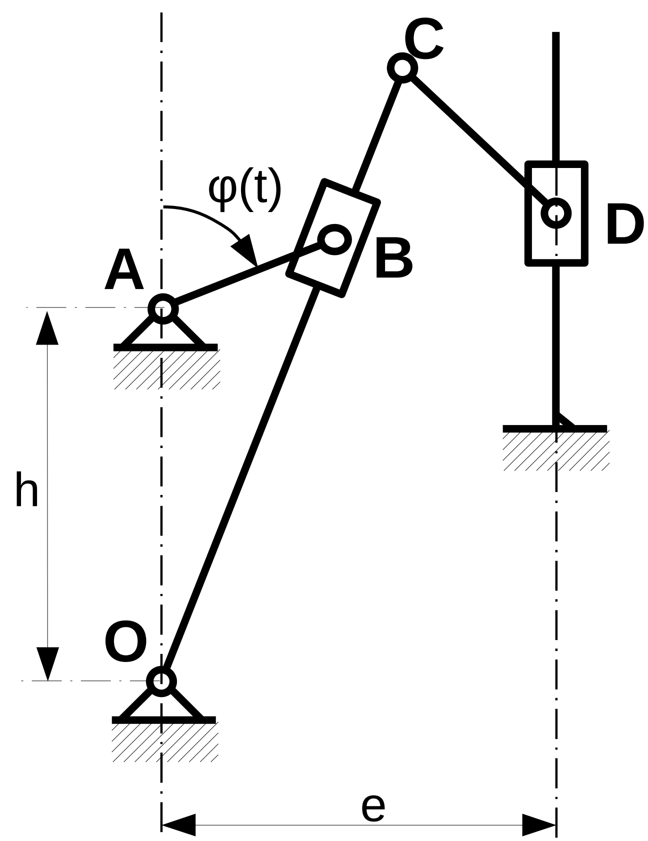
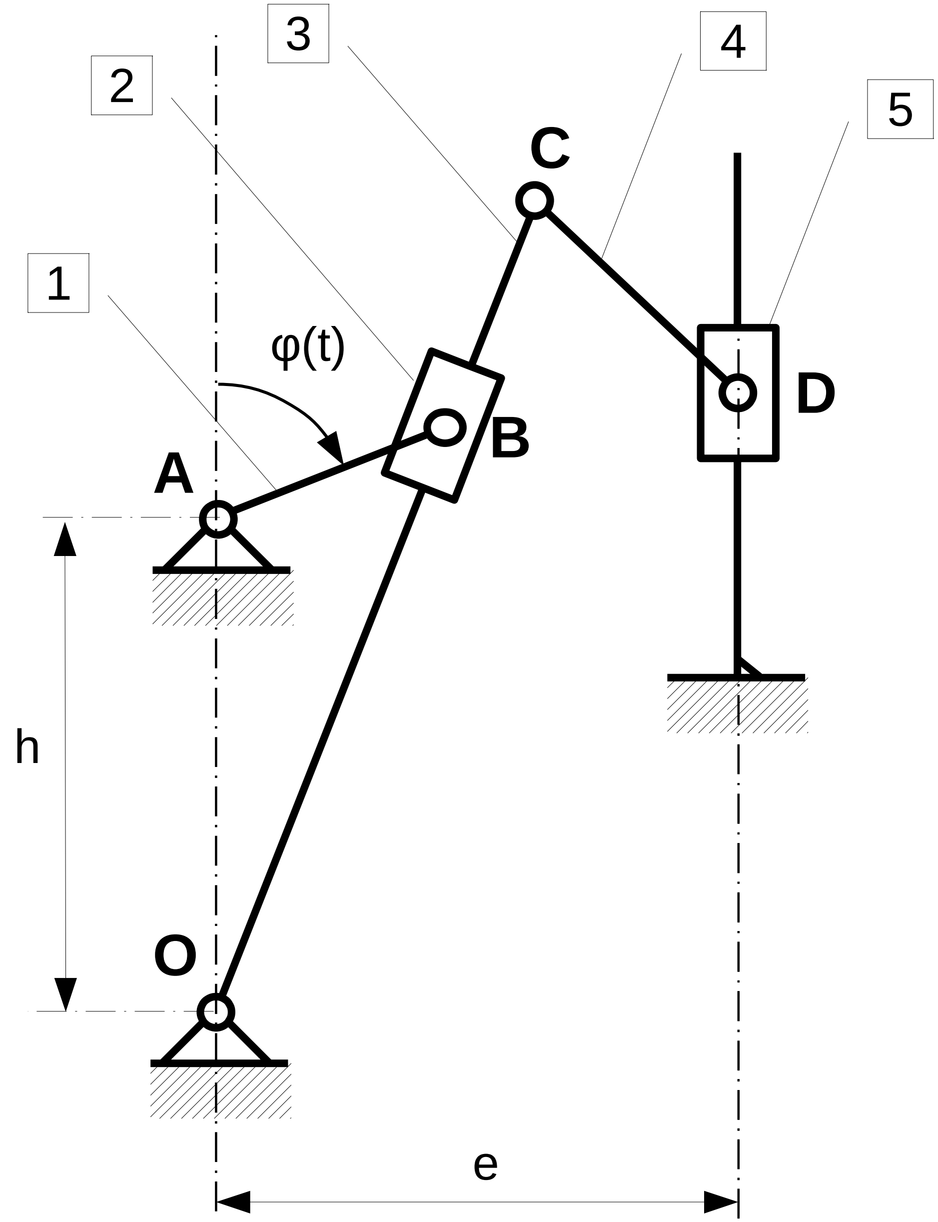
PAiTM - zima 2016/2017

Prędkości i przyspieszenia w mechanizmach płaskich
Przykład – metoda analityczna

Dane: geometria mechanizmu (długości członów)
oraz kąt φ(t) członu napędowego. $\varphi(t)=\omega t$, $\omega=const.$
Szukane: prędkość i przyspieszenie suwaka D jako funkcja kąta φ(t).



Wydział Samochodów i Maszyn Roboczych, Politechnika Warszawska,
Instytut Podstaw Budowy Maszyn, Zakład Mechaniki, dr inż. Sebastian Korczak


3
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φ(t)
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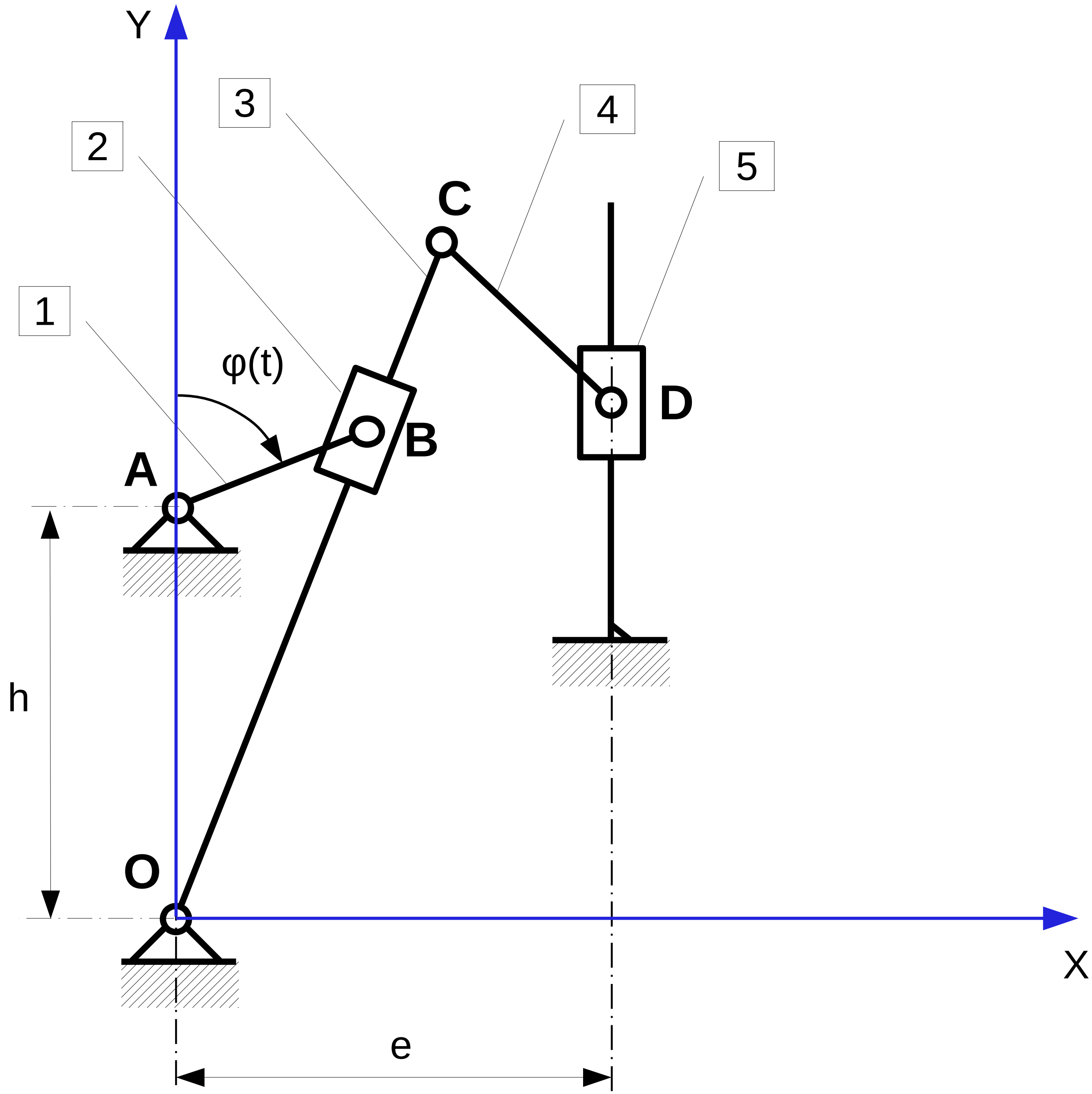
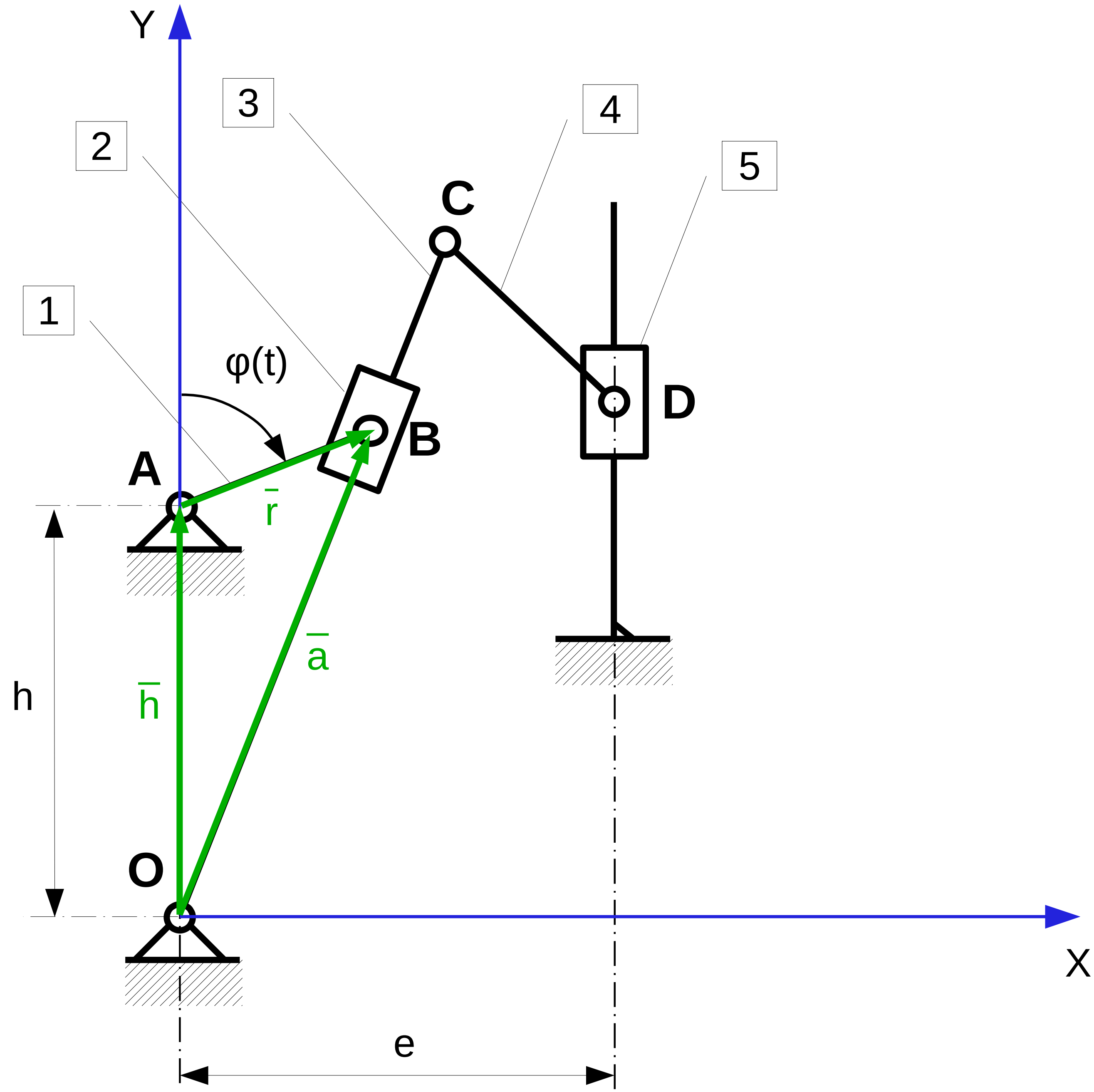
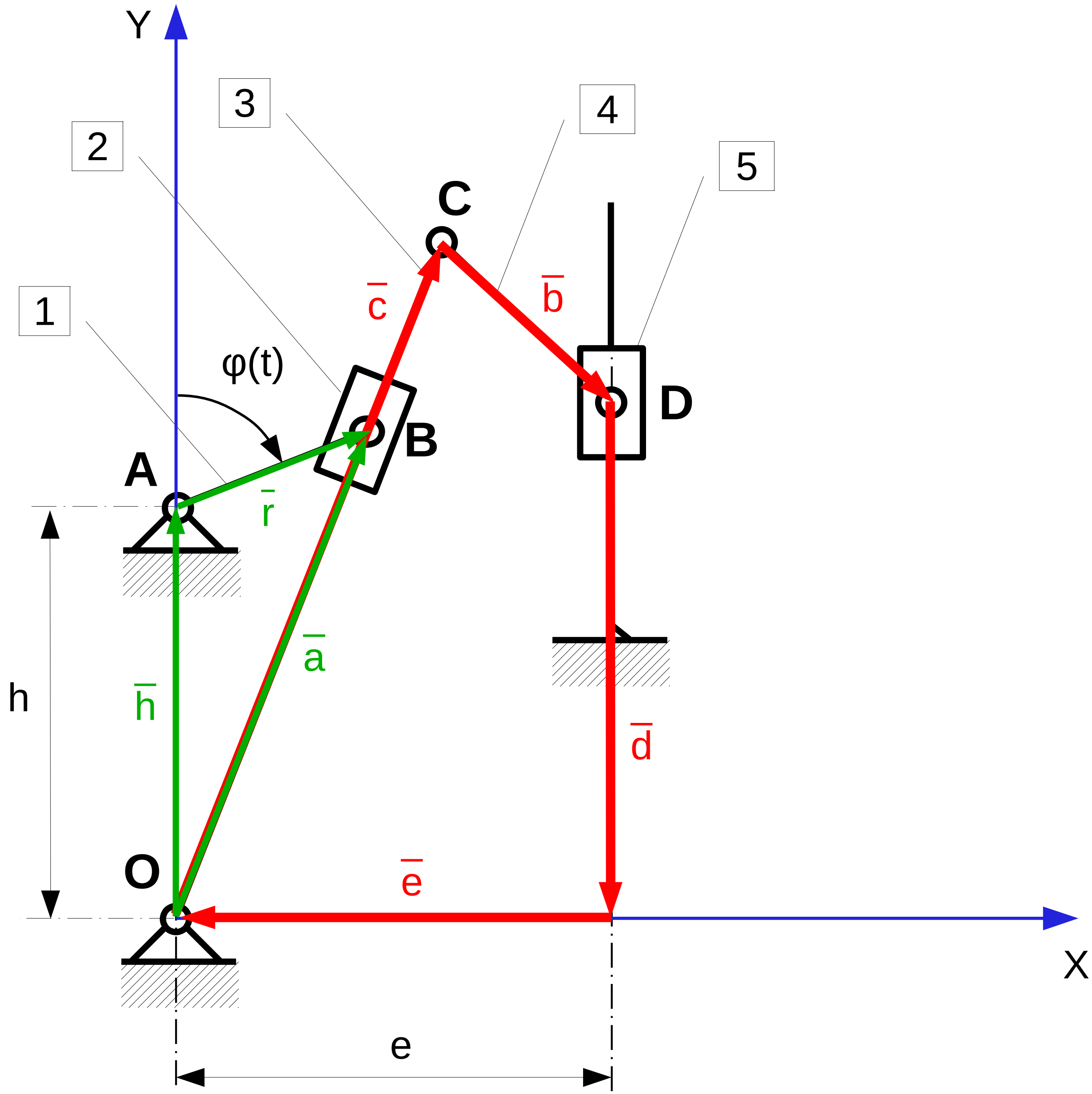
1. Wprowadzamy układ współrzędnych

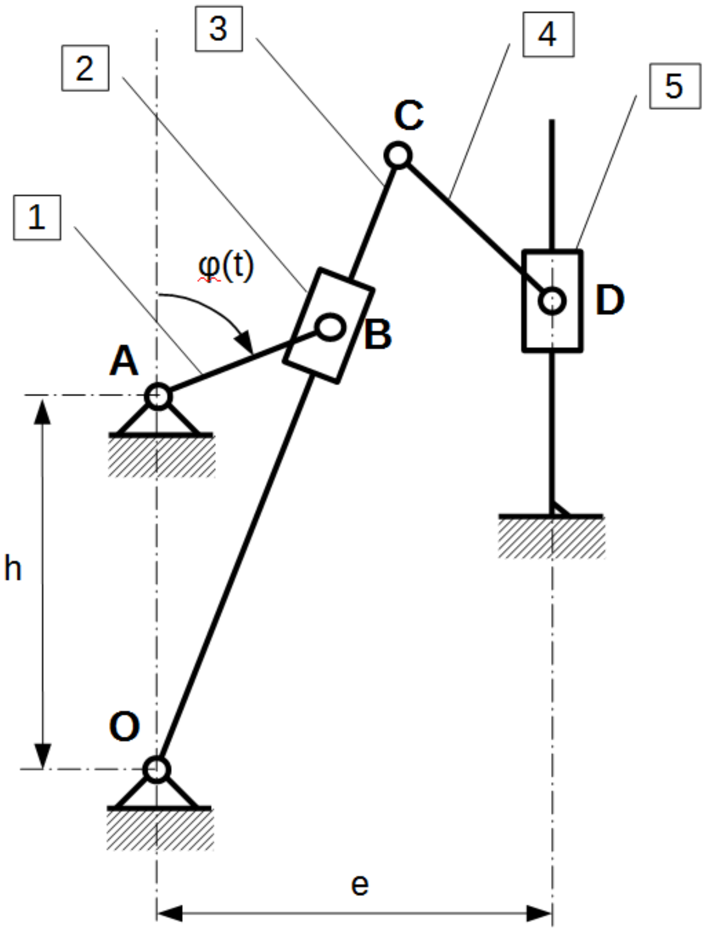


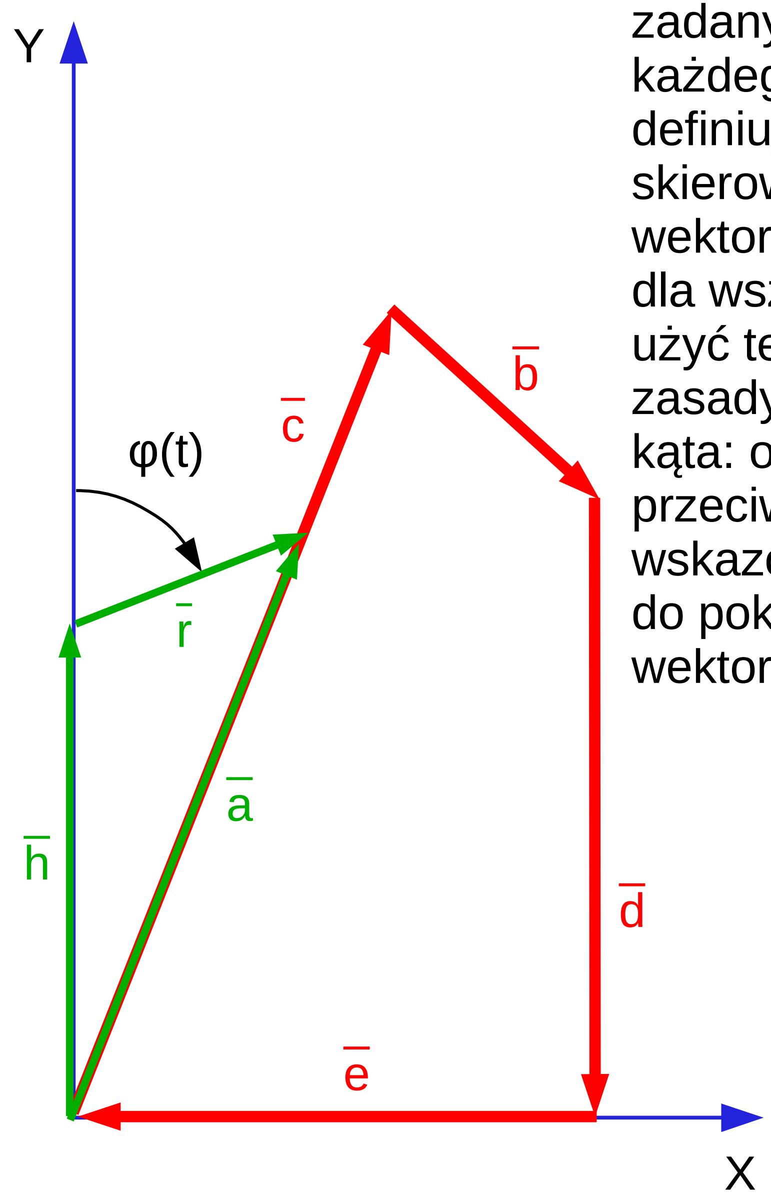
2. Wprowadzamy zamknięte wieloboki wektorów. Wektory te rozpinane są pomiędzy punktami charakterystycznymi mechanizmu. Kiedy mechanizm pracuje to wektory te mogą zmieniać swoje położenie, orientację i długość.



