Warsaw University of Technology
The Faculty of Automotive
and Construction Machinery Engineering
Institute of Machine Design Fundamentals
Department of Mechanics
http://www.ipbm.simr.pw.edu.pl/

Theory of Machines and Automatic Control

Winter 2017/2018

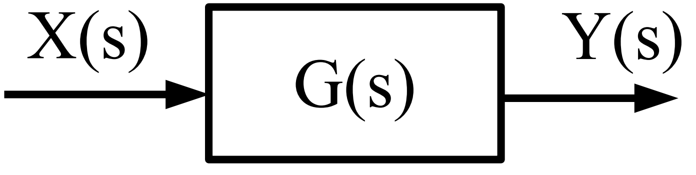
Lecturer: Sebastian Korczak, PhD Eng.

Lecture 11

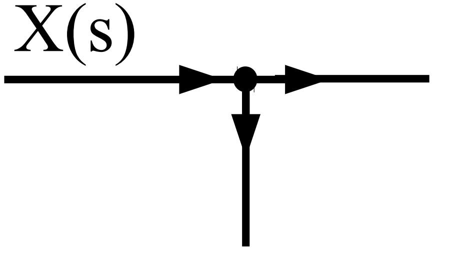
Block diagram algebra.
Control and controllers.

BLOCK DIAGRAM ALGEBRA

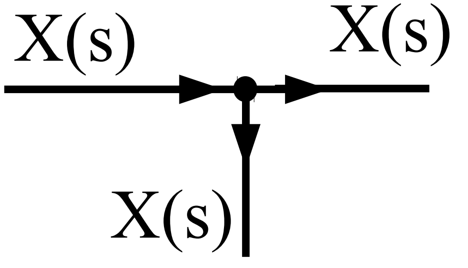
BLOCK DIAGRAM ALGEBRA
Transfer function



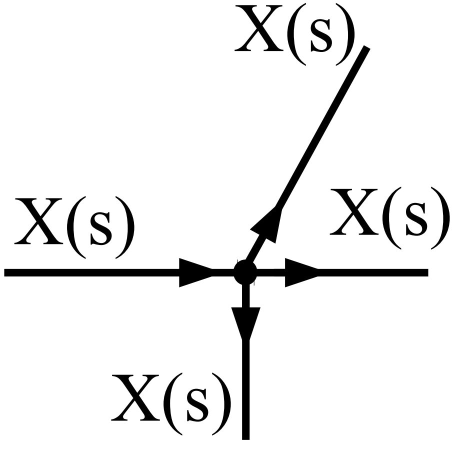
BLOCK DIAGRAM ALGEBRA
information node



BLOCK DIAGRAM ALGEBRA
information node

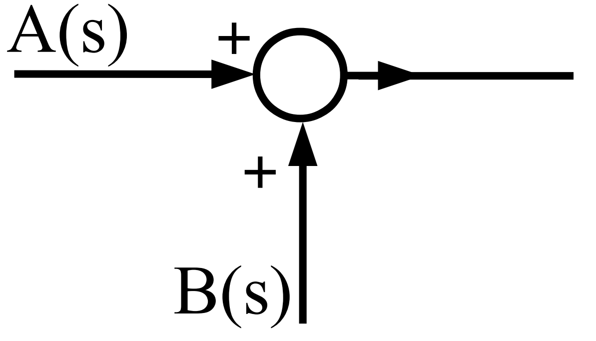


BLOCK DIAGRAM ALGEBRA
information node



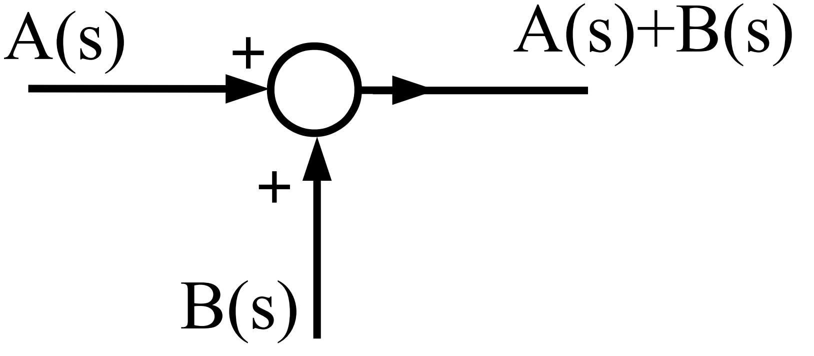
one input,
a few outputs,

BLOCK DIAGRAM ALGEBRA
sum node

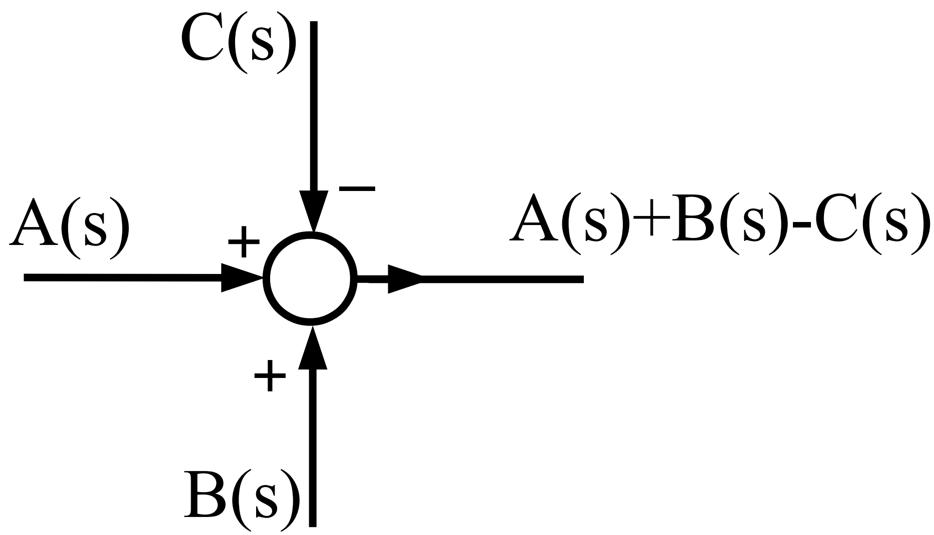


a few inputs,
one output,

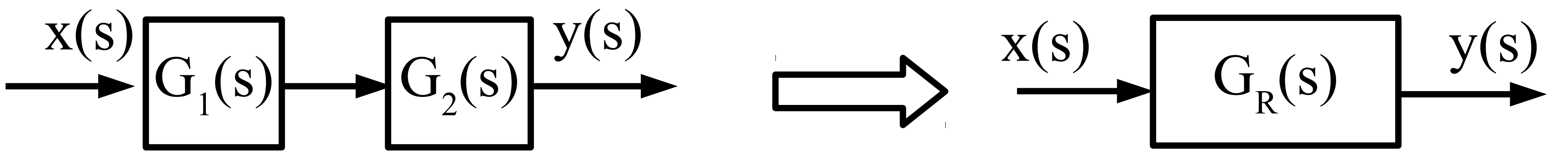
