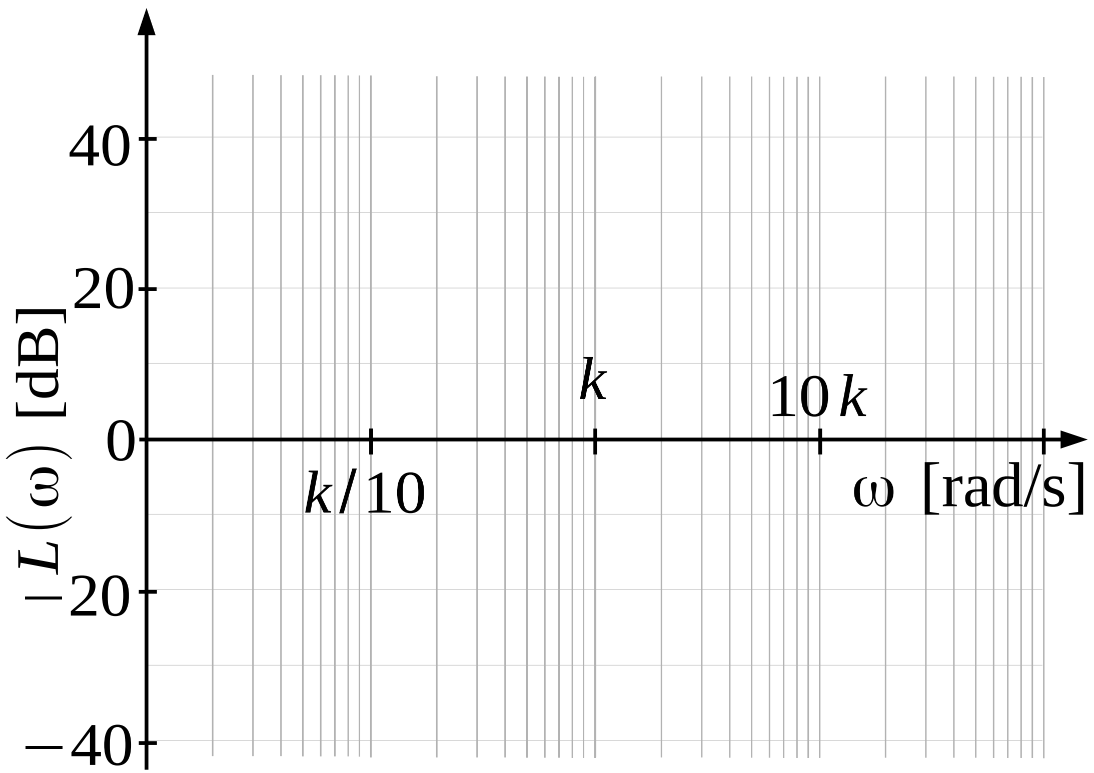
7. Bode plot:

$$P(\omega)=0,\quad Q(\omega)=k\,\omega$$

Differentiator

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k\omega|$

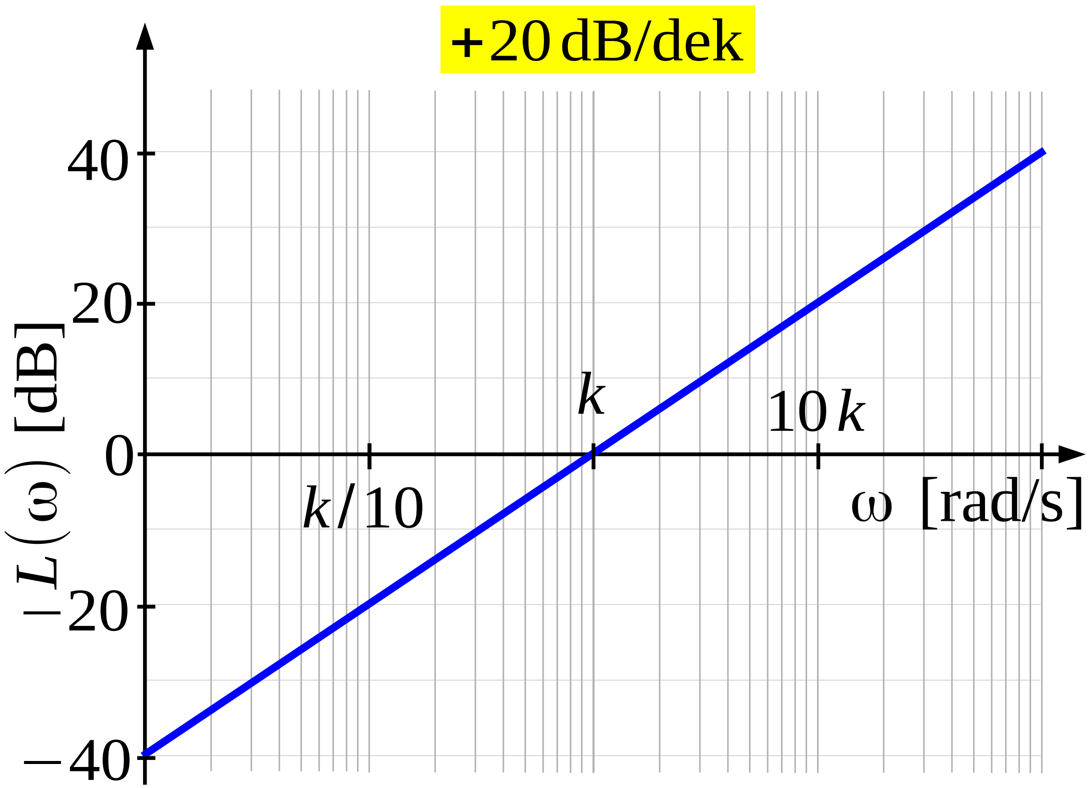
$$L(\omega)=20\log A(\omega)=20\log|k\omega| \qquad \varphi(\omega)=\arctan\frac{Q}{P}=\arctan(\infty)$$



Differentiator

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k\,\omega|$

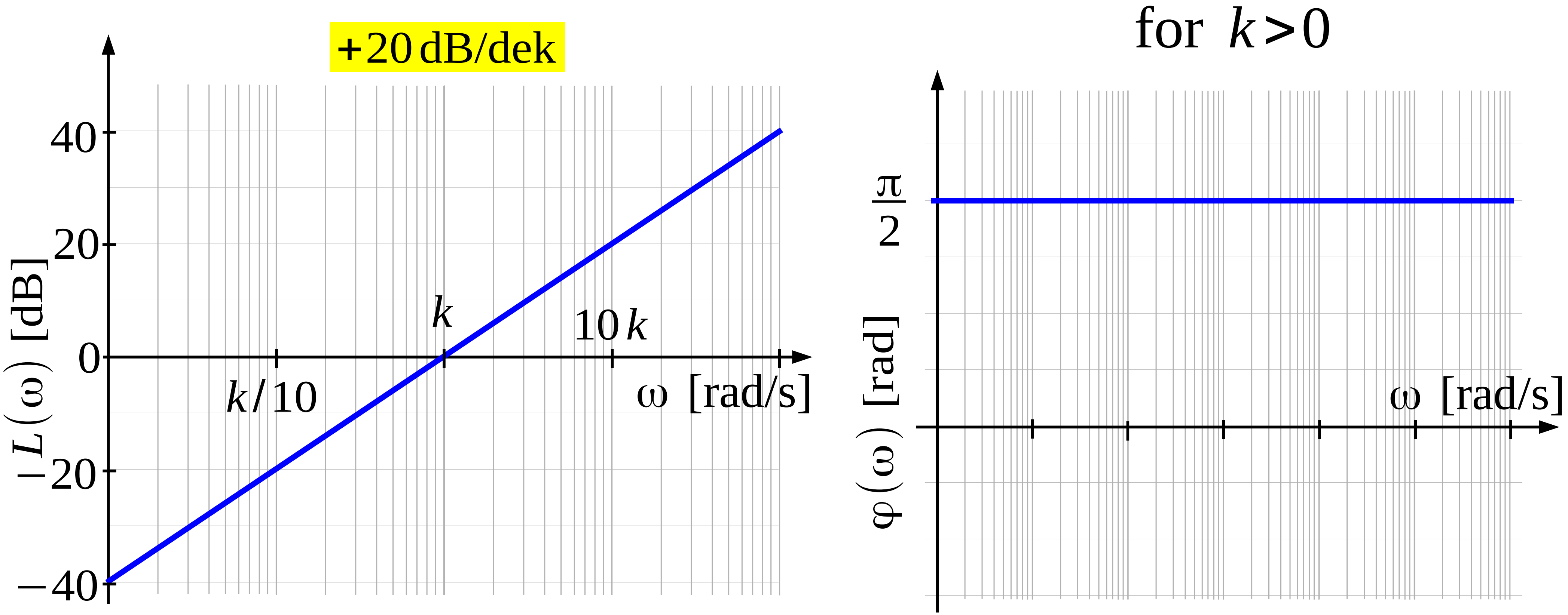
$$L(\omega)=20\log A(\omega)=20\log|k\,\omega| \qquad \varphi(\omega)=\arctan\frac{Q}{P}=\arctan(\infty)$$



Differentiator

7. Bode plot: $A(\omega) = \sqrt{P^2 + Q^2} = |k\,\omega|$

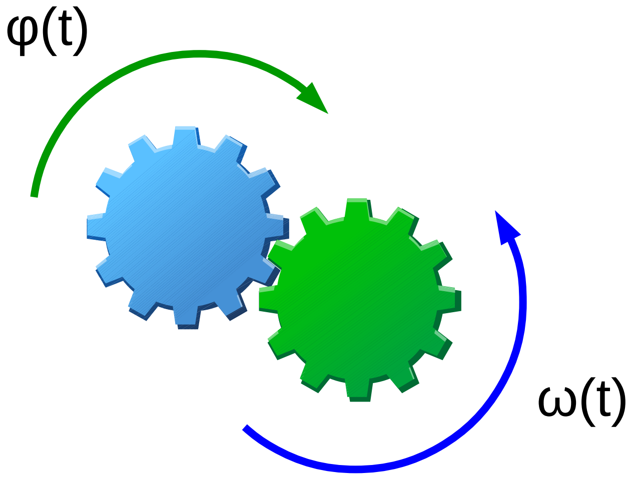
$$L(\omega) = 20 \log A(\omega) = 20 \log |k\,\omega| \qquad \varphi(\omega) = \arctan\frac{Q}{P} = \arctan(\infty)$$