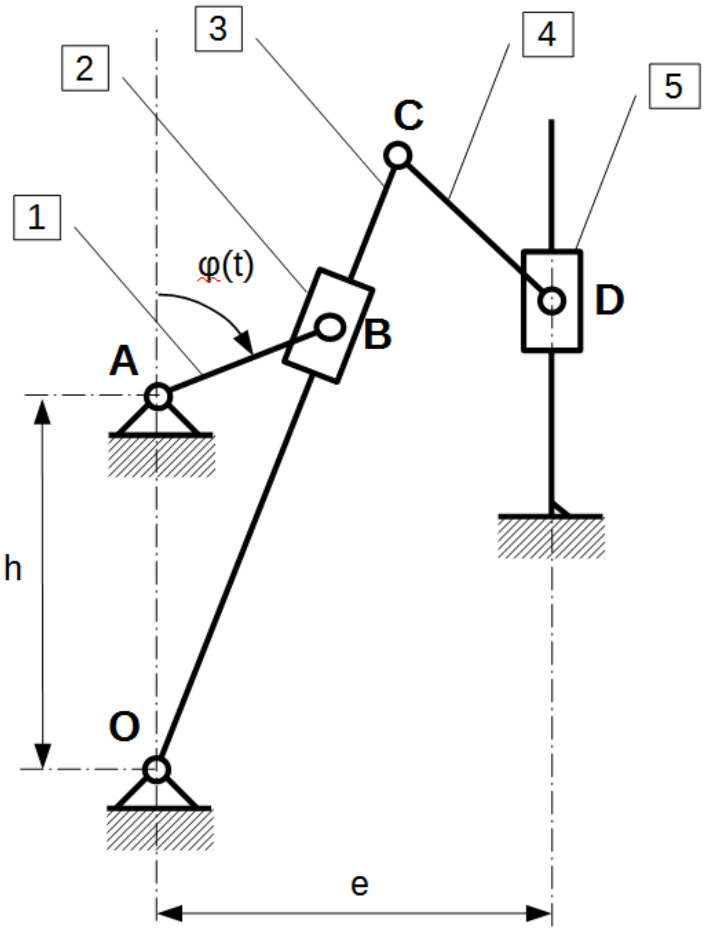
3. Tworzymy jak najłatwiejsze wieloboki wektorów, których będzie tyle, ile potrzeba, aby każdy punkt mechanizmu był uwzględniony.







4. Oznaczamy zadany kąt φ(t). Dla każdego wektora definiujemy kąt skierowania tego wektora. Najlepiej dla wszystkich kątów użyć tej samej zasady określania kąta: obrót osi X przeciwnie do ruchu wskazówek zegara do pokrycia z wektorem.



1
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3
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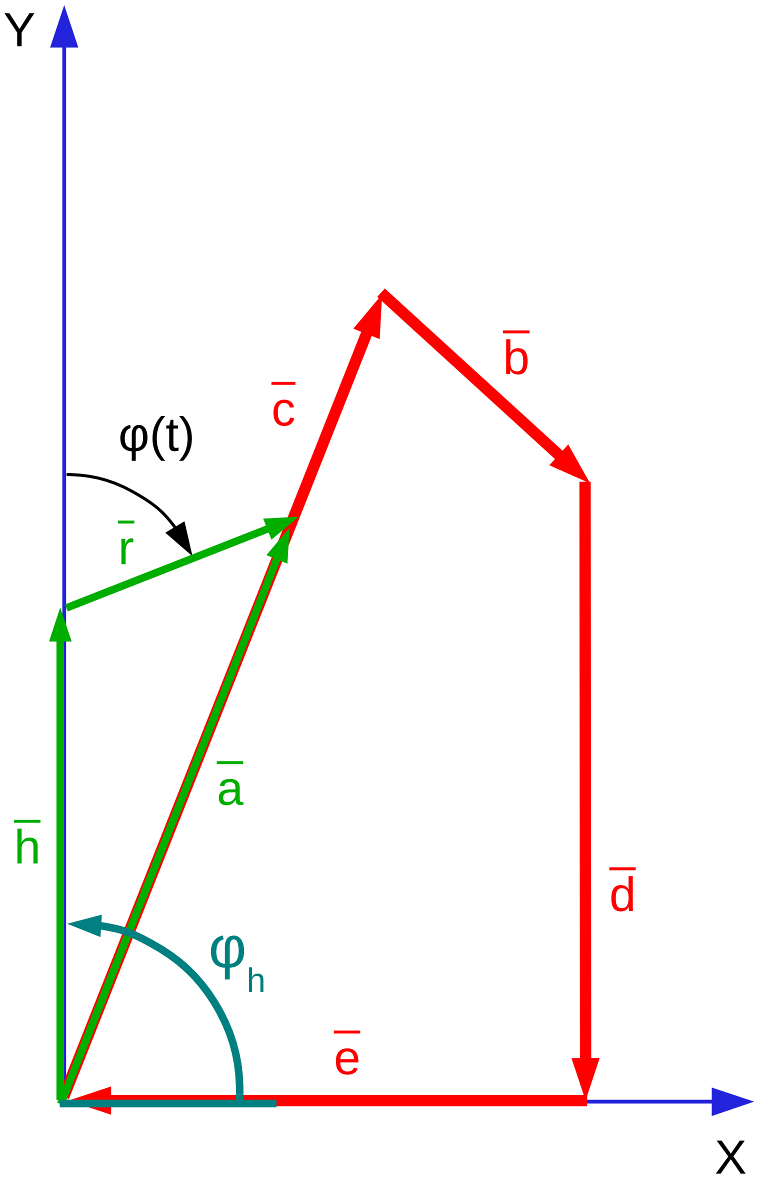

Y
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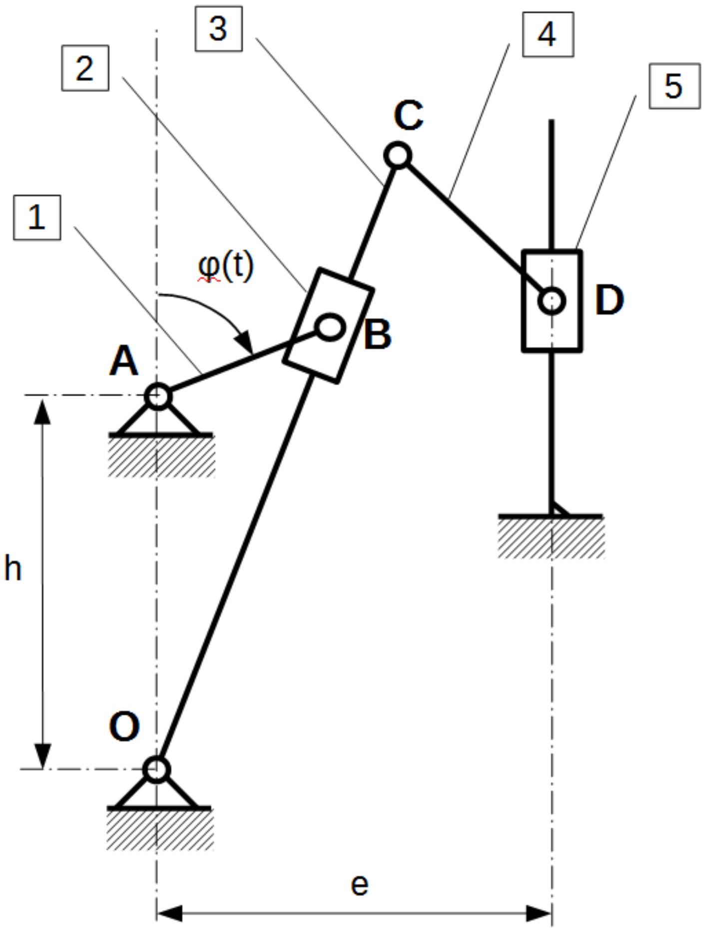
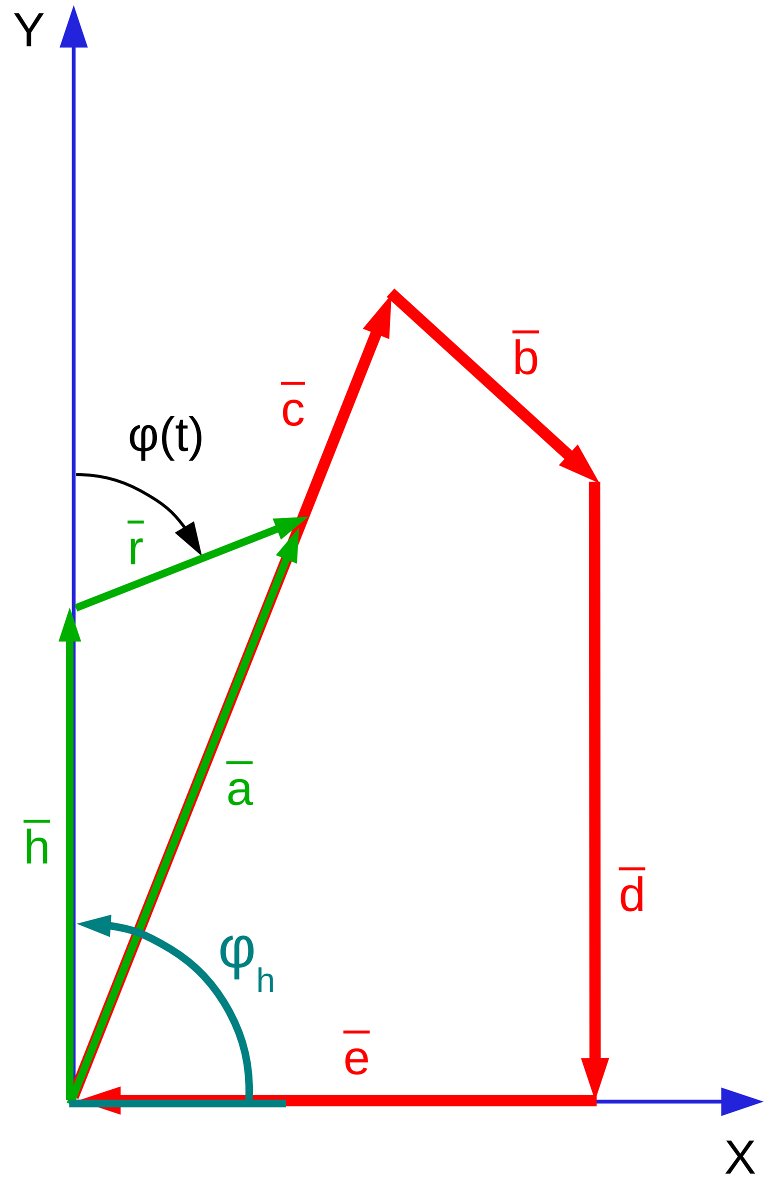
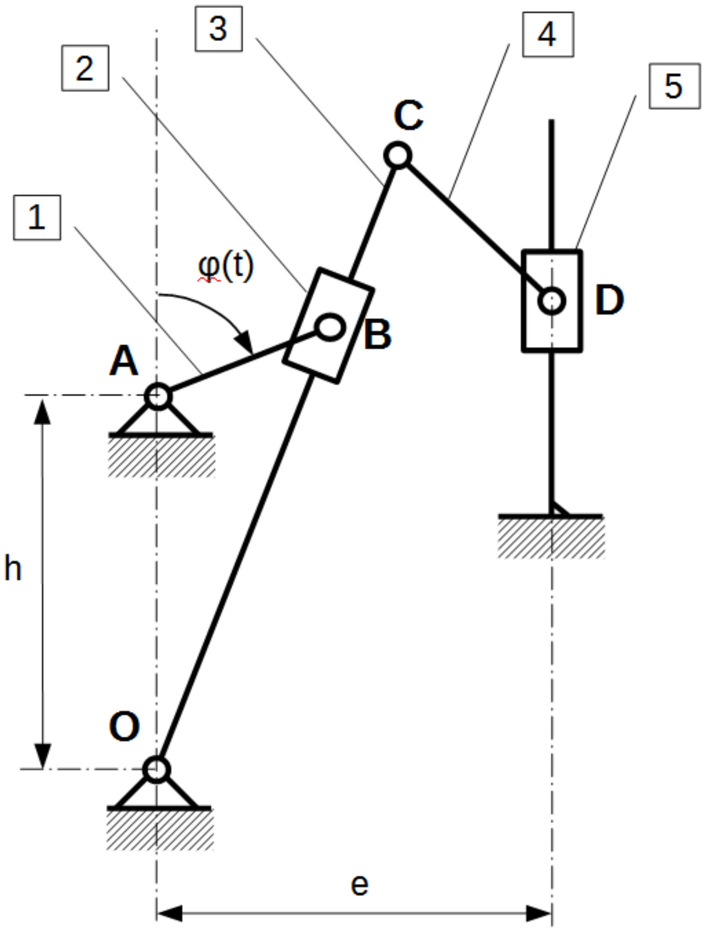


$\varphi_h = 90^o$





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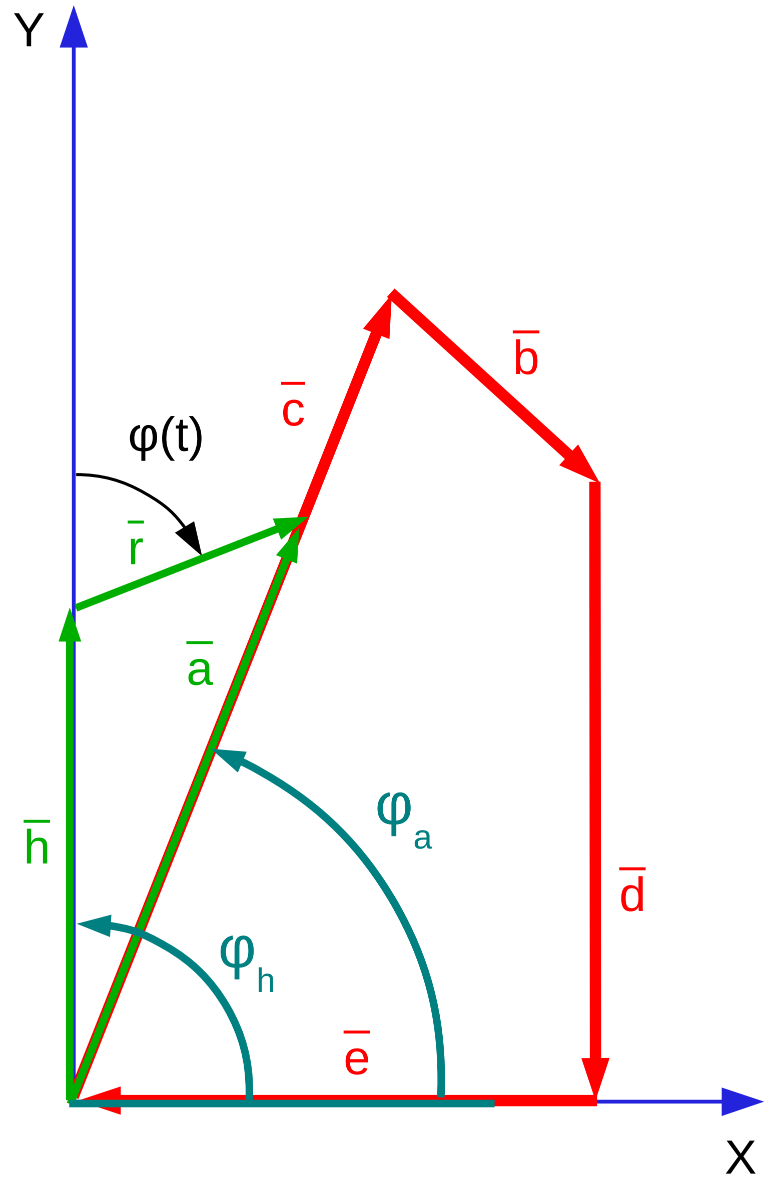
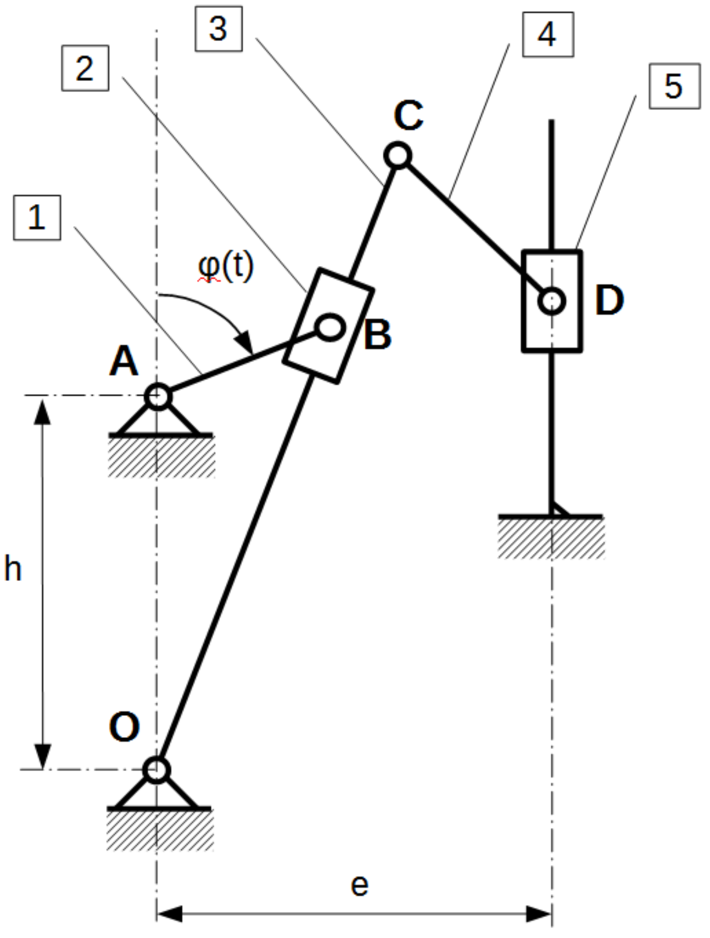
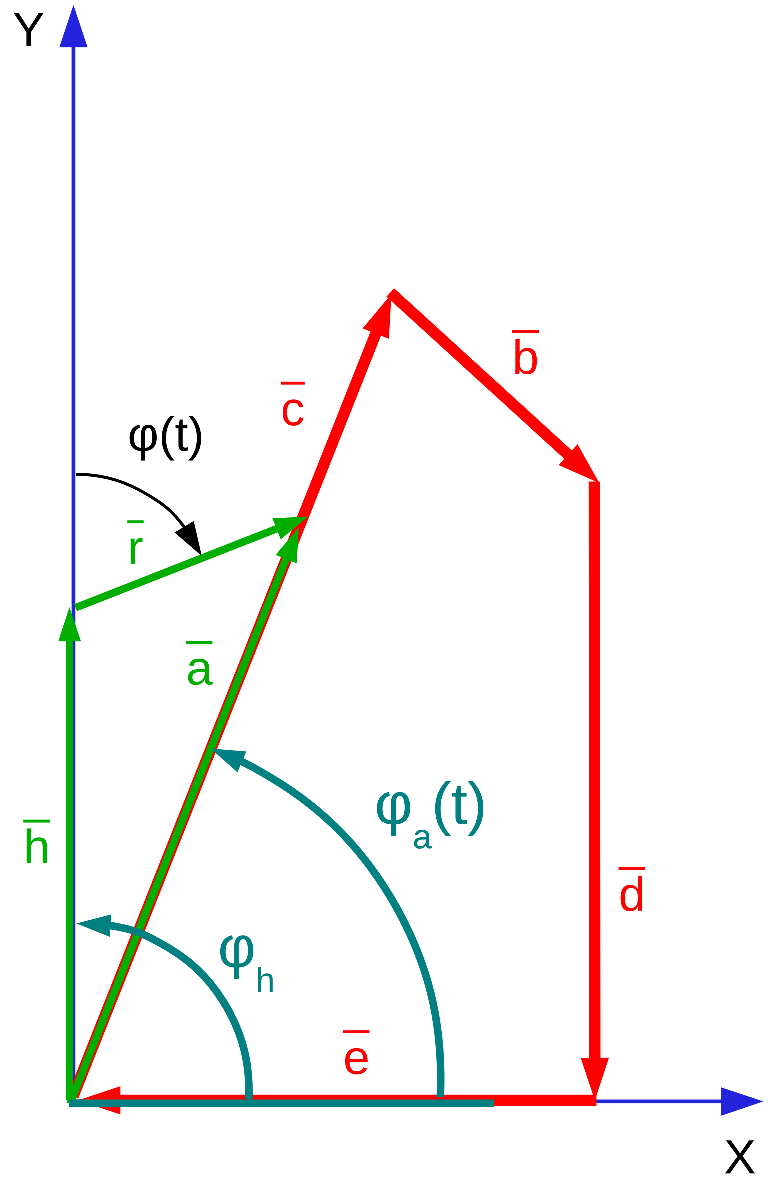
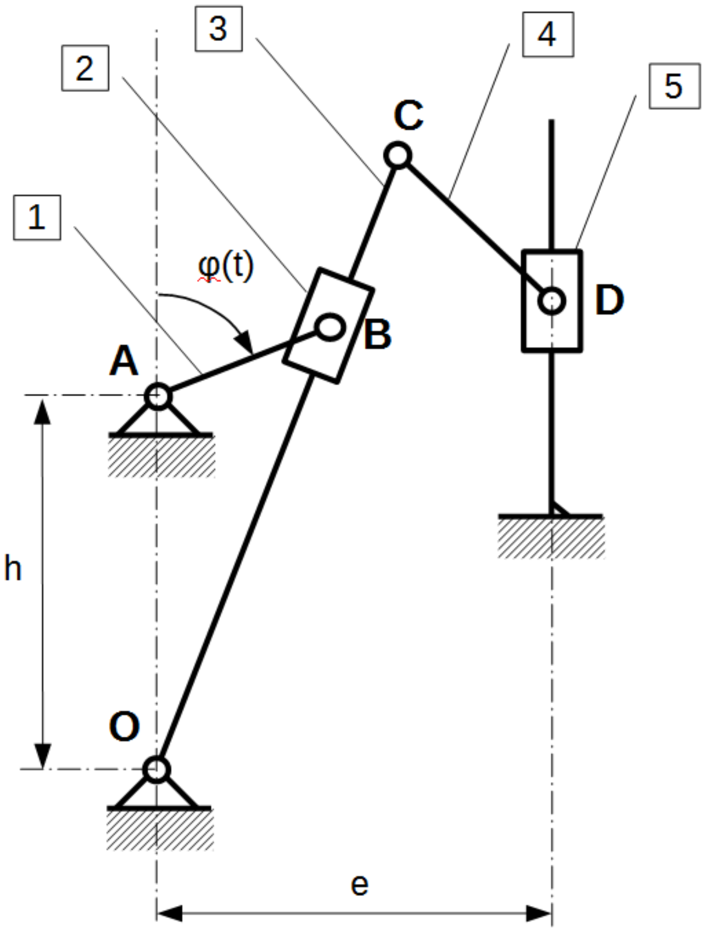