BLOCK DIAGRAM ALGEBRA
sum node



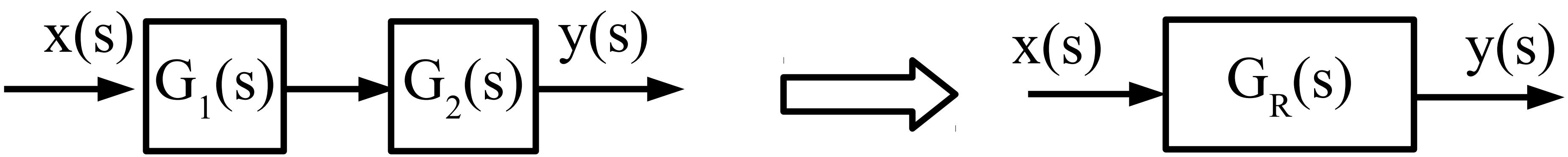
BLOCK DIAGRAM ALGEBRA
sum node



BLOCK DIAGRAM ALGEBRA
serial connection

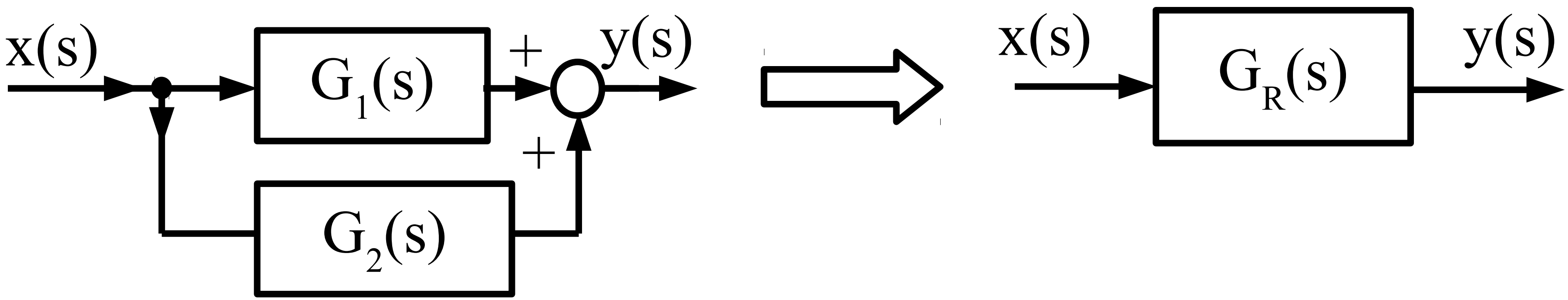


BLOCK DIAGRAM ALGEBRA
serial connection

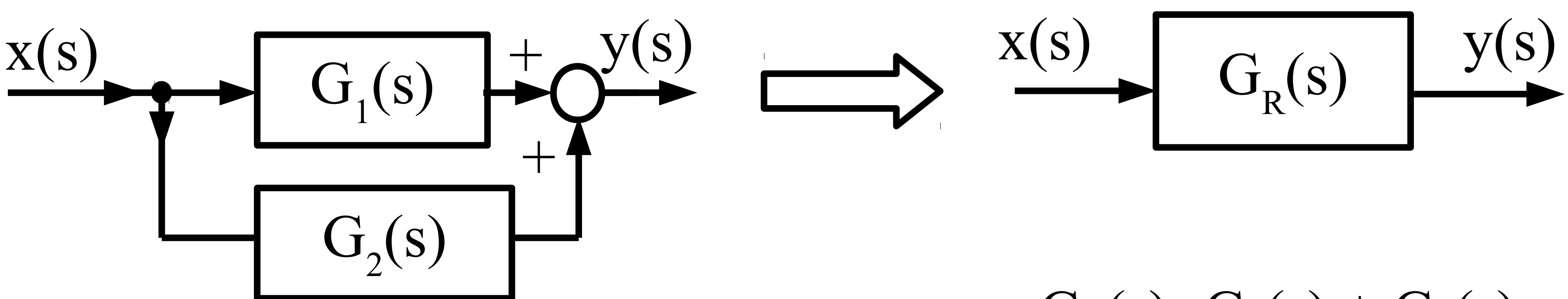


$$G_R(s)=G_1(s)\ G_2(s)$$

BLOCK DIAGRAM ALGEBRA
parallel connection

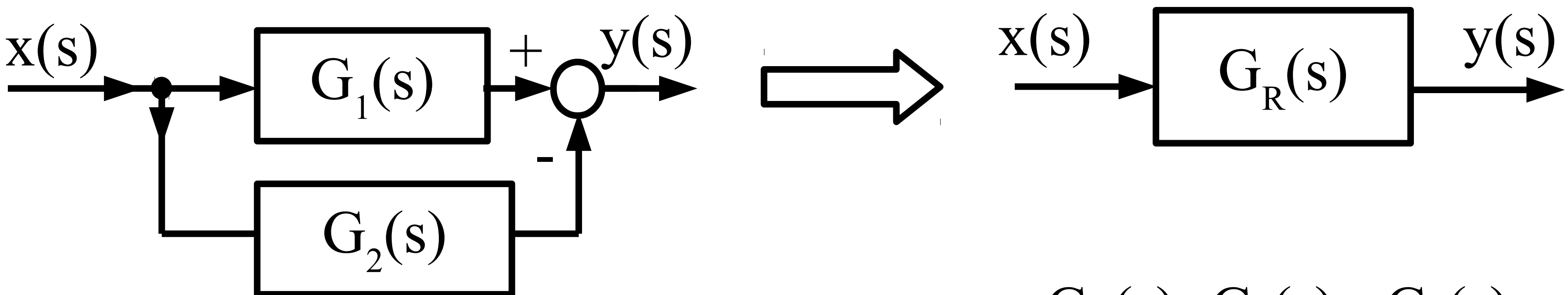


BLOCK DIAGRAM ALGEBRA
parallel connection



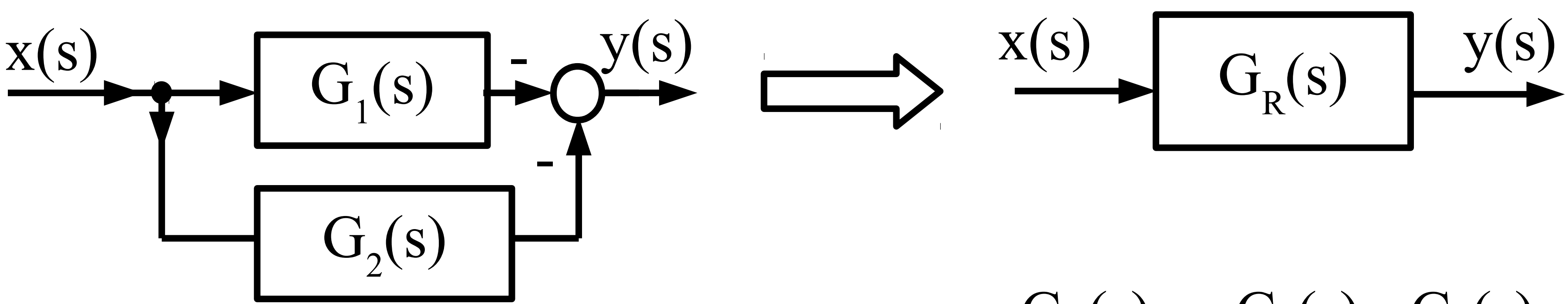
$$G_R(s)=G_1(s) + G_2(s)$$

BLOCK DIAGRAM ALGEBRA
parallel connection



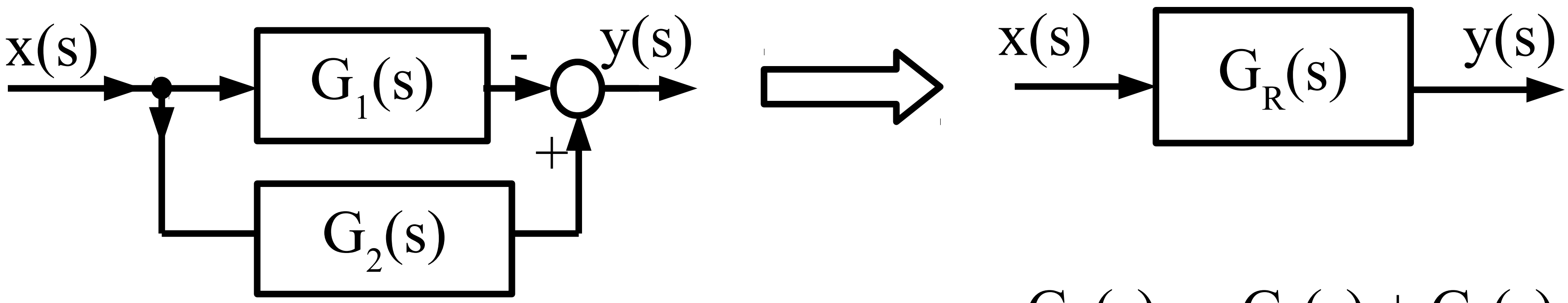
$$G_R(s)=G_1(s) - G_2(s)$$

BLOCK DIAGRAM ALGEBRA
parallel connection



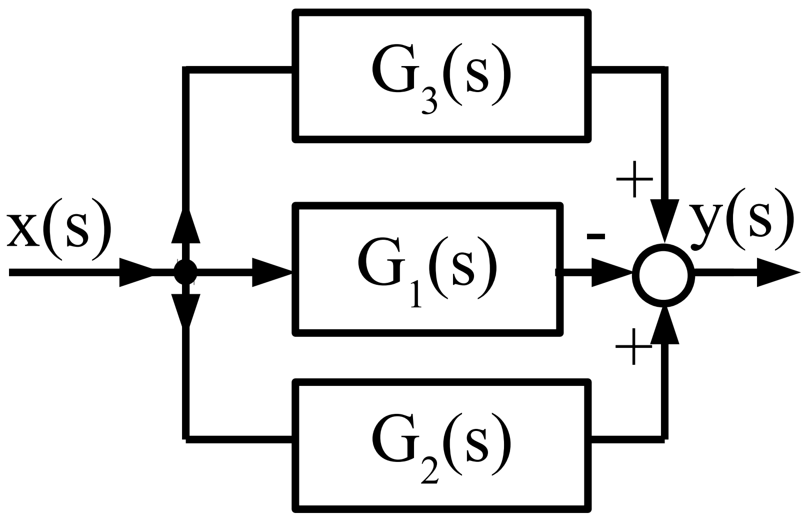
$$G_R(s) = -G_1(s) - G_2(s)$$

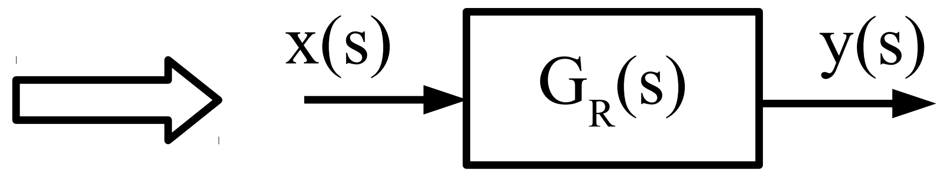
BLOCK DIAGRAM ALGEBRA
parallel connection



$$G_R(s) = -G_1(s) + G_2(s)$$

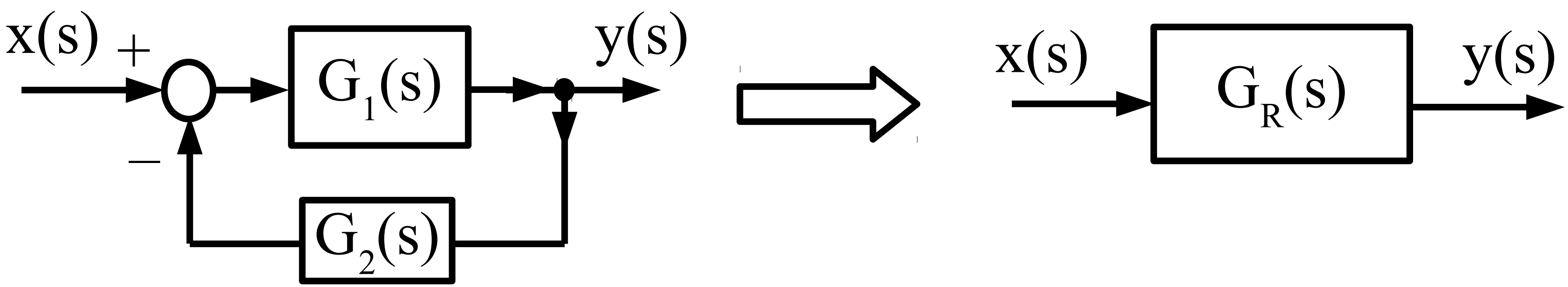
BLOCK DIAGRAM ALGEBRA
parallel connection



