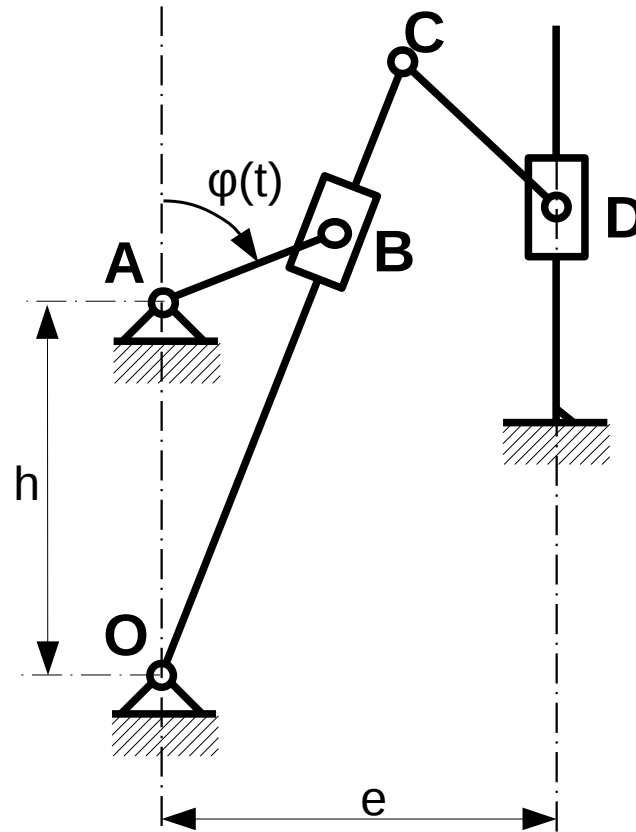


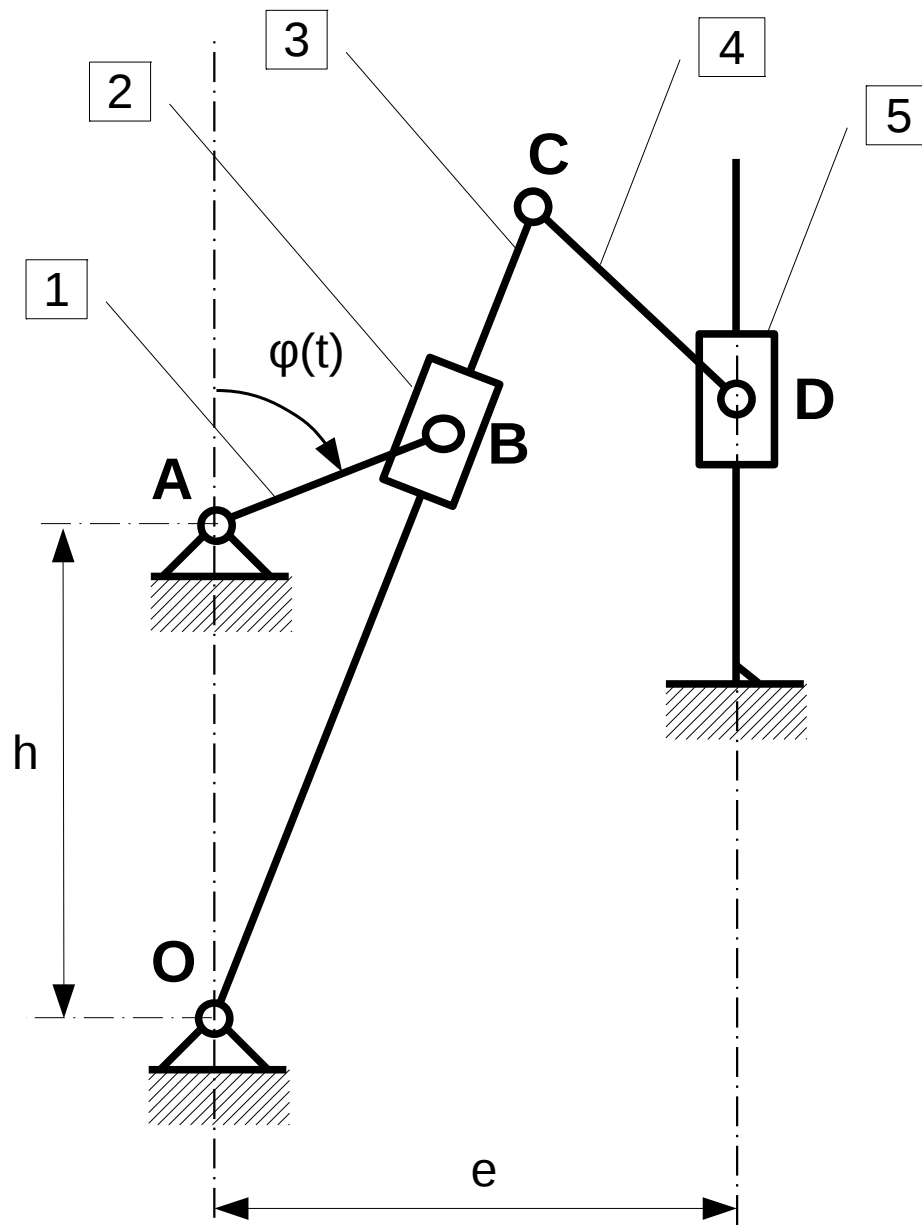
# TM&AC - Winter 2017/2018

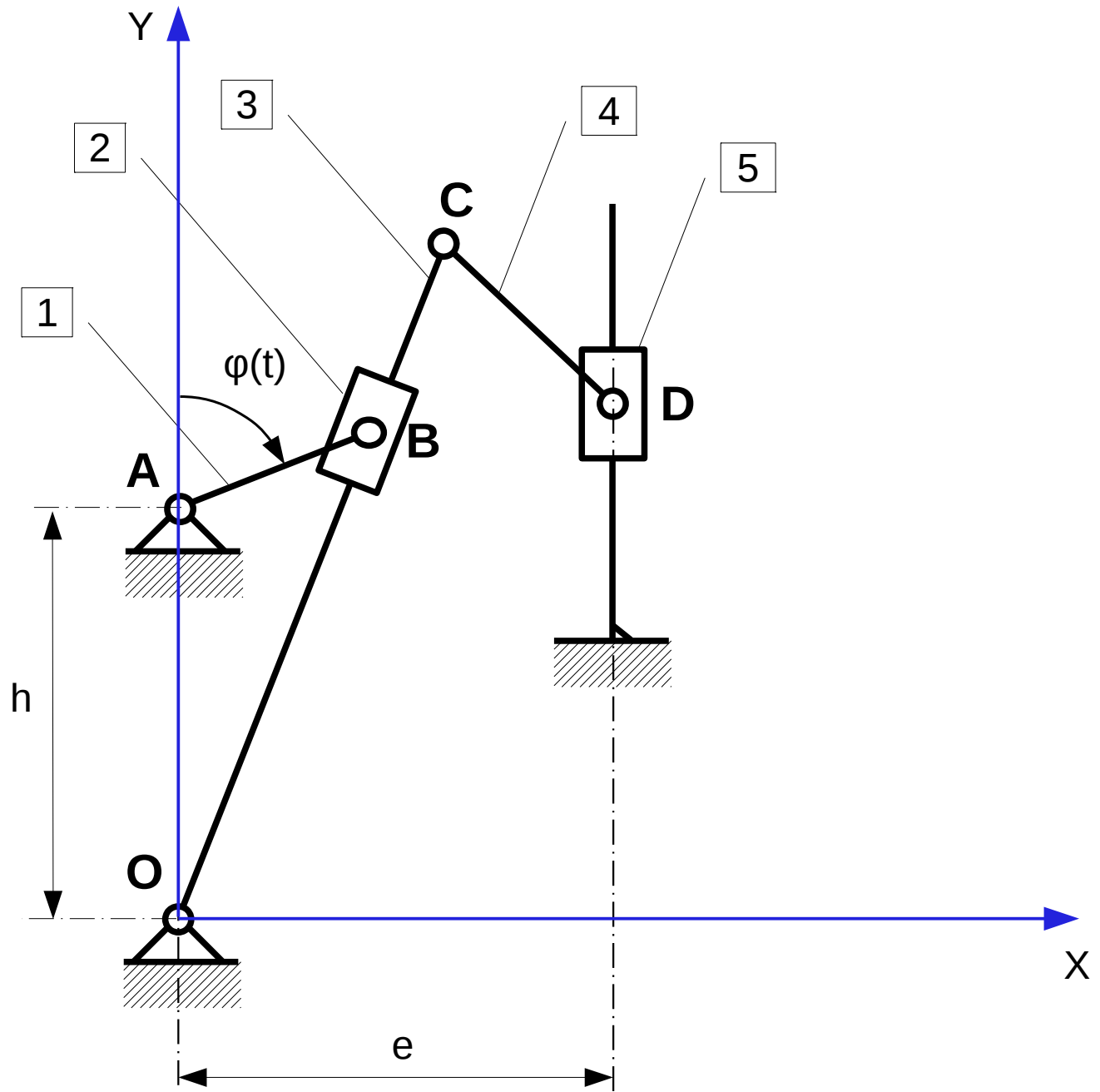
## velocities and accelerations in planar mechanisms

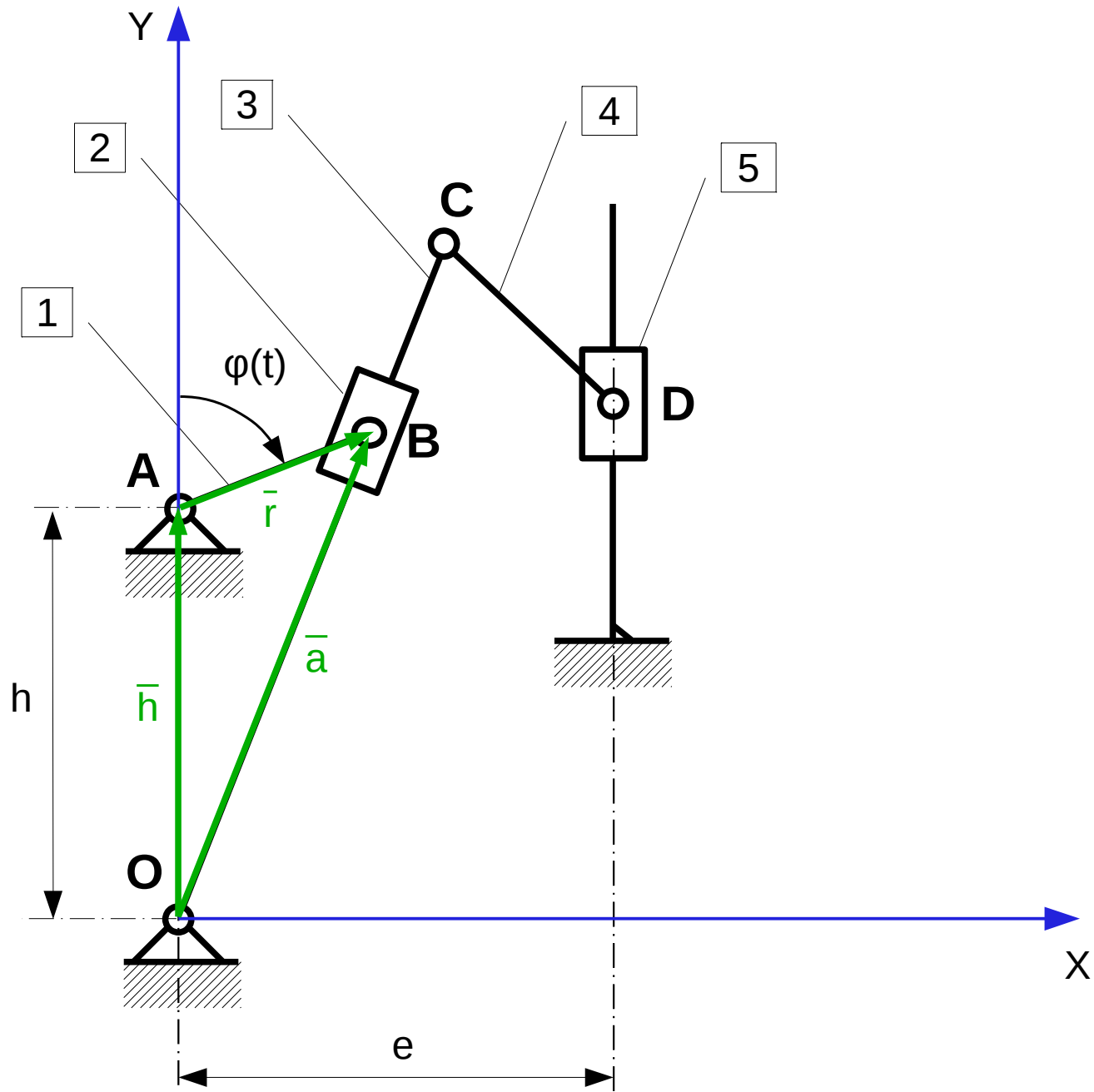
### EXAMPLE

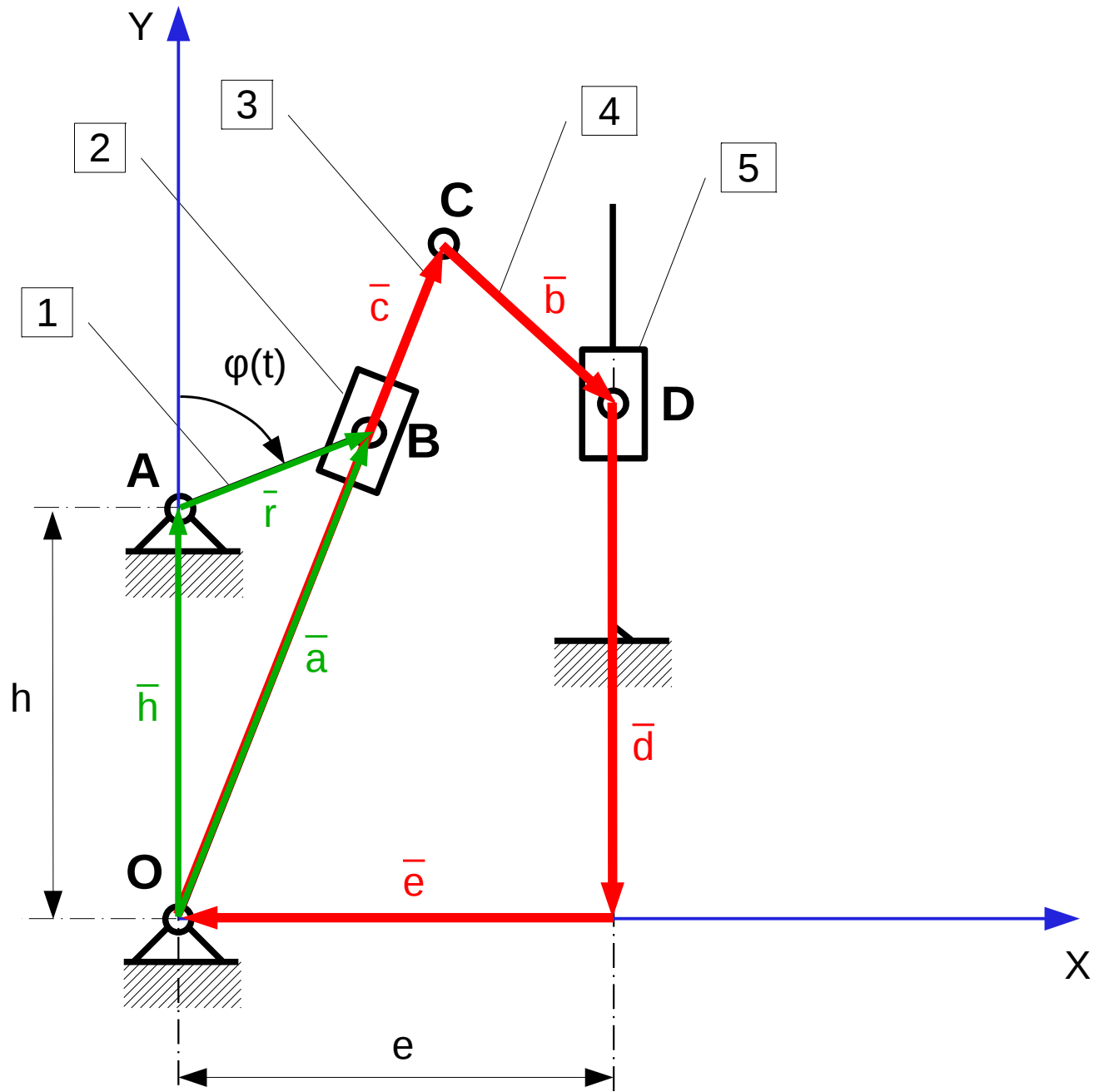
Given: mechanism geometry and angular displacement  $\varphi(t)$  of driven element.

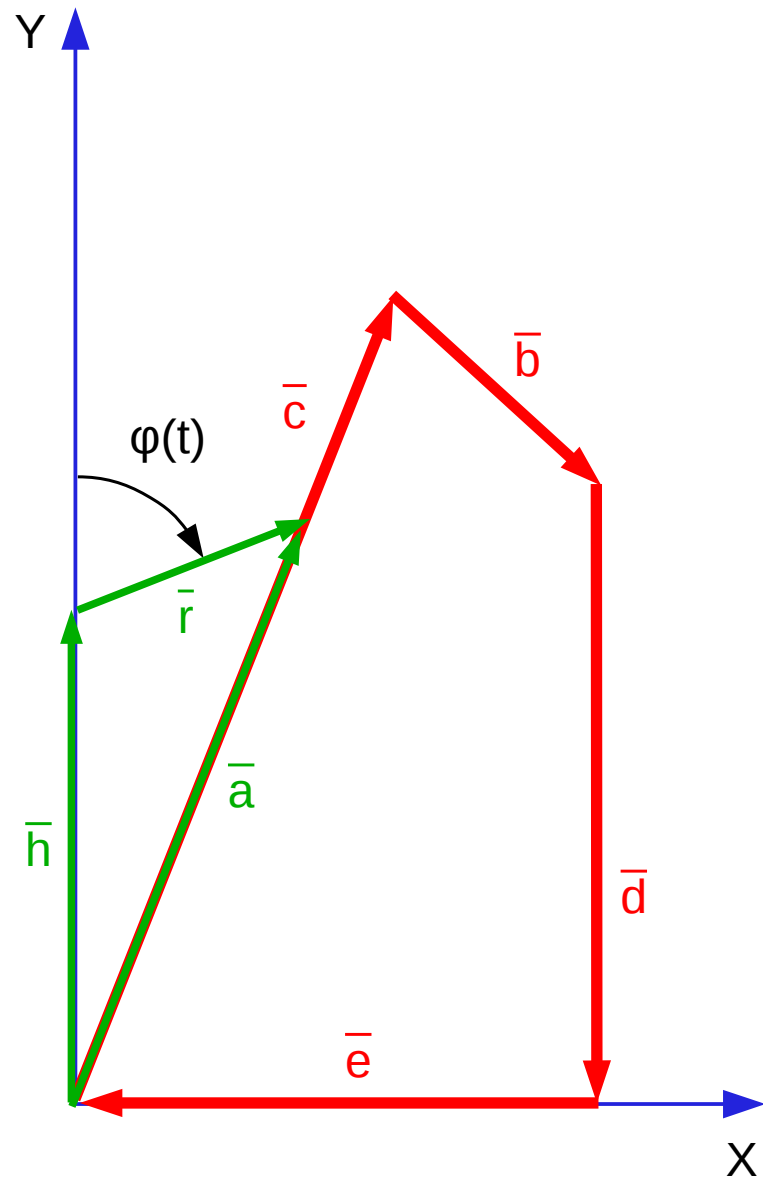
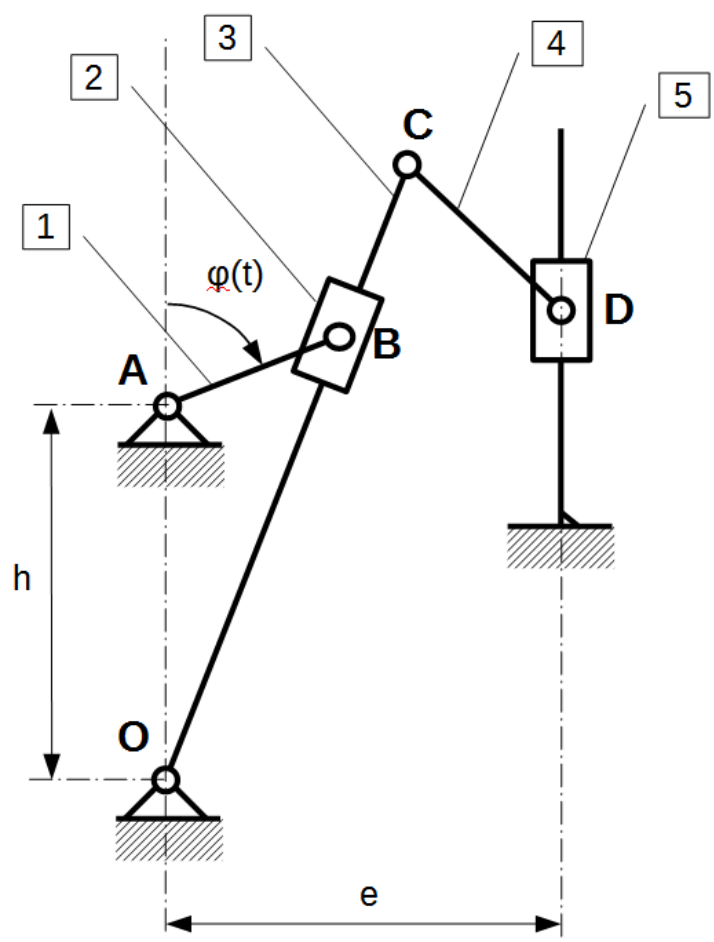