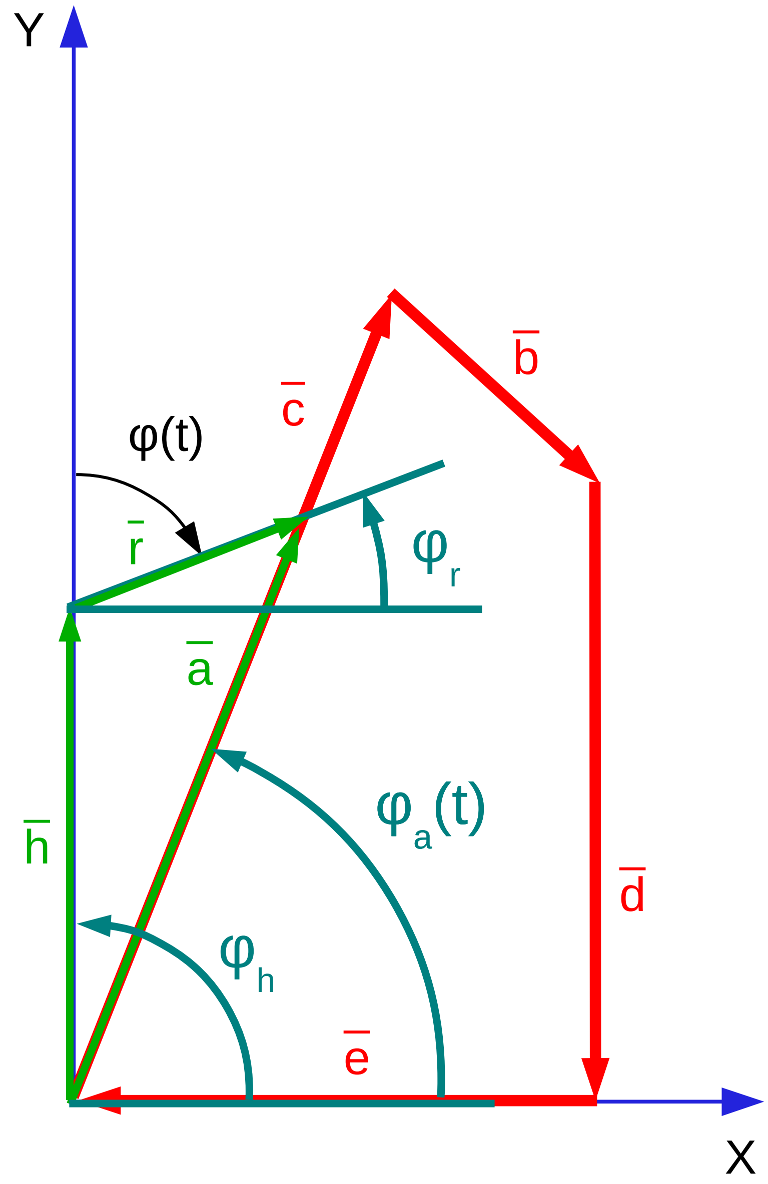




$\varphi_h = 90^o$

$\varphi_a(t) \neq const.$





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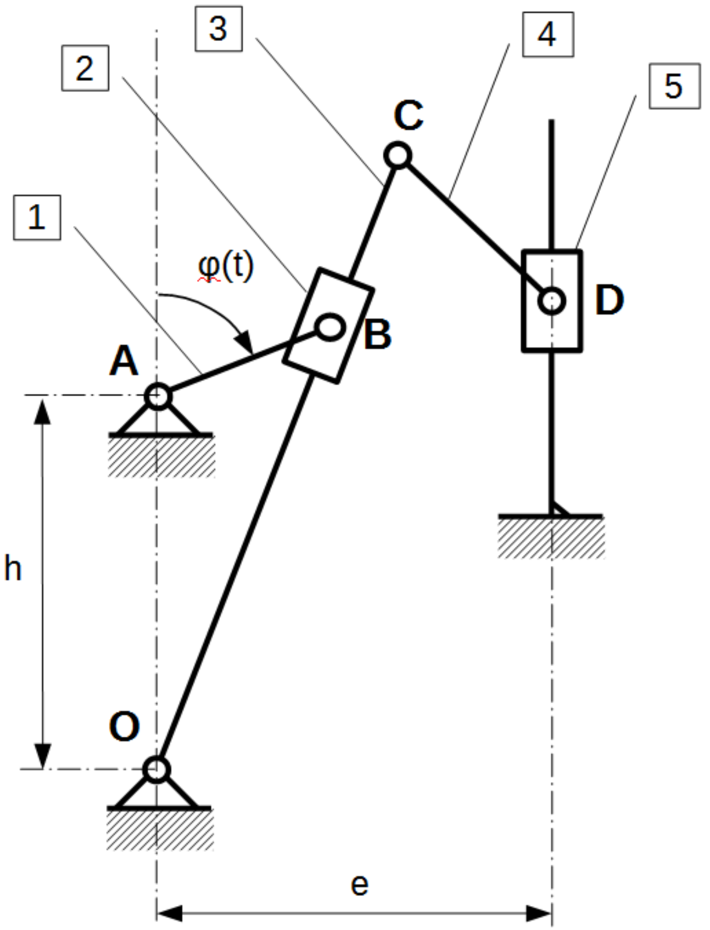

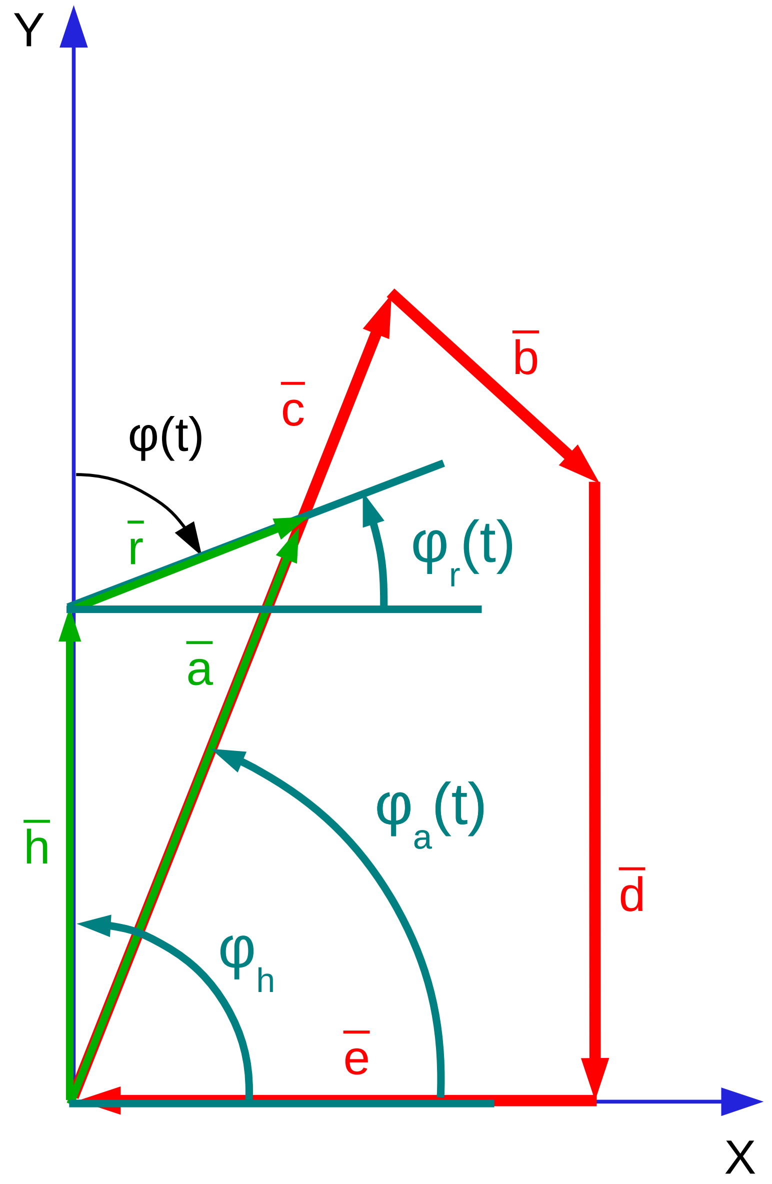

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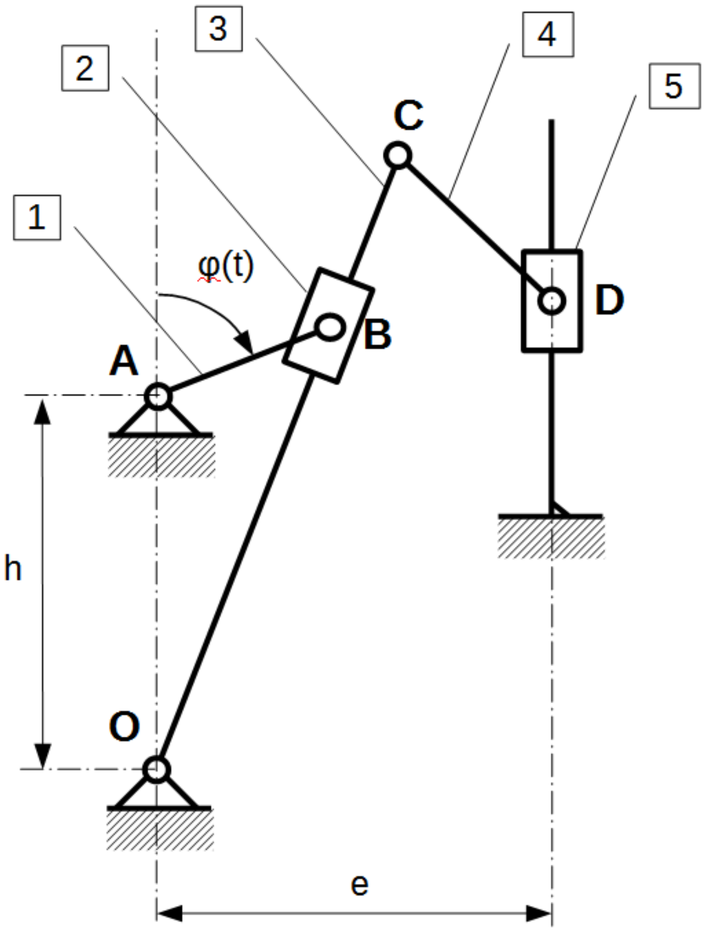
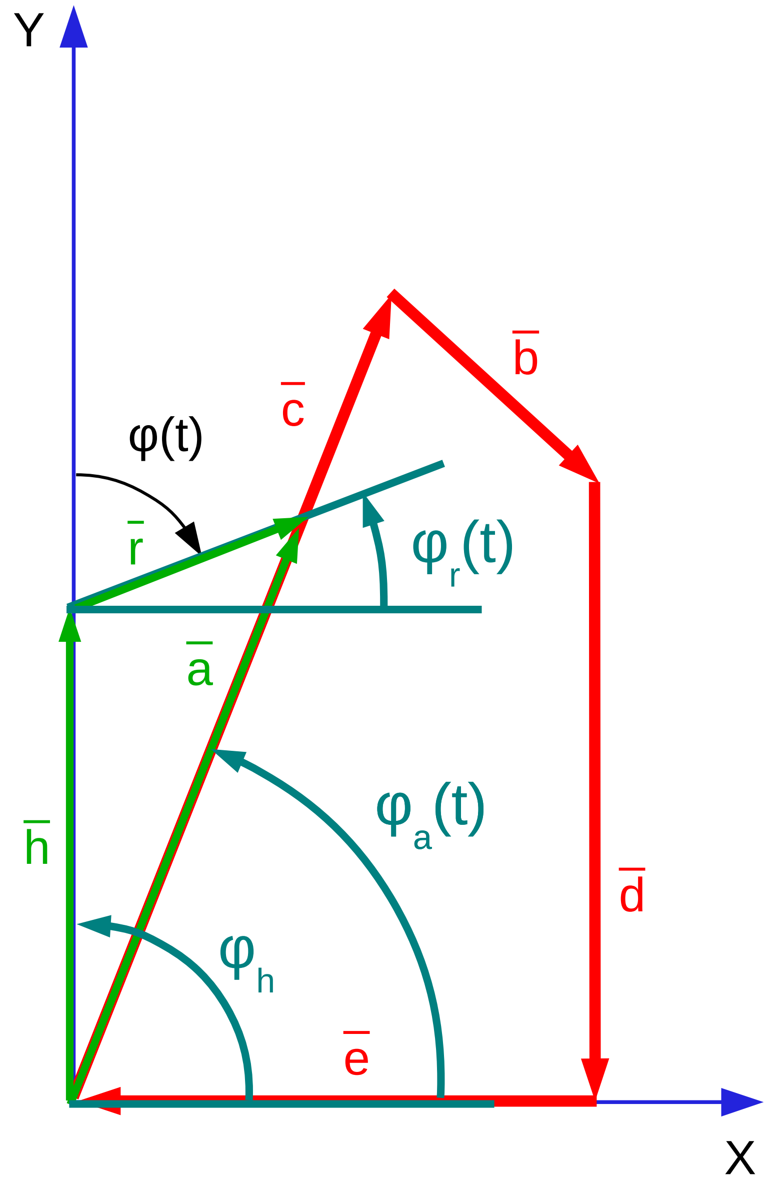
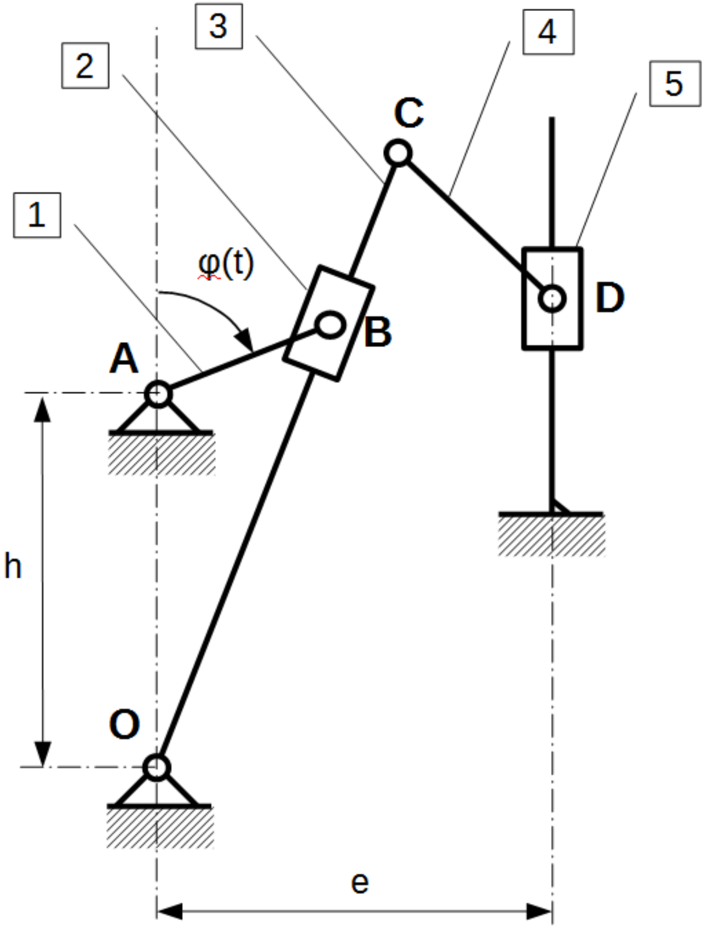