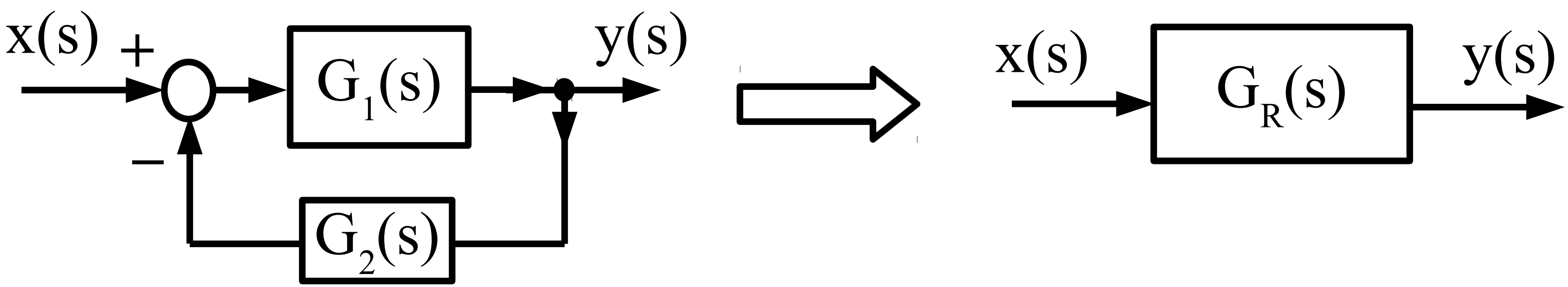


$$G_R(s) = -G_1(s) + G_2(s) + G_3(s)$$

BLOCK DIAGRAM ALGEBRA
feedback

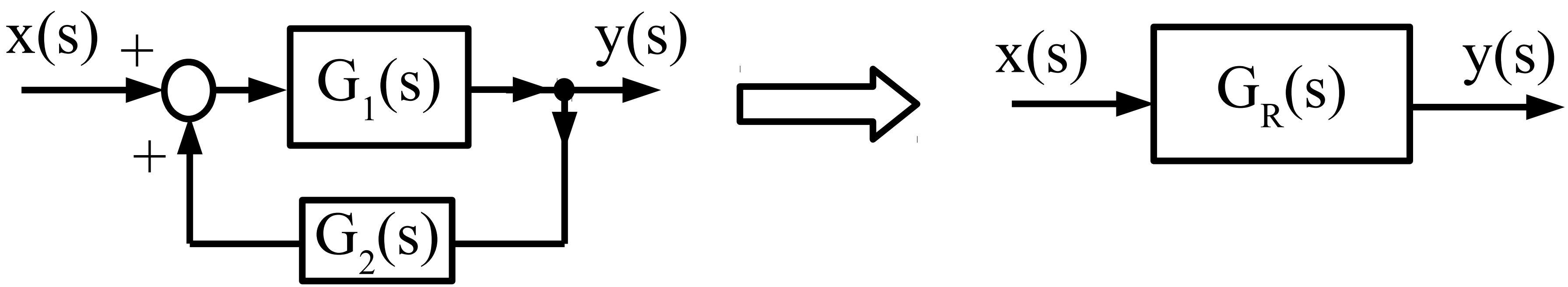


BLOCK DIAGRAM ALGEBRA
feedback



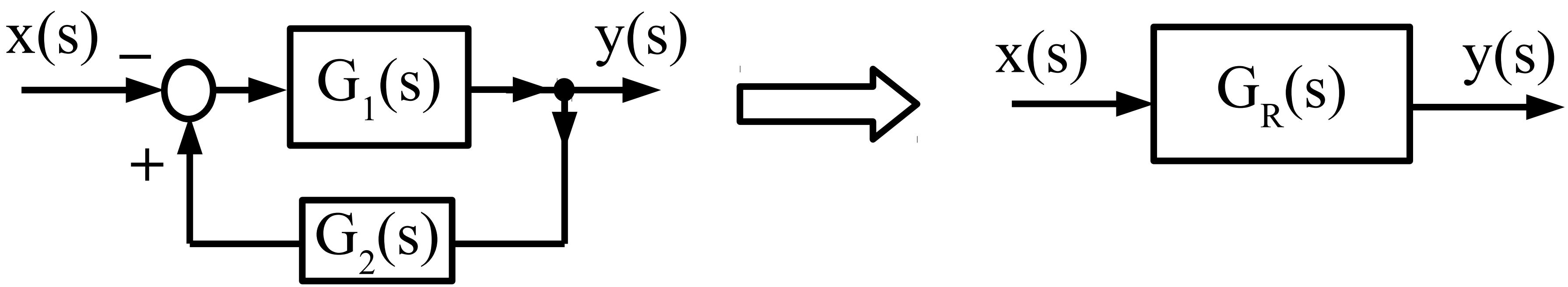
$$G_R = \frac{G_1}{1 + G_1 G_2}$$

BLOCK DIAGRAM ALGEBRA
feedback



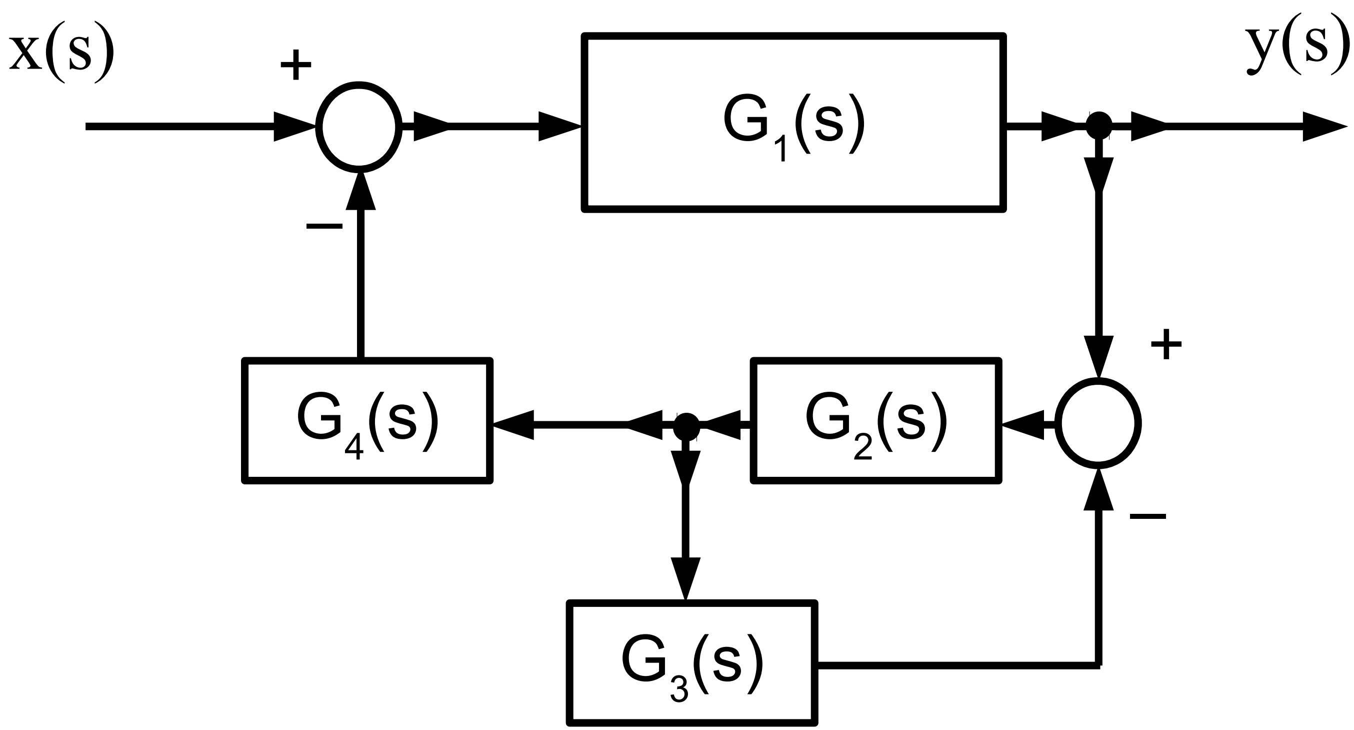
$$G_R = \frac{G_1}{1 - G_1 G_2}$$

BLOCK DIAGRAM ALGEBRA
