Warsaw University of Technology
The Faculty of Automotive and Construction Machinery Engineering
Institute of Machine Design Fundamentals
Department of Mechanics
http://www.ipbm.simr.pw.edu.pl/

Theory of Machines and Automatic Control

Winter 2017/2018

Lecturer: Sebastian Korczak, PhD Eng.

Lecture 5

Cam-follower mechanisms cont.
Dynamics of planar mechanisms.

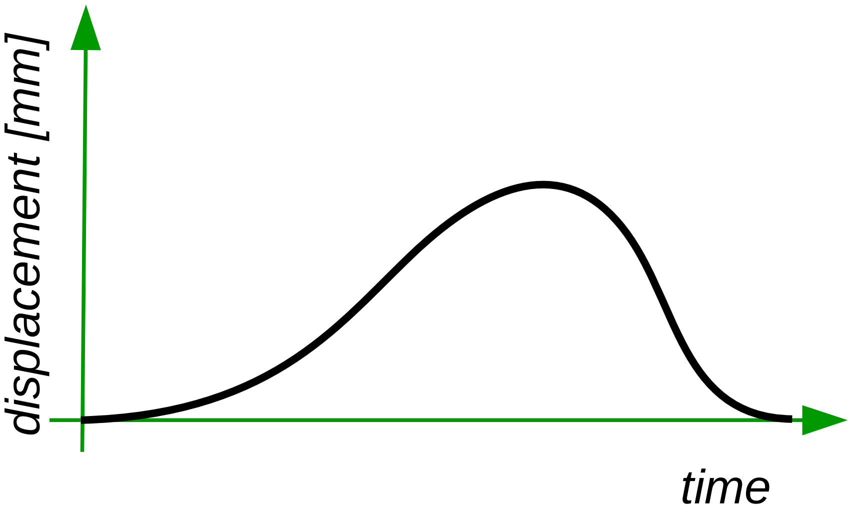
Cam-follower mechanisms cont.

Analysis and synthesis of cam-follower mechanisms

Analysis	Syntesis
• substitution of IV. class kinematic pair with V. class kinematic pairs + graphical method (velocity and acceleration scheme) • graphical determination of a follower movement and graphical differentiation • analytical method (substitution with polygones of vectors)	• graphical determination of cam outline by a base circle rotation with follower movement • analytical designing with a function description

Synthesis of cam-follower mechanisms

Graphical method



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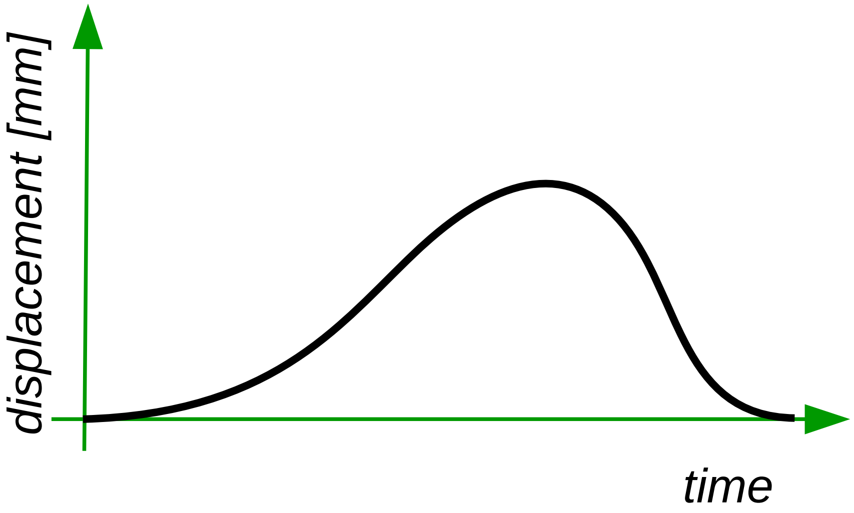

assumption: $\varphi(t)=\omega t$

$\omega=\text{const.}$

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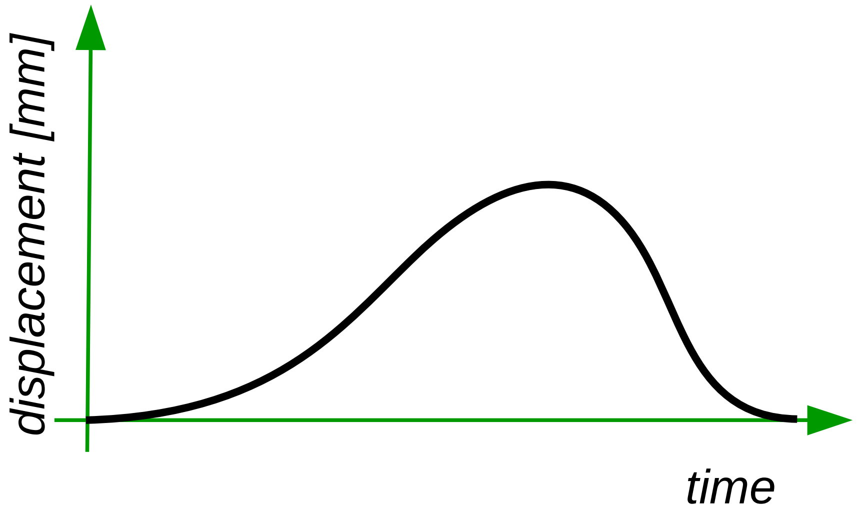

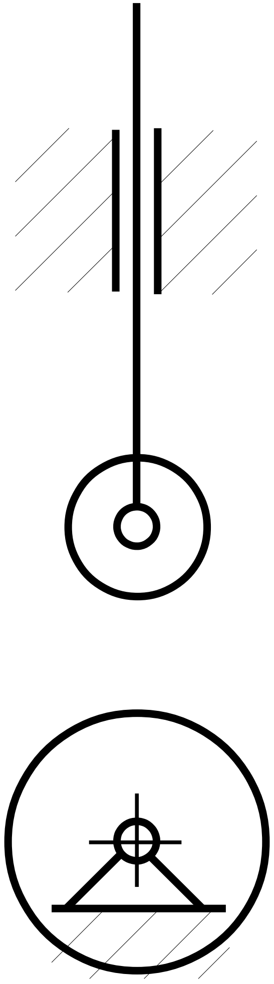

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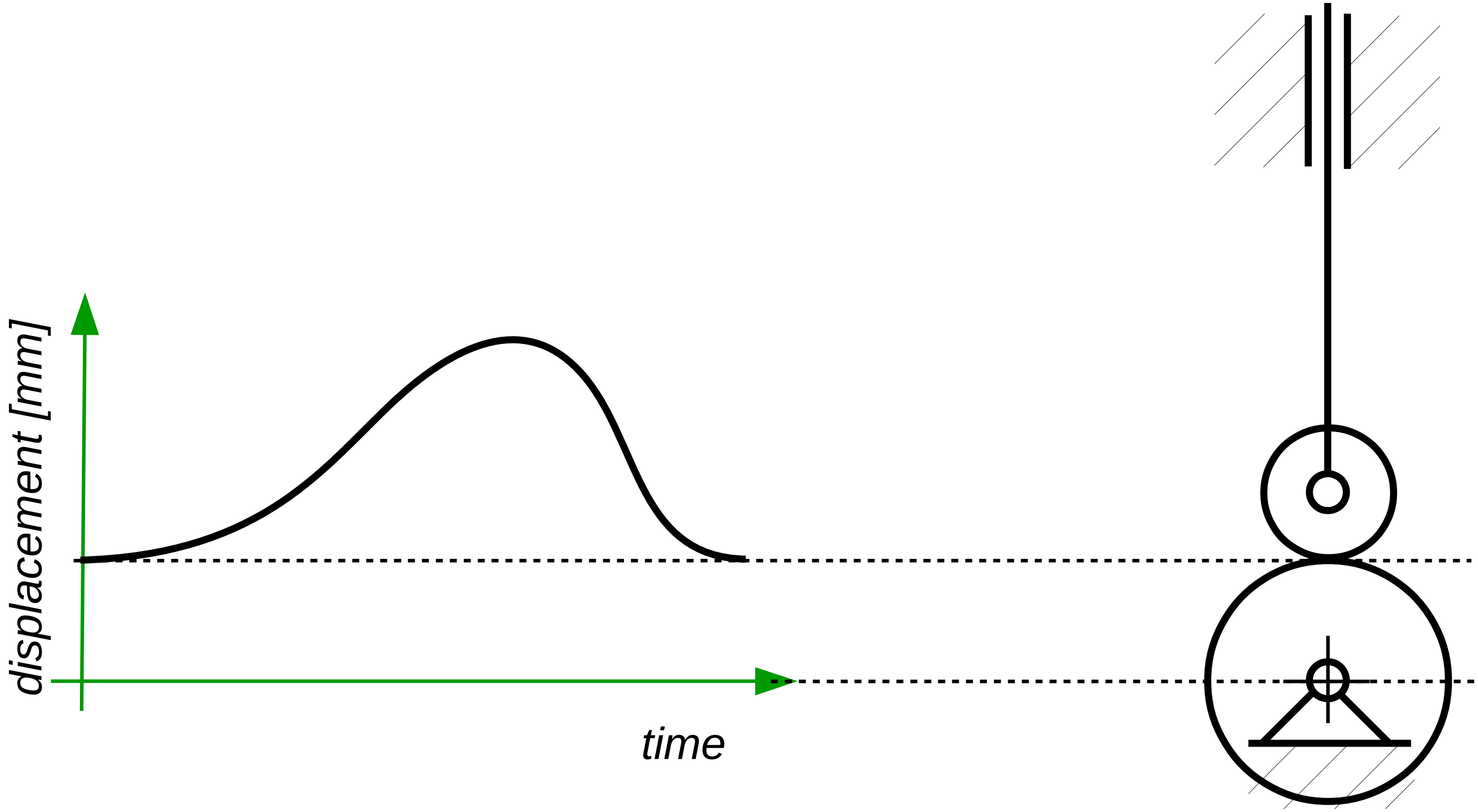

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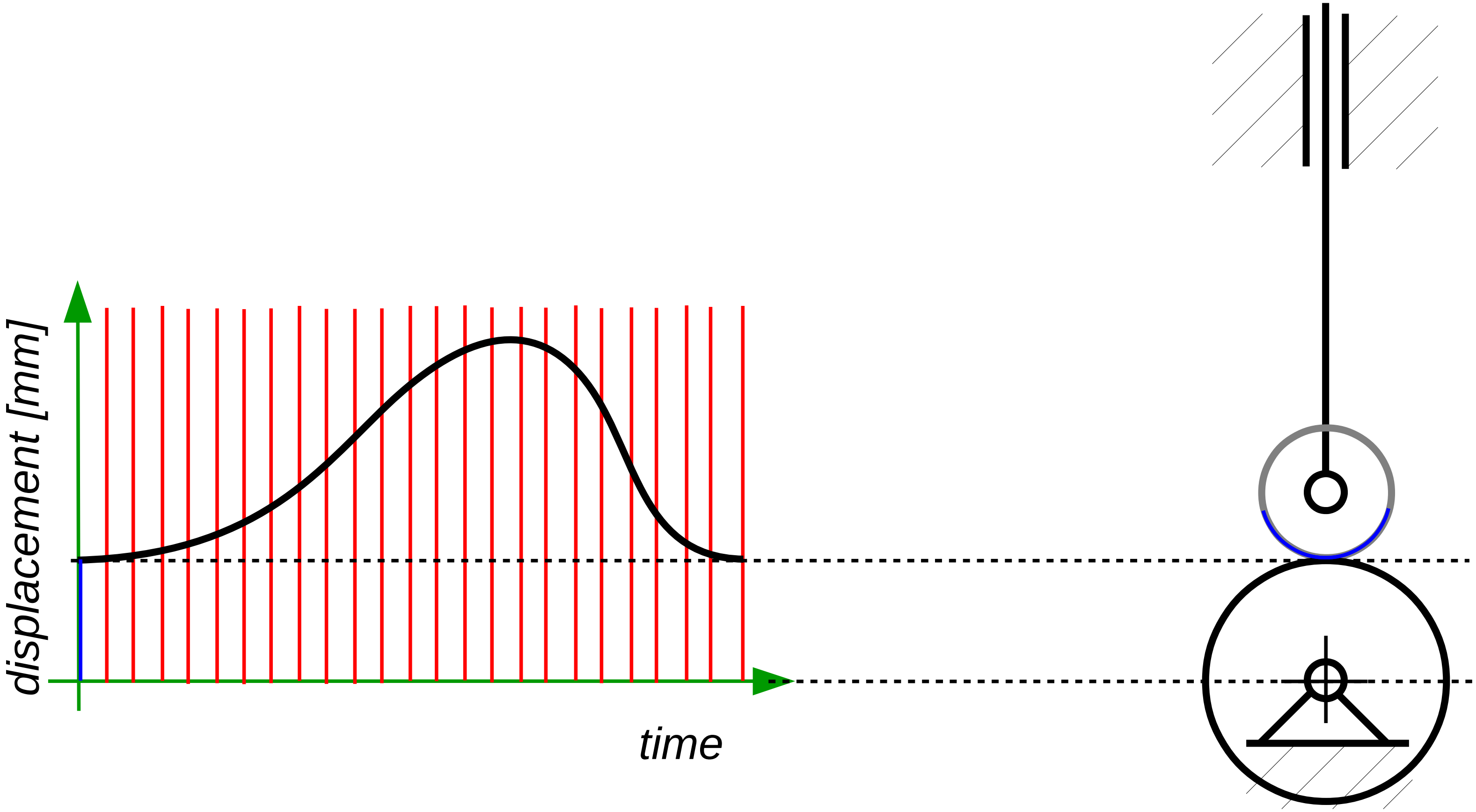

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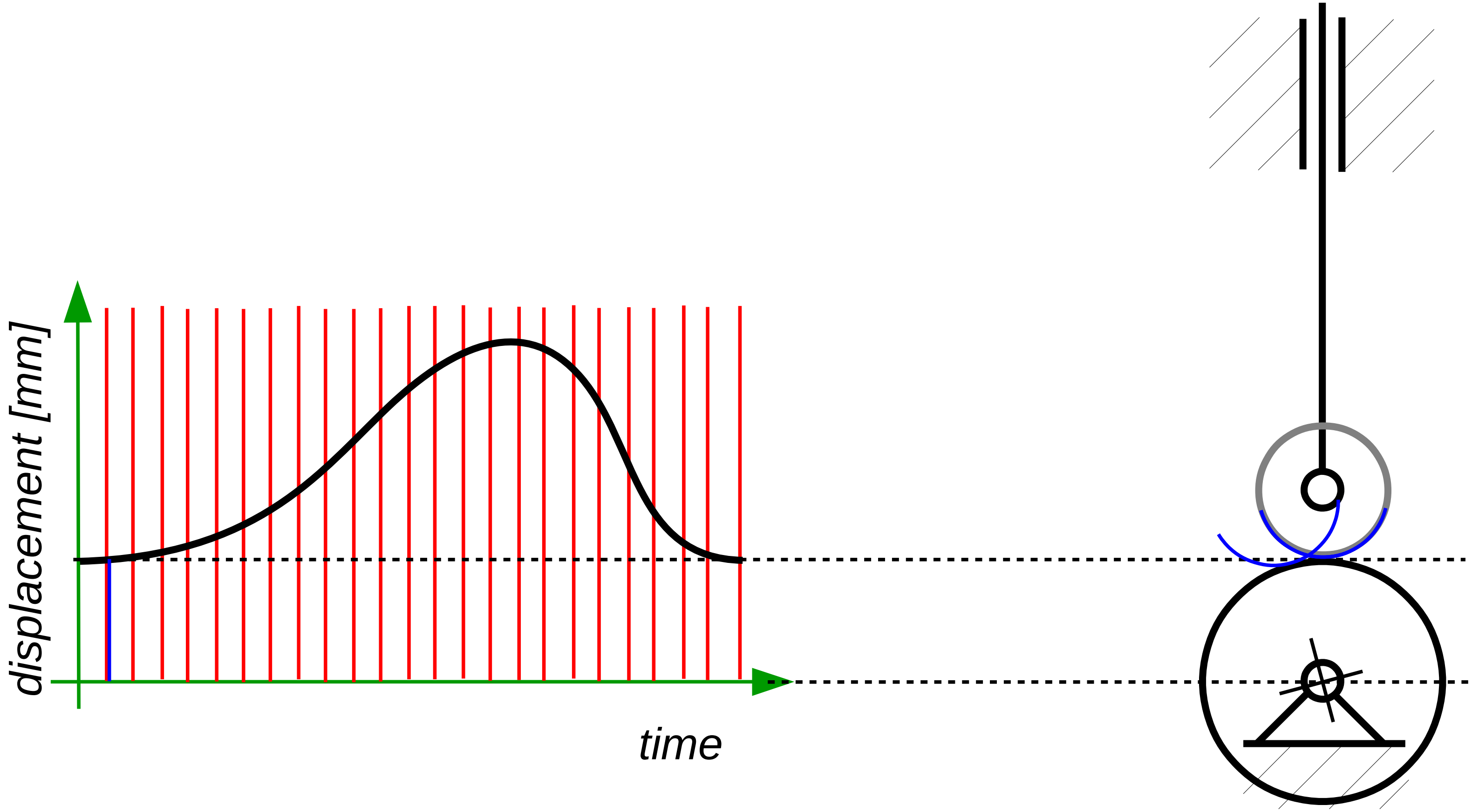

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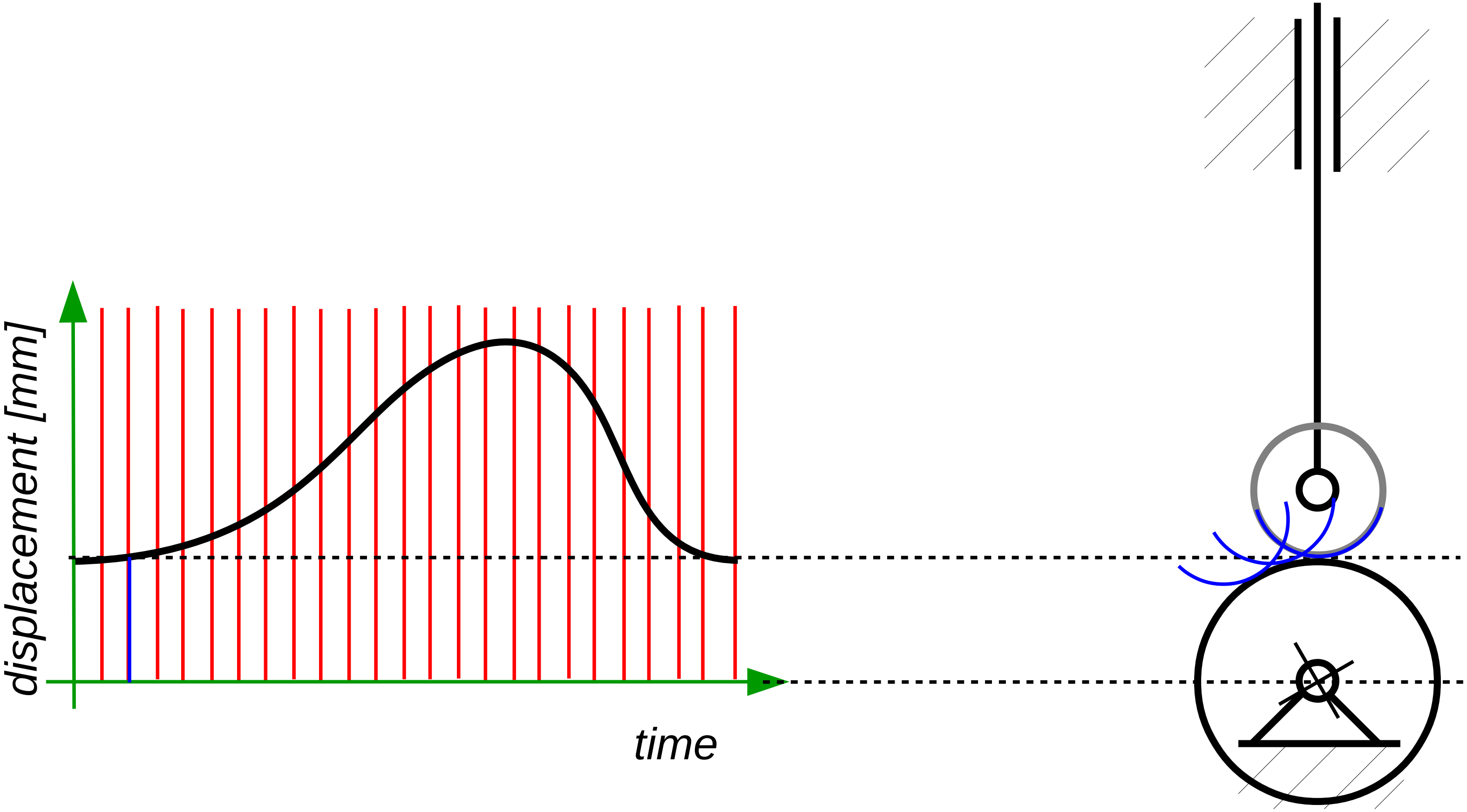

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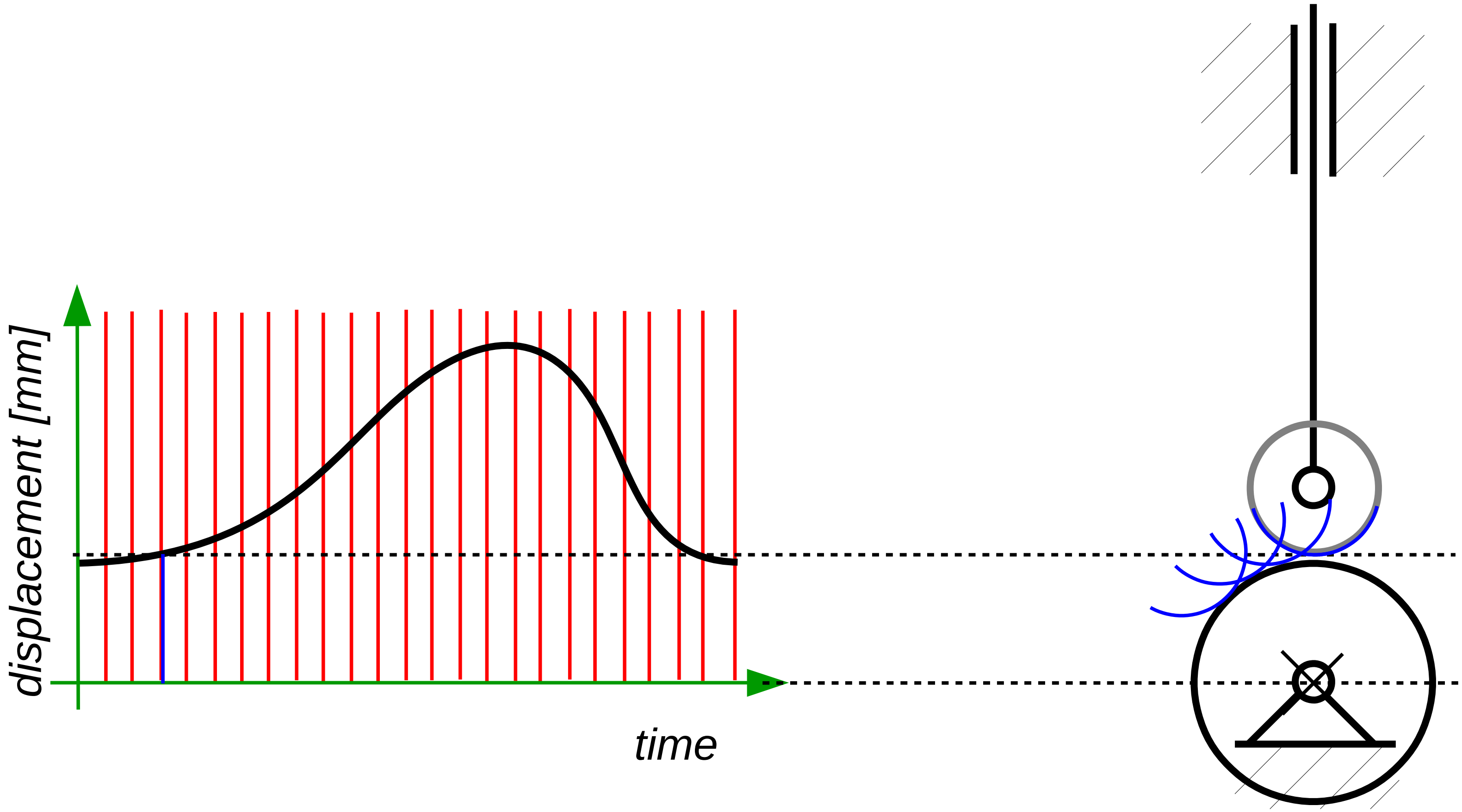

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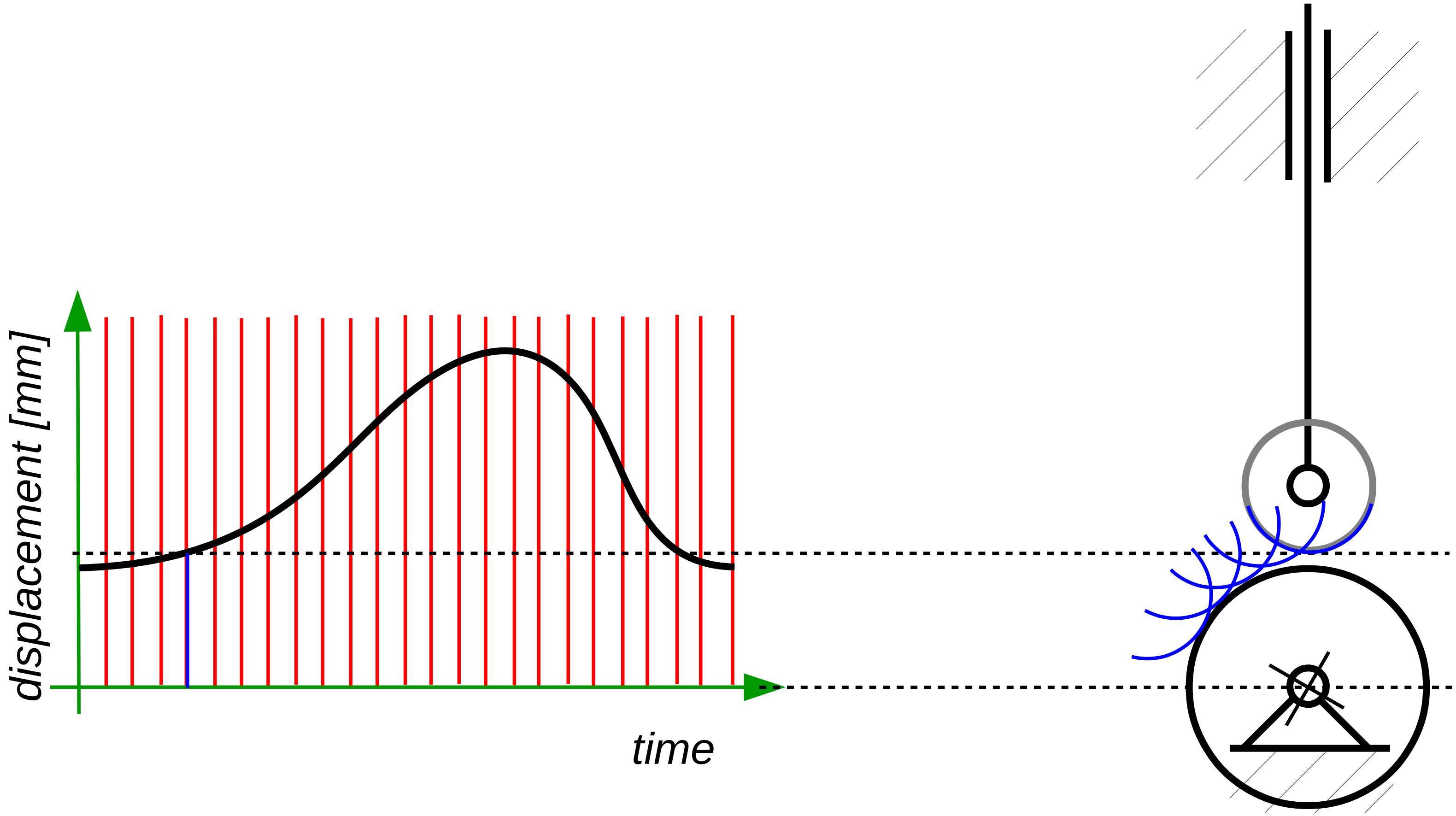

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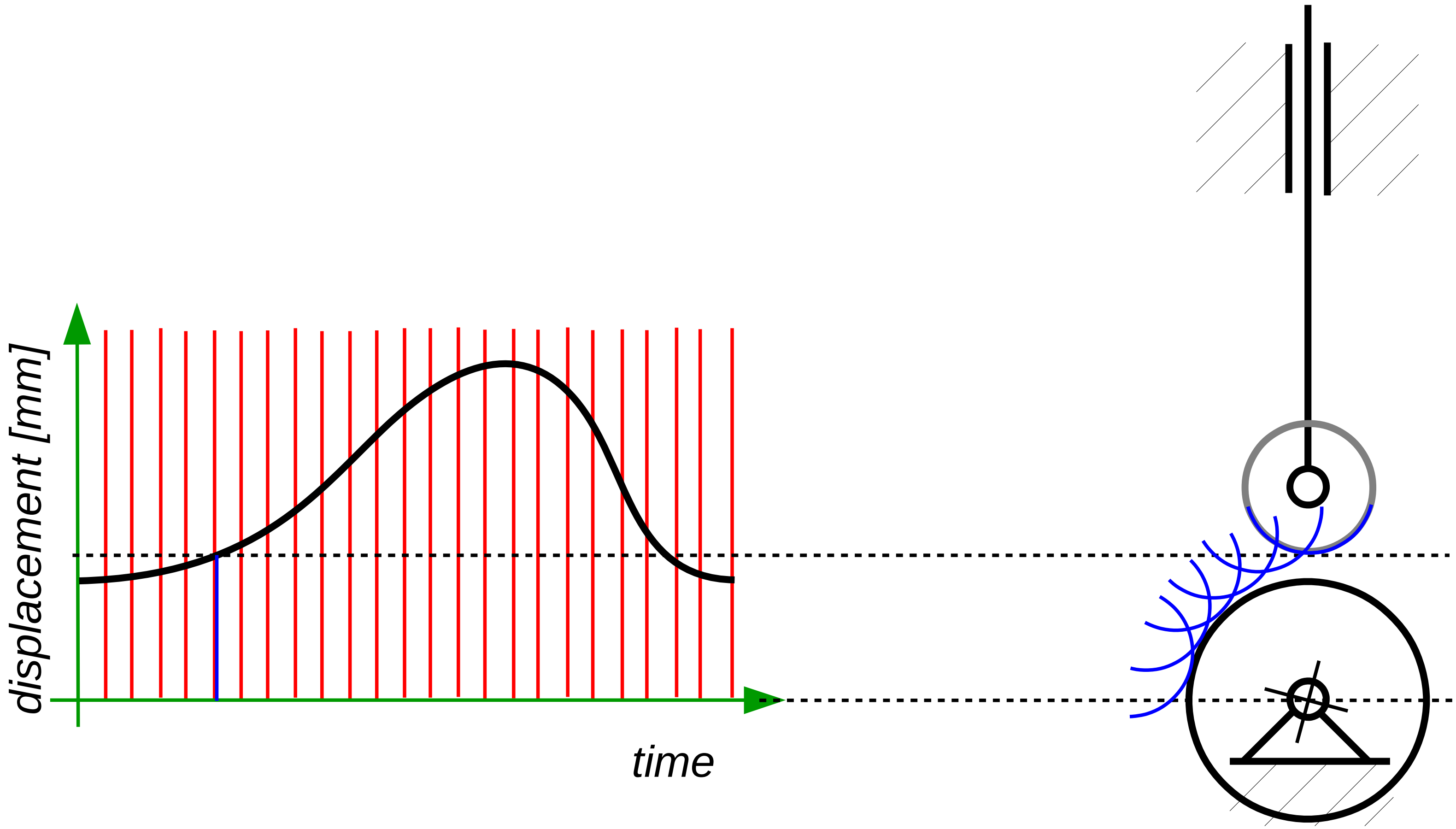

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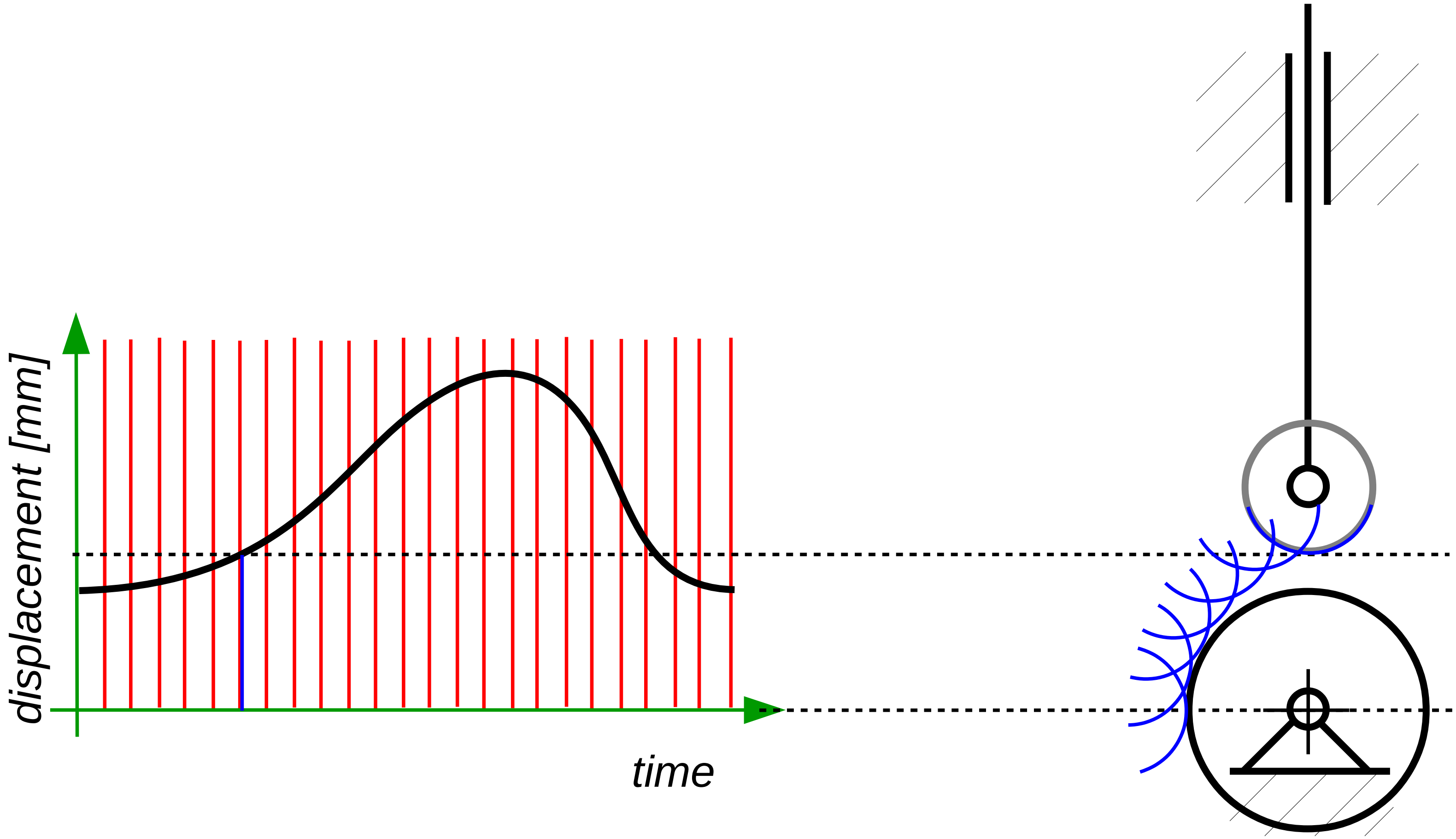