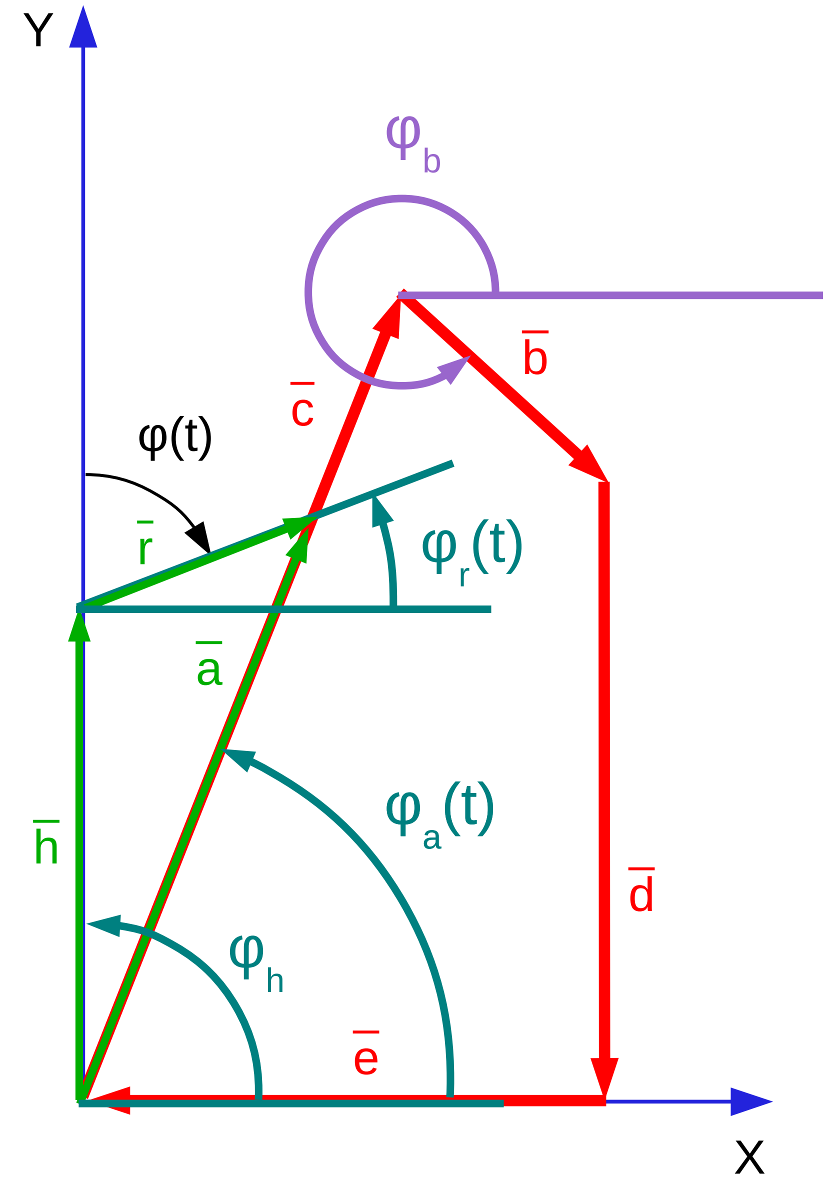


$\varphi_h = 90^o$

$\varphi_a(t) \neq const.$

$\varphi_r(t) = 90^o - \varphi(t)$





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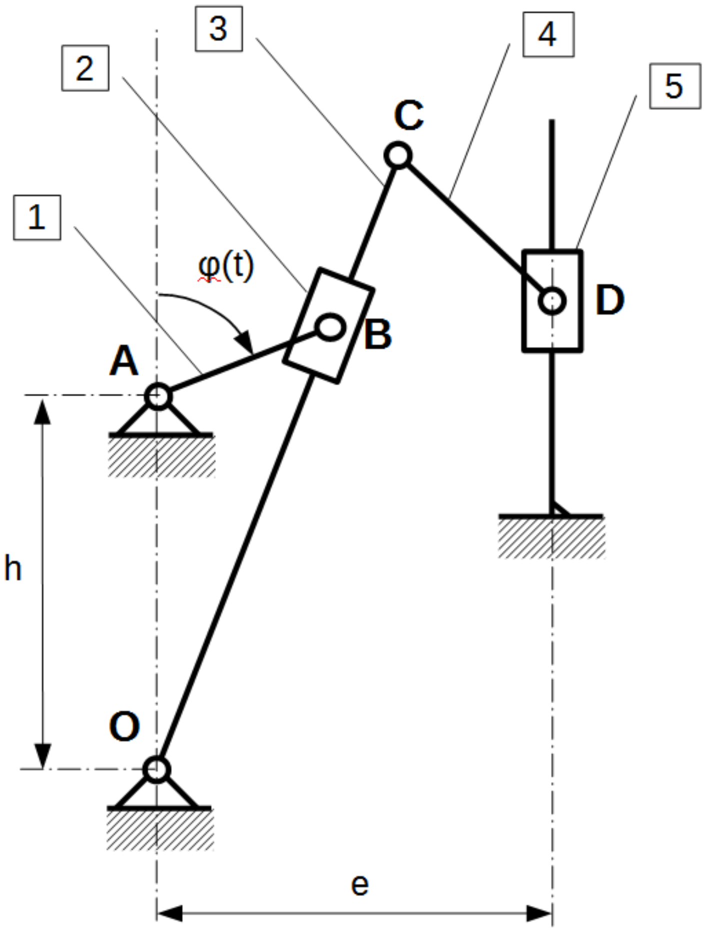
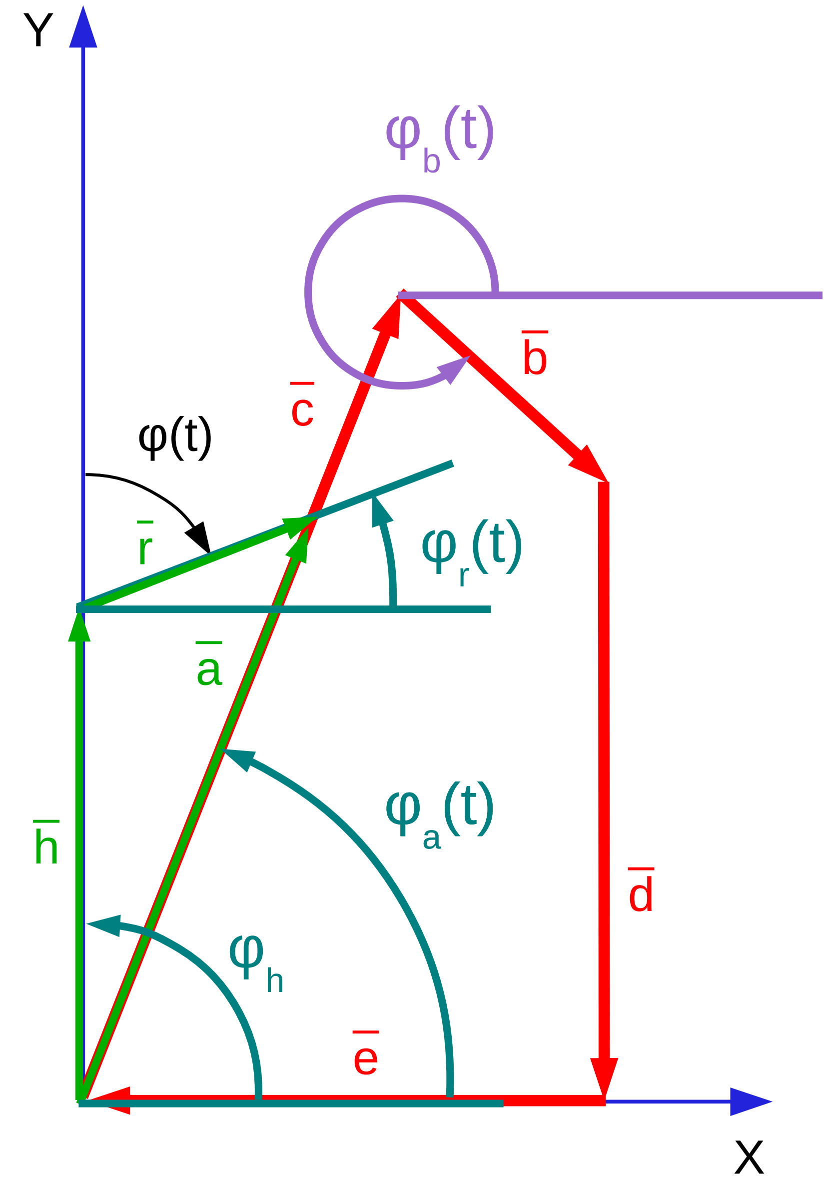


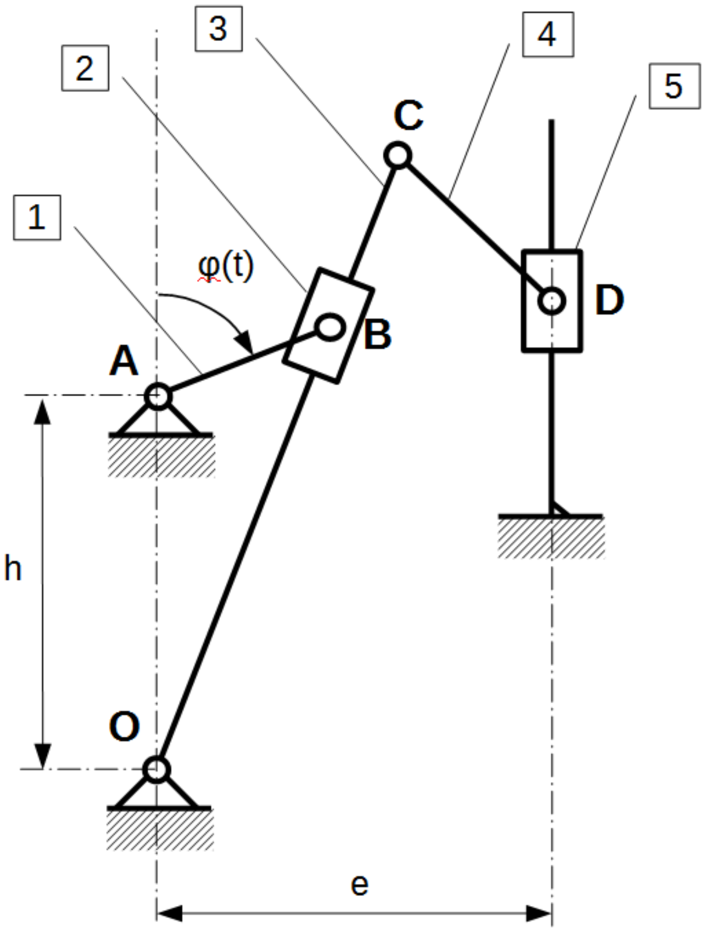


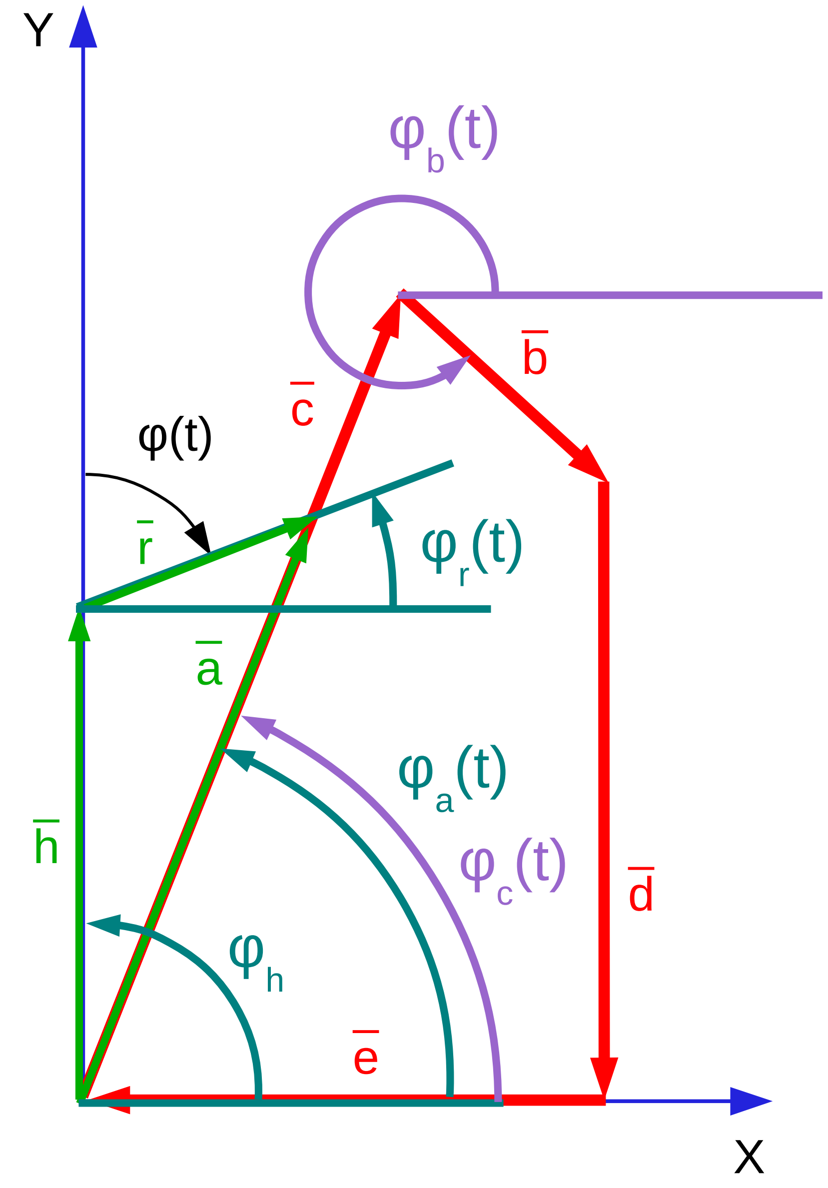
$\varphi_h = 90^o$

$\varphi_a(t) \neq \text{const.}$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_b(t) \neq \text{const.}$





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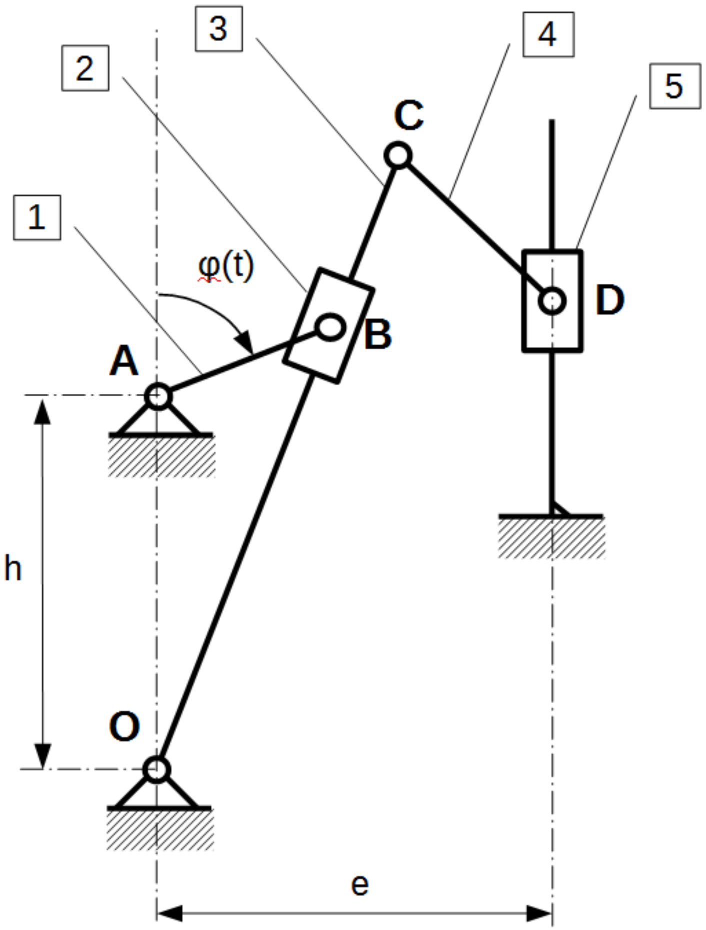
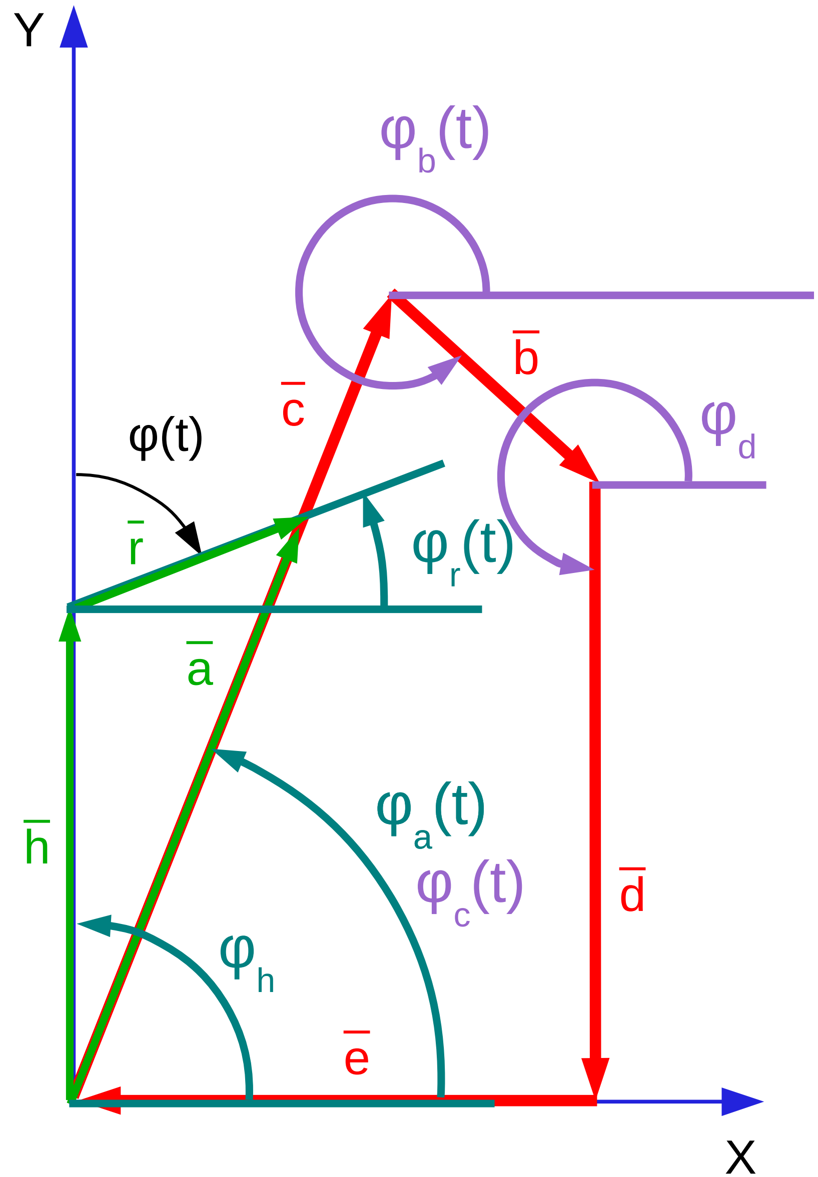
$\varphi_c(t) = \varphi_a(t)$





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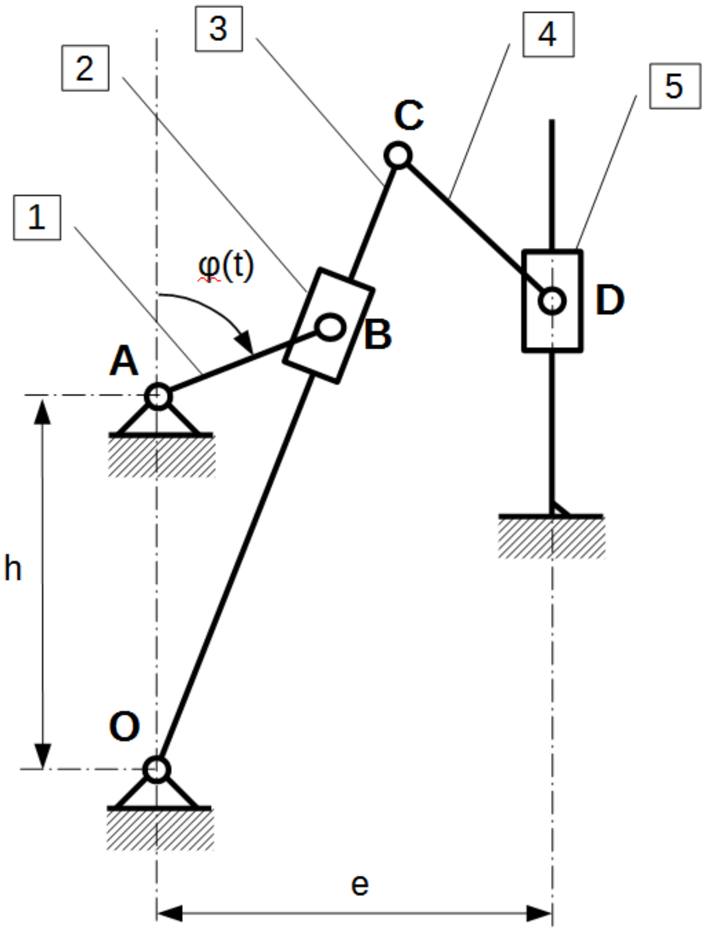
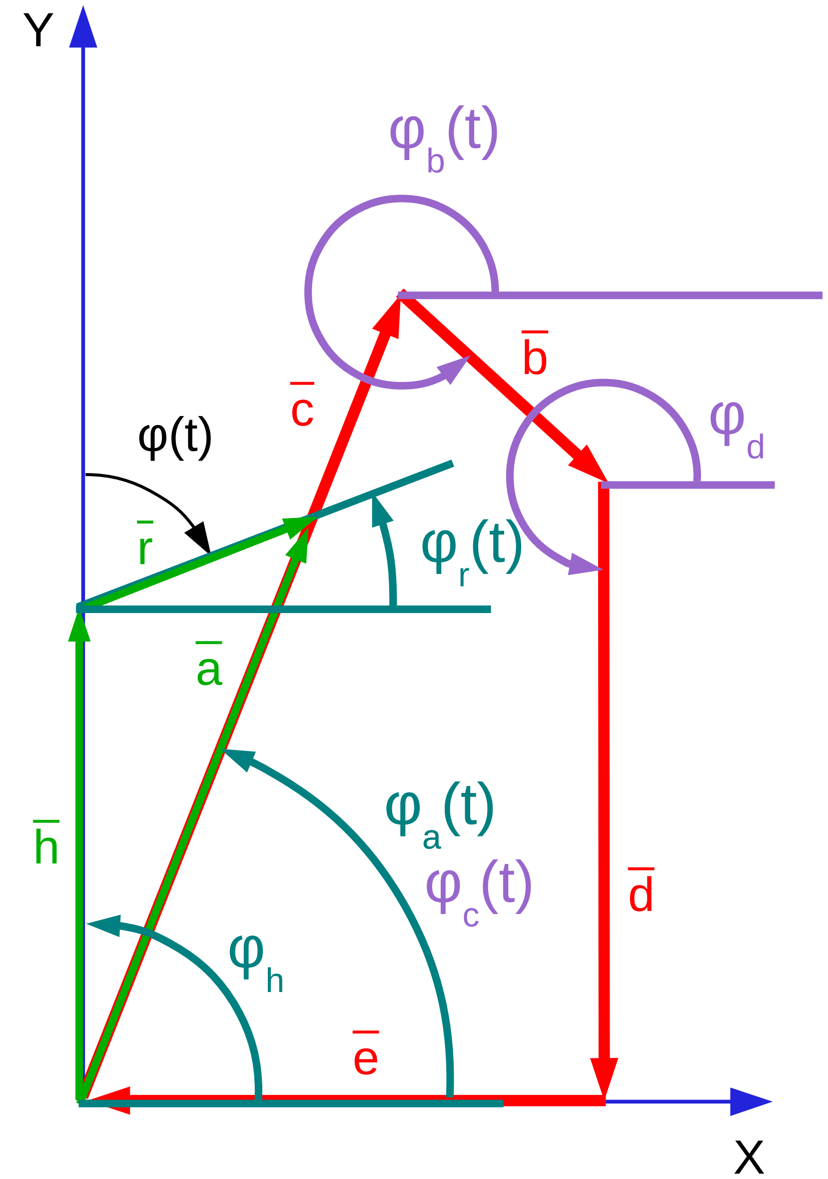