feedback

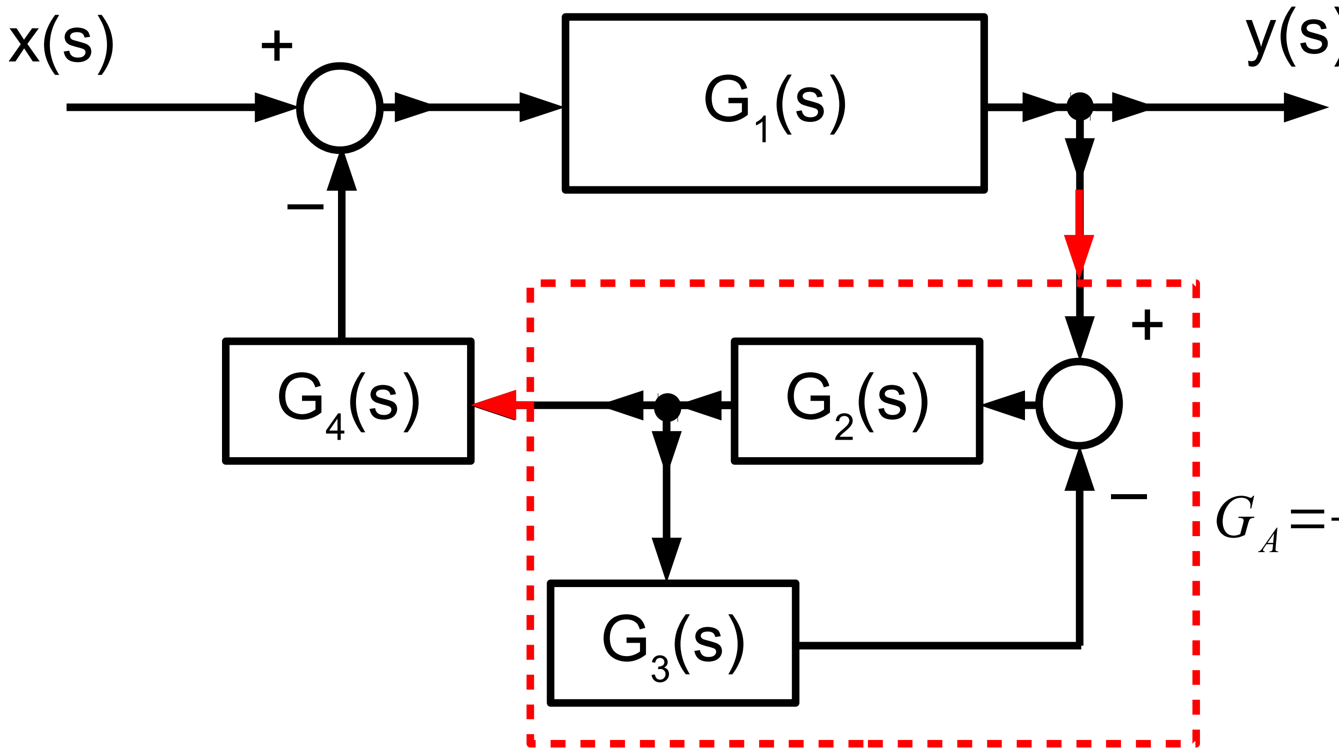


$$G_R = \frac{-G_1}{1 - G_1 G_2}$$

EXAMPLE 1



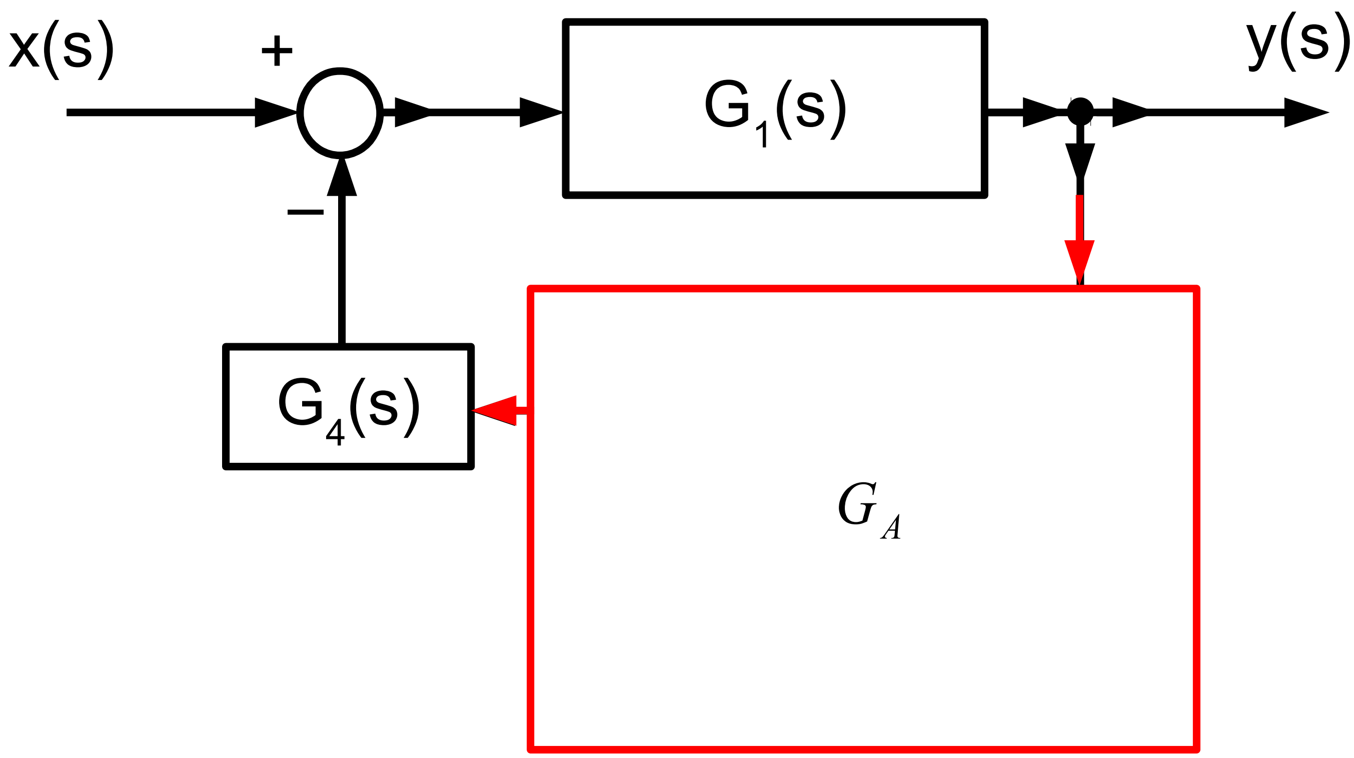
EXAMPLE 1



$$G_A = \frac{G_2}{1 + G_2 G_3}$$

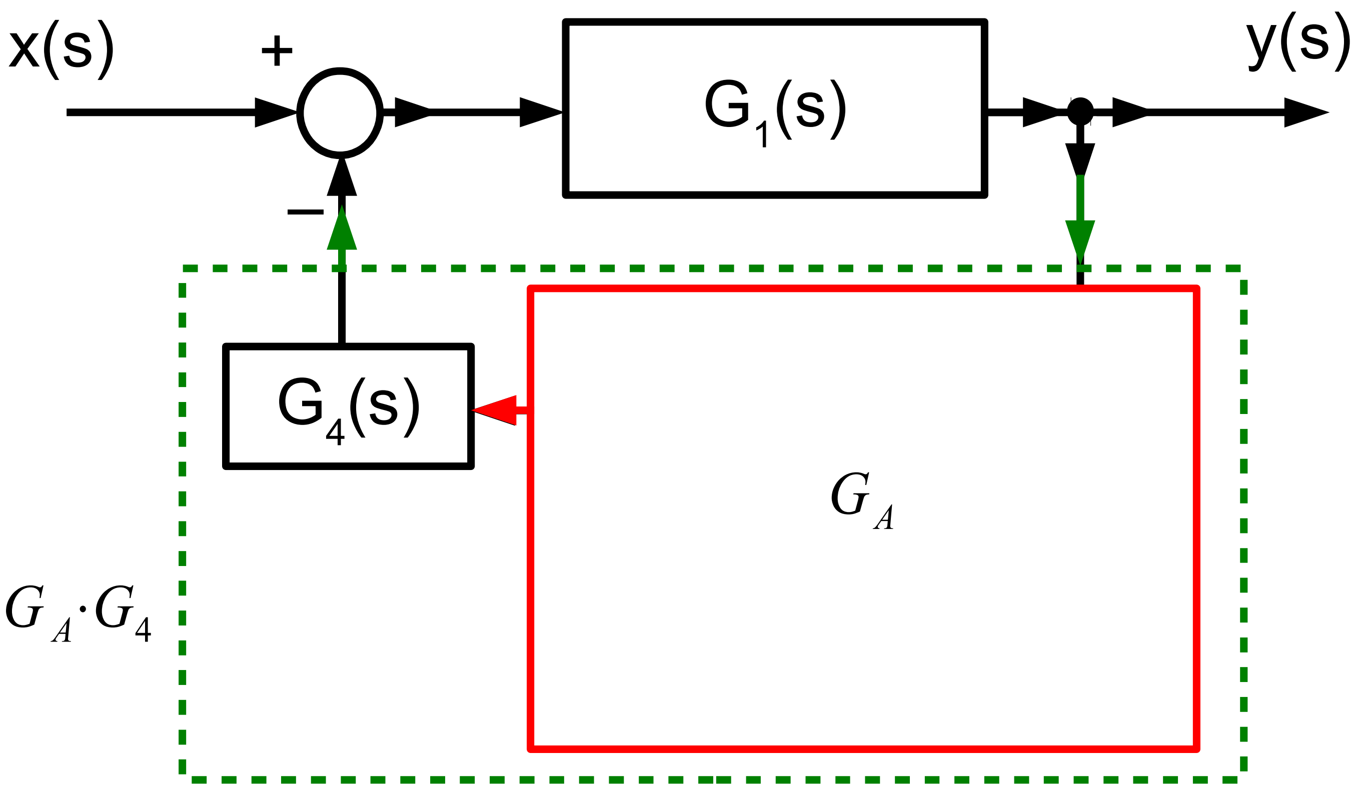
$$G_A = \frac{G_2}{1 + G_2 G_3}$$

EXAMPLE 1



$$G_A = \frac{G_2}{1+G_2G_3}$$

EXAMPLE 1

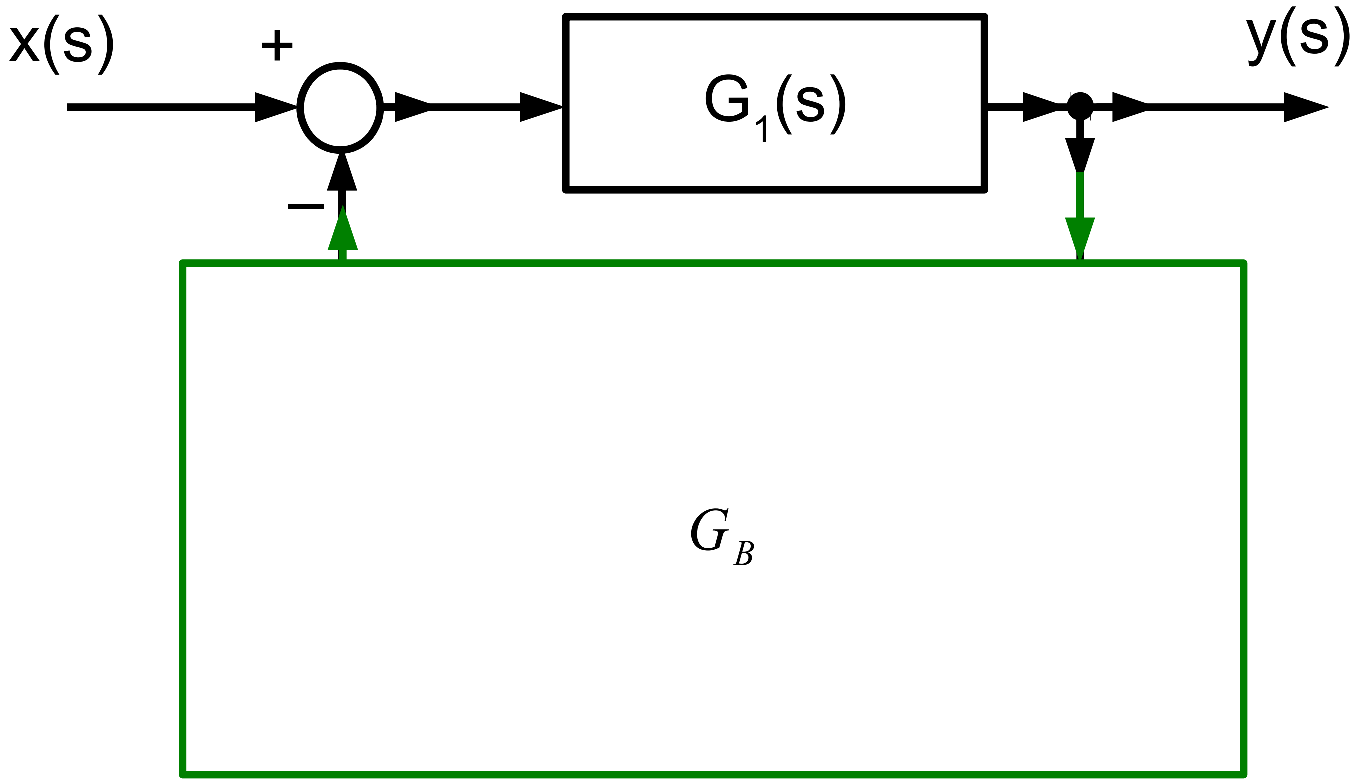


$$G_B = G_A \cdot G_4$$