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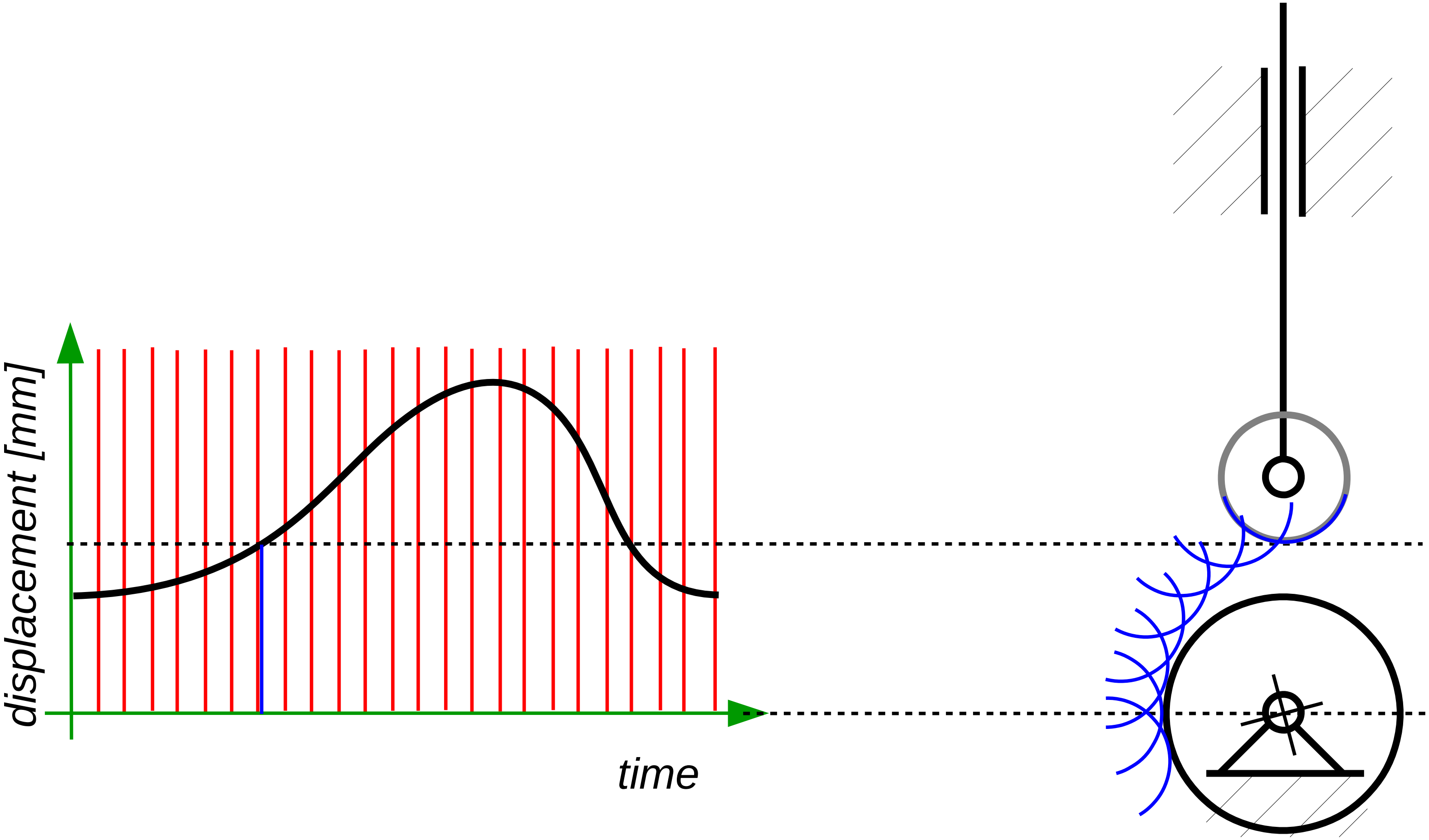

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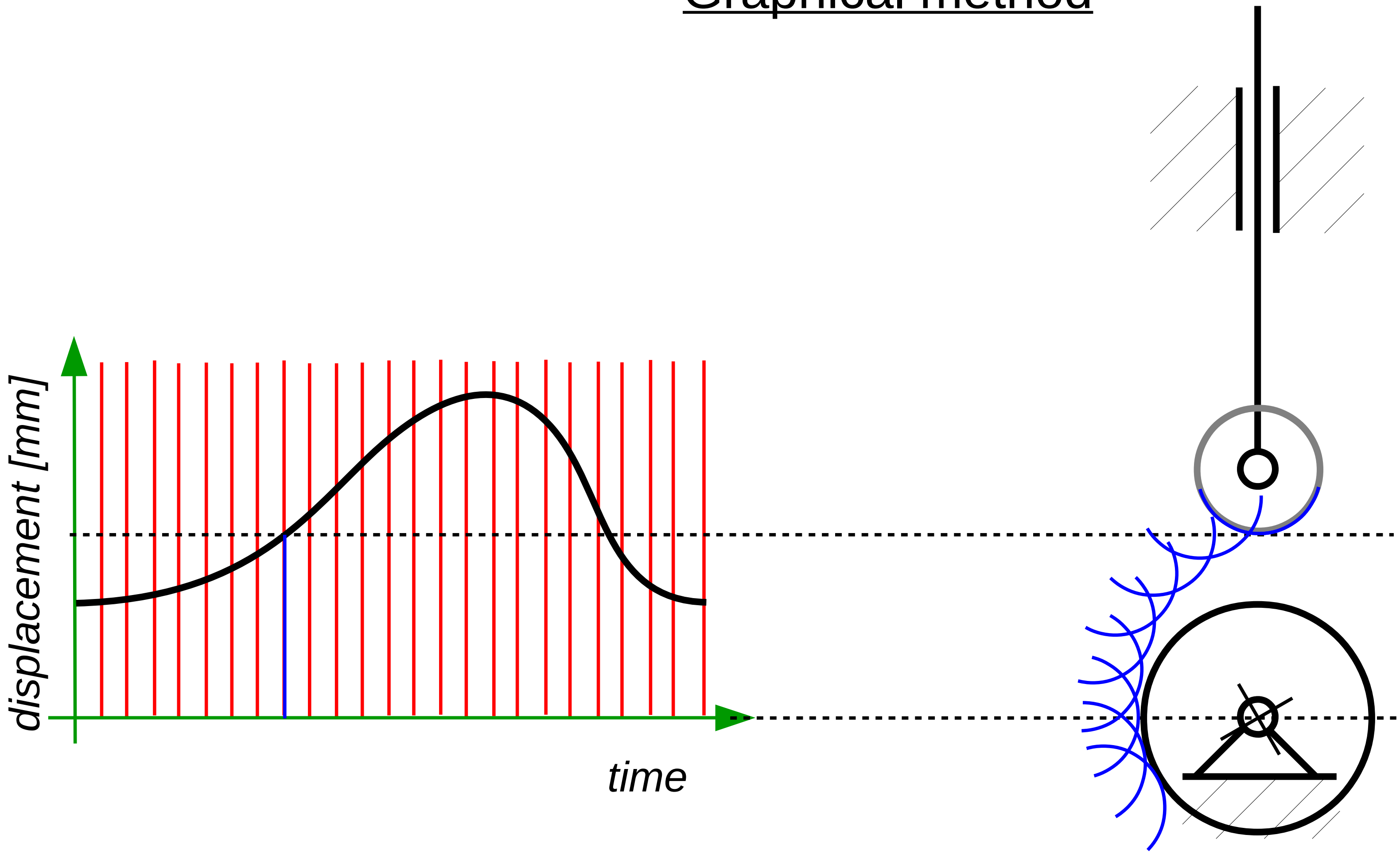

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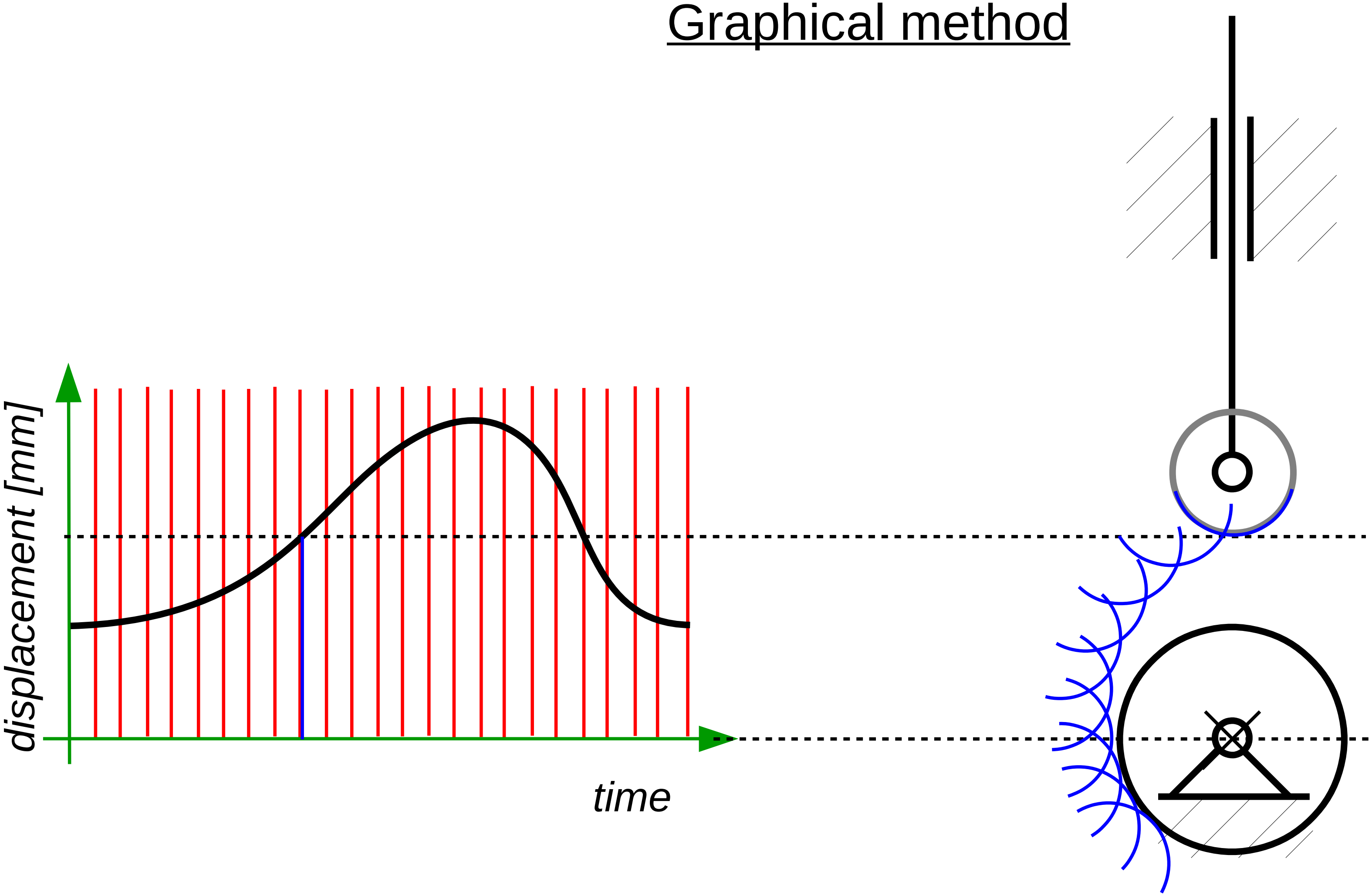

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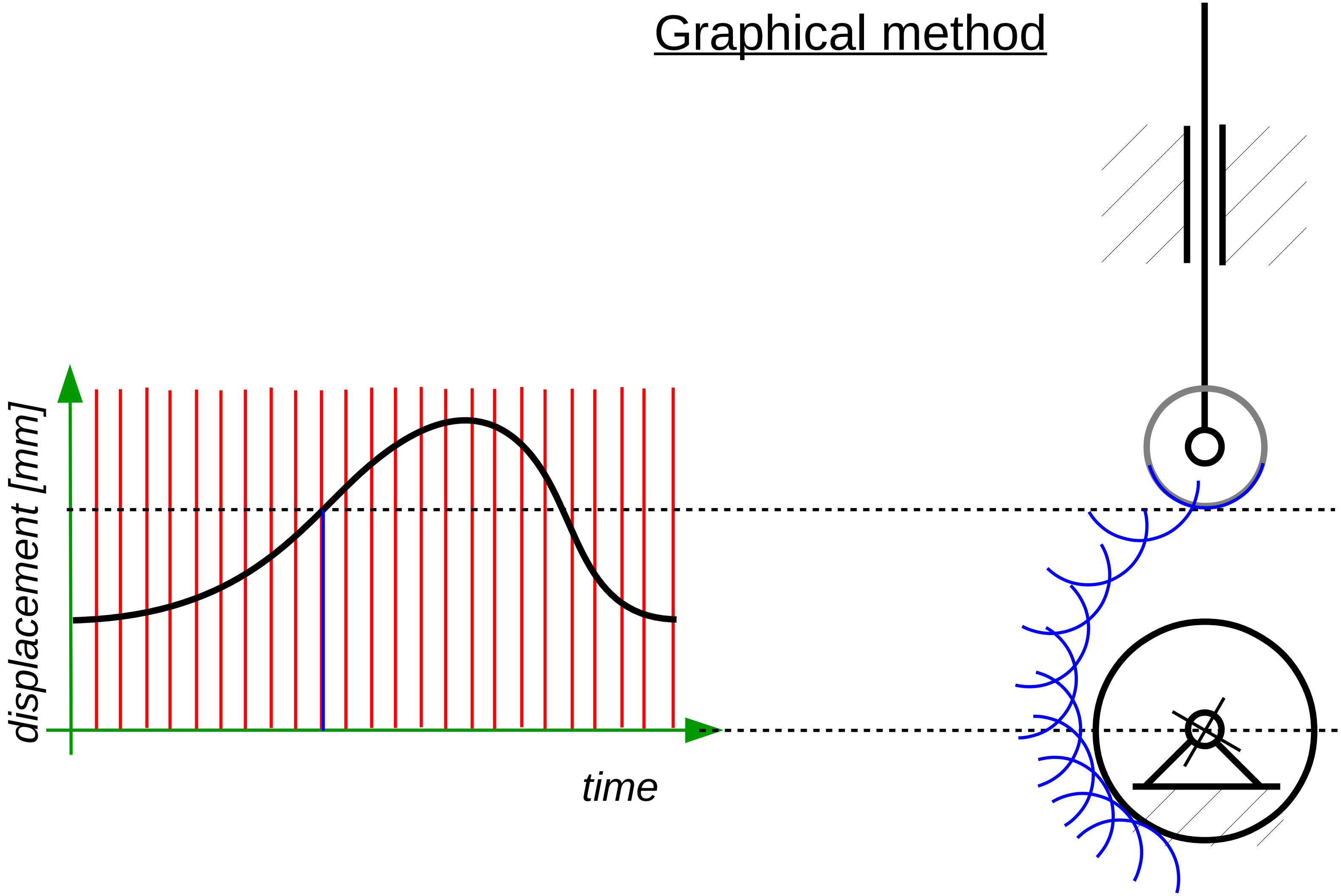

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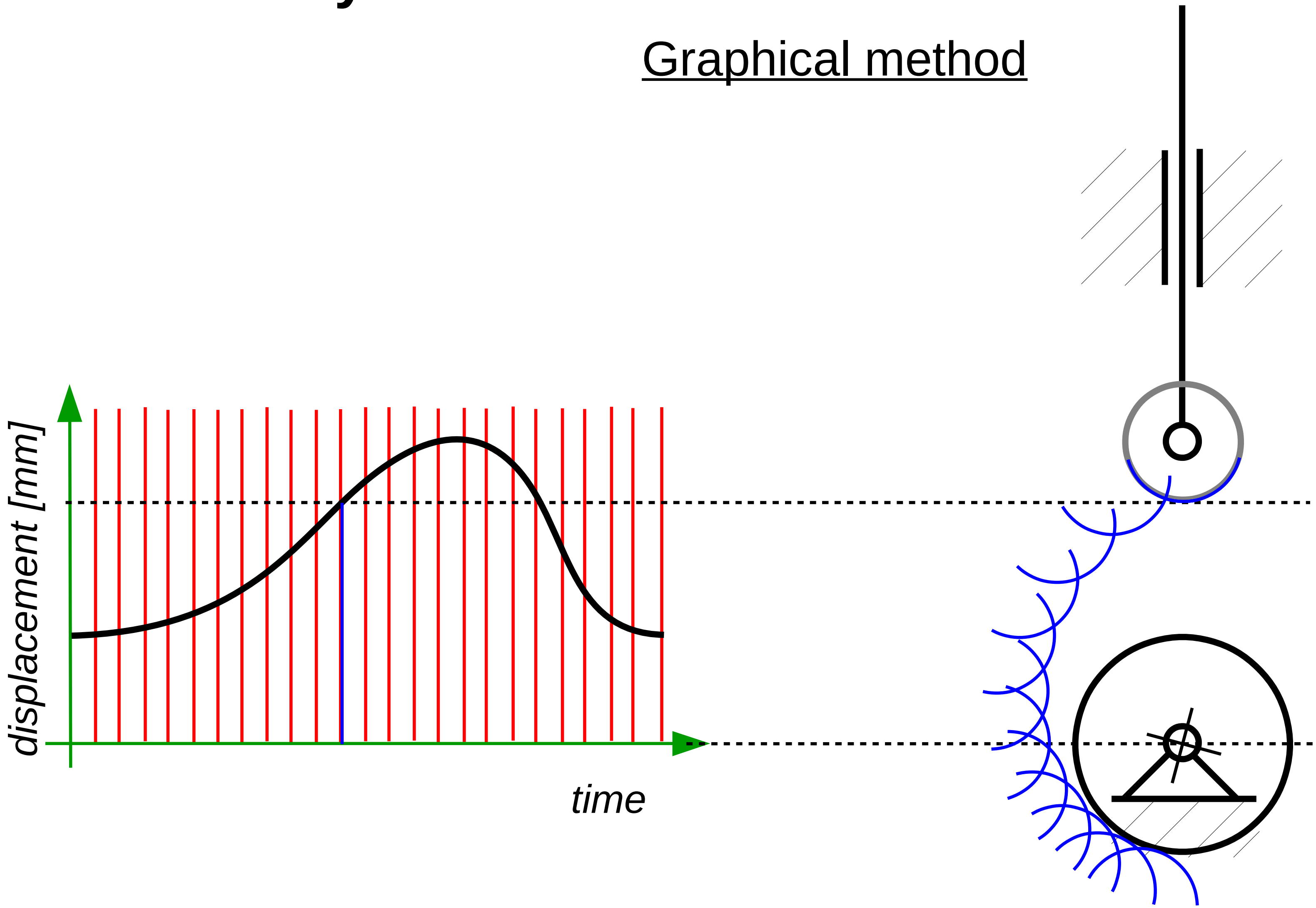

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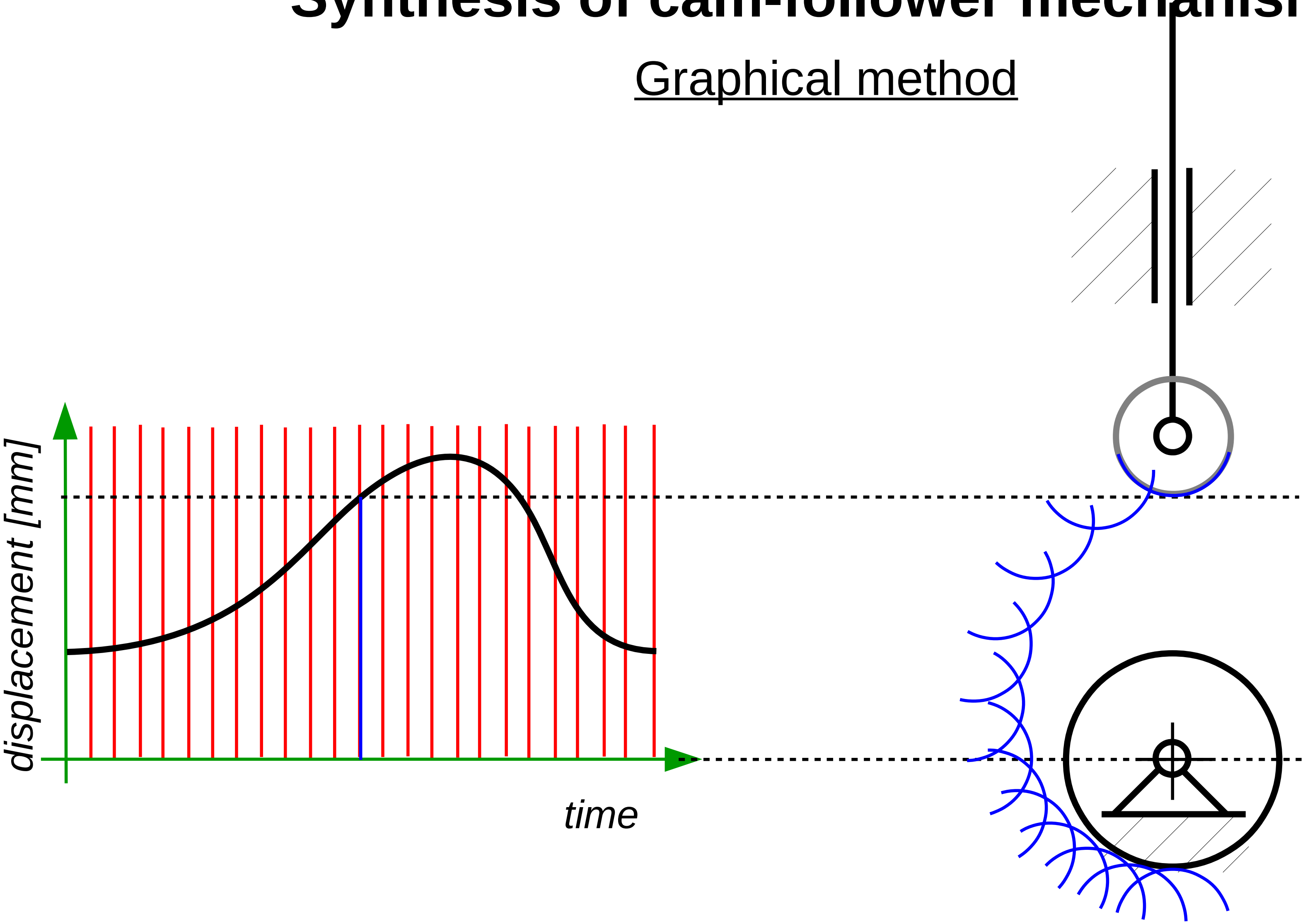

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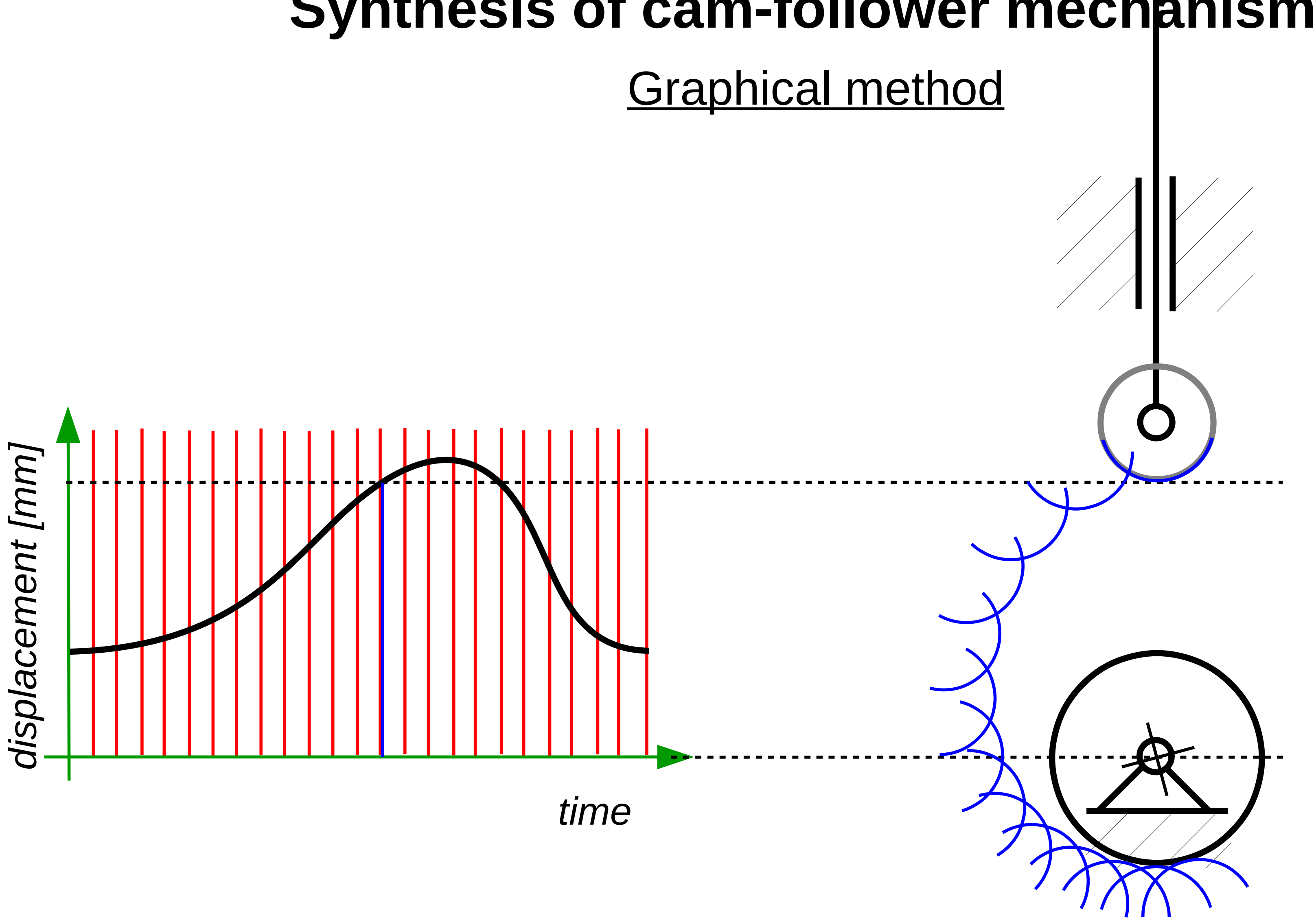

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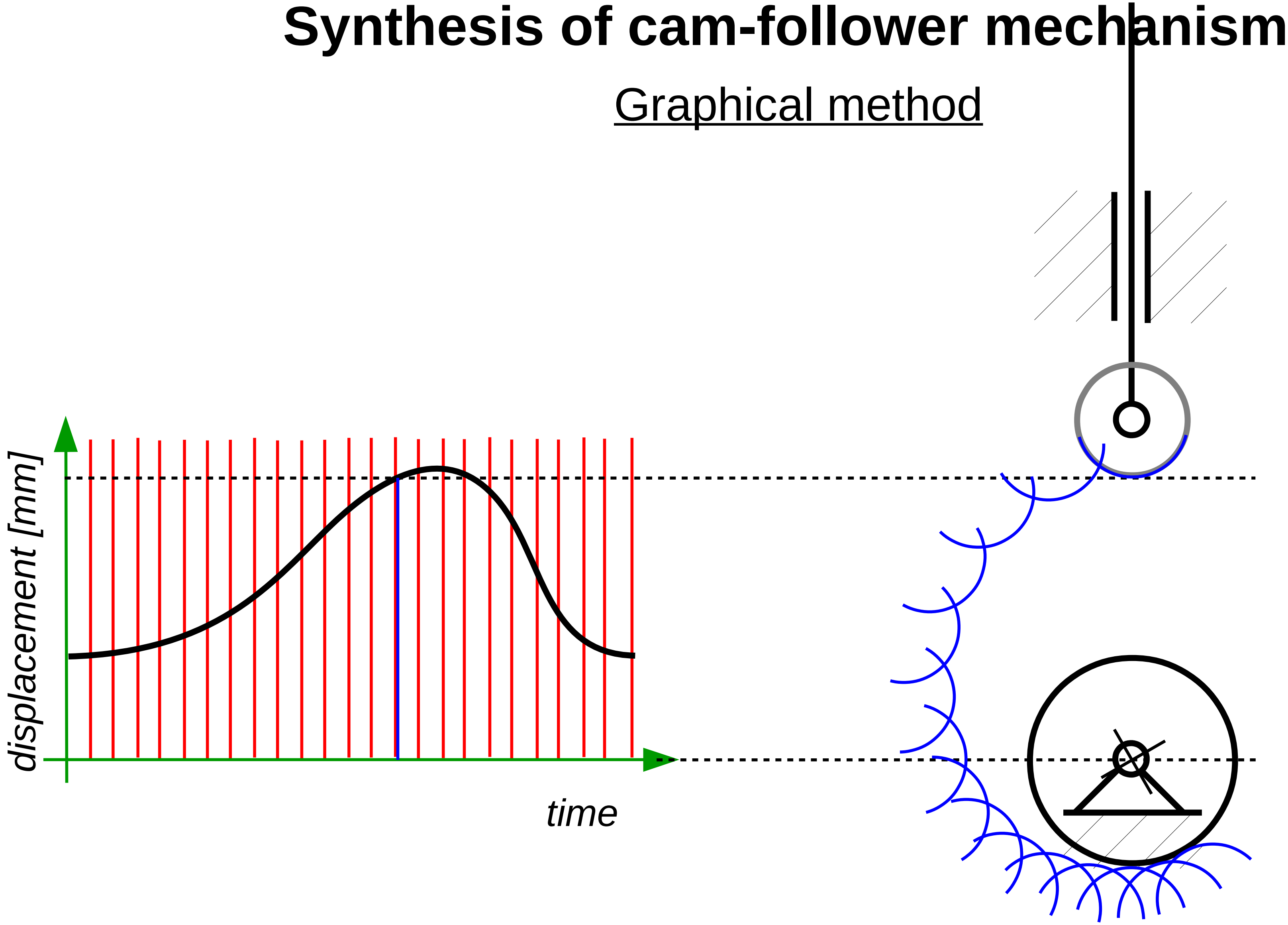

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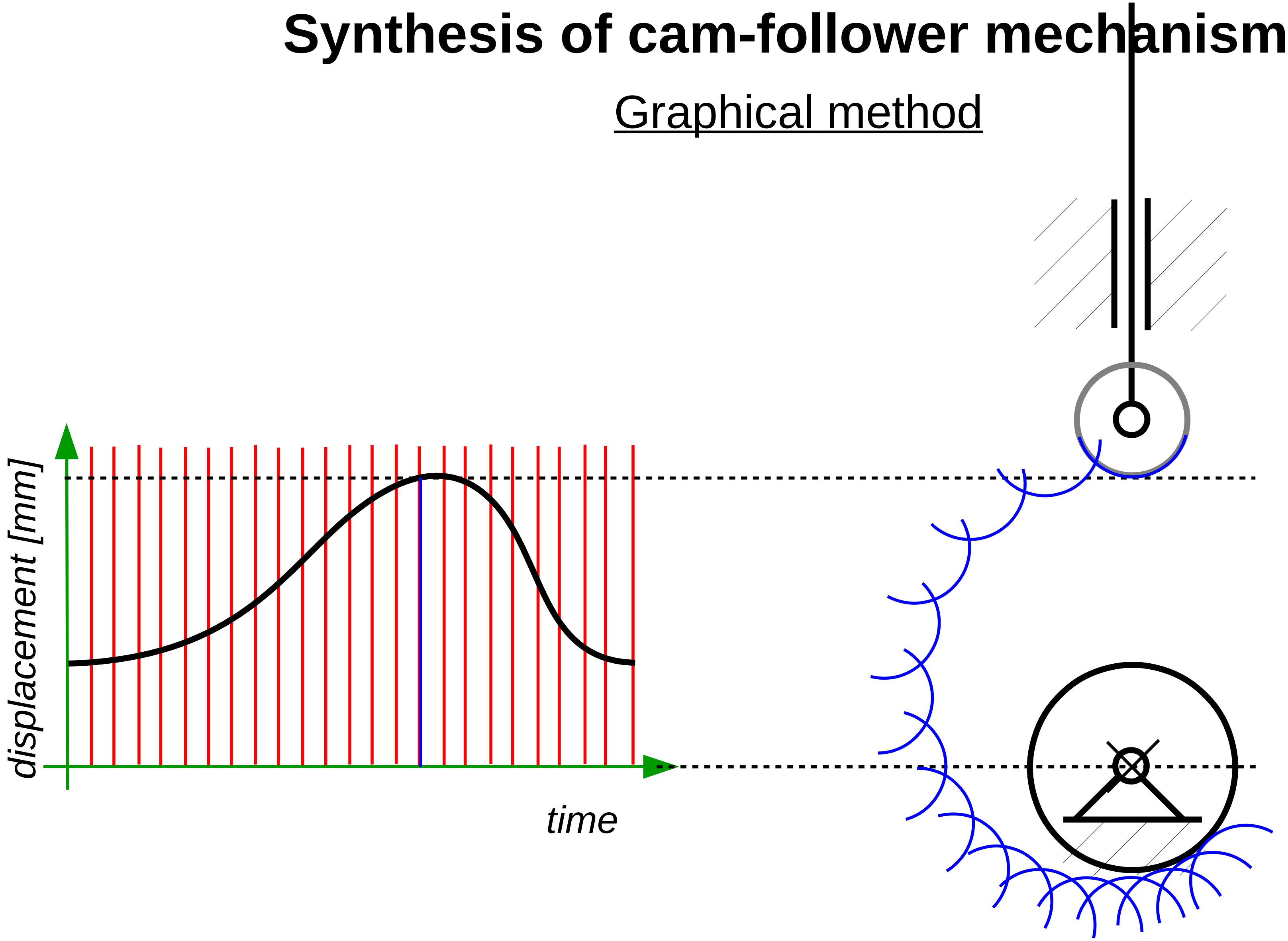

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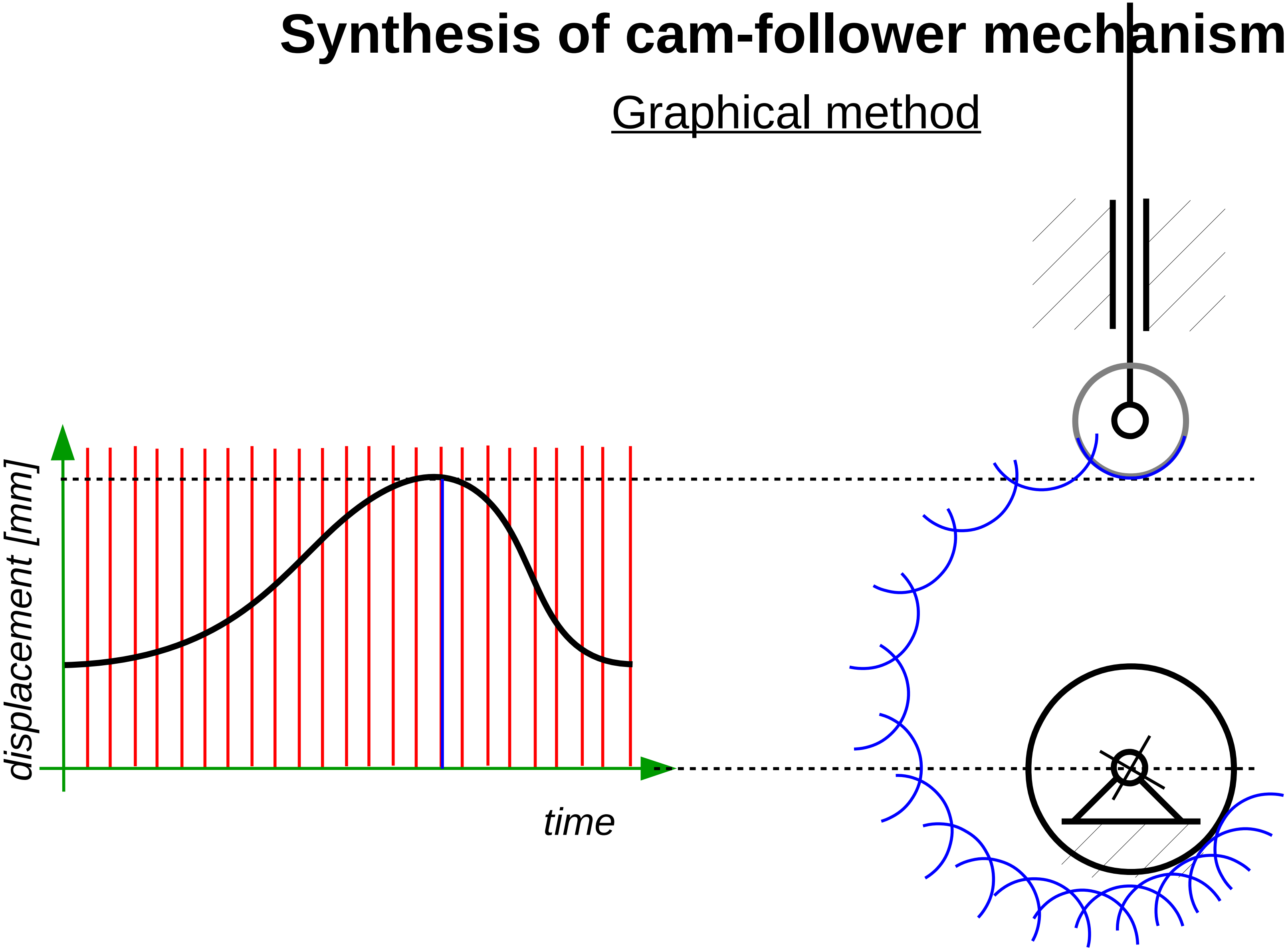