$\varphi_h = 90^o$

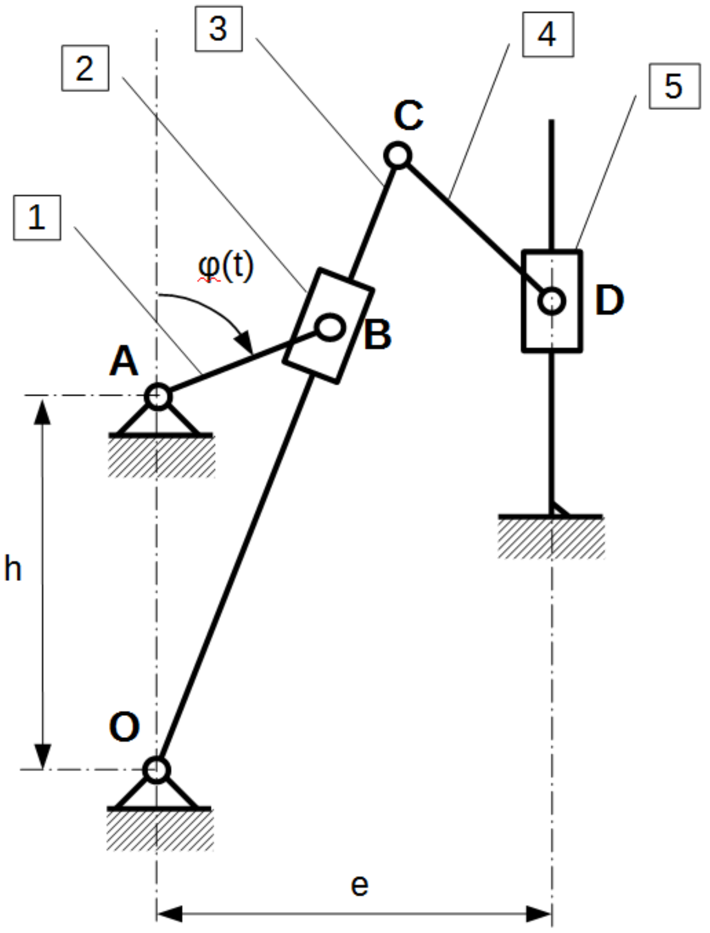
$\varphi_d = 270^o$

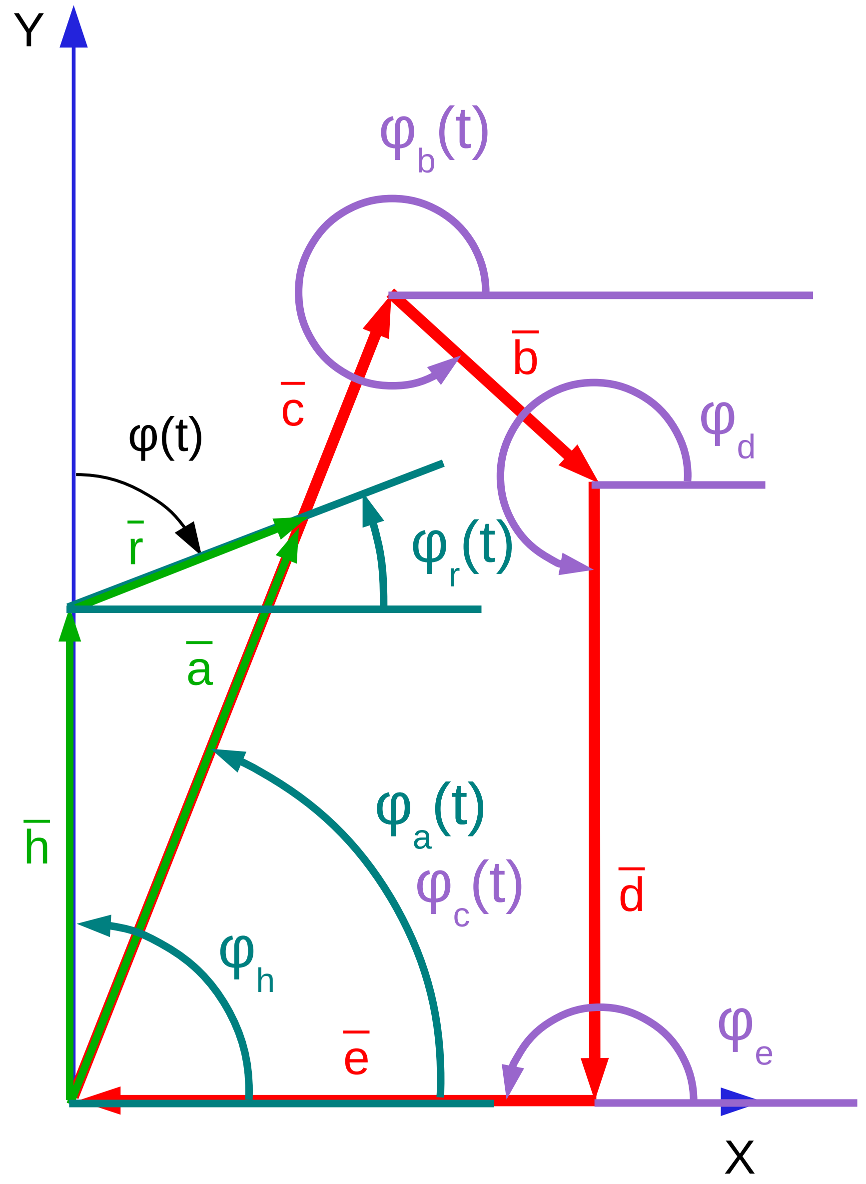
$\varphi_a(t) \neq \text{const.}$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_b(t) \neq \text{const.}$

$\varphi_c(t) = \varphi_a(t)$





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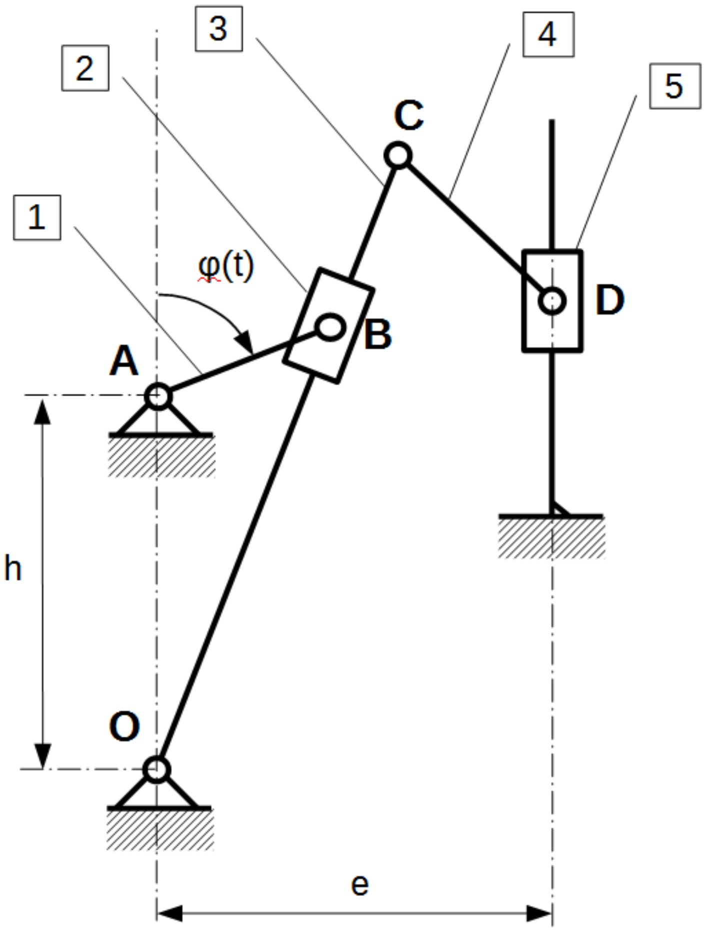
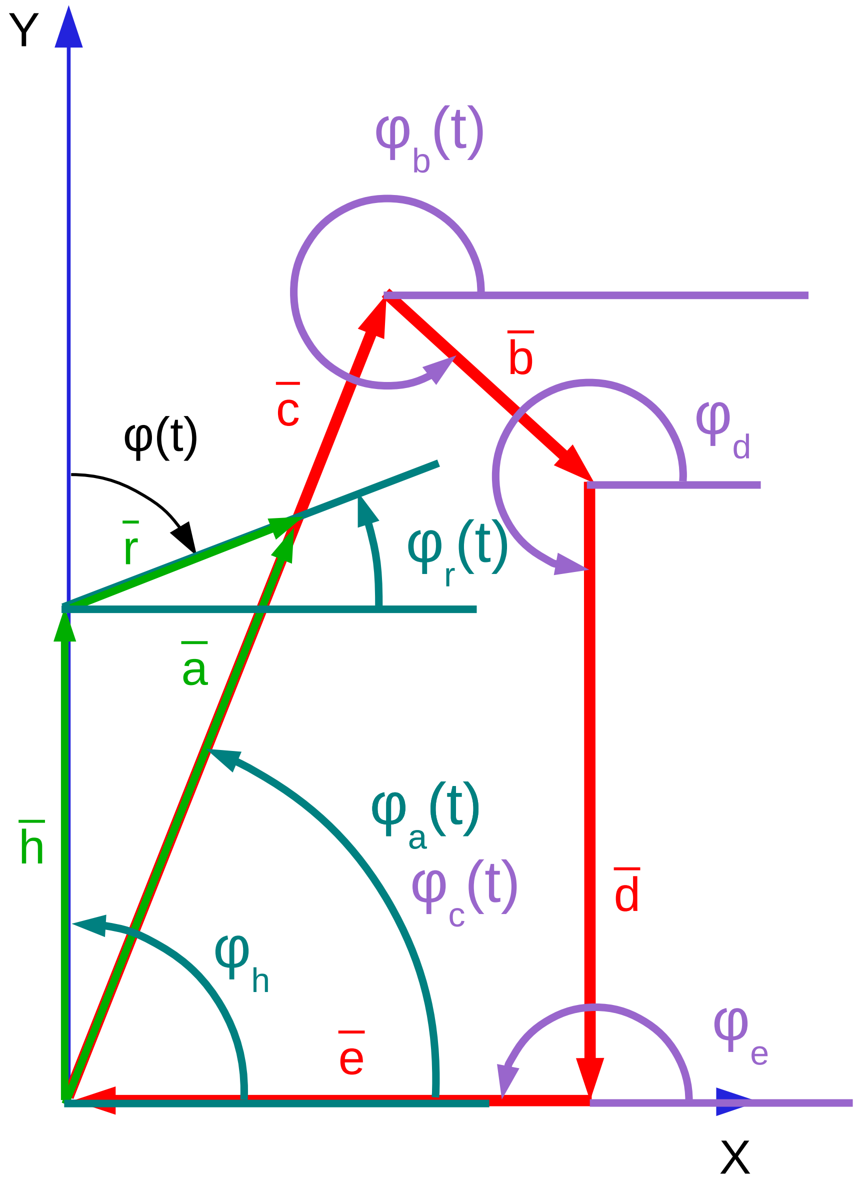




$\varphi_h = 90^o$

$\varphi_a(t) \neq \text{const.}$

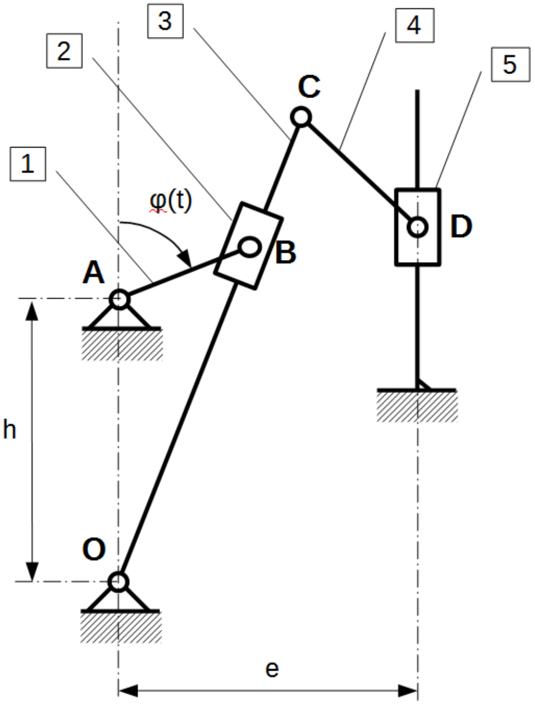
$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_b(t) \neq \text{const.}$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_d = 270^o$

$\varphi_e = 180^o$



$\varphi_h = 90^o$

$\varphi_a(t) \neq const.$

$\varphi_b(t) \neq const.$

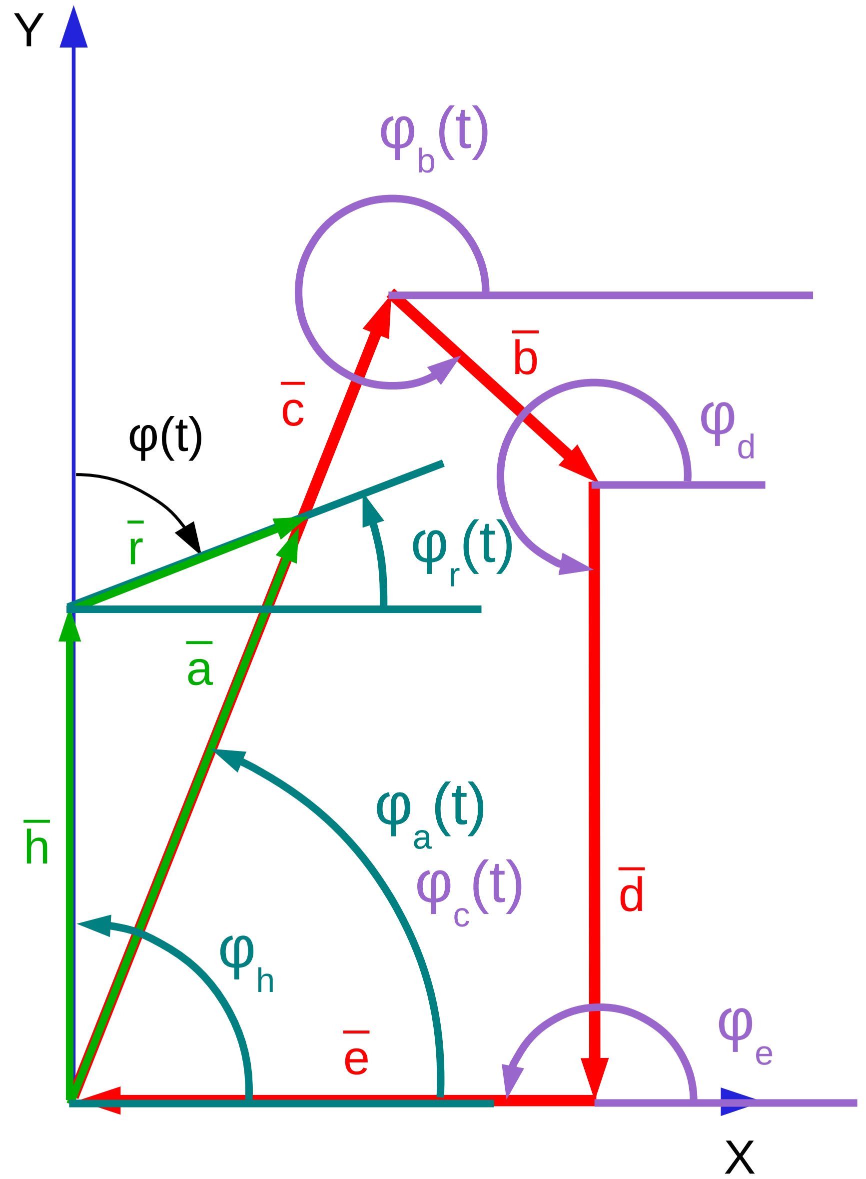
$\varphi_c(t) = \varphi_a(t)$

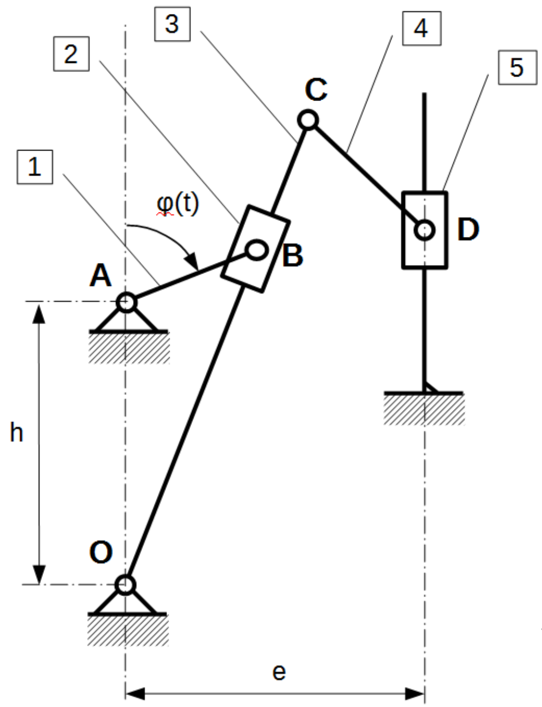
$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_d = 270^o$

$\varphi_e = 180^o$

5. Określiliśmy wszystkie kąty, podkreślając, które mają stała wartość a które zmieniają się w czasie pracy mechanizmu.





6. Teraz określamy długości wszystkich wektorów:

$|\bar{r}| = |AB| = r = \text{const.}$

$\varphi_h = 90^o$

$\varphi_a(t) \neq \text{const.}$

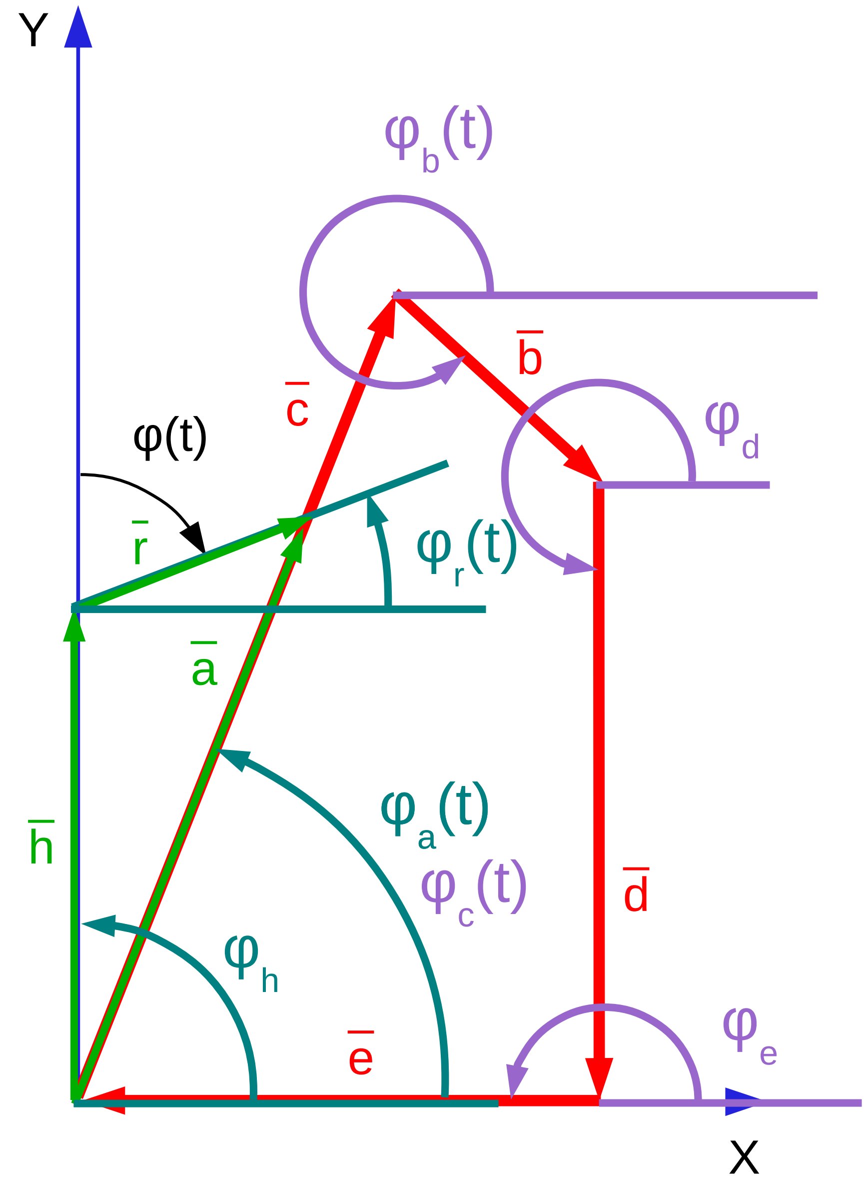
$\varphi_b(t) \neq \text{const.}$

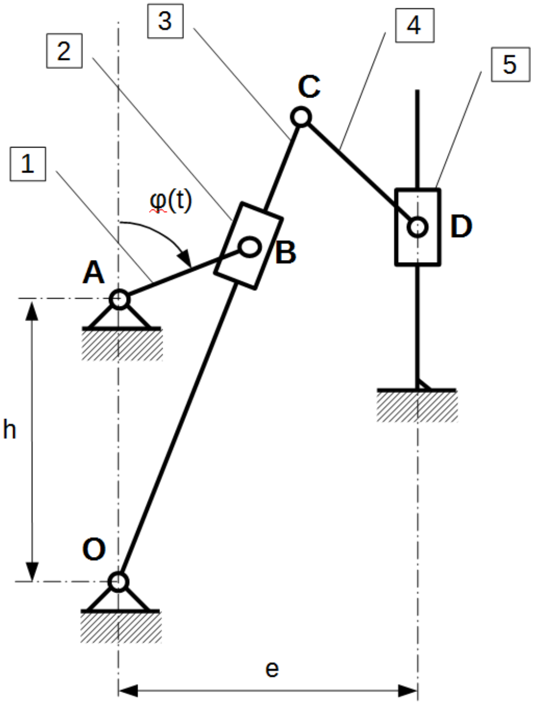
$\varphi_c(t) = \varphi_a(t)$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_d = 270^o$

