Warsaw University of Technology
The Faculty of Automotive
and Construction Machinery Engineering
Institute of Machine Design Fundamentals
Department of Mechanics
http://www.ipbm.simr.pw.edu.pl/

Theory of Machines and Automatic Control

Winter 2017/2018

Lecturer: Sebastian Korczak, PhD, Eng.

Theory of Machines and Automatic Control

Winter 2017/2018

Field of studies: Electric and Hybrid Vehicle Engineering (full-time)

form of studies: 30 hrs lecture, 15 hrs project class

ECTS: 4

Lecture: Tuesdays at 8:15 (room 3.3)

Projects: Wednesdays at 9:15 (room 3.8)
1st meeting on 8th November

Lecturer: Sebastian Korczak, PhD, Eng.
room: 2.8b
e-mails: sebastian.korczak@simr.pw.edu.pl, admin@myinventions.pl
consultations: Tuesdays at 10:00-11:00 and Thursdays at 12:00-13:00
website with materials and marks: http://myinventions.pl/lectures/

Lecture 3

Accelerations in planar mechanisms.

Kinematics of mechanisms

Kinematic analysis of a mechanism – determination of velocities and accelerations of selected mechanism members' points at considered configuration. Mechanism structure must be given (geometry of members, kinematic pairs) and drive method must be known.

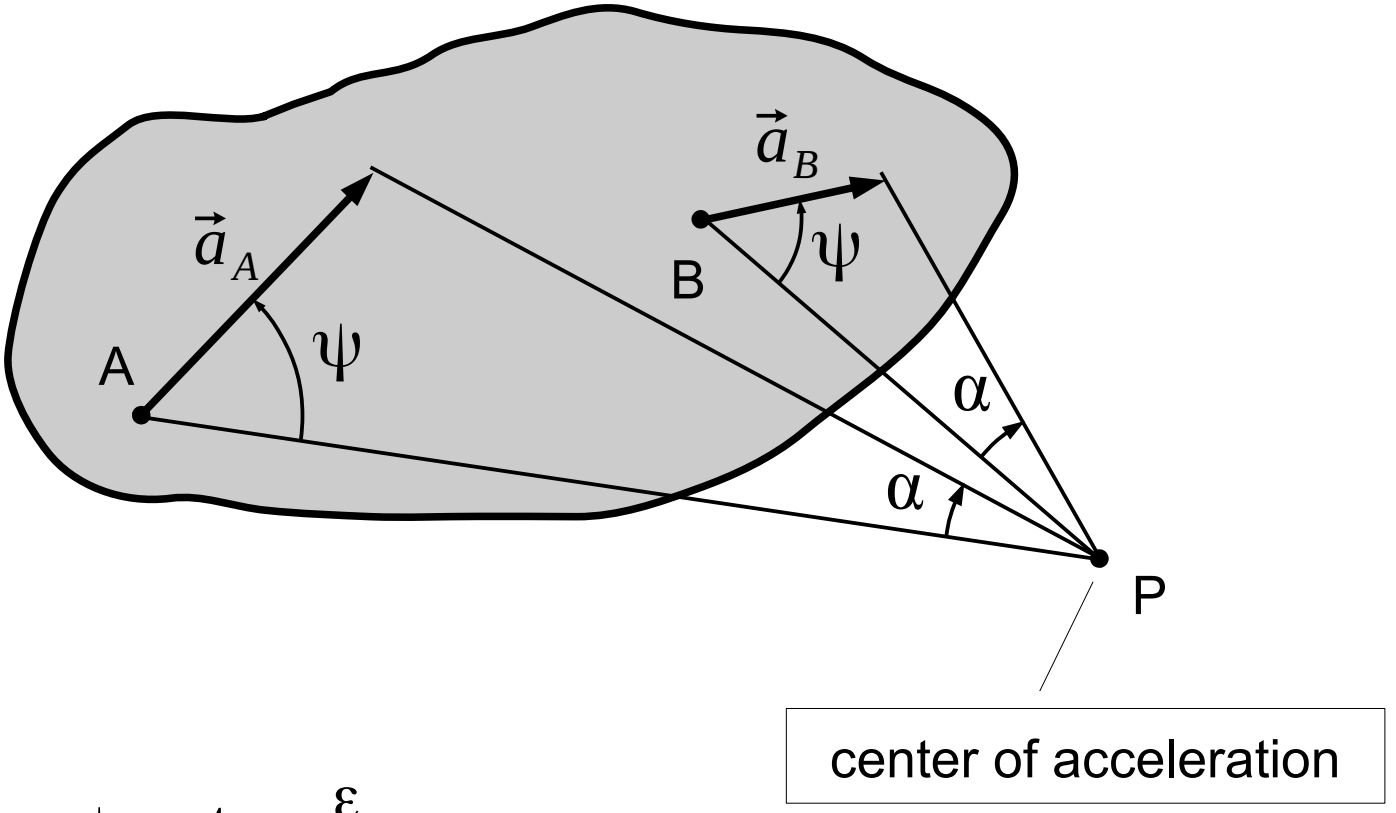
Methods of velocities and acceleration determination

Graphical methods

- velocity projection method,
- instantaneous center of rotation method,
- instantaneous center of acceleration method,
- method of rotated velocities,
- velocity decomposition method,
- acceleration decomposition method,
- velocity scheme (diagram) method,
- accelerations scheme (diagram) method.

Analytical method

Instantaneous center of acceleration



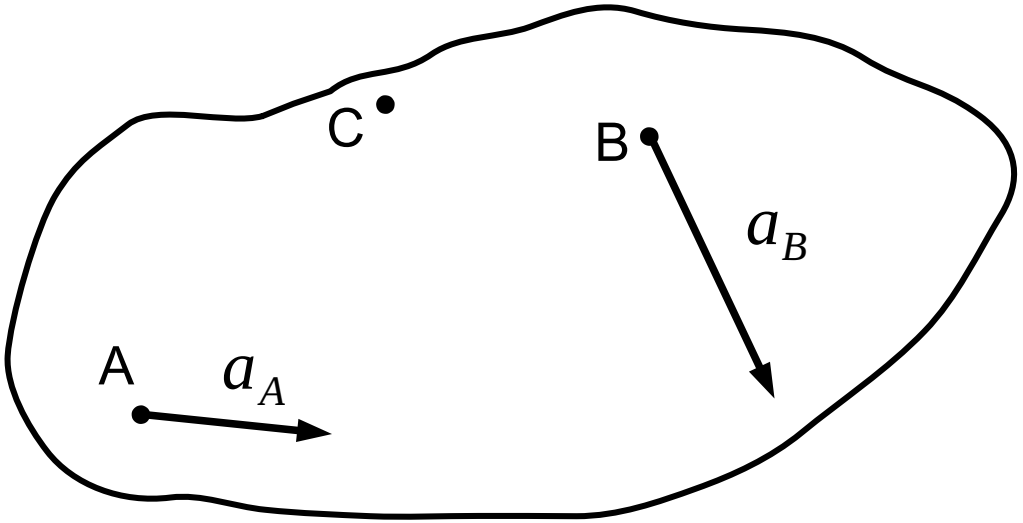
$$\psi = \operatorname{atan} \frac{\varepsilon}{\omega^2}$$

Instantaneous center of acceleration method

Example

Given: $\bar{a}_A$ and $\bar{a}_B$

Searched: $\bar{a}_C$



Instantaneous center of acceleration method

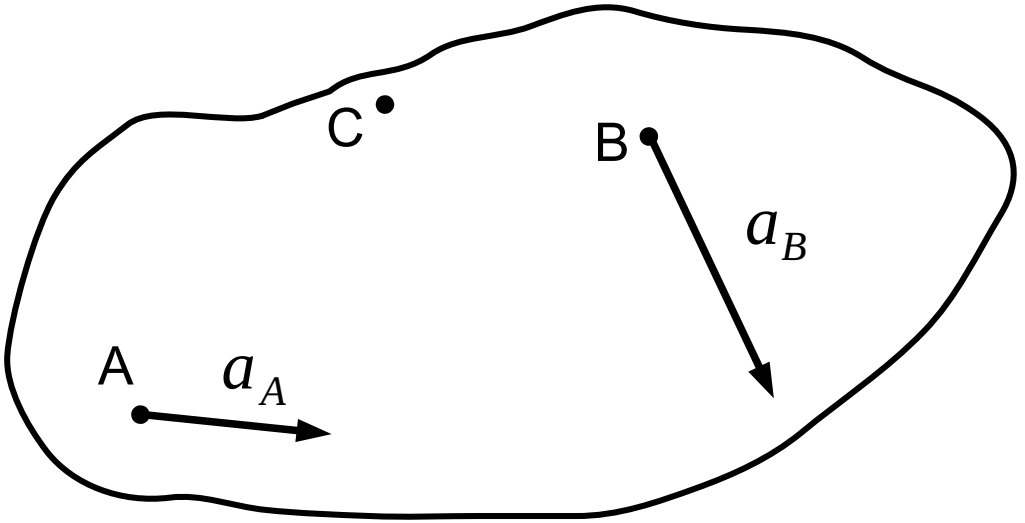
Example

Given: a_A and a_B

Searched: a_C

1. STEP:

construction of ψ



Instantaneous center of acceleration method

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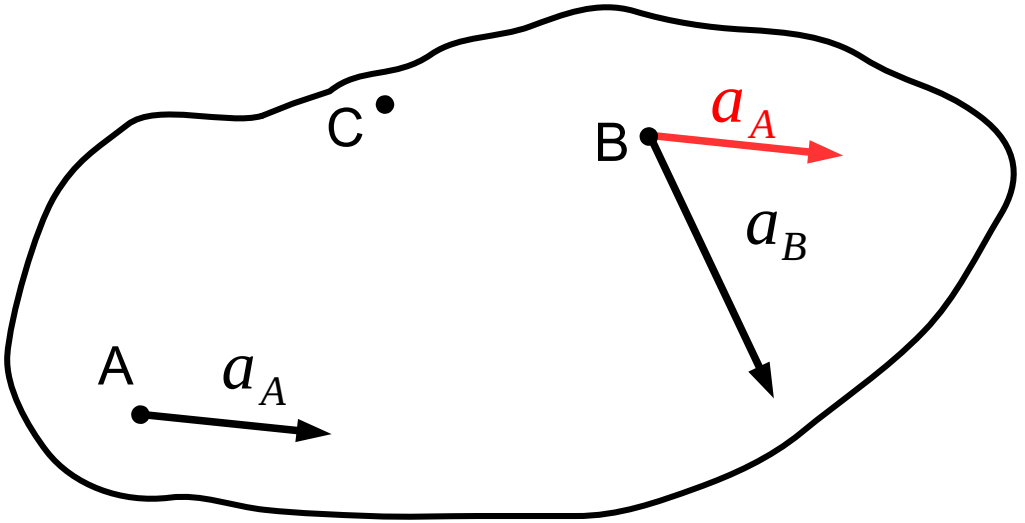