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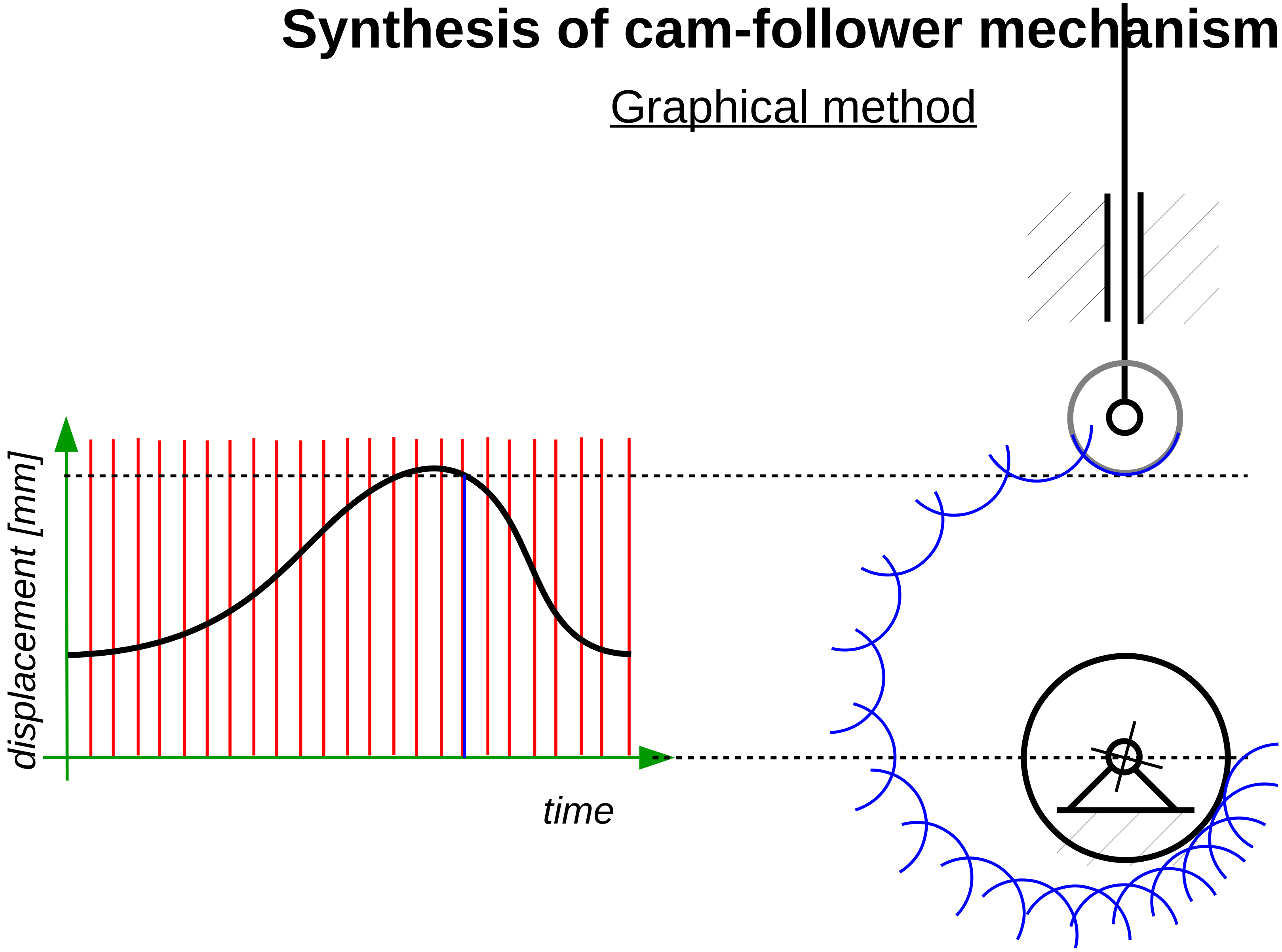

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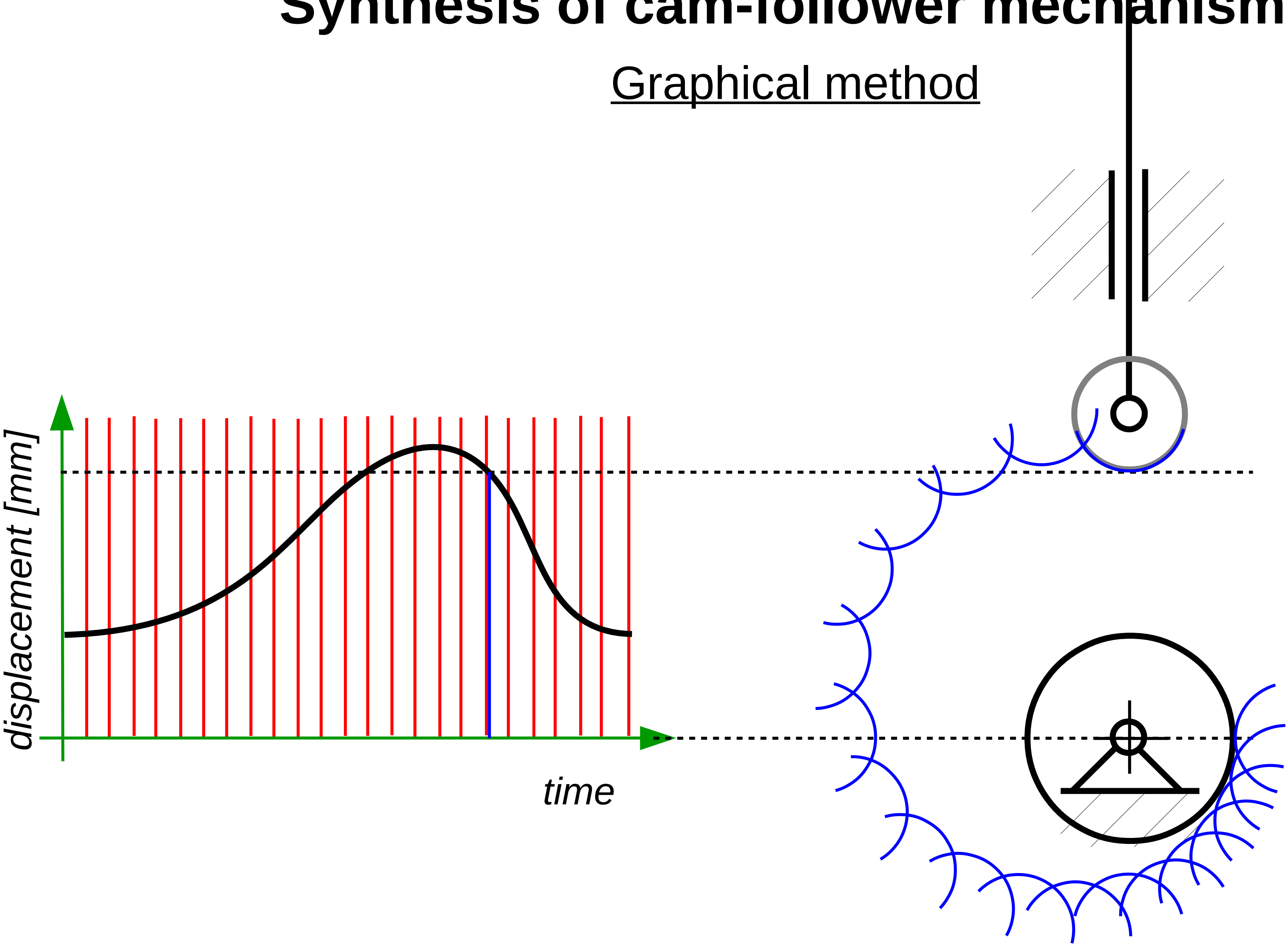

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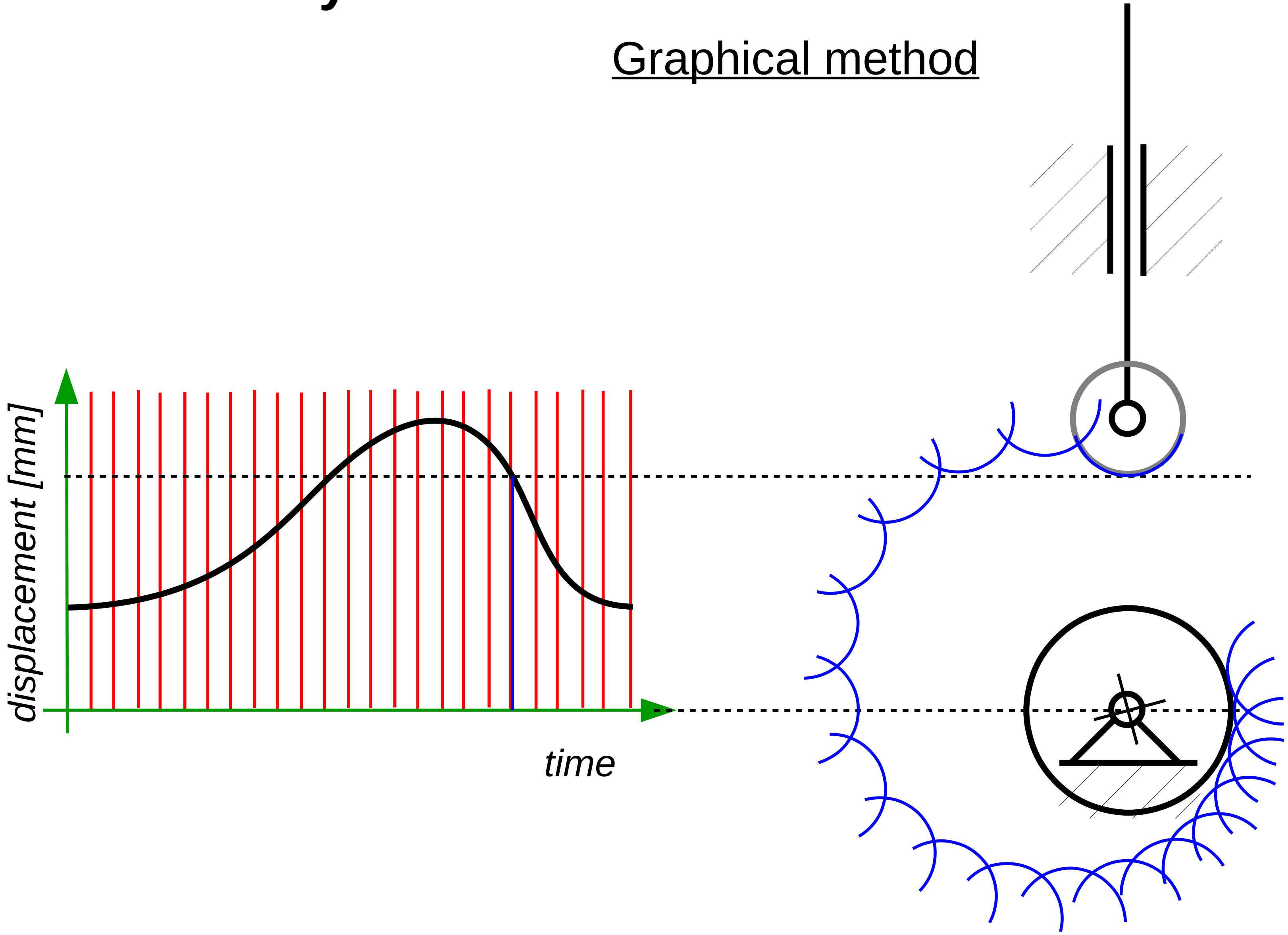

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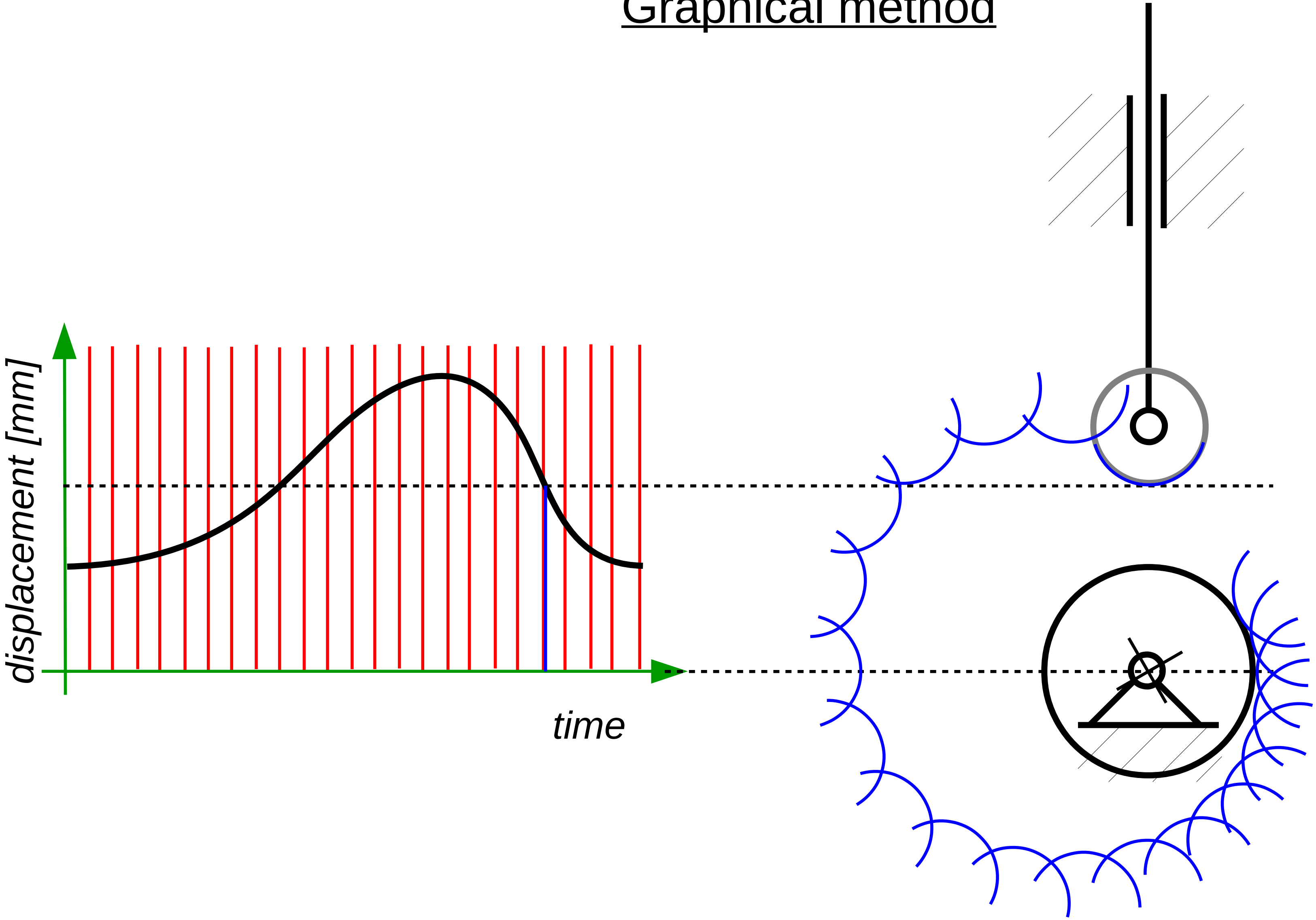

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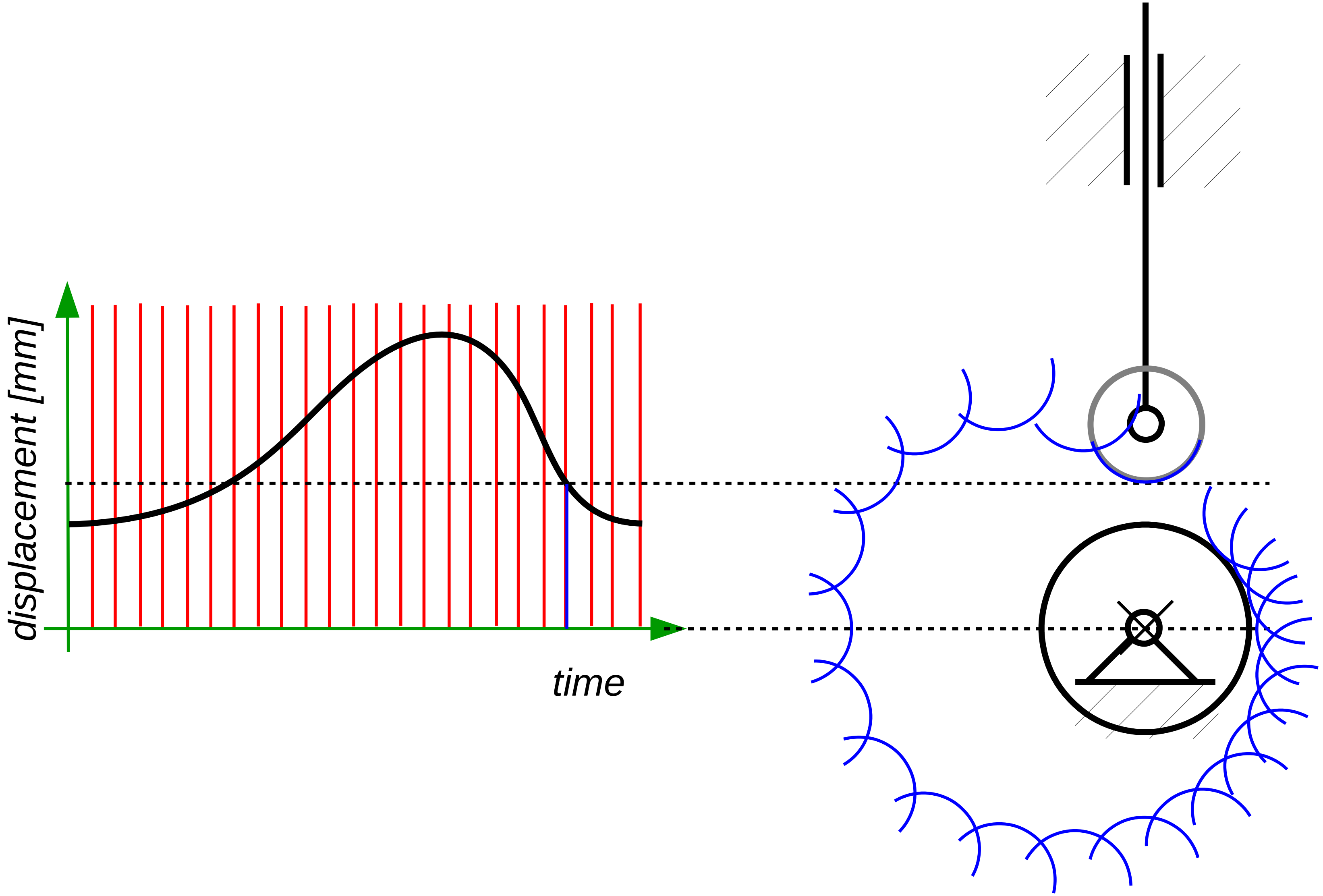

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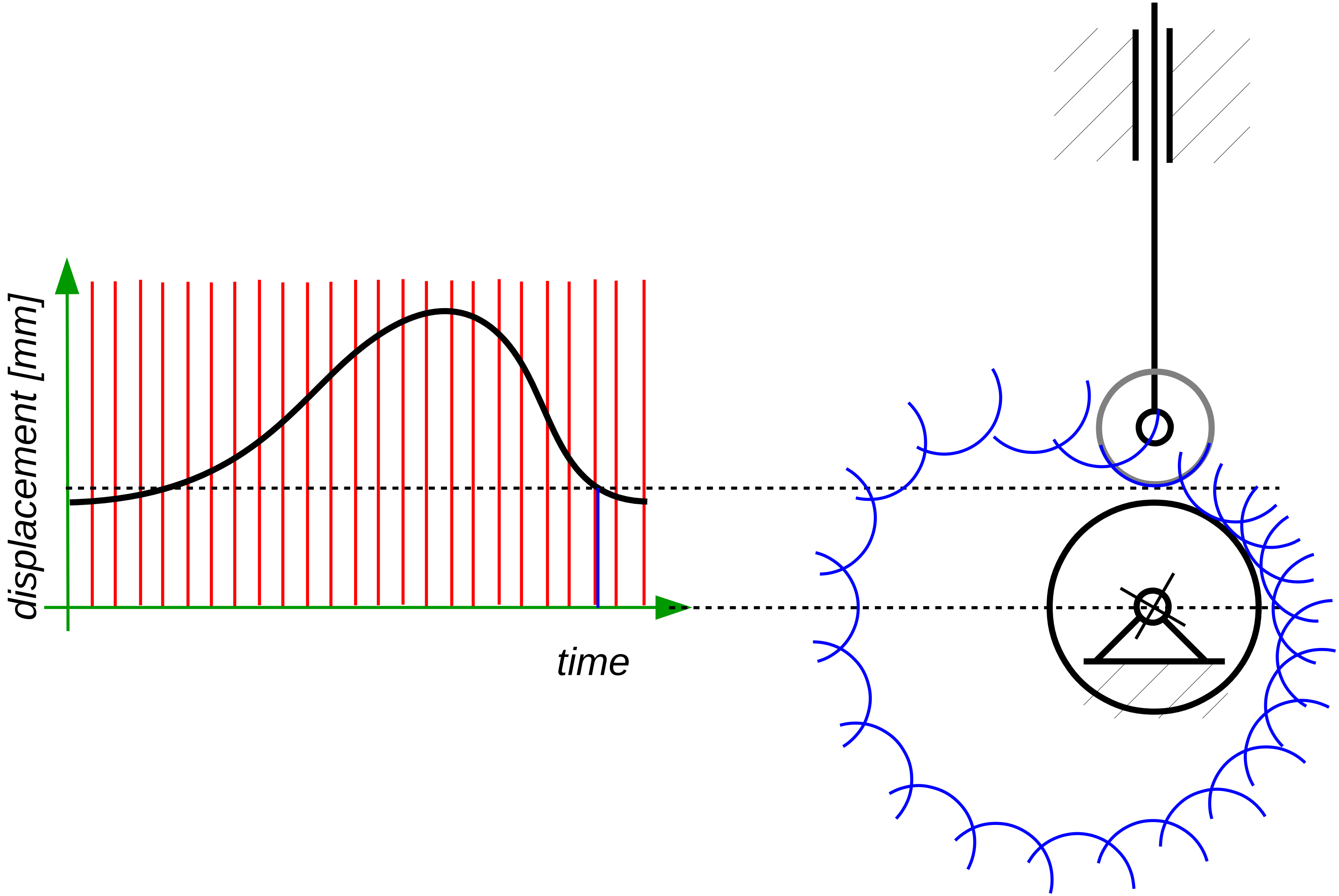

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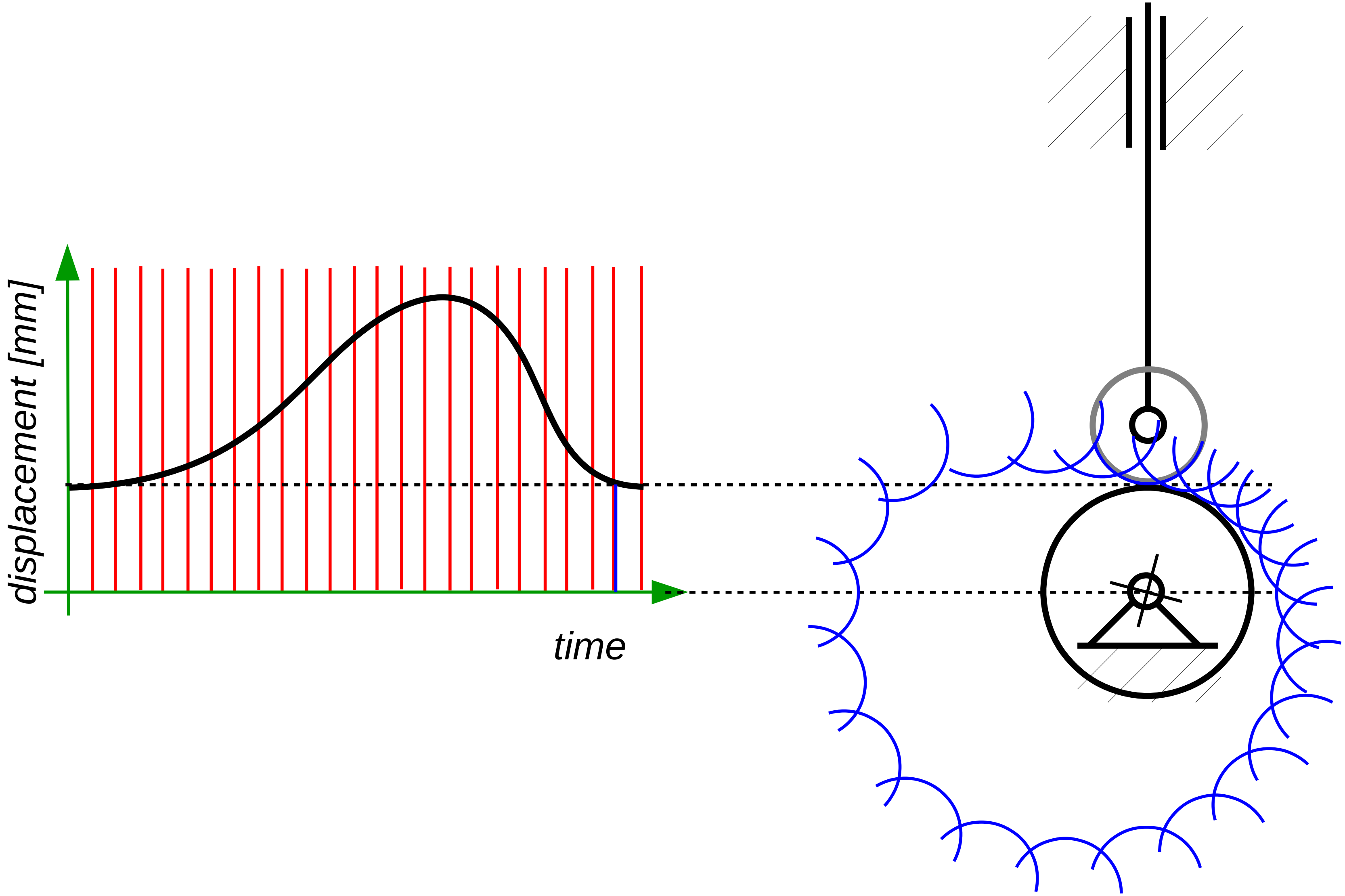

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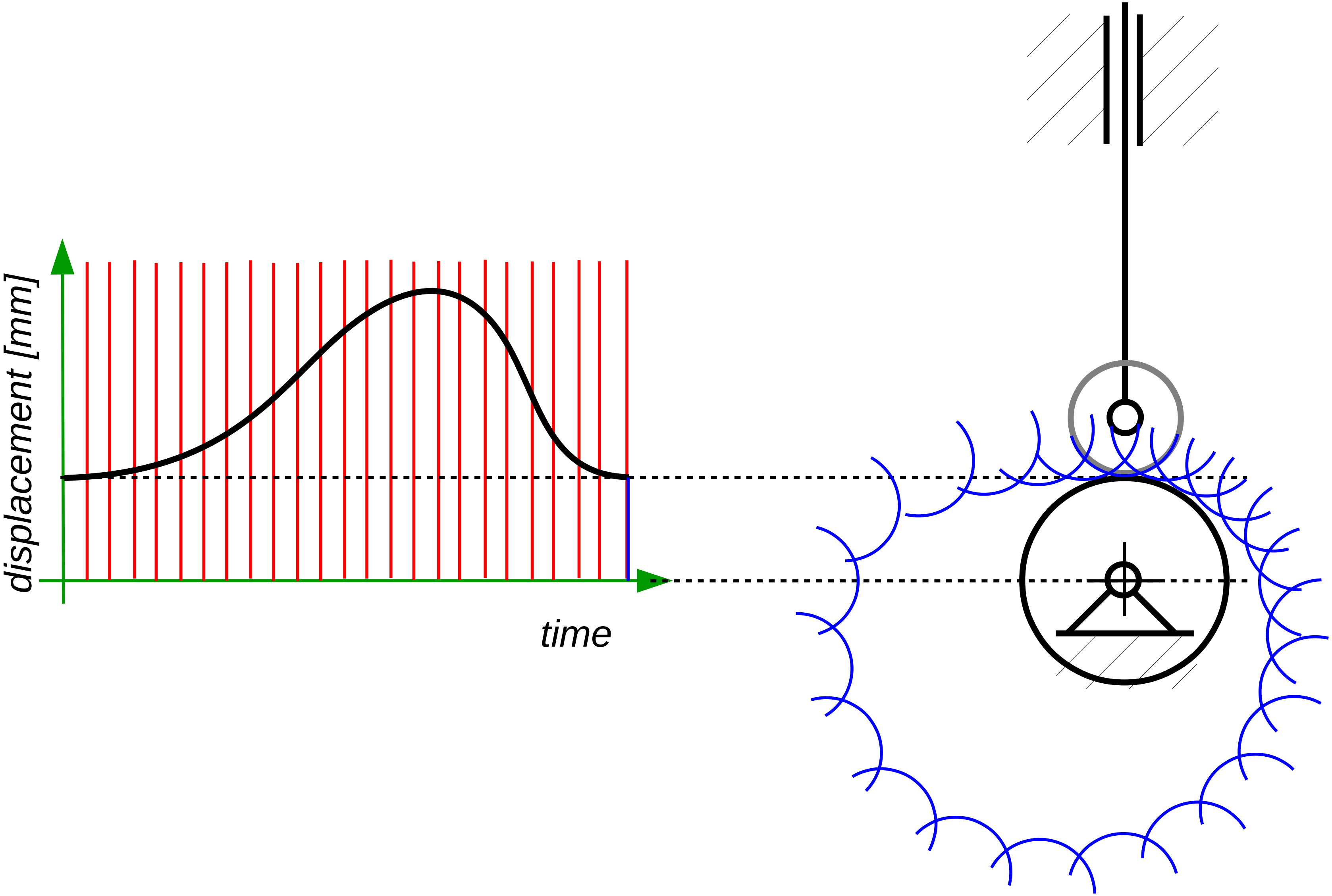

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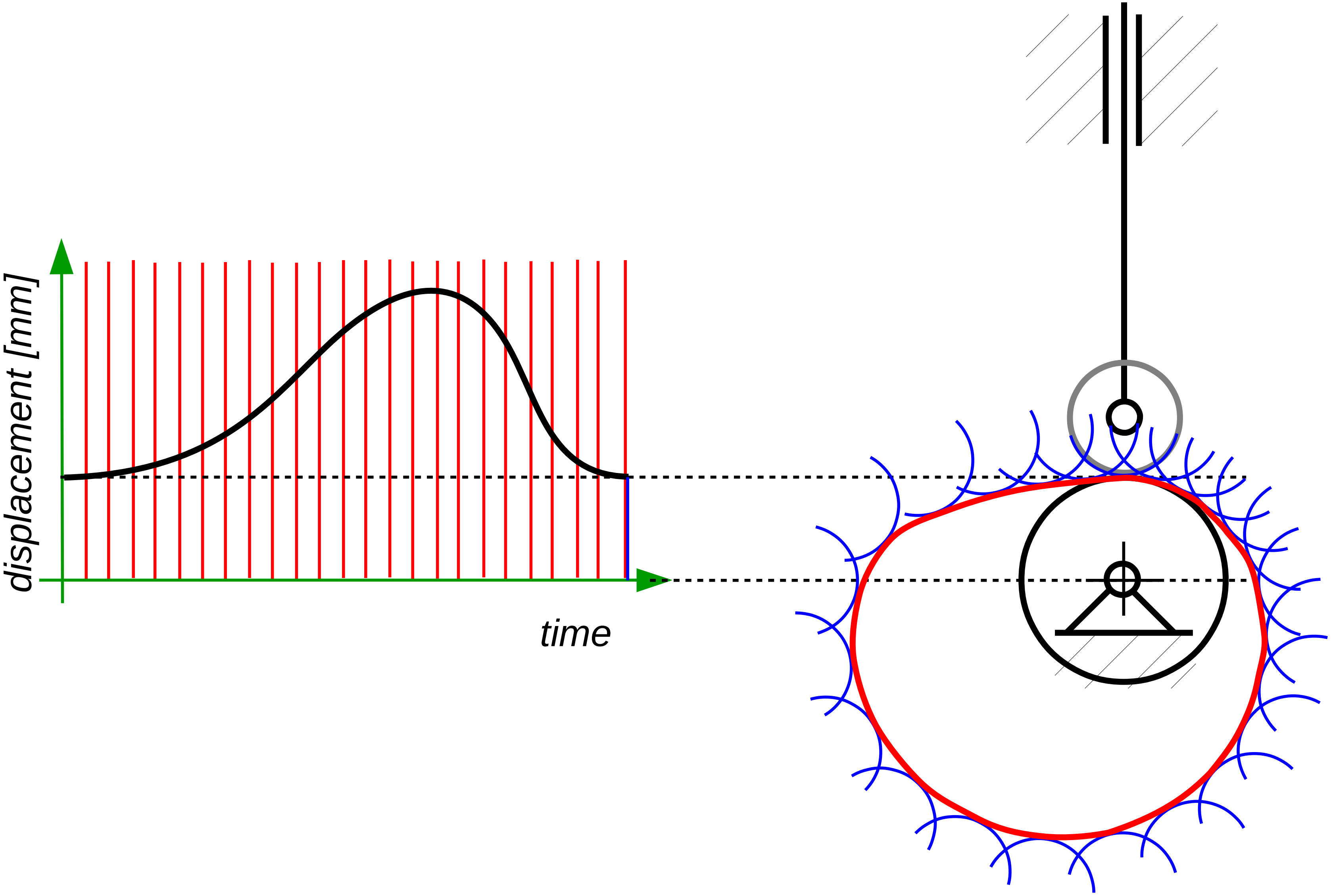

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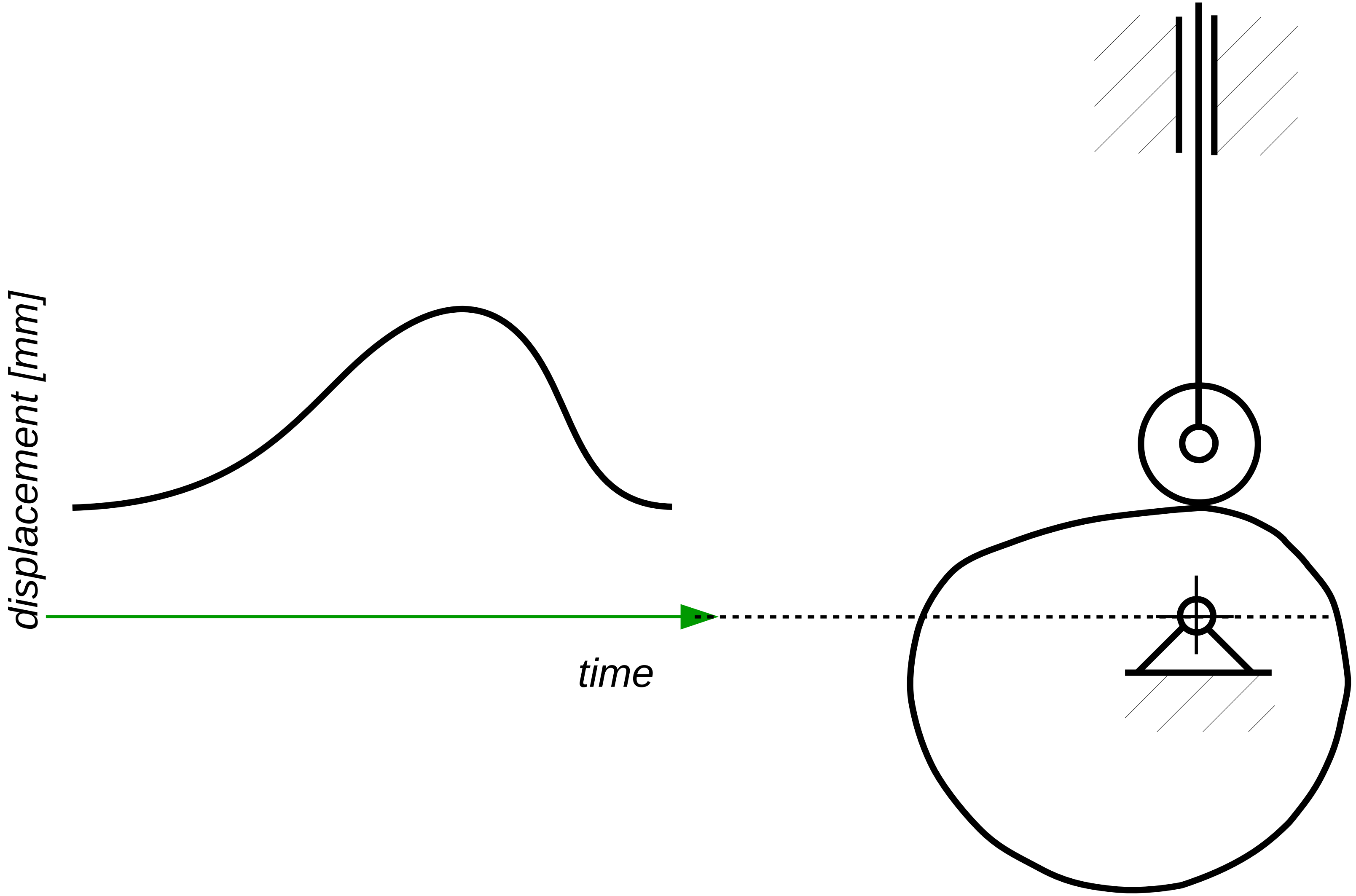

Synthesis of cam-follower mechanisms

Analytical method

Synthesis of cam-follower mechanisms

Analytical method

For a given function of velocity or acceleration, function of a follower displacement can be found by integration.