$\varphi_e = 180^o$





$\varphi_h = 90^o$

$\varphi_a(t) \neq const.$

$\varphi_b(t) \neq const.$

$\varphi_c(t) = \varphi_a(t)$

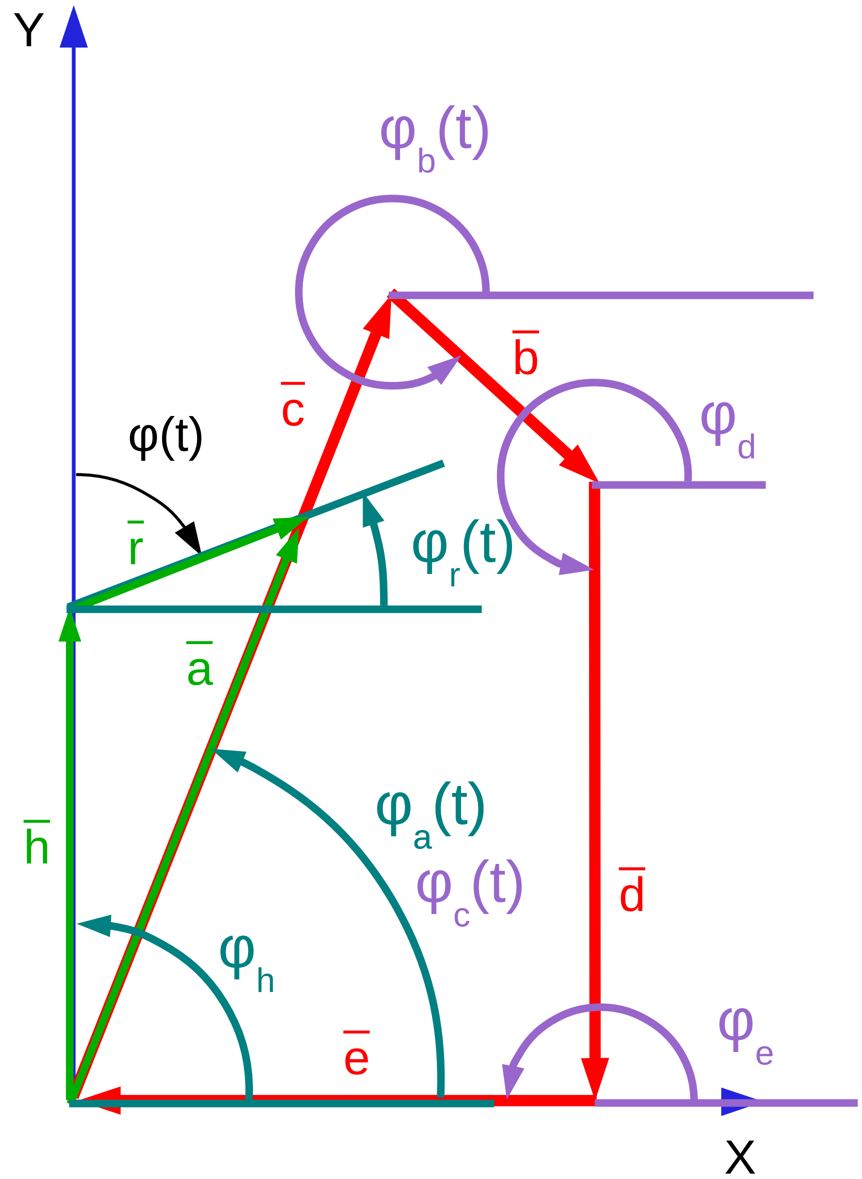
$\varphi_r(t) = 90^o - \varphi(t)$

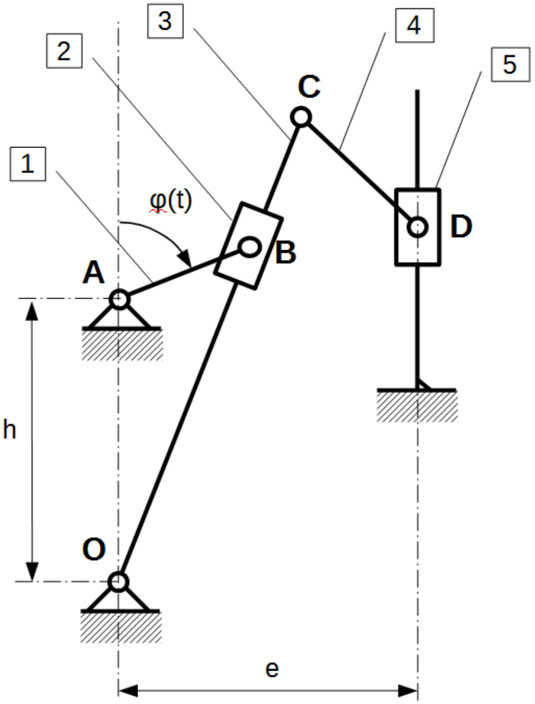
$\varphi_d = 270^o$

$\varphi_e = 180^o$

$|\bar{r}| = |AB| = r = const.$

$|\bar{h}| = |OA| = h = const.$





$\varphi_h = 90^o$

$\varphi_a(t) \neq const.$

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$\varphi_c(t) = \varphi_a(t)$

$\varphi_r(t) = 90^o - \varphi(t)$

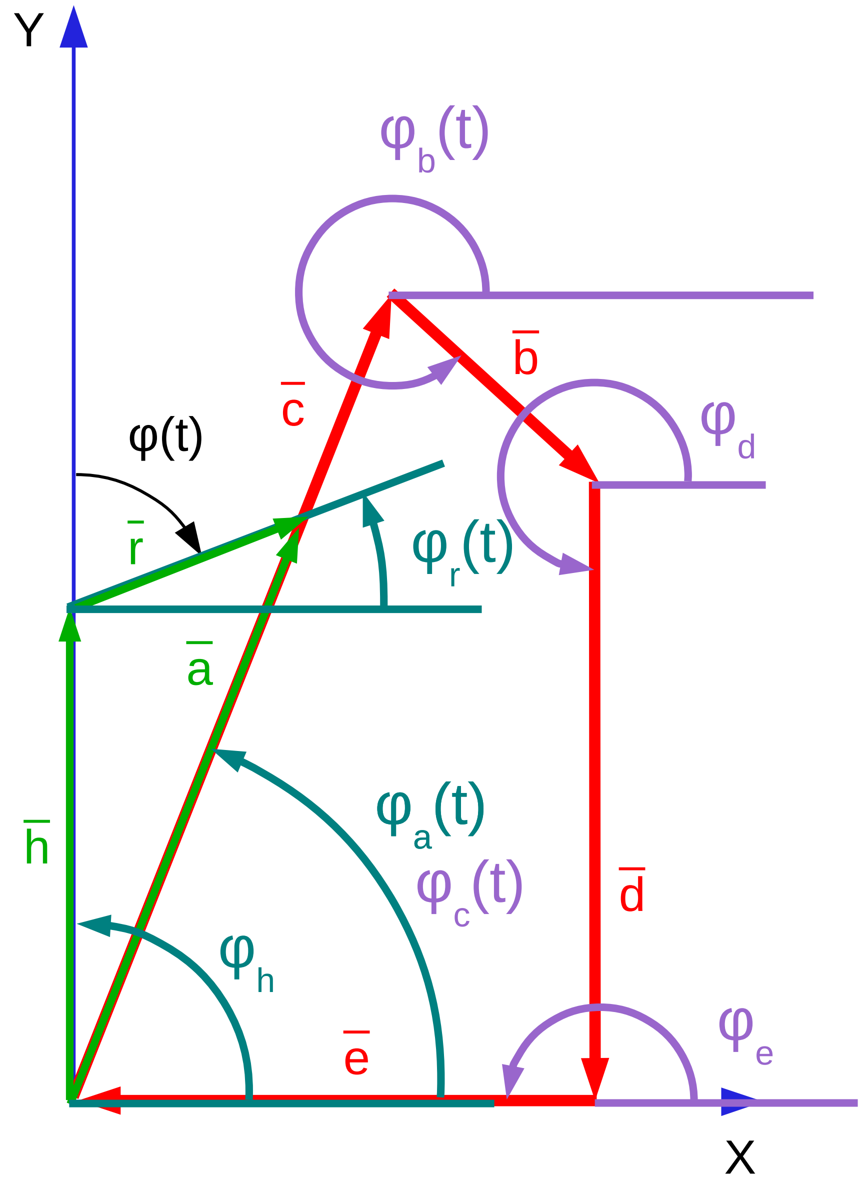
$\varphi_d = 270^o$

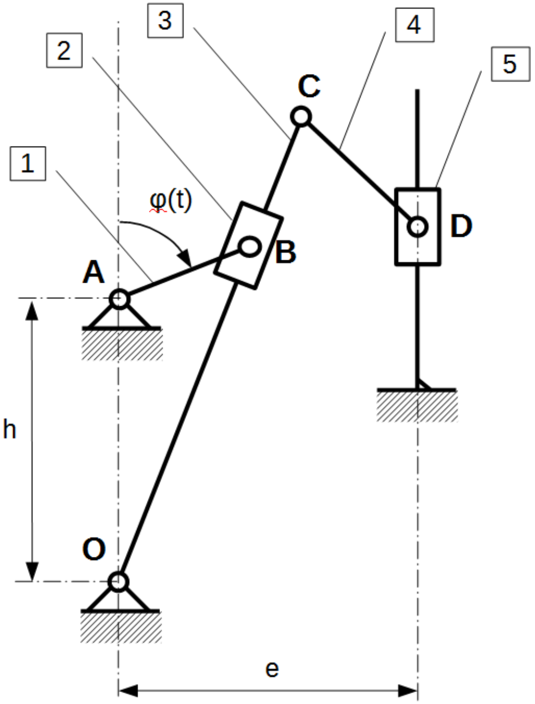
$\varphi_e = 180^o$

$|\bar{r}| = |AB| = r = const.$

$|\bar{h}| = |OA| = h = const.$

$|\bar{a}| = |OB| = a(t)$





$\varphi_h = 90^o$

$\varphi_a(t) \neq \text{const.}$

$\varphi_b(t) \neq \text{const.}$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_d = 270^o$

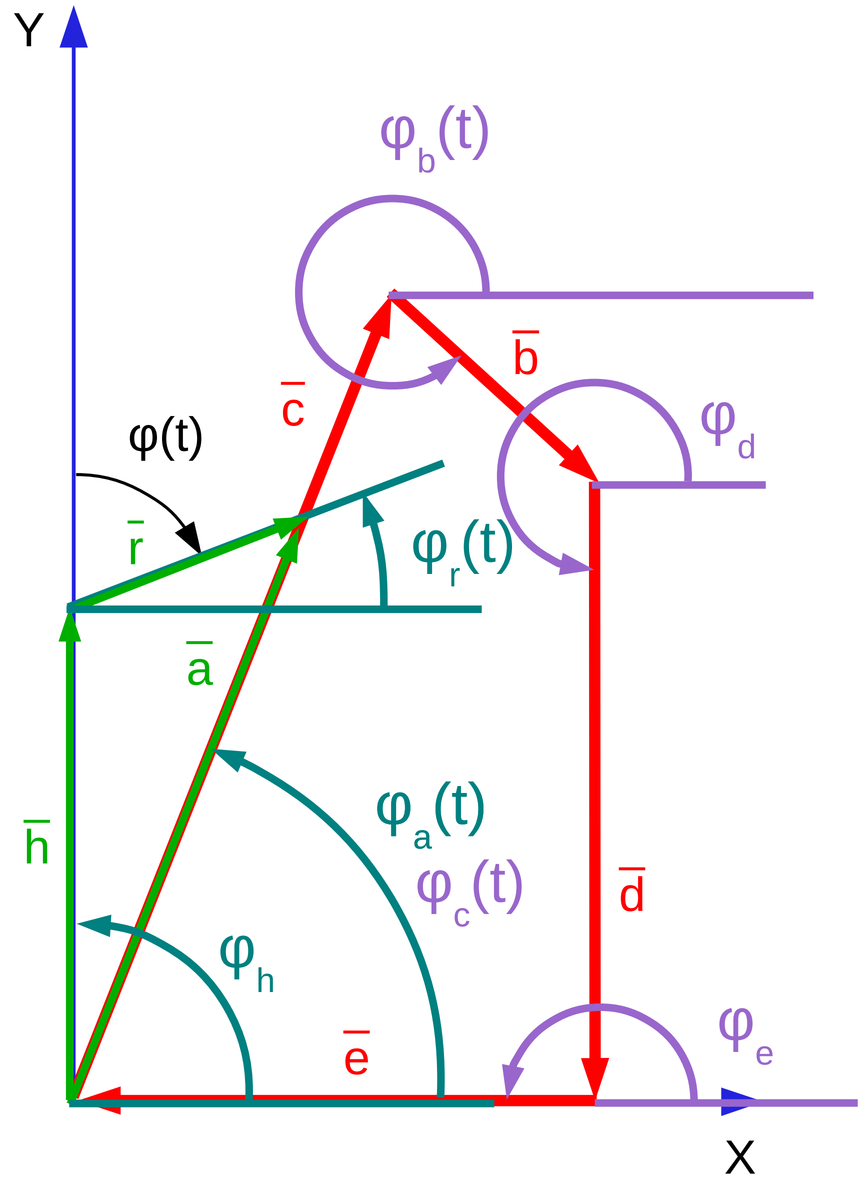
$\varphi_e = 180^o$

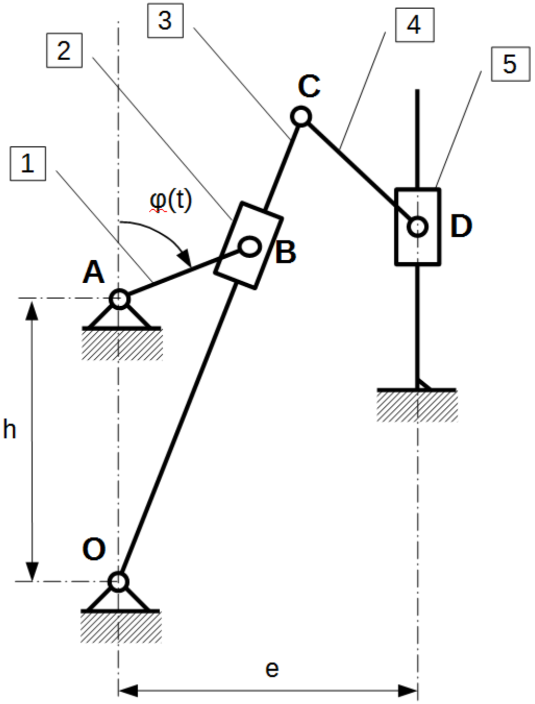
$|\bar{r}| = |AB| = r = \text{const.}$

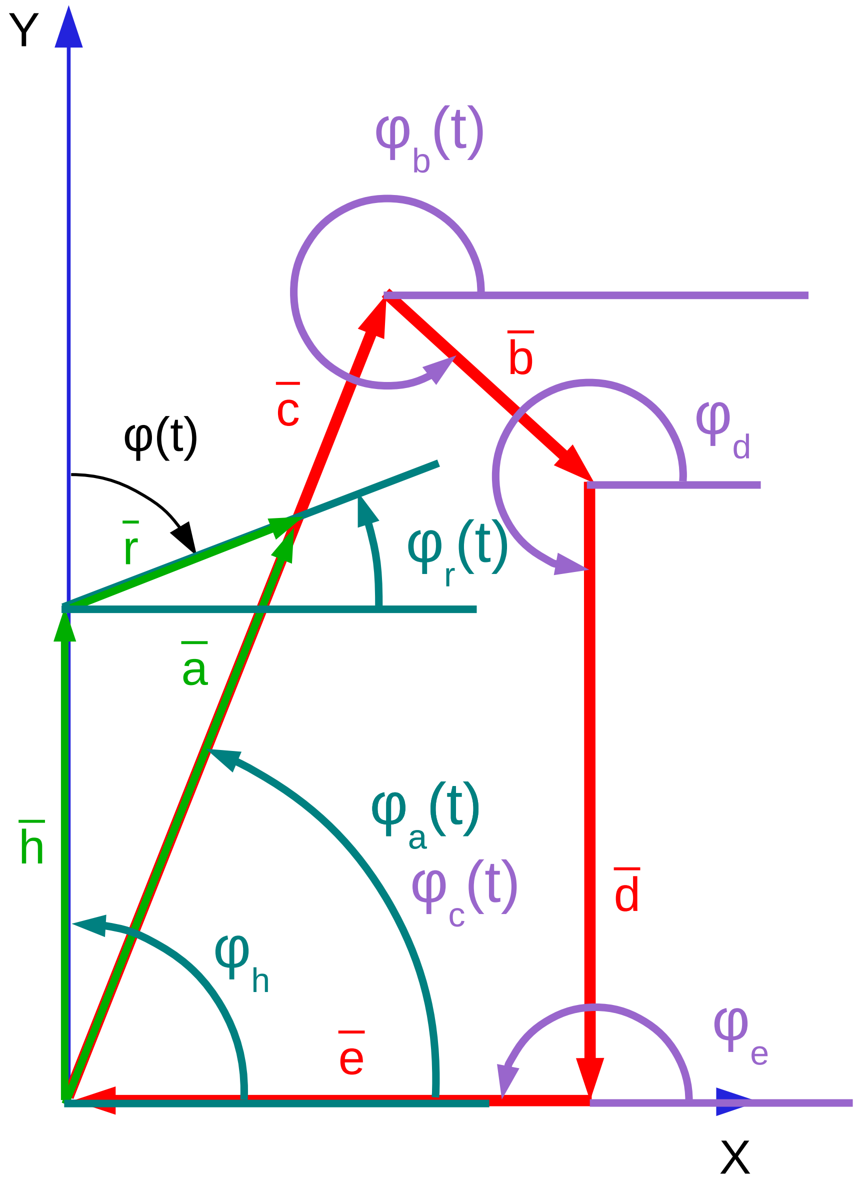
$|\bar{h}| = |OA| = h = \text{const.}$

$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = \text{const.}$







$\varphi_h = 90^o$

$\varphi_a(t) \neq \text{const.}$

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$\varphi_d = 270^o$

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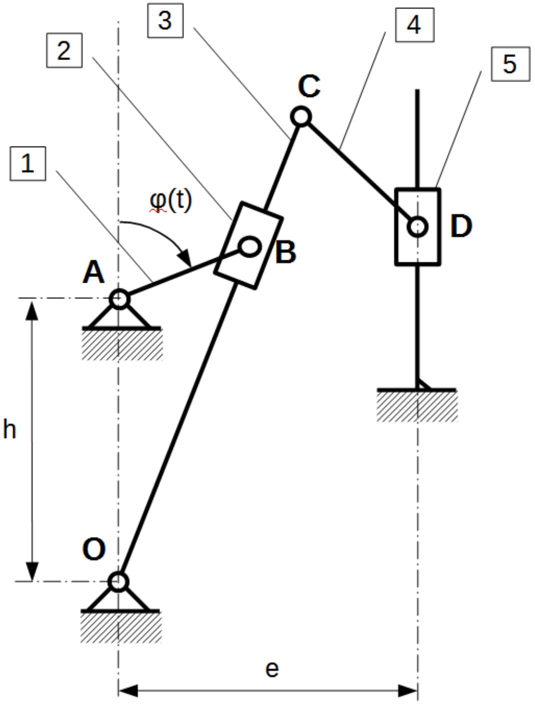
$|\bar{r}| = |AB| = r = \text{const.}$

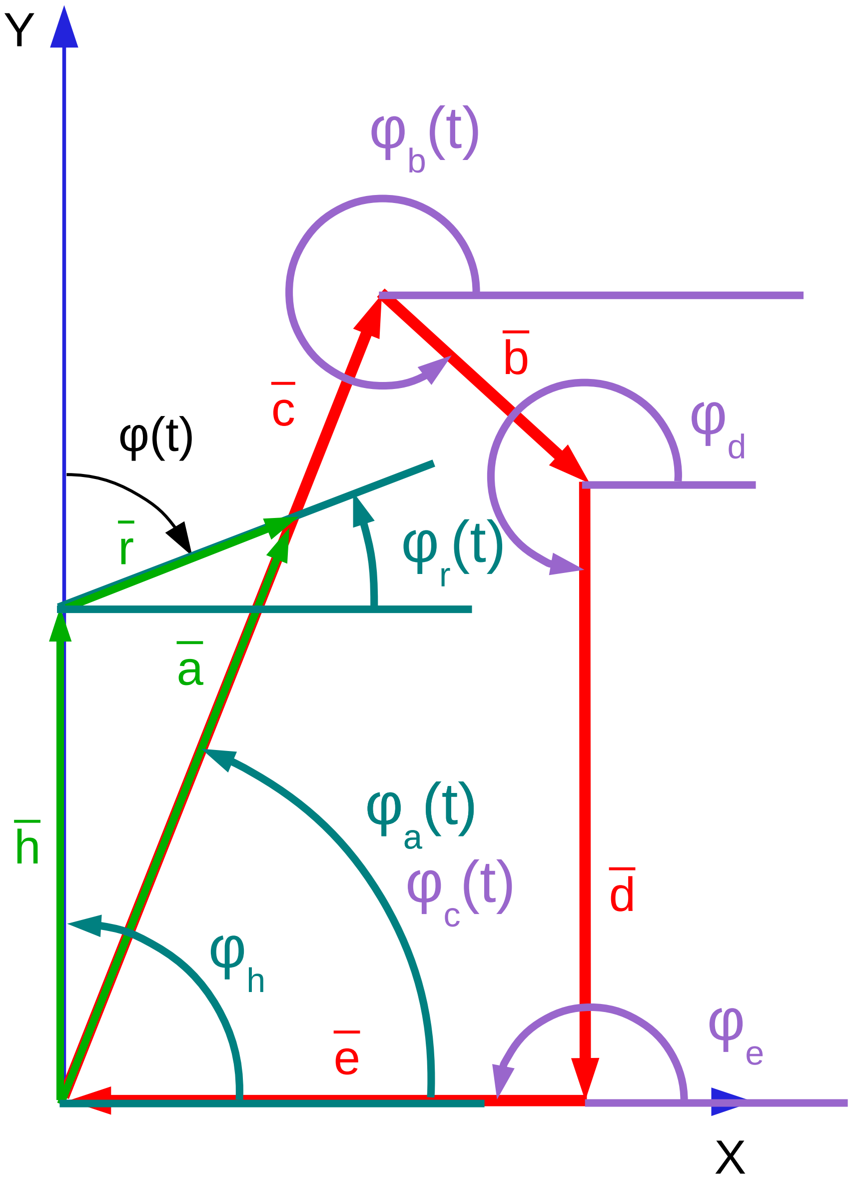
$|\bar{h}| = |OA| = h = \text{const.}$

$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = \text{const.}$

$|b| = |CD| = b = \text{const.}$





$\varphi_h = 90^o$

$\varphi_a(t) \neq$ const.

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$\varphi_c(t) = \varphi_a(t)$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_d = 270^o$

$\varphi_e = 180^o$

$|\bar{r}| = |AB| = r =$ const.