Instantaneous center of acceleration method

Example

$$a_B = a_A + a_{BA}$$

Given: a_A and a_B

Searched: a_C

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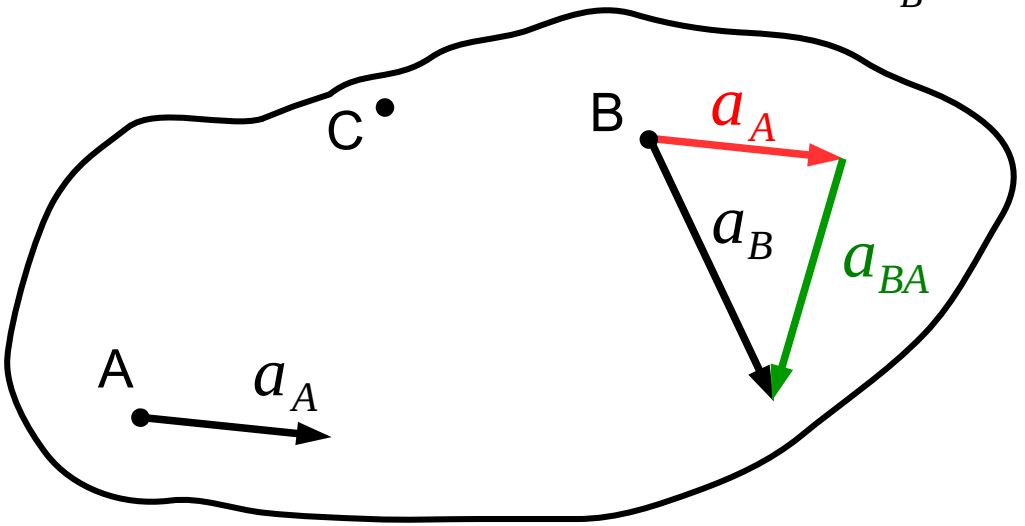
construction of ψ



Instantaneous center of acceleration method

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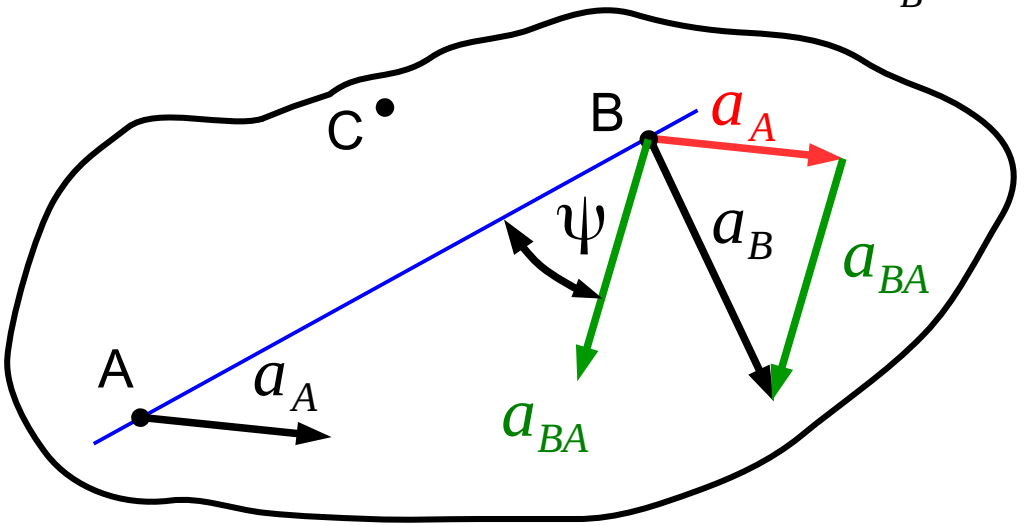

Instantaneous center of acceleration method

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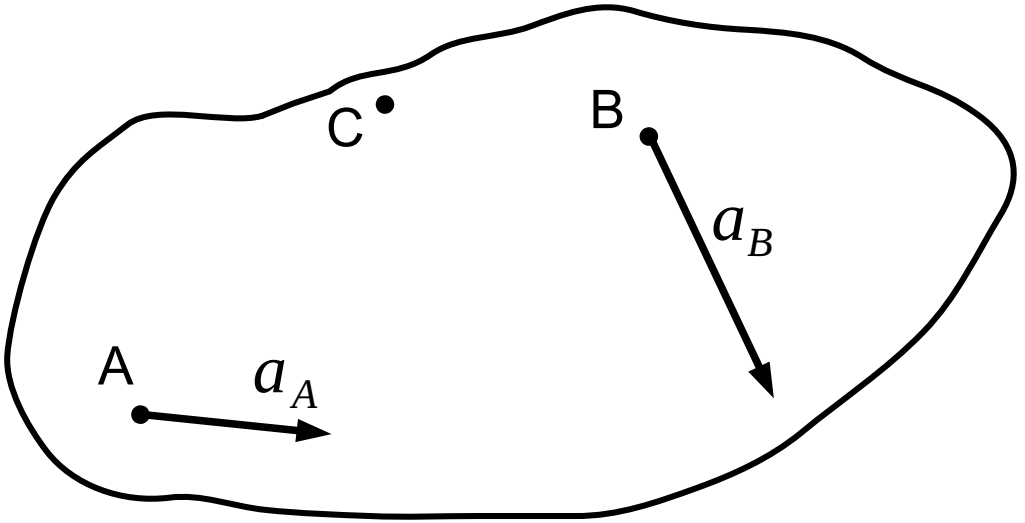

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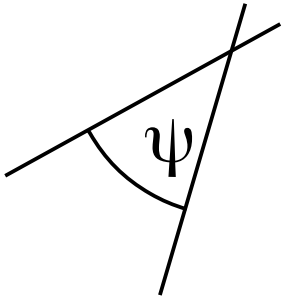

Instantaneous center of acceleration method

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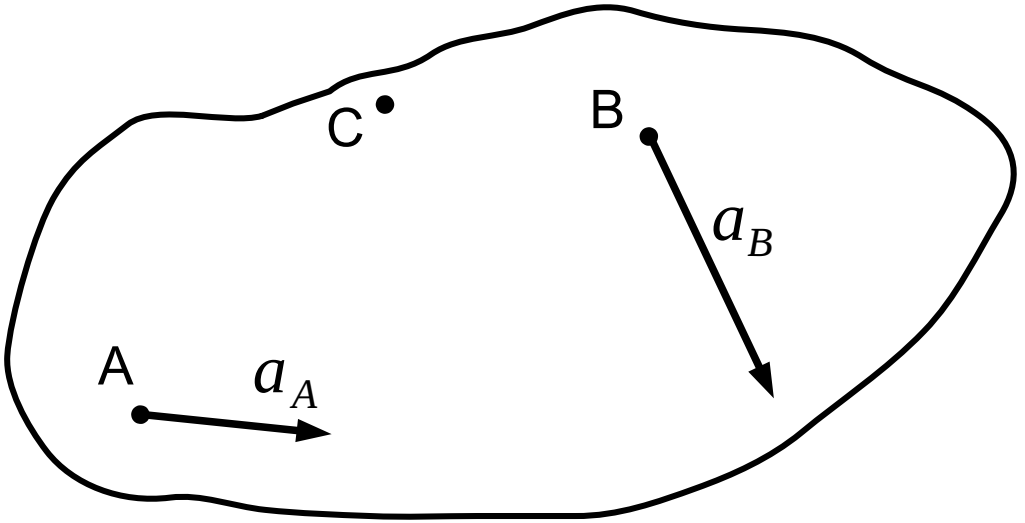