$$G_A = \frac{G_2}{1+G_2 G_3}$$

$$G_B = G_A \cdot G_4$$

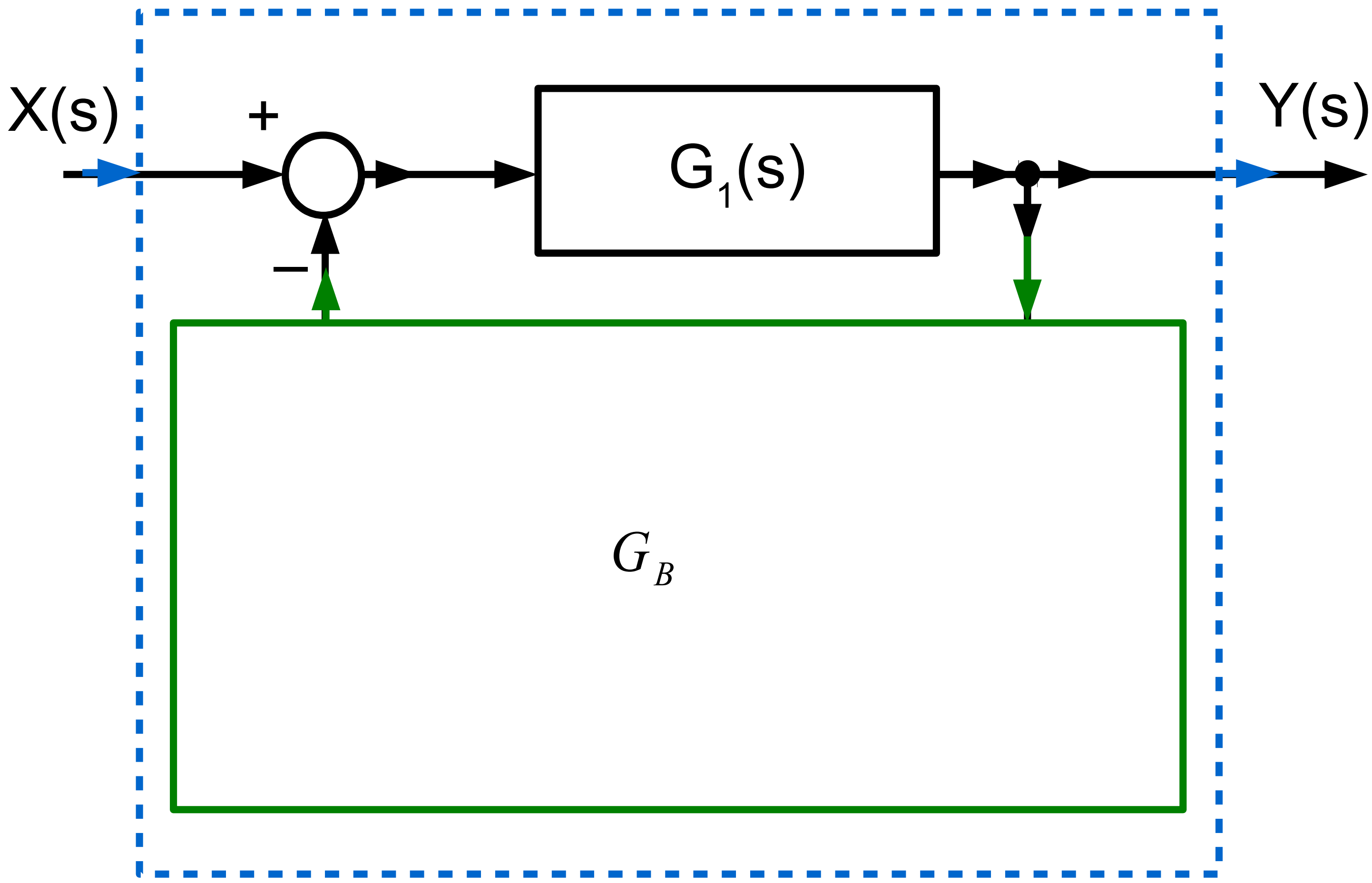
EXAMPLE 1



EXAMPLE 1

$$G_A = \frac{G_2}{1 + G_2 G_3}$$

$$G_B = G_A \cdot G_4$$

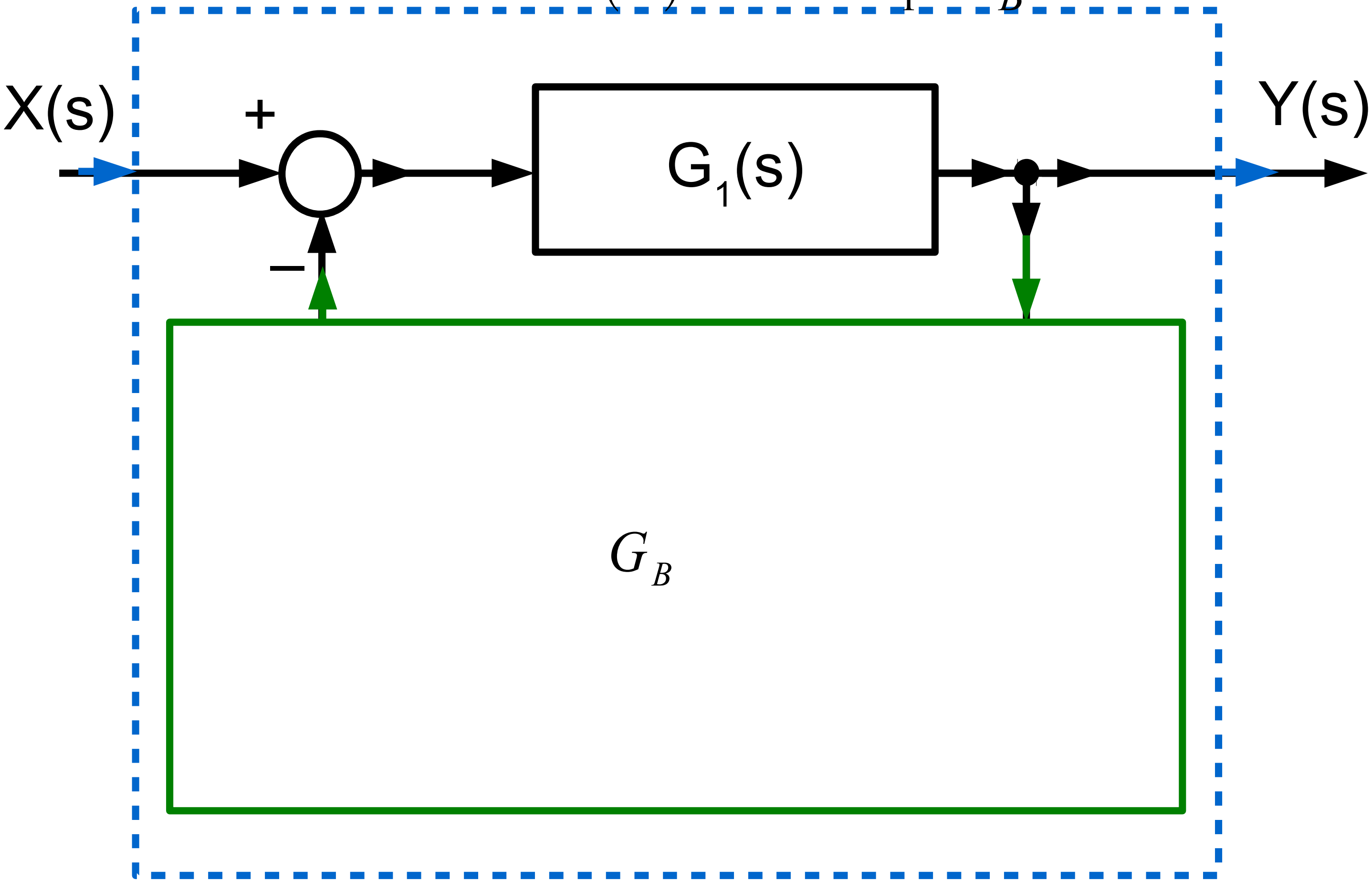


$$G_A = \frac{G_2}{1+G_2 G_3}$$

$$G_B = G_A \cdot G_4$$

EXAMPLE 1

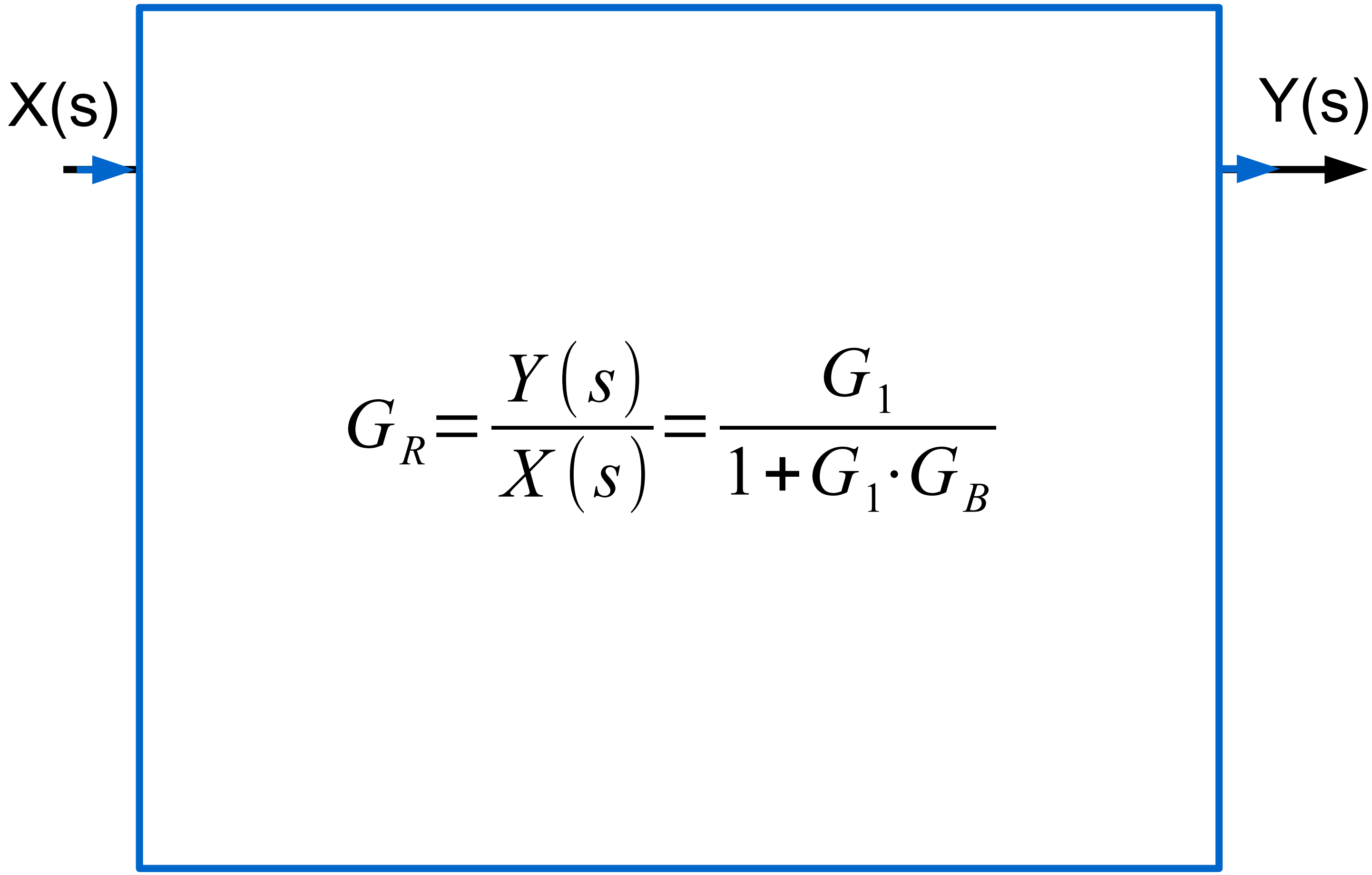
$$G_R = \frac{Y(s)}{X(s)} = \frac{G_1}{1+G_1 \cdot G_B}$$