Follower displacement as a function of cam angle could be used to obtain cam outline directly (or after change of coordinates).

For a knife-edge follower we will obtain exact real displacement.
For a roller-ended follower some errors are possible.

Roller-ended follower give us limitation of a maximum velocity (there is a relation between roller radius and cam size).

Usually we are designing symmetric and smooth cams.

Synthesis of cam-follower mechanisms

Analytical method – example

Design profile of a cam cooperating with eccentric knife-edge follower to obtain constant acceleration of uplift and downlift with constant angular velocity.

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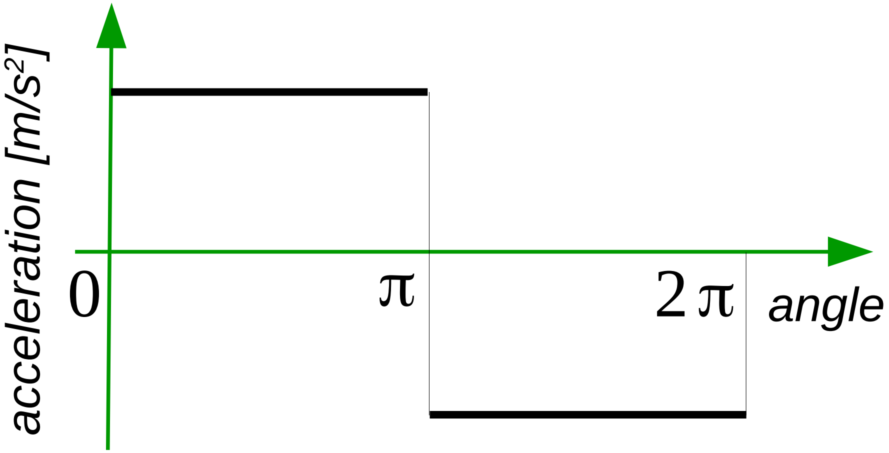

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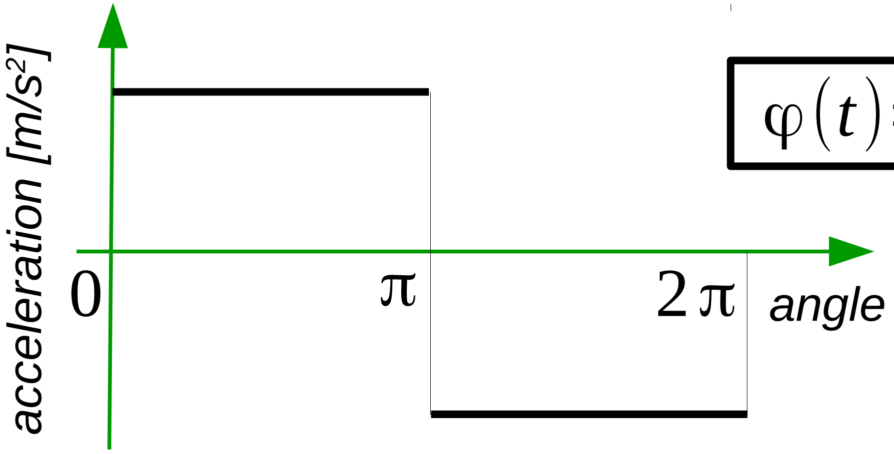

$$\varphi(t)=\omega t$$

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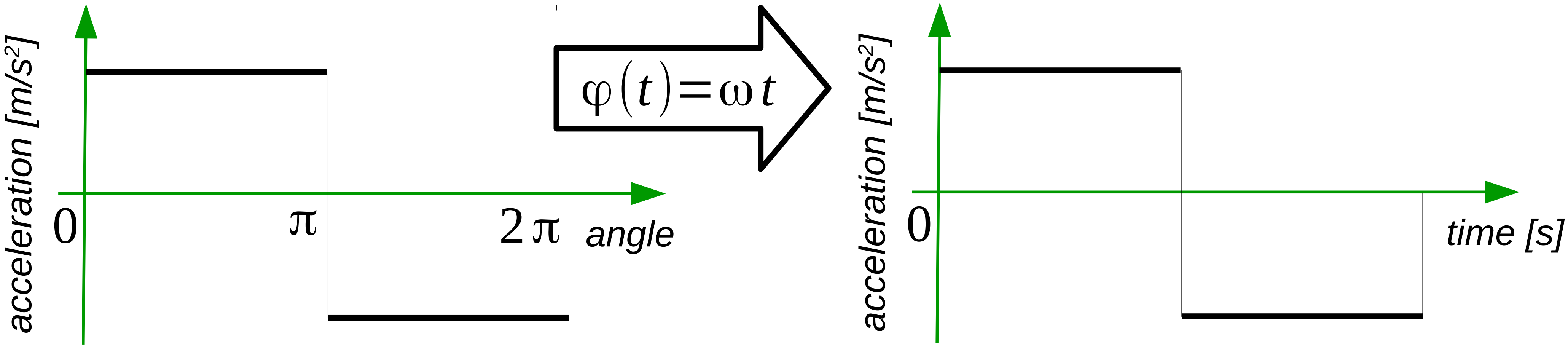

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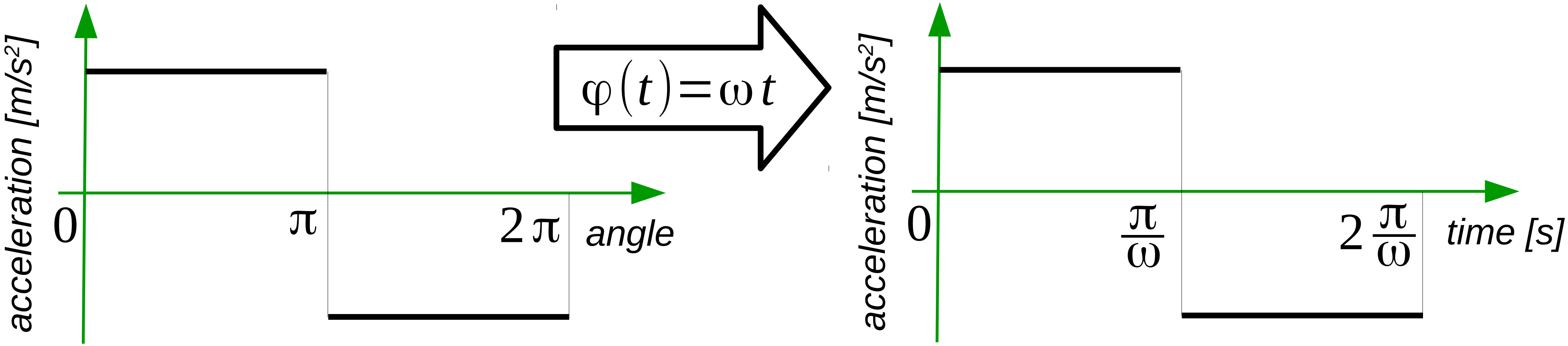

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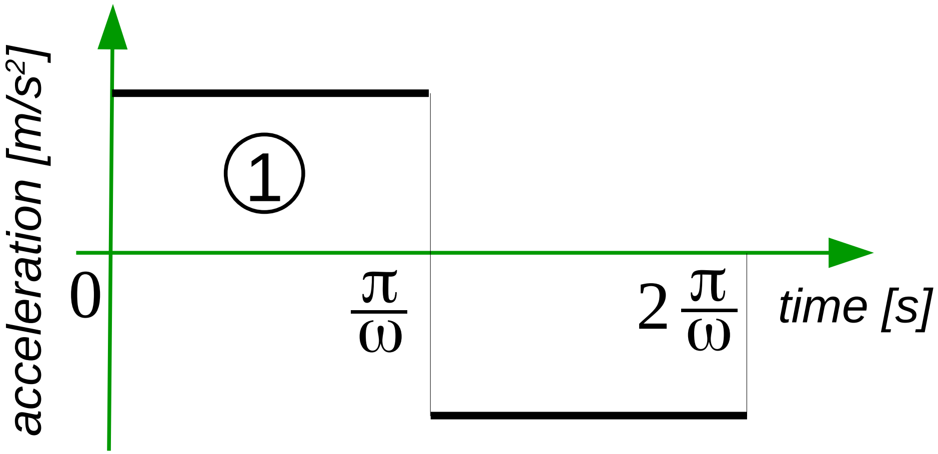

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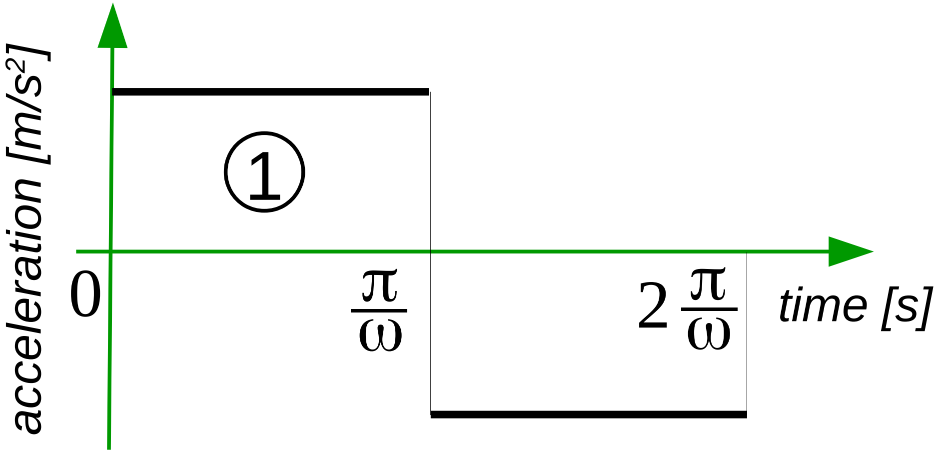

$$p_1 = a$$

$$v_1(t) = at + C_1$$

$$h_1(t) = \frac{at^2}{2} + C_1 t + C_2$$

Synthesis of cam-follower mechanisms

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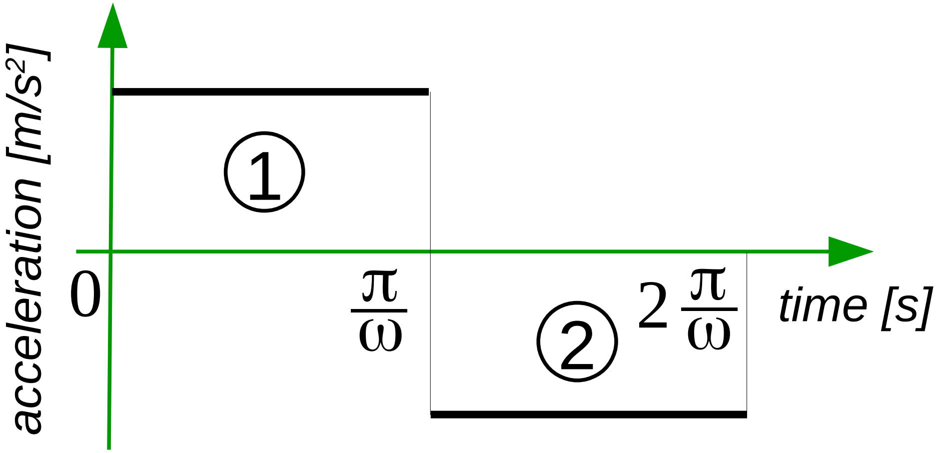

$$p_1 = a$$

$$v_1(t) = at + C_1$$

$$h_1(t) = \frac{at^2}{2} + C_1 t + C_2$$

$$p_2 = -a$$

$$v_2(t) = -at + C_3$$

$$h_2(t) = \frac{-at^2}{2} + C_3 t + C_4$$

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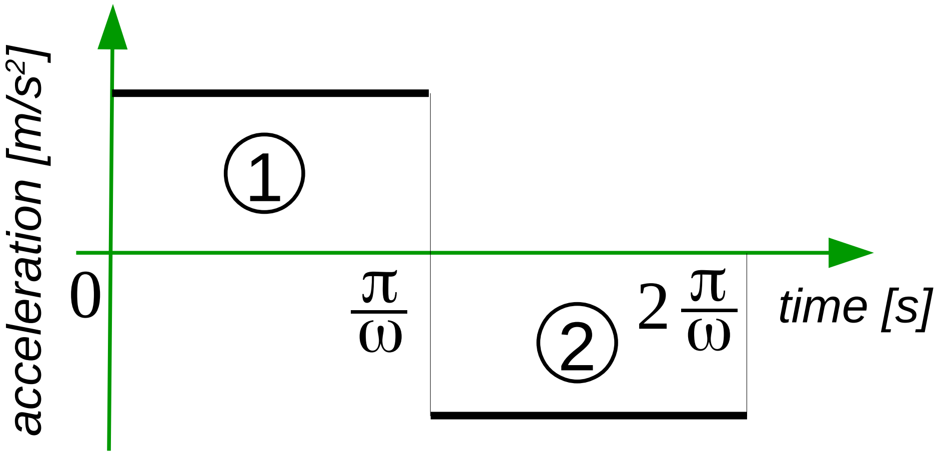

Cam base-radius

$$h_1(t=0)=h_2\left(t=2\frac{\pi}{\omega}\right)=R$$

$$p_1=a$$

$$v_1(t)=at+C_1$$

$$h_1(t)=\frac{at^2}{2}+C_1t+C_2$$

$$p_2=-a$$

$$v_2(t)=-at+C_3$$

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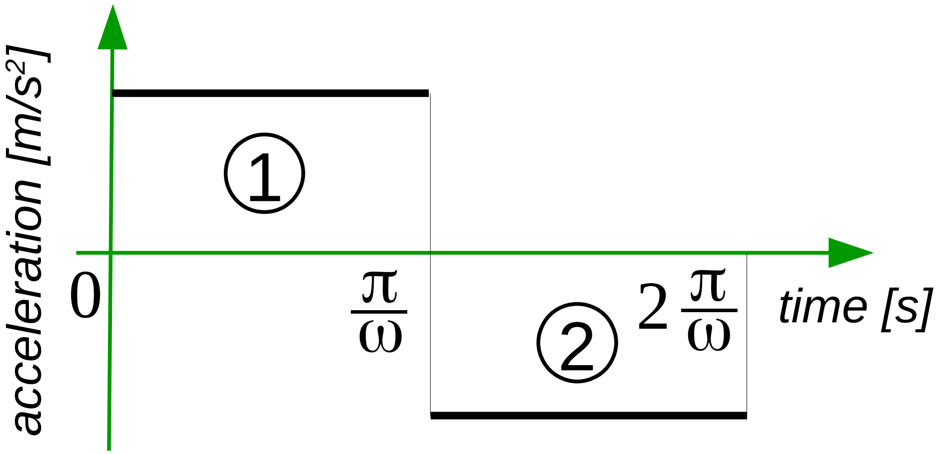

Cam base-radius

$$h_1(t=0)=h_2\left(t=2\frac{\pi}{\omega}\right)=R$$

$$C_2=R$$

$$C_4=R+2a\frac{\pi^2}{\omega^2}-2C_3\frac{\pi}{\omega}$$

$$p_1=a$$

$$v_1(t)=at+C_1$$

$$h_1(t)=\frac{at^2}{2}+C_1t+C_2$$

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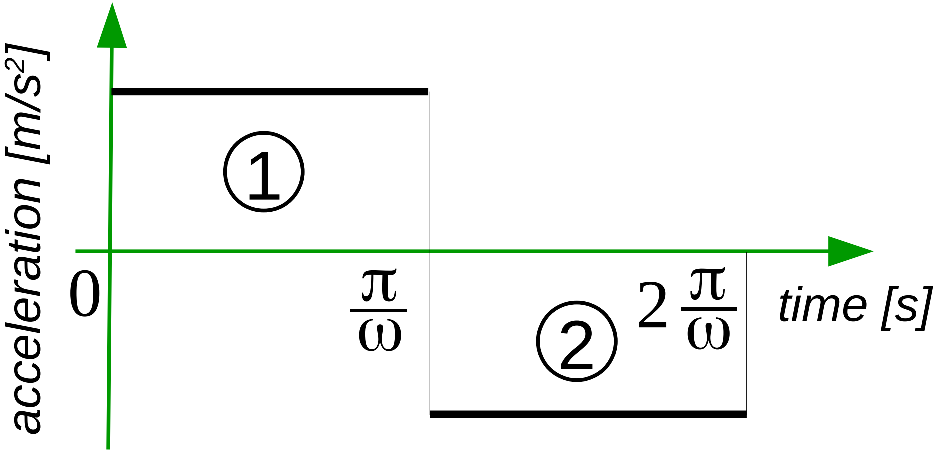

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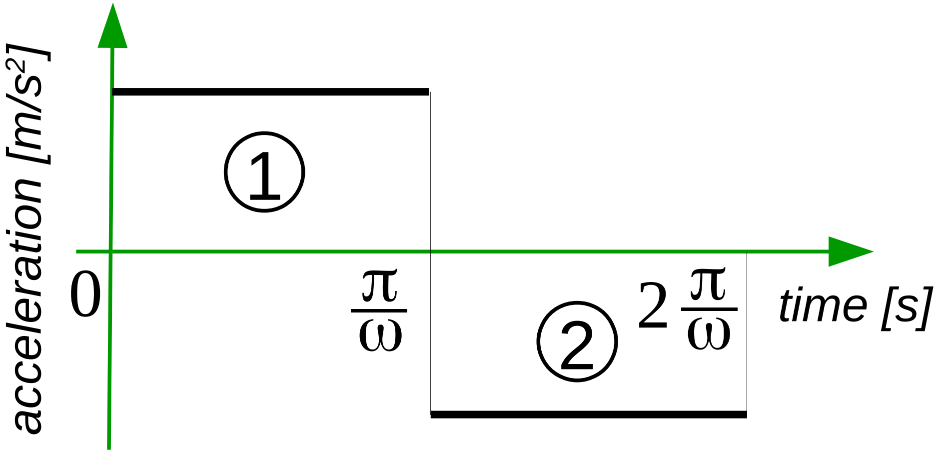

Cam outline continuity

$$h_2\left(t=\frac{\pi}{\omega}\right)=h_1\left(t=\frac{\pi}{\omega}\right)$$

$$p_1=a$$

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$$h_1(t)=\frac{at^2}{2}+C_1t+R$$

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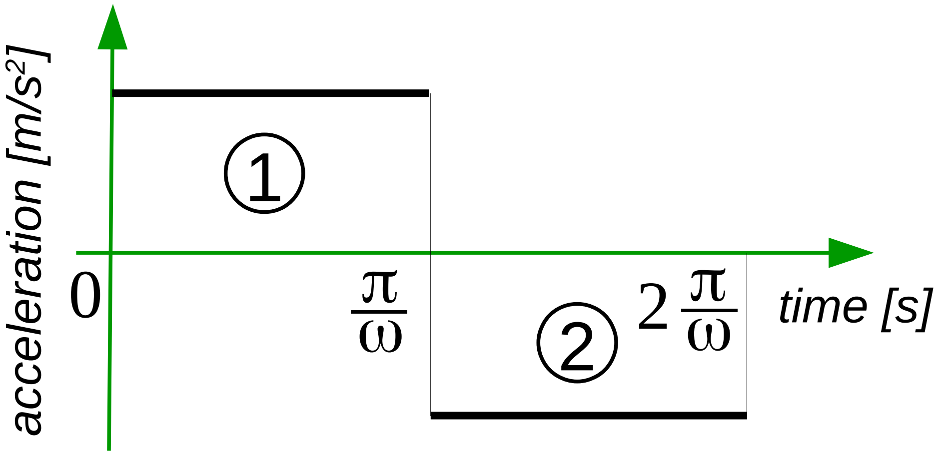

Cam outline continuity

$$h_2\left(t=\frac{\pi}{\omega}\right)=h_1\left(t=\frac{\pi}{\omega}\right)$$

$$C_3=a\frac{\pi}{\omega}-C_1$$

$$p_1=a$$

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$$h_1(t)=\frac{at^2}{2}+C_1t+R$$

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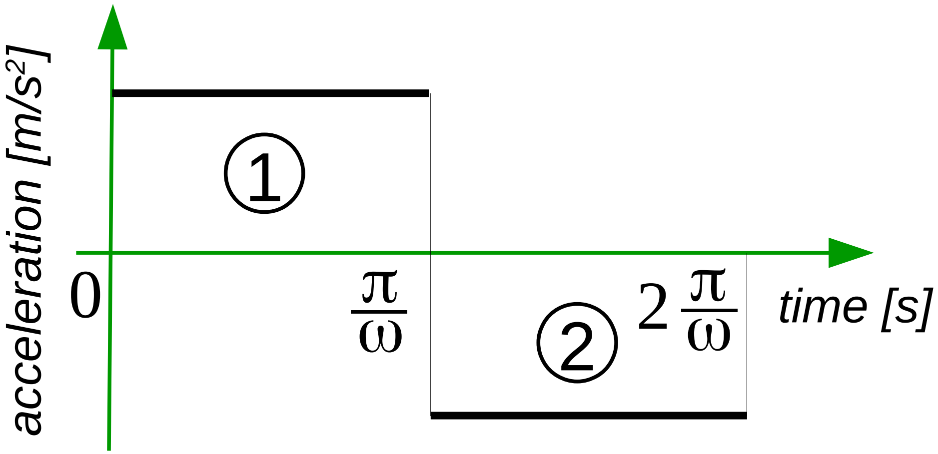

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$$v_2(t)=-at+a\frac{\pi}{\omega}-C_1$$

$$h_2(t)=\frac{-at^2}{2}+R+a\frac{\pi}{\omega}t+C_1\left(2\frac{\pi}{\omega}-t\right)$$

Synthesis of cam-follower mechanisms

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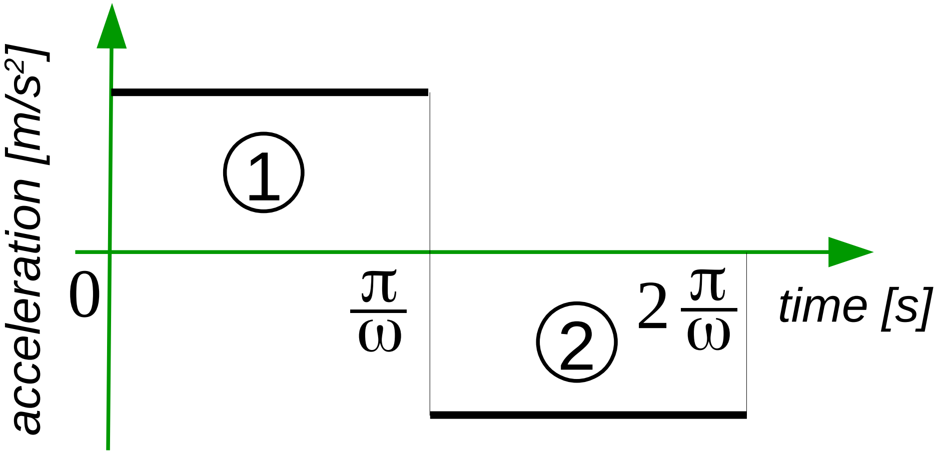
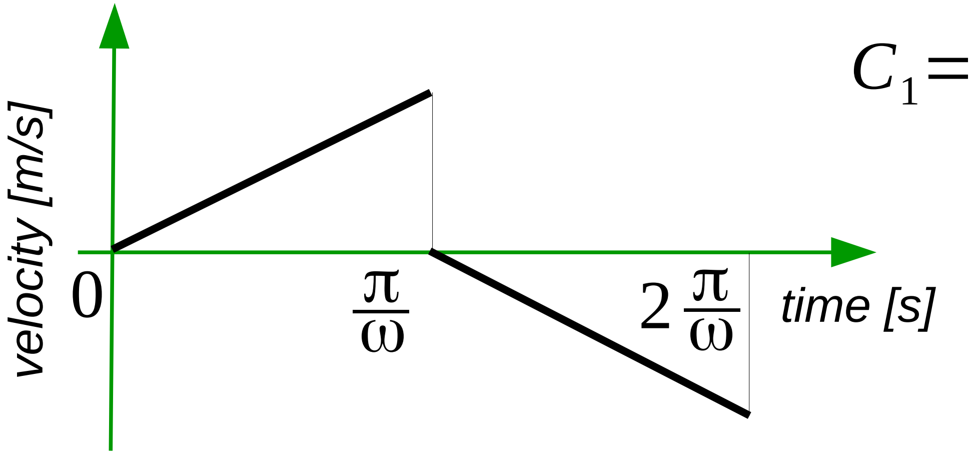
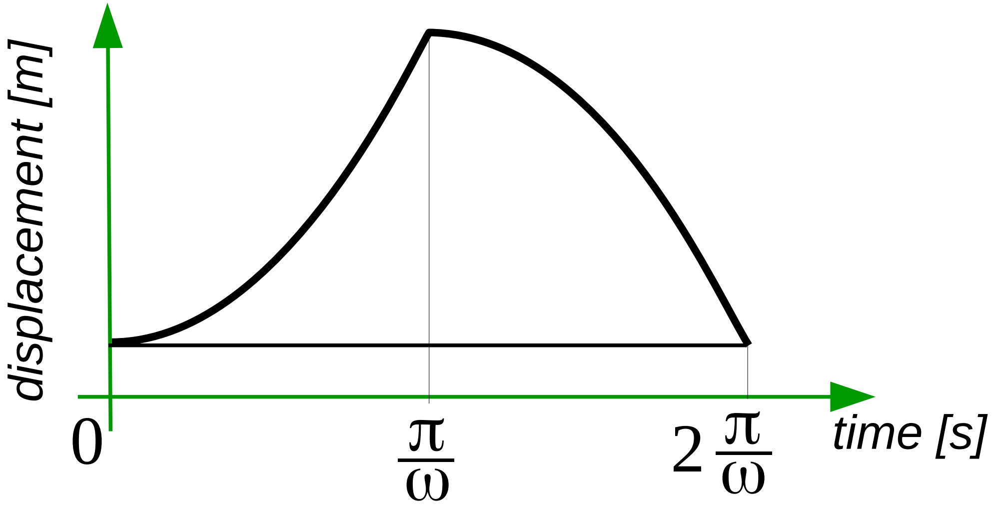




$C_1 = 0$



Synthesis of cam-follower mechanisms

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Design profile of a cam cooperating with eccentric knife-edge follower to obtain constant acceleration of uplift and downlift with constant angular velocity.

$$h(t)=\begin{cases} \dfrac{a t^2}{2}+C_1 t+R\text{, dla } t\in\left(0,\dfrac{\pi}{\omega}\right) \\ \dfrac{-a t^2}{2}+R+a\dfrac{\pi}{\omega}t+C_1\left(2\dfrac{\pi}{\omega}-t\right)\text{, dla } t\in\left(\dfrac{\pi}{\omega},2\dfrac{\pi}{\omega}\right) \end{cases}$$

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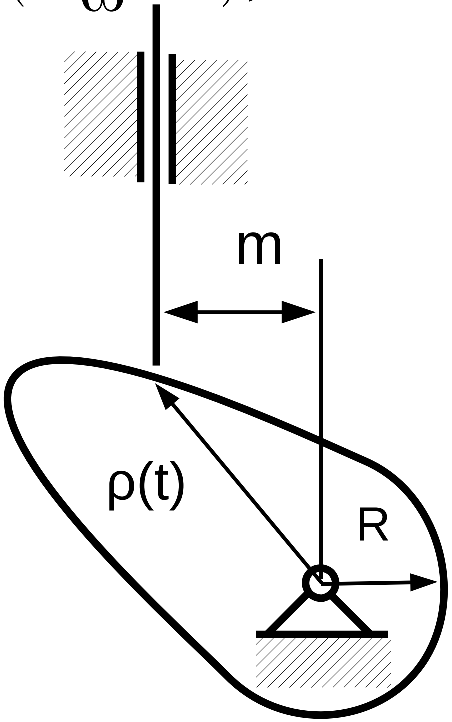
Design profile of a cam cooperating with eccentric knife-edge follower to obtain constant acceleration of uplift and downlift with constant angular velocity.

$$h(t)=\begin{cases} \dfrac{a t^2}{2}+C_1 t+R\,, \text{ dla } t\in\left(0,\dfrac{\pi}{\omega}\right) \\ \dfrac{-a t^2}{2}+R+a\dfrac{\pi}{\omega}t+C_1\left(2\dfrac{\pi}{\omega}-t\right), \text{ dla } t\in\left(\dfrac{\pi}{\omega},2\dfrac{\pi}{\omega}\right) \end{cases}$$

With follower offset consideration

$$\rho(t)=\sqrt{R^2-m^2}+h(t)$$



Synthesis of cam-follower mechanisms

Analytical method – example

Design profile of a cam cooperating with eccentric knife-edge follower to obtain constant acceleration of uplift and downlift with constant angular velocity.

$$h(t)=\begin{cases} \dfrac{a t^2}{2}+C_1 t+R, \text{ dla } t\in\left(0,\dfrac{\pi}{\omega}\right) \\ \dfrac{-a t^2}{2}+R+a\dfrac{\pi}{\omega}t+C_1\left(2\dfrac{\pi}{\omega}-t\right), \text{ dla } t\in\left(\dfrac{\pi}{\omega},2\dfrac{\pi}{\omega}\right) \end{cases}$$

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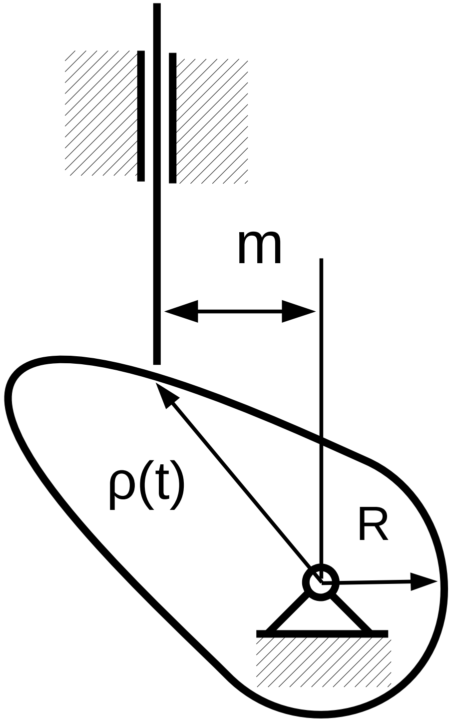
$$\rho(t)=\sqrt{R^2-m^2}+h(t)$$



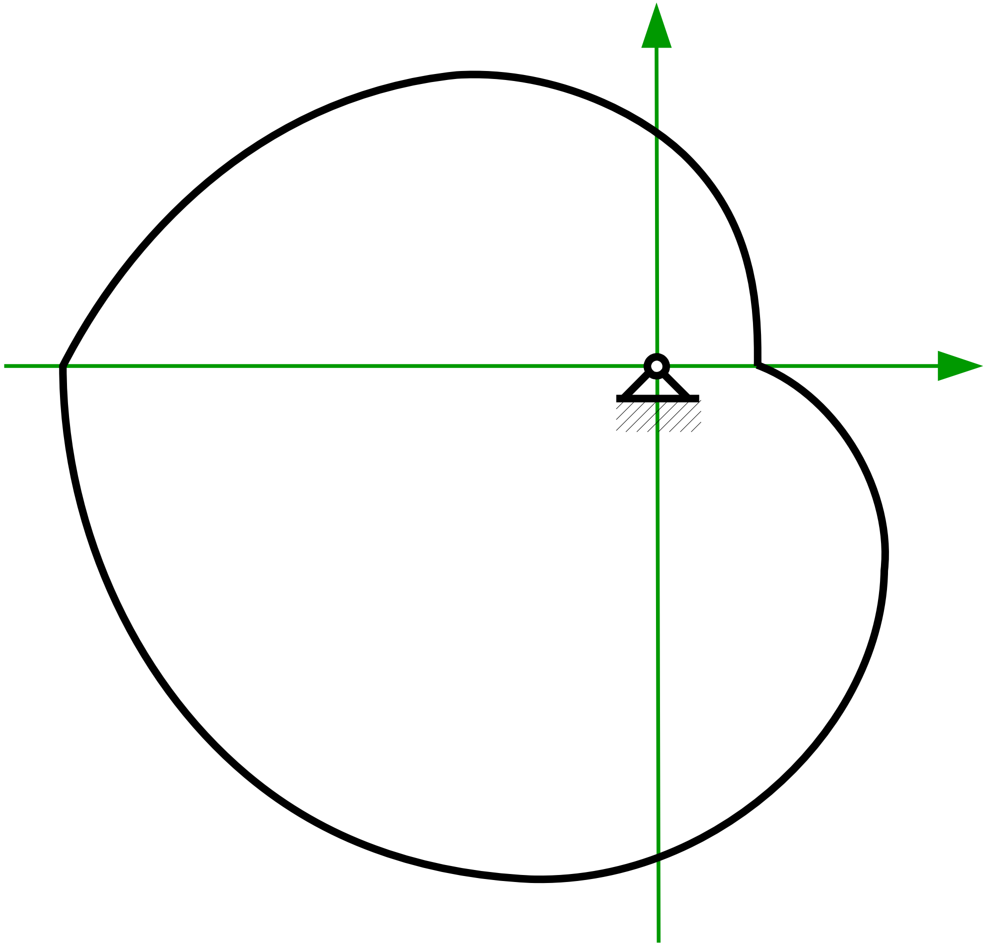
From polar to Cartesian coordinate

$$x(t)=\rho(t)\cos\omega t$$
$$y(t)=\rho(t)\sin\omega t$$

Synthesis of cam-follower mechanisms

Analytical method – example

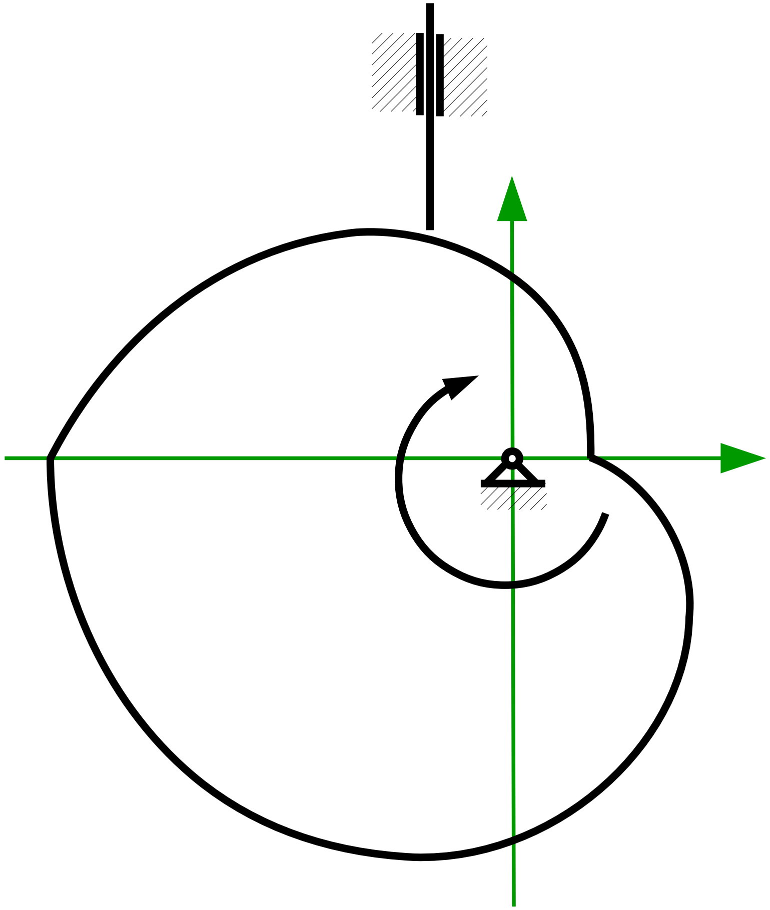
Design profile of a cam cooperating with eccentric knife-edge follower to obtain constant acceleration of uplift and downlift with constant angular velocity.