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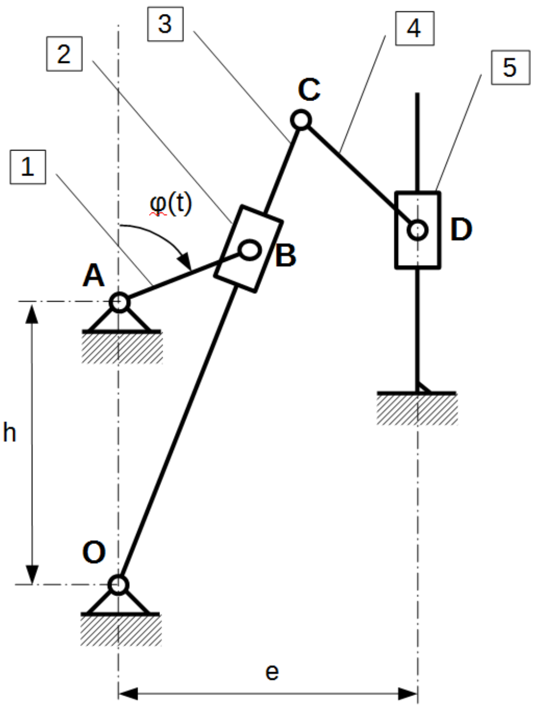
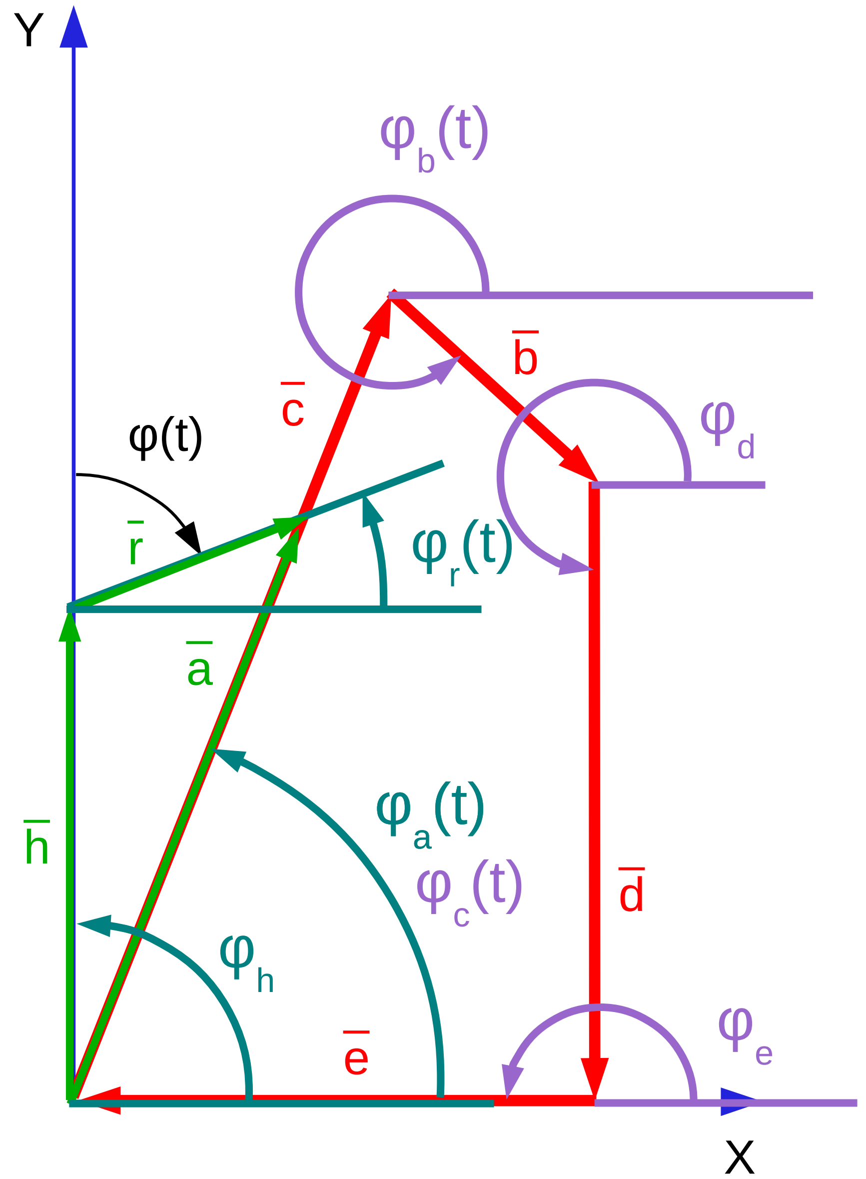
$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c =$ const.

$|b| = |CD| = b =$ const.

$|d| = d(t)$





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$|\bar{b}| = |CD| = b = const.$

$|\bar{d}| = d(t)$

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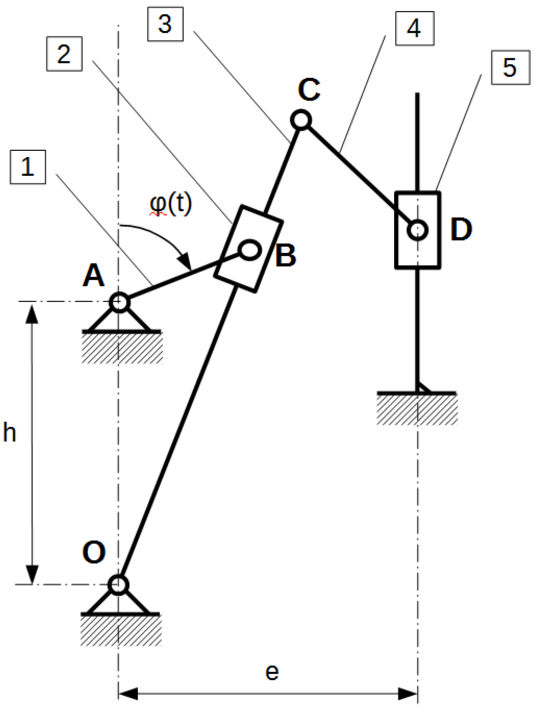

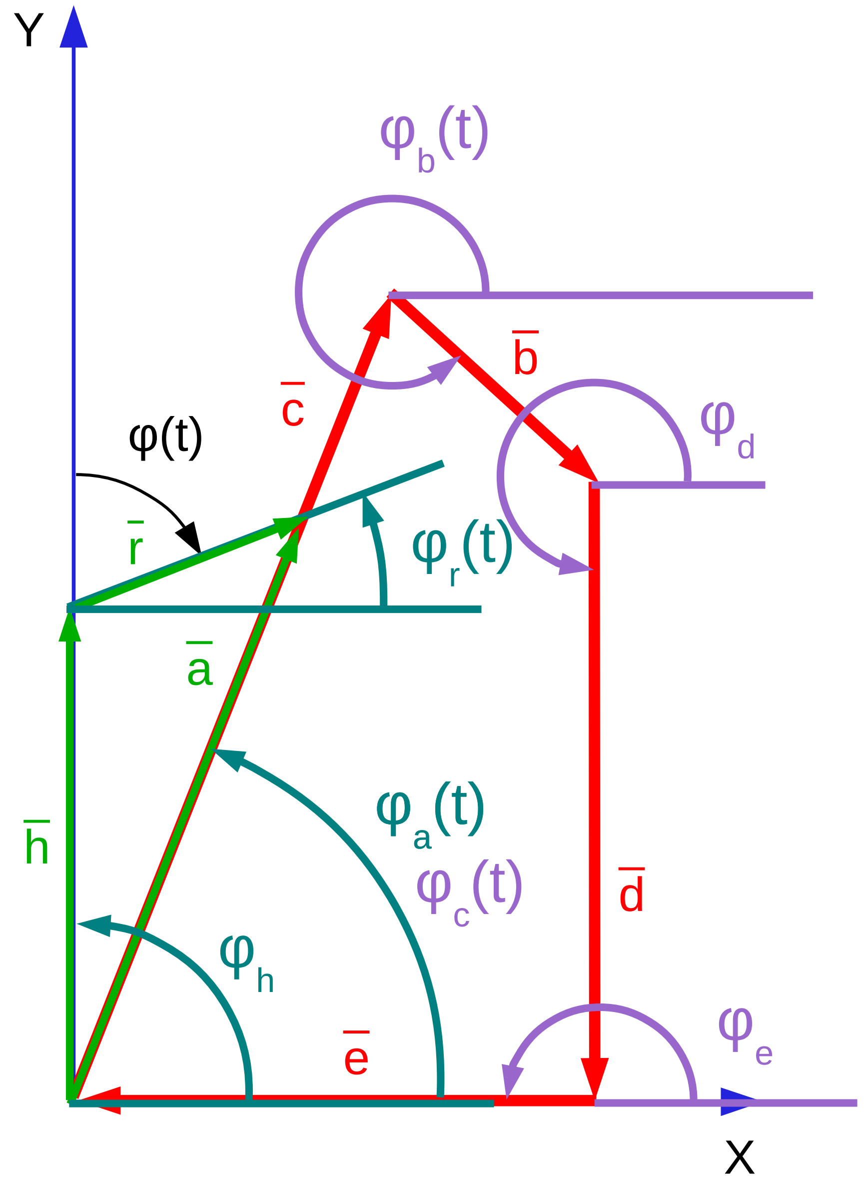

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$\varphi_b(t) \neq const.$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_d = 270^o$

$\varphi_e = 180^o$

$|\bar{r}| = |AB| = r = const.$

$|\bar{h}| = |OA| = h = const.$

$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = const.$

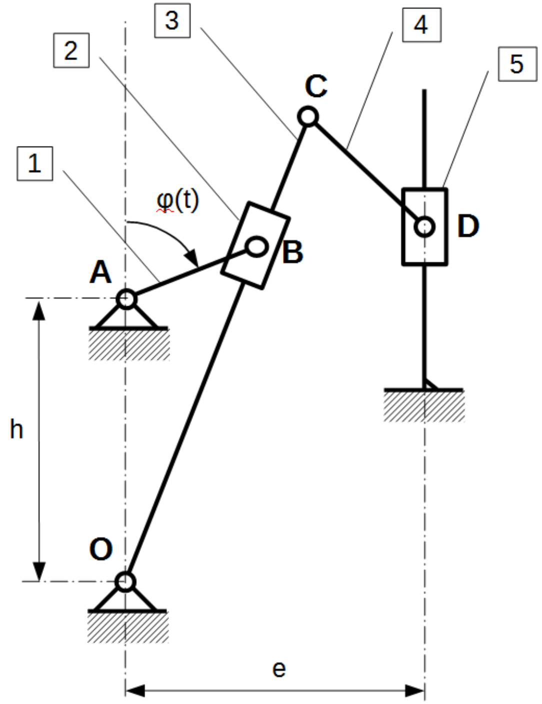
$|b| = |CD| = b = const.$

$|d| = d(t)$

$|\bar{e}| = e = const.$

$$\bar{h} + \bar{r} = \bar{a}$$
$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

7. Piszemy równania wektorowe dla wieloboków



$\varphi_h = 90^o$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_a(t) \neq const.$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_b(t) \neq const.$

$\varphi_d = 270^o$

$\varphi_e = 180^o$

$|\overline{h}| = |OA| = h = const.$

$|\overline{r}| = |AB| = r = const.$

$|\overline{a}| = |OB| = a(t)$

$|\overline{c}| = |OC| = c = const.$

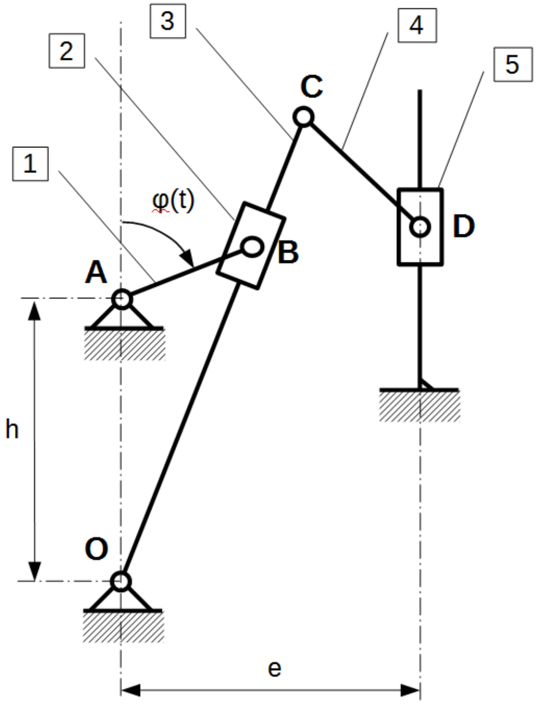
$|b| = |CD| = b = const.$

$|d| = d(t)$

$|\overline{e}| = e = const.$

$$\overline{h} + \overline{r} = \overline{a}$$
$$\overline{c} + \overline{b} + \overline{d} + \overline{e} = \overline{0}$$

8. Rozwiązujemy to równanie wektorowe rzutując wektory na osie układu współrzędnych.



$\varphi_h = 90^o$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_a(t) \neq const.$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_b(t) \neq const.$

$\varphi_d = 270^o$

$\varphi_e = 180^o$

$|\bar{h}| = |OA| = h = const.$

$|\bar{r}| = |AB| = r = const.$

$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = const.$

$|b| = |CD| = b = const.$

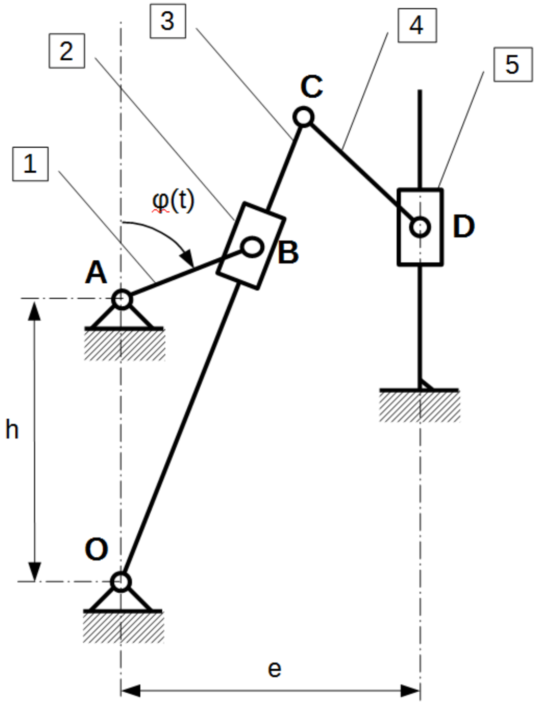
$|d| = d(t)$

$|\bar{e}| = e = const.$

$$\bar{h} + \bar{r} = \bar{a}$$
$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

$$x:\quad h\cos\phi_h + r\cos\phi_r(t) = a(t)\cos\phi_a(t)$$
$$y:\quad h\sin\phi_h + r\sin\phi_r(t) = a(t)\sin\phi_a(t)$$
$$x:\quad c\cos\phi_c(t) + b\cos\phi_b(t) + d(t)\cos\phi_d + e\cos\phi_e = 0$$
$$y:\quad c\sin\phi_c(t) + b\sin\phi_b(t) + d(t)\sin\phi_d + e\sin\phi_e = 0$$

9. Dzięki temu, że wszystkie kąty oznaczyliśmy tą samą metodą, rzuty są odpowiednio z kosinusami i sinusami.



$\varphi_h = 90^o$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_a(t) \neq const.$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_b(t) \neq const.$

$\varphi_d = 270^o$

$\varphi_e = 180^o$

$|\bar{h}| = |OA| = h = const.$

$|\bar{r}| = |AB| = r = const.$

$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = const.$

$|b| = |CD| = b = const.$

$|d| = d(t)$

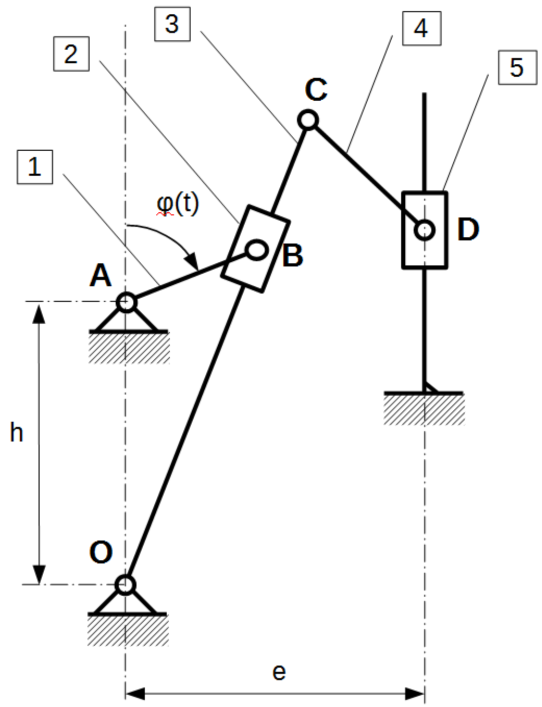
$|\bar{e}| = e = const.$

$$\bar{h} + \bar{r} = \bar{a}$$
$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

10. Upraszczamy równania rzutów z użyciem informacji o kątach.

$$x:\ h\cos\phi_h + r\cos\phi_r(t) = a(t)\cos\phi_a(t)$$
$$y:\ h\sin\phi_h + r\sin\phi_r(t) = a(t)\sin\phi_a(t)$$
$$x:\ c\cos\phi_c(t) + b\cos\phi_b(t) + d(t)\cos\phi_d + e\cos\phi_e = 0$$
$$y:\ c\sin\phi_c(t) + b\sin\phi_b(t) + d(t)\sin\phi_d + e\sin\phi_e = 0$$

$$x:\ h\cos 90^o + r\cos(90^o - \phi(t)) = a(t)\cos\phi_a(t)$$
$$y:\ h\sin 90^o + r\sin(90^o - \phi(t)) = a(t)\sin\phi_a(t)$$
$$x:\ c\cos\phi_a(t) + b\cos\phi_b(t) + d(t)\cos 270^o + e\cos 180^o = 0$$
$$y:\ c\sin\phi_a(t) + b\sin\phi_b(t) + d(t)\sin 270^o + e\sin 180^o = 0$$



$\varphi_h = 90^o$

$\varphi_r(t) = 90^o - \varphi(t)$

$\varphi_a(t) \neq const.$

$\varphi_c(t) = \varphi_a(t)$

$\varphi_b(t) \neq const.$

$\varphi_d = 270^o$

$\varphi_e = 180^o$

$|\bar{h}| = |OA| = h = const.$

$|\bar{r}| = |AB| = r = const.$

$|\bar{a}| = |OB| = a(t)$

$|\bar{c}| = |OC| = c = const.$

$|b| = |CD| = b = const.$

$|d| = d(t)$

$|\bar{e}| = e = const.$

$$\begin{aligned} \bar{h} + \bar{r} &= \bar{a} \\ \bar{c} + \bar{b} + \bar{d} + \bar{e} &= \bar{0} \end{aligned}$$

$$x:\ h\cos\phi_h + r\cos\phi_r(t) = a(t)\cos\phi_a(t)$$

$$y:\ h\sin\phi_h + r\sin\phi_r(t) = a(t)\sin\phi_a(t)$$

$$x:\ c\cos\phi_c(t) + b\cos\phi_b(t) + d(t)\cos\phi_d + e\cos\phi_e = 0$$

$$y:\ c\sin\phi_c(t) + b\sin\phi_b(t) + d(t)\sin\phi_d + e\sin\phi_e = 0$$

$$x:\ h\cos 90^o + r\cos(90^o - \phi(t)) = a(t)\cos\phi_a(t)$$

$$y:\ h\sin 90^o + r\sin(90^o - \phi(t)) = a(t)\sin\phi_a(t)$$

$$x:\ c\cos\phi_a(t) + b\cos\phi_b(t) + d(t)\cos 270^o + e\cos 180^o = 0$$

$$y:\ c\sin\phi_a(t) + b\sin\phi_b(t) + d(t)\sin 270^o + e\sin 180^o = 0$$

$$x:\ r\sin\phi(t) = a(t)\cos\phi_a(t)$$

$$y:\ h + r\cos\phi(t) = a(t)\sin\phi_a(t)$$

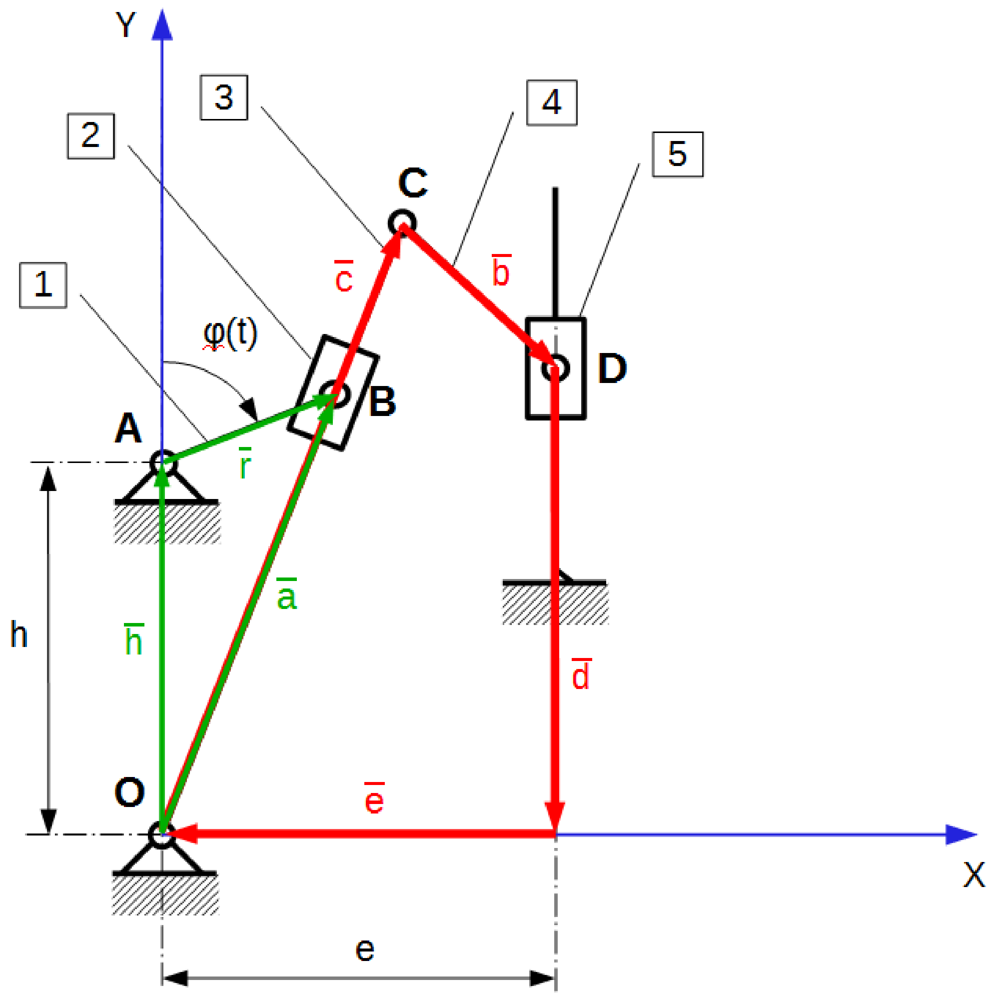
$$x:\ c\cos\phi_a(t) + b\cos\phi_b(t) - e = 0$$

$$y:\ c\sin\phi_a(t) + b\sin\phi_b(t) - d(t) = 0$$

11. Po wszystkich uproszczeniach mamy układ 4 równań z 4 niewiadomymi. Rozwiązujemy go, uważając na funkcje typu *arcus*, które nie zawsze zwracają wartości w interesującym nas zakresie kątów.