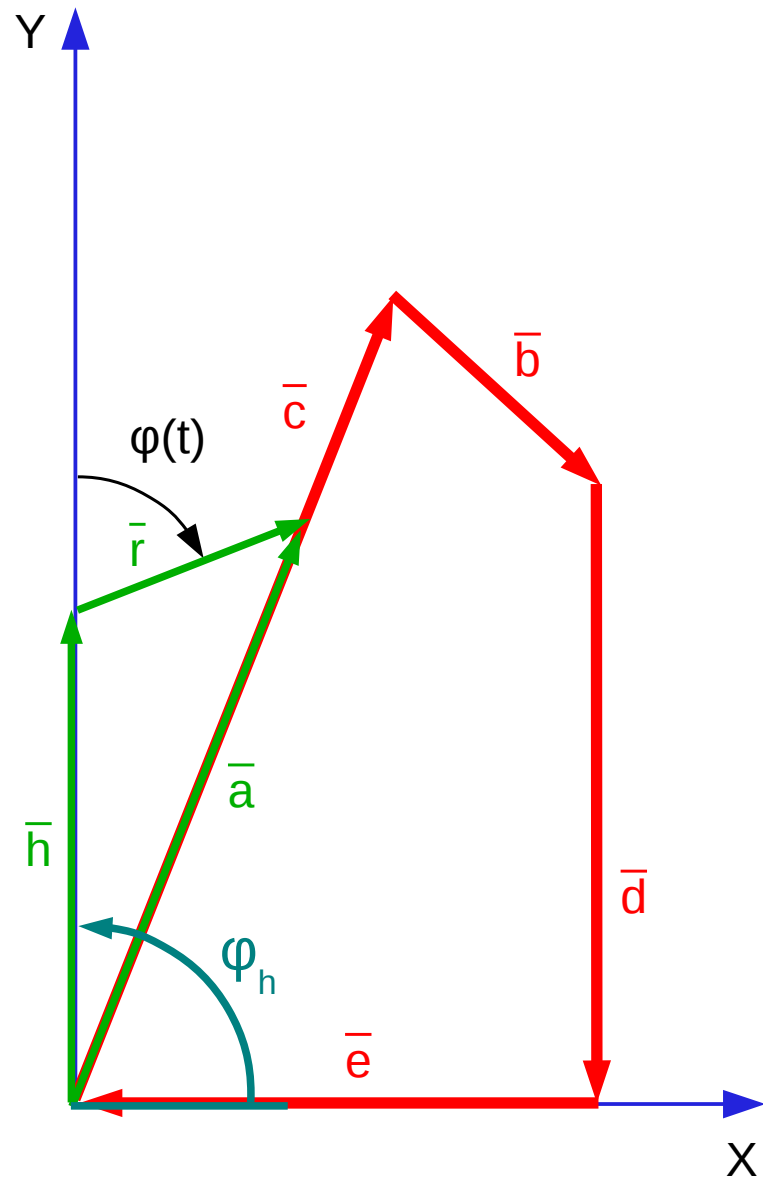
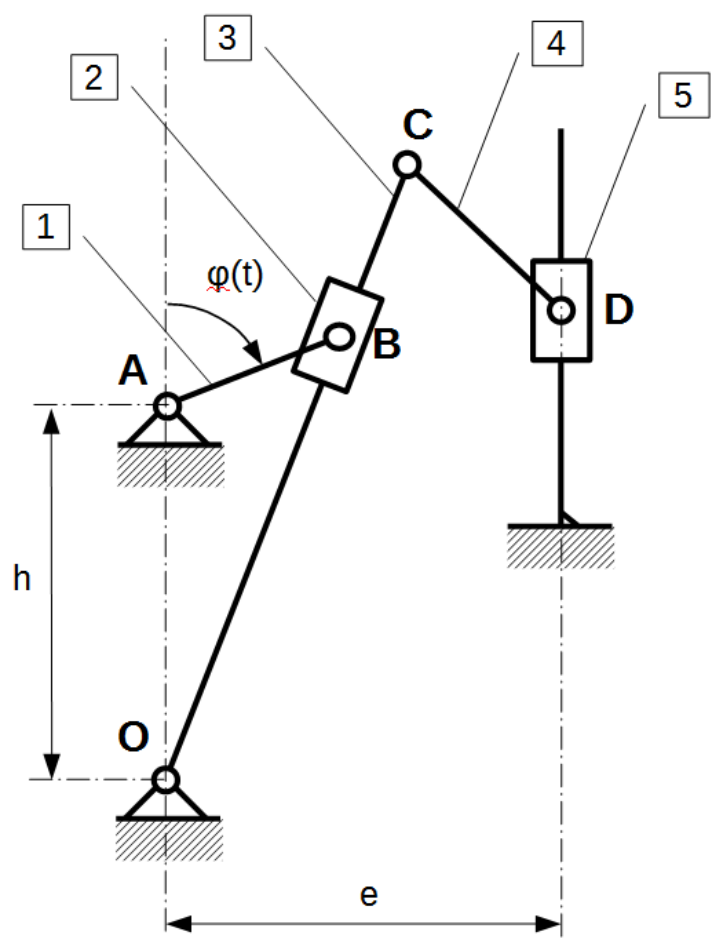


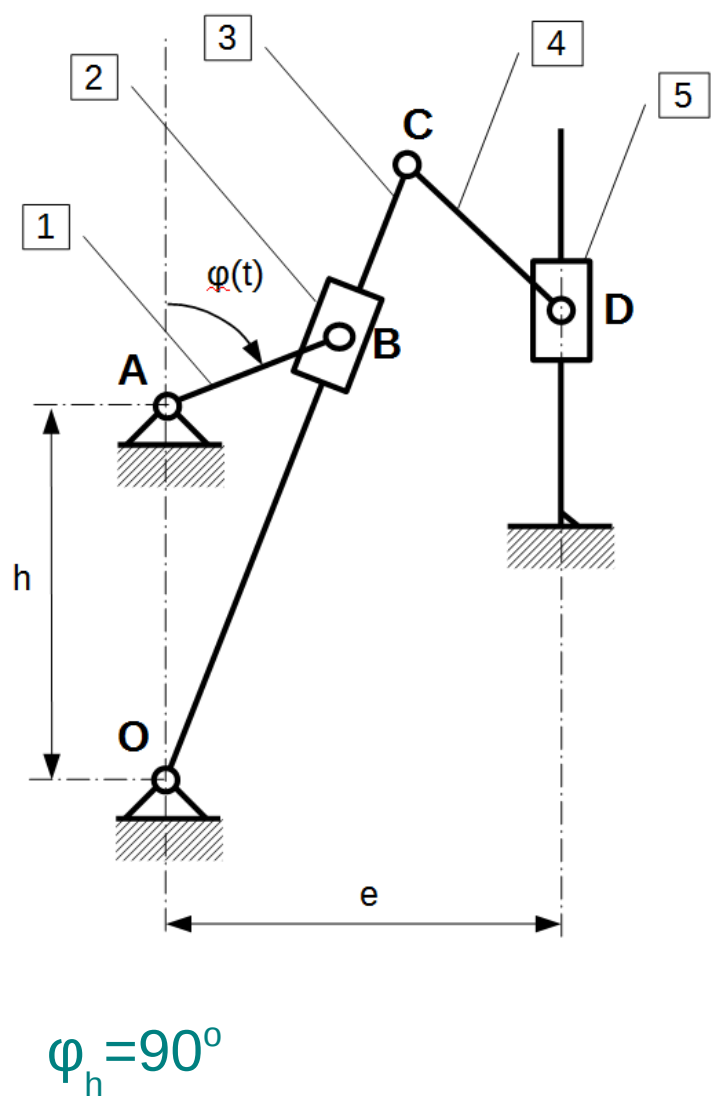




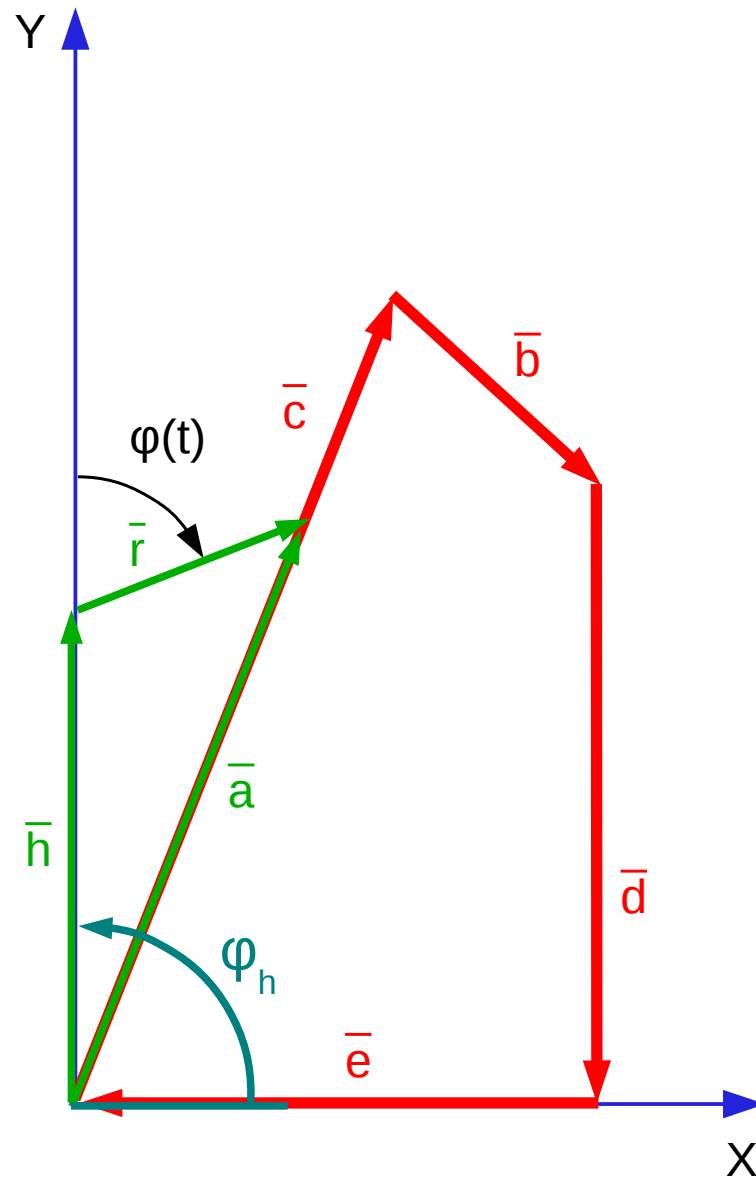




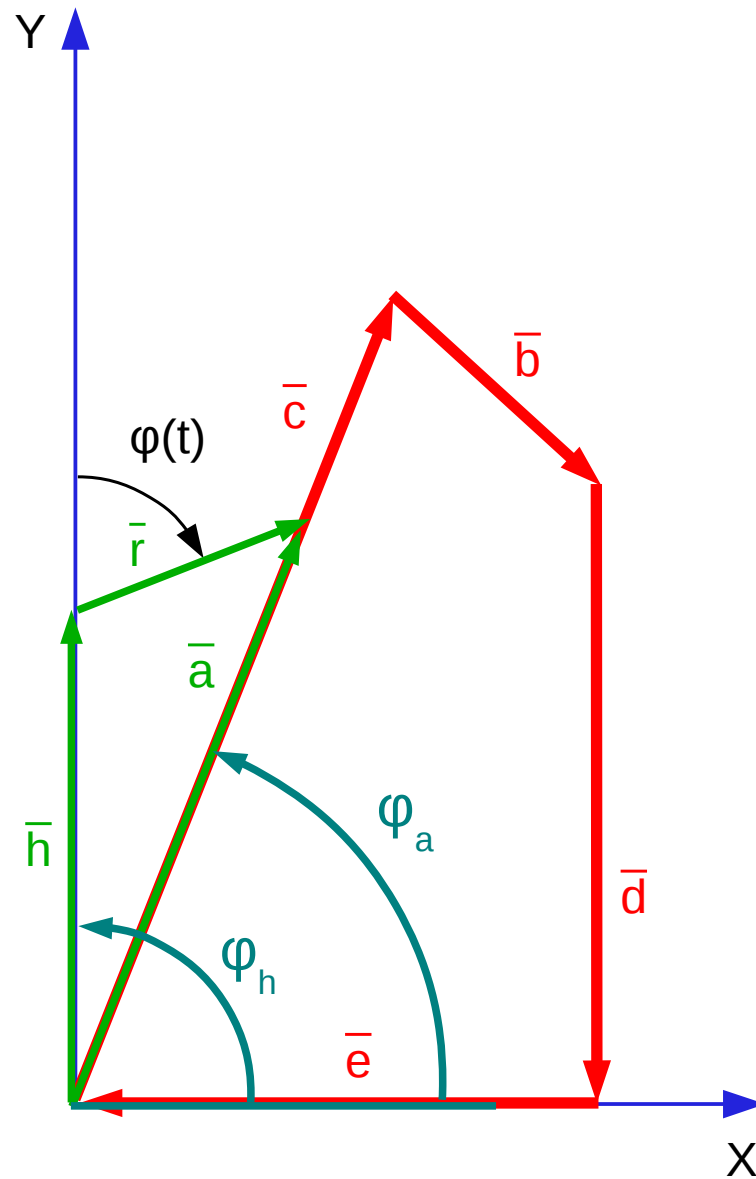
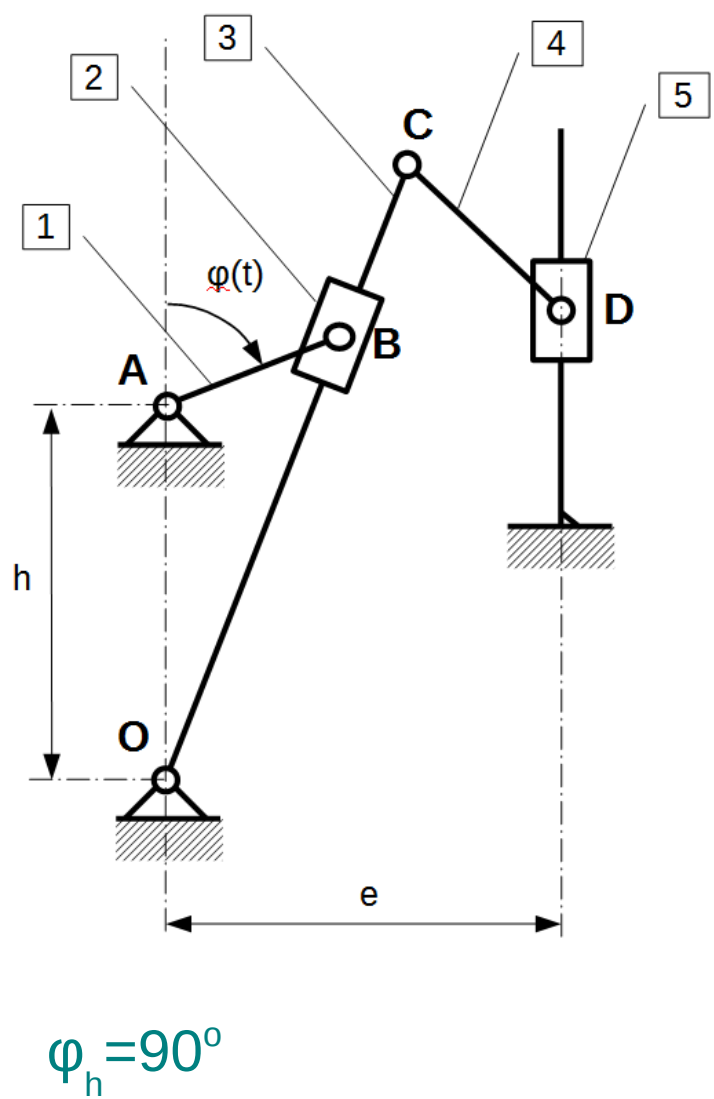