Differentiator

Examples

1

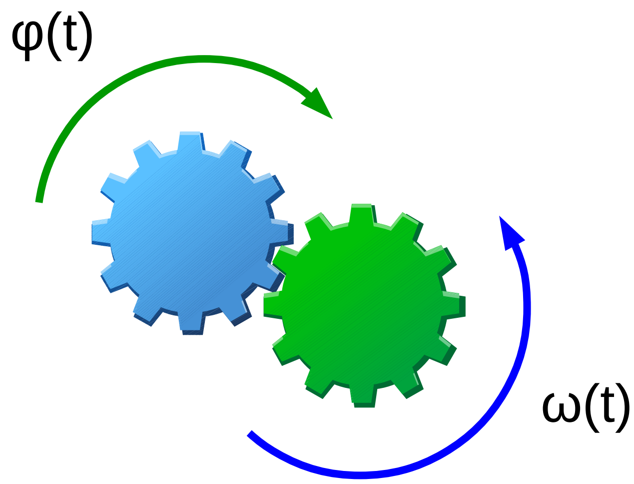


GEARBOX:
input – rotation angle φ(t)
output – angular velocity ω(t)

Differentiator

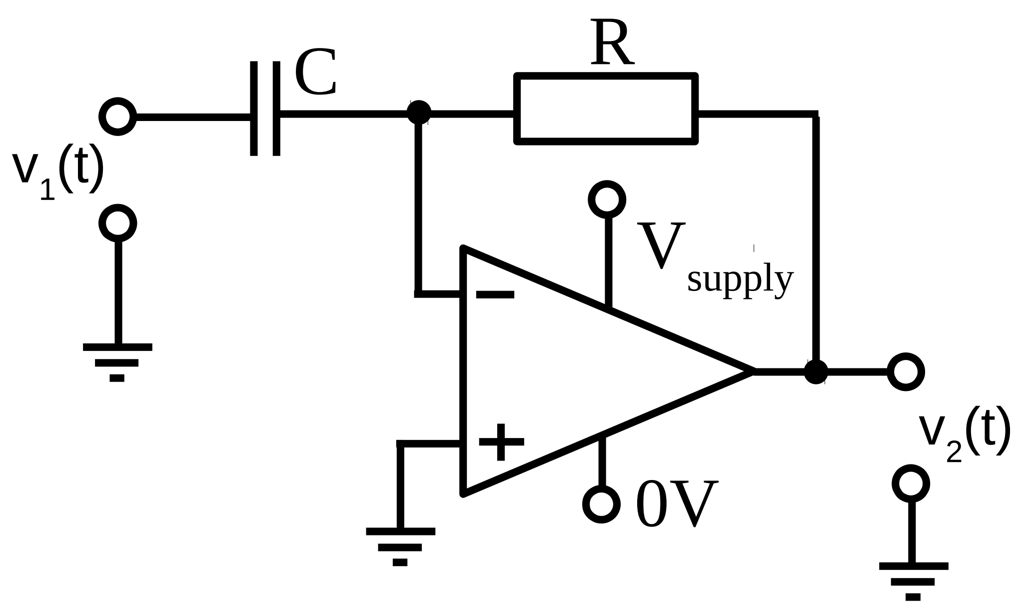
Examples

1



GEARBOX:
input – rotation angle φ(t)
output – angular velocity ω(t)

2



OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = -RC \frac{dv_1(t)}{dt}$$

Real differentiator (derivative+1st order)

1. Element equation: $T\frac{dy(t)}{dt}+y(t)=k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

Real differentiator (derivative+1st order)

1. Element equation: $T\frac{dy(t)}{dt}+y(t)=k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

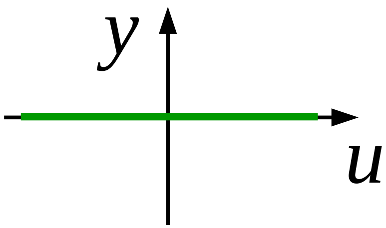
2. Static characteristic (steady state):

Real differentiator (derivative+1st order)

1. Element equation: $T\frac{dy(t)}{dt}+y(t)=k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $y=0$ for $\frac{dy}{dt}=0\wedge\frac{du}{dt}=0$



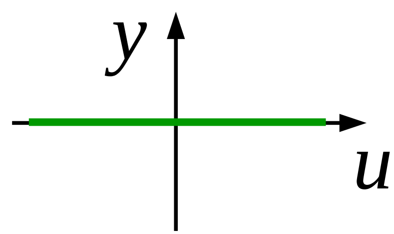
3. Transfer function:

Real differentiator (derivative+1st order)

1. Element equation: $T\frac{dy(t)}{dt}+y(t)=k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $y=0$ for $\frac{dy}{dt}=0\wedge\frac{du}{dt}=0$



3. Transfer function: $H(s)=\frac{k\,s}{Ts+1}$

Real differentiator (derivative+1st order)

input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0 \frac{1}{s}$

4. Step response:

Real differentiator (derivative+1st order)

4. Step response:

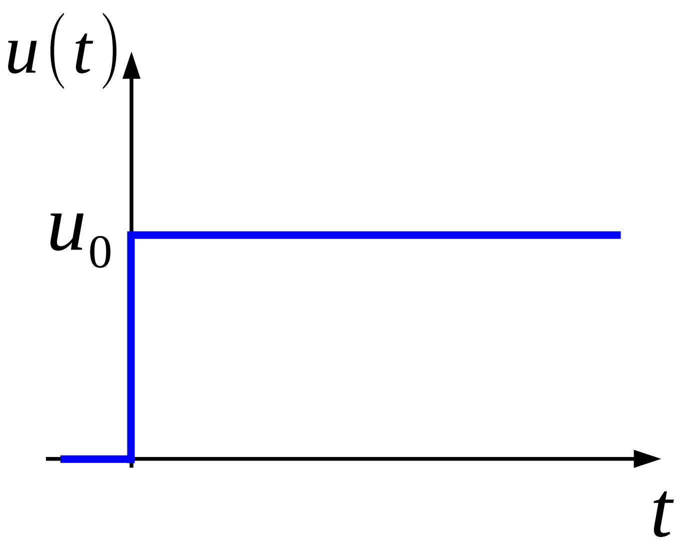
input: $u(t)=u_0 1(t)$

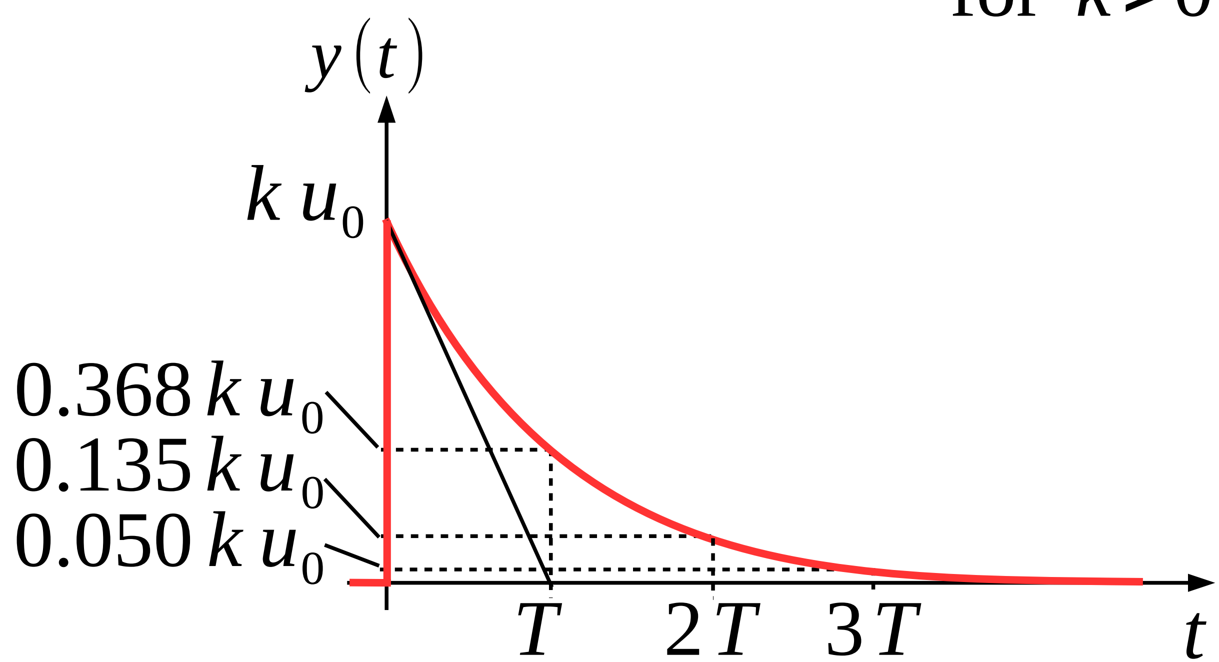
Laplace of input: $U(s)=u_0 \frac{1}{s}$

Laplace of output: $Y(s)=H(s)U(s)=\frac{k u_0}{Ts+1}$

output: $y(t)=L^{-1}\{Y(s)\}=k u_0 e^{-t/T}$

for $k>0$





Real differentiator (derivative+1st order)

5. Frequency response:

Real differentiator (derivative+1st order)

5. Frequency response: $H(j\omega)=\dfrac{k\ j\omega}{Tj\omega+1}$

$$P(\omega)=\frac{kT\omega^2}{T^2\omega^2+1},\quad Q(\omega)=\frac{k\omega}{T^2\omega^2+1}$$