$$G_A = \frac{G_2}{1 + G_2 G_3}$$

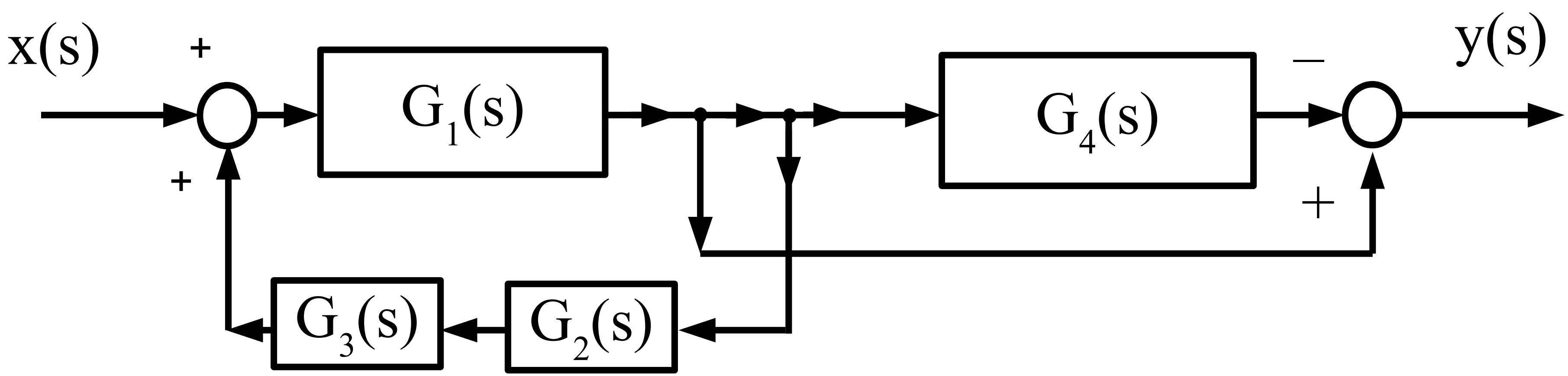
$$G_B = G_A \cdot G_4$$

EXAMPLE 1

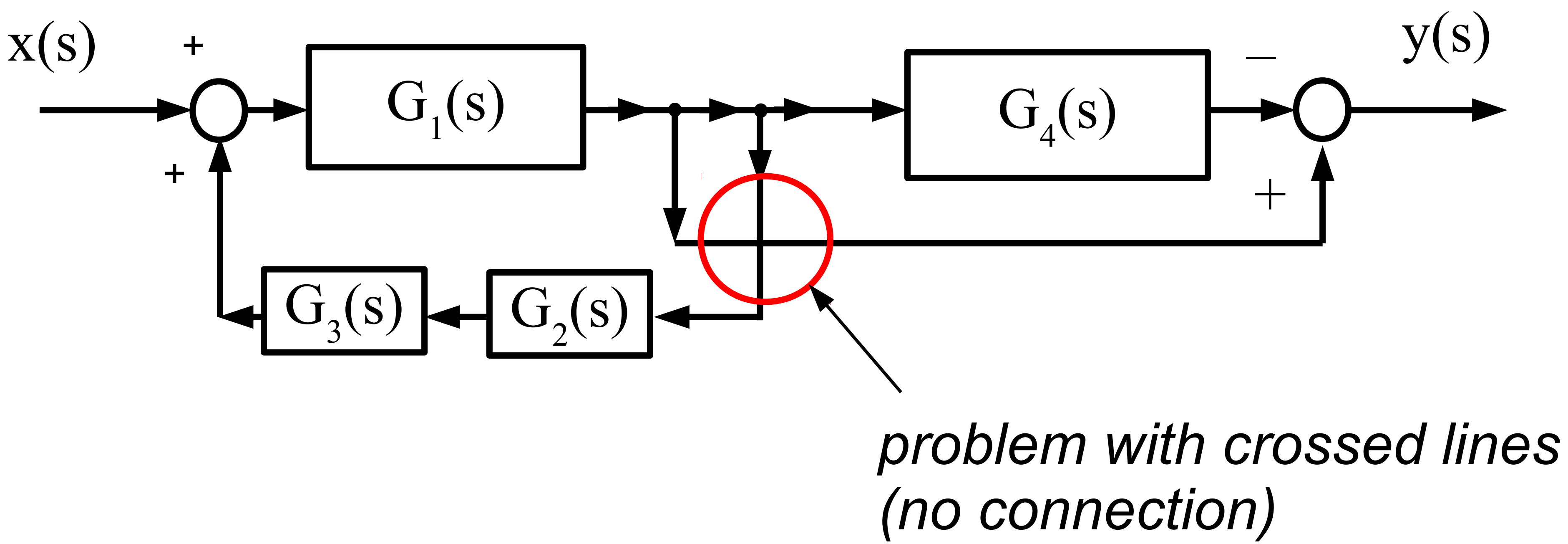


$$G_R = \frac{Y(s)}{X(s)} = \frac{G_1}{1 + G_1 \cdot G_B}$$

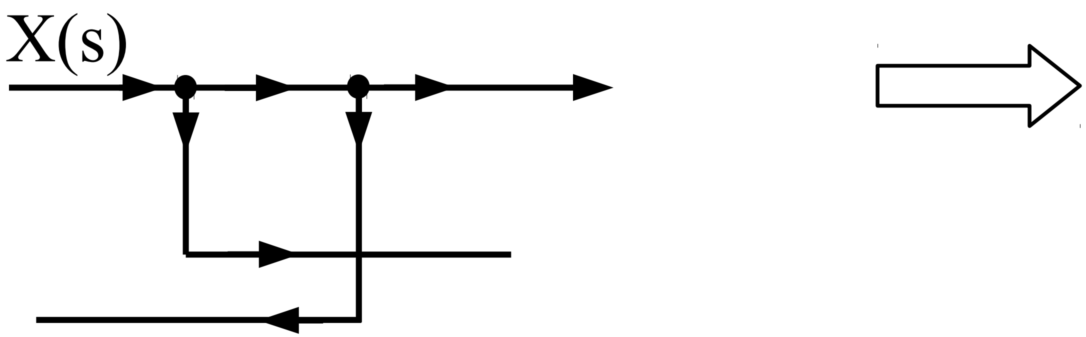
EXAMPLE 2



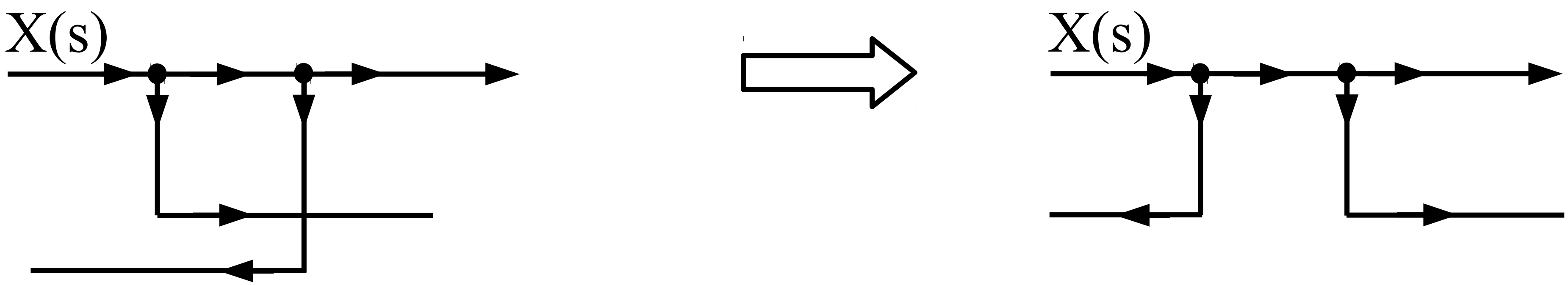
EXAMPLE 2


x(s)
+
+
G1(s)
G4(s)
–
y(s)
+
G3(s)
G2(s)
problem with crossed lines
(no connection)

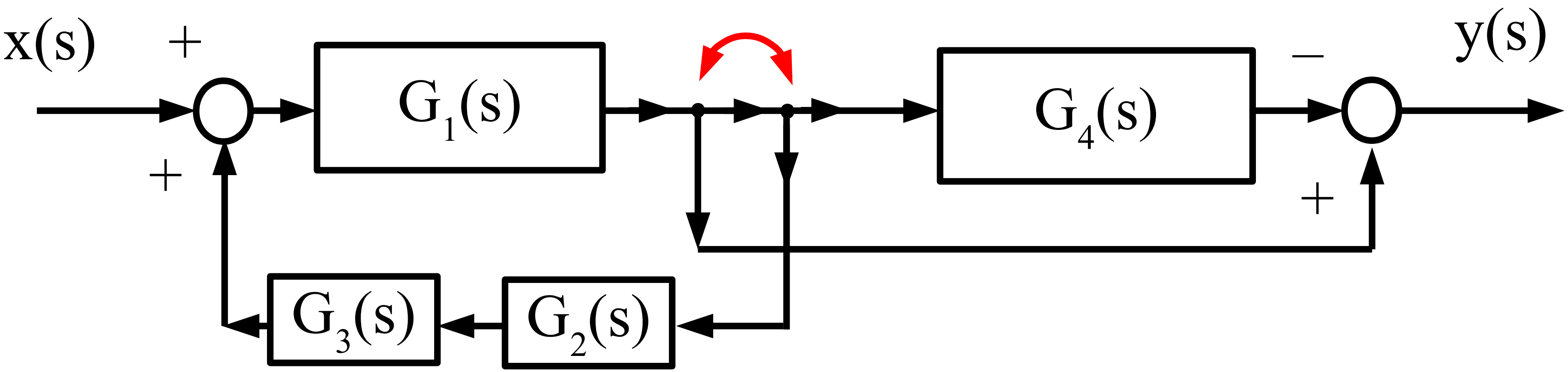
BLOCK DIAGRAM ALGEBRA
change of information points order



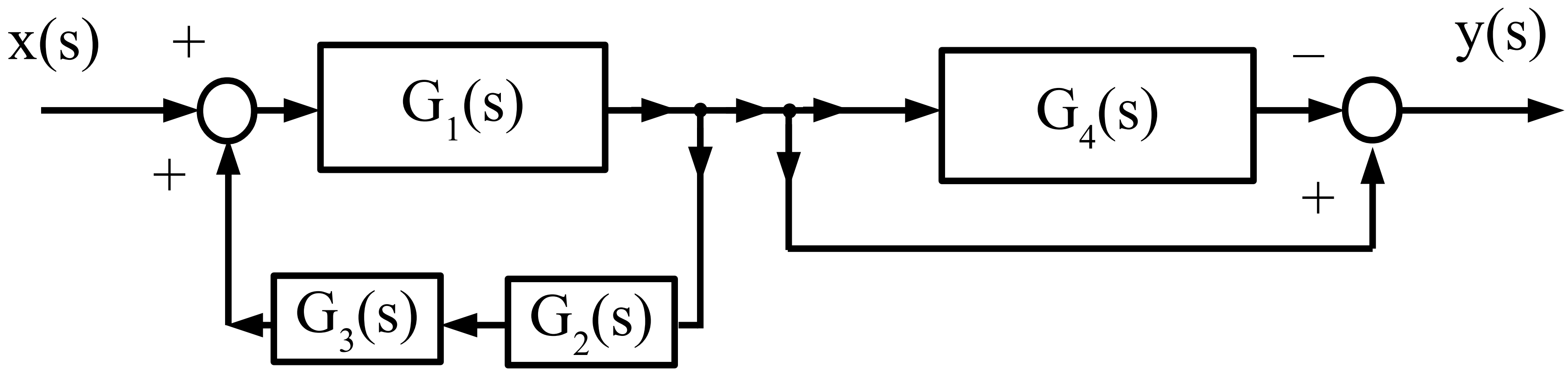
BLOCK DIAGRAM ALGEBRA
change of information points order