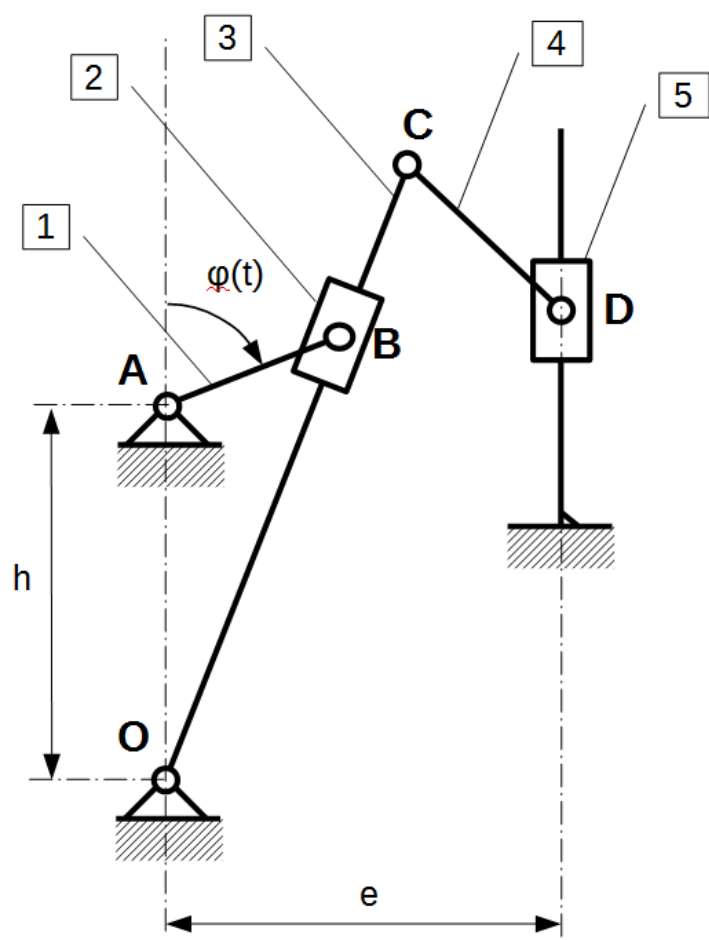


$\varphi_h = 90^\circ$

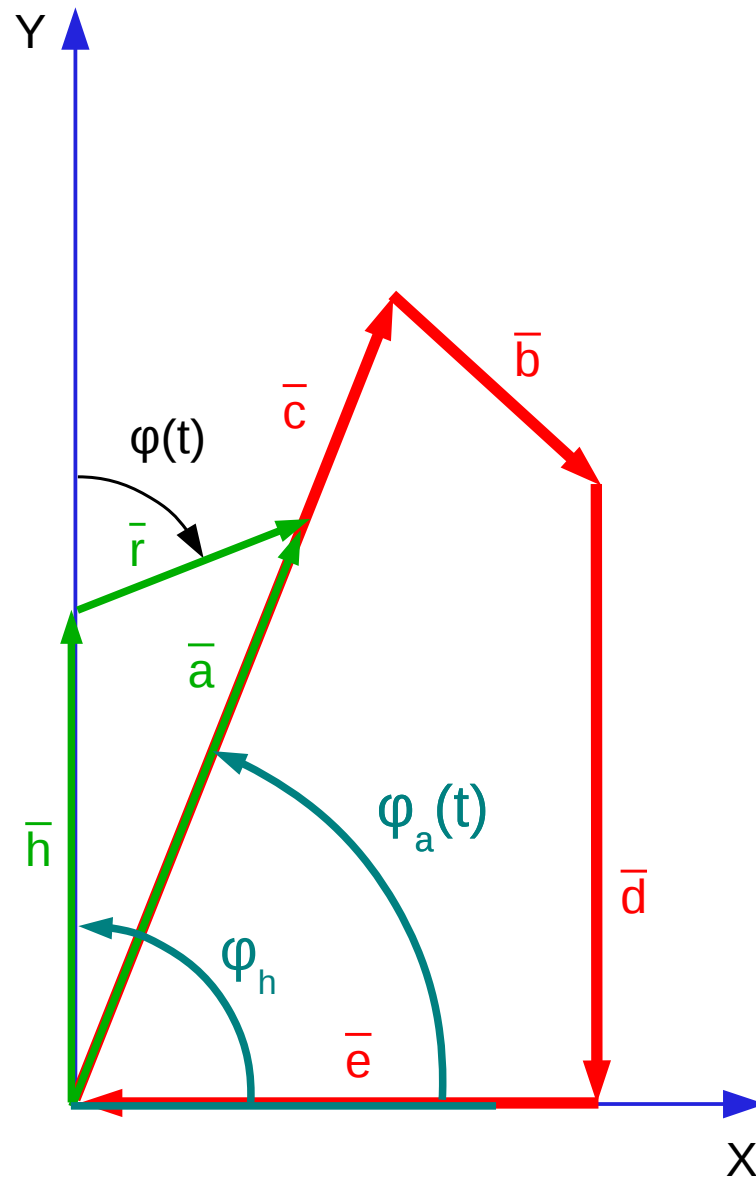


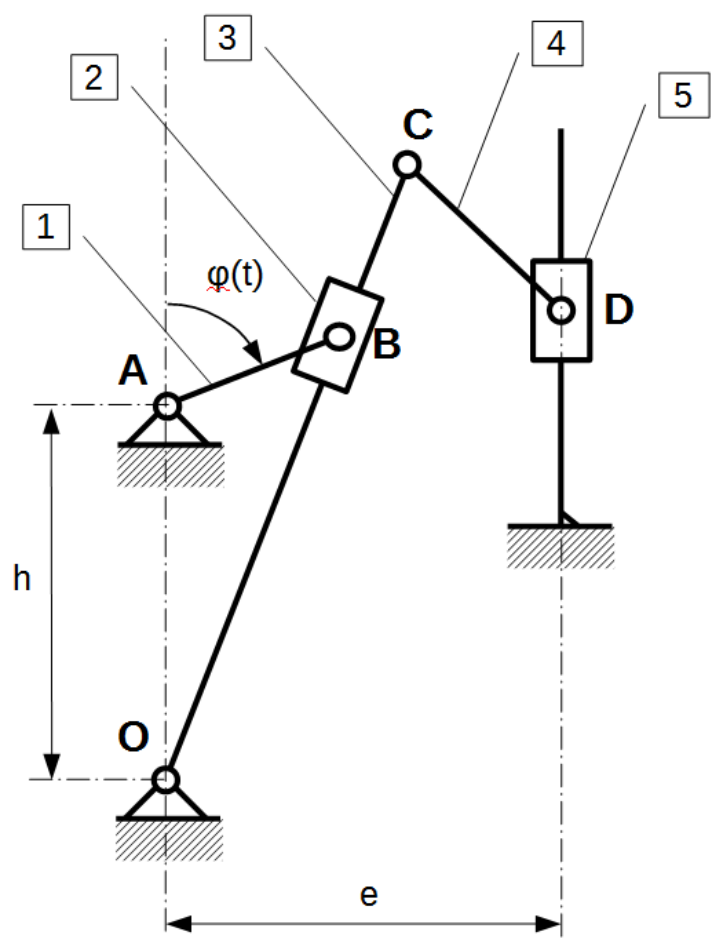




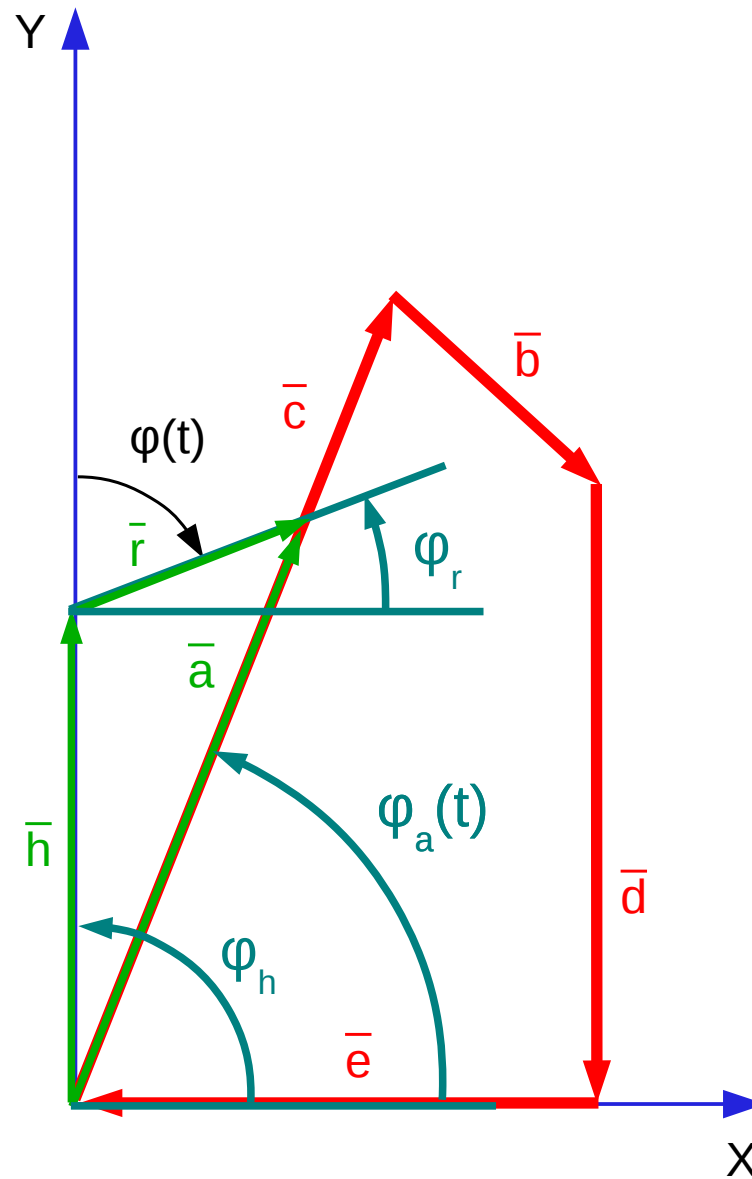


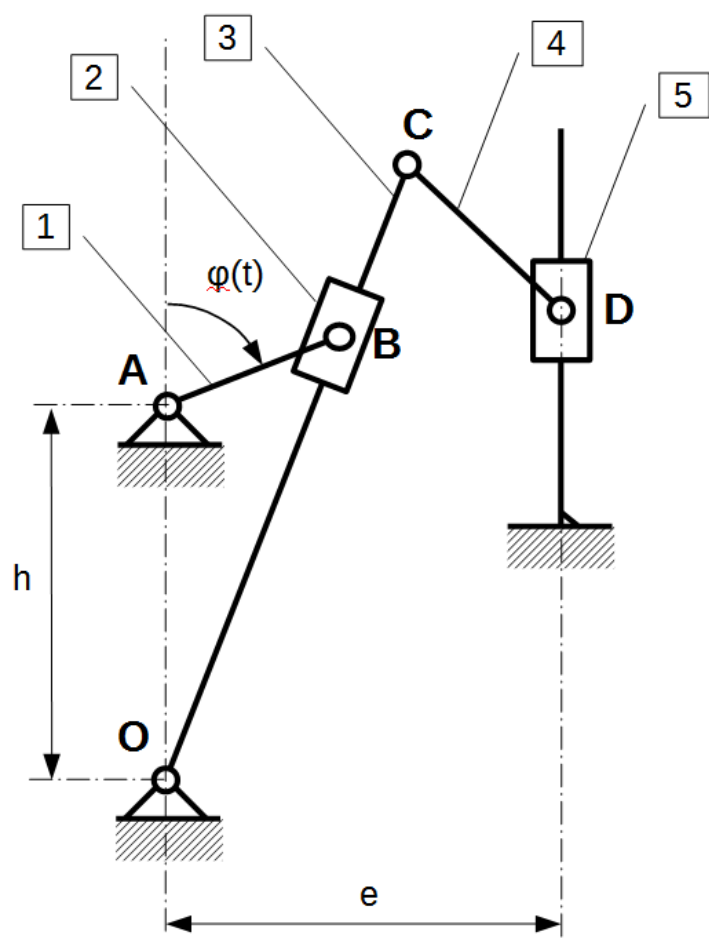
$\varphi_h = 90^\circ$   
 $\varphi_a(t) \neq \text{const.}$



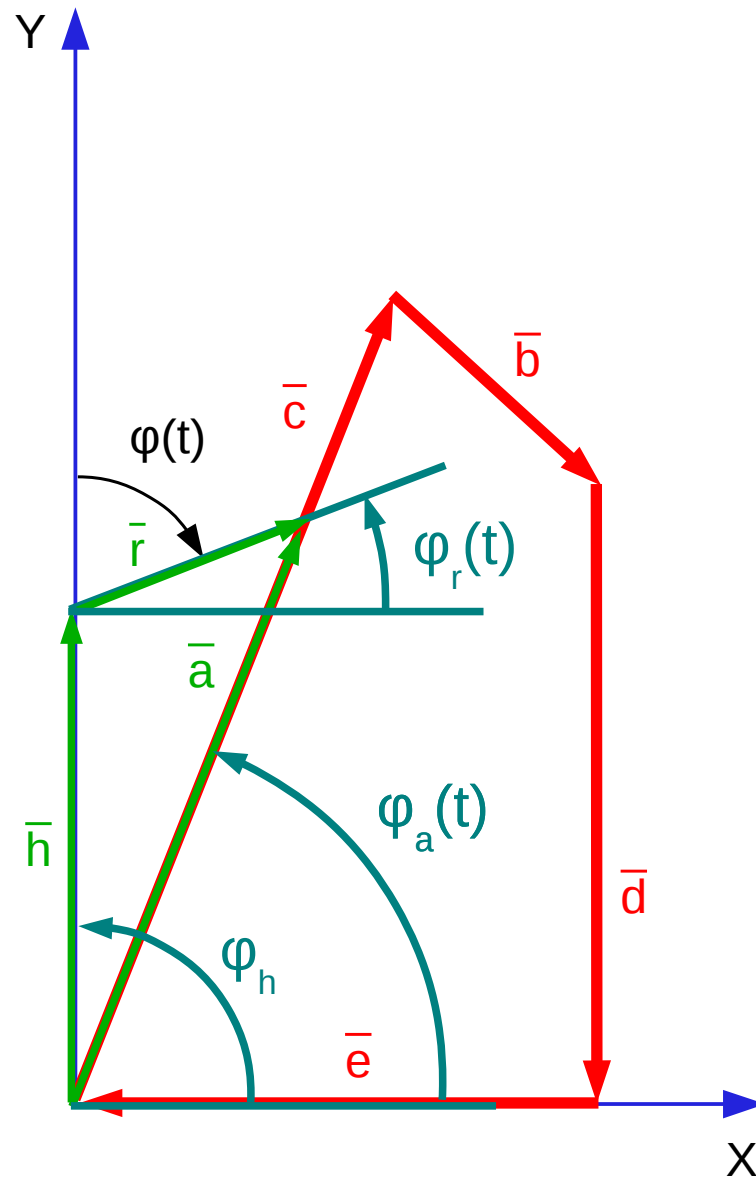


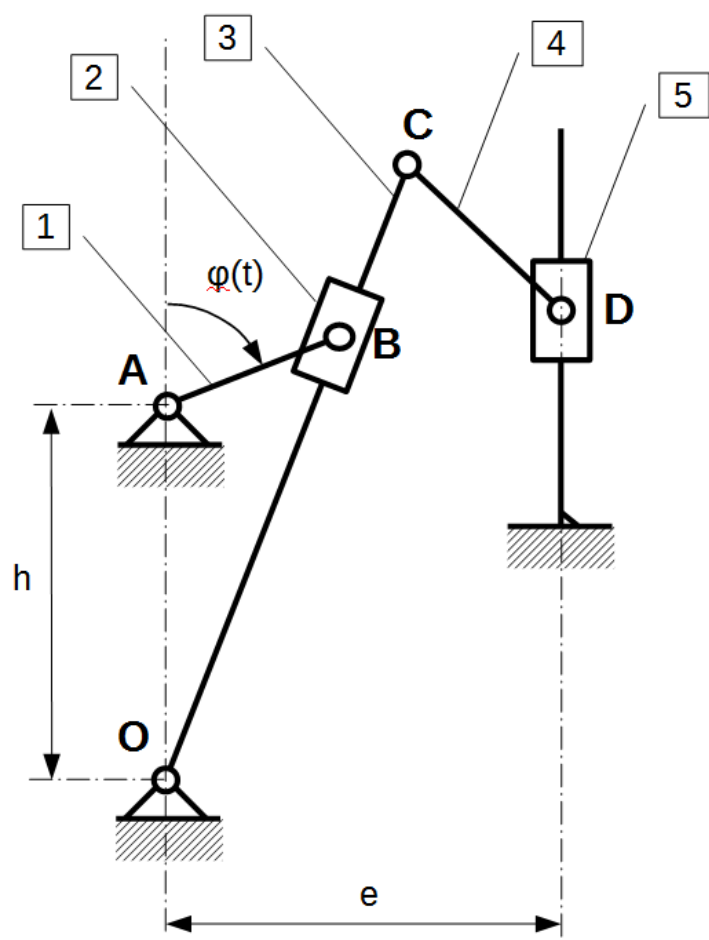
$\varphi_h = 90^\circ$   
 $\varphi_a(t) \neq \text{const.}$



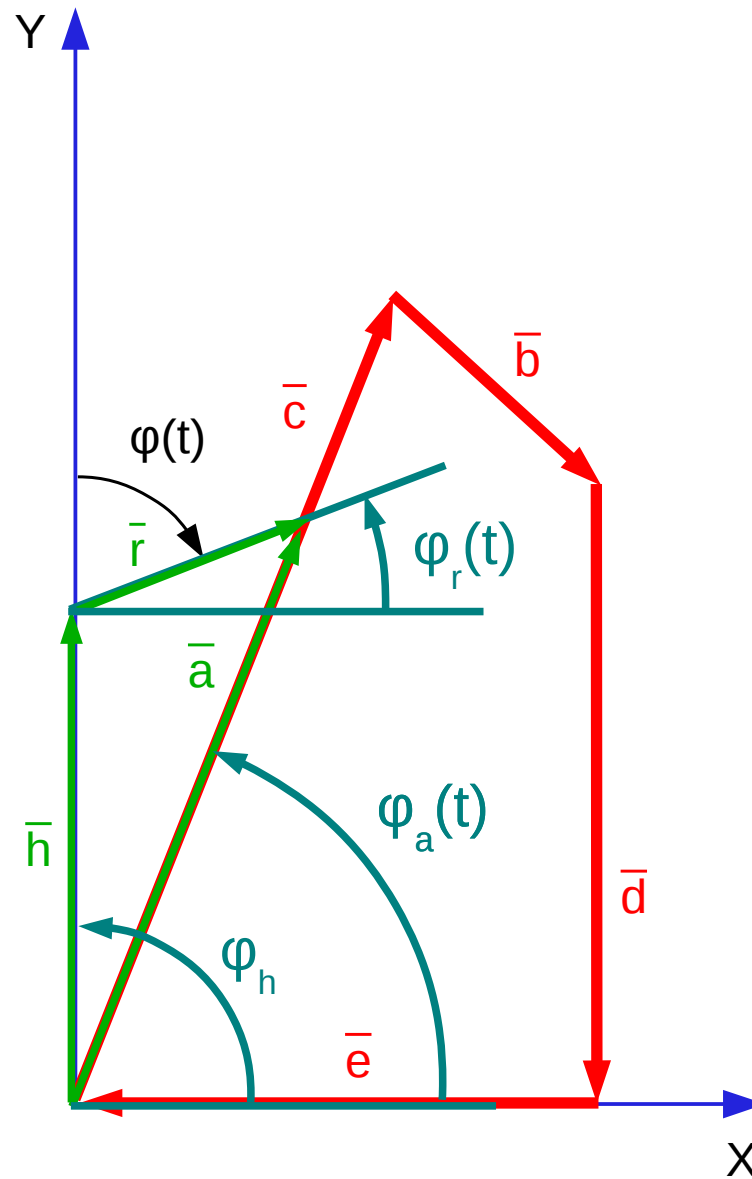


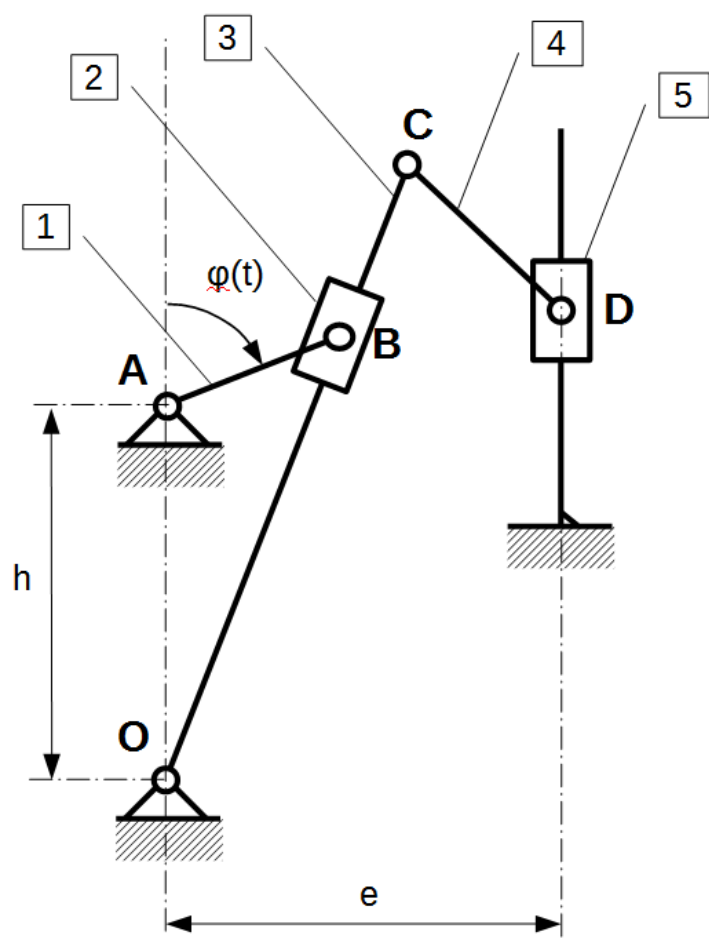
$\varphi_h = 90^\circ$   
 $\varphi_a(t) \neq \text{const.}$





$\varphi_h = 90^\circ$   
 $\varphi_a(t) \neq \text{const.}$   
 $\varphi_r(t) = 90^\circ - \varphi(t)$