6. Nyquist plot:

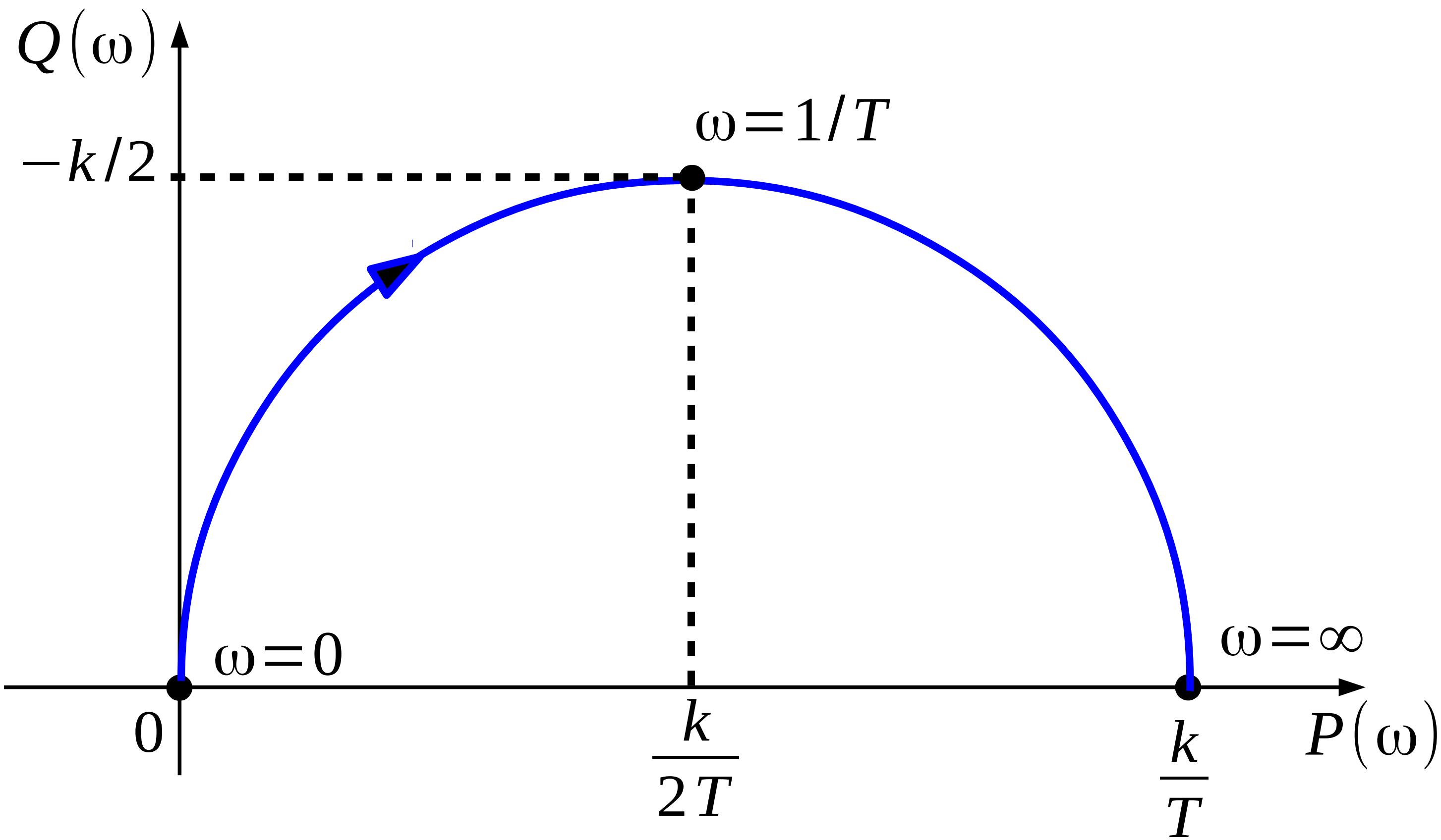
Real differentiator (derivative+1st order)

5. Frequency response: $H(j\omega)=\dfrac{k\,j\omega}{Tj\omega+1}$

$$P(\omega)=\frac{kT\omega^2}{T^2\omega^2+1},\quad Q(\omega)=\frac{k\omega}{T^2\omega^2+1}$$

6. Nyquist plot:

for $k>0$



Real differentiator (derivative+1st order)

7. Bode plot:

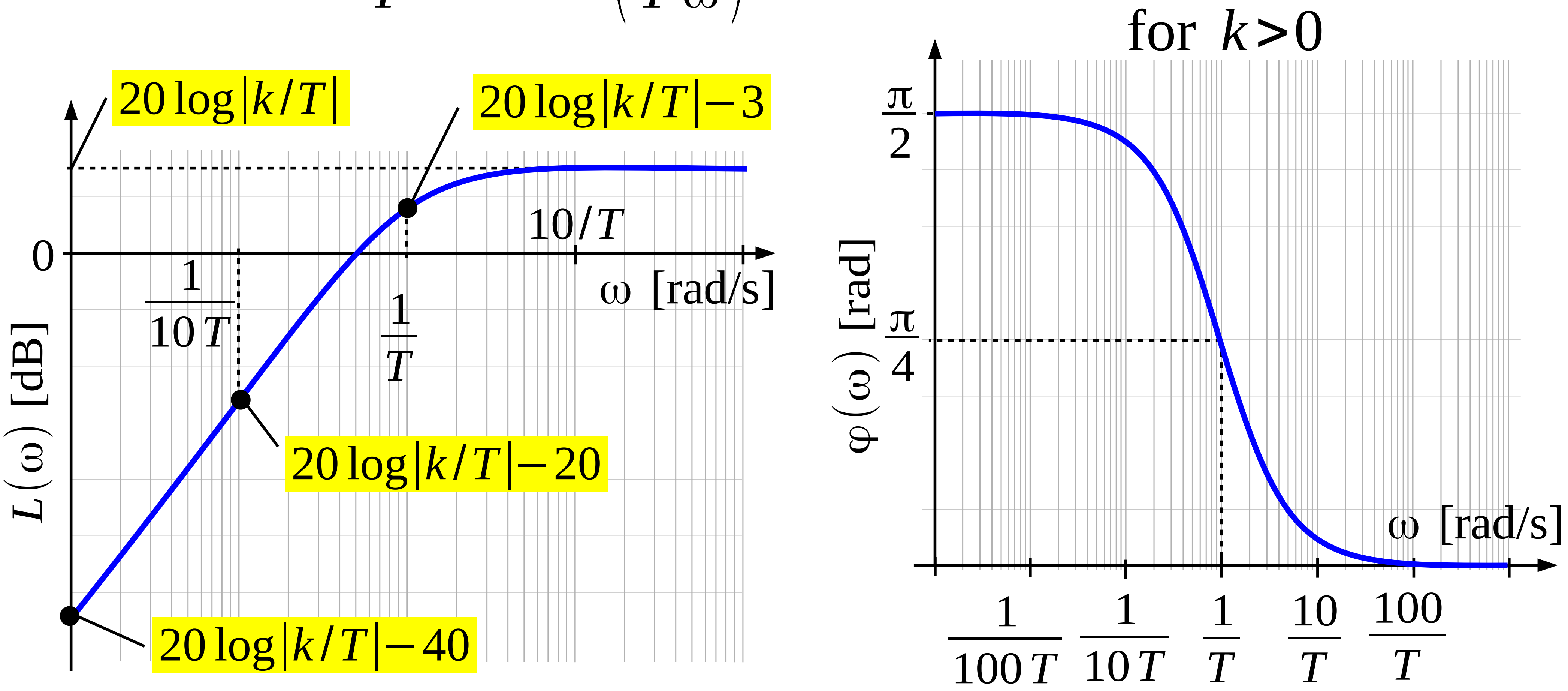
$$P(\omega)=\frac{k\,T\,\omega^2}{T^2\omega^2+1},\quad Q(\omega)=\frac{k\,\omega}{T^2\omega^2+1}$$

Real differentiator (derivative+1st order)

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k\,\omega|/\sqrt{T^2\omega^2+1}$

$$L(\omega)=20\log A(\omega)=20\log|k\,\omega|-20\log\sqrt{T^2\omega^2+1}$$

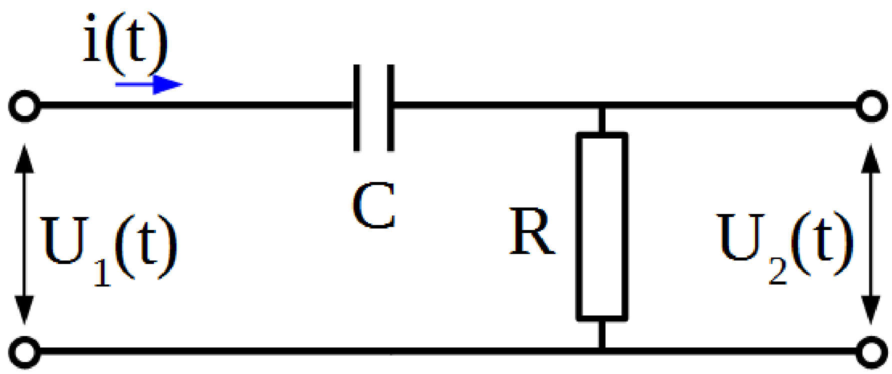
$$\varphi(\omega)=\arctan\frac{Q}{P}=\arctan\left(\frac{1}{T\,\omega}\right)$$



Real differentiator (derivative+1st order)

Examples

1



RC CIRCUIT:
input – voltage $u_1(t)$
output – voltage $u_2(t)$

Delay

$u(t)$ - input
$y(t)$ - output

1. Element equation: $y(t)=u(t-\tau)$

Delay

1. Element equation: $y(t)=u(t-\tau)$

$u(t)$ - input
$y(t)$ - output

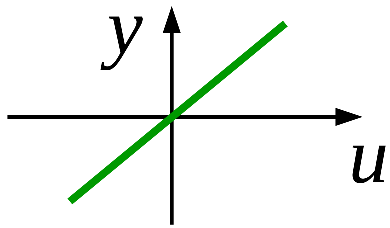
2. Static characteristic (steady state):

Delay

1. Element equation: $y(t)=u(t-\tau)$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $y=u$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$



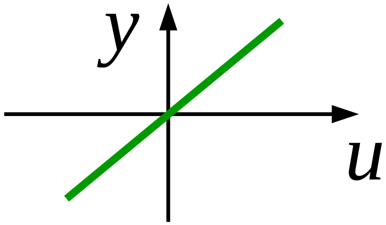
3. Transfer function:

Delay

1. Element equation: $y(t)=u(t-\tau)$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $y=u$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$



3. Transfer function: $H(s)=e^{-\tau s}$

Delay

4. Step response:

input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0\frac{1}{s}$

Delay

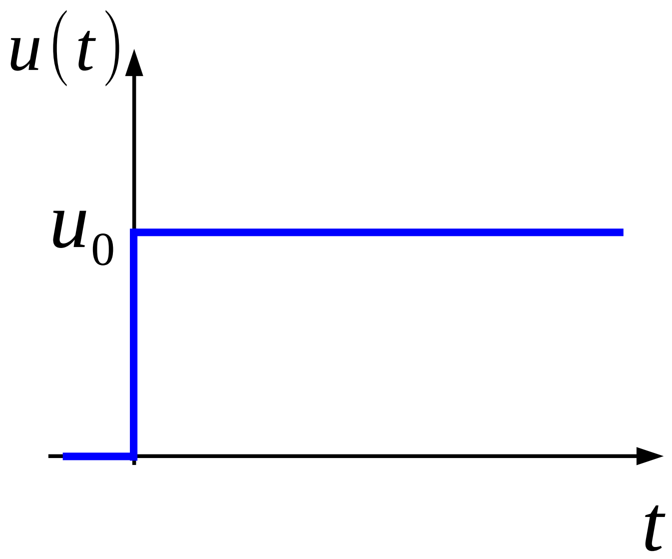
4. Step response:

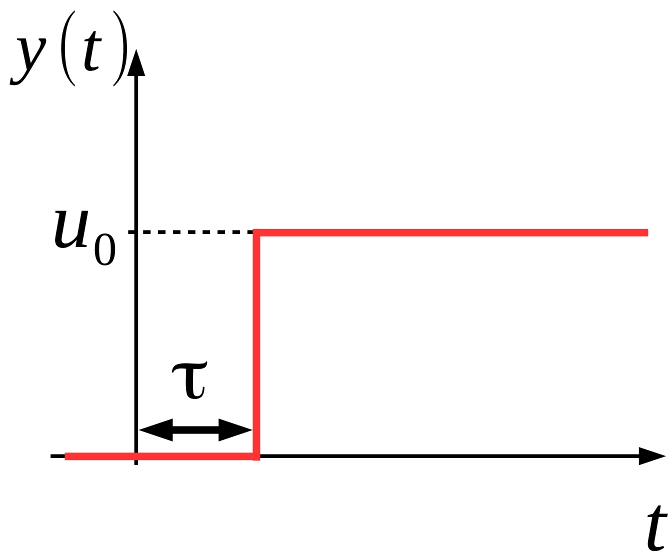
input: $u(t) = u_0 1(t)$

Laplace of input: $U(s) = u_0 \frac{1}{s}$

Laplace of output: $Y(s) = H(s) U(s) = \frac{u_0}{s} e^{-\tau s}$

output: $y(t) = L^{-1}\{Y(s)\} = u_0 1(t-\tau)$





Delay

5. Frequency response:

Delay

5. Frequency response: $H(j\omega)=e^{-\tau j\omega}$

$$e^{-jx}=\cos x-j\sin x$$

$$P(\omega)=\cos(\tau\omega),\quad Q(\omega)=-\sin(\tau\omega)$$

6. Nyquist plot:

Delay

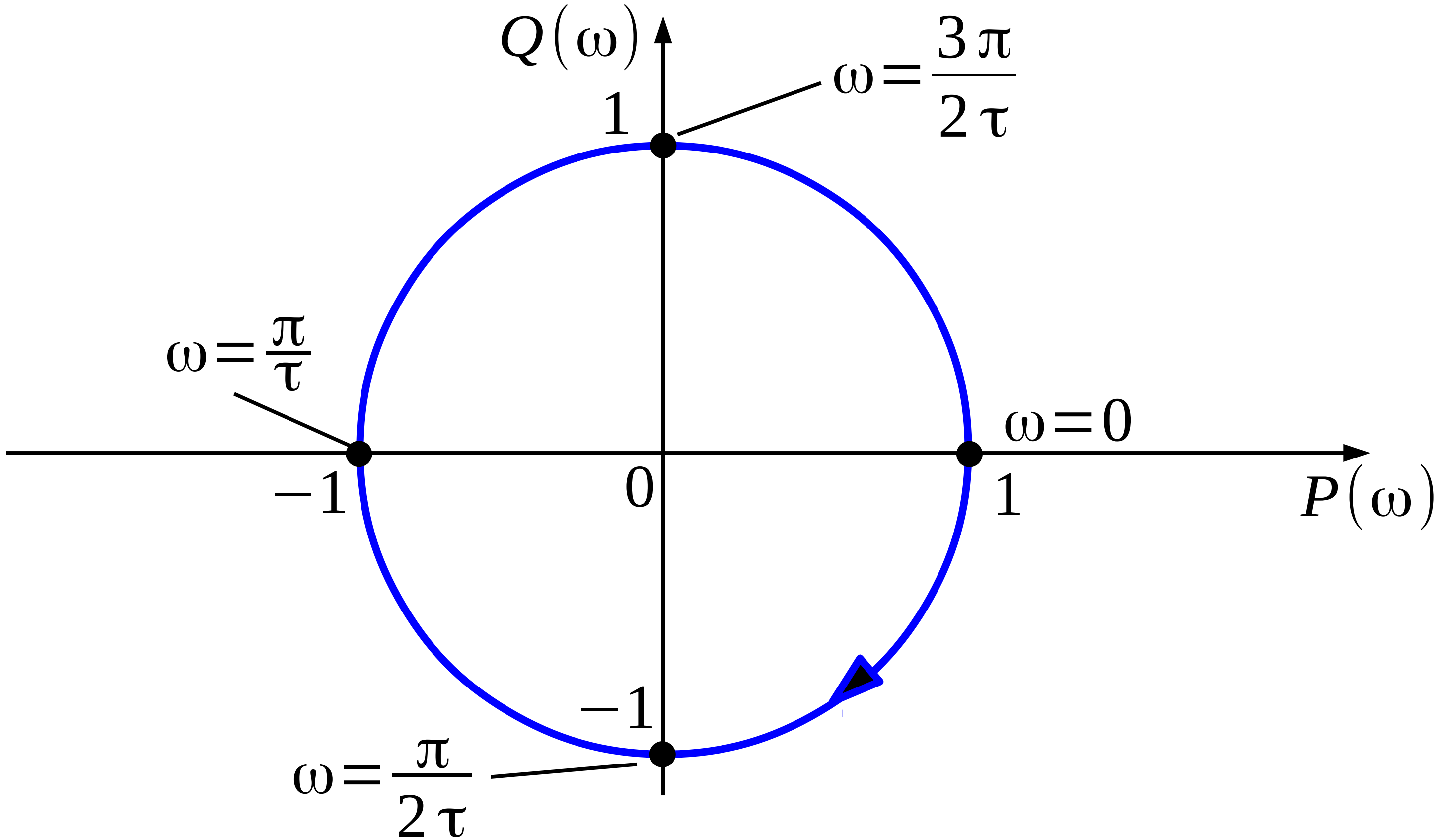
5. Frequency response: $H(j\omega)=e^{-\tau j\omega}$

$$e^{-jx}=\cos x - j\sin x$$

$$P(\omega)=\cos(\tau\omega),\;\; Q(\omega)=-\sin(\tau\omega)$$

6. Nyquist plot:

for $k>0$



Delay

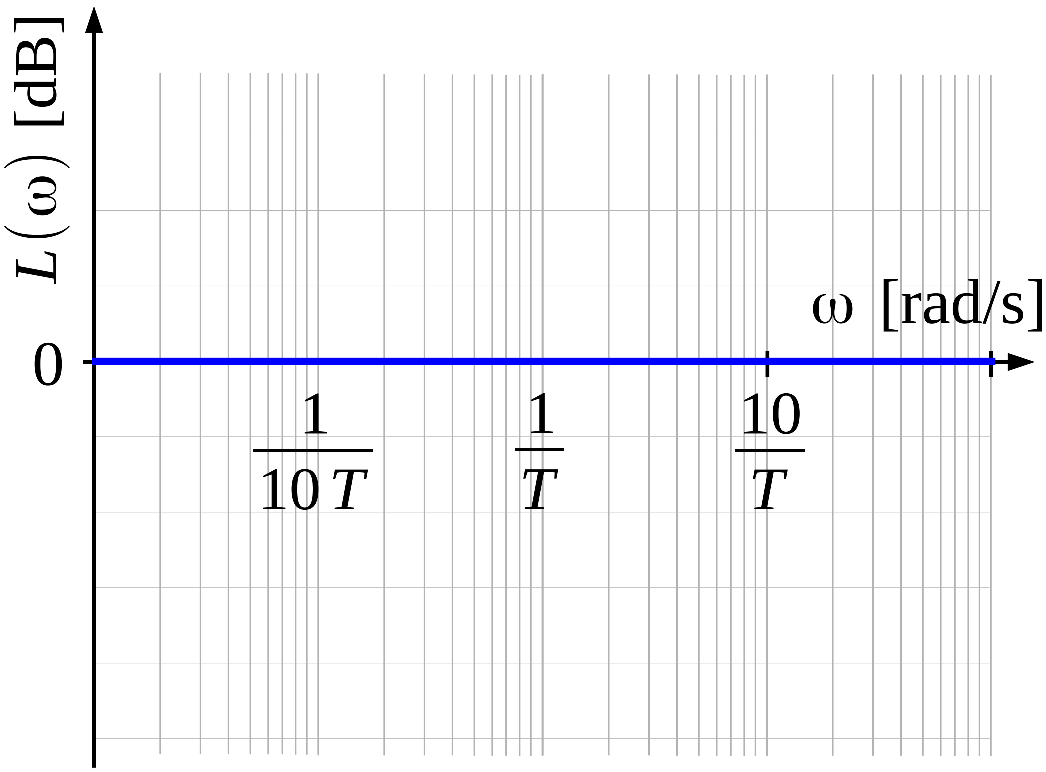
7. Bode plot: $P(\omega) = \cos(\tau\omega), \quad Q(\omega) = -\sin(\tau\omega)$

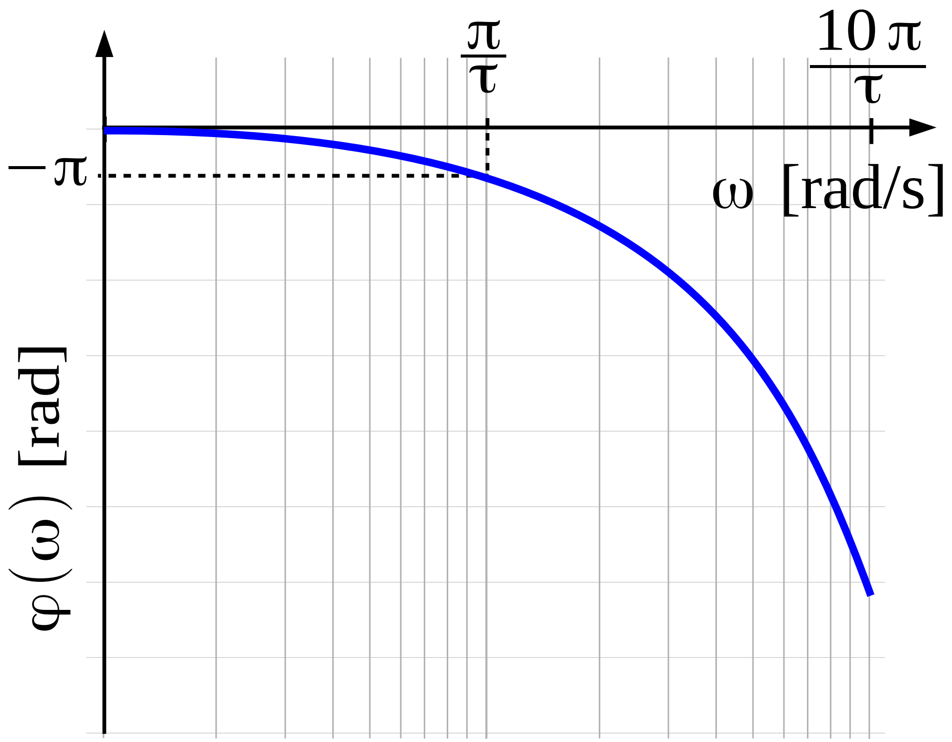
Delay

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=1$

$$L(\omega)=20\log A(\omega)=20\log 1=0$$

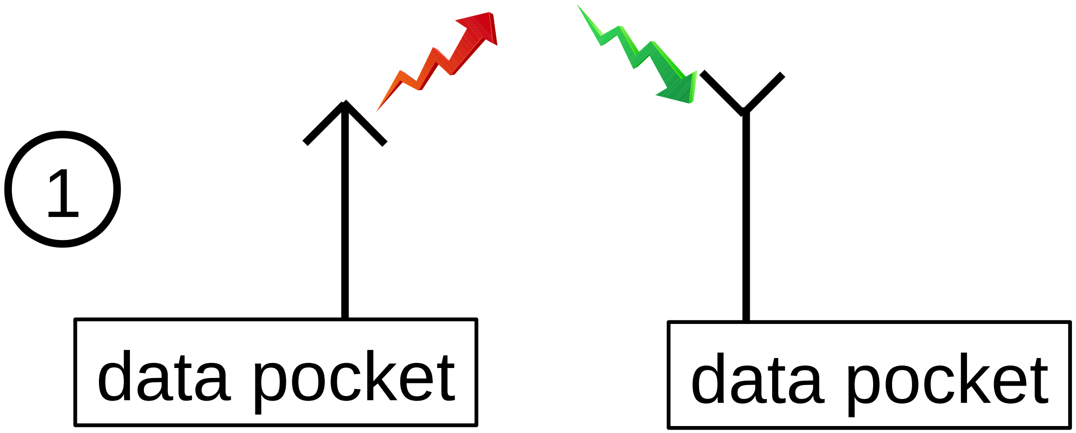
$$\varphi(\omega)=\arctan\frac{Q}{P}=\arctan\left(-\tan(\tau\omega)\right)=-\tau\omega$$