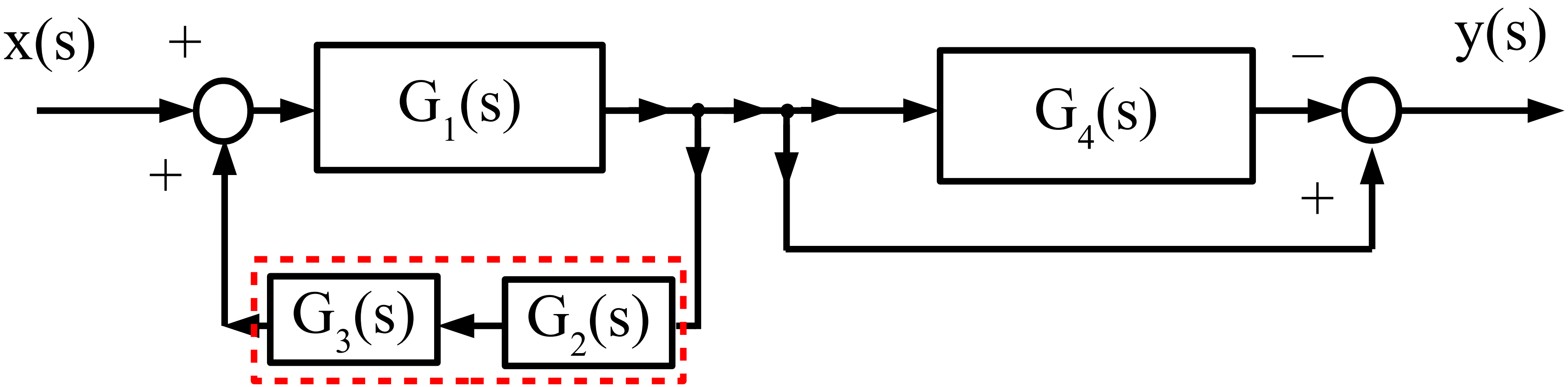
EXAMPLE 2



EXAMPLE 2



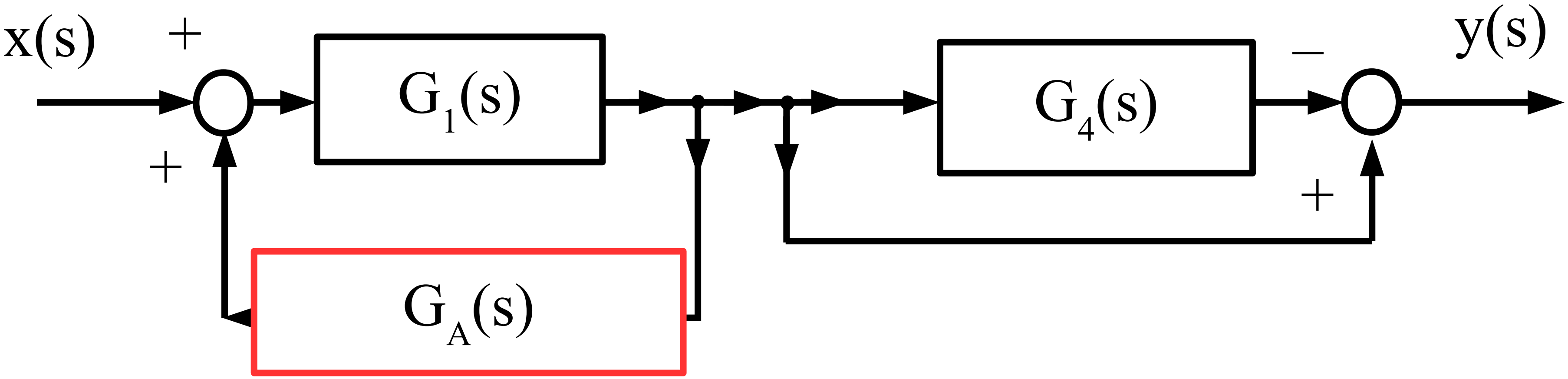
EXAMPLE 2



$G_A = G_2 G_3$

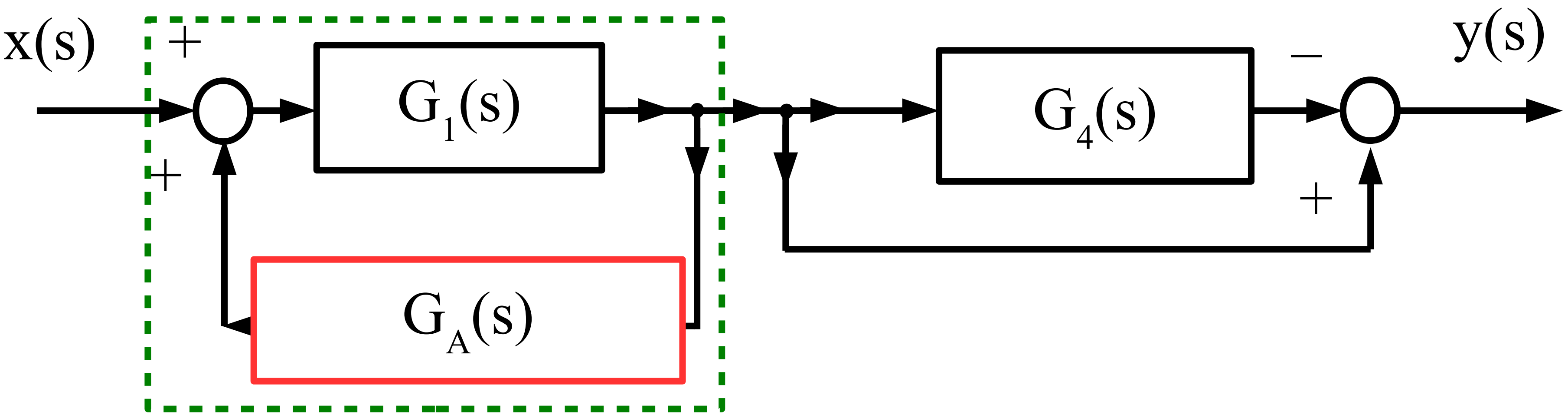
$G_A = G_2 G_3$

EXAMPLE 2



$G_A = G_2 G_3$

EXAMPLE 2

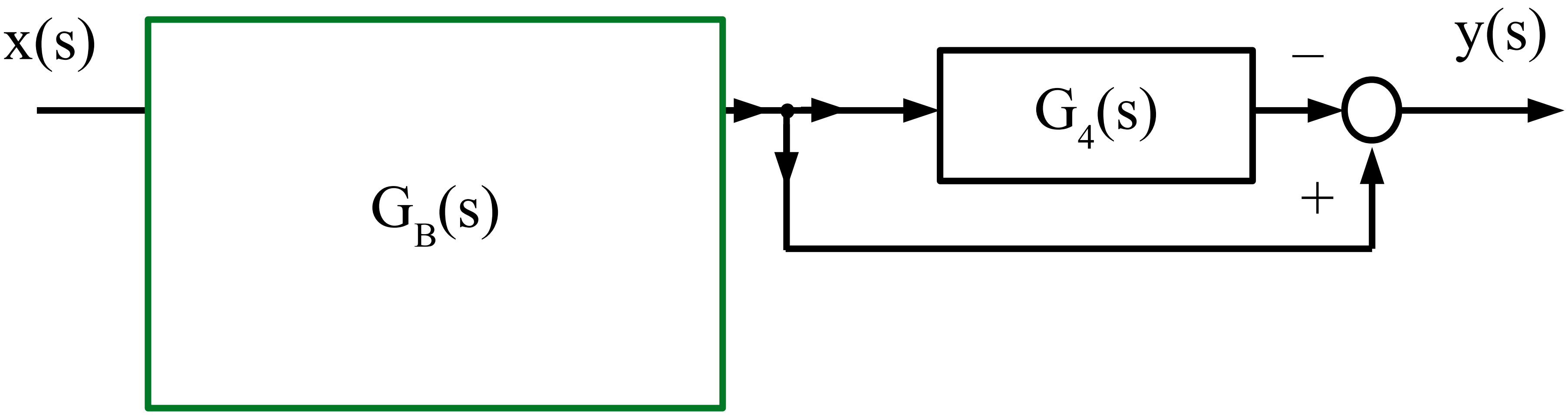


$$G_B = \frac{G_1}{1 - G_1 G_A}$$

$$G_A = G_2 G_3$$

EXAMPLE 2

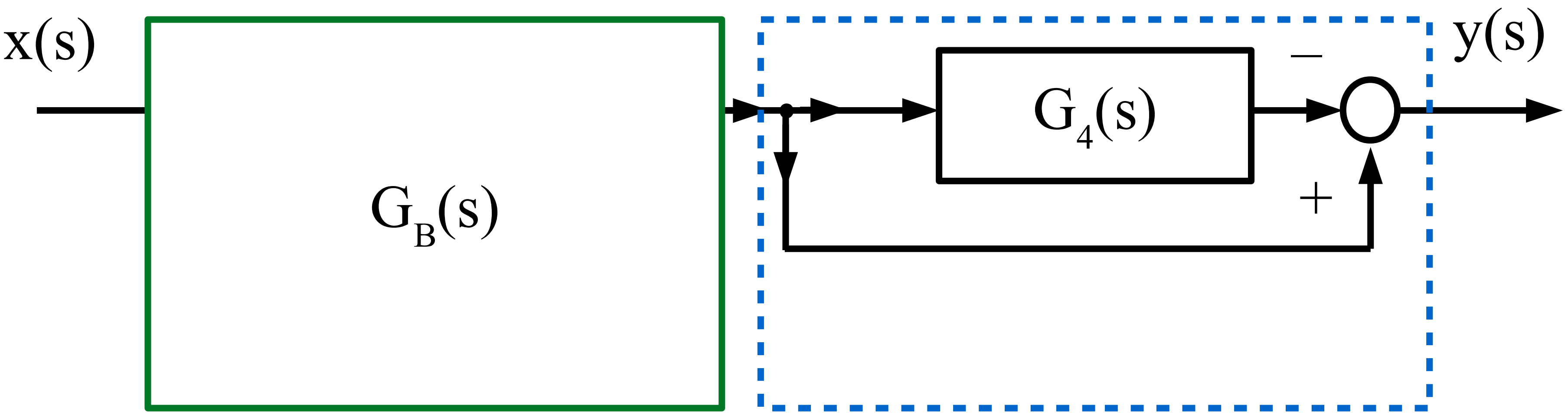
$$G_B = \frac{G_1}{1 - G_1 G_A}$$



$$G_A = G_2 G_3$$

EXAMPLE 2

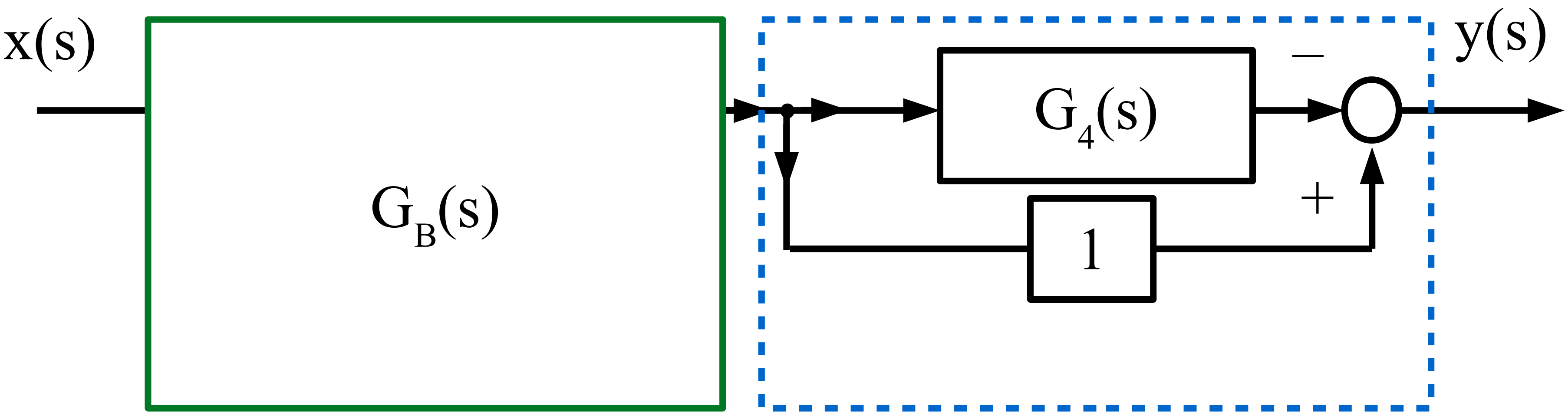
$$G_B = \frac{G_1}{1 - G_1 G_A}$$