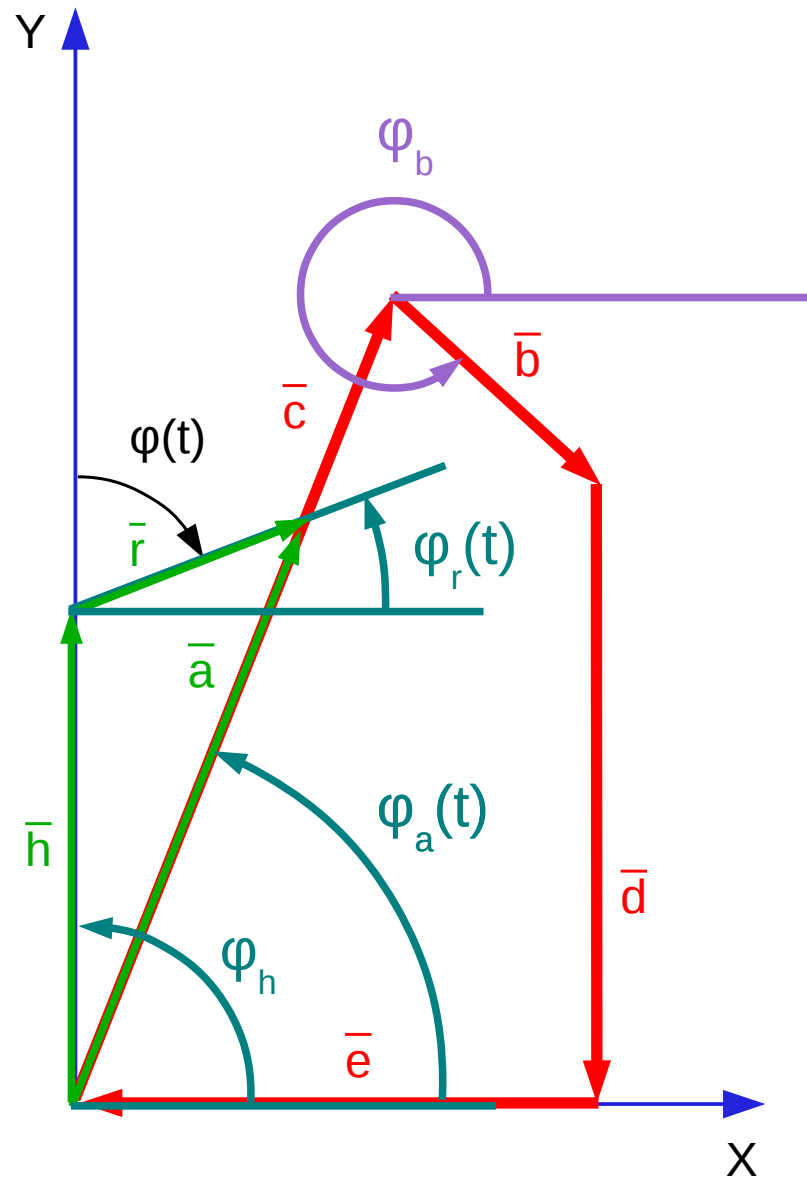


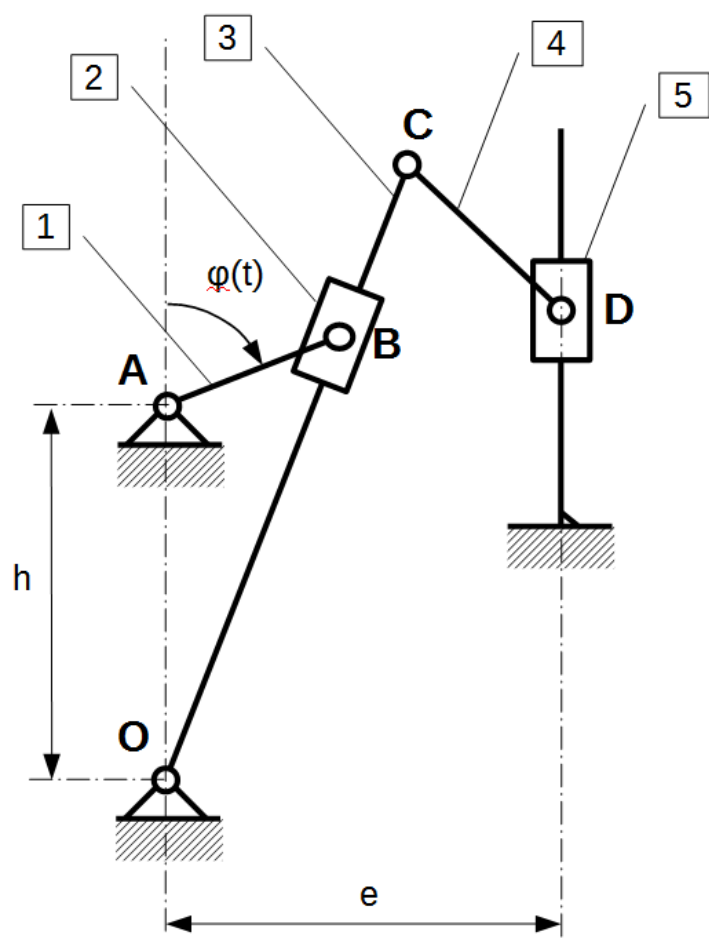


$$\varphi_h = 90^\circ$$

$$\varphi_a(t) \neq \text{const.}$$

$$\varphi_r(t) = 90^\circ - \varphi(t)$$



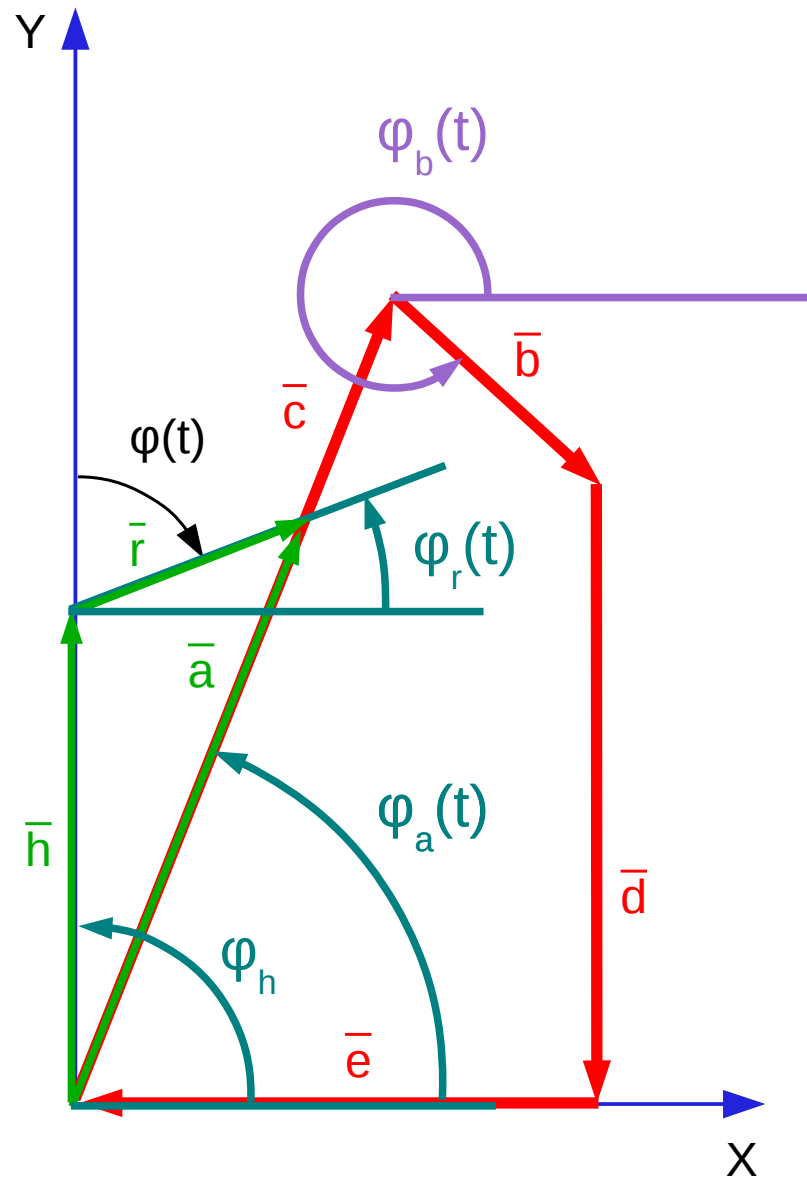


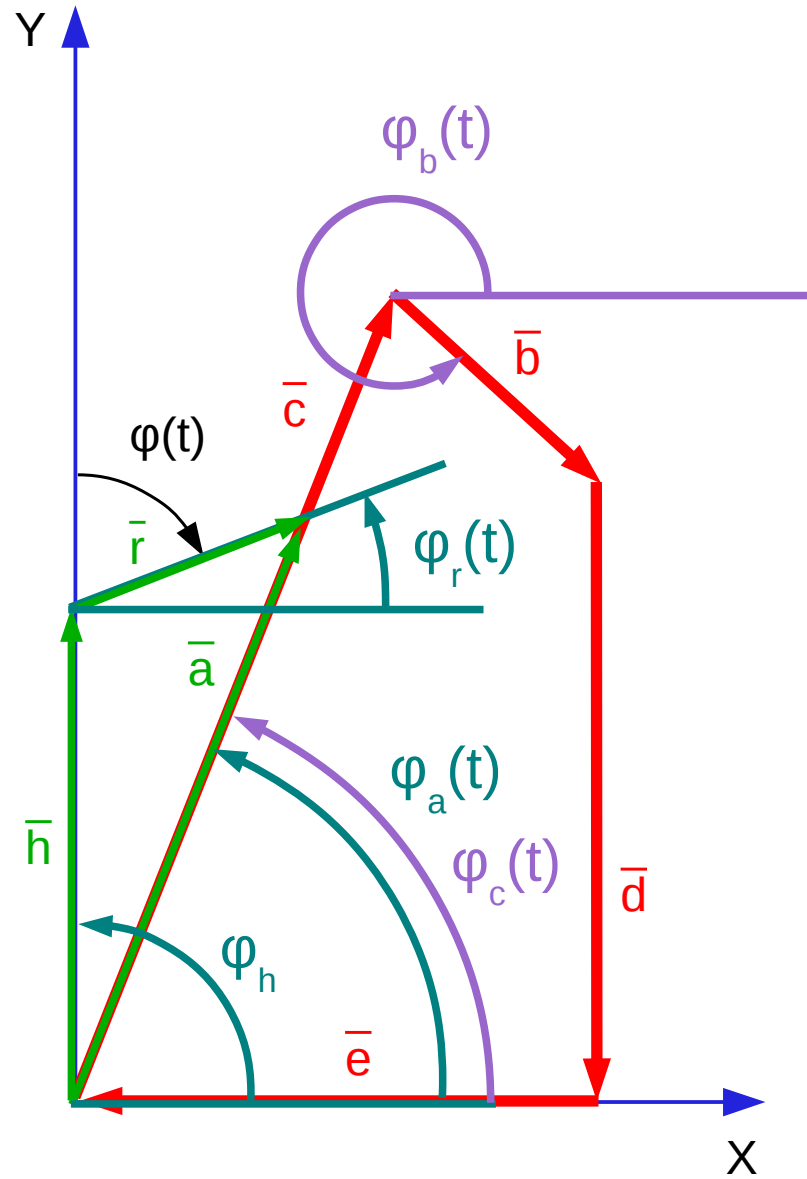
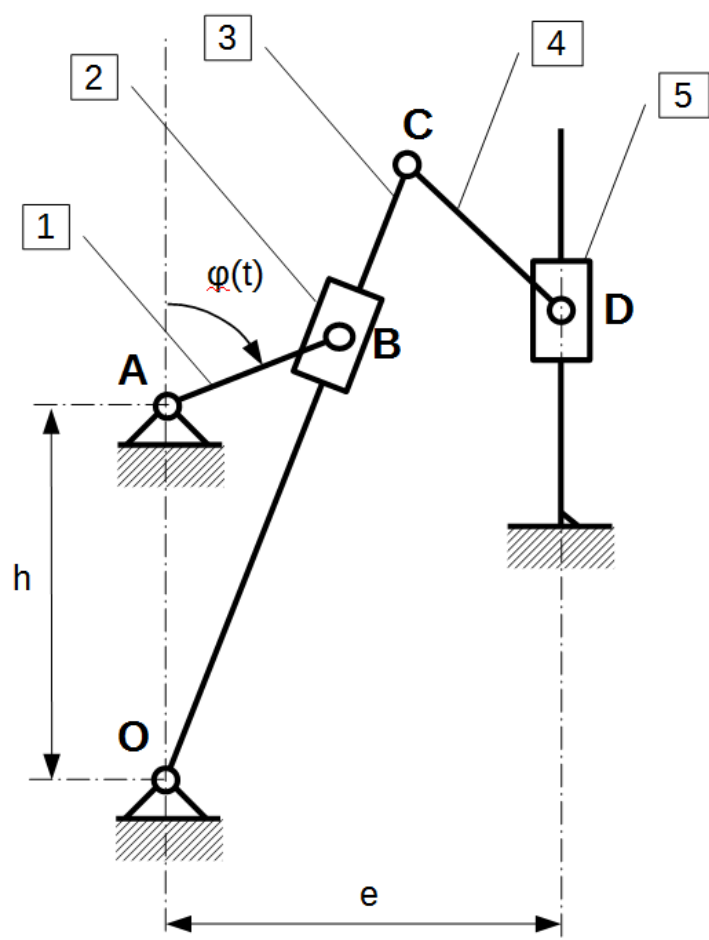
$$\varphi_h = 90^\circ$$

$$\varphi_a(t) \neq \text{const.}$$

$$\varphi_r(t) = 90^\circ - \varphi(t)$$

$$\varphi_b(t) \neq \text{const.}$$





$$\varphi_h = 90^\circ$$

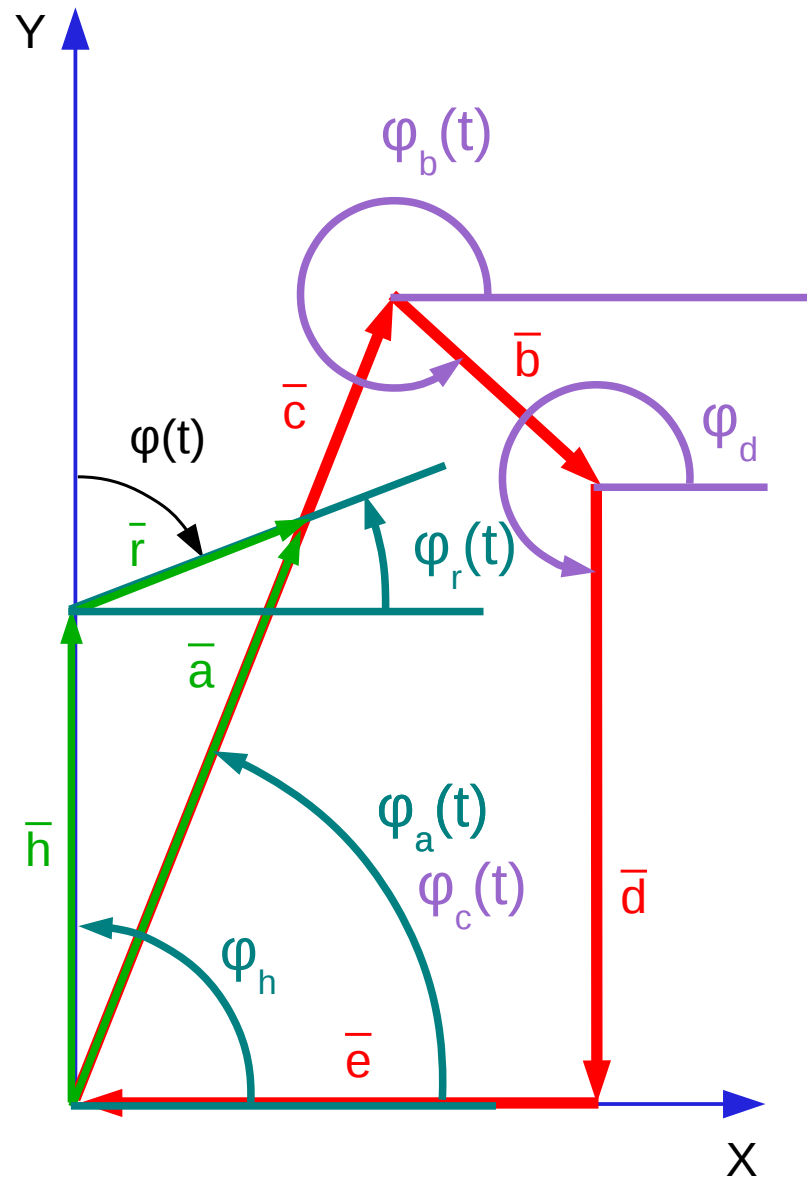
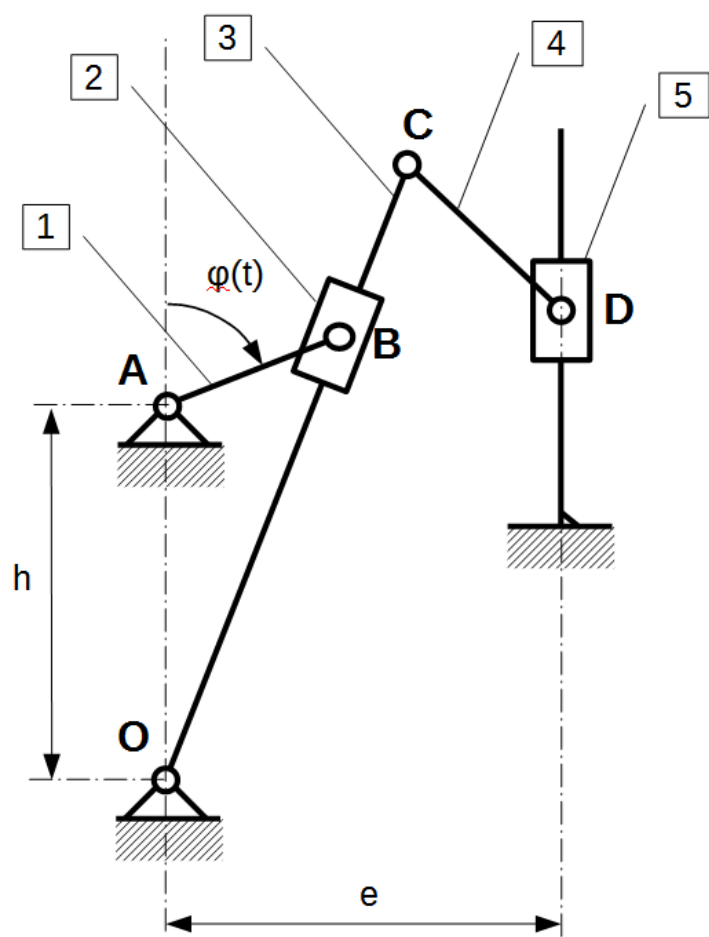
$$\varphi_a(t) \neq \text{const.}$$

$$\varphi_r(t) = 90^\circ - \varphi(t)$$

$$\varphi_b(t) \neq \text{const.}$$

$$\varphi_c(t) = \varphi_a(t)$$





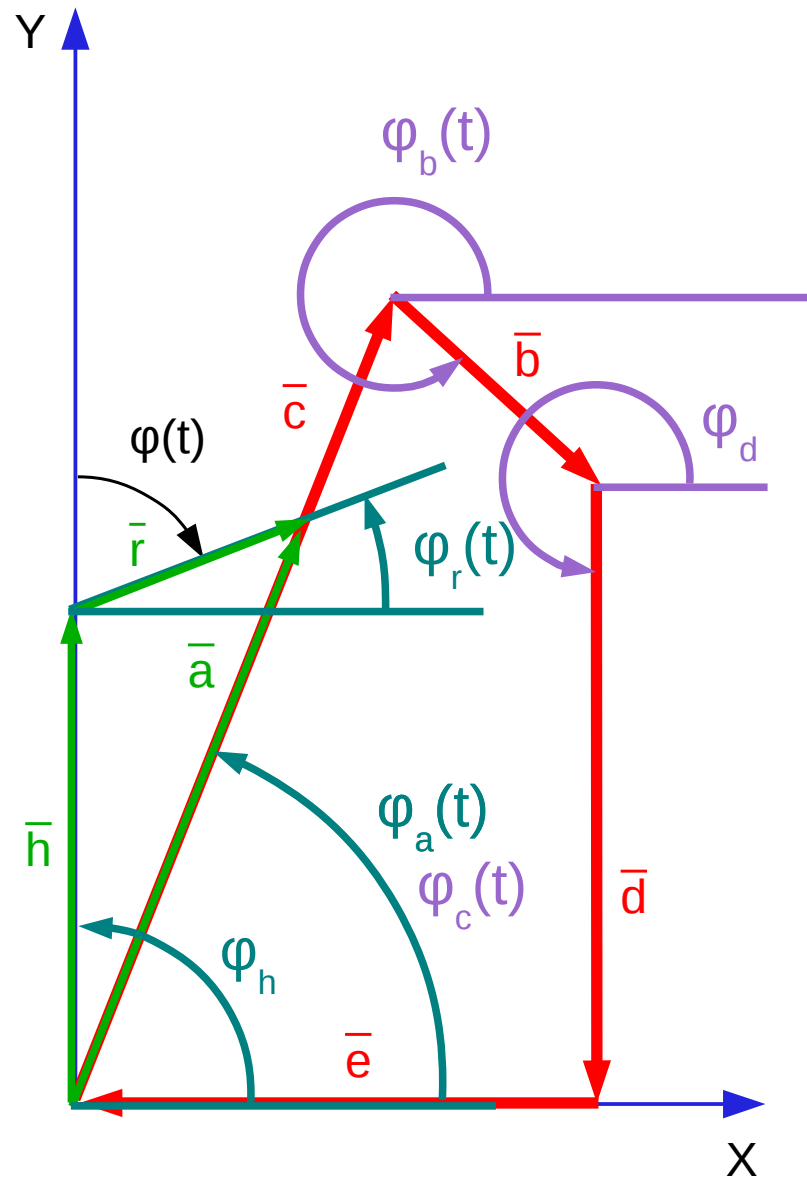
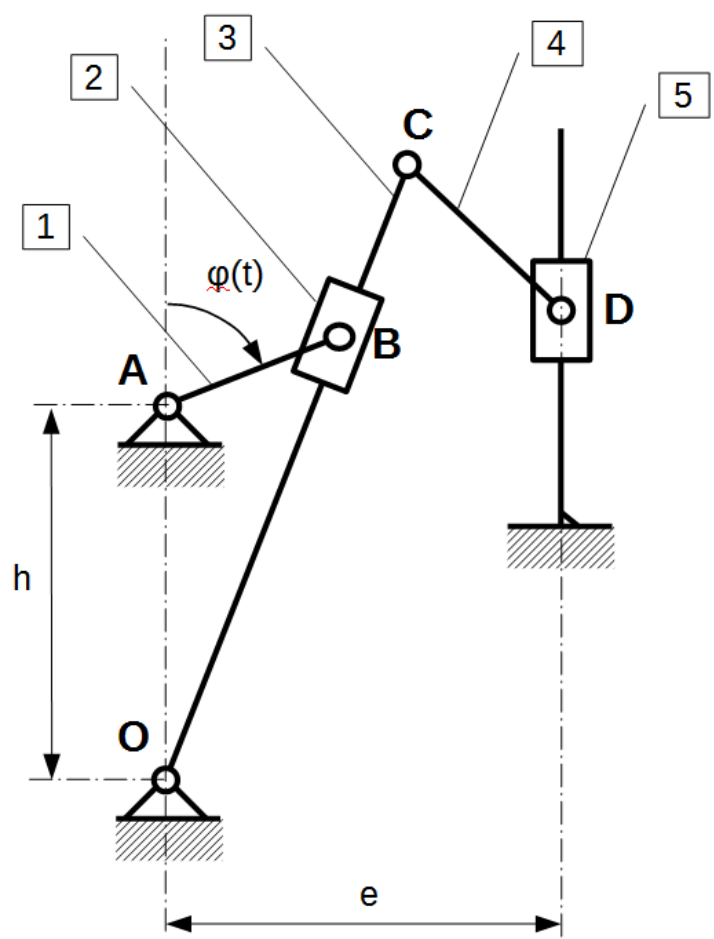
$$\varphi_h = 90^\circ$$