1. $x:\ \ r\sin\phi(t)=a(t)\cos\phi_a(t)$
2. $y:\ \ h+r\cos\phi(t)=a(t)\sin\phi_a(t)$
3. $x:\ \ c\cos\phi_a(t)+b\cos\phi_b(t)-e=0$
4. $y:\ \ c\sin\phi_a(t)+b\sin\phi_b(t)-d(t)=0$

12. Po rozwiązaniu tego układu skupiamy się na funkcji d(t) która opisuje odległość suwaka od podpory.



$$\text{1} \quad x:\ r\sin\phi(t)=a(t)\cos\phi_a(t)$$

$$\text{2} \quad y:\ h+r\cos\phi(t)=a(t)\sin\phi_a(t)$$

$$\text{3} \quad x:\ c\cos\phi_a(t)+b\cos\phi_b(t)-e=0$$

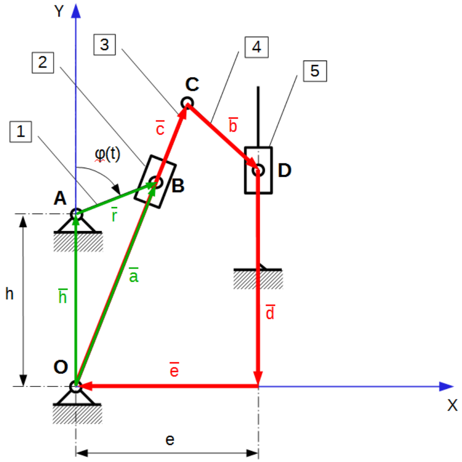
$$\text{4} \quad y:\ c\sin\phi_a(t)+b\sin\phi_b(t)-d(t)=0$$

$$a(t)=\sqrt{r^2\sin^2(t)+(h+r\cos\phi(t))^2}$$

$$\phi_a(t)=atan\left(\frac{h+r\cos\phi(t)}{r\sin\phi(t)}\right)$$

$$\phi_b(t)=\arccos\frac{e-c\cos\phi_a(t)}{b}$$

$$d(t)=c\sin\phi_a(t)+b\sqrt{1-\left(\frac{e-c\cos\phi_a(t)}{b}\right)^2}$$

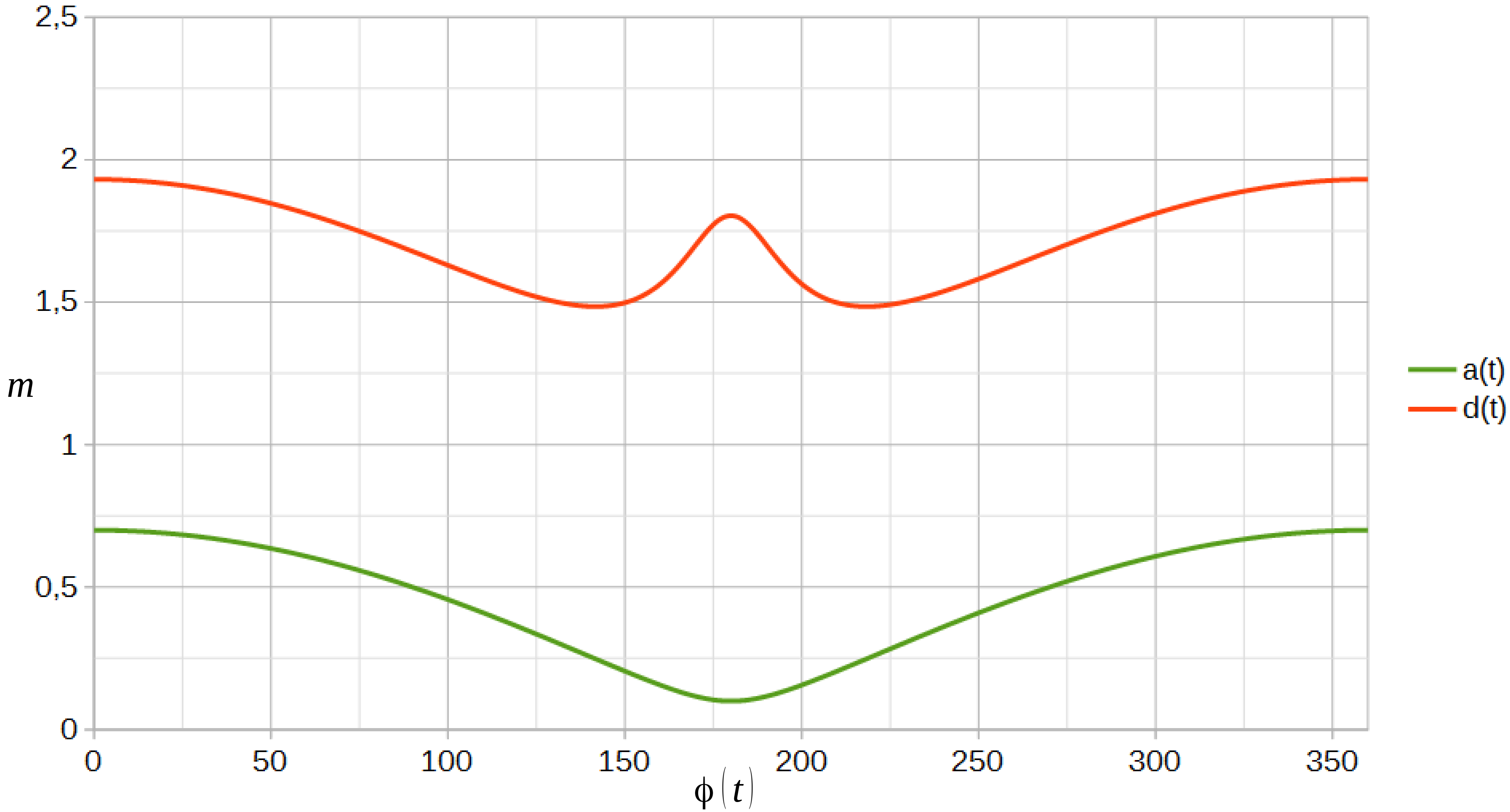


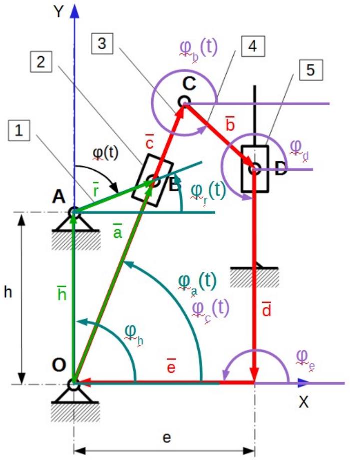
13. Dla przykładowych danych (tabela obok) przedstawiono wykresy dla funkcji:

$$a(t)=\sqrt{r^2\sin^2(t)+(h+r\cos\phi(t))^2}$$

$$d(t)=c\sin\phi_a(t)+b\sqrt{1-\left(\frac{e-c\cos\phi_a(t)}{b}\right)^2}$$

r=	0,3	[m]
h=	0,4	[m]
c=	1	[m]
e=	0,4	[m]
b=	1	[m]
ω=	1	[rad/s]

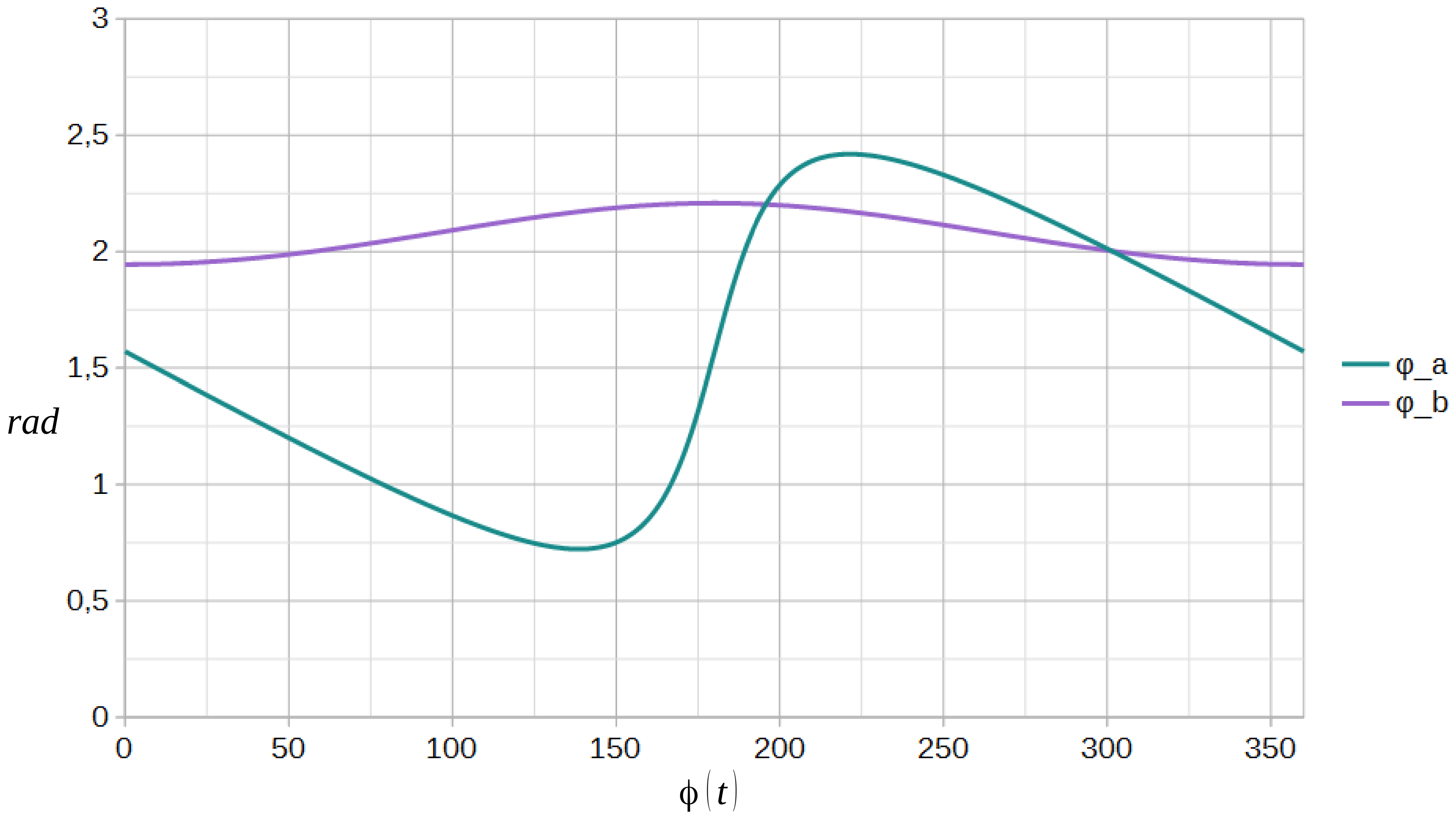




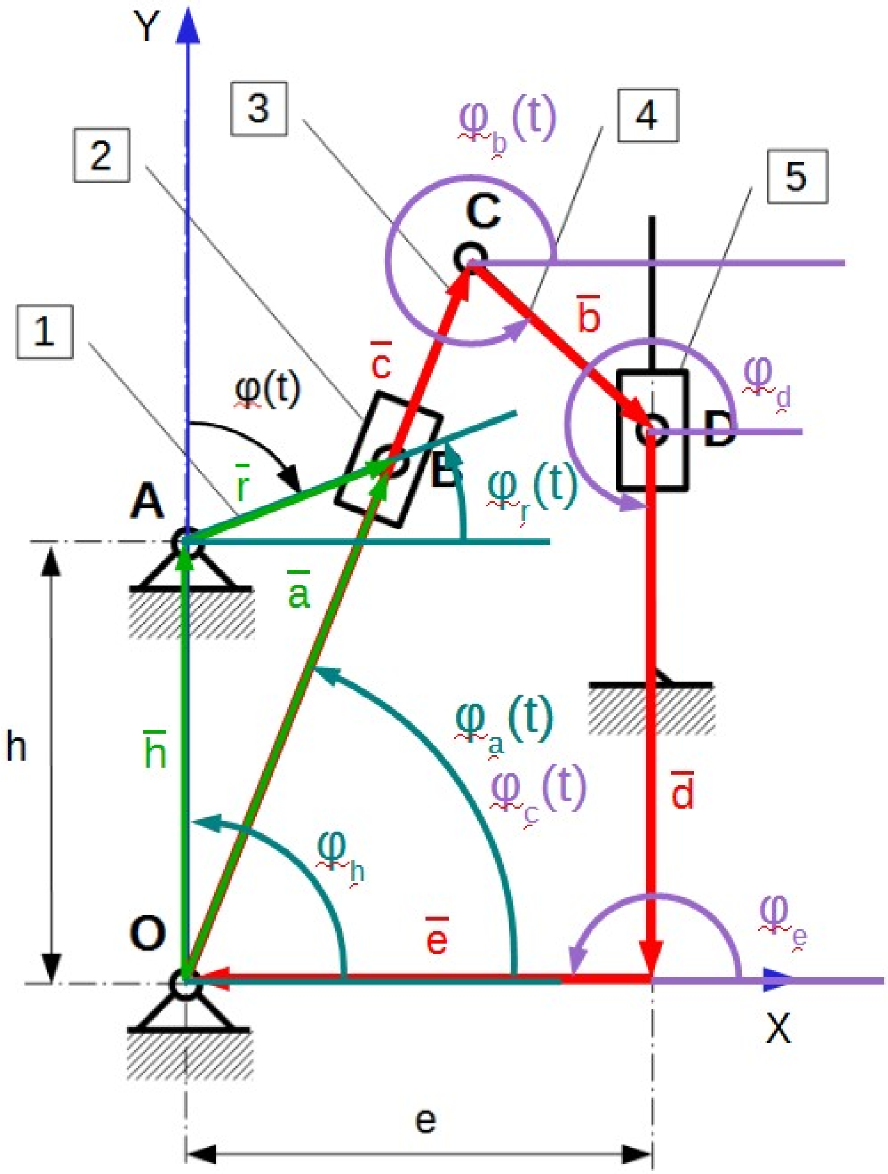
$$\phi_a(t) = atan\left(\frac{h + r\cos\phi(t)}{r\sin\phi(t)}\right)$$

$$\phi_b(t) = \arccos\frac{e - c\cos\phi_a(t)}{b}$$

r=	0,3	[m]
h=	0,4	[m]
c=	1	[m]
e=	0,4	[m]
b=	1	[m]
ω=	1	[rad/s]



14. Poszukiwane: prędkość i przyspieszenie suwaka D otrzymamy z różniczkowań po czasie funkcji d(t). Różniczkowanie to, można wykonać “na papierze”, ale ze względu na skomplikowaną formę d(t) lepiej użyć oprogramowania do obliczeń symbolicznych (Mathematica, wxMaxima, Mathcad). Można też wykonać przybliżone różniczkowanie numeryczne metodą ilorazu różnicowego.



(1) $x: \; r\sin\phi(t) = a(t)\cos\phi_a(t)$

(2) $y: \; h + r\cos\phi(t) = a(t)\sin\phi_a(t)$

(3) $x: \; c\cos\phi_a(t) + b\cos\phi_b(t) - e = 0$

(4) $y: \; c\sin\phi_a(t) + b\sin\phi_b(t) - d(t) = 0$

$$a(t) = \sqrt{r^2\sin^2(t) + (h + r\cos\phi(t))^2}$$

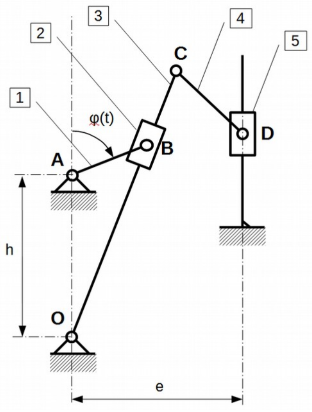
$$\phi_a(t) = atan\left(\frac{h + r\cos\phi(t)}{r\sin\phi(t)}\right)$$

$$\phi_b(t) = \arccos\frac{e - c\cos\phi_a(t)}{b}$$

$$d(t) = c\sin\phi_a(t) + b\sqrt{1 - \left(\frac{e - c\cos\phi_a(t)}{b}\right)^2}$$

$$v_D(t) = \dot{d}(t)$$

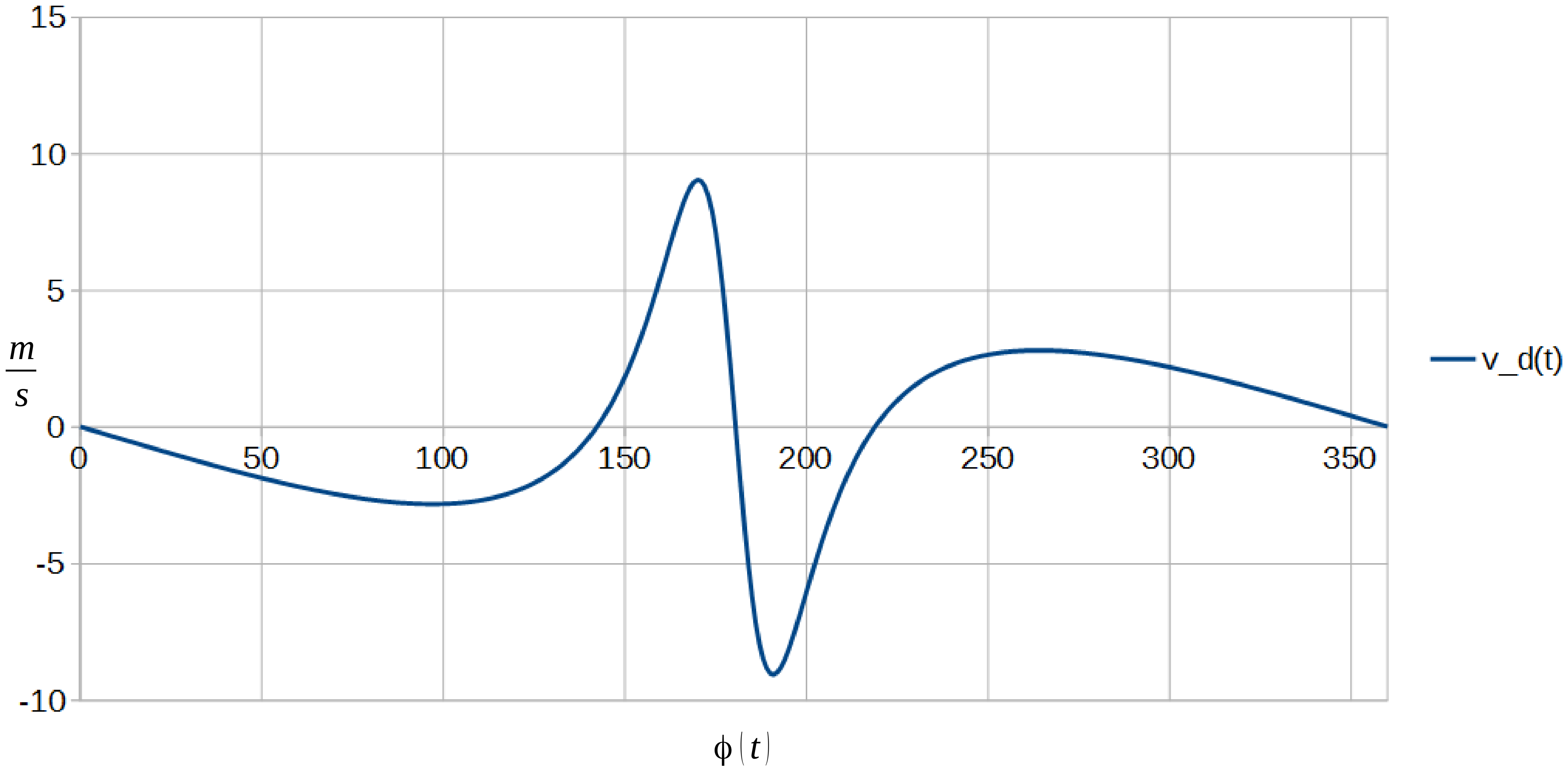
$$a_D(t) = \ddot{d}(t)$$

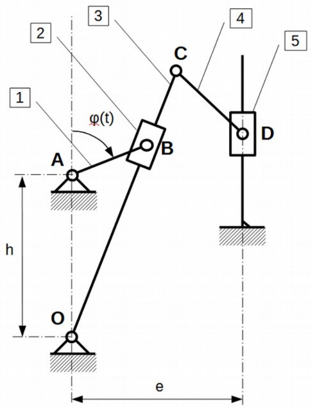


15. Wykres prędkości

r=	0,3	[m]
h=	0,4	[m]
c=	1	[m]
e=	0,4	[m]
b=	1	[m]
ω=	1	[rad/s]

$$v_D(t) = \dot{d}(t)$$





16. Wykres przyspieszenia

r=	0,3	[m]
h=	0,4	[m]
c=	1	[m]
e=	0,4	[m]
b=	1	[m]
ω=	1	[rad/s]

$$a_D(t) = \ddot{d}(t)$$