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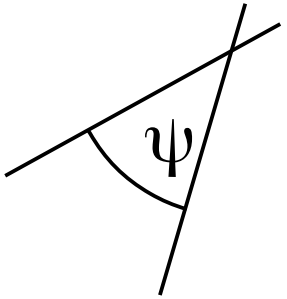
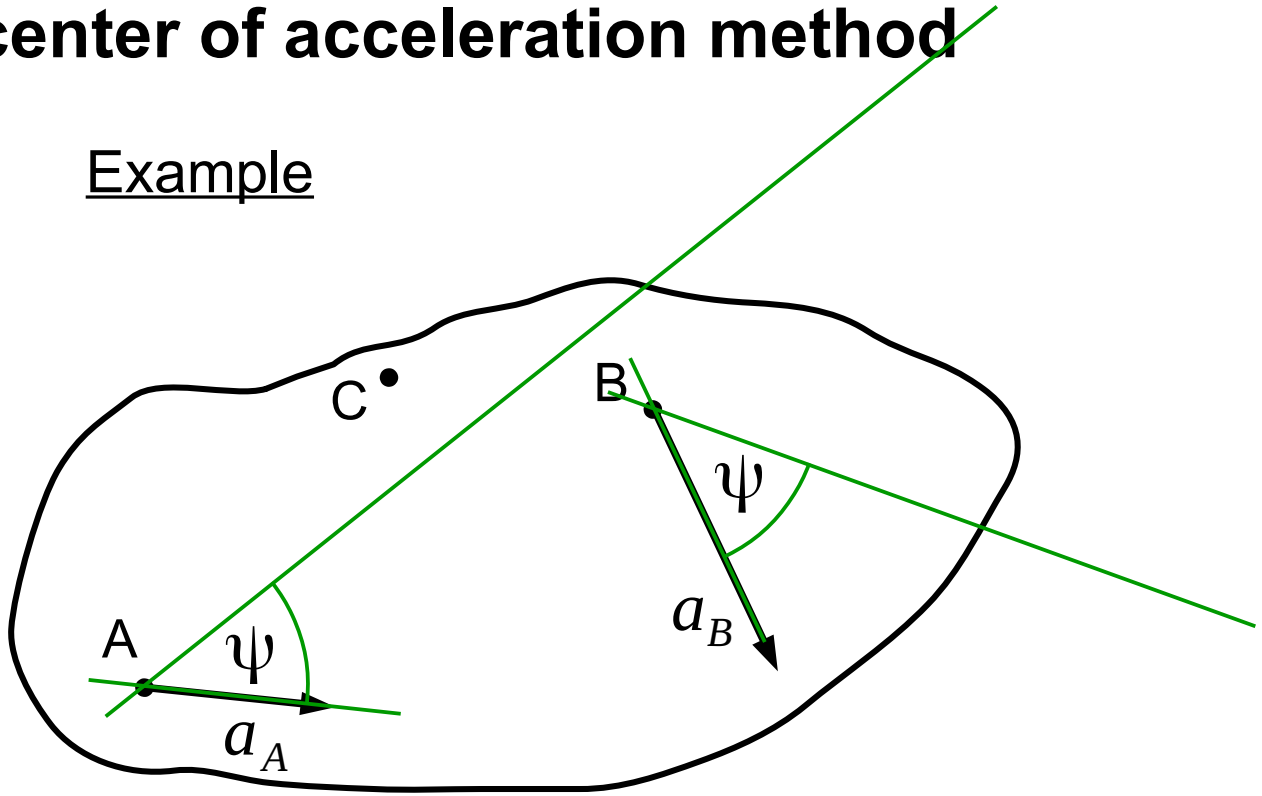


2. STEP: find out the center of acceleration

Instantaneous center of acceleration method

Example



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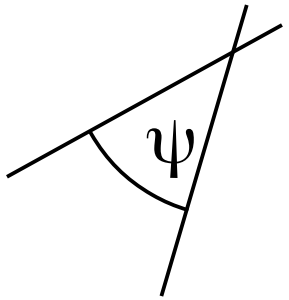
construction of ψ



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Instantaneous center of acceleration method

Example

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Searched: a_C

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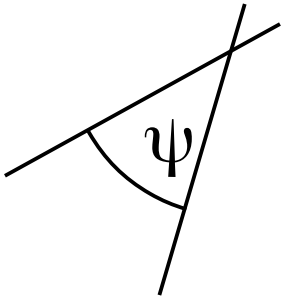

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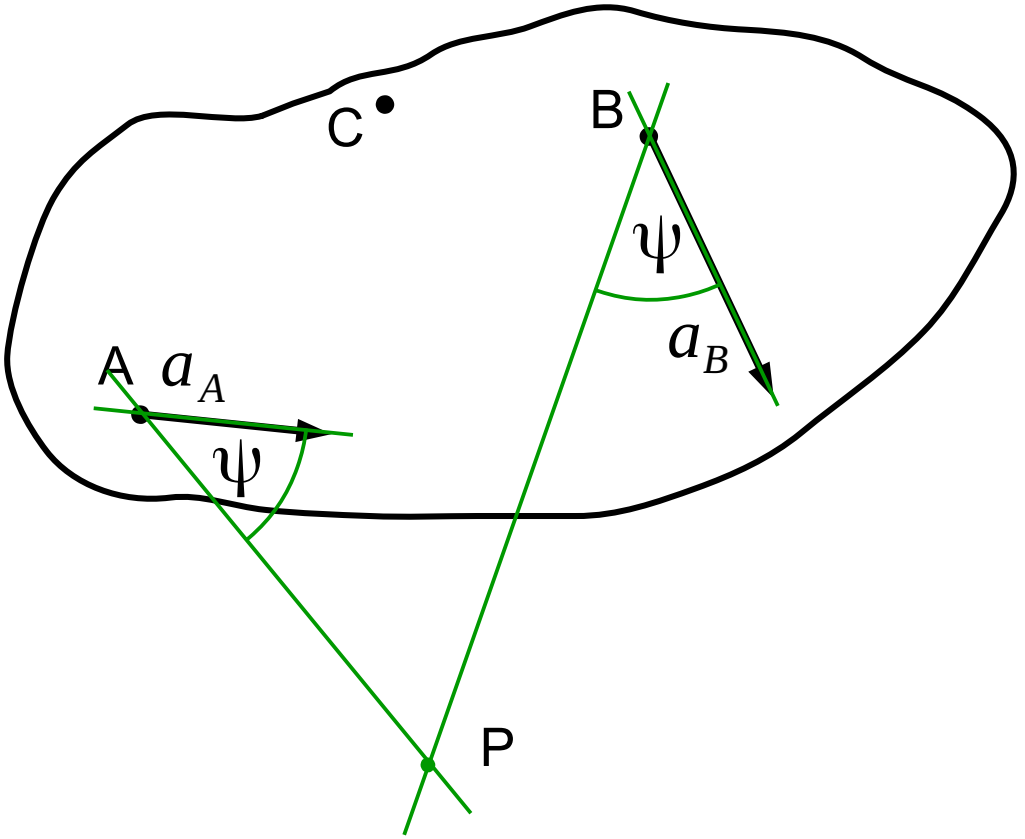

Instantaneous center of acceleration method

Example

Given: a_A and a_B

Searched: a_C

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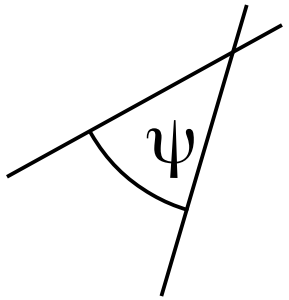
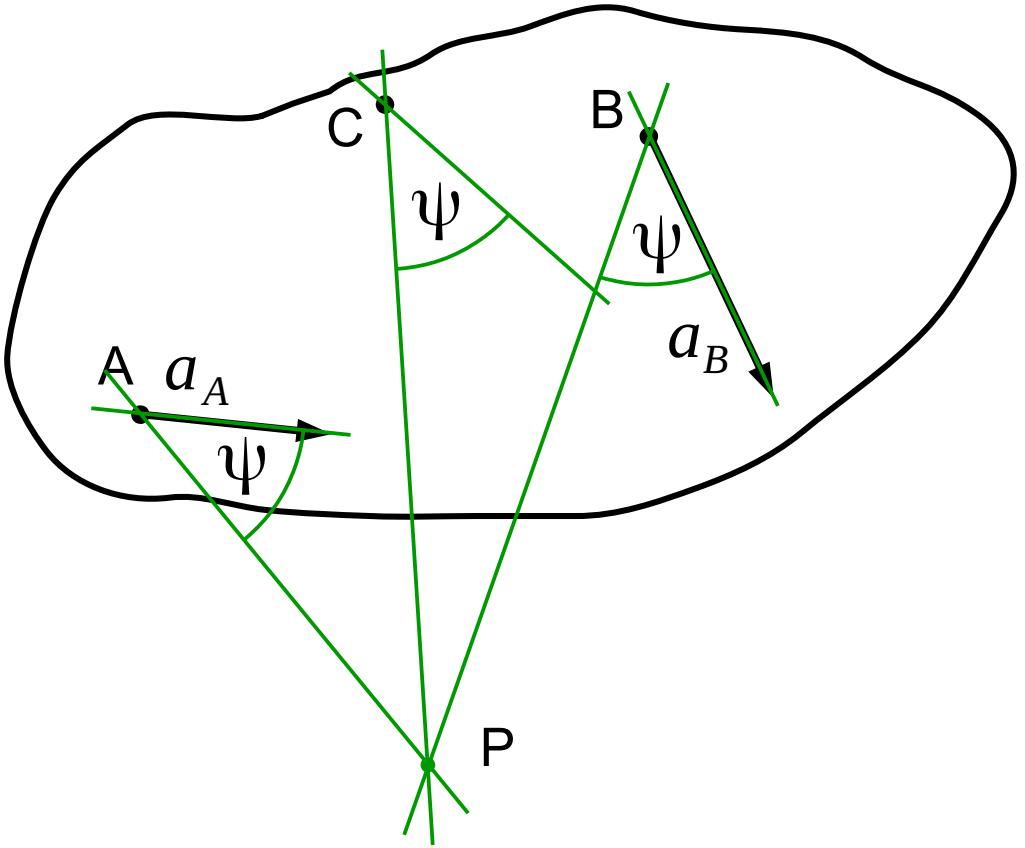
construction of ψ