$$\varphi_a(t) \neq \text{const.}$$

$$\varphi_r(t) = 90^\circ - \varphi(t)$$

$$\varphi_b(t) \neq \text{const.}$$

$$\varphi_c(t) = \varphi_a(t)$$



$$\varphi_h = 90^\circ$$

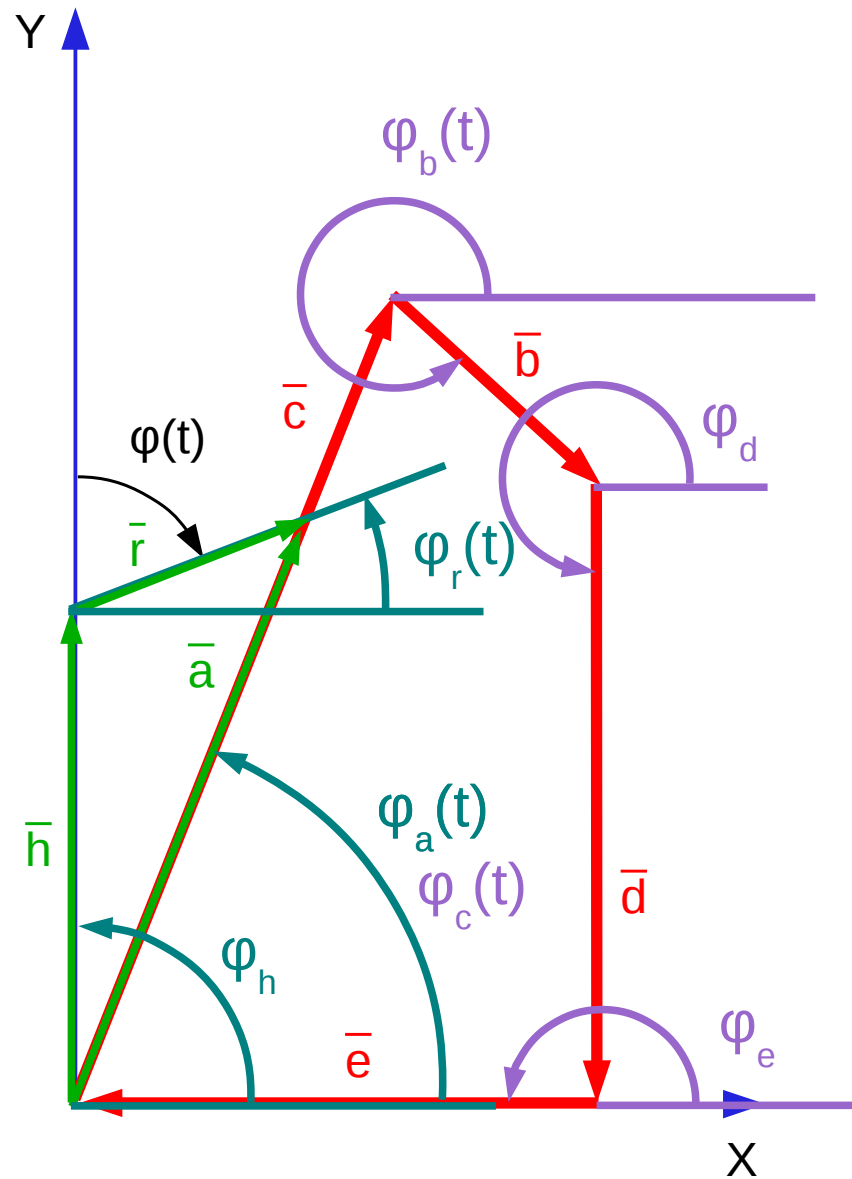
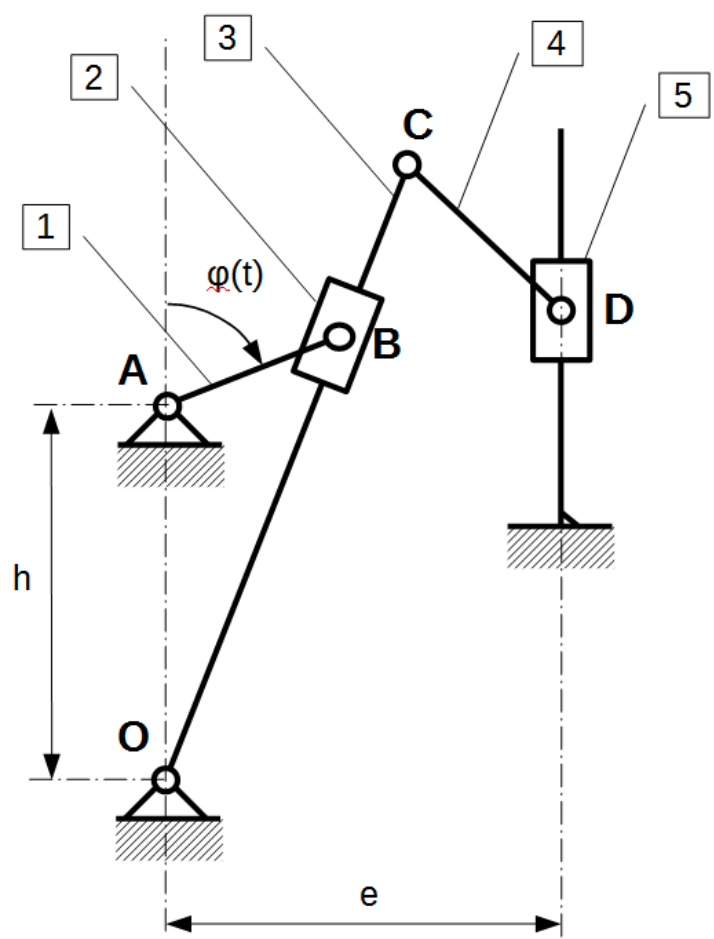
$$\varphi_d = 270^\circ$$

$$\varphi_a(t) \neq \text{const.}$$

$$\varphi_r(t) = 90^\circ - \varphi(t)$$

$$\varphi_b(t) \neq \text{const.}$$

$$\varphi_c(t) = \varphi_a(t)$$



$$\varphi_h = 90^\circ$$

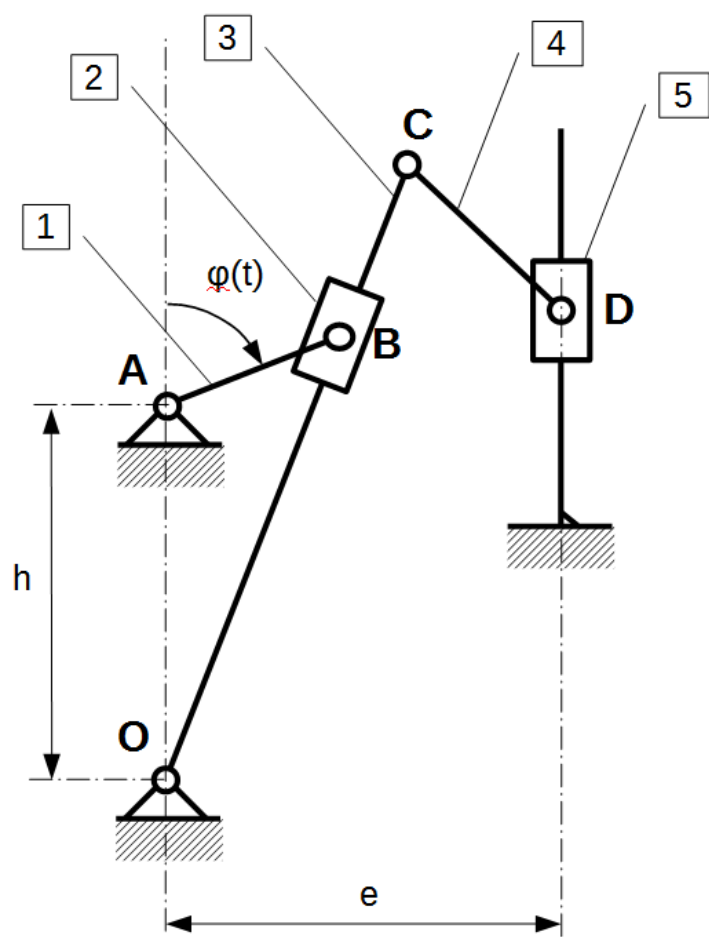
$$\varphi_a(t) \neq \text{const.}$$

$$\varphi_r(t) = 90^\circ - \varphi(t)$$

$$\varphi_b(t) \neq \text{const.}$$

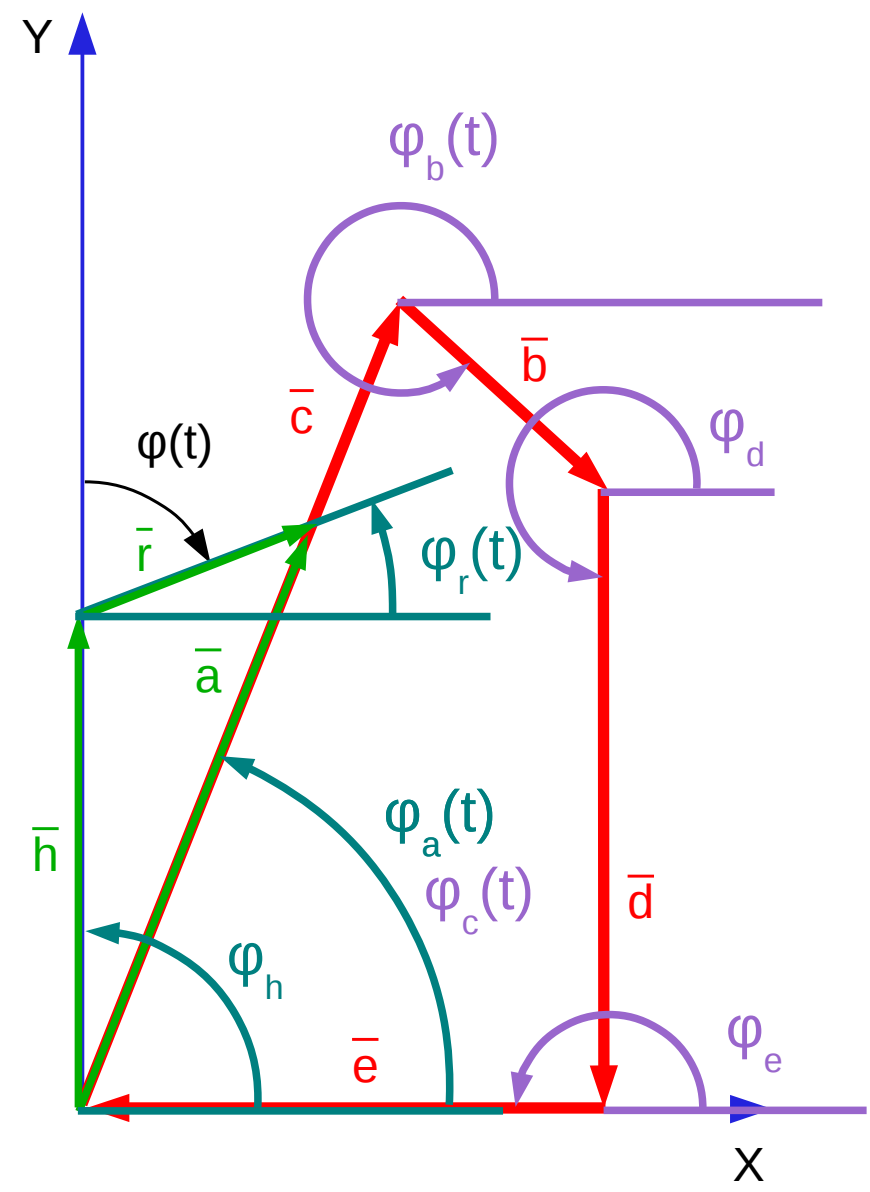
$$\varphi_c(t) = \varphi_a(t)$$

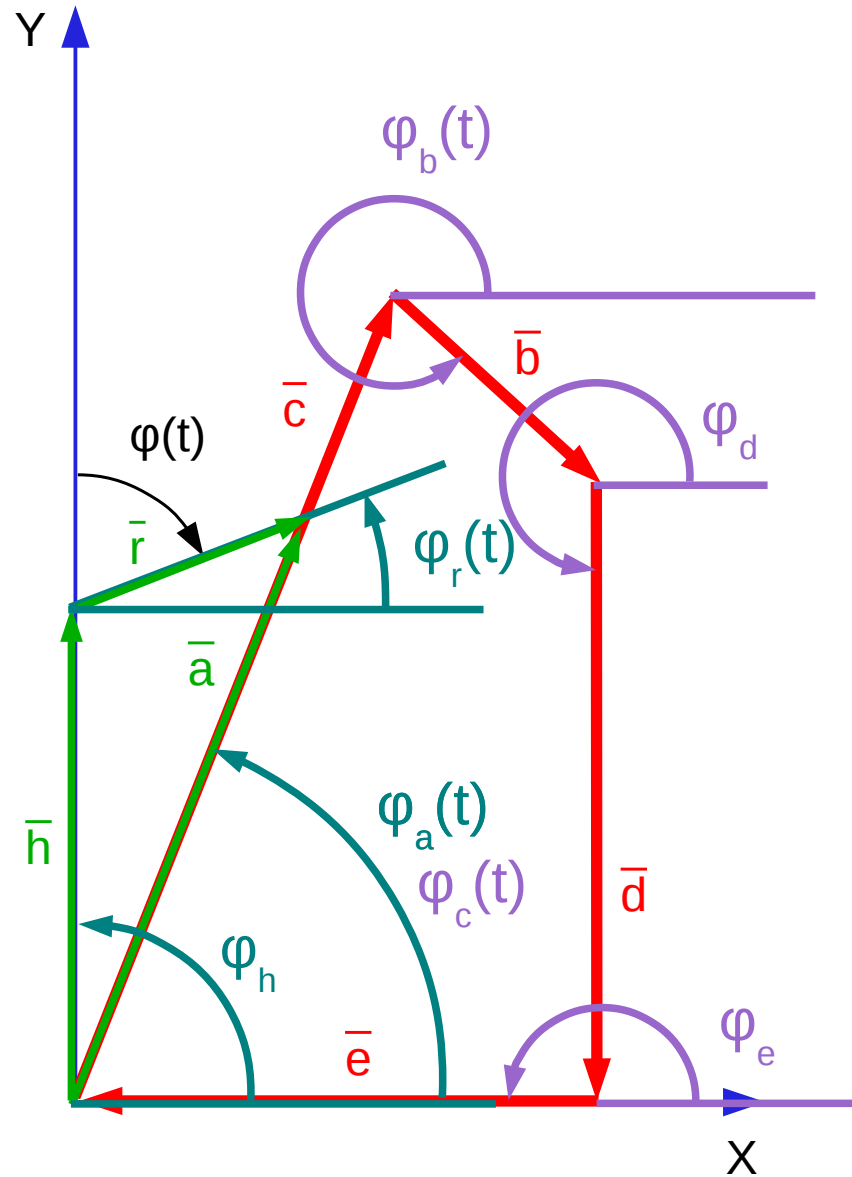
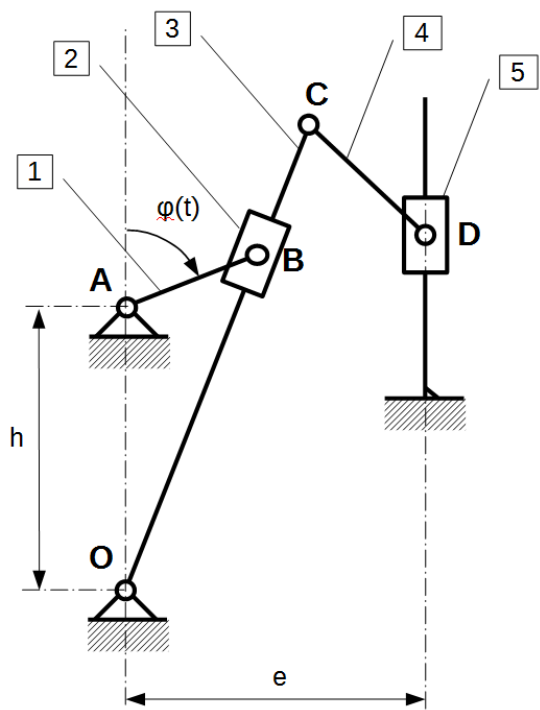
$$\varphi_d = 270^\circ$$



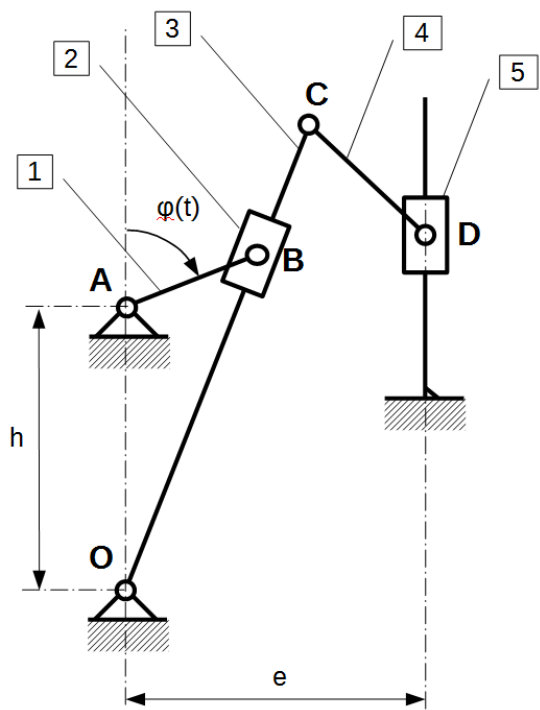
$\varphi_h = 90^\circ$   
 $\varphi_a(t) \neq \text{const.}$   
 $\varphi_r(t) = 90^\circ - \varphi(t)$   
 $\varphi_b(t) \neq \text{const.}$   
 $\varphi_c(t) = \varphi_a(t)$

$\varphi_d = 270^\circ$   
 $\varphi_e = 180^\circ$



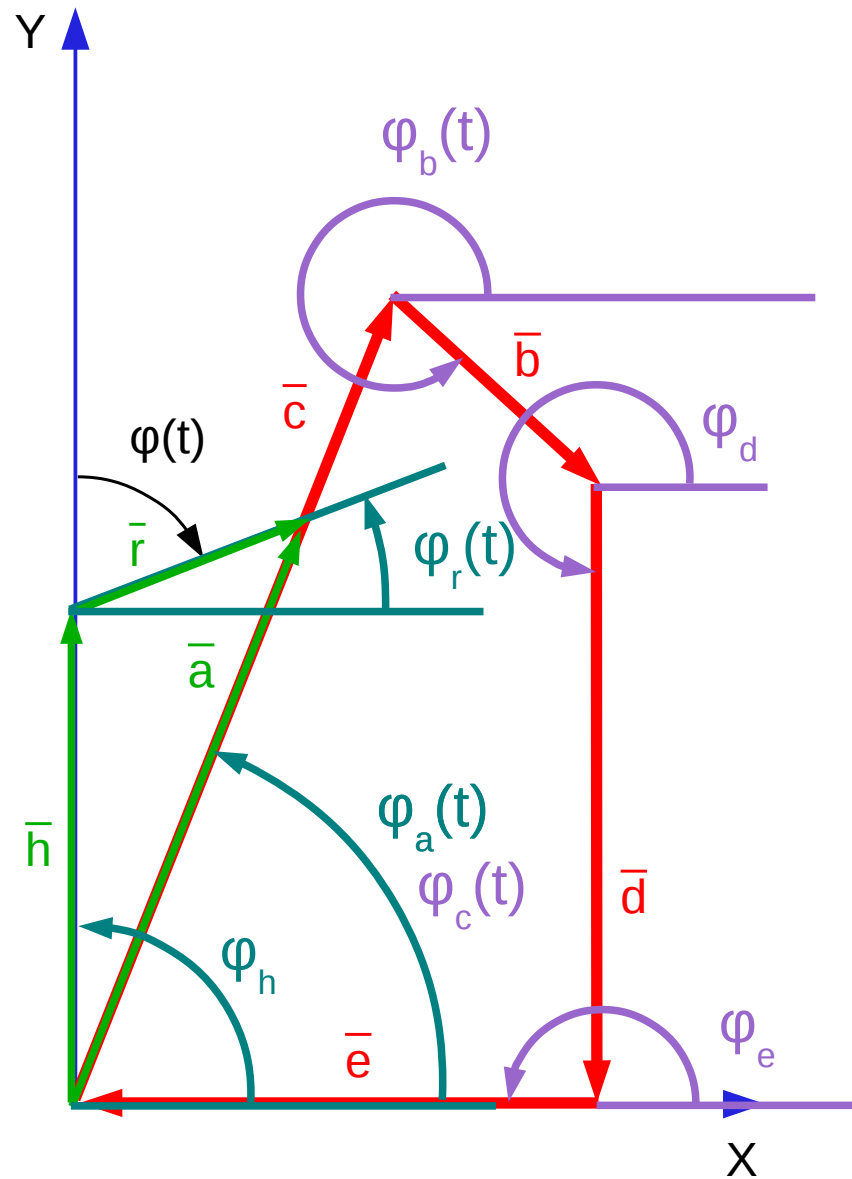


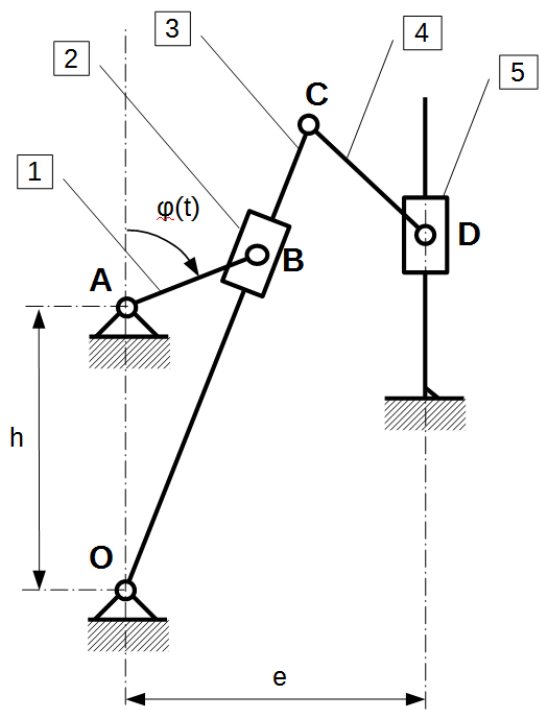
- $\varphi_h = 90^\circ$
- $\varphi_a(t) \neq \text{const.}$
- $\varphi_b(t) \neq \text{const.}$
- $\varphi_c(t) = \varphi_a(t)$
- $\varphi_r(t) = 90^\circ - \varphi(t)$
- $\varphi_d = 270^\circ$
- $\varphi_e = 180^\circ$



$$|\bar{r}| = |AB| = r = \text{const.}$$

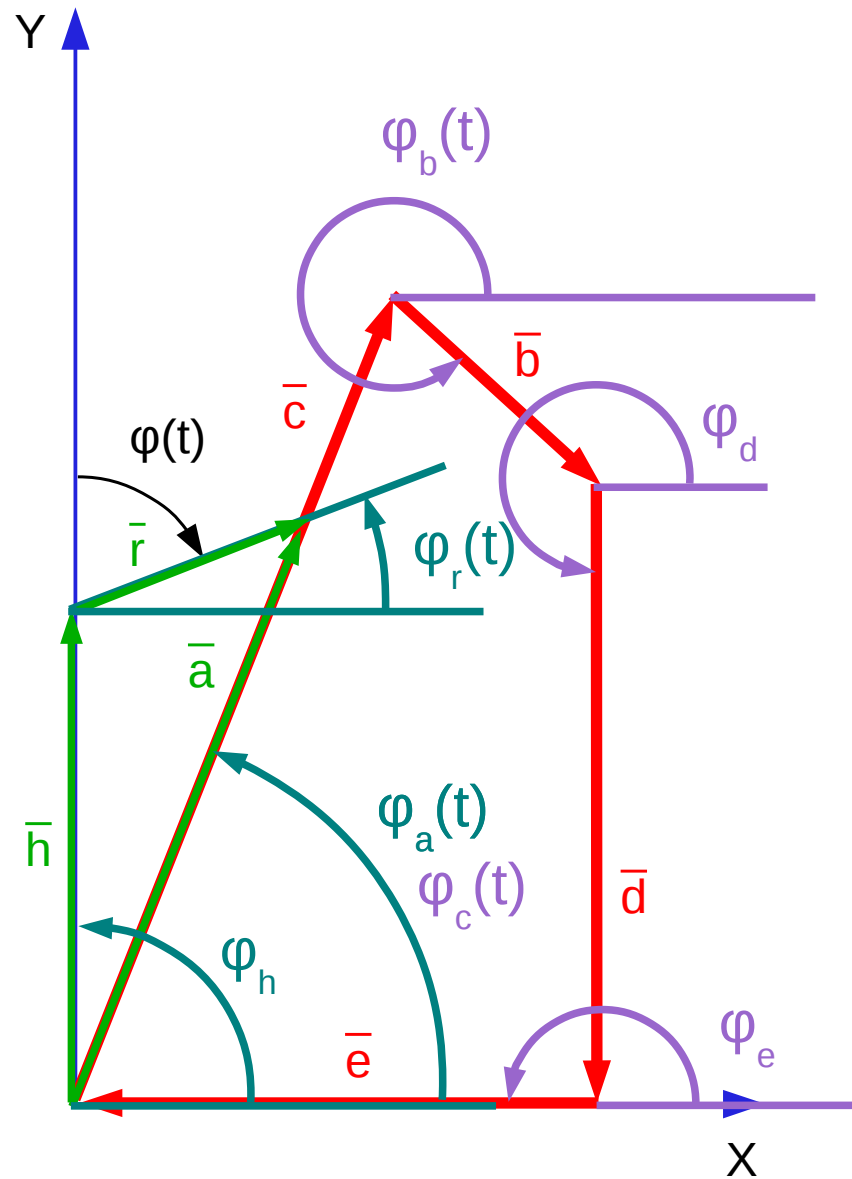
- $\varphi_h = 90^\circ$
- $\varphi_a(t) \neq \text{const.}$
- $\varphi_b(t) \neq \text{const.}$
- $\varphi_c(t) = \varphi_a(t)$
- $\varphi_r(t) = 90^\circ - \varphi(t)$
- $\varphi_d = 270^\circ$
- $\varphi_e = 180^\circ$