Delay

Examples



WIRELESS TRANSMISSION:
input – sent data
output – received data

Second-order inertial element

1. Element equation: $T_1^2 \frac{d^2 y(t)}{dt^2} + T_2 \frac{dy(t)}{dt} + y(t) = k\,u(t)$

Second-order inertial element

1. Element equation: $$T_1^2 \frac{d^2 y(t)}{dt^2} + T_2 \frac{dy(t)}{dt} + y(t) = k\,u(t)$$

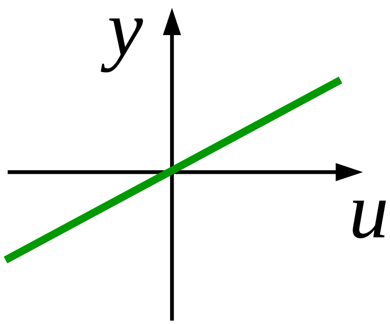
---

2. Static characteristic (steady state):

Second-order inertial element

1. Element equation: $T_1^2 \frac{d^2 y(t)}{dt^2} + T_2 \frac{dy(t)}{dt} + y(t) = k\,u(t)$

2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$

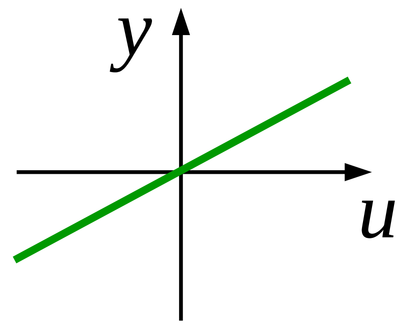


3. Transfer function:

Second-order inertial element

1. Element equation: $$T_1^2 \frac{d^2 y(t)}{dt^2} + T_2 \frac{dy(t)}{dt} + y(t) = k\,u(t)$$

2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



3. Transfer function: $$H(s) = \frac{k}{T_1^2 s^2 + T_2 s + 1}$$

Second-order inertial element

4. Step response:

Second-order inertial element

4. Step response:

input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0 \frac{1}{s}$

Laplace of output: $Y(s)=H(s)U(s)=\frac{k u_0}{s\left(T_1^2 s^2+T_2 s+1\right)}$

Second-order inertial element

4. Step response:

input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0\frac{1}{s}$

Laplace of output: $Y(s)=H(s)U(s)=\frac{k u_0}{s\left(T_1^2 s^2+T_2 s+1\right)}$

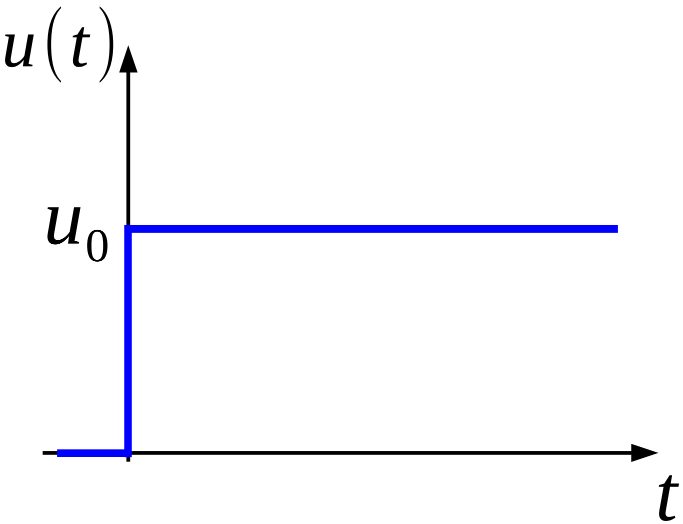
output: $y(t)=L^{-1}\{Y(s)\}=$

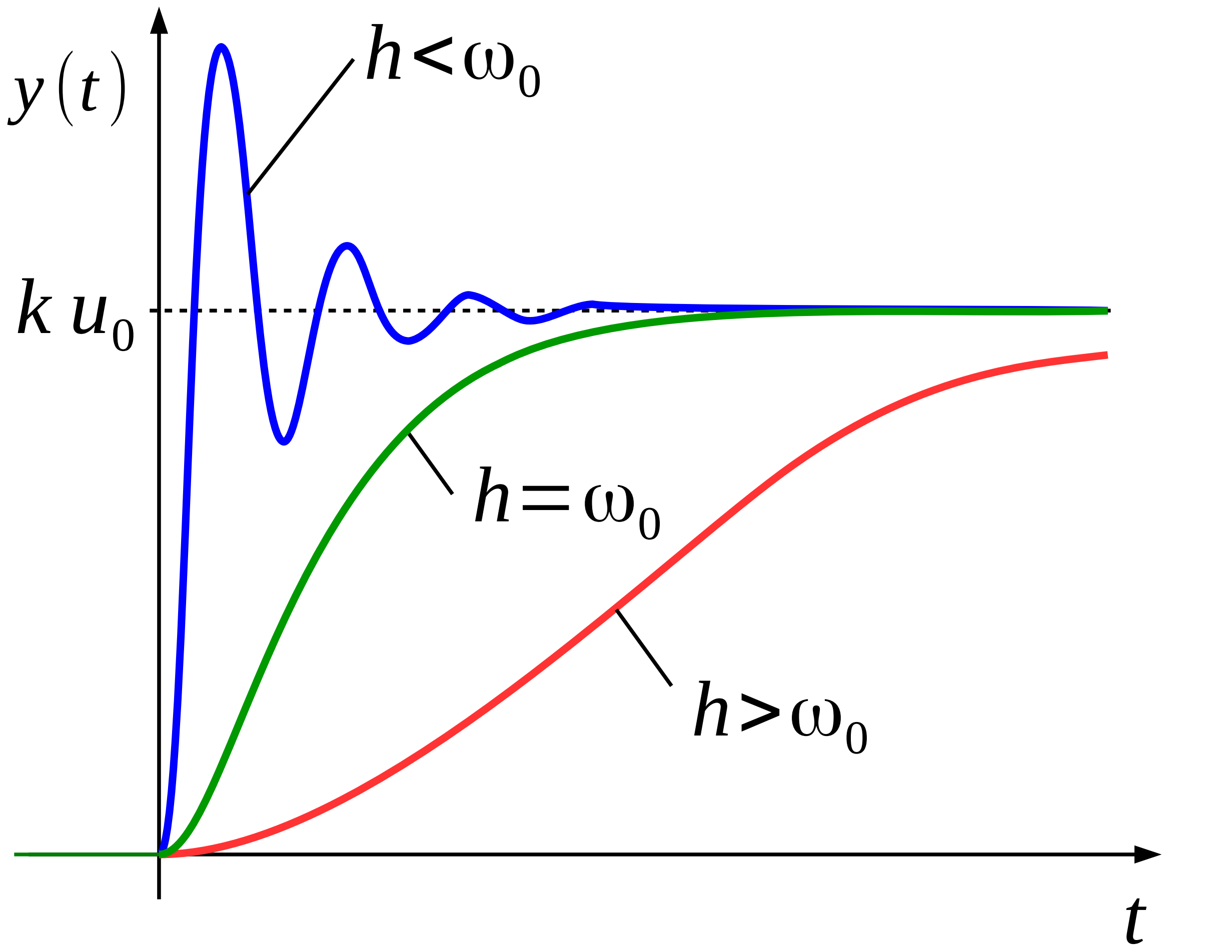
$$=\begin{cases} k u_0 \omega_0^2\left(1-e^{-ht}\left(\cos\omega t+\frac{h}{\omega}\sin\omega t\right)\right), \text{ for } h\le\omega_0 \\ k u_0 \omega_0^2\left(1+e^{-ht}\left(\left(\frac{h+w}{2w}-1\right)e^{-wt}-\frac{h+w}{2w}e^{wt}\right)\right), \text{ for } h\ge\omega_0 \end{cases}$$

where: $h=\frac{T_2}{2T_1^2},\ \omega_0=\frac{1}{T_1},\ \omega=\sqrt{\omega_0^2-h^2},\ w=\sqrt{h^2-\omega_0^2}$

Second-order inertial element

4. Step response:





Second-order inertial element

5. Frequency response:

Second-order inertial element

5. Frequency response: $H(j\omega)=\dfrac{k}{-T_1^2\omega^2+T_2 j\omega+1}$

$$P(\omega)=\frac{k\left(1-T_1^2\omega^2\right)}{\left(1-T_1^2\omega^2\right)^2+T_2^2\omega^2},\quad Q(\omega)=\frac{-k\,T_2\,\omega}{\left(1-T_1^2\omega^2\right)^2+T_2^2\omega^2}$$