2. STEP: find out the center of acceleration

3. STEP: find out a_C



Instantaneous center of acceleration method

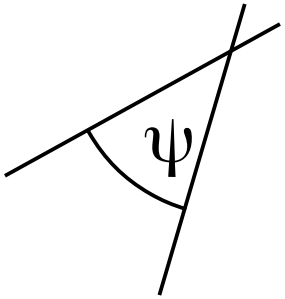
Example

Given: $\bar{a}_A$ and $\bar{a}_B$

Searched: $\bar{a}_C$

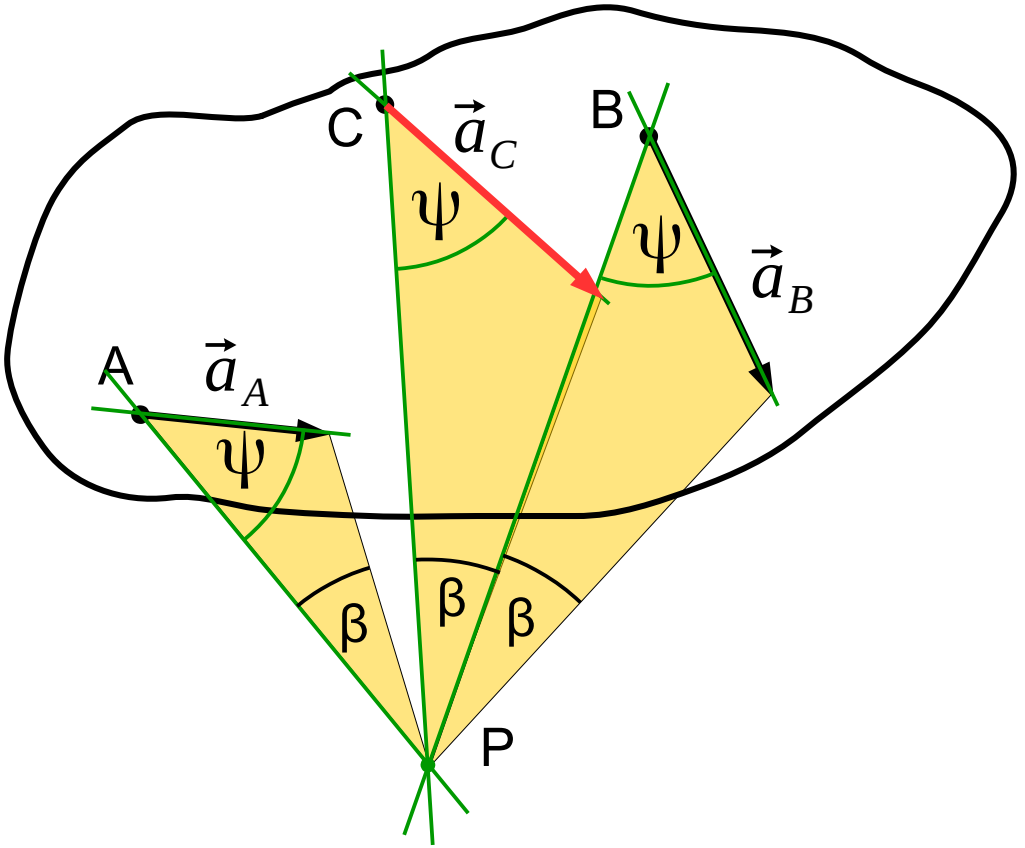
1. STEP:

construction of ψ



2. STEP: find out the center of acceleration

3. STEP: find out $\bar{a}_C$

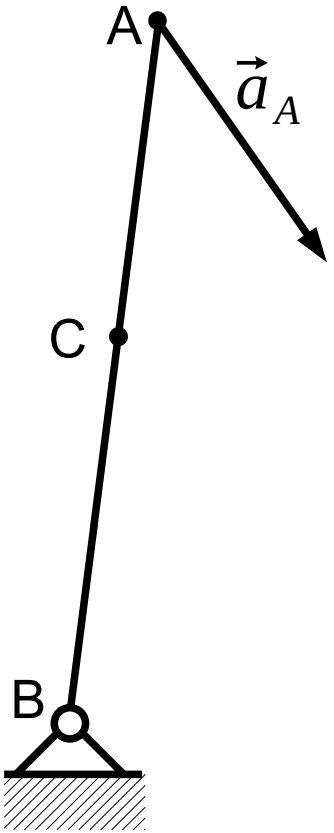


Instantaneous center of acceleration method

Example 2

Given: $\bar{a}_A$

Searched: $\bar{a}_C$

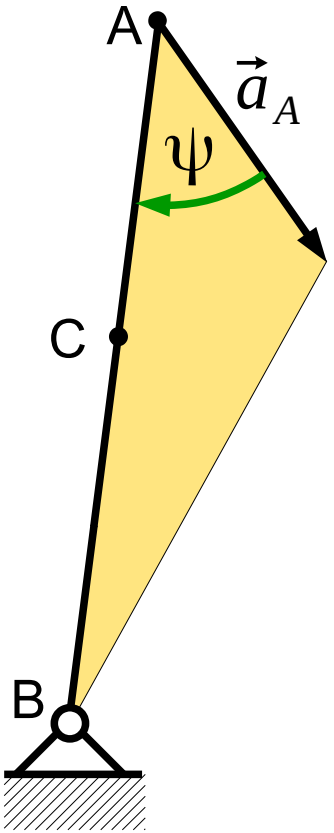


Instantaneous center of acceleration method

Example 2

Given: $\bar{a}_A$

Searched: $\bar{a}_C$

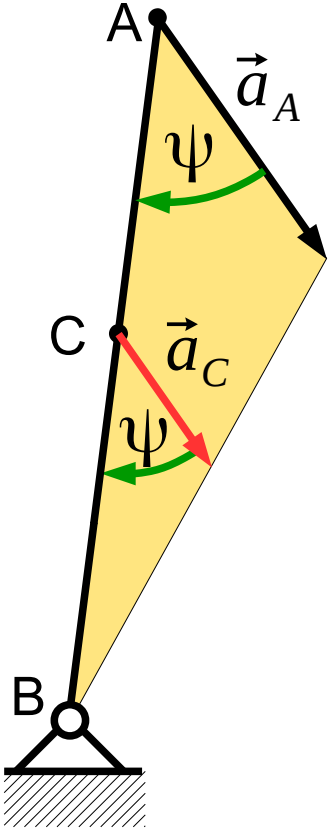


Instantaneous center of acceleration method

Example 2

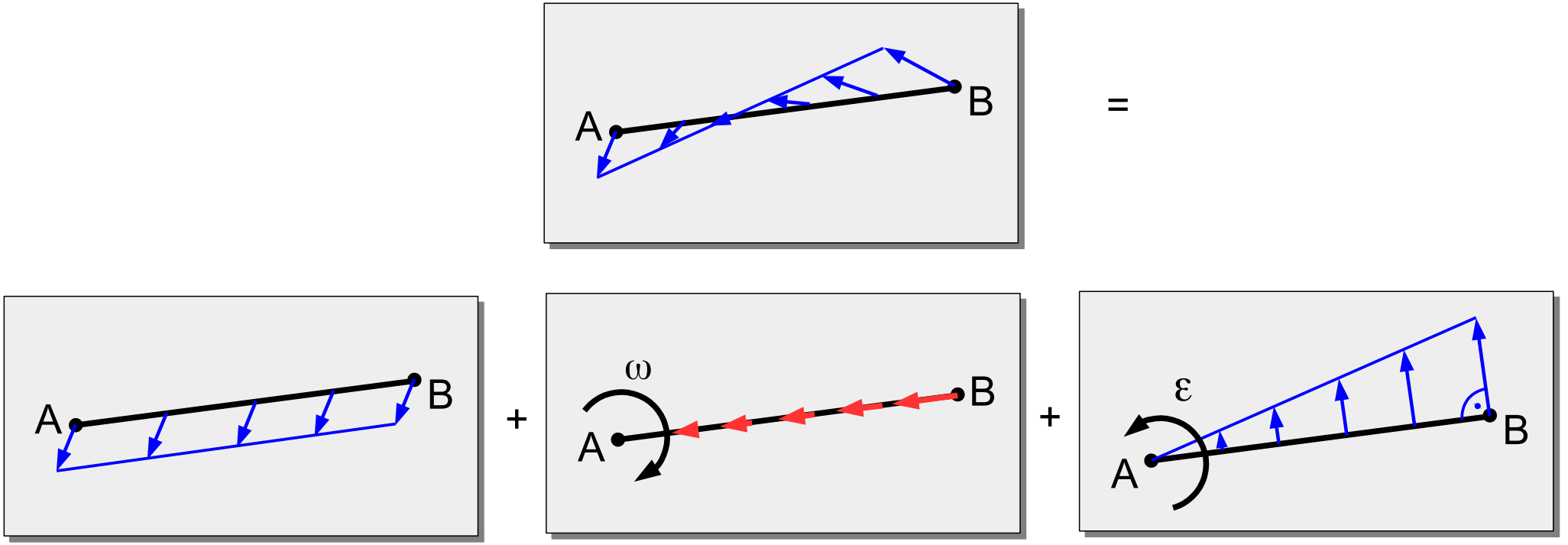
Given: $\bar{a}_A$

Searched: $\bar{a}_C$



Acceleration decomposition method

Example



Acceleration decomposition method

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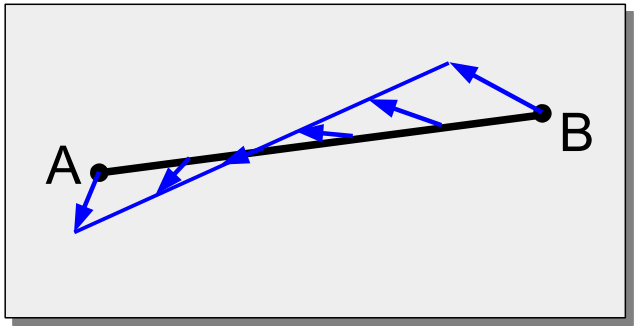