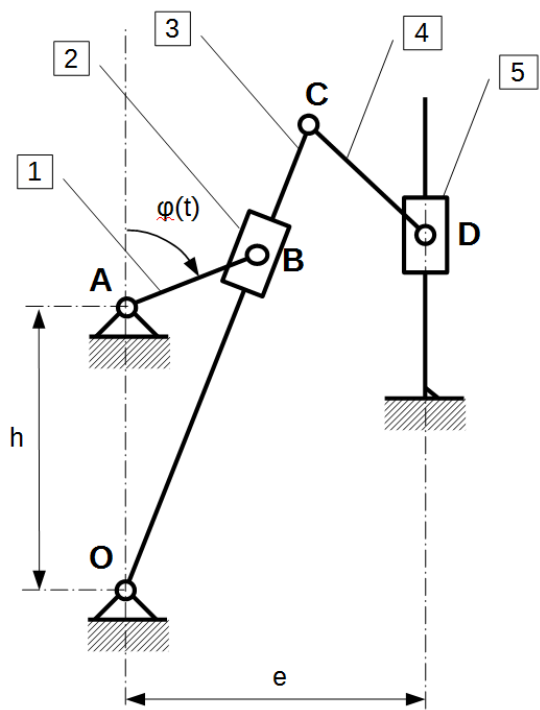




- $\varphi_h = 90^\circ$
- $\varphi_a(t) \neq \text{const.}$
- $\varphi_b(t) \neq \text{const.}$
- $\varphi_c(t) = \varphi_a(t)$
- $\varphi_r(t) = 90^\circ - \varphi(t)$
- $\varphi_d = 270^\circ$
- $\varphi_e = 180^\circ$

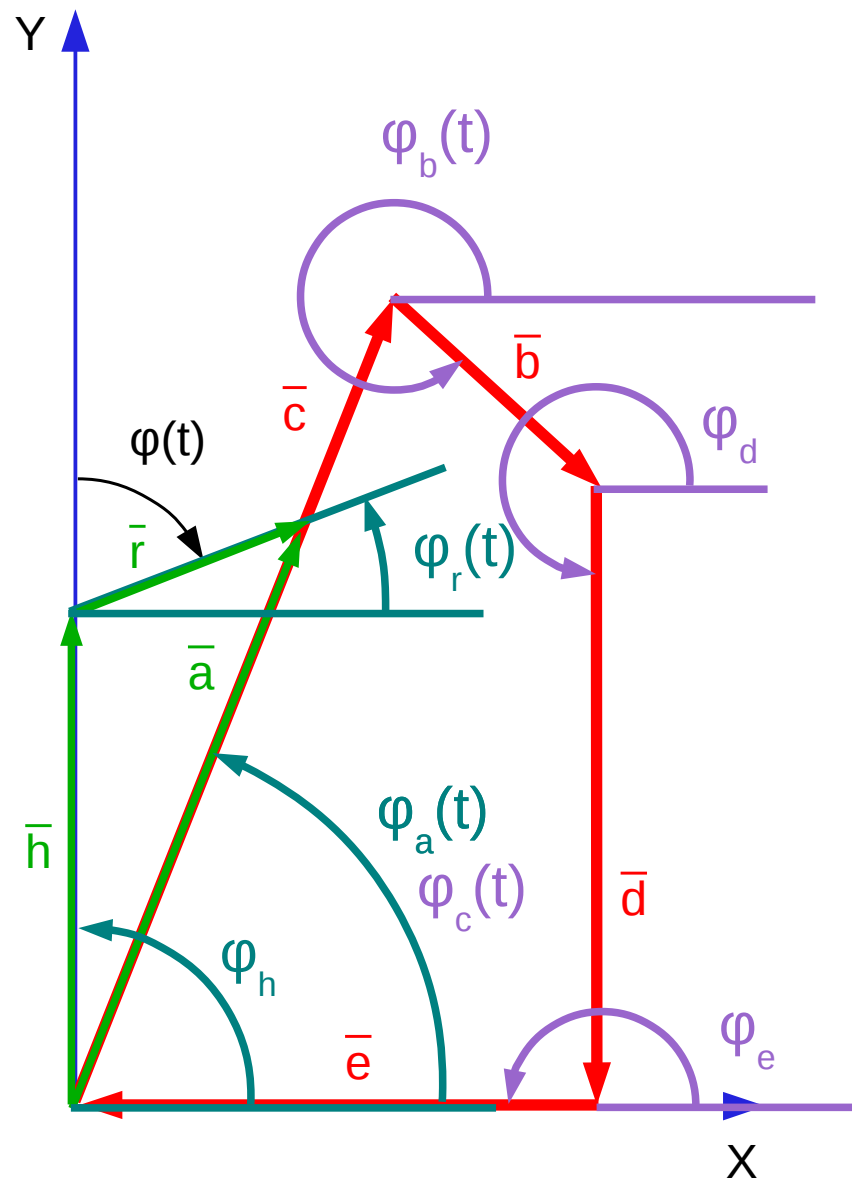
$|\bar{r}| = |AB| = r = \text{const.}$   
 $|\bar{h}| = |OA| = h = \text{const.}$



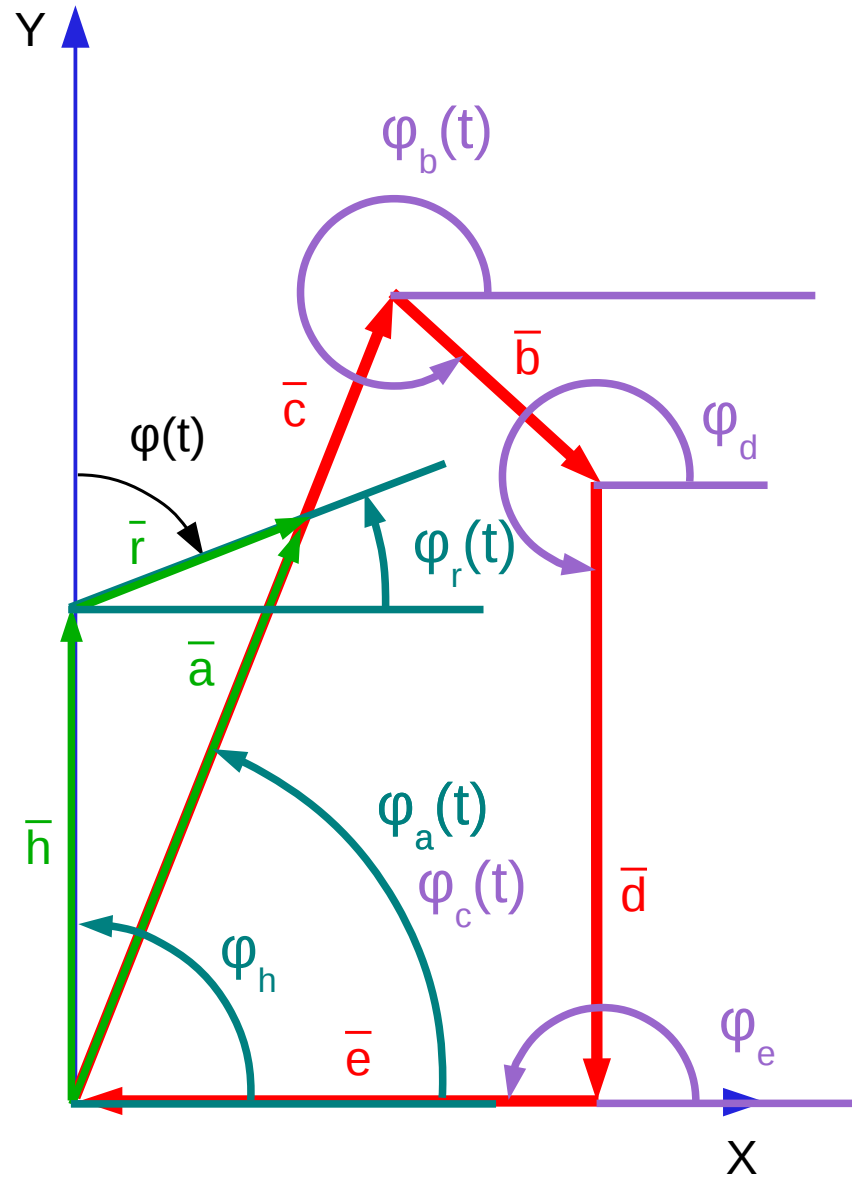
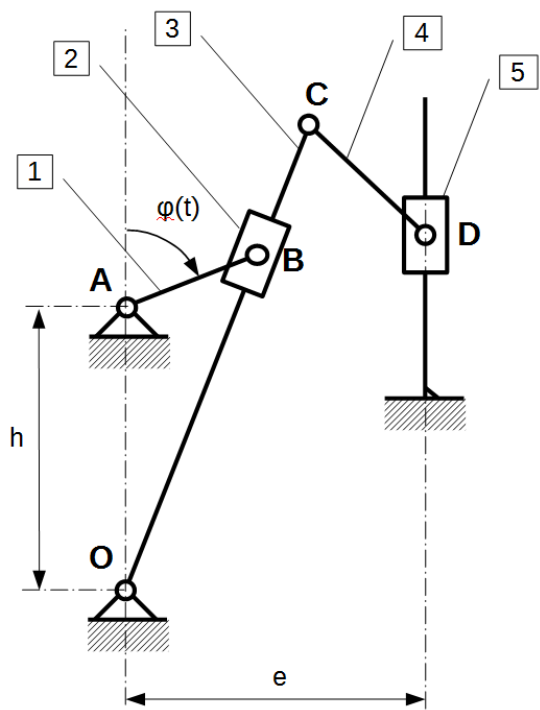


- $\varphi_h = 90^\circ$
- $\varphi_a(t) \neq \text{const.}$
- $\varphi_b(t) \neq \text{const.}$
- $\varphi_c(t) = \varphi_a(t)$
- $\varphi_r(t) = 90^\circ - \varphi(t)$
- $\varphi_d = 270^\circ$
- $\varphi_e = 180^\circ$

- $|\bar{r}| = |AB| = r = \text{const.}$
- $|\bar{h}| = |OA| = h = \text{const.}$
- $|\bar{a}| = |OB| = a(t)$

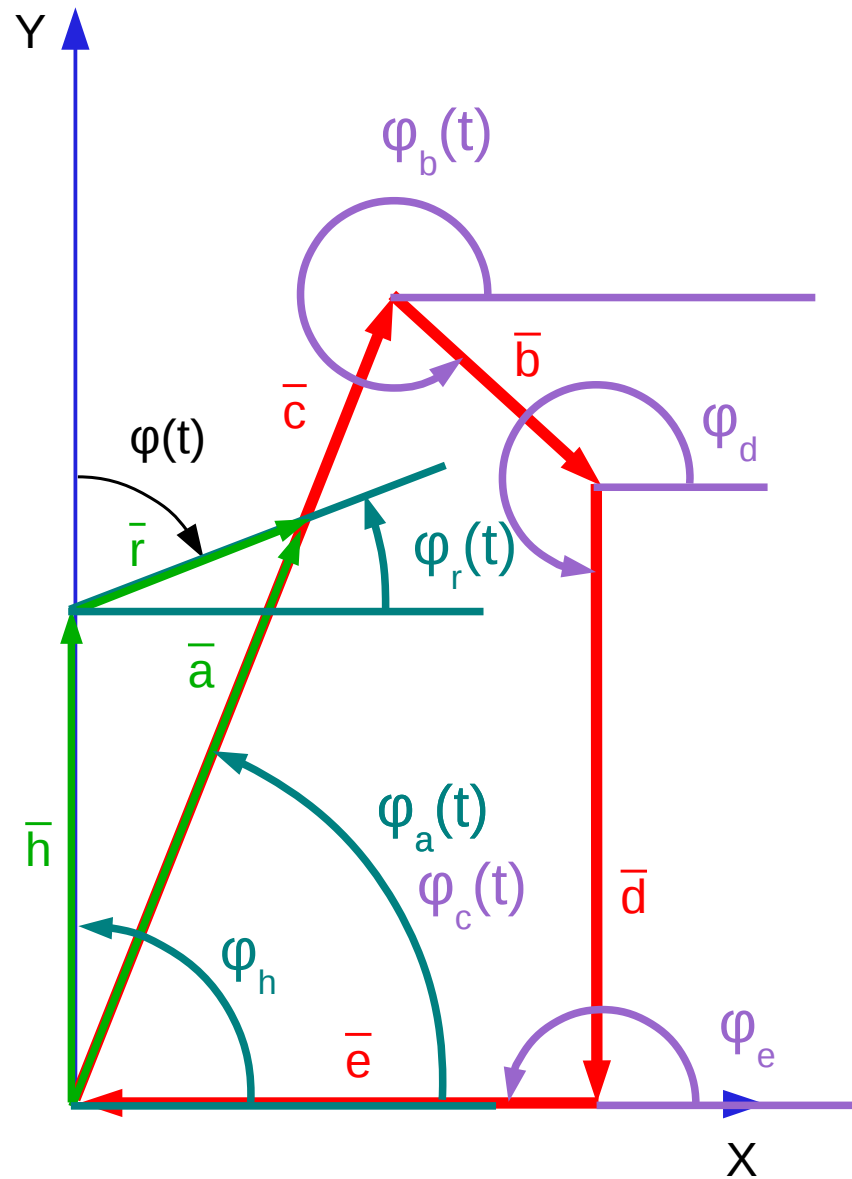
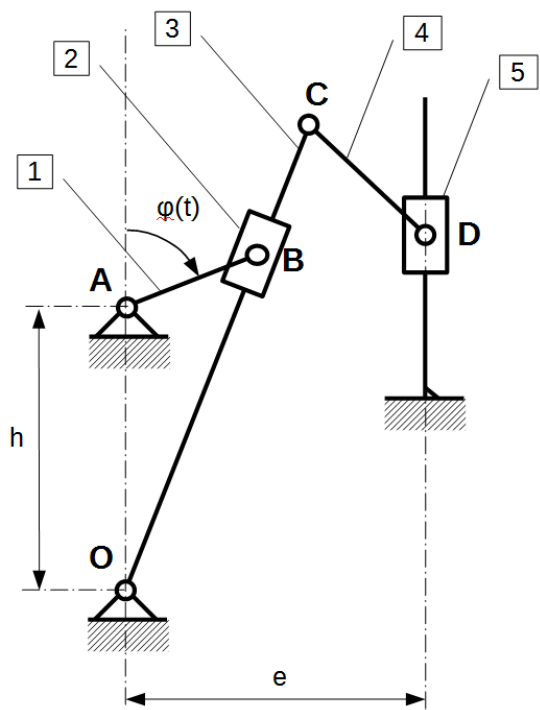






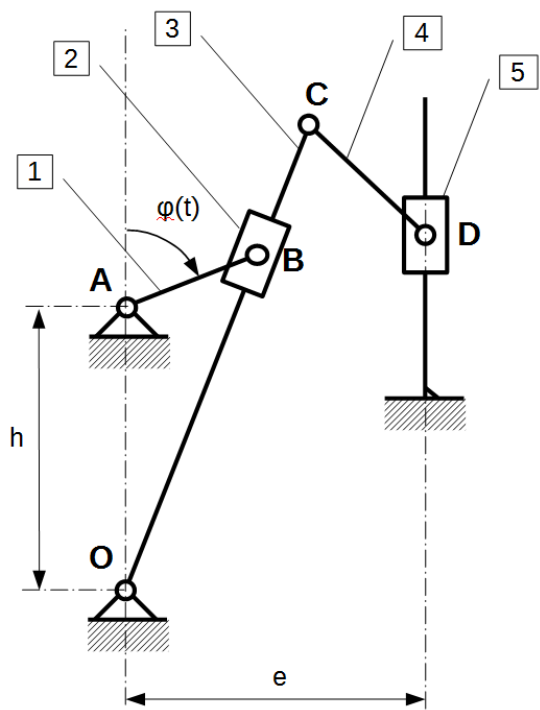
- $\varphi_h = 90^\circ$
- $\varphi_a(t) \neq \text{const.}$
- $\varphi_b(t) \neq \text{const.}$
- $\varphi_c(t) = \varphi_a(t)$
- $\varphi_r(t) = 90^\circ - \varphi(t)$
- $\varphi_d = 270^\circ$
- $\varphi_e = 180^\circ$

- $|\vec{r}| = |AB| = r = \text{const.}$
- $|\vec{h}| = |OA| = h = \text{const.}$
- $|\vec{a}| = |OB| = a(t)$
- $|\vec{c}| = |OC| = c = \text{const.}$



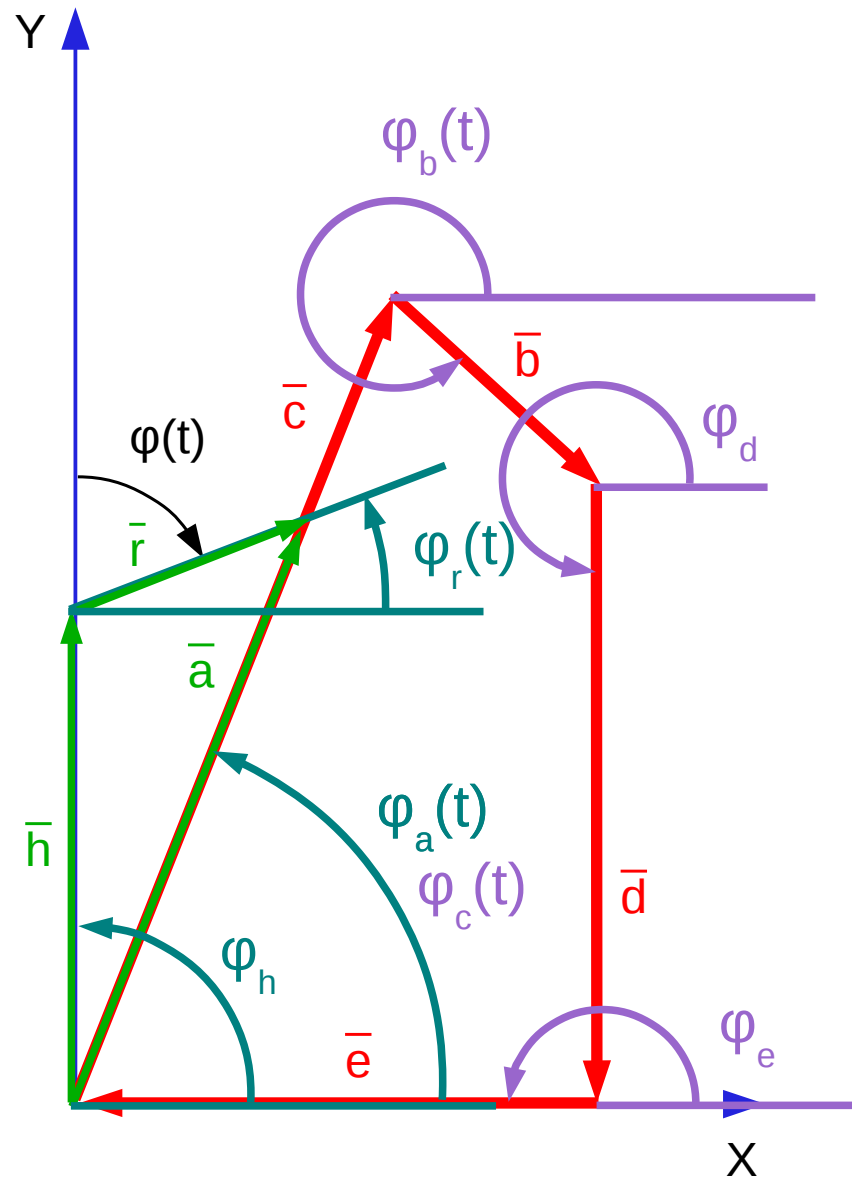
- $\varphi_h = 90^\circ$
- $\varphi_a(t) \neq \text{const.}$
- $\varphi_b(t) \neq \text{const.}$
- $\varphi_c(t) = \varphi_a(t)$
- $\varphi_r(t) = 90^\circ - \varphi(t)$
- $\varphi_d = 270^\circ$
- $\varphi_e = 180^\circ$