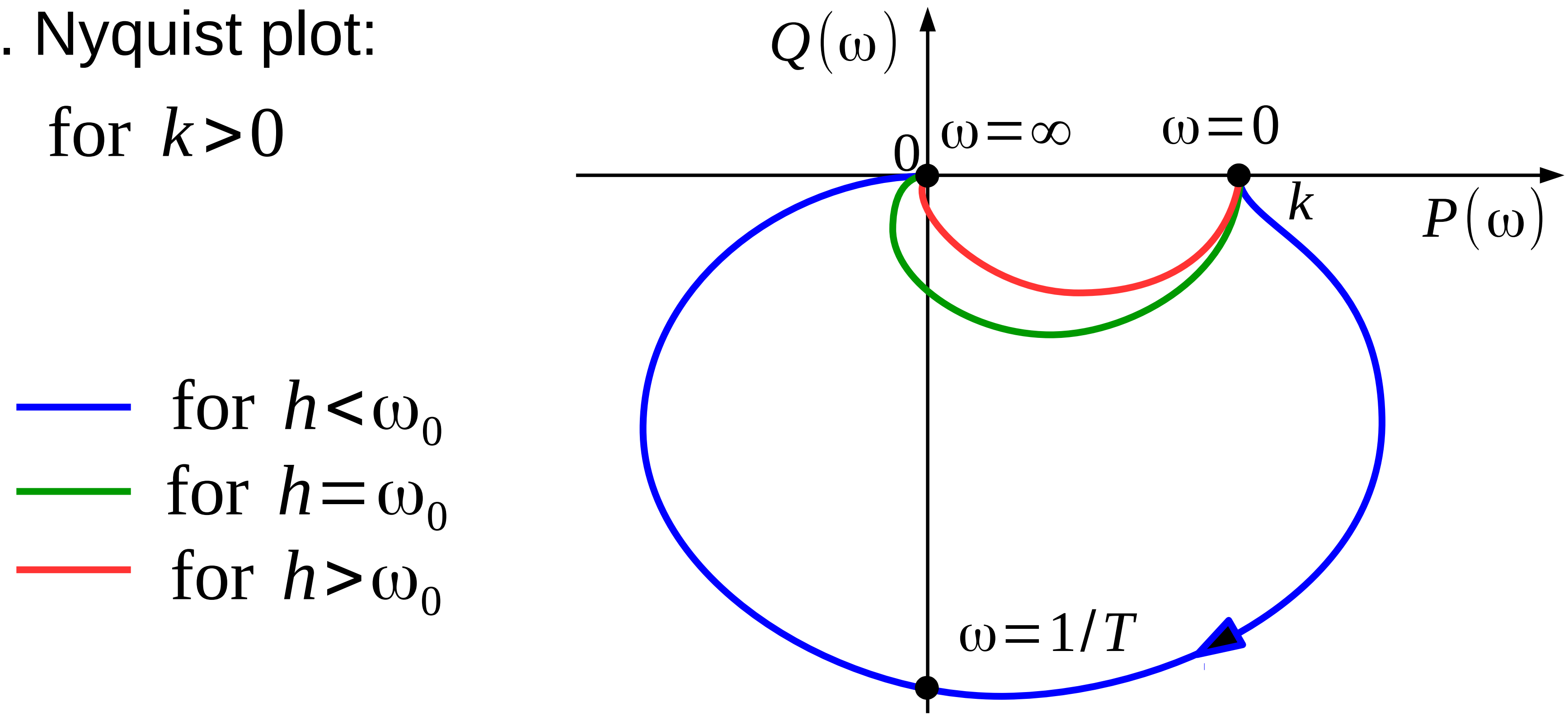
Second-order inertial element

5. Frequency response: $H(j\omega)=\frac{k}{-T_1^2\omega^2+T_2 j\omega+1}$

$$P(\omega)=\frac{k(1-T_1^2\omega^2)}{(1-T_1^2\omega^2)^2+T_2^2\omega^2},\quad Q(\omega)=\frac{-kT_2\omega}{(1-T_1^2\omega^2)^2+T_2^2\omega^2}$$