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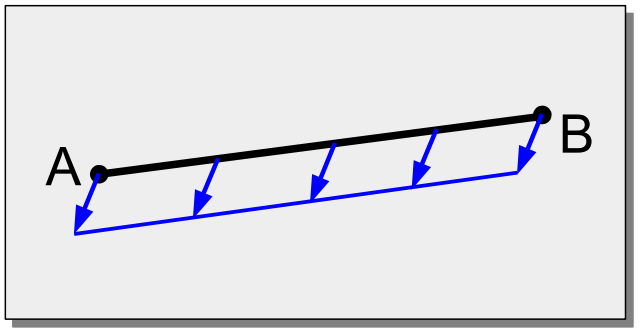

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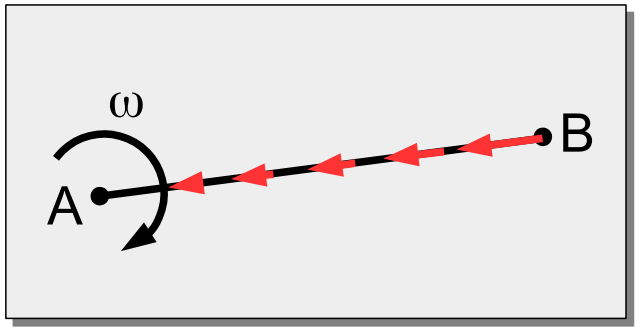

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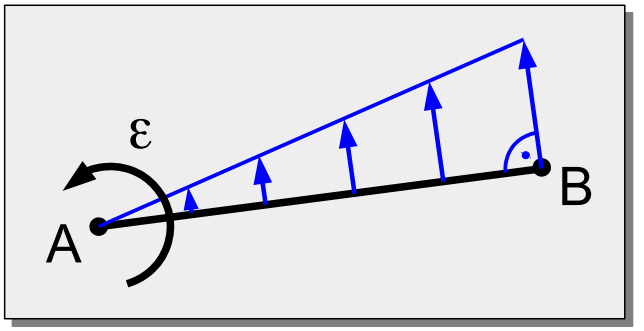


$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}^{\,n}_{BA} + \vec{a}^{\,t}_{BA}$$

absolute acceleration of point B

absolute acceleration of point A

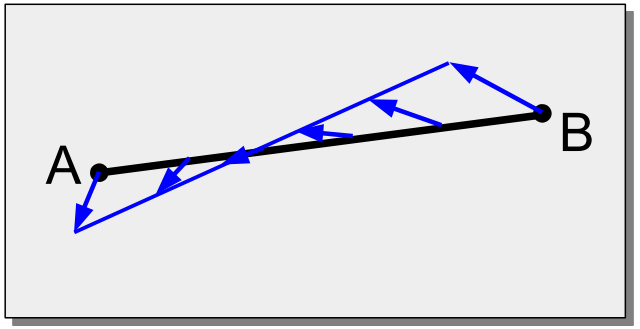
Angular acceleration of point B in rotation around point A.

Centripetal acceleration (normal)

Rotary acceleration (tangential)

Acceleration decomposition method

Example



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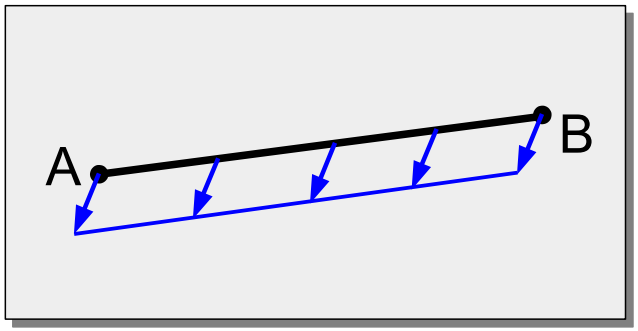

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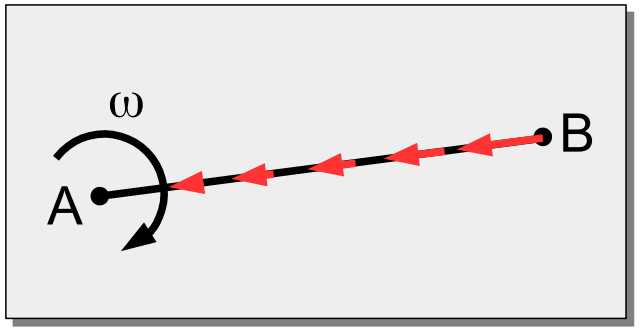

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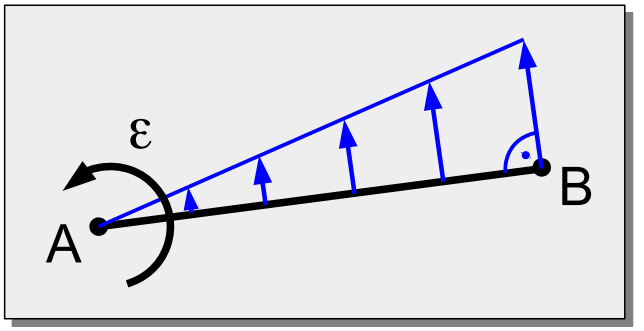


$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}^{\,n}_{BA} + \vec{a}^{\,t}_{BA}$$

Centripetal acceleration (normal)

Rotary acceleration (tangential)

$$\vec{a}_{BA} = \vec{\varepsilon} \times \overrightarrow{AB}$$

$$\vec{a}_{BA} = \vec{\omega} \times \left(\vec{\omega} \times \overrightarrow{AB} \right) = -\omega^2 \overrightarrow{AB}$$

Acceleration scheme (diagram)

Acceleration scheme of a rigid body – geometry created by the ends of it's acceleration vectors moved to the common starting point (acceleration scheme's pole).

Acceleration scheme is similar to the corresponding rigid body:

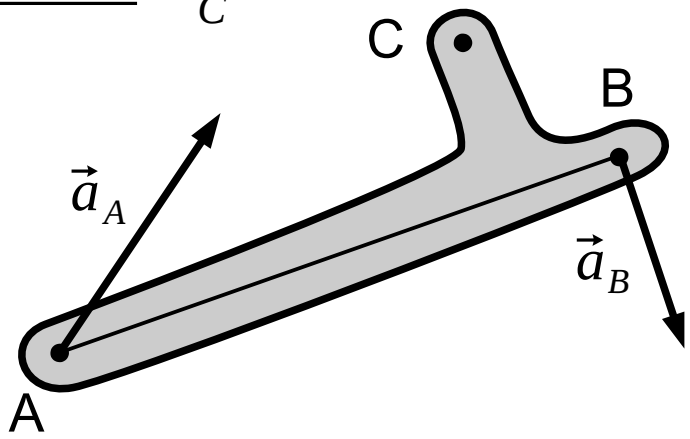
it is scaled and rotated by $(180^{o}-\psi)$ *angle in the direction of body's angular velocity if* $\mathrm{sgn}\omega=\mathrm{sgn}\varepsilon$ *(or opposite direction if* $\mathrm{sgn}\omega\neq\mathrm{sgn}\varepsilon$*).*

Acceleration scheme method

Example

Given: $\bar{a}_A$ and $\bar{a}_B$ + geometry

Searched: $\bar{a}_C$



Acceleration scheme method

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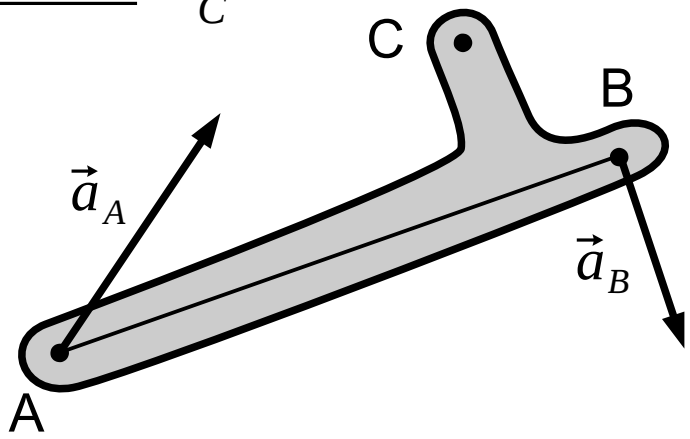
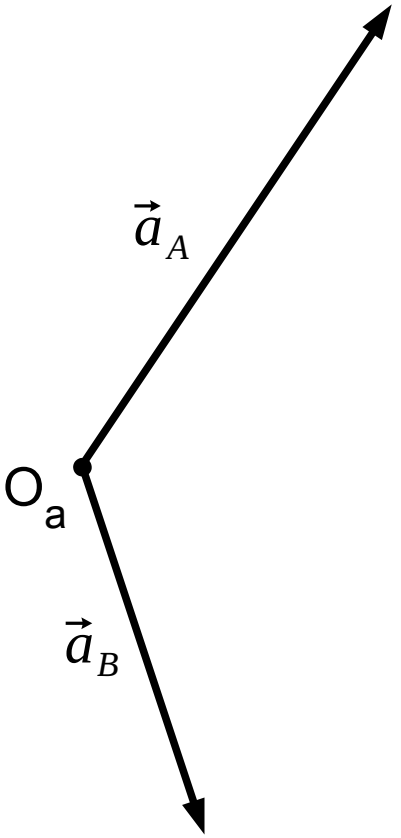