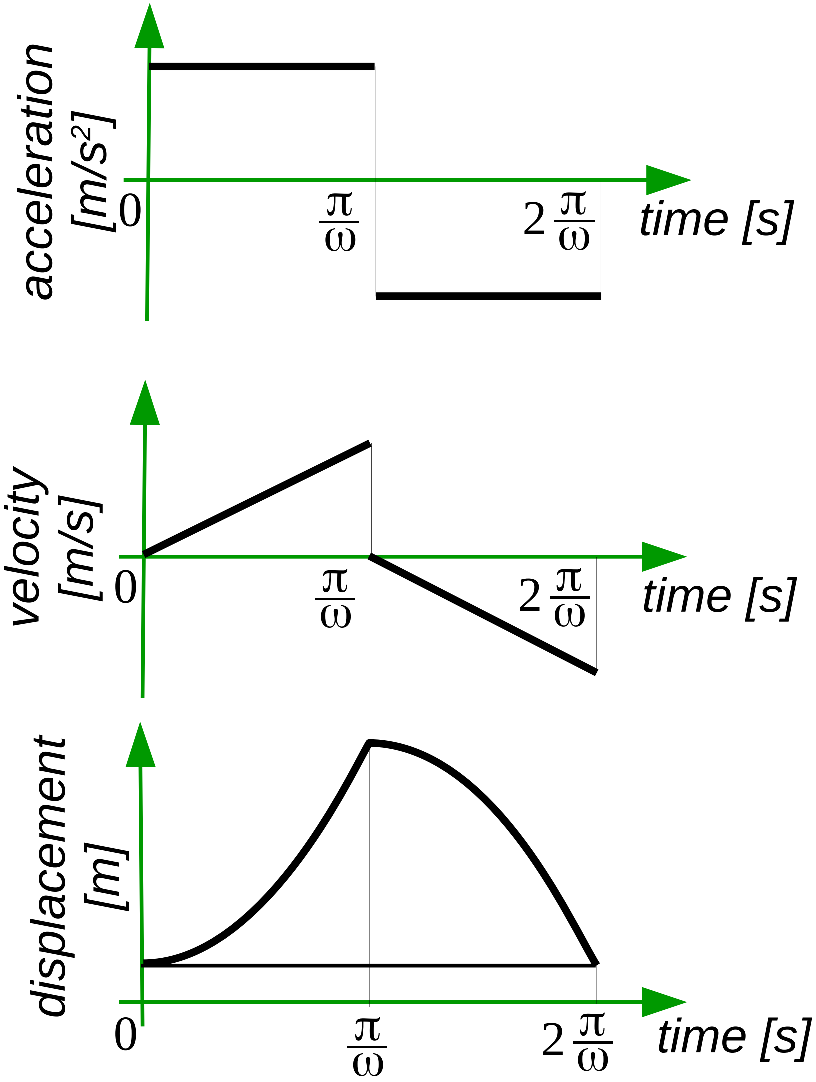


Synthesis of cam-follower mechanisms

Analytical method – example

Design profile of a cam cooperating with eccentric knife-edge follower to obtain constant acceleration of uplift and downlift with constant angular velocity.

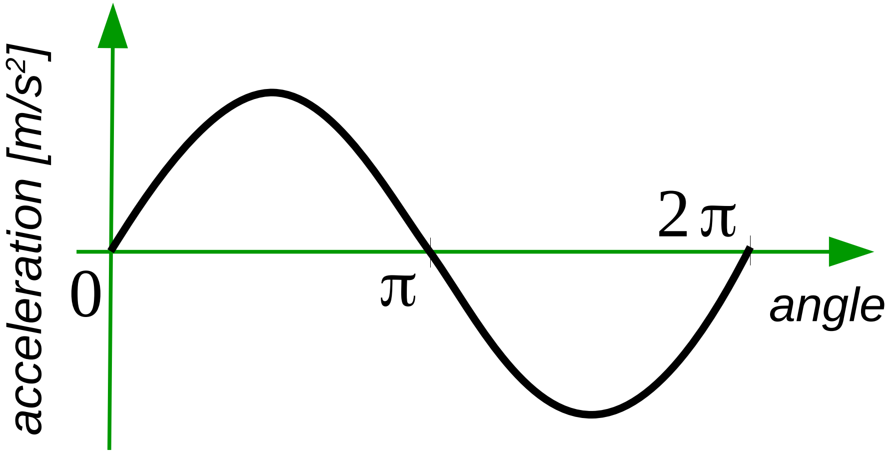




Synthesis of cam-follower mechanisms

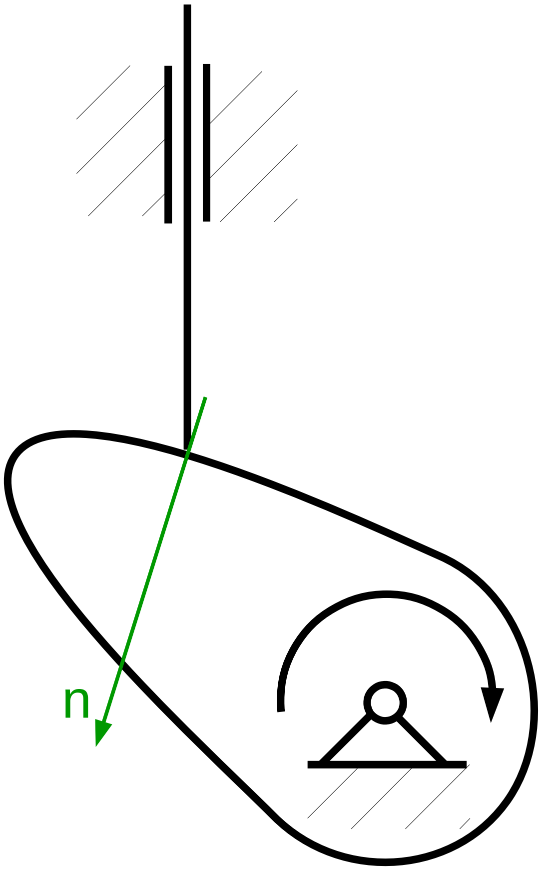
Analytical method – example 2

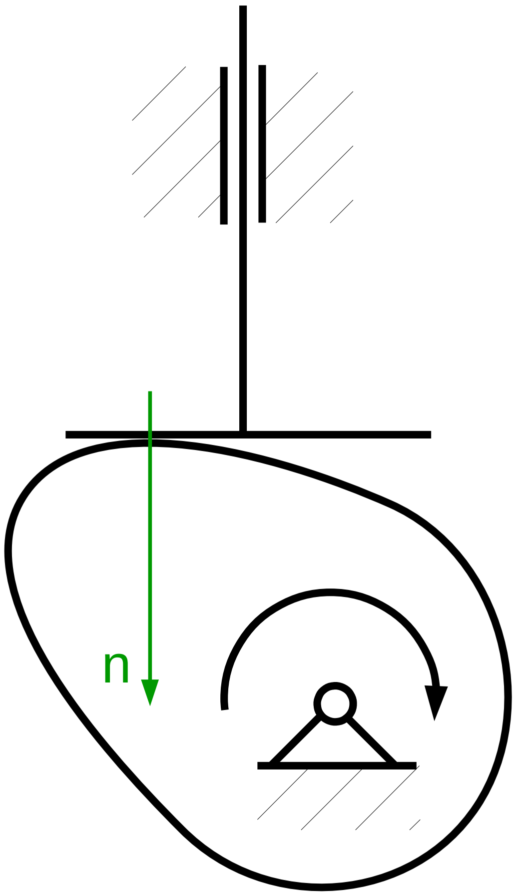
Design profile of a cam cooperating with centric knife-edge follower to obtain harmonic acceleration with constant angular velocity.

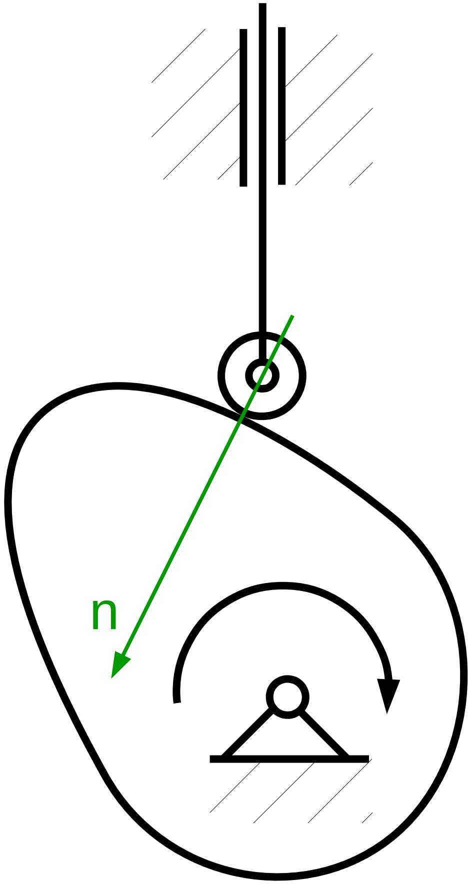


Cam-follower mechanisms

Distance of contact and angle of contact

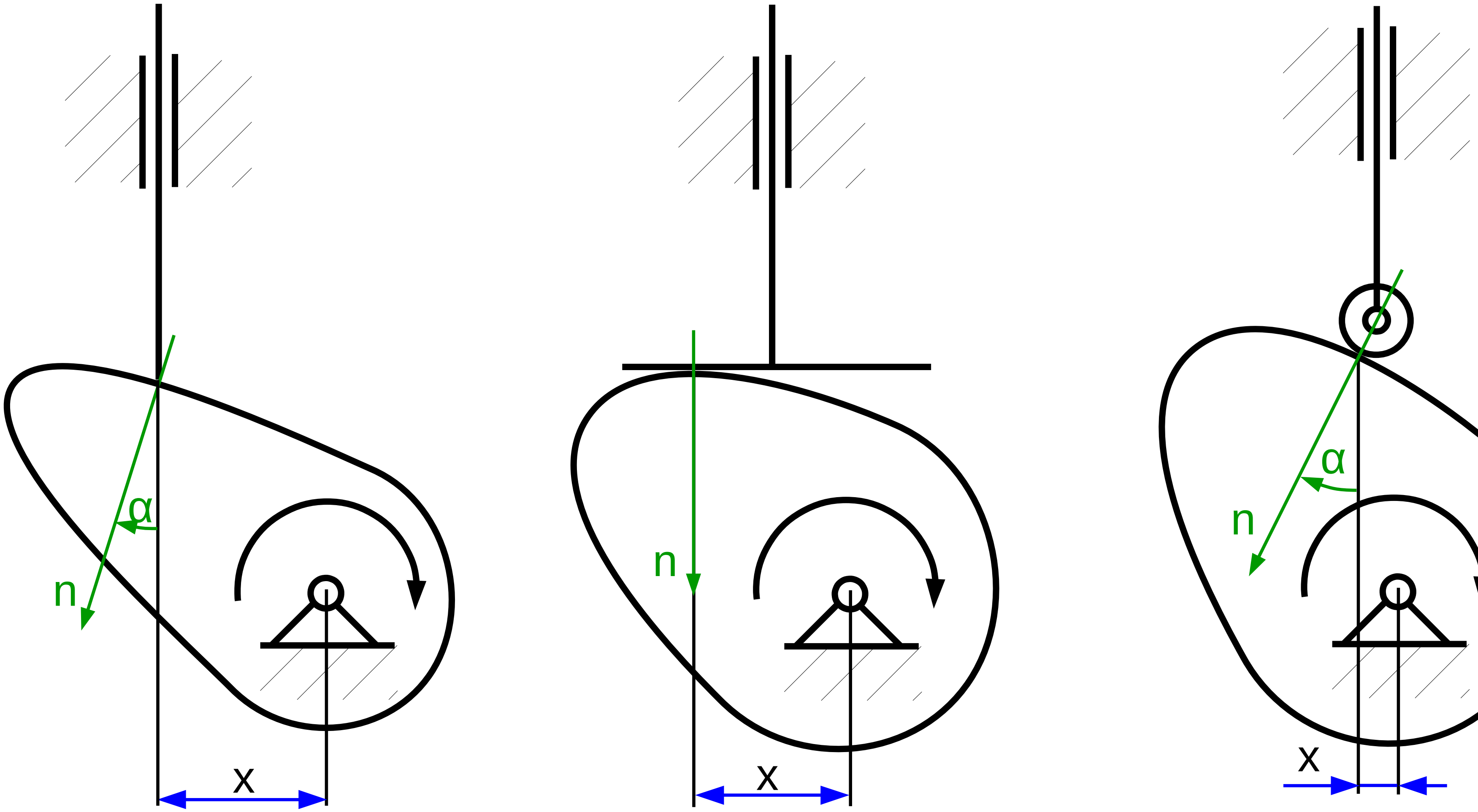






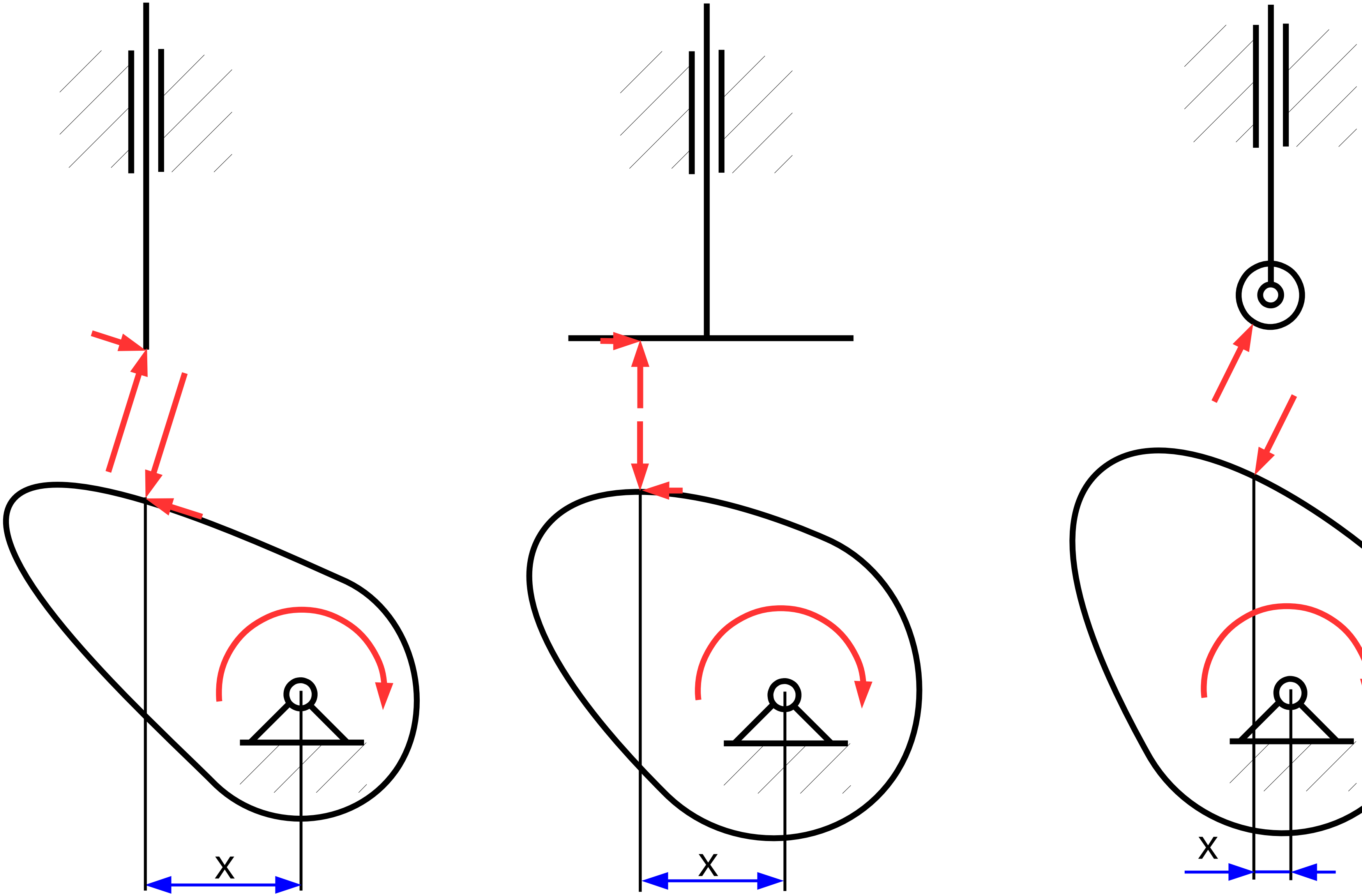
Cam-follower mechanisms

Distance of contact and angle of contact



Cam-follower mechanisms

Distance of contact and angle of contact



Cam-follower mechanisms

Minimal cam size

1st condition: minimum cam radius in case of material strength and wear resistance.

Cam-follower mechanisms

Minimal cam size

2nd condition: maximum angle of contact in case of follower bending resistance and pressure inside the socket.

Cam-follower mechanisms

Minimal cam size

3rd condition: maximum distance of contact in case of follower's stem bending (for a flat-faced followers).

Cam-follower mechanisms

Minimal cam size

Follower offset towards direction opposite to direction of rotation decrease angle of contact

Dynamics of planar mechanisms

Dynamics of planar mechanisms

Overview

Members description as rigid bodies and and material points.

Graphical determination of inertial forces and torques.

Reaction forces in kinematic pairs.

Driving and operating forces/torques.

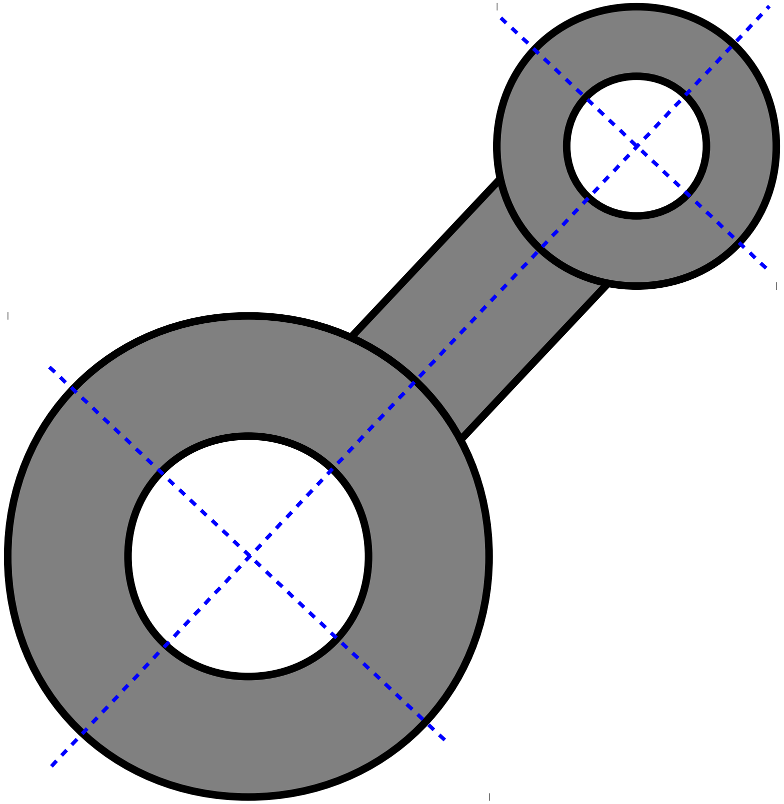
Inverse and direct dynamics problems.

Graphical, analytical and graphical-analytical method.

Friction in kinematic pairs.

Dynamics of planar mechanisms

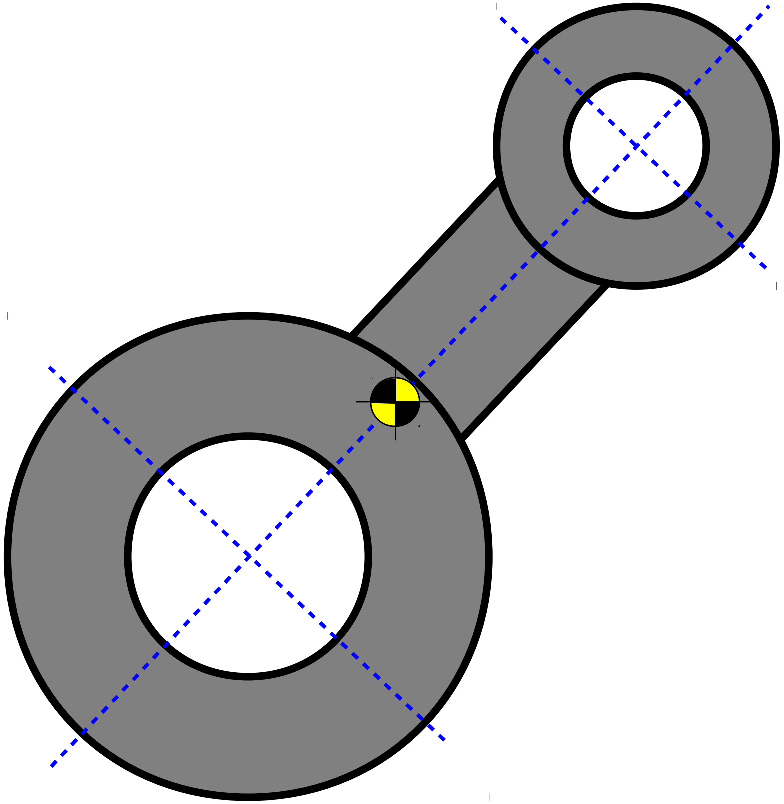
Members description



For a planar mechanism member represented by a rigid body:

Dynamics of planar mechanisms

Members description



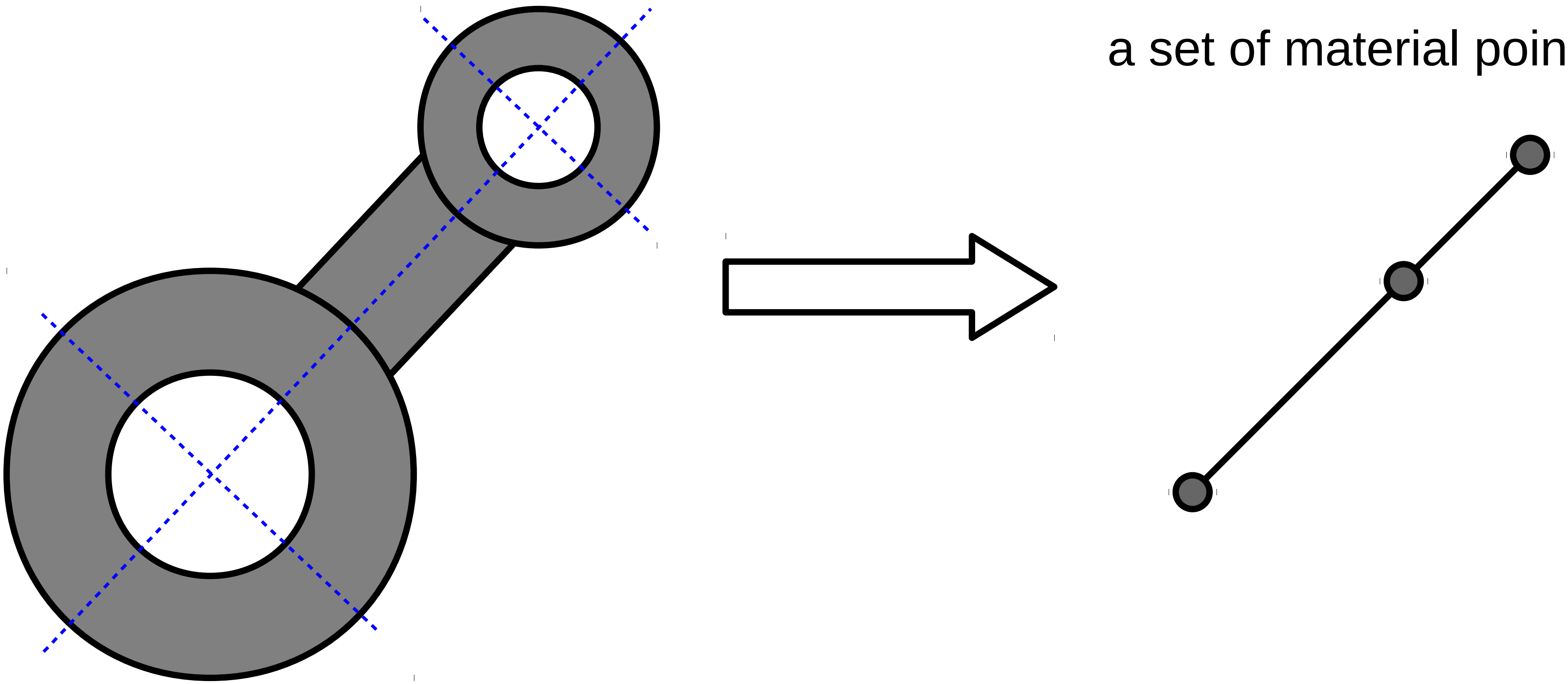
For a planar mechanism member represented by a rigid body:

- mass
- location of a center of a mass
- mass moment of inertia wrt the axis perpendicular to the motion plan in center of a mass
- location of connection points

Dynamics of planar mechanisms

Members description

Material points method

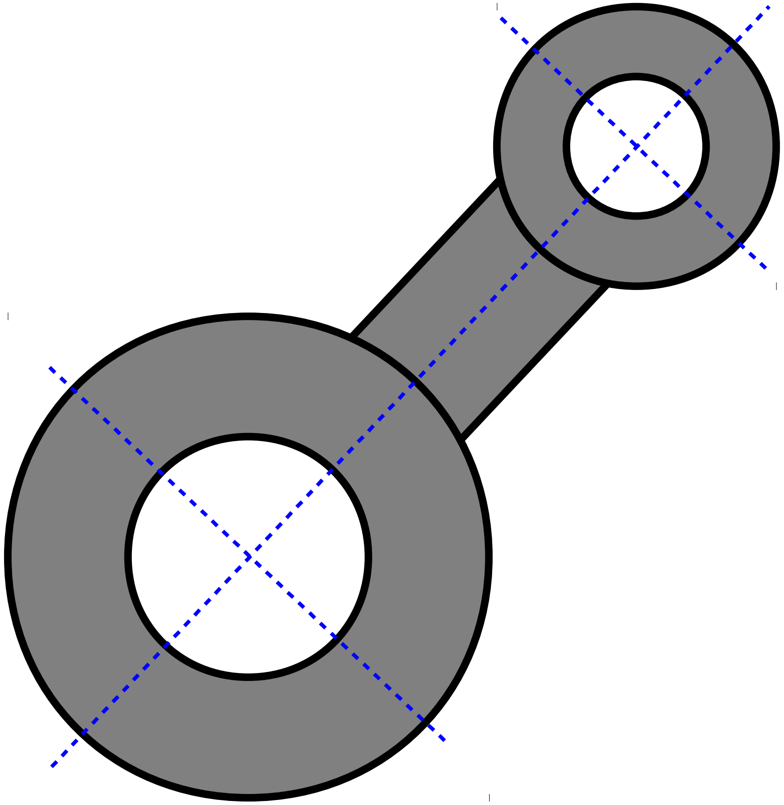


a set of material points

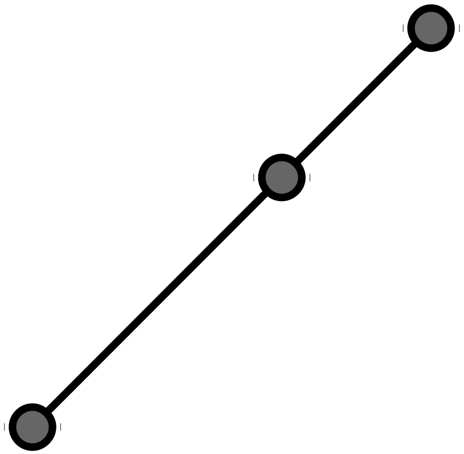
Dynamics of planar mechanisms

Members description

Material points method



a set of material points



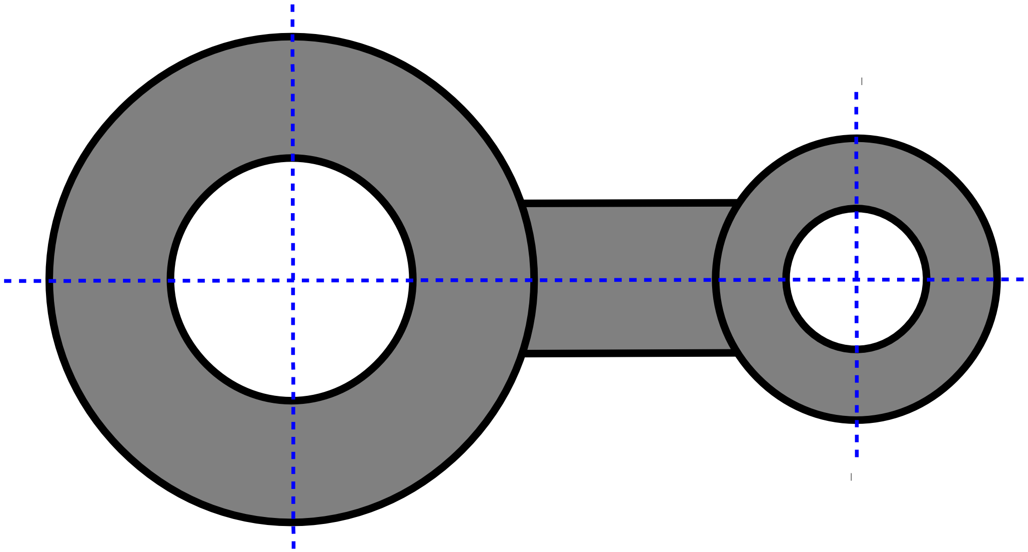
- same masses
- same center of mass
- same inertia

Dynamics of planar mechanisms

Members description

Material points method - example

Given:
Geometry, mass *m*,
center of a mass location
(pt. C) and mass moment
of inertia I_C



Dynamics of planar mechanisms

Members description

Material points method - example

Given:
Geometry, mass *m*,
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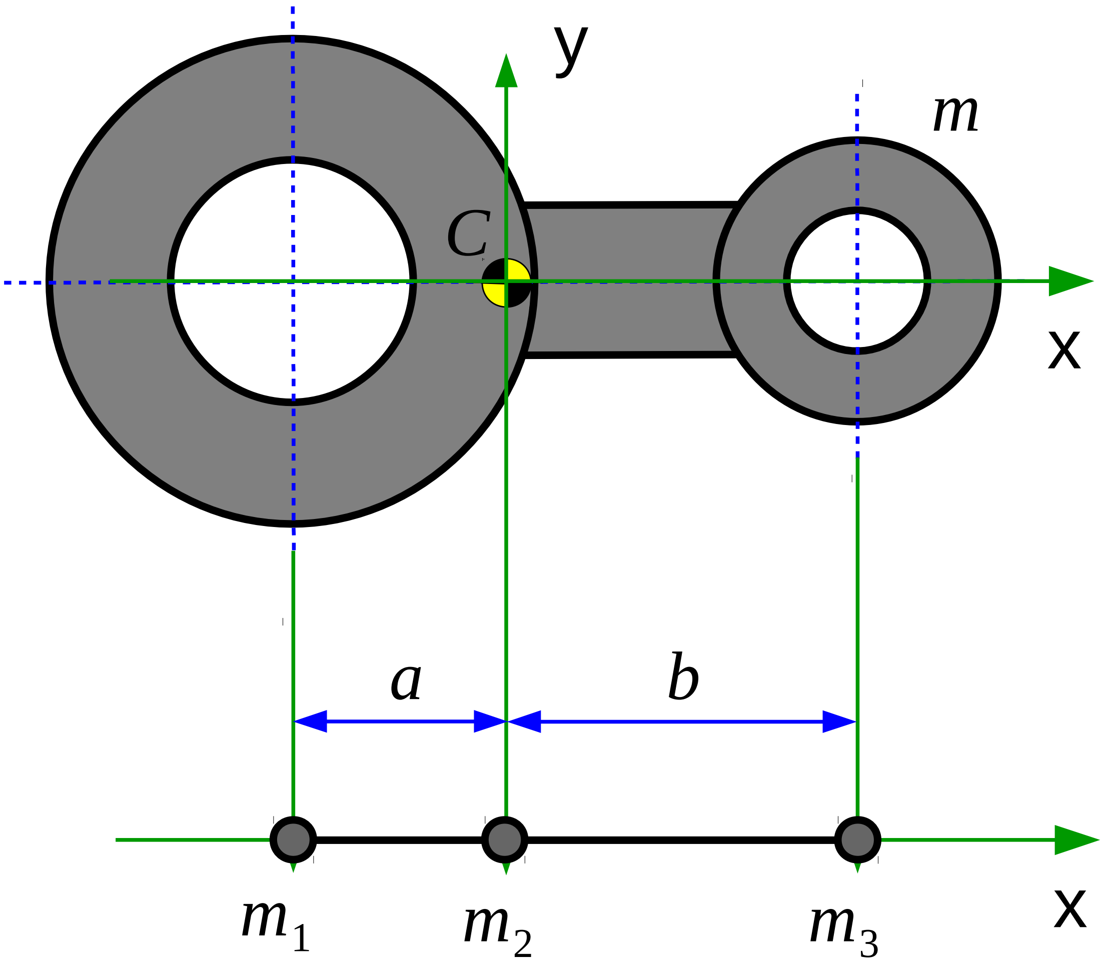