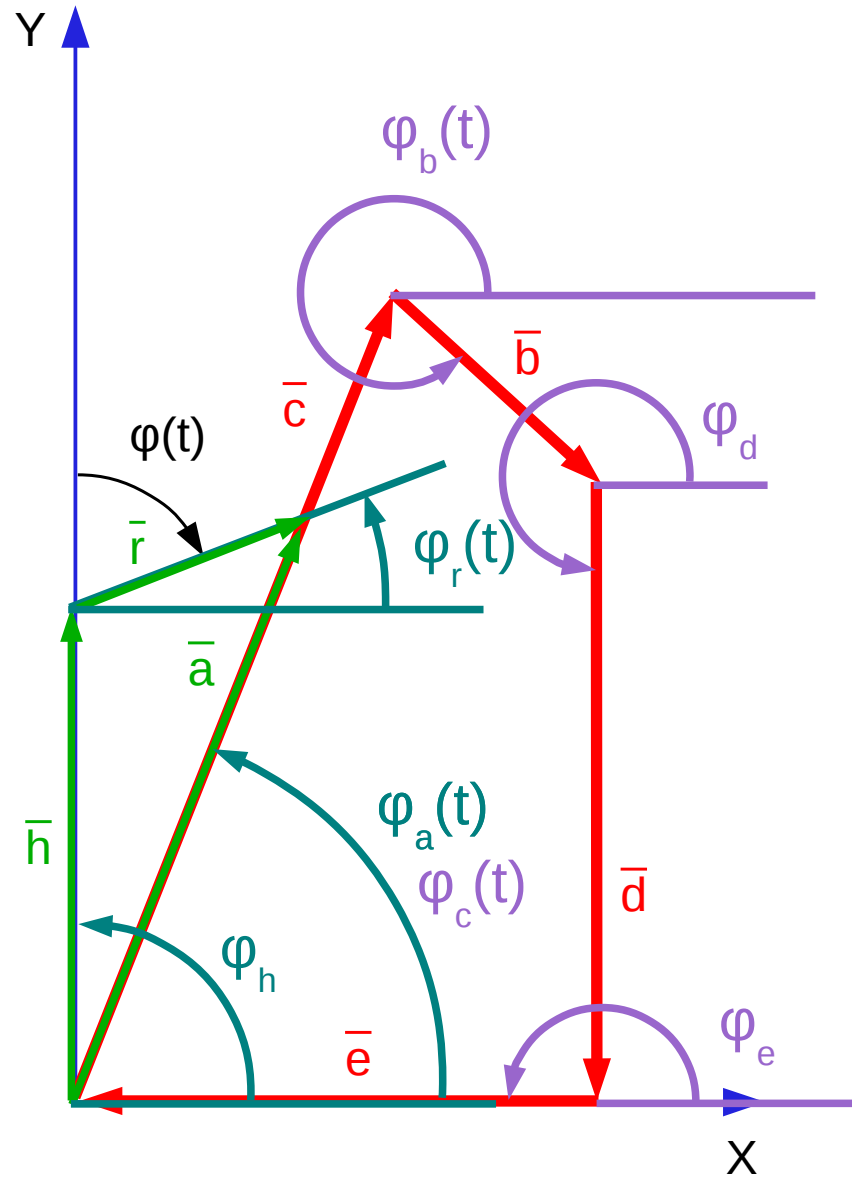
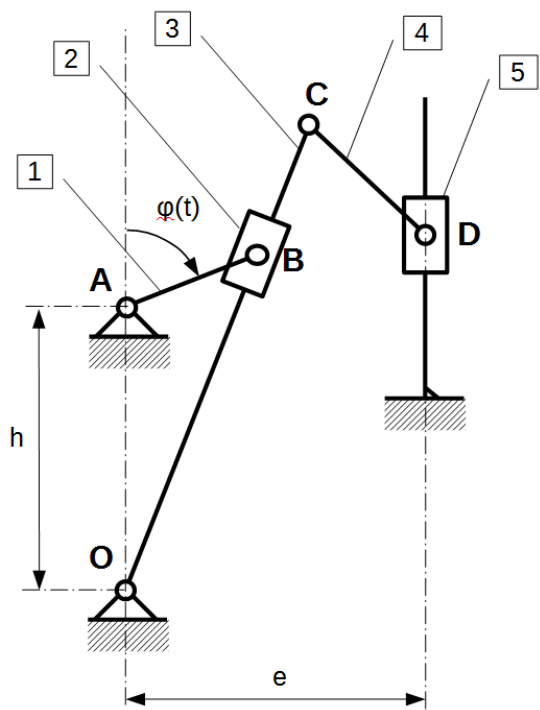
- $|\vec{r}| = |AB| = r = \text{const.}$
- $|\vec{h}| = |OA| = h = \text{const.}$
- $|\vec{a}| = |OB| = a(t)$
- $|\vec{c}| = |OC| = c = \text{const.}$
- $|\vec{b}| = |CD| = b = \text{const.}$



$\varphi_h = 90^\circ$   
 $\varphi_a(t) \neq \text{const.}$   
 $\varphi_b(t) \neq \text{const.}$   
 $\varphi_c(t) = \varphi_a(t)$   
 $\varphi_r(t) = 90^\circ - \varphi(t)$   
 $\varphi_d = 270^\circ$   
 $\varphi_e = 180^\circ$

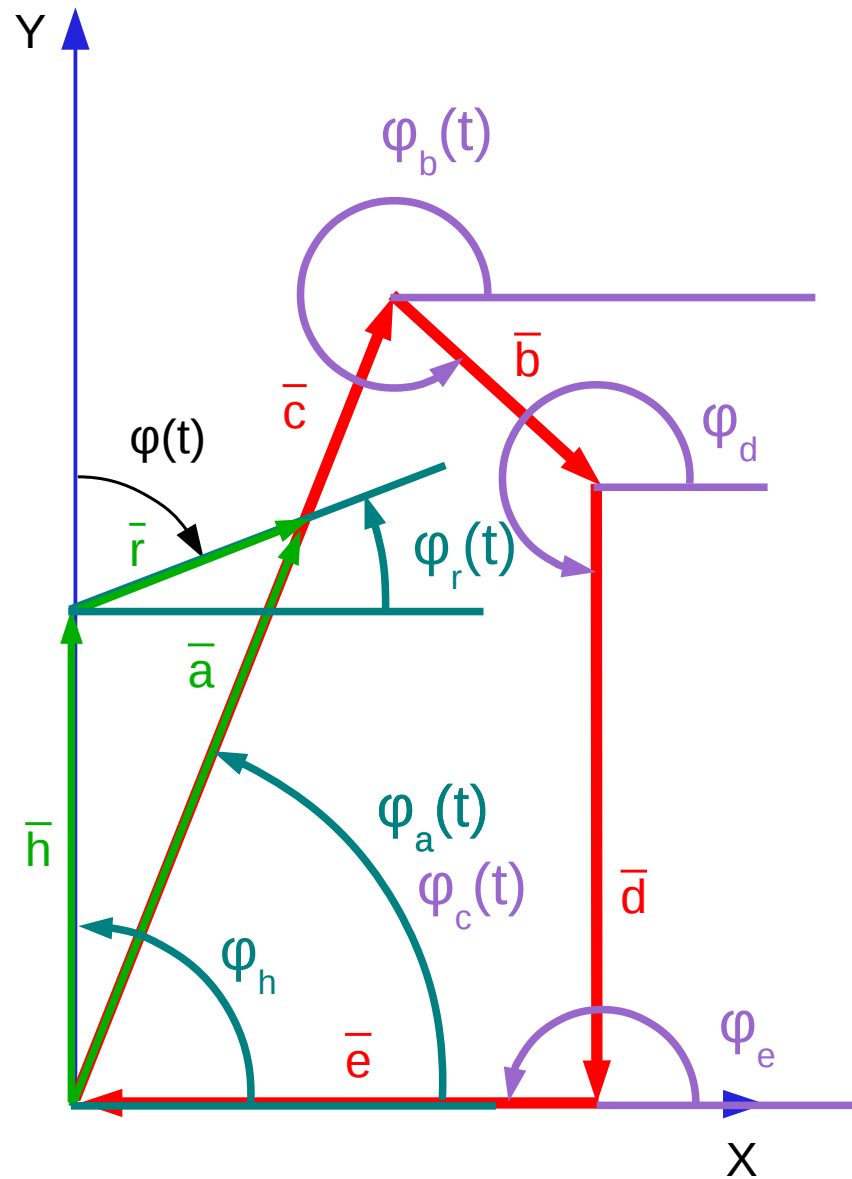
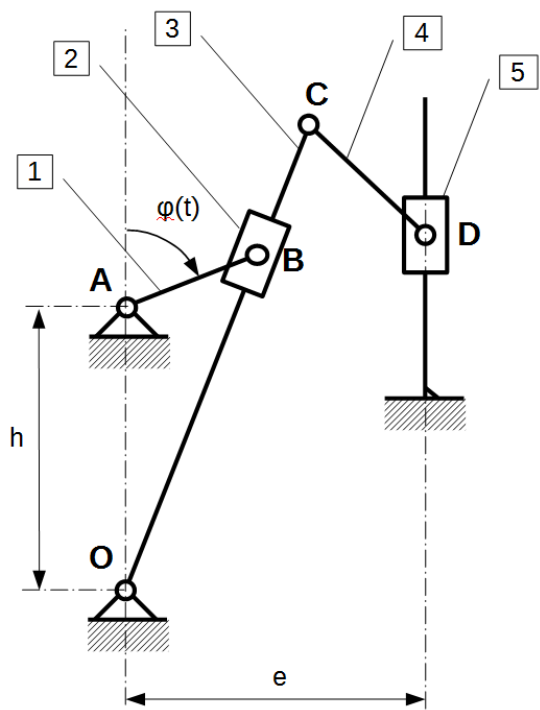
$|\bar{r}| = |AB| = r = \text{const.}$   
 $|\bar{h}| = |OA| = h = \text{const.}$   
 $|\bar{a}| = |OB| = a(t)$   
 $|\bar{c}| = |OC| = c = \text{const.}$   
 $|b| = |CD| = b = \text{const.}$   
 $|d| = d(t)$





- $\varphi_h = 90^\circ$
- $\varphi_a(t) \neq \text{const.}$
- $\varphi_b(t) \neq \text{const.}$
- $\varphi_c(t) = \varphi_a(t)$
- $\varphi_r(t) = 90^\circ - \varphi(t)$
- $\varphi_d = 270^\circ$
- $\varphi_e = 180^\circ$

- $|\vec{r}| = |AB| = r = \text{const.}$
- $|\vec{h}| = |OA| = h = \text{const.}$
- $|\vec{a}| = |OB| = a(t)$
- $|\vec{c}| = |OC| = c = \text{const.}$
- $|b| = |CD| = b = \text{const.}$
- $|d| = d(t)$
- $|\vec{e}| = e = \text{const.}$

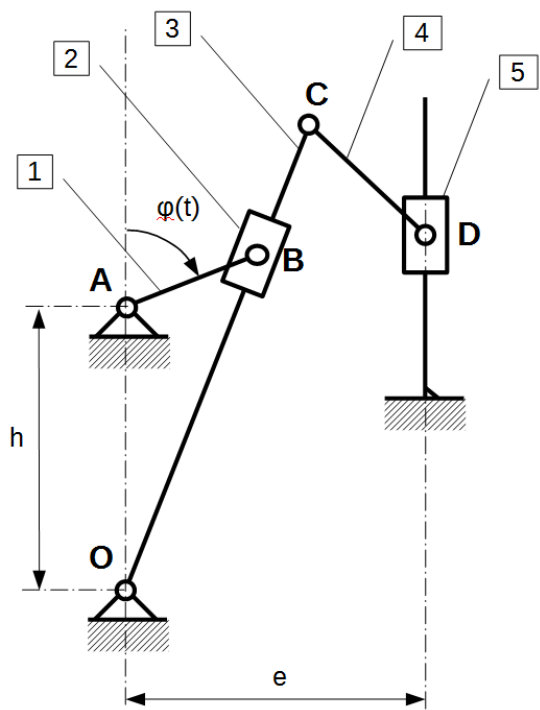


$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

$$\bar{c} + \bar{b} + \frac{\bar{h}}{d} + \frac{\bar{r}}{d} = \frac{\bar{a}}{d}$$

- $\varphi_h = 90^\circ$
- $\varphi_a(t) \neq \text{const.}$
- $\varphi_b(t) \neq \text{const.}$
- $\varphi_c(t) = \varphi_a(t)$
- $\varphi_r(t) = 90^\circ - \varphi(t)$
- $\varphi_d = 270^\circ$
- $\varphi_e = 180^\circ$

- $|\bar{r}| = |AB| = r = \text{const.}$
- $|\bar{h}| = |OA| = h = \text{const.}$
- $|\bar{a}| = |OB| = a(t)$
- $|\bar{c}| = |OC| = c = \text{const.}$
- $|b| = |CD| = b = \text{const.}$
- $|d| = d(t)$
- $|\bar{e}| = e = \text{const.}$



$$\varphi_h = 90^\circ$$

$$\varphi_r(t) = 90^\circ - \varphi(t)$$

$$\varphi_a(t) \neq \text{const.}$$

$$\varphi_c(t) = \varphi_a(t)$$

$$\varphi_b(t) \neq \text{const.}$$

$$\varphi_d = 270^\circ$$

$$\varphi_e = 180^\circ$$

$$|\bar{h}| = |OA| = h = \text{const.}$$

$$|\bar{r}| = |AB| = r = \text{const.}$$

$$|\bar{a}| = |OB| = a(t)$$

$$|\bar{c}| = |OC| = c = \text{const.}$$

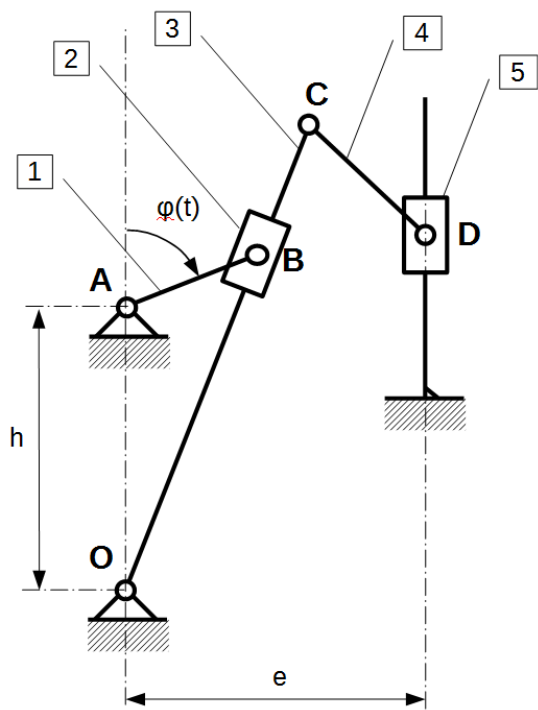
$$|b| = |CD| = b = \text{const.}$$

$$|d| = d(t)$$

$$|\bar{e}| = e = \text{const.}$$

$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

$$\bar{h} + \bar{r} = \bar{a}$$



$$\varphi_h = 90^\circ$$

$$\varphi_r(t) = 90^\circ - \varphi(t)$$

$$\varphi_a(t) \neq \text{const.}$$

$$\varphi_c(t) = \varphi_a(t)$$

$$\varphi_b(t) \neq \text{const.}$$

$$\varphi_d = 270^\circ$$

$$\varphi_e = 180^\circ$$

$$|\bar{h}| = |OA| = h = \text{const.}$$

$$|\bar{r}| = |AB| = r = \text{const.}$$

$$|\bar{a}| = |OB| = a(t)$$

$$|\bar{c}| = |OC| = c = \text{const.}$$

$$|b| = |CD| = b = \text{const.}$$

$$|d| = d(t)$$

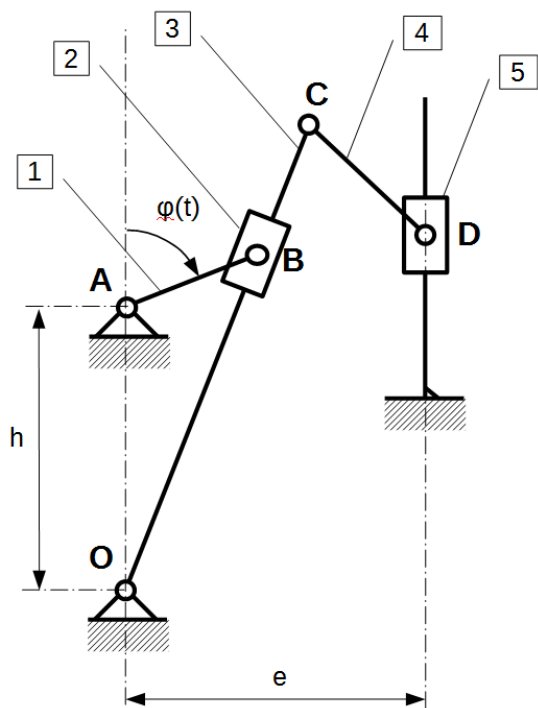
$$|\bar{e}| = e = \text{const.}$$

$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

$$\bar{h} + \bar{r} = \bar{a}$$

$$x: h \cos \phi_h + r \cos \phi_r(t) = a(t) \cos \phi_a(t)$$

$$y: h \sin \phi_h + r \sin \phi_r(t) = a(t) \sin \phi_a(t)$$



$$\varphi_h = 90^\circ$$

$$\varphi_r(t) = 90^\circ - \varphi(t)$$

$$\varphi_a(t) \neq \text{const.}$$

$$\varphi_c(t) = \varphi_a(t)$$

$$\varphi_b(t) \neq \text{const.}$$

$$\varphi_d = 270^\circ$$

$$\varphi_e = 180^\circ$$

$$|\bar{h}| = |OA| = h = \text{const.}$$

$$|\bar{r}| = |AB| = r = \text{const.}$$

$$|\bar{a}| = |OB| = a(t)$$

$$|\bar{c}| = |OC| = c = \text{const.}$$

$$|b| = |CD| = b = \text{const.}$$

$$|d| = d(t)$$

$$|\bar{e}| = e = \text{const.}$$

$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

$$\bar{h} + \bar{r} = \bar{a}$$

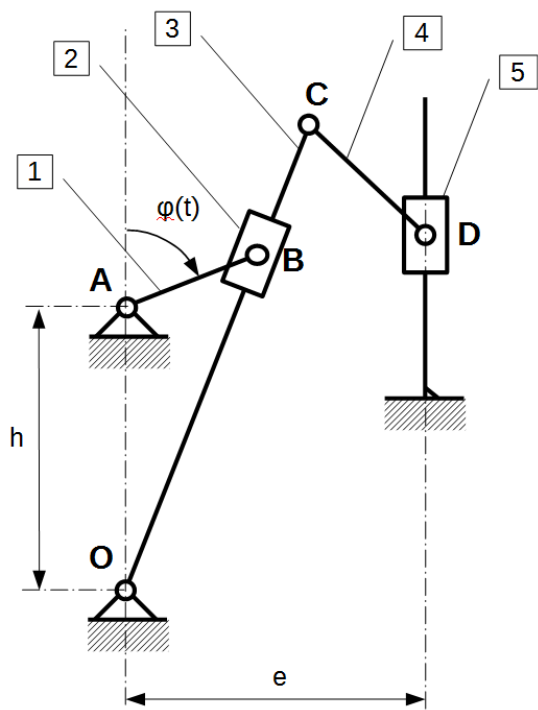
$$x: h \cos \phi_h + r \cos \phi_r(t) = a(t) \cos \phi_a(t)$$

$$y: h \sin \phi_h + r \sin \phi_r(t) = a(t) \sin \phi_a(t)$$

$$x: c \cos \phi_c(t) + b \cos \phi_b(t) + d(t) \cos \phi_d + e \cos \phi_e = 0$$

$$y: c \sin \phi_c(t) + b \sin \phi_b(t) + d(t) \sin \phi_d + e \sin \phi_e = 0$$





$$\varphi_h = 90^\circ$$

$$\varphi_r(t) = 90^\circ - \varphi(t)$$

$$\varphi_a(t) \neq \text{const.}$$

$$\varphi_c(t) = \varphi_a(t)$$

$$\varphi_b(t) \neq \text{const.}$$

$$\varphi_d = 270^\circ$$

$$\varphi_e = 180^\circ$$

$$|\bar{h}| = |OA| = h = \text{const.}$$

$$|\bar{r}| = |AB| = r = \text{const.}$$

$$|\bar{a}| = |OB| = a(t)$$

$$|\bar{c}| = |OC| = c = \text{const.}$$

$$|b| = |CD| = b = \text{const.}$$

$$|d| = d(t)$$

$$|\bar{e}| = e = \text{const.}$$

$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

$$x: h \cos \phi_h + r \cos \phi_r(t) = a(t) \cos \phi_a(t)$$

$$y: h \sin \phi_h + r \sin \phi_r(t) = a(t) \sin \phi_a(t)$$

$$x: c \cos \phi_c(t) + b \cos \phi_b(t) + d(t) \cos \phi_d + e \cos \phi_e = 0$$