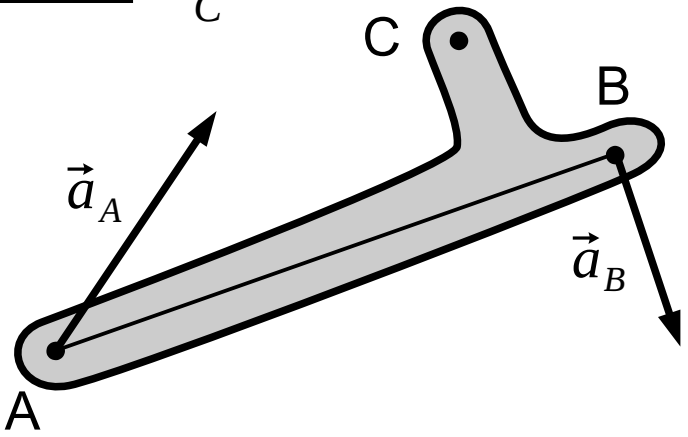
acceleration scale, e.g.: 1cm → 1m/s

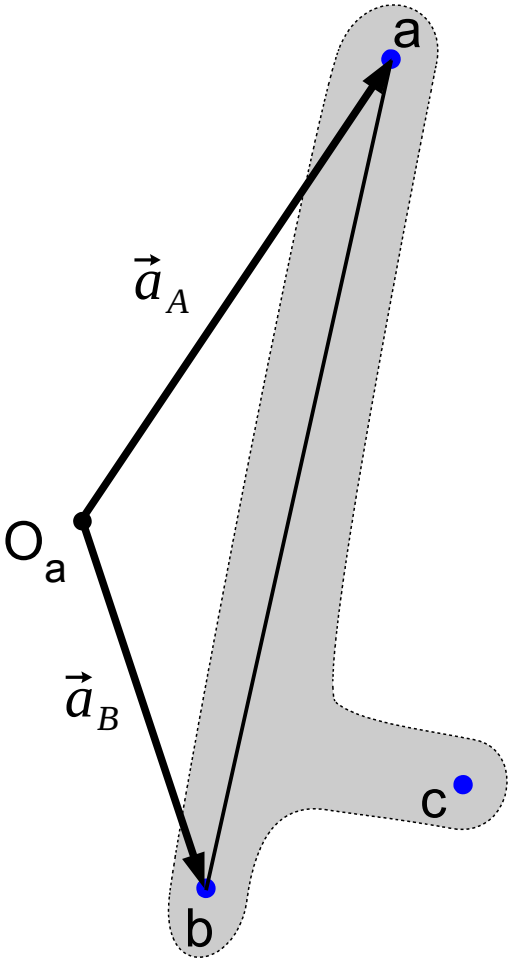
Acceleration scheme method

Example

Given: $\bar{a}_A$ and $\bar{a}_B$ + geometry

Searched: $\bar{a}_C$





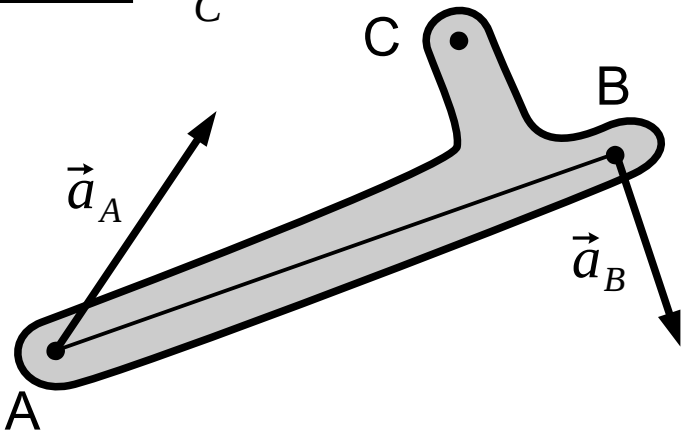
acceleration scale, e.g.: 1cm → 1m/s
geometry scale wrt. original dimensions

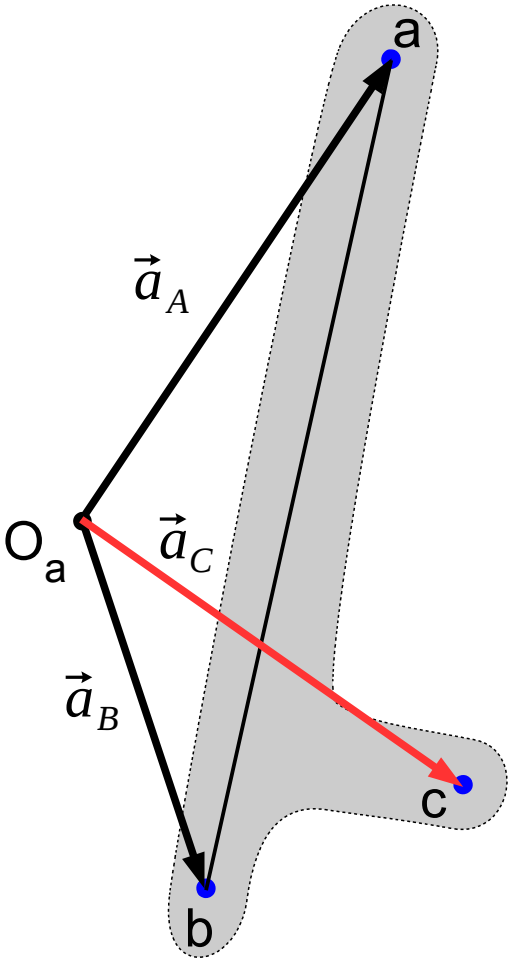
Acceleration scheme method

Example

Given: $\bar{a}_A$ and $\bar{a}_B$ + geometry

Searched: $\bar{a}_C$





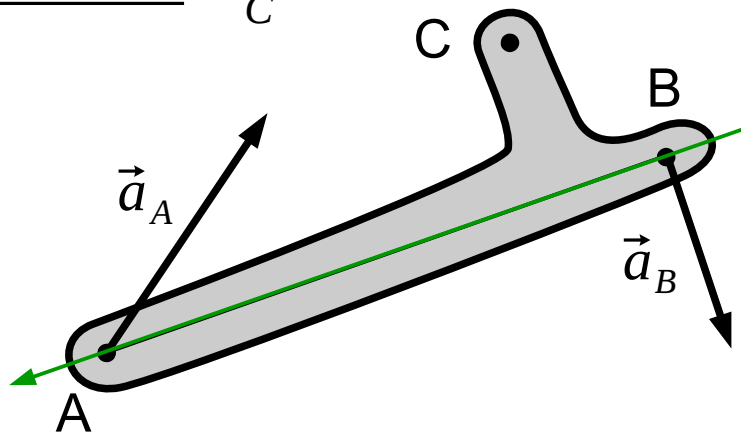
acceleration scale, e.g.: 1cm → 1m/s²
geometry scale wrt. original dimensions