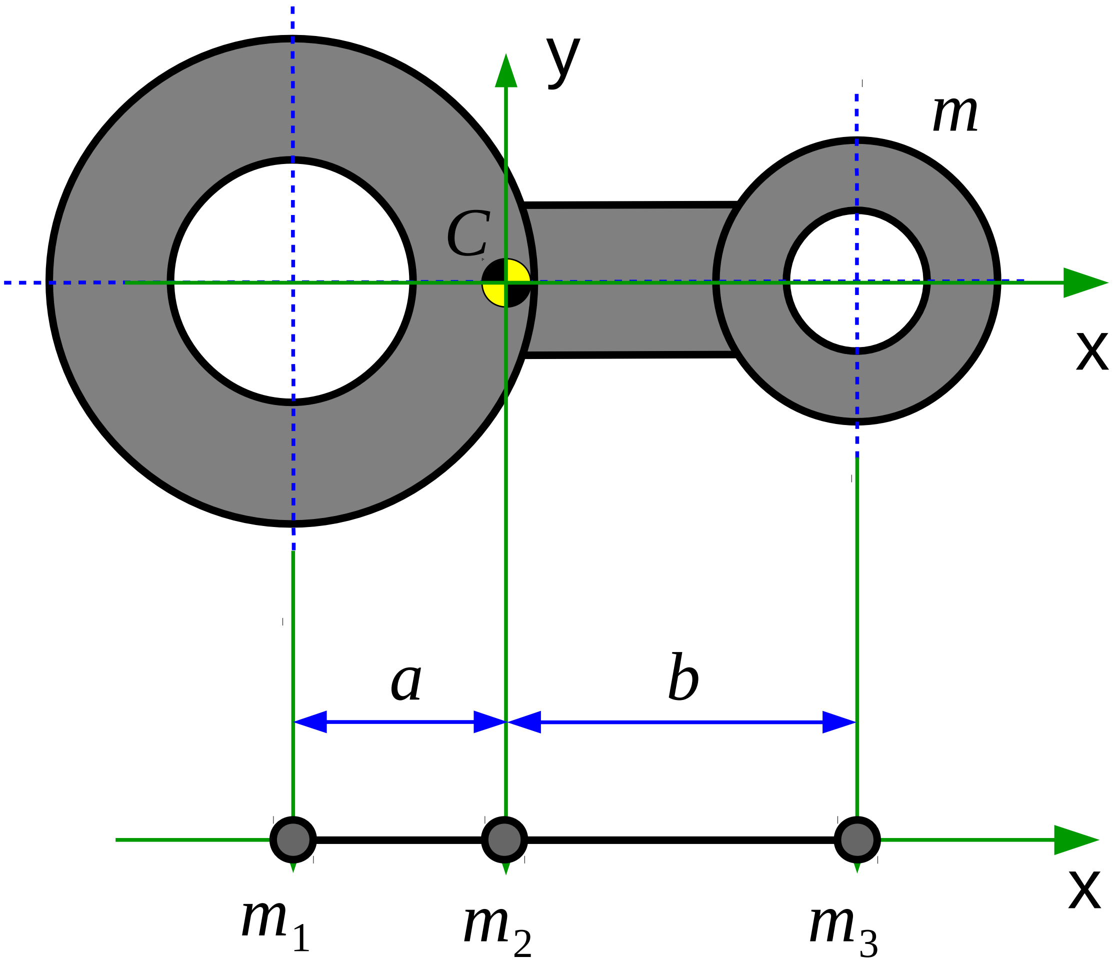
Dynamics of planar mechanisms

Members description

Material points method - example

Given:
Geometry, mass *m*,
center of a mass location
(pt. C) and mass moment
of inertia I_C

$$m_1+m_2+m_3=m$$

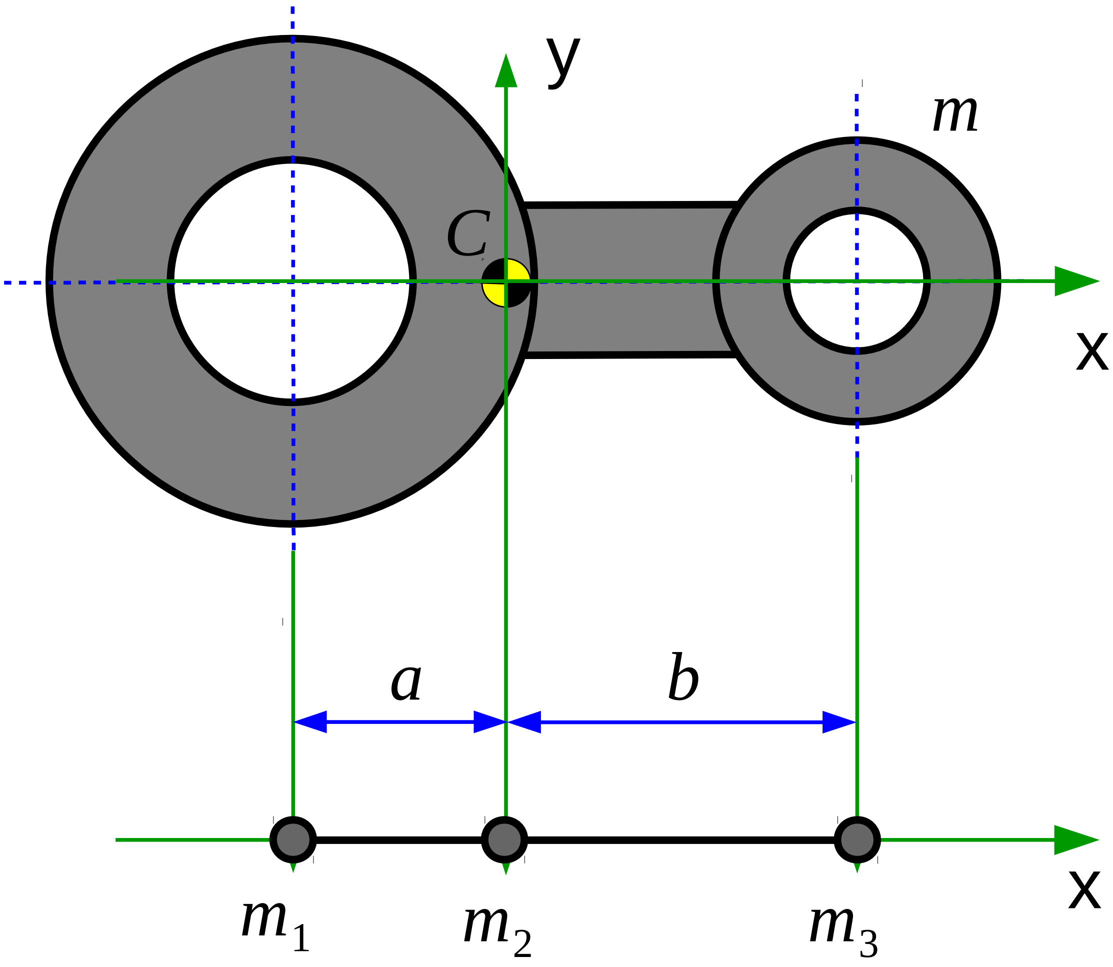


Dynamics of planar mechanisms

Members description

Material points method - example

Given:
Geometry, mass *m*,
center of a mass location
(pt. C) and mass moment
of inertia I_C



$$m_1+m_2+m_3=m$$

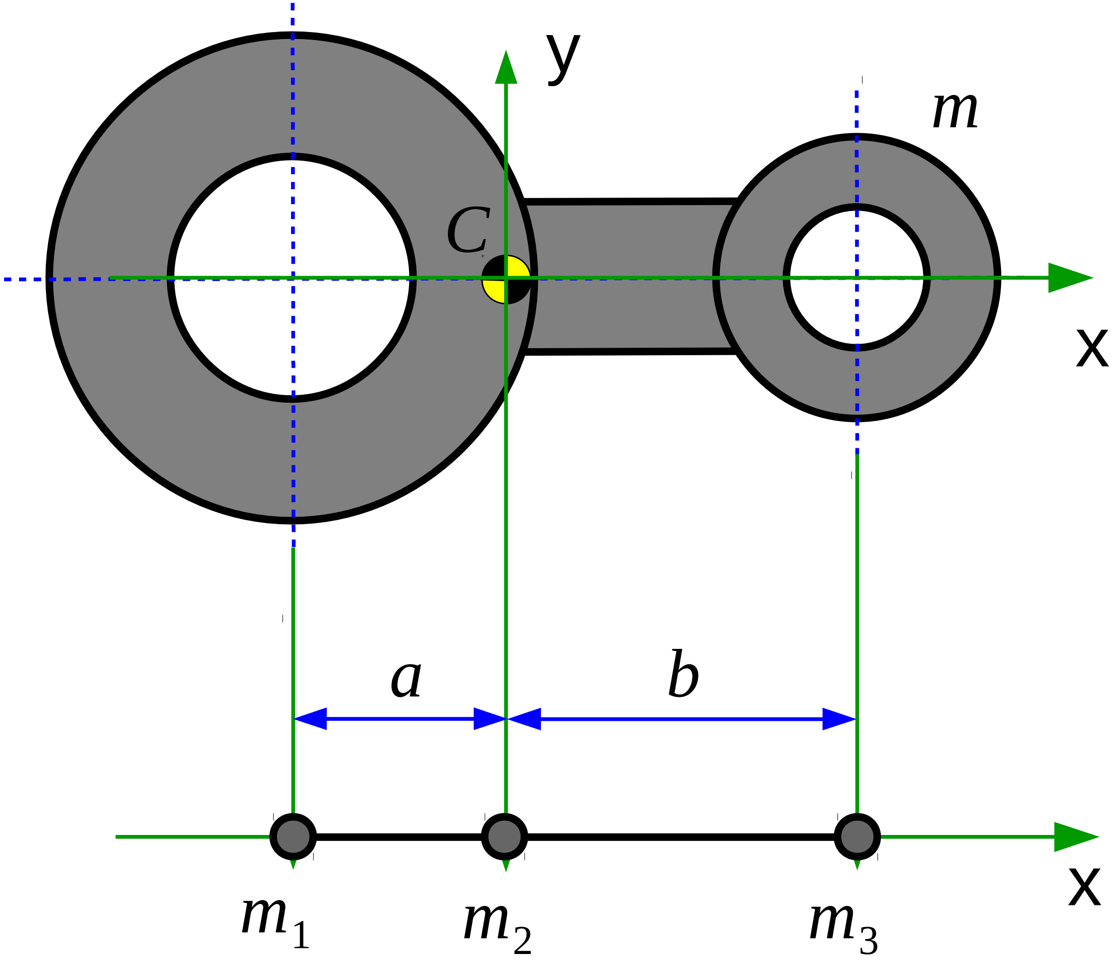
$$\frac{-a\,m_1+b\,m_3}{m_1+m_2+m_3}=0$$

Dynamics of planar mechanisms

Members description

Material points method - example

Given:
Geometry, mass *m*,
center of a mass location
(pt. C) and mass moment
of inertia I_C



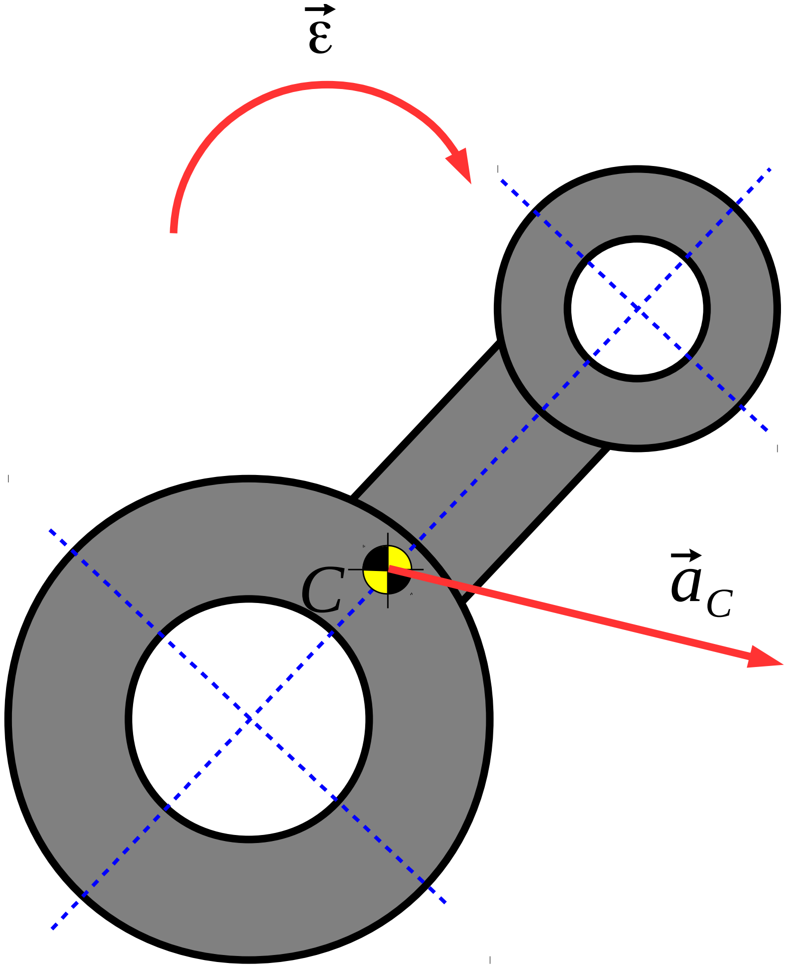
$$m_1+m_2+m_3=m$$

$$\frac{-a\,m_1+b\,m_3}{m_1+m_2+m_3}=0$$

$$m_1a^2+m_3b^2=I_C$$

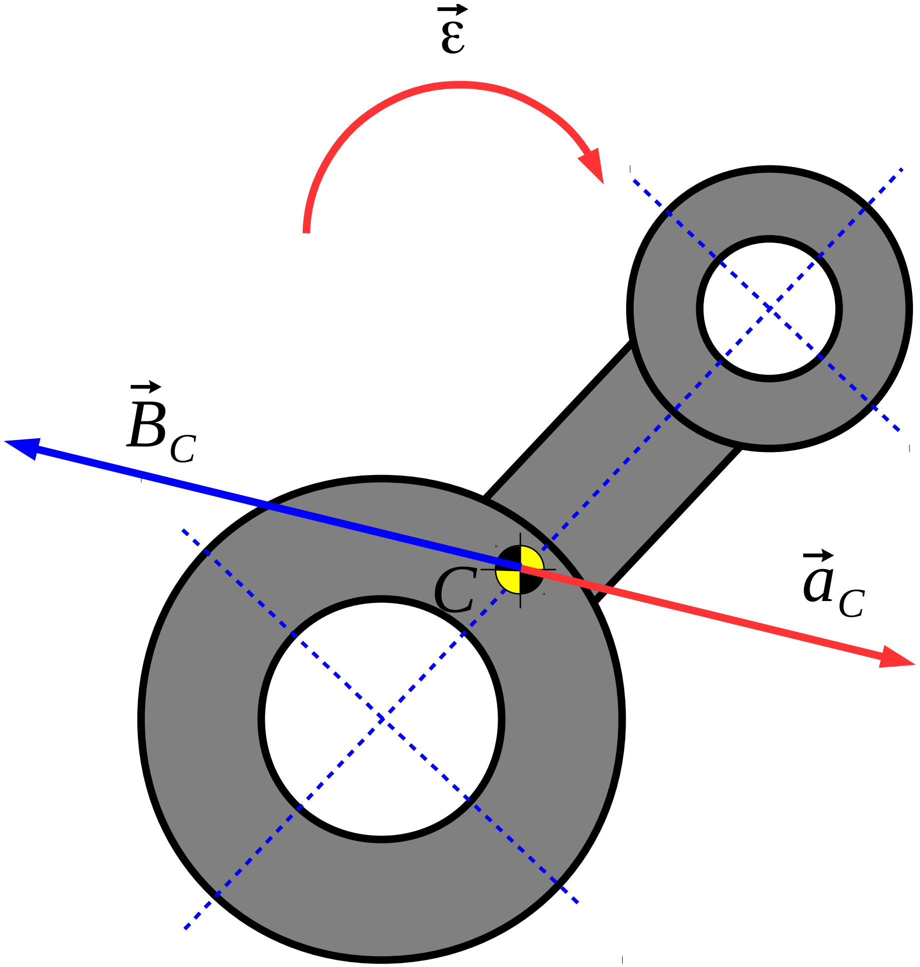
Dynamics of planar mechanisms

Inertia forces and torques



Dynamics of planar mechanisms

Inertia forces and torques

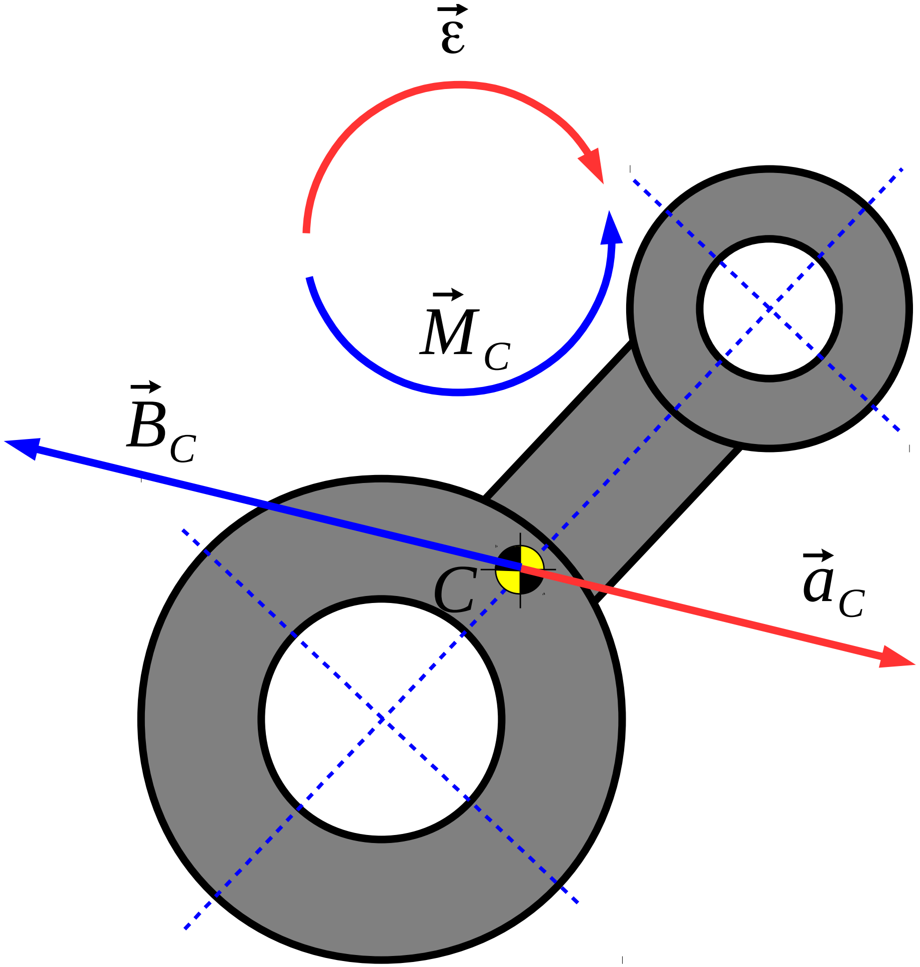


inertia force

$$\vec{B}_C = -m\,\vec{a}_C$$

Dynamics of planar mechanisms

Inertia forces and torques



inertia force

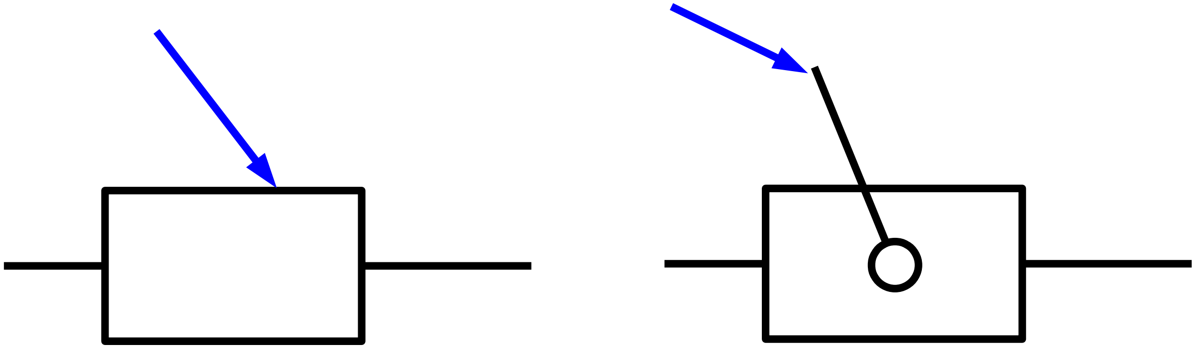
$$\vec{B}_C = -m\vec{a}_C$$

Inertia torque

$$\vec{M}_C = -I_C\vec{\varepsilon}$$

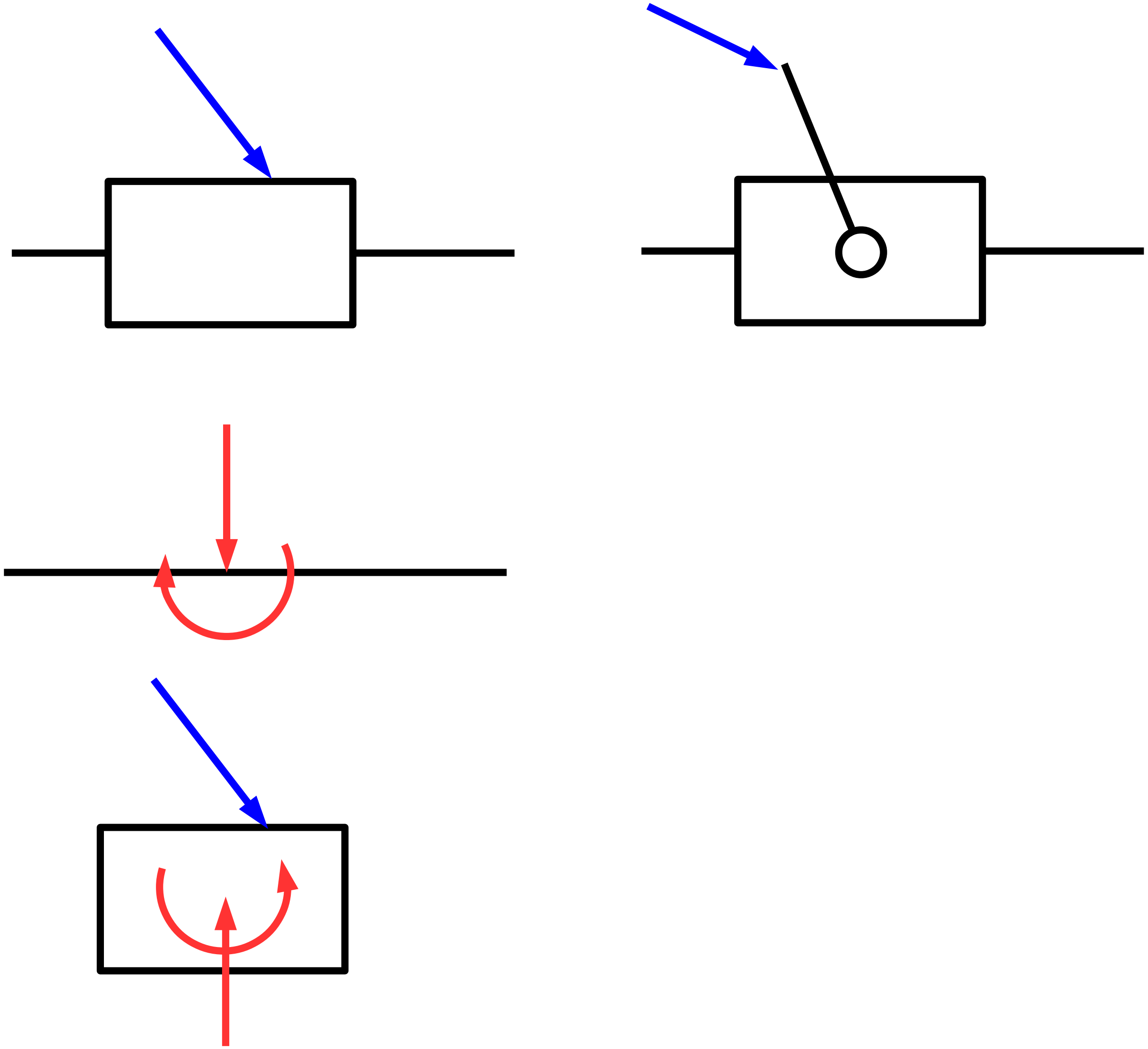
Dynamics of planar mechanisms

Reaction forces inside a kinematic pairs (without friction)



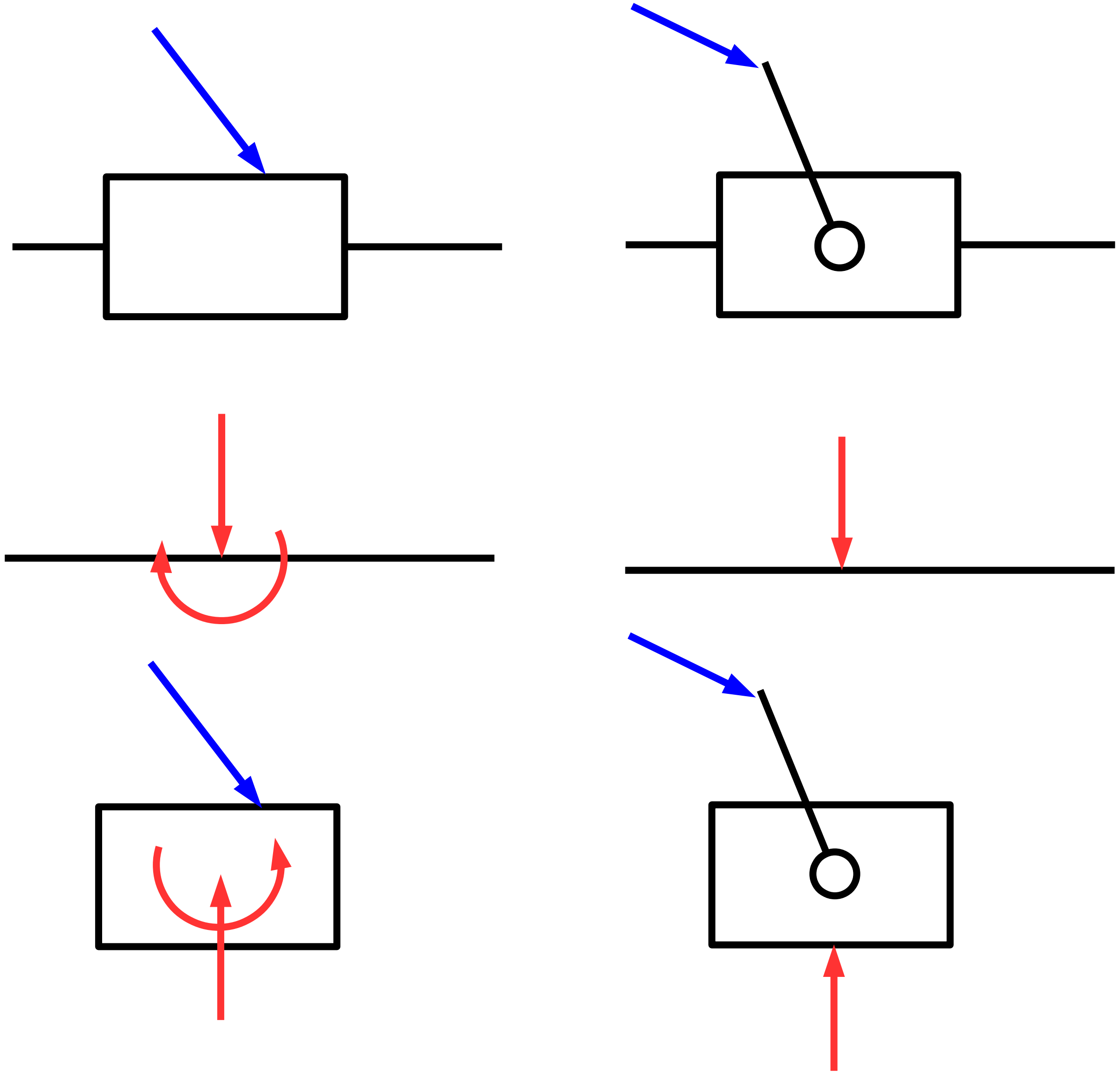
Dynamics of planar mechanisms

Reaction forces inside a kinematic pairs (without friction)



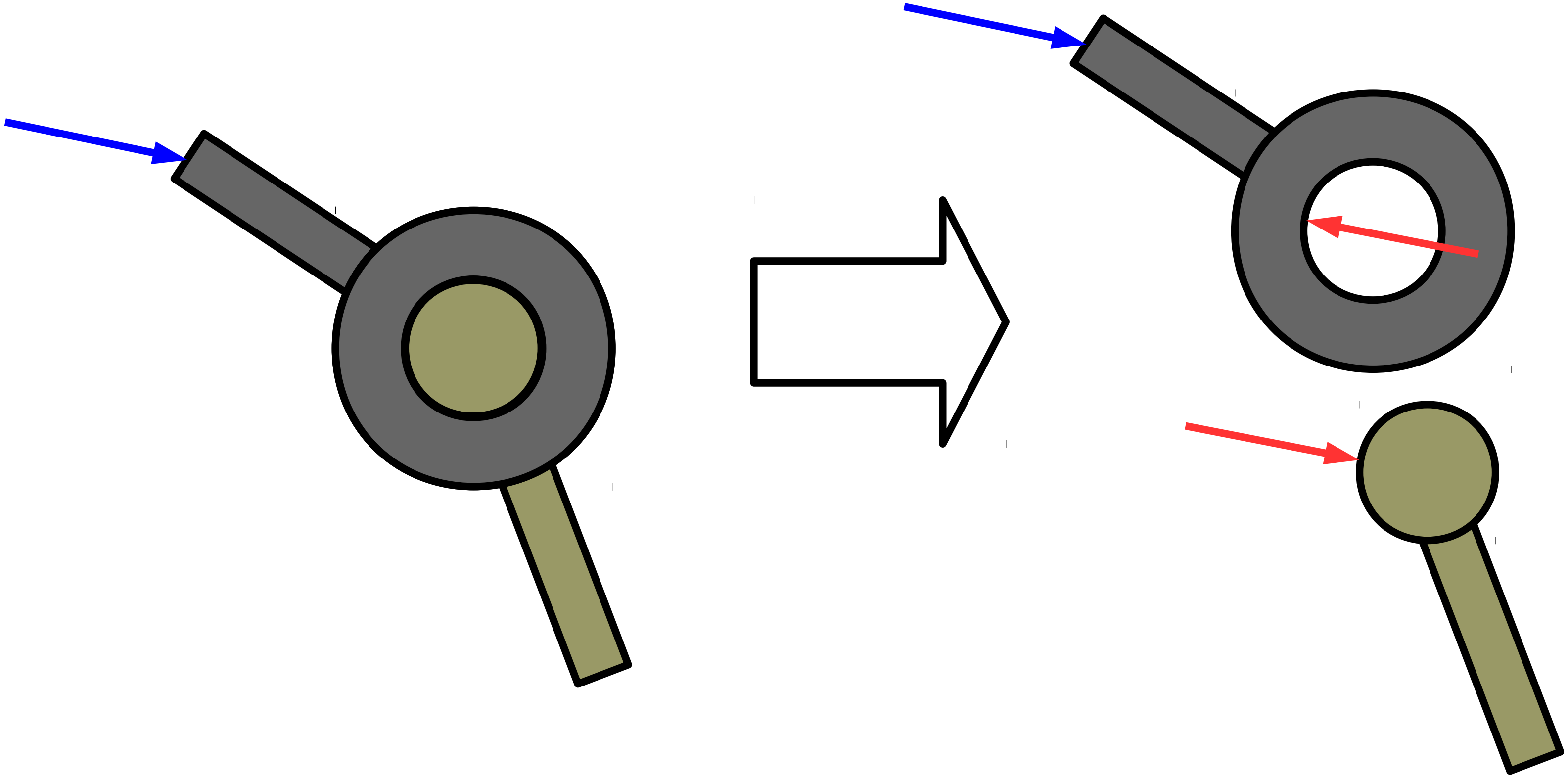
Dynamics of planar mechanisms

Reaction forces inside a kinematic pairs (without friction)



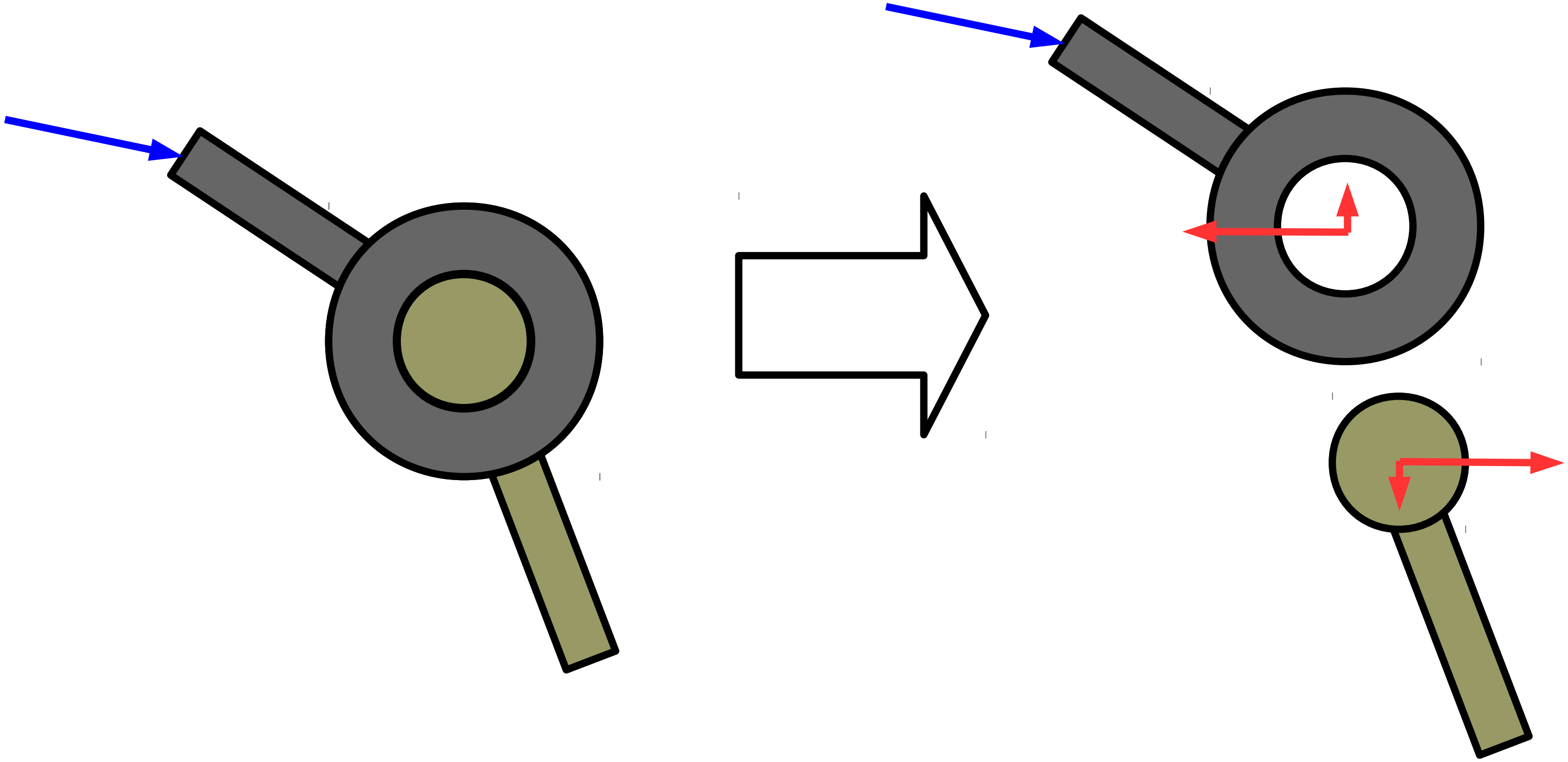
Dynamics of planar mechanisms

Reaction forces inside a kinematic pairs (without friction)



Dynamics of planar mechanisms

Reaction forces inside a kinematic pairs (without friction)