6. Nyquist plot:

for $k>0$

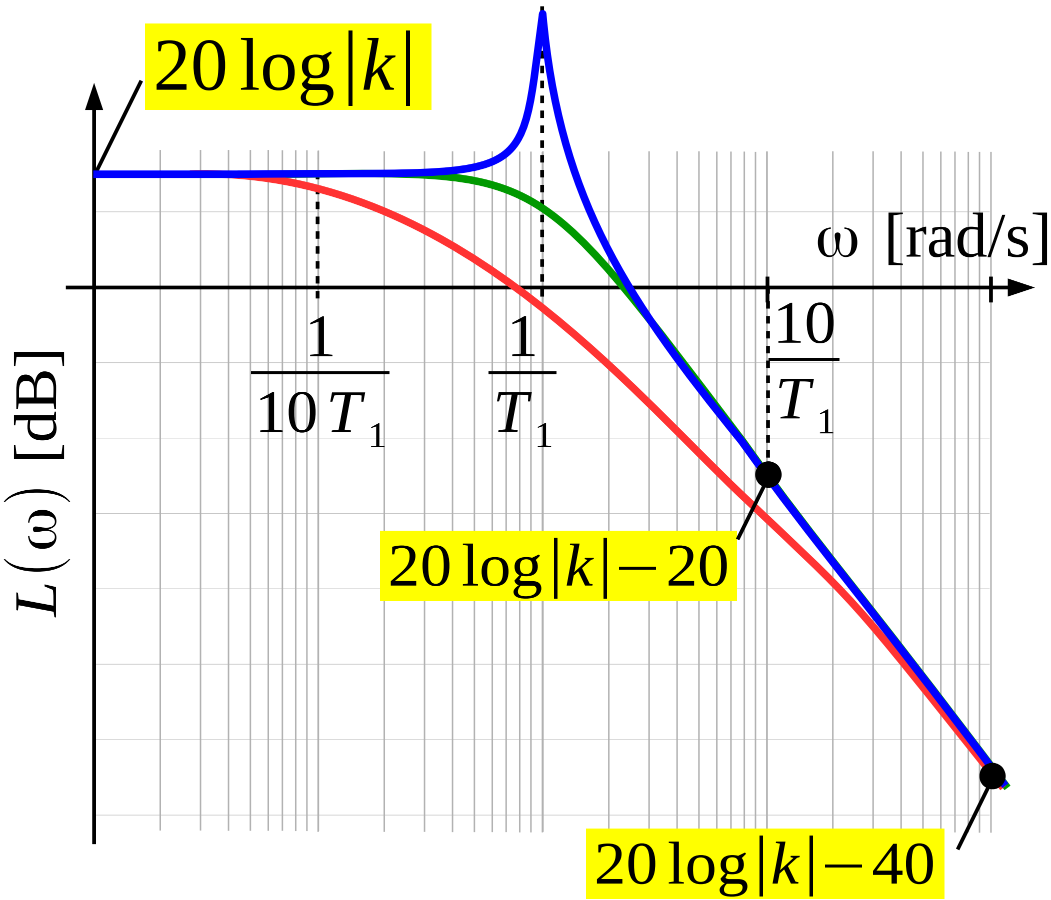


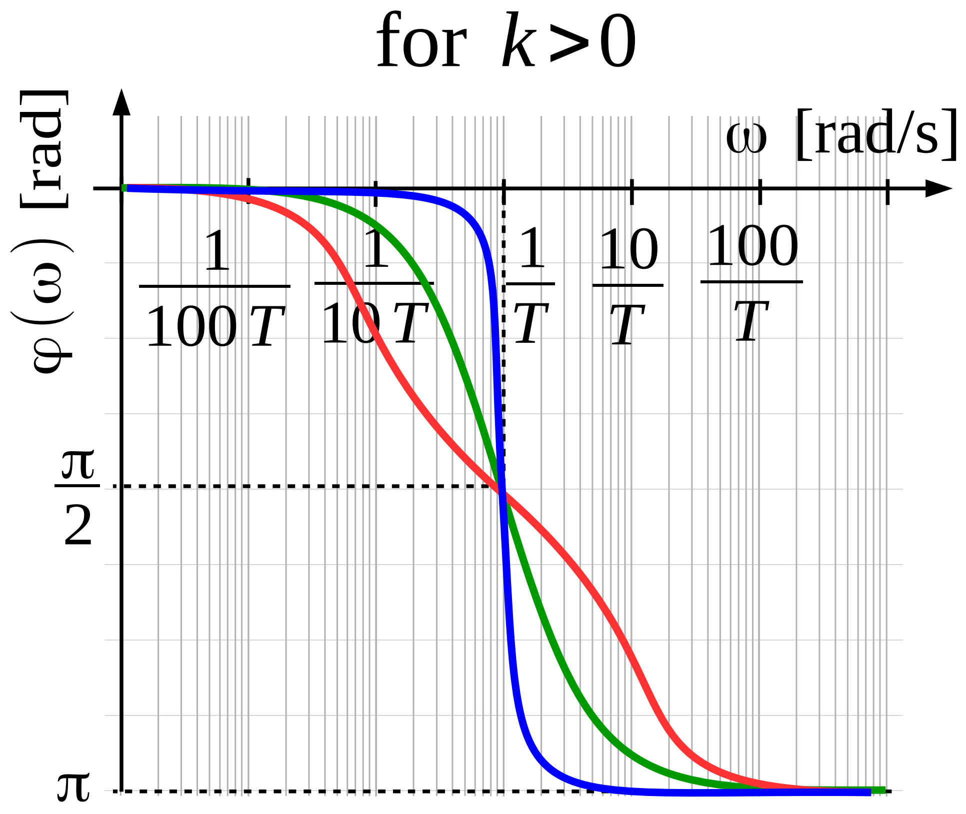
Second-order inertial element

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}$

$L(\omega)=20\log A(\omega)$

$\varphi(\omega)=\arctan\frac{Q}{P}$



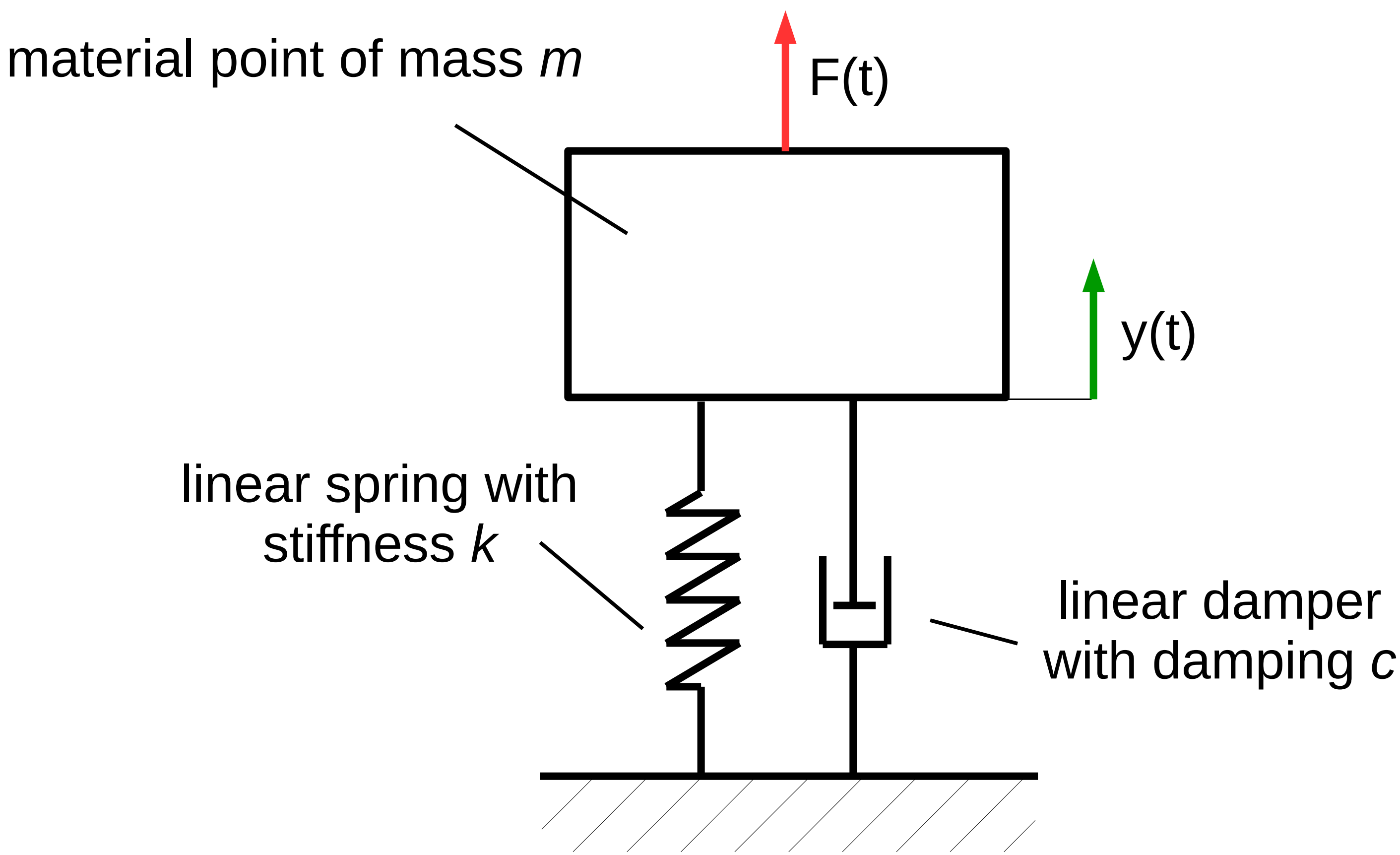


Second-order inertial element

<u>Examples</u>

1

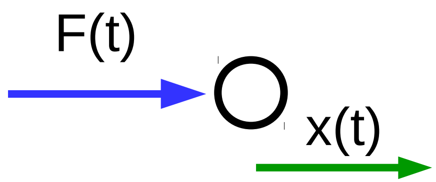
VIBRATING SYSTEM:
input – force F(t)
output – displacement y(t)



Second-order inertial element

Examples

2



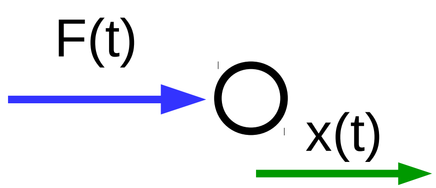
LINEAR MOTION OF A MATERIAL POINT WITH LINEAR DAMPING:
input – force F(t)
output – displacement x(t)

example: car driving on a flat surface with air resistance proportional to velocity, described using machine equation of motion, with assumption of constant reduced mass.

Second-order inertial element

Examples

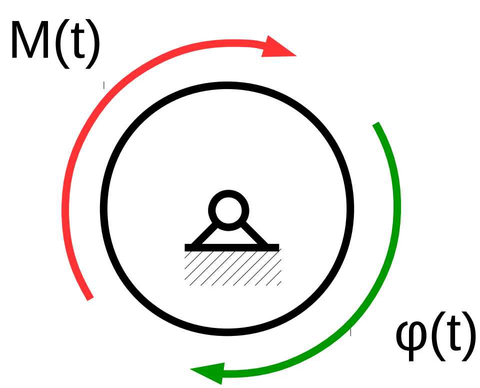
2



LINEAR MOTION OF A MATERIAL POINT WITH LINEAR DAMPING:
input – force F(t)
output – displacement x(t)

example: car driving on a flat surface with air resistance proportional to velocity, described using machine equation of motion, with assumption of constant reduced mass.

3

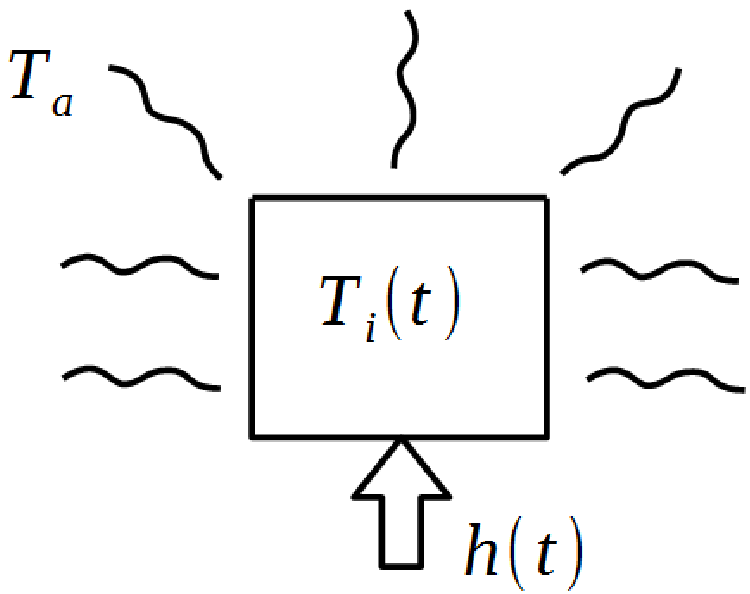


ANGULAR MOTION OF A RIGID BODY WITH LINEAR DAMPING:
input – torque M(t)
output – angle φ(t)

Second-order inertial element

Examples

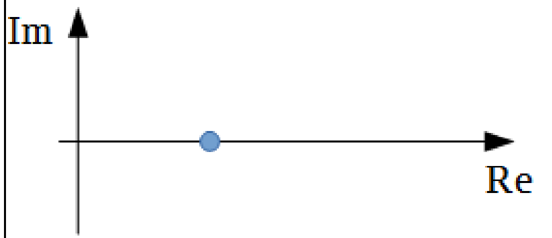
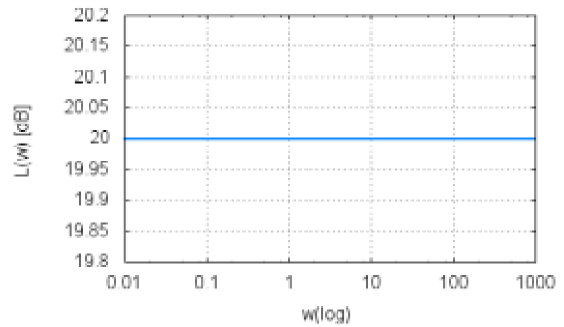
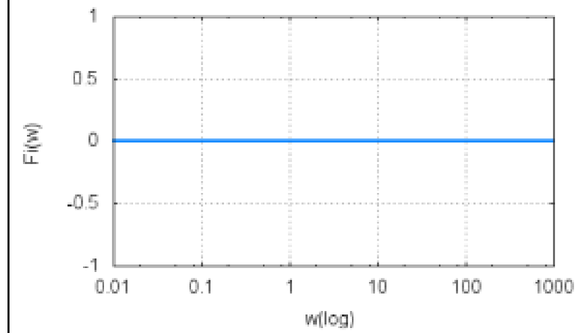
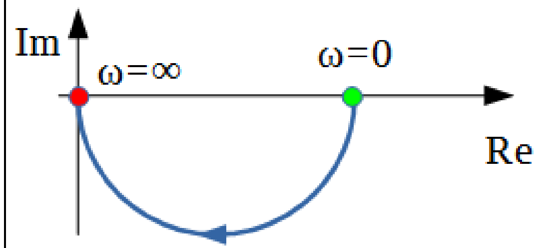
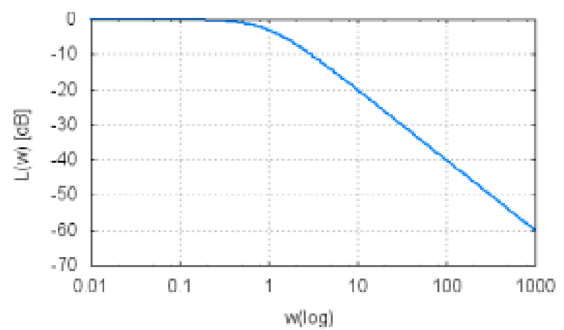
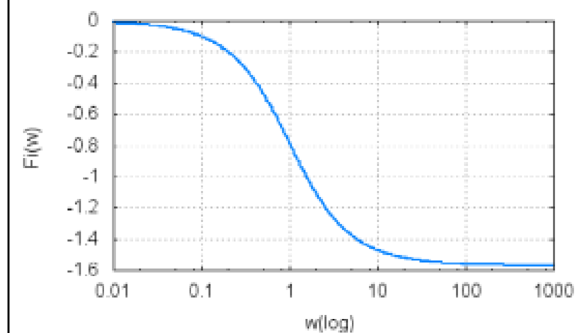
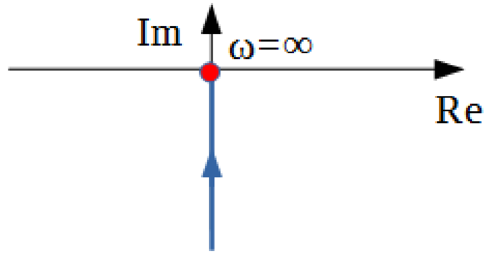
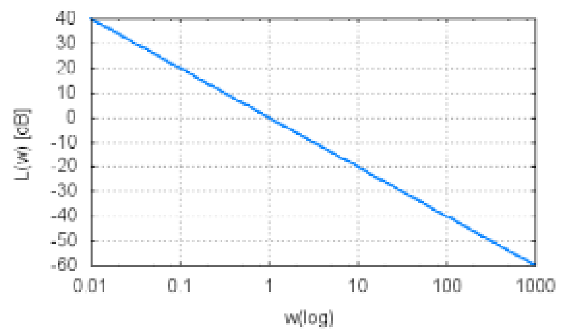
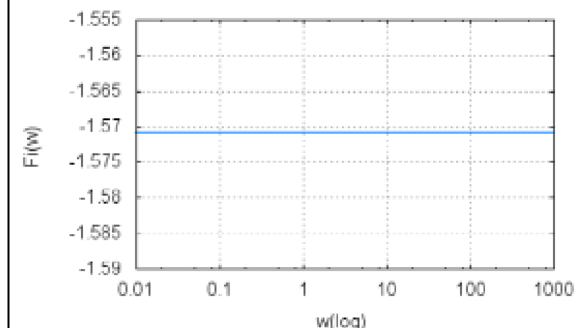
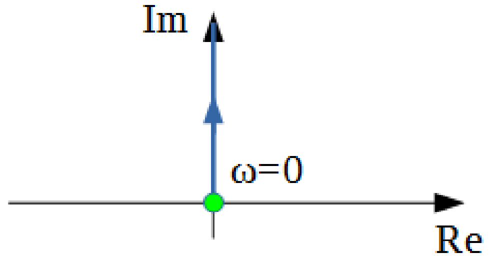
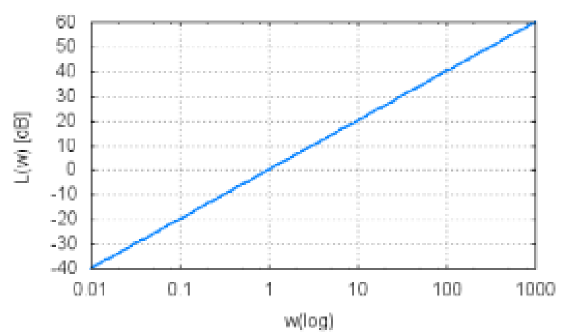
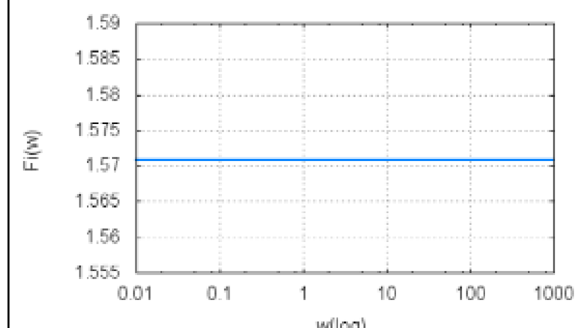
4