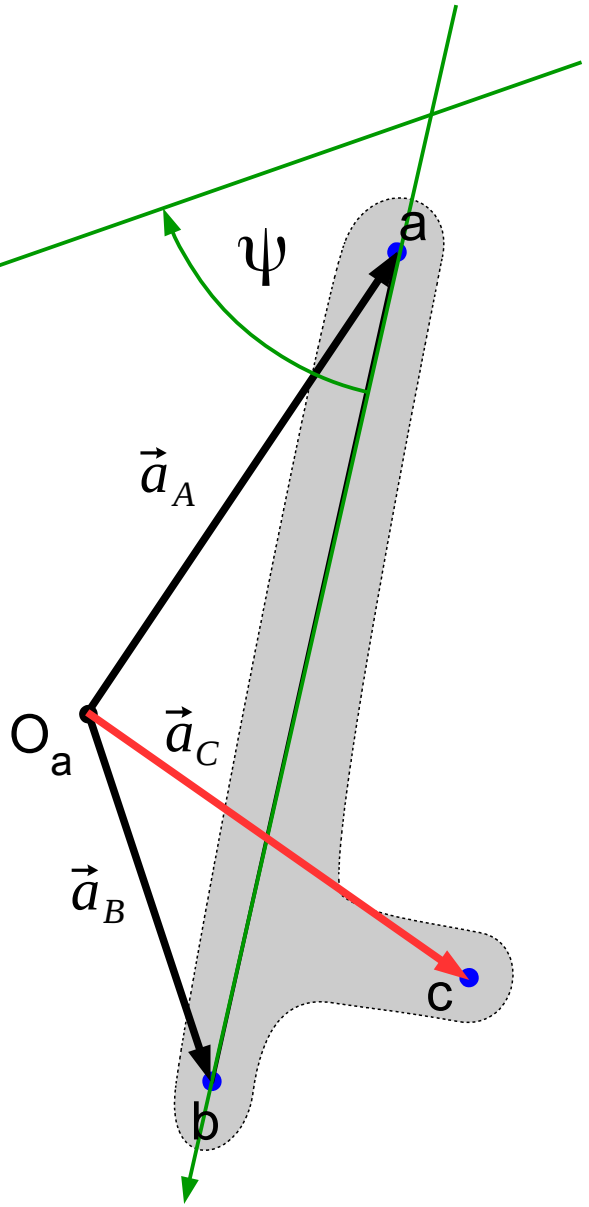
Acceleration scheme method

Example

Given: $\bar{a}_A$ and $\bar{a}_B$ + geometry

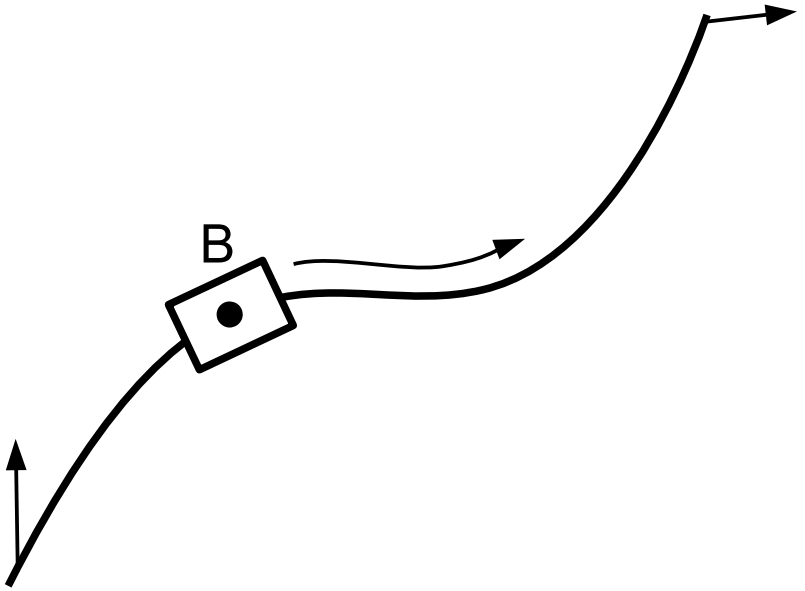
Searched: $\bar{a}_C$



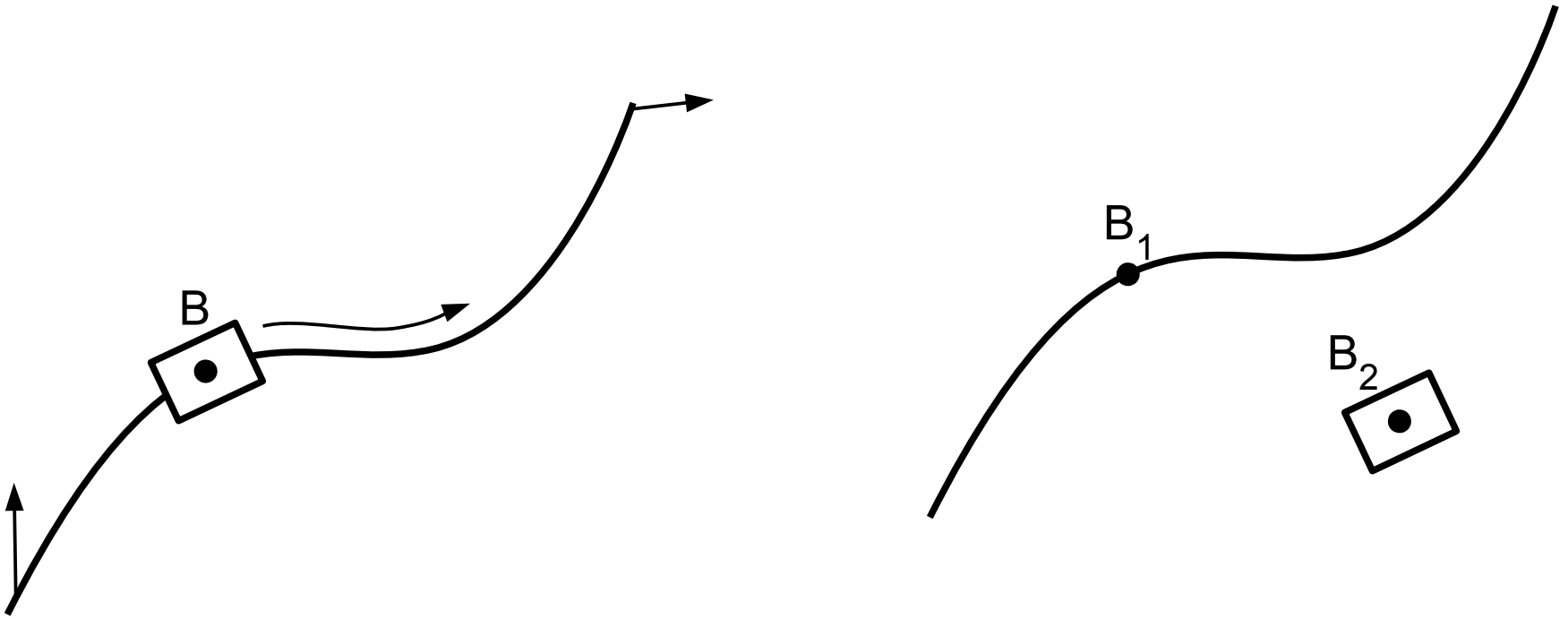


acceleration scale, e.g.: 1cm → 1m/s
geometry scale wrt. original dimensions

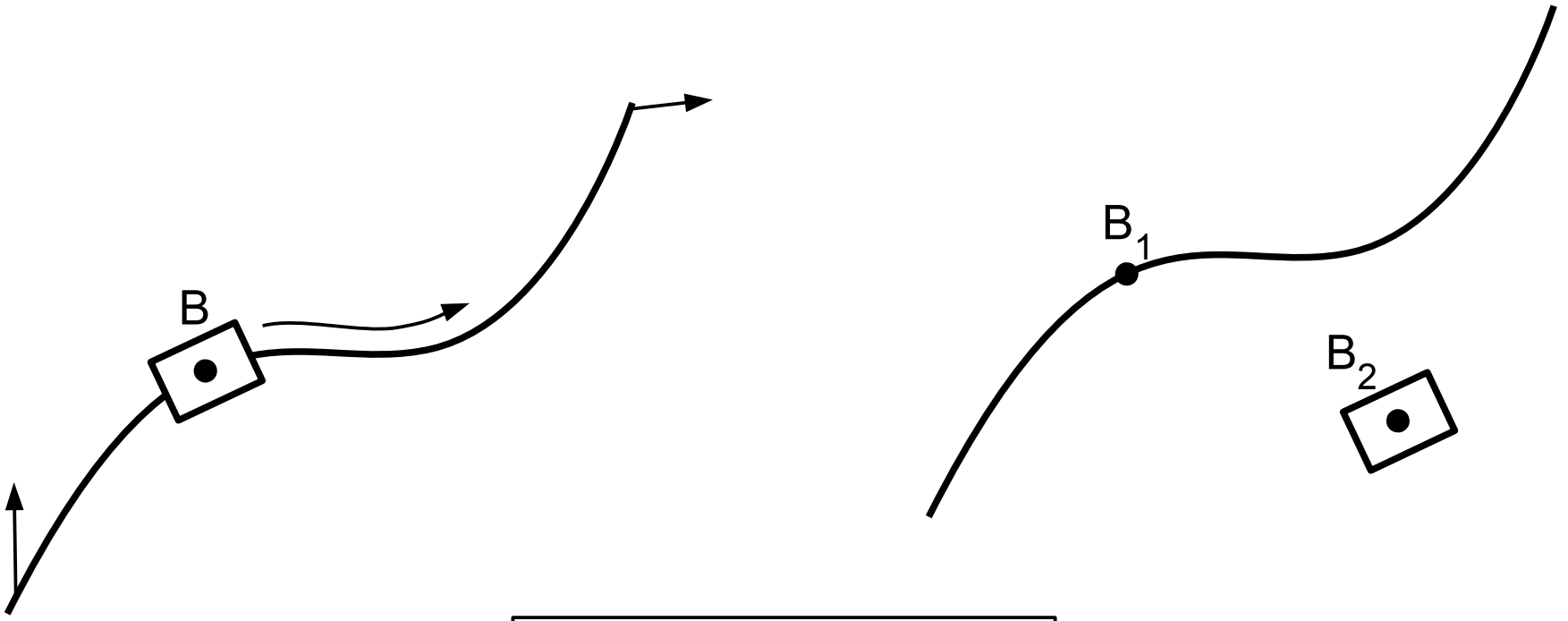
Accelerations in relative motion



Accelerations in relative motion



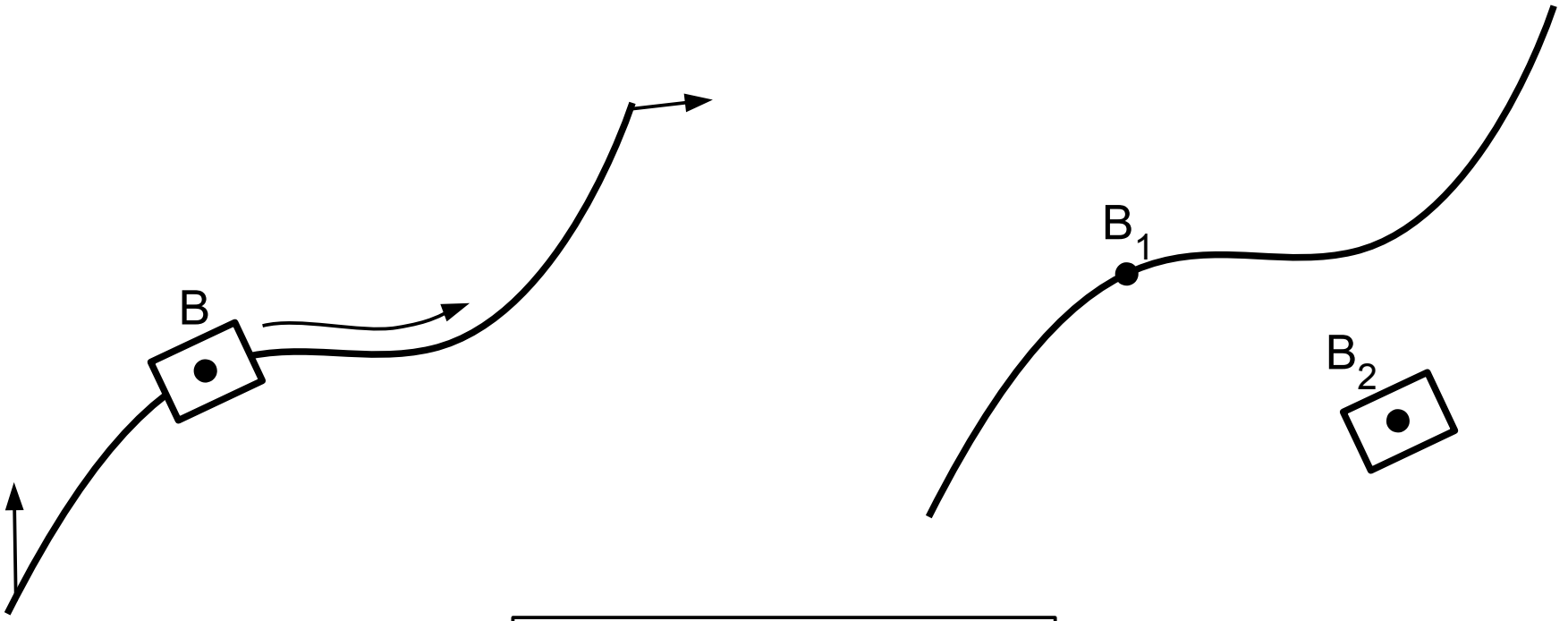
Accelerations in relative motion



$$\vec{a}_{B2} = \vec{a}^{u}_{B1} + \vec{a}^{w}_{B2B1} + \vec{a}^{c}$$

- $\vec{a}_{B2}$ — absolute acceleration of point B2