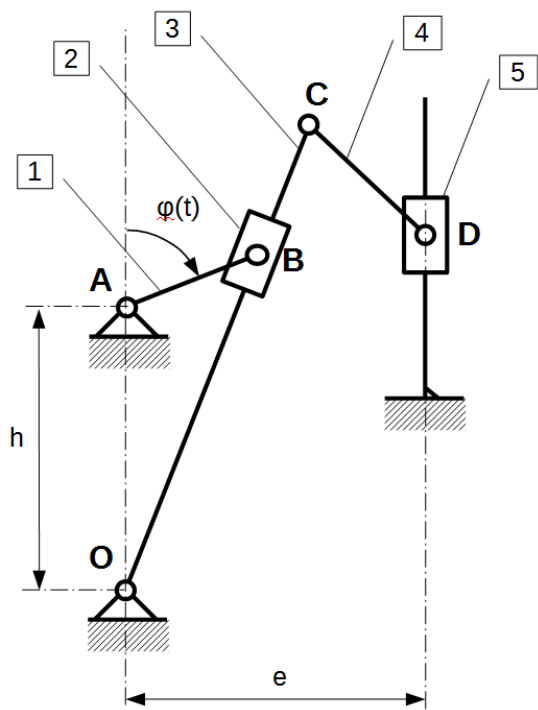
$$y: c \sin \phi_c(t) + b \sin \phi_b(t) + d(t) \sin \phi_d + e \sin \phi_e = 0$$

$$x: h \cos 90^\circ + r \cos(90^\circ - \phi(t)) = a(t) \cos \phi_a(t)$$

$$y: h \sin 90^\circ + r \sin(90^\circ - \phi(t)) = a(t) \sin \phi_a(t)$$

$$x: c \cos \phi_a(t) + b \cos \phi_b(t) + d(t) \cos 270^\circ + e \cos 180^\circ = 0$$

$$y: c \sin \phi_a(t) + b \sin \phi_b(t) + d(t) \sin 270^\circ + e \sin 180^\circ = 0$$



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$$|\bar{c}| = |OC| = c = \text{const.}$$

$$|b| = |CD| = b = \text{const.}$$

$$|d| = d(t)$$

$$|\bar{e}| = e = \text{const.}$$

$$\bar{c} + \bar{b} + \bar{d} + \bar{e} = \bar{0}$$

$$x: h \cos \phi_h + r \cos \phi_r(t) = a(t) \cos \phi_a(t)$$

$$y: h \sin \phi_h + r \sin \phi_r(t) = a(t) \sin \phi_a(t)$$

$$x: c \cos \phi_c(t) + b \cos \phi_b(t) + d(t) \cos \phi_d + e \cos \phi_e = 0$$

$$y: c \sin \phi_c(t) + b \sin \phi_b(t) + d(t) \sin \phi_d + e \sin \phi_e = 0$$

$$x: h \cos 90^\circ + r \cos(90^\circ - \phi(t)) = a(t) \cos \phi_a(t)$$

$$y: h \sin 90^\circ + r \sin(90^\circ - \phi(t)) = a(t) \sin \phi_a(t)$$

$$x: c \cos \phi_a(t) + b \cos \phi_b(t) + d(t) \cos 270^\circ + e \cos 180^\circ = 0$$

$$y: c \sin \phi_a(t) + b \sin \phi_b(t) + d(t) \sin 270^\circ + e \sin 180^\circ = 0$$

$$x: r \sin \phi(t) = a(t) \cos \phi_a(t)$$

$$y: h + r \cos \phi(t) = a(t) \sin \phi_a(t)$$

$$x: c \cos \phi_a(t) + b \cos \phi_b(t) - e = 0$$

$$y: c \sin \phi_a(t) + b \sin \phi_b(t) - d(t) = 0$$

- ①  $x: r \sin \phi(t) = a(t) \cos \phi_a(t)$
- ②  $y: h + r \cos \phi(t) = a(t) \sin \phi_a(t)$
- ③  $x: c \cos \phi_a(t) + b \cos \phi_b(t) - e = 0$
- ④  $y: c \sin \phi_a(t) + b \sin \phi_b(t) - d(t) = 0$

- ①  $x: r \sin \phi(t) = a(t) \cos \phi_a(t)$
- ②  $y: h + r \cos \phi(t) = a(t) \sin \phi_a(t)$
- ③  $x: c \cos \phi_a(t) + b \cos \phi_b(t) - e = 0$
- ④  $y: c \sin \phi_a(t) + b \sin \phi_b(t) - d(t) = 0$

$$a(t) = \sqrt{r^2 \sin^2(t) + (h + r \cos \phi(t))^2}$$

$$\phi_a(t) = \operatorname{atan} \left( \frac{h + r \cos \phi(t)}{r \sin \phi(t)} \right)$$

$$\phi_b(t) = \arccos \frac{e - c \cos \phi_a(t)}{b}$$

$$d(t) = c \sin \phi_a(t) + b \sqrt{1 - \left( \frac{e - c \cos \phi_a(t)}{b} \right)^2}$$

- ①  $x: r \sin \phi(t) = a(t) \cos \phi_a(t)$
- ②  $y: h + r \cos \phi(t) = a(t) \sin \phi_a(t)$
- ③  $x: c \cos \phi_a(t) + b \cos \phi_b(t) - e = 0$
- ④  $y: c \sin \phi_a(t) + b \sin \phi_b(t) - d(t) = 0$

$$a(t) = \sqrt{r^2 \sin^2(t) + (h + r \cos \phi(t))^2}$$

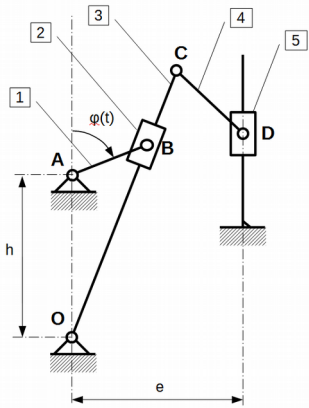
$$\phi_a(t) = \operatorname{atan}\left(\frac{h + r \cos \phi(t)}{r \sin \phi(t)}\right)$$

$$\phi_b(t) = \arccos \frac{e - c \cos \phi_a(t)}{b}$$

$$d(t) = c \sin \phi_a(t) + b \sqrt{1 - \left(\frac{e - c \cos \phi_a(t)}{b}\right)^2}$$

$$v_D(t) = \dot{d}(t)$$

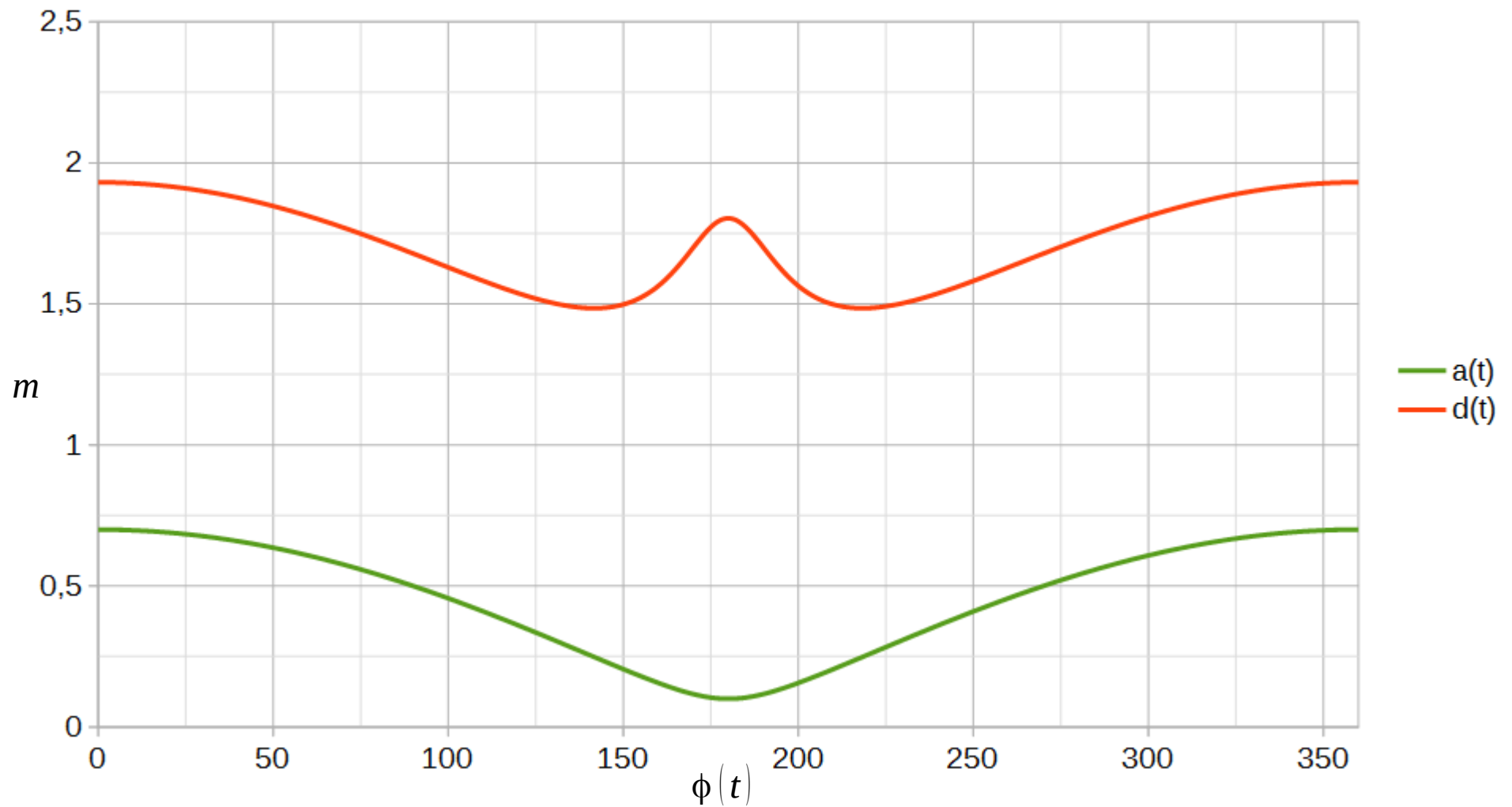
$$a_D(t) = \ddot{d}(t)$$

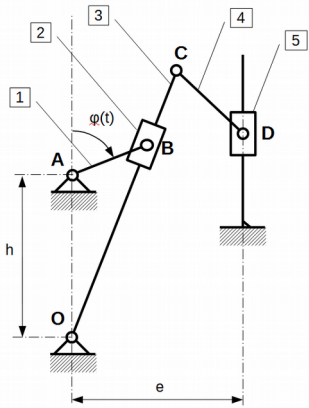


$$a(t) = \sqrt{r^2 \sin^2(t) + (h + r \cos \phi(t))^2}$$

$$d(t) = c \sin \phi_a(t) + b \sqrt{1 - \left( \frac{e - c \cos \phi_a(t)}{b} \right)^2}$$

<b>r=</b>	0,3	[m]
<b>h=</b>	0,4	[m]
<b>c=</b>	1	[m]
<b>e=</b>	0,4	[m]
<b>b=</b>	1	[m]
<b>ω=</b>	1	[rad/s]

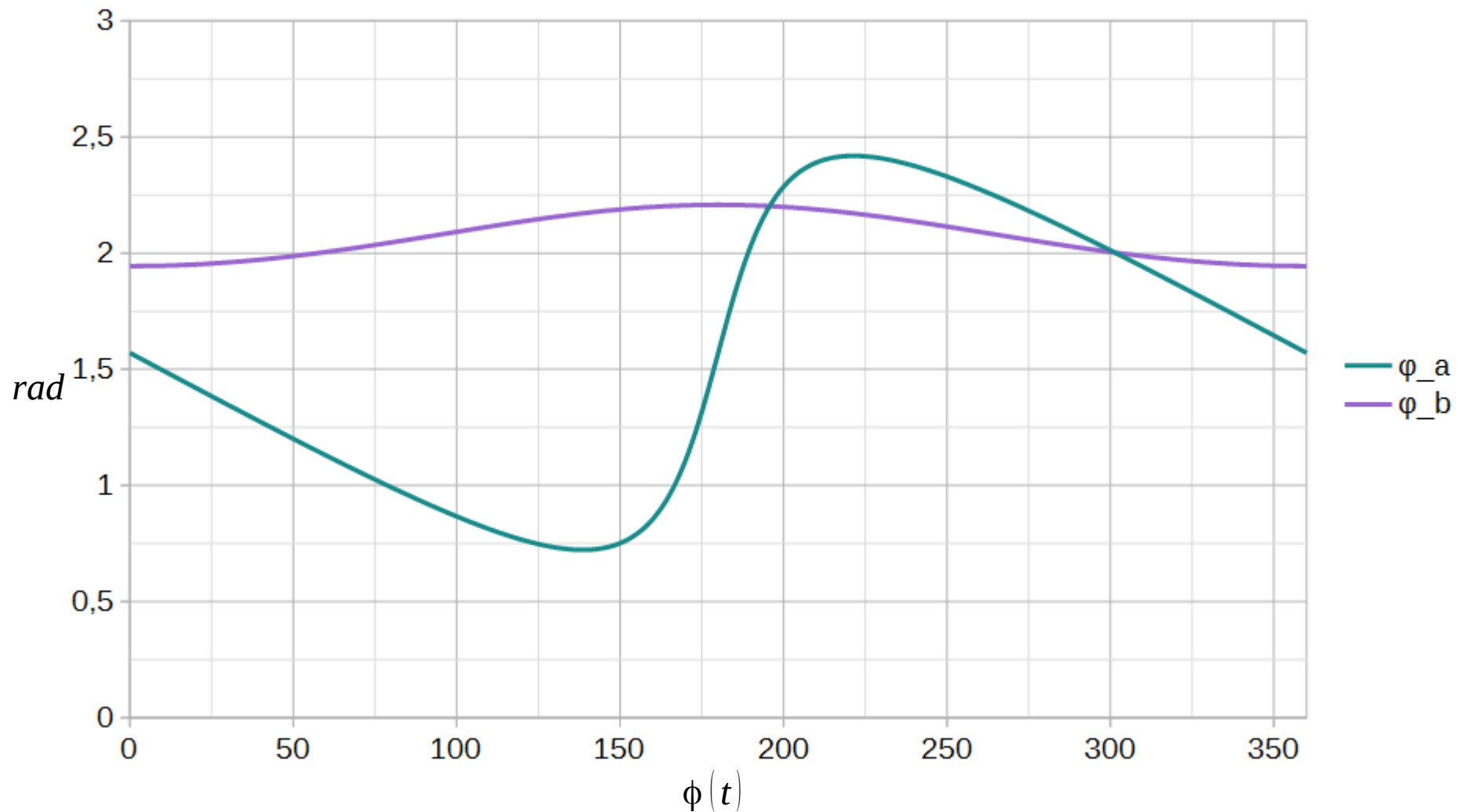


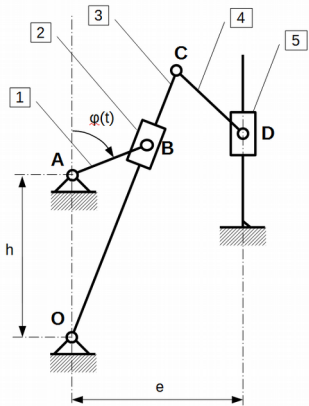


$$\phi_a(t) = \text{atan} \left( \frac{h + r \cos \phi(t)}{r \sin \phi(t)} \right)$$

$$\phi_b(t) = \arccos \frac{e - c \cos \phi_a(t)}{b}$$

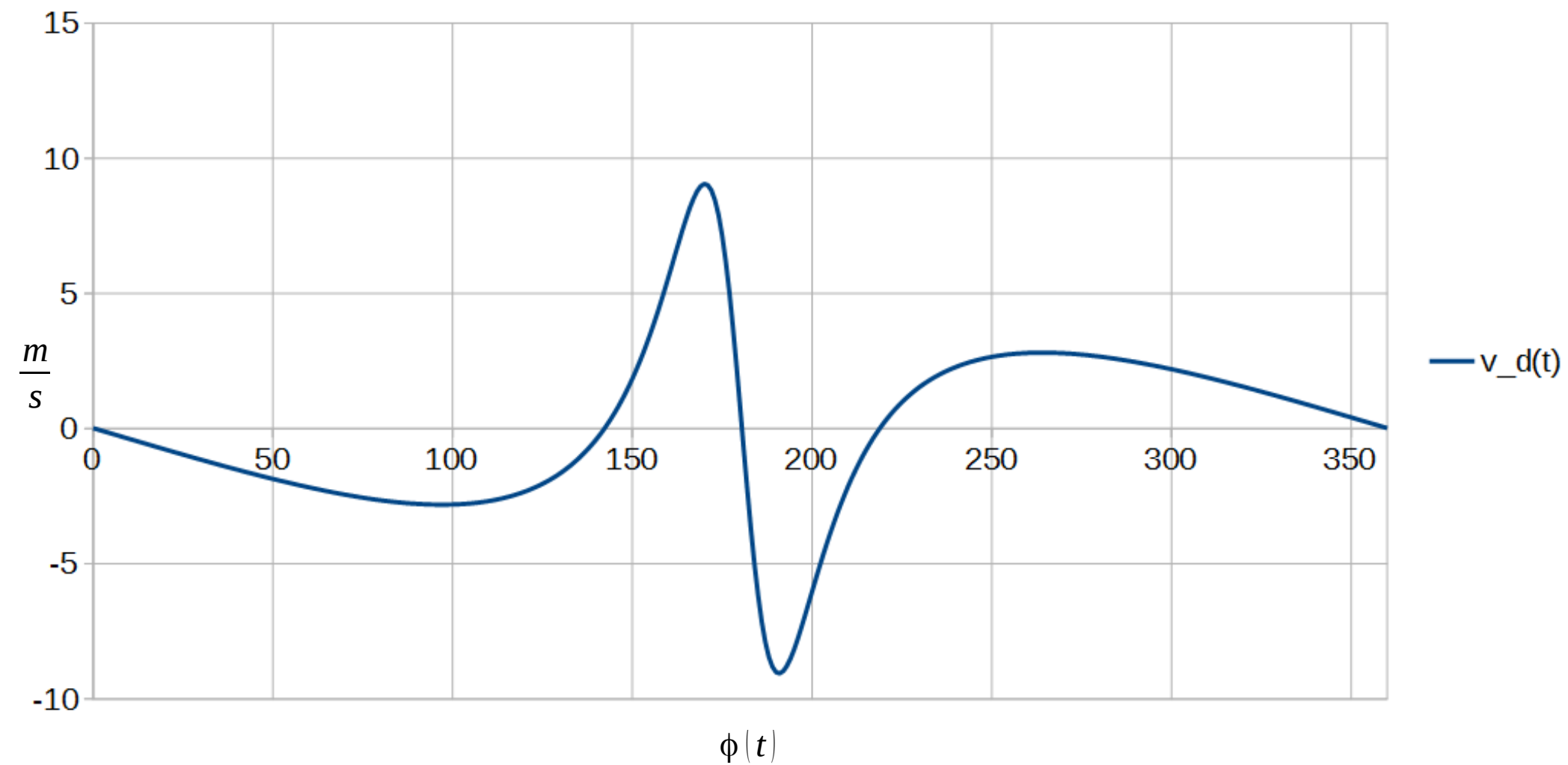
<b>r=</b>	0,3	[m]
<b>h=</b>	0,4	[m]
<b>c=</b>	1	[m]
<b>e=</b>	0,4	[m]
<b>b=</b>	1	[m]
<b>ω=</b>	1	[rad/s]



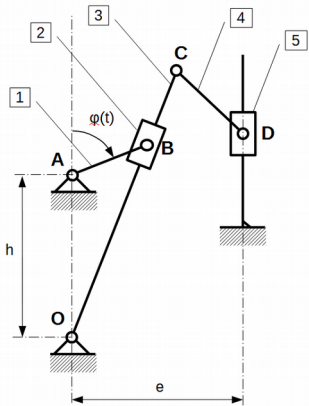


<b>r=</b>	0,3	[m]
<b>h=</b>	0,4	[m]
<b>c=</b>	1	[m]
<b>e=</b>	0,4	[m]
<b>b=</b>	1	[m]
<b>ω=</b>	1	[rad/s]

$$v_D(t) = \dot{d}(t)$$







<b>r=</b>	0,3	[m]
<b>h=</b>	0,4	[m]
<b>c=</b>	1	[m]
<b>e=</b>	0,4	[m]
<b>b=</b>	1	[m]
<b>ω=</b>	1	[rad/s]

$$a_D(t) = \ddot{d}(t)$$