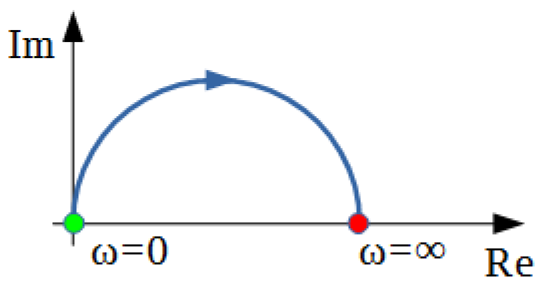
HEATED OBJECT WITH HIGH INERTIA:
input – heater power h(t)
output – object temperature $T_i(t)$

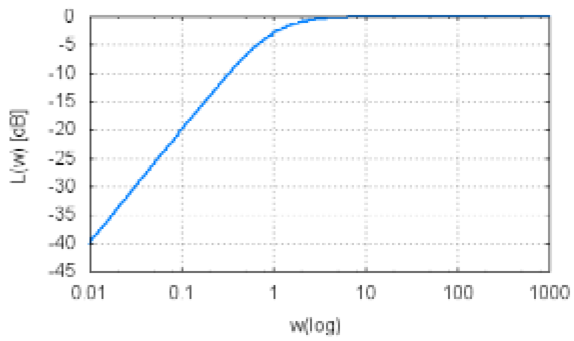
Classification of basic automatic systems

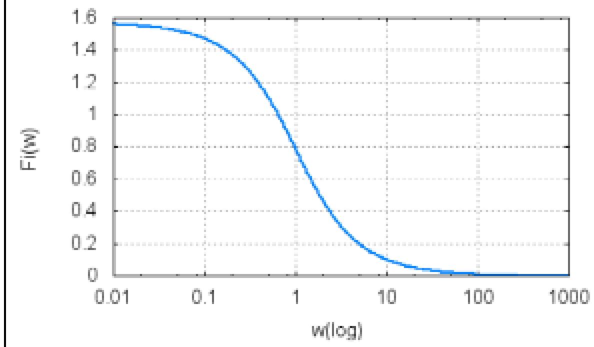
Element name	Transfer function
proportional	k
first order (inertial)	$\frac{k}{Ts+1}$
integrator	$\frac{k}{s}$
differentiator	ks
differentiator with inertia	$\frac{ks}{Ts+1}$
delay	$e^{-\tau s}$
second order (oscillator)	$\frac{k}{T_1^2 s^2 + T_2 s + 1}$

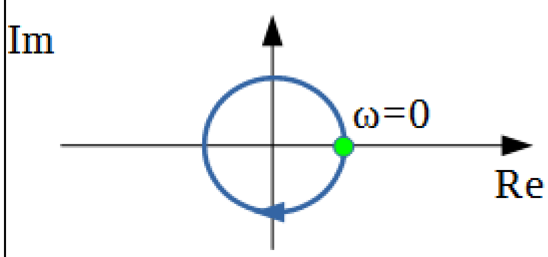
Nyquist plot	Bode Plot (example)	
		
		
		
		

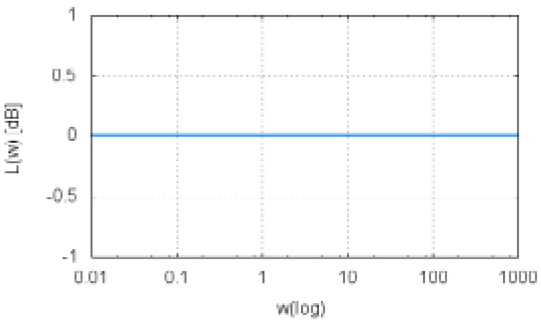
Nyquist plot	Bode Plot (example)	

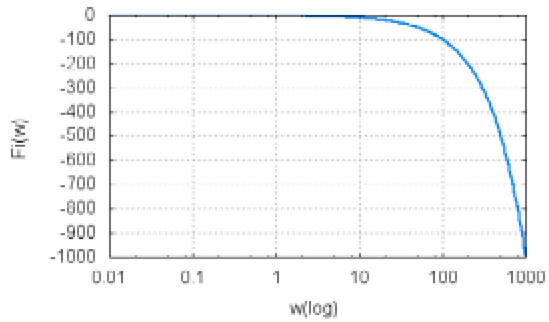


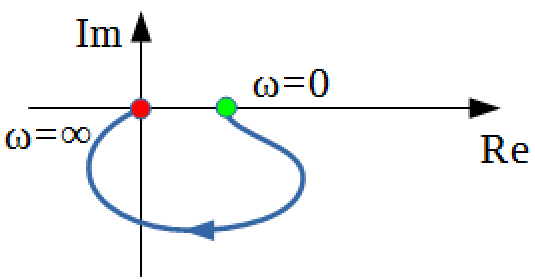


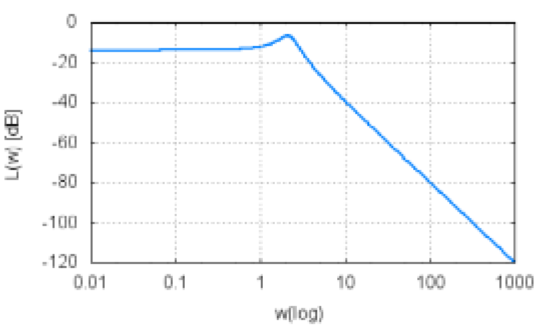


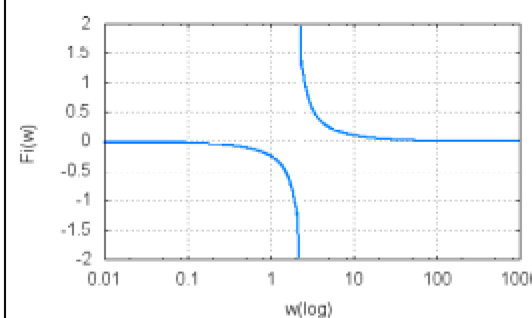












Example

Calculate and sketch step response and Bode Plots for a system with transfer function $H(s) = 3/s$.

Example

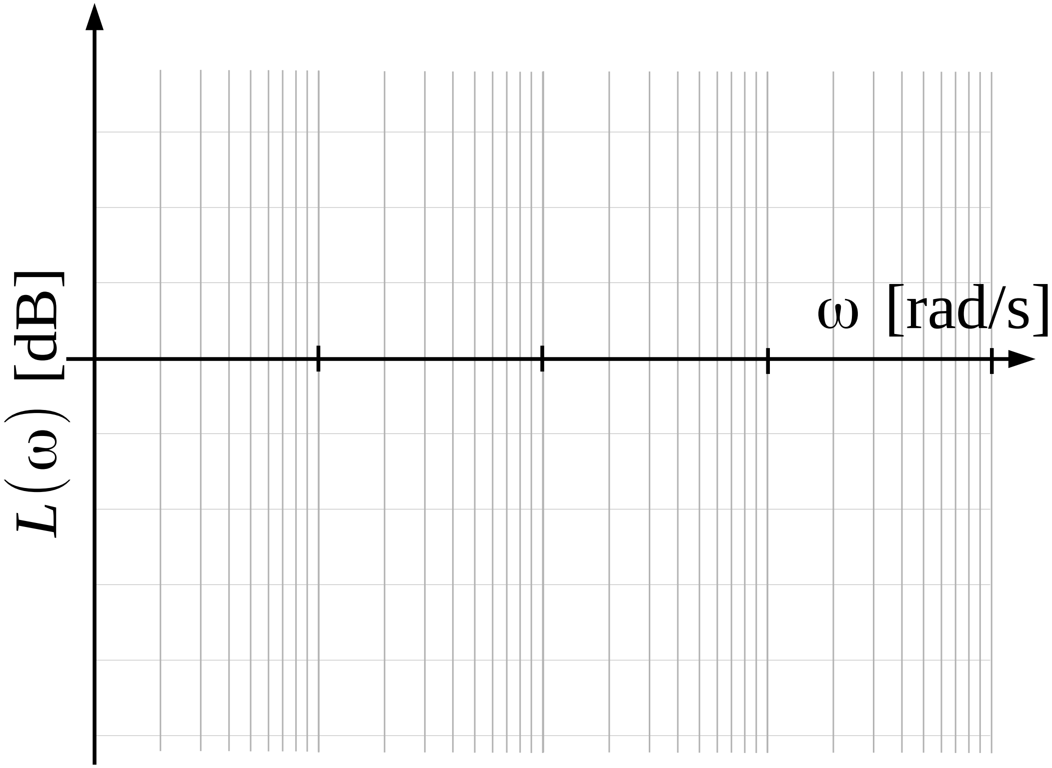
Calculate and sketch step response and Bode Plots for a system with transfer function $H(s) = 3/s$.

Example

Calculate and sketch step response and Bode Plots for a system with transfer function H(s) = 3/s.

Example

Calculate and sketch step response and Bode Plots for a system with transfer function H(s) = 3/s.



Example

Calculate and sketch step response and Bode Plots for a system with transfer function H(s) = 3/s.