- $\vec{a}^{u}_{B1}$ — Transportation acceleration (absolute acceleration of point B1)
- $\vec{a}^{w}_{B2B1}$ — Relative acceleration
- $\vec{a}^{c}$ — Coriolis acceleration

Accelerations in relative motion



$$\vec{a}_{B2} = \vec{a}^{u}_{B1} + \vec{a}^{w}_{B2B1} + \vec{a}^{c}$$

absolute acceleration of point B2

Transportation acceleration (absolute acceleration of point B1)

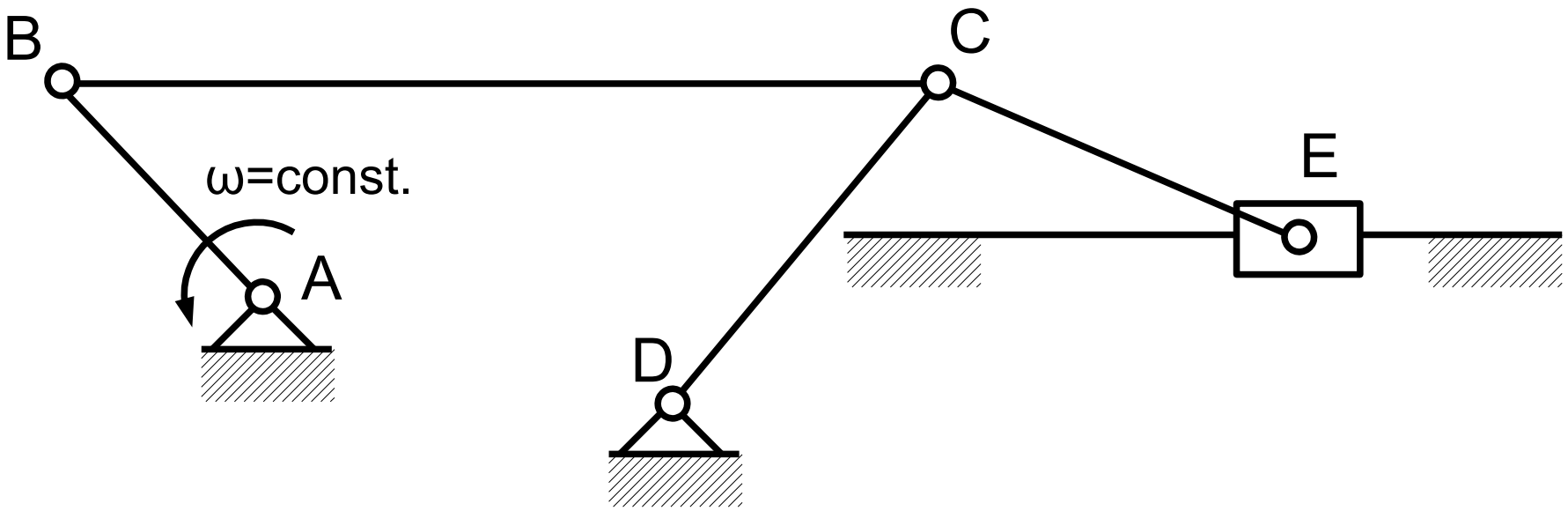
Relative acceleration

Coriolis acceleration

$$\vec{a}^{c} = 2\vec{\omega}_{u} \times \vec{v}_{B2B1}$$

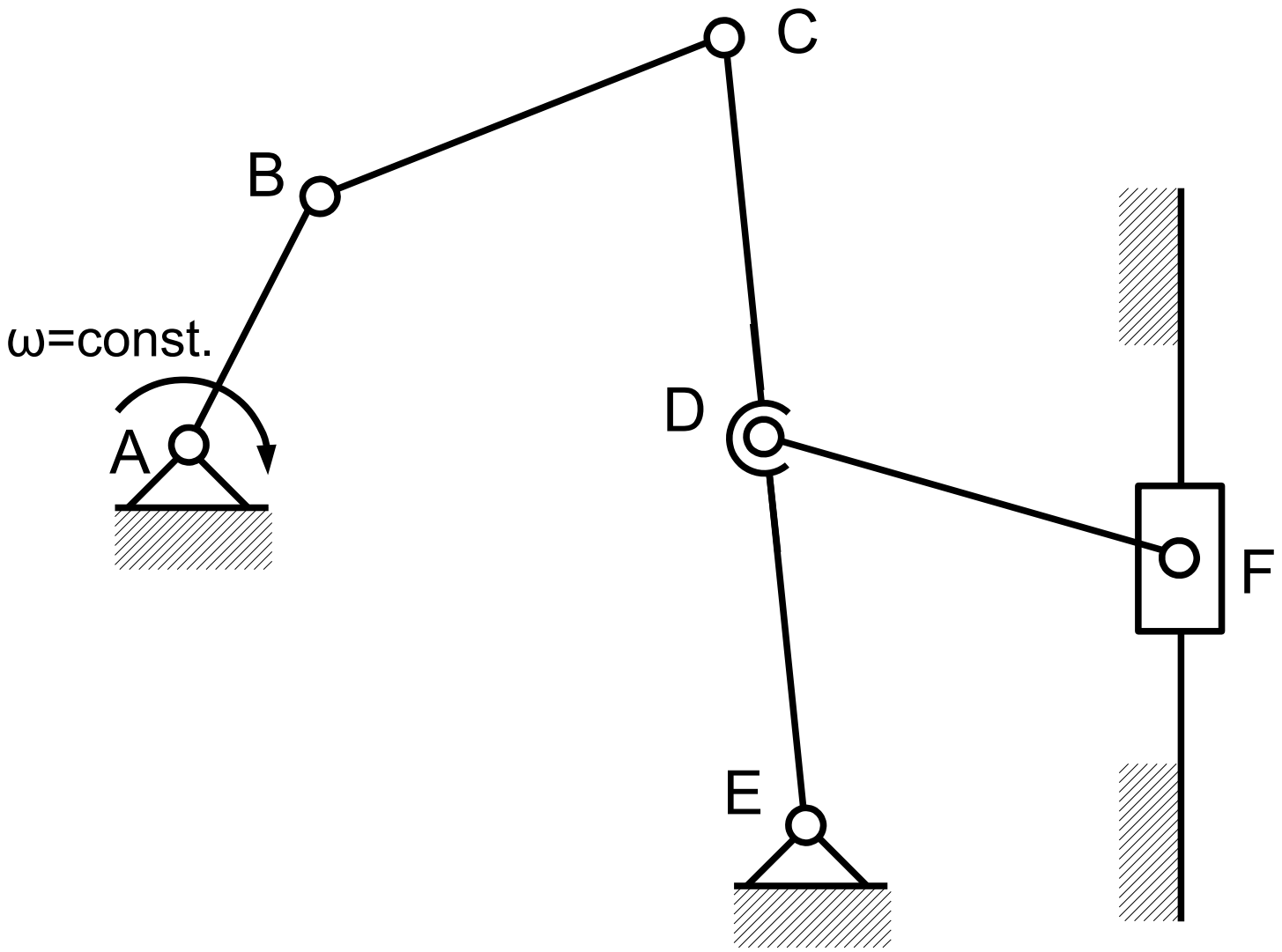
Accelerations

Example



Accelerations

Example



Accelerations in relative motion

Example