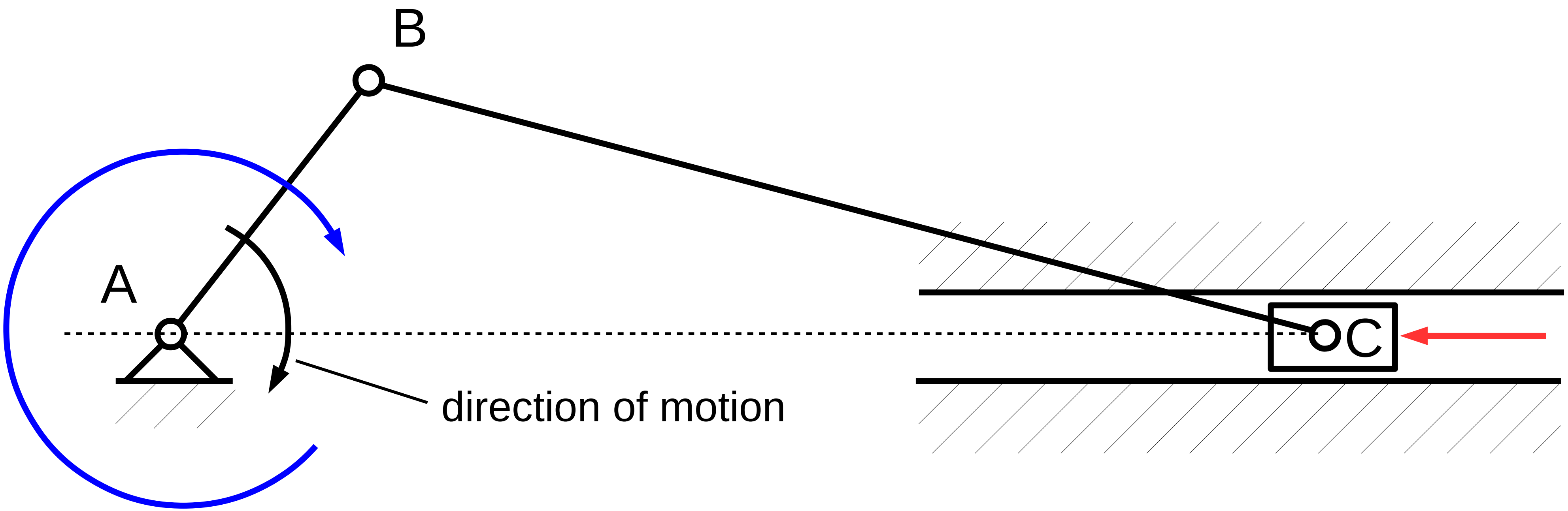
Dynamics of planar mechanisms

Driving and operating forces/torques

Dynamics of planar mechanisms

Driving and operating forces/torques

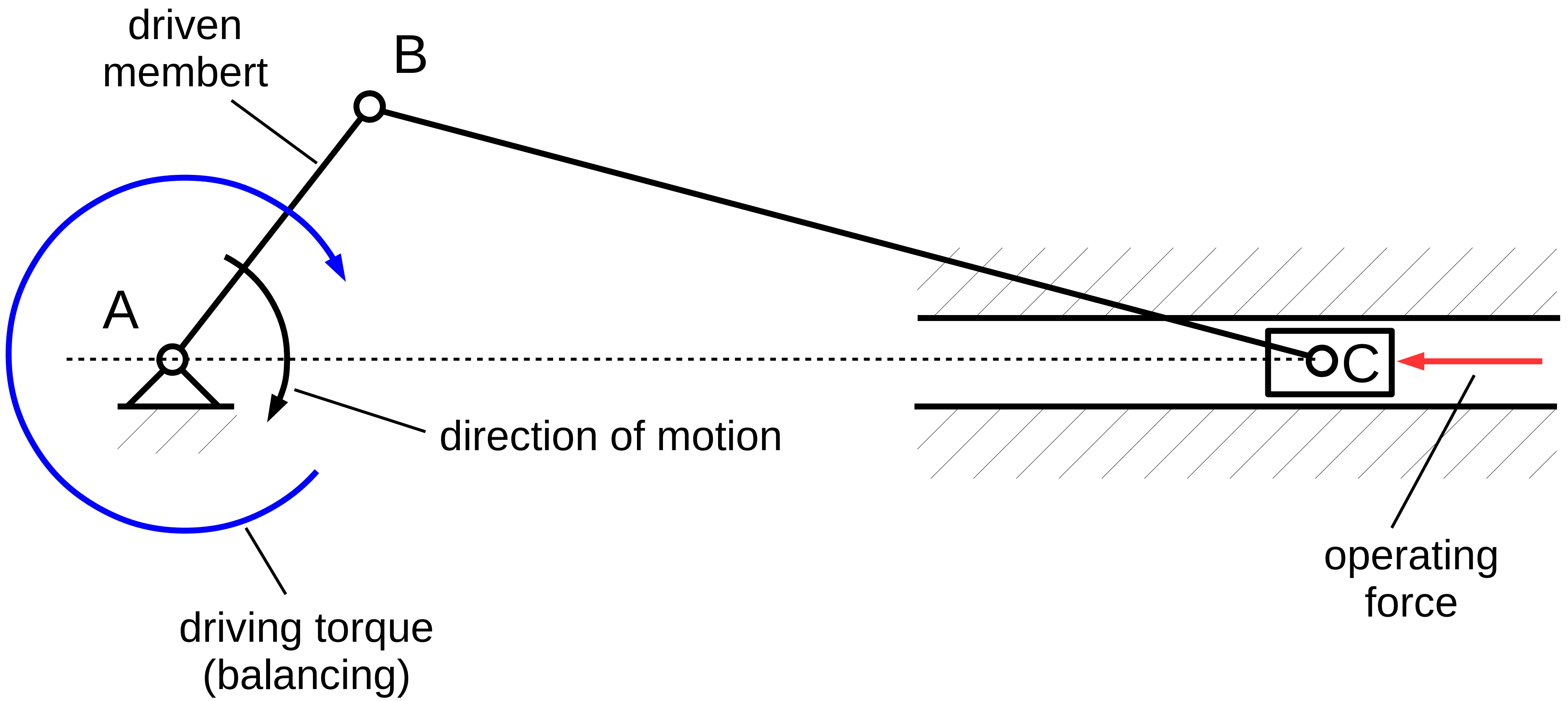
example – compressor



Dynamics of planar mechanisms

Driving and operating forces/torques

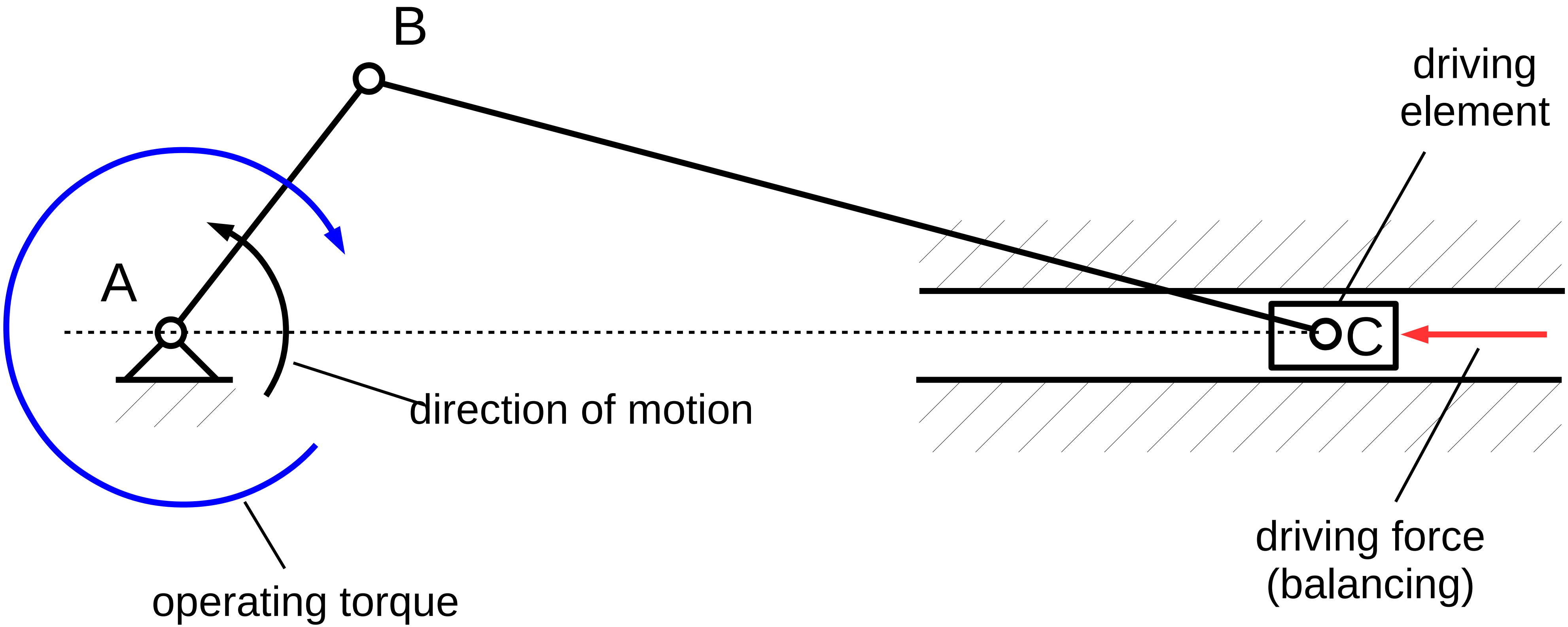
example – compressor



Dynamics of planar mechanisms

Driving and operating forces/torques

example – engine



Dynamics of planar mechanisms

Inverse dynamics problem – calculation of forces and torques that cause given motion of a mechanism.

Direct dynamics problem – calculation of mechanism's motion caused by external forces and torques.

Dynamics of planar mechanisms

Inverse dynamics problem

Calculation of forces and torques that cause given motion of a mechanism (kinetostatics)

0. Mechanism and it's geometry, driving and operating forces/torques, displacement, velocity and acceleration functions are given.

1. Calculation of inertia forces and torques acting moving members of the mechanism.

2. Decomposition of the mechanism with reaction disclosure.

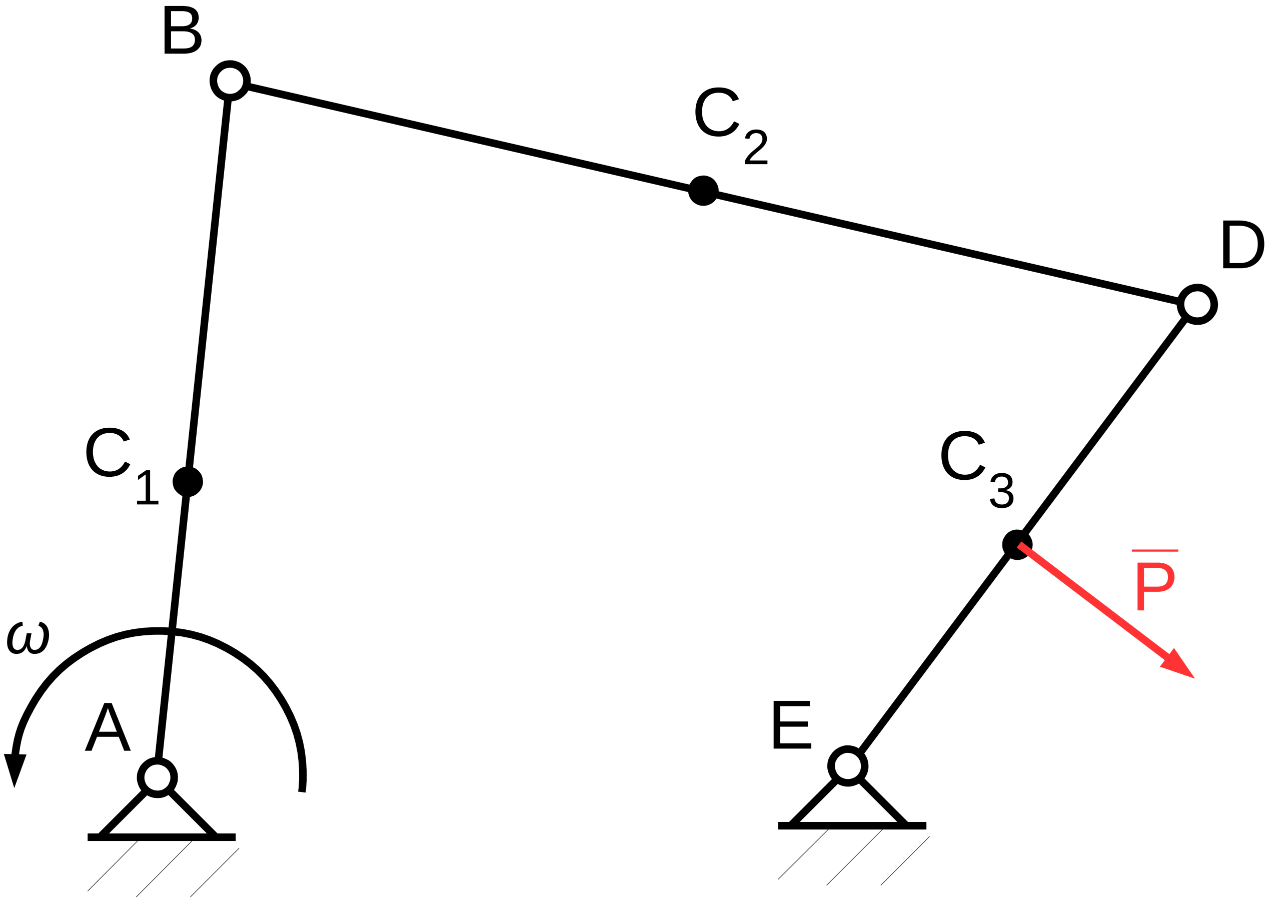
3. Write down vector sums of external forces, reactions and inertia forces (d'Alembert equations).

4. Solve the equations with graphical and/or analytical method.

Dynamics of planar mechanisms

Inverse dynamic problem – example

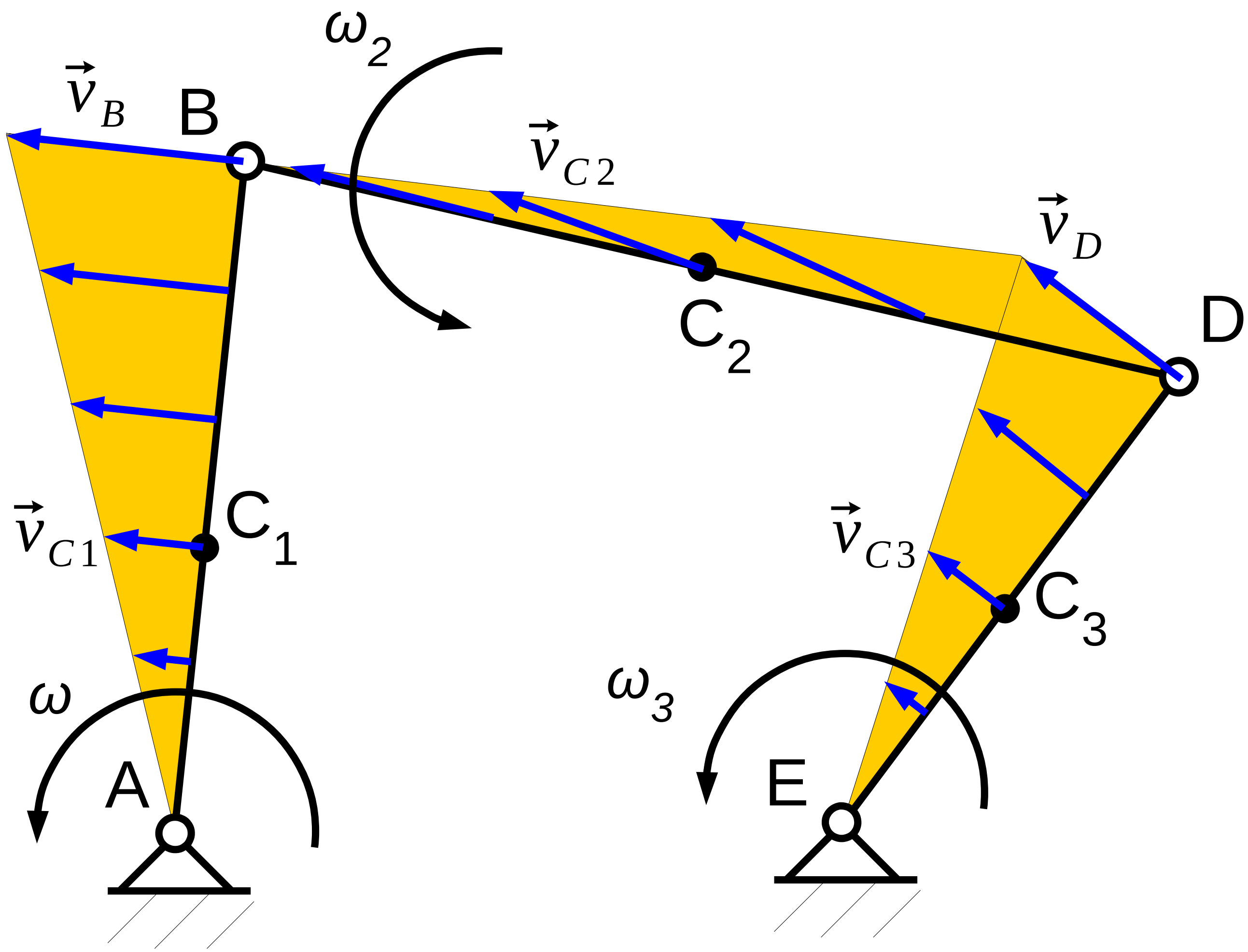
Given:
geometry, mass, center of a mass locations, mass moments of inertia for all mechanism members, constant angular velocity ω of a driven element, operating force P.



Dynamics of planar mechanisms

Inverse dynamic problem – example

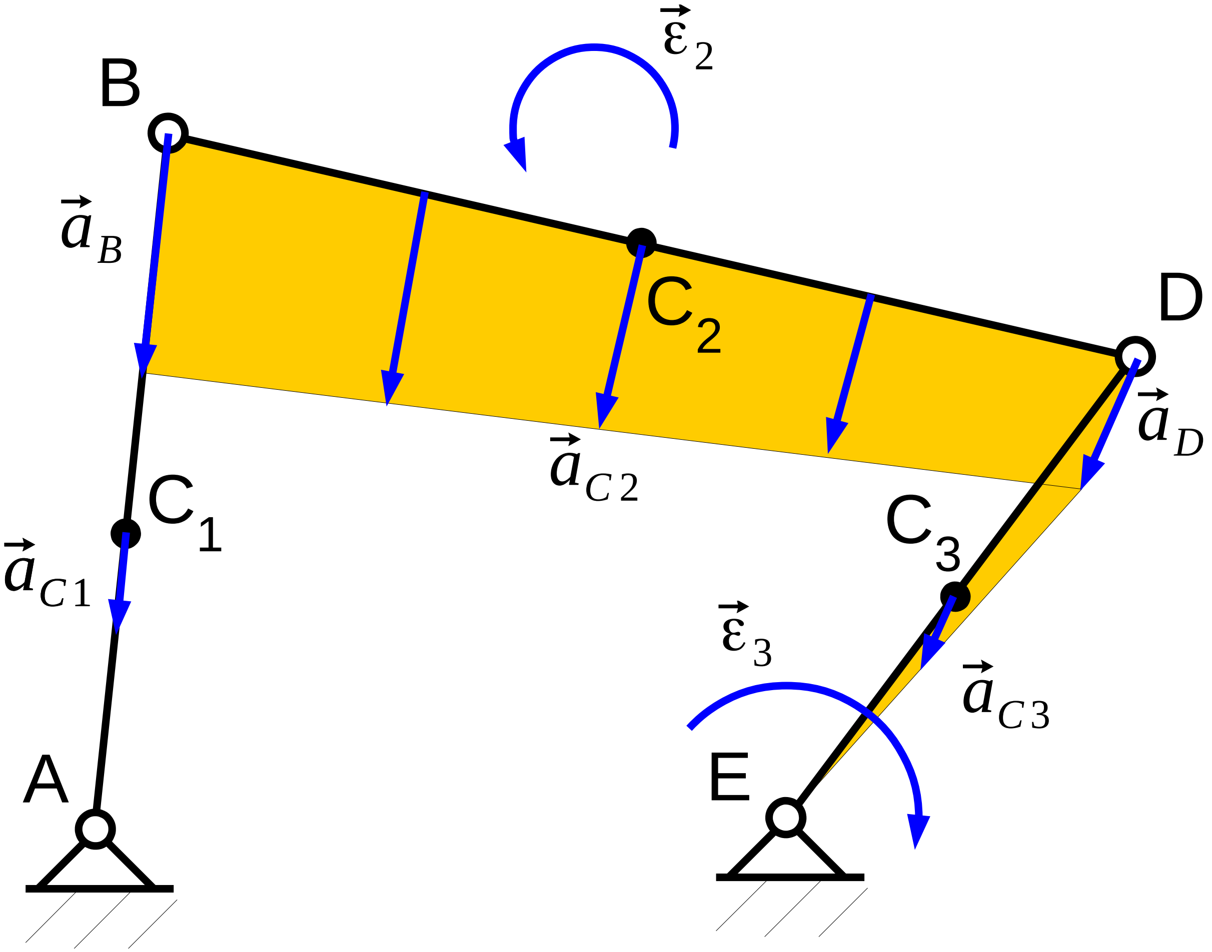
velocity scheme



Dynamics of planar mechanisms

Inverse dynamic problem – example

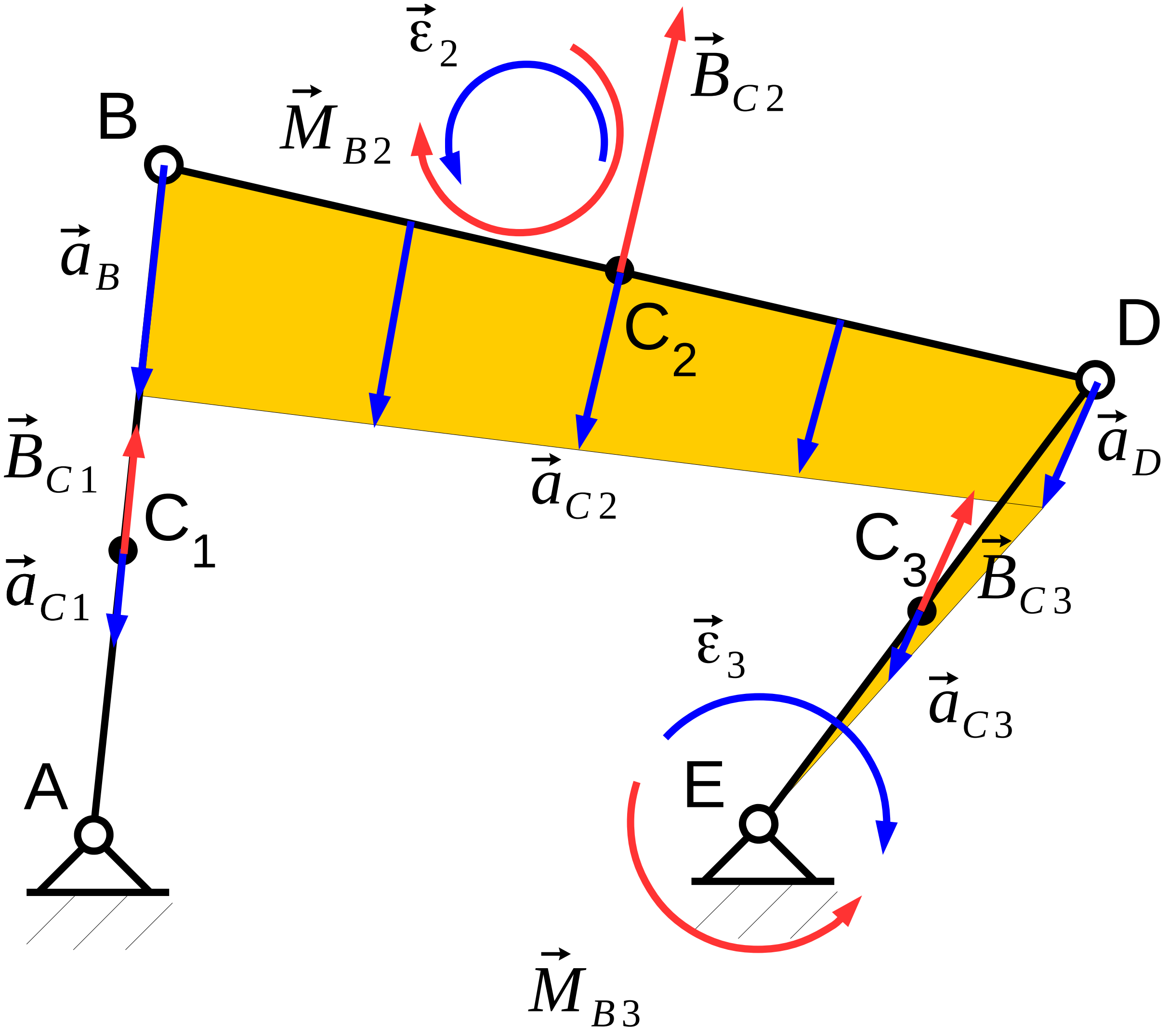
acceleration scheme



Dynamics of planar mechanisms

Inverse dynamic problem – example

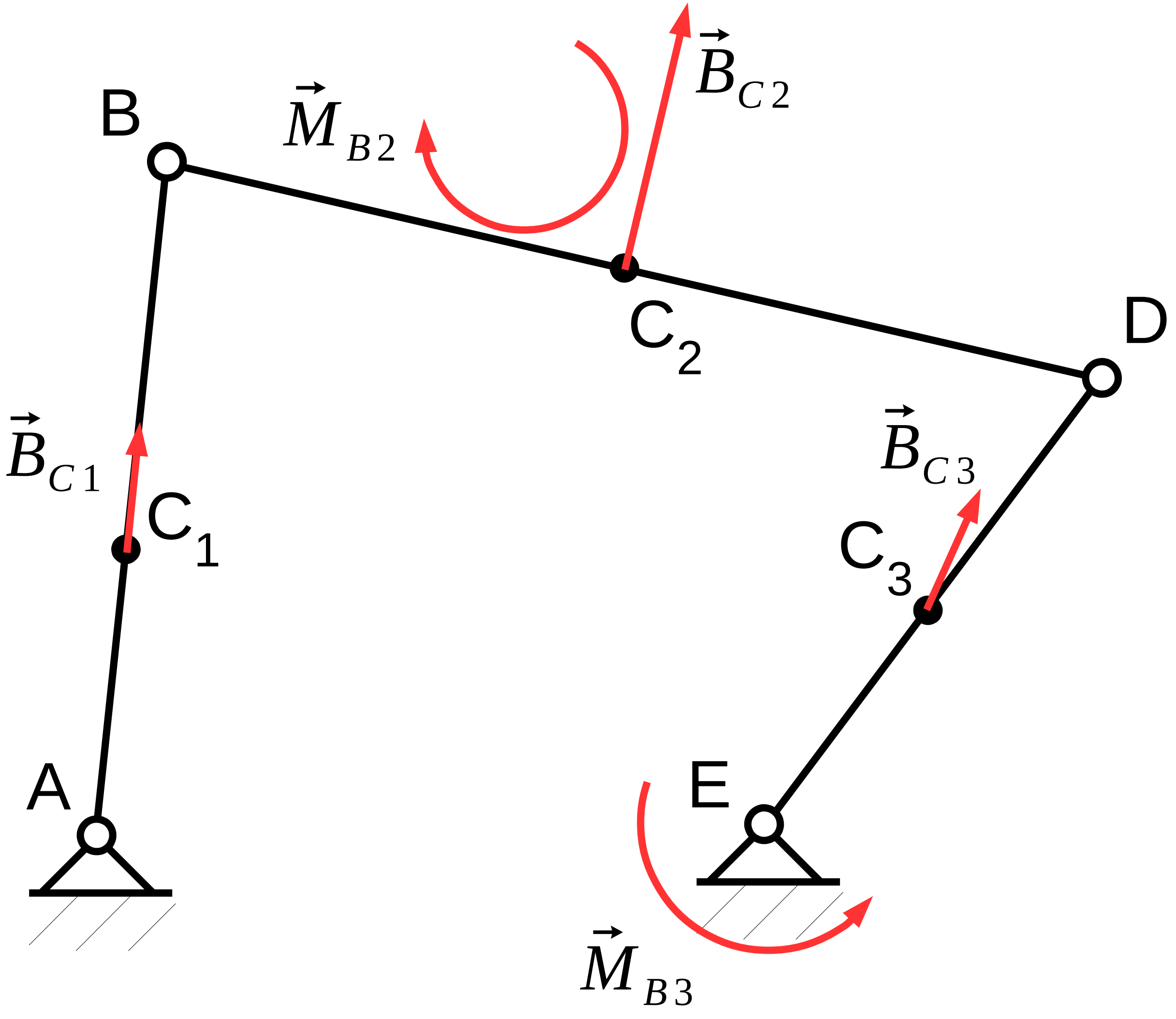
inertial forces and torques



Dynamics of planar mechanisms

Inverse dynamic problem – example

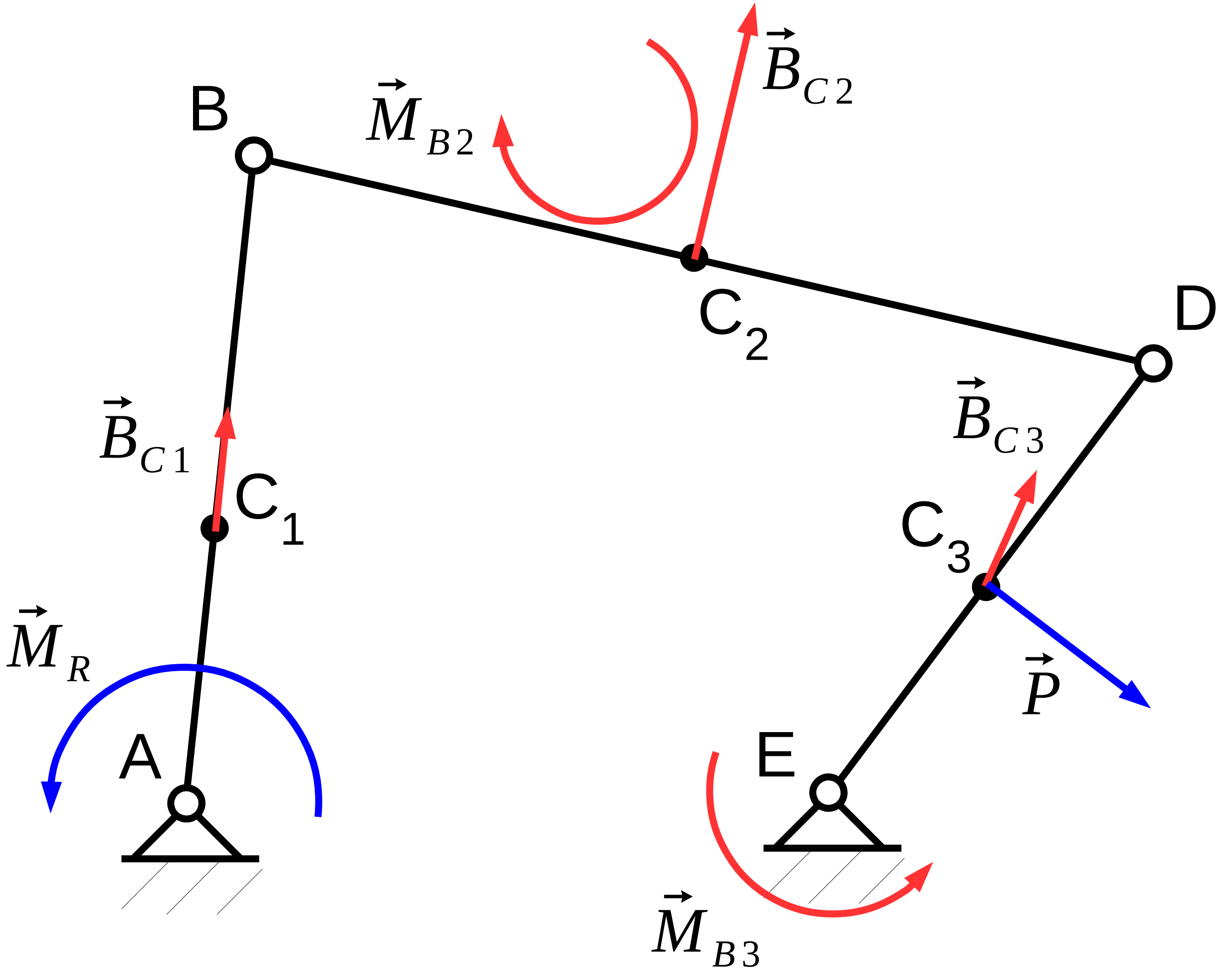
inertial forces and torques



Dynamics of planar mechanisms

Inverse dynamic problem – example

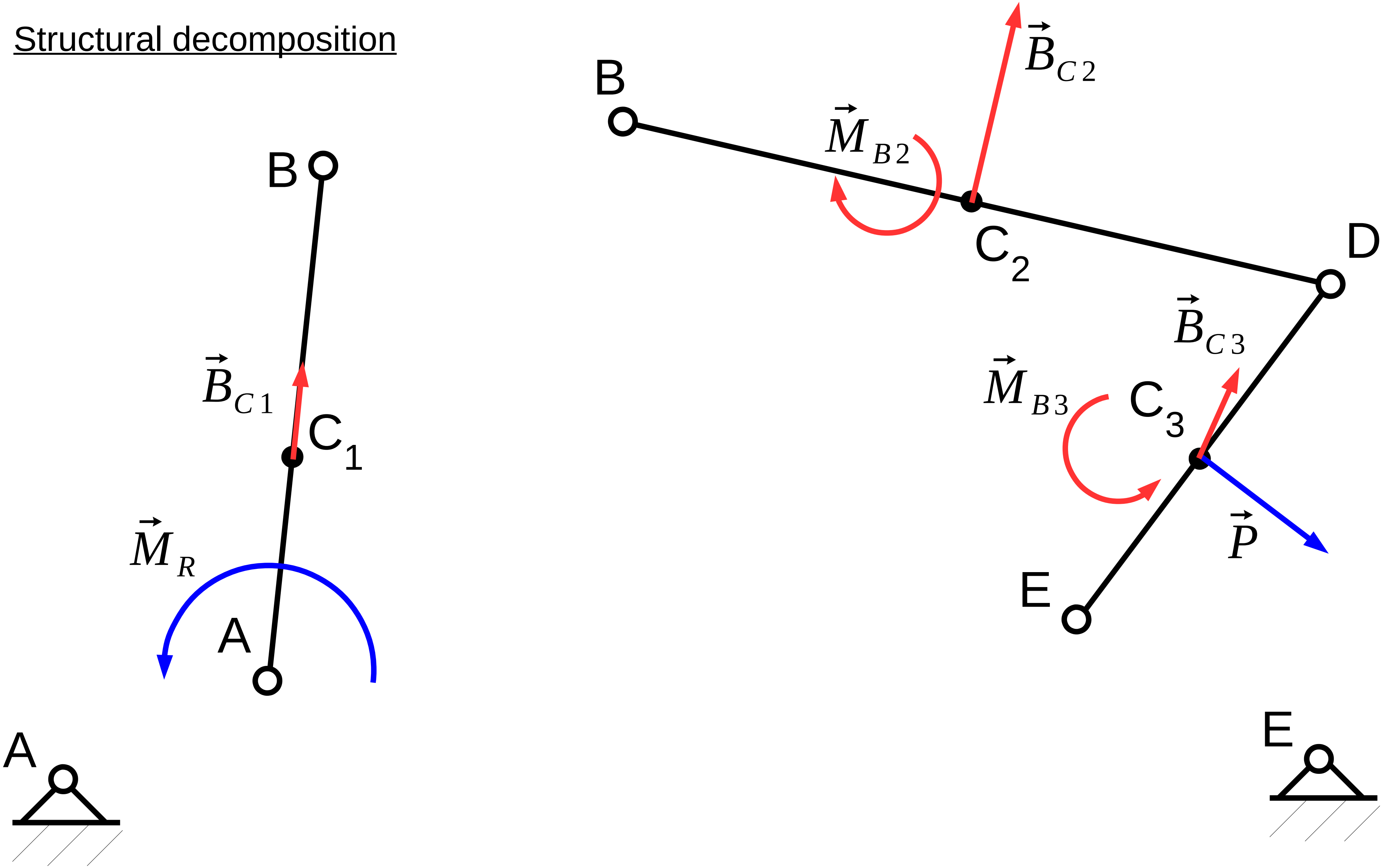
inertial forces and torques
+
external and operating forces and torques



Dynamics of planar mechanisms

Inverse dynamic problem – example

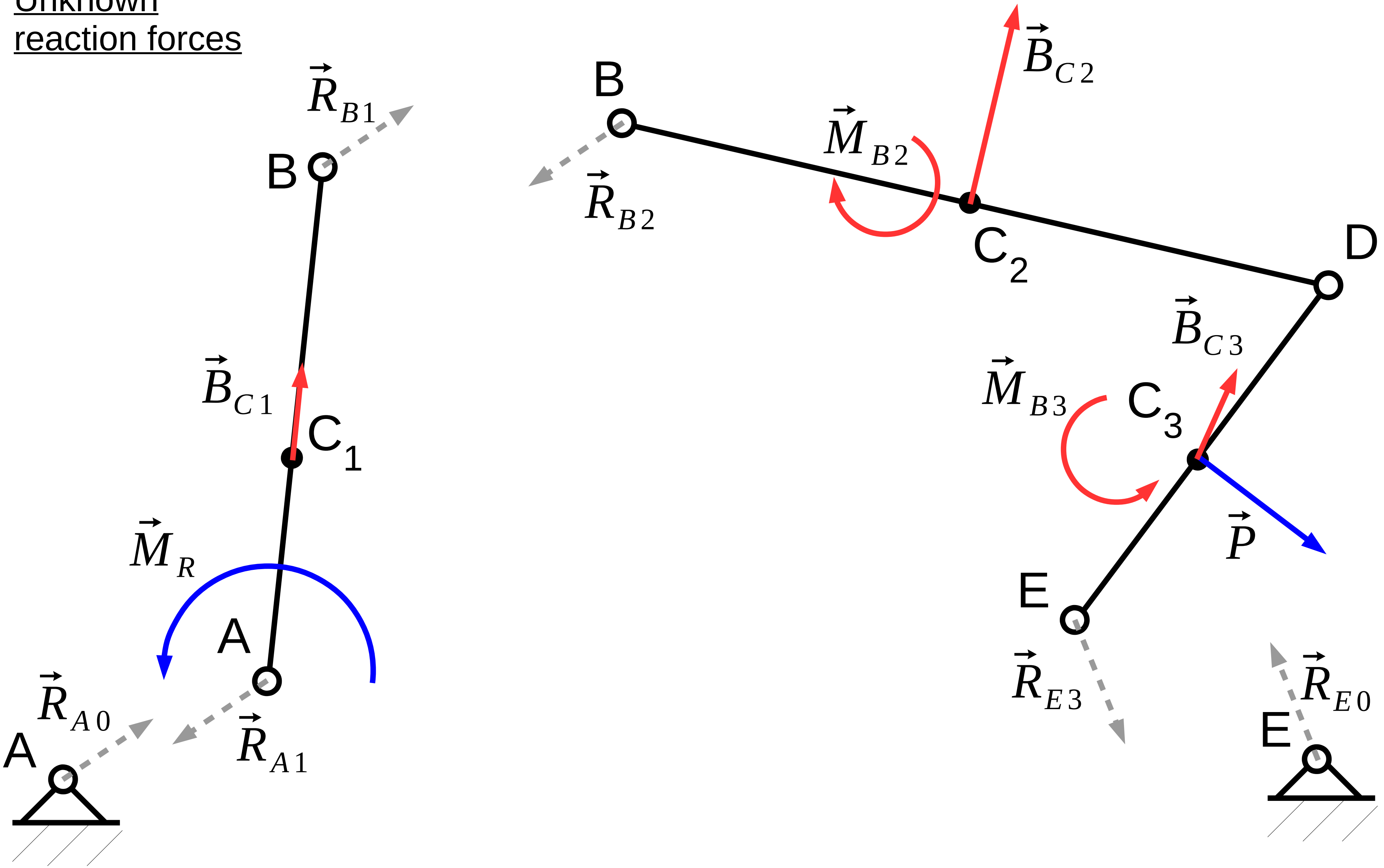
Structural decomposition



Dynamics of planar mechanisms

Inverse dynamic problem – example

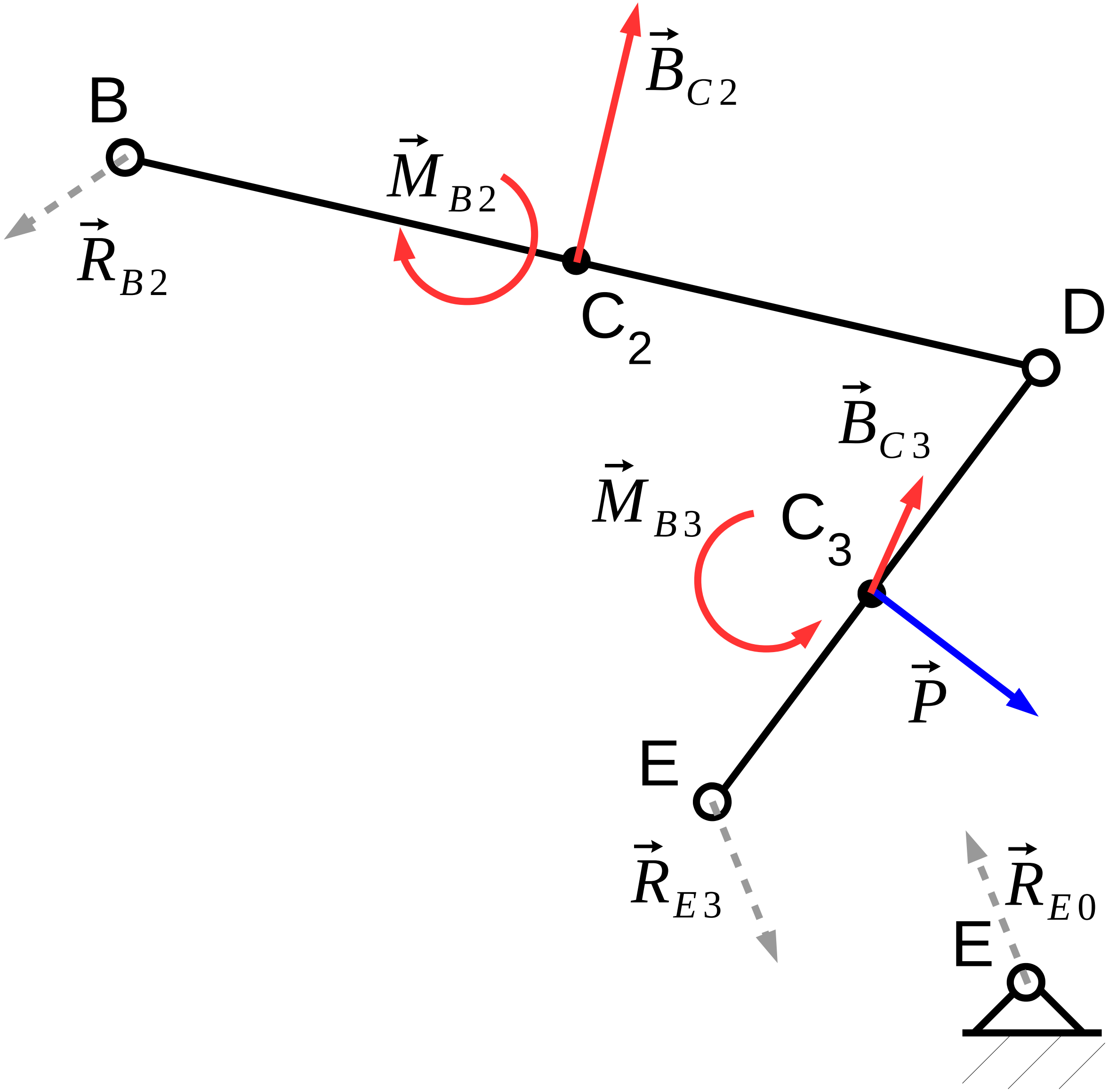
Unknown
reaction forces



Dynamics of planar mechanisms

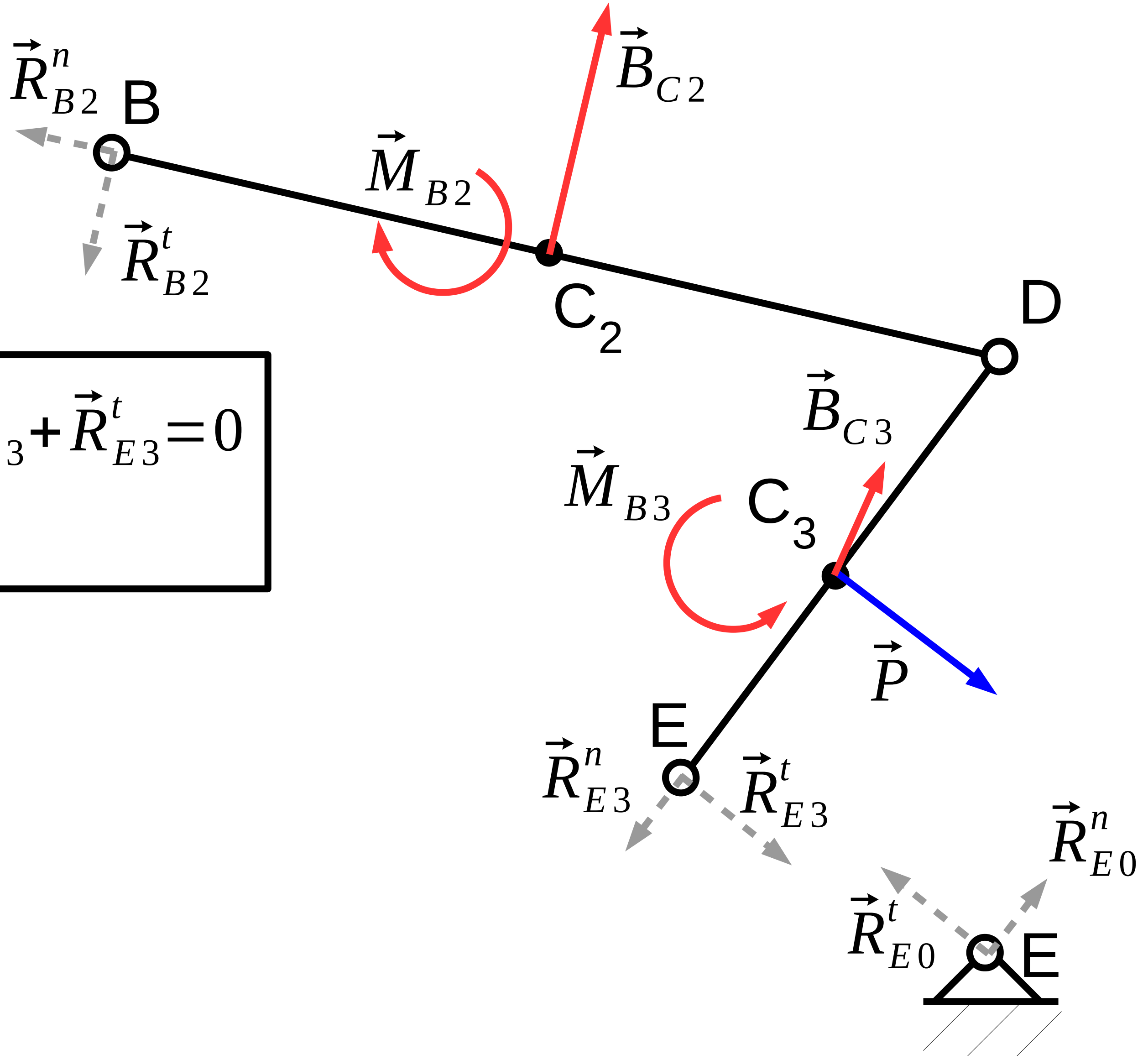
Inverse dynamic problem – example

$$\vec{R}_{B2}+\vec{B}_{C2}+\vec{B}_{C3}+\vec{P}+\vec{R}_{E3}=0$$



Dynamics of planar mechanisms

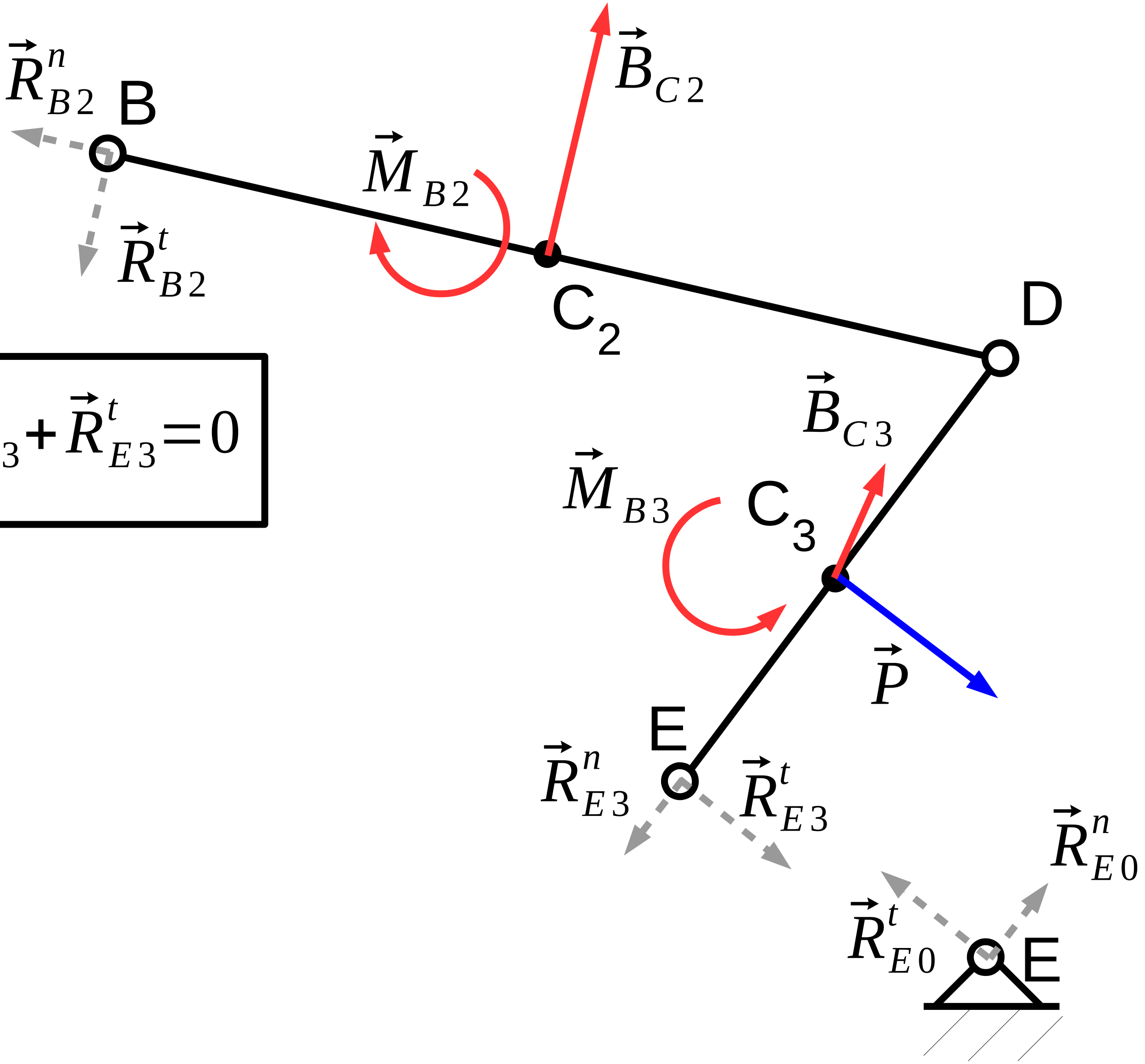
Inverse dynamic problem – example



$$\vec{R}^n_{B2}+\vec{R}^t_{B2}+\vec{B}_{C2}+\vec{B}_{C3}+\vec{P}+\vec{R}^n_{E3}+\vec{R}^t_{E3}=0$$

Dynamics of planar mechanisms

Inverse dynamic problem – example



$$\boxed{\vec{R}^n_{B2}+\vec{R}^t_{B2}+\vec{B}_{C2}+\vec{B}_{C3}+\vec{P}+\vec{R}^n_{E3}+\vec{R}^t_{E3}=0}$$

$$part\ BD:\ \sum M_D: \ldots.$$

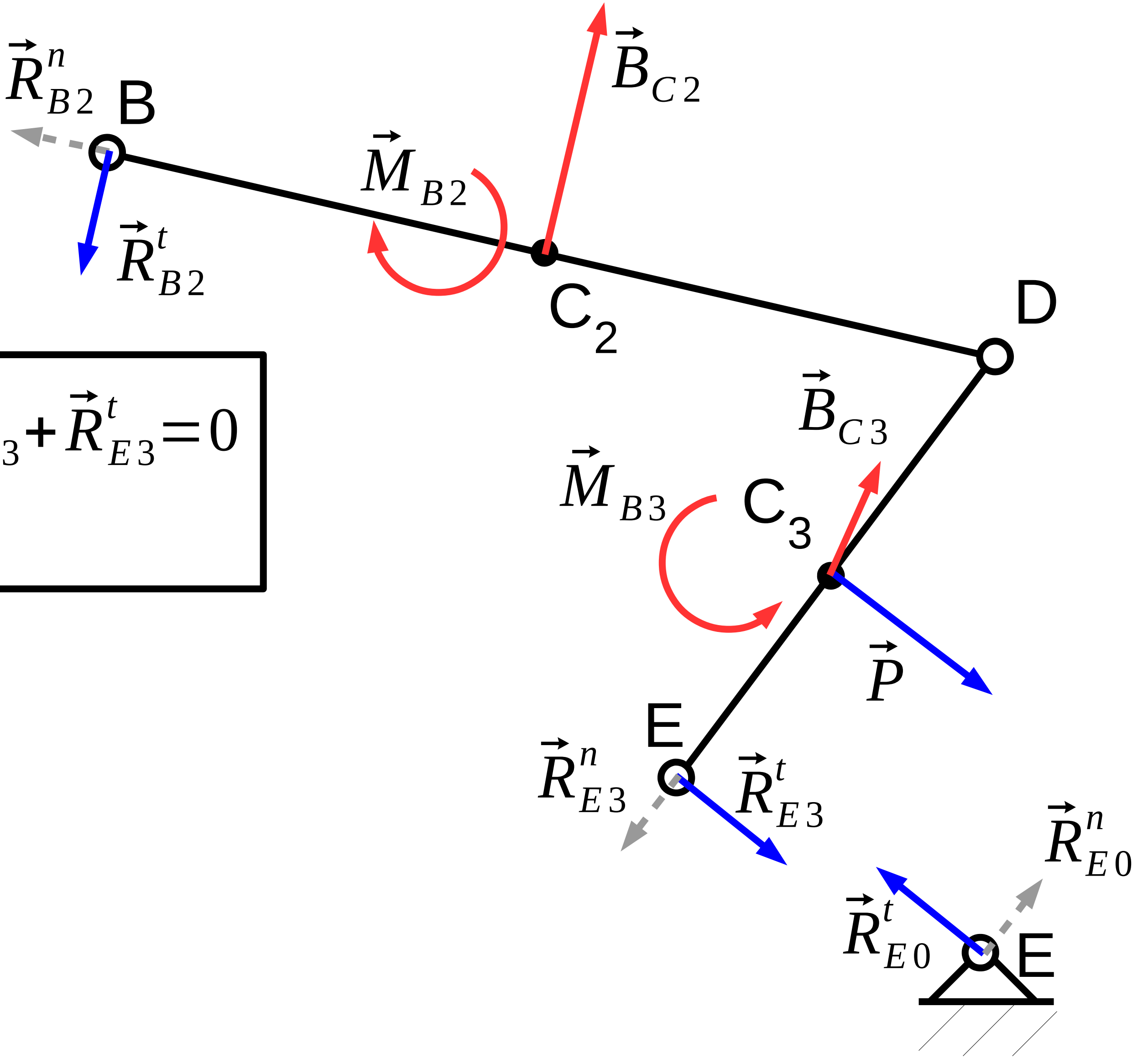
$$\vec{R}^t_{B2}=\ldots$$

$$part\ ED:\ \sum M_D: \ldots.$$

$$\vec{R}^t_{E3}=\ldots$$

Dynamics of planar mechanisms

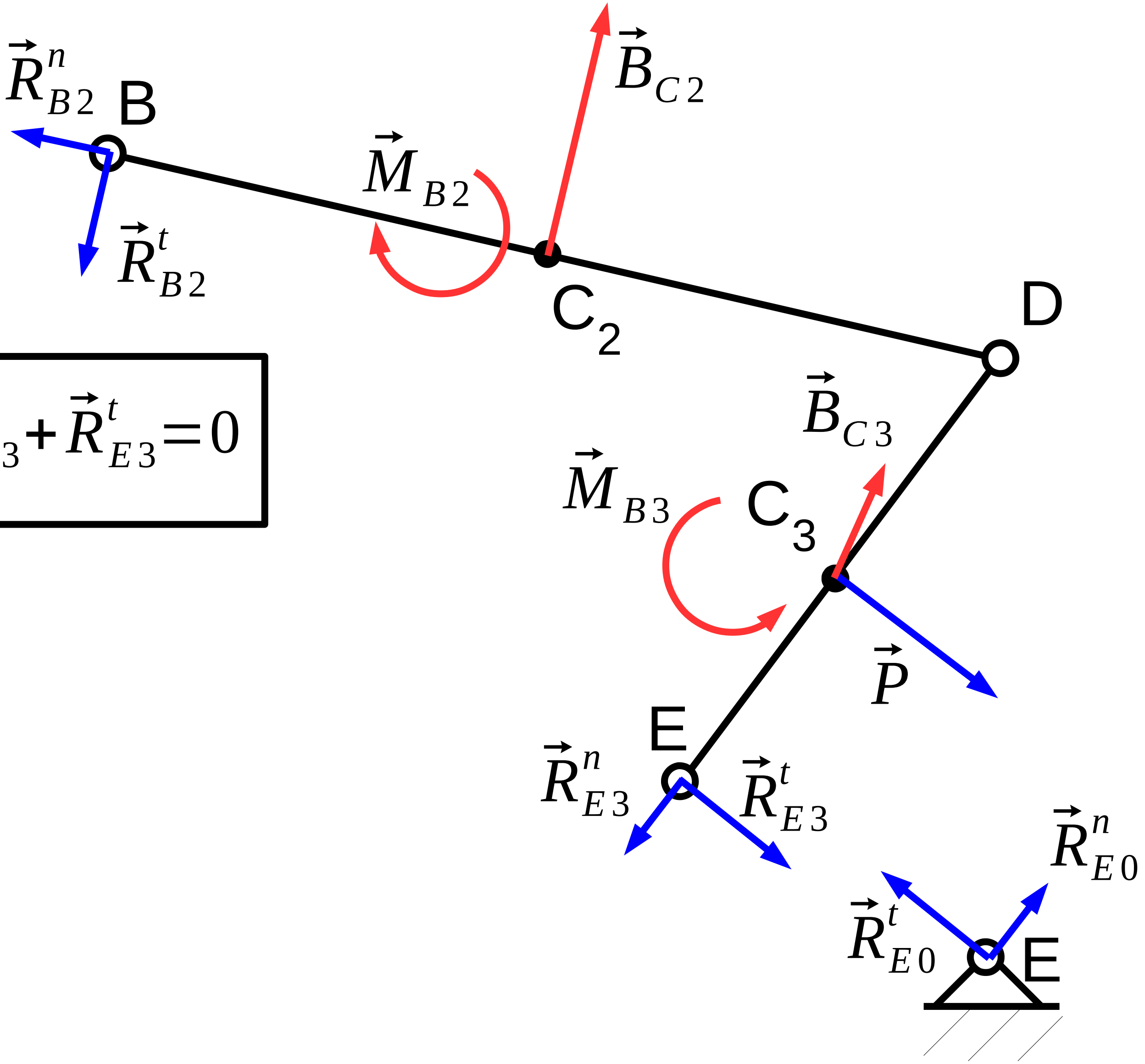
Inverse dynamic problem – example



$$\vec{R}^n_{B2}+\vec{R}^t_{B2}+\vec{B}_{C2}+\vec{B}_{C3}+\vec{P}+\vec{R}^n_{E3}+\vec{R}^t_{E3}=0$$

Dynamics of planar mechanisms

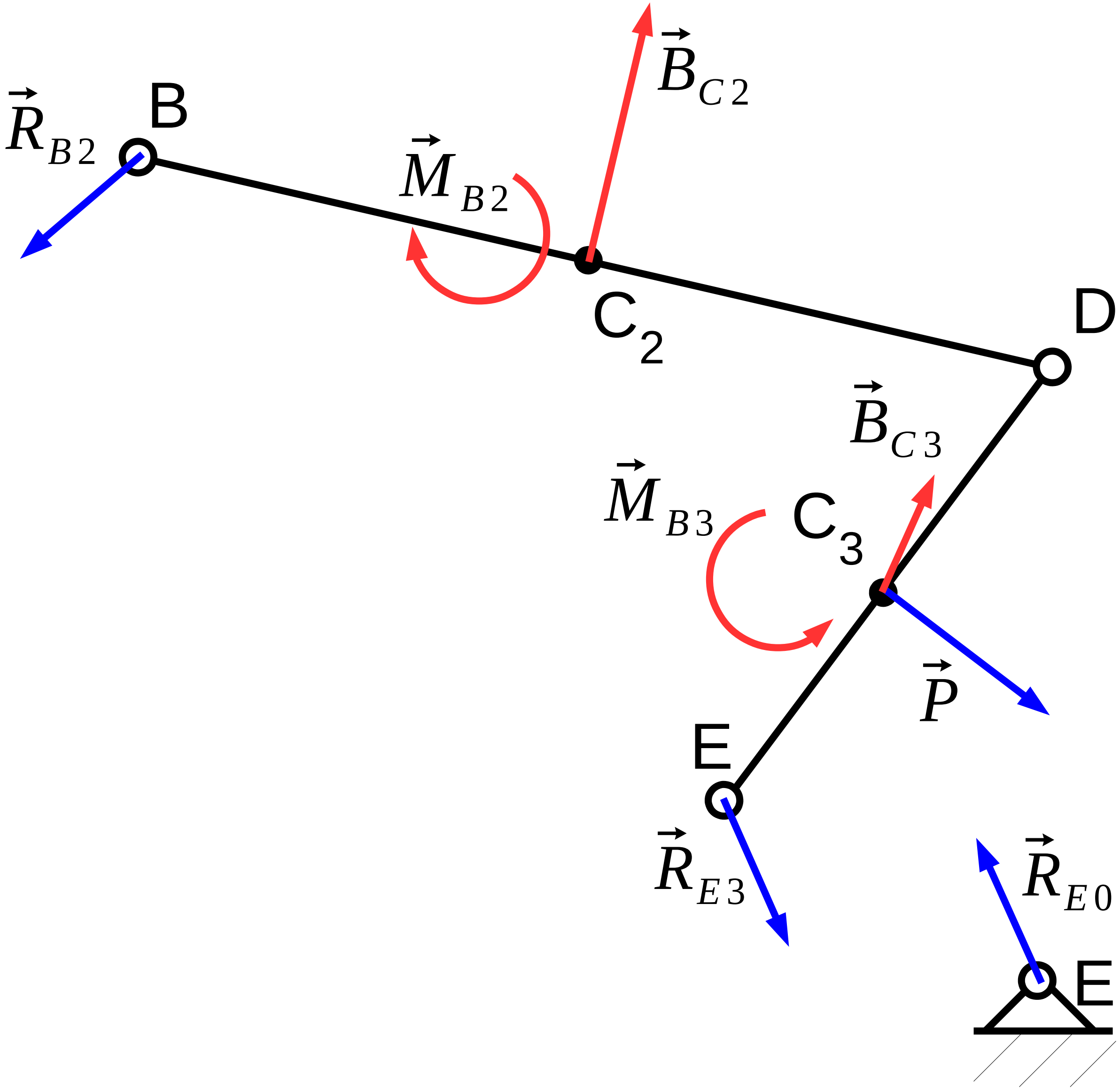
Inverse dynamic problem – example



$$\boxed{\vec{R}_{B2}^n+\vec{R}_{B2}^t+\vec{B}_{C2}+\vec{B}_{C3}+\vec{P}+\vec{R}_{E3}^n+\vec{R}_{E3}^t=0}$$

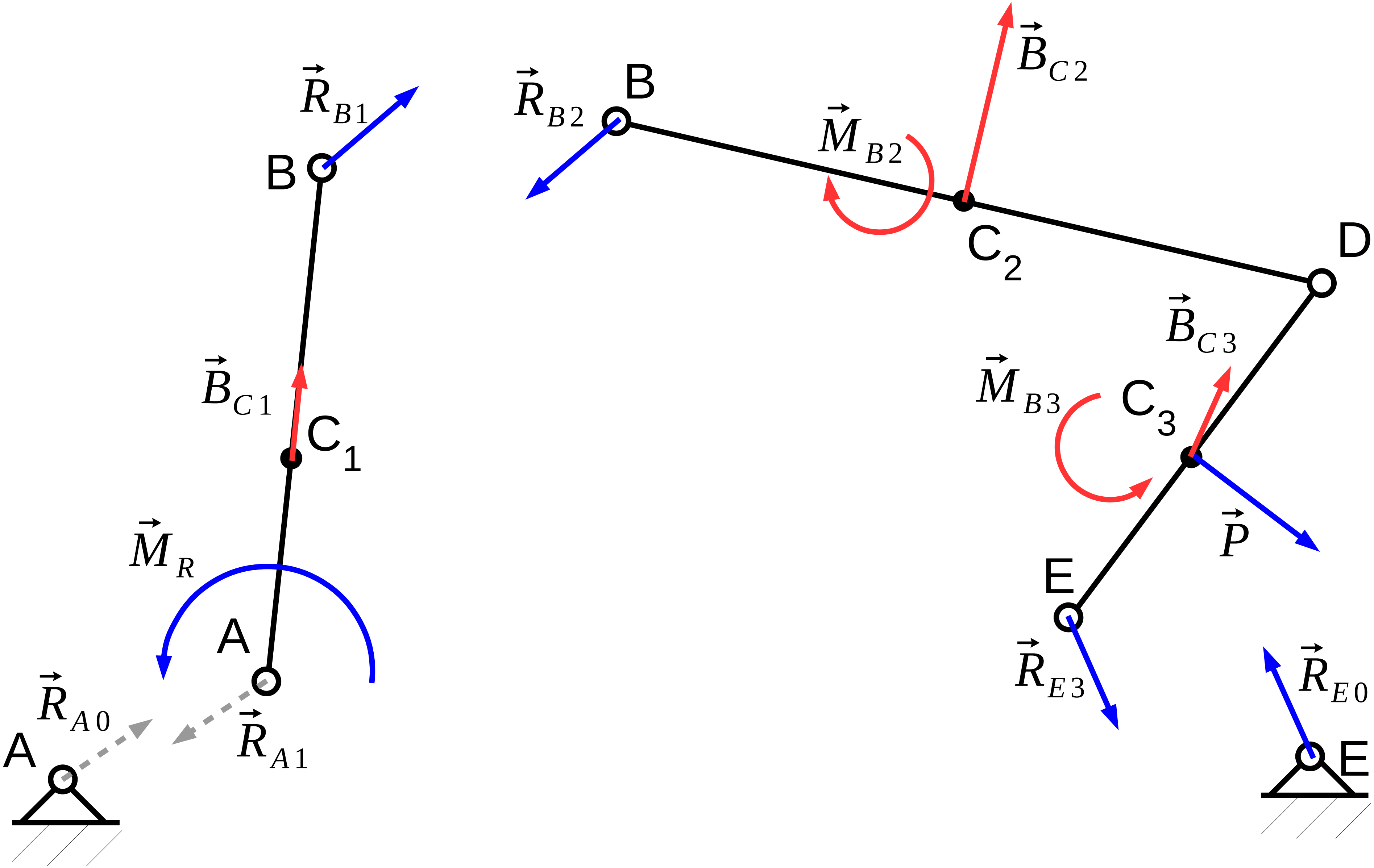
Dynamics of planar mechanisms

Inverse dynamic problem – example



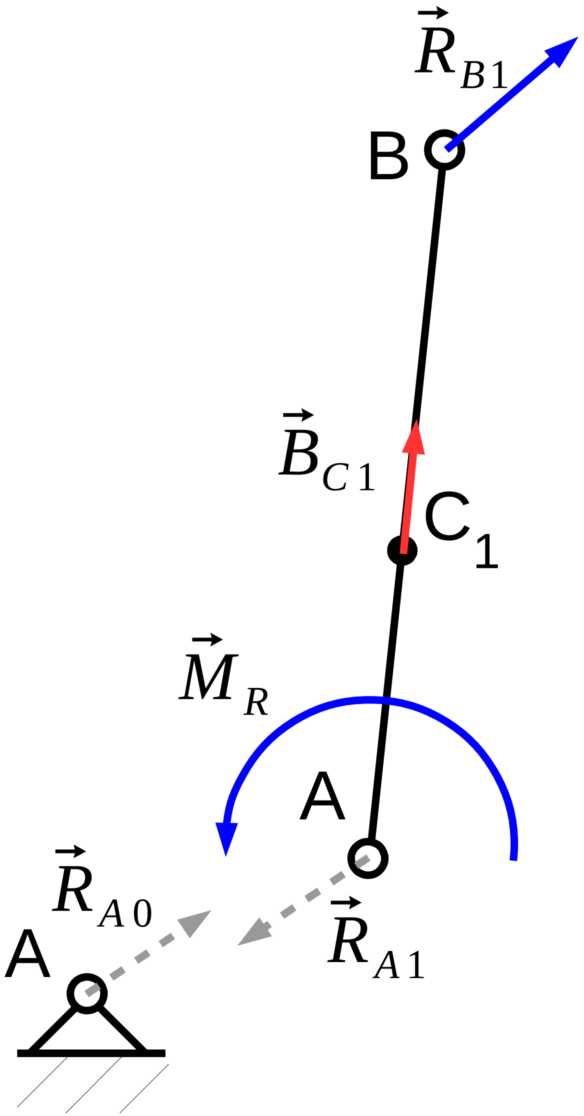
Dynamics of planar mechanisms

Inverse dynamic problem – example



Dynamics of planar mechanisms

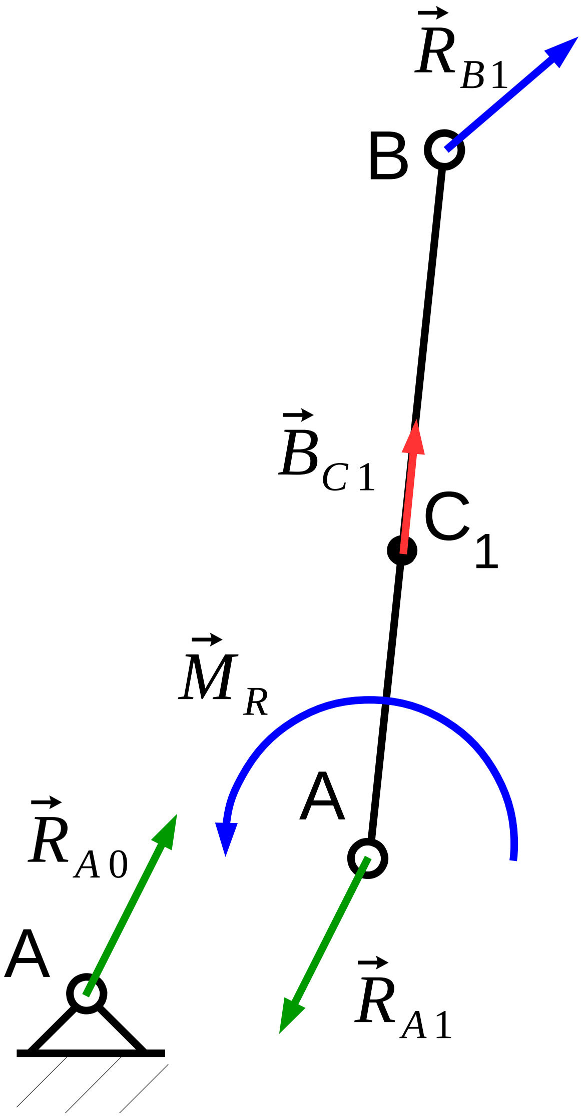
Inverse dynamic problem – example



$$part\ AB:\ \vec{R}_{A1}+\vec{B}_{C1}+\vec{R}_{B1}=0$$

Dynamics of planar mechanisms

Inverse dynamic problem – example



$$\sum M_A : \ldots.$$

$$M_R = \ldots$$