$$G_A = G_2 G_3$$

$$G_B = \frac{G_1}{1 - G_1 G_A}$$

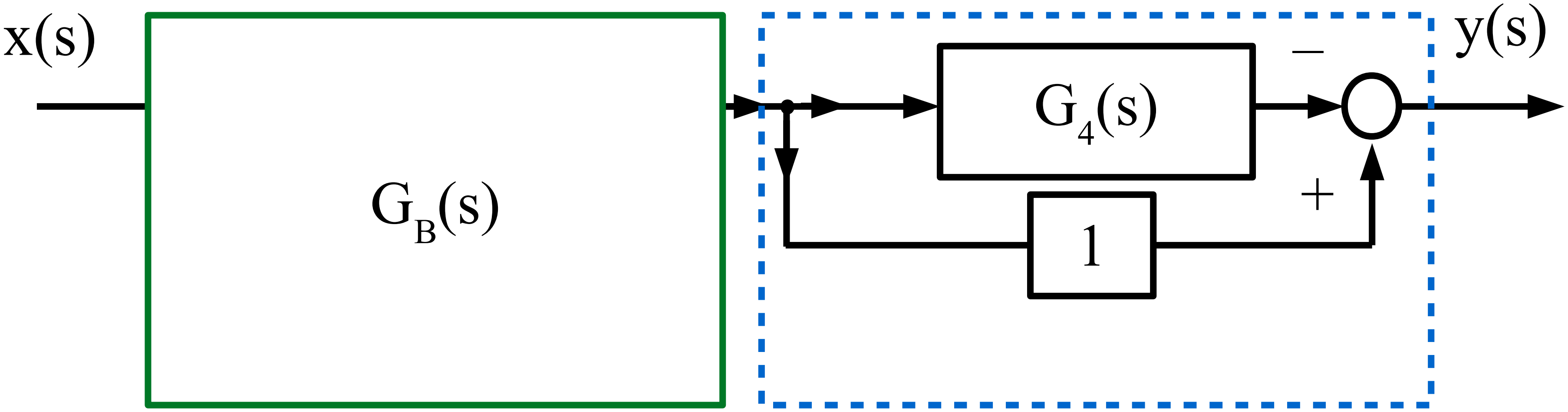
EXAMPLE 2



$$G_A = G_2 G_3$$

$$G_B = \frac{G_1}{1 - G_1 G_A}$$

EXAMPLE 2



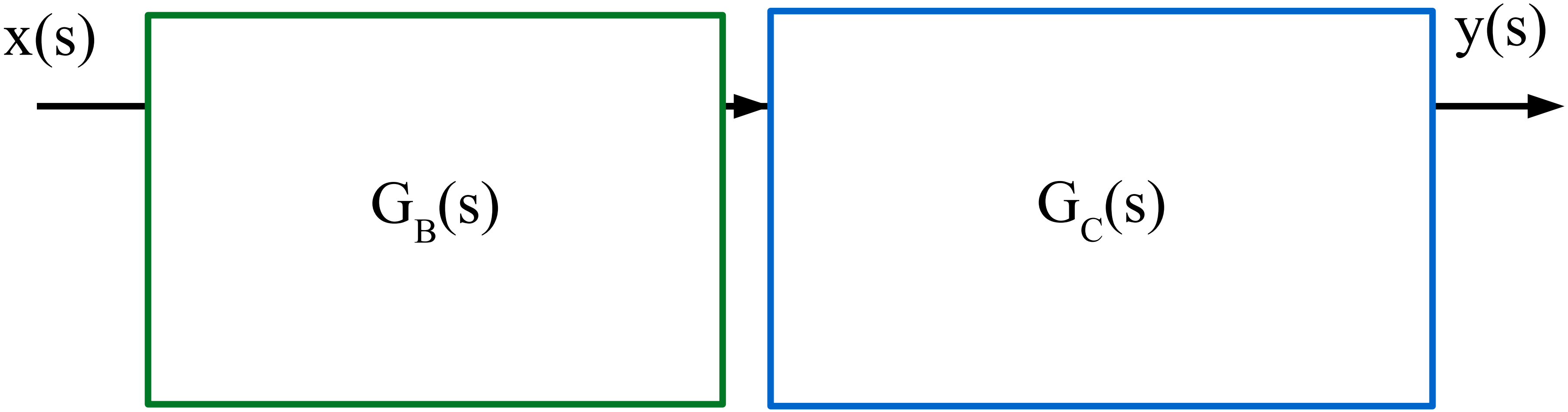
$$G_C = -G_4 + 1$$

$$G_A = G_2 G_3$$

$$G_B = \frac{G_1}{1 - G_1 G_A}$$

$$G_C = -G_4 + 1$$

EXAMPLE 2

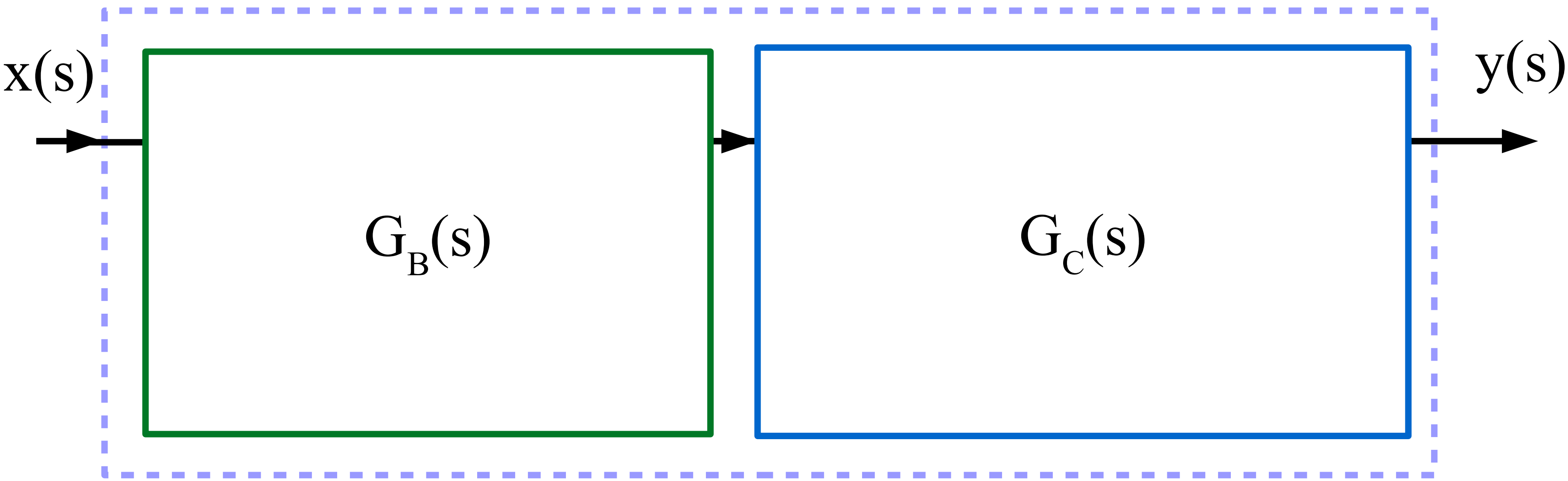


$$G_A = G_2 G_3$$

$$G_B = \frac{G_1}{1 - G_1 G_A}$$

$$G_C = -G_4 + 1$$

EXAMPLE 2



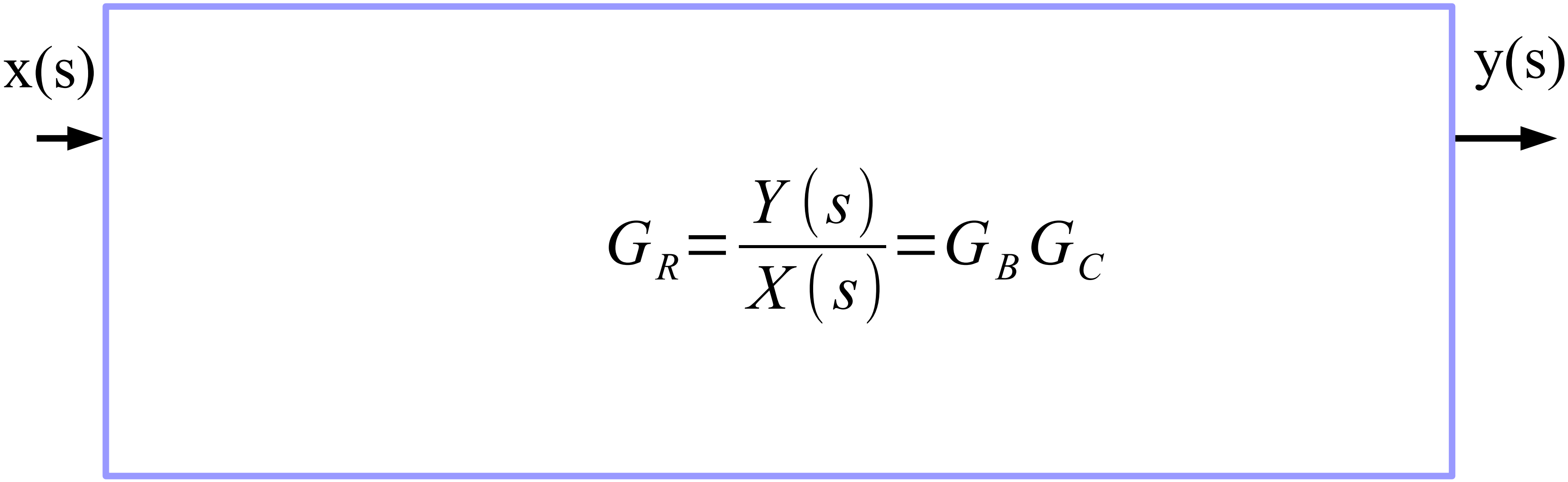
$$G_R = \frac{Y(s)}{X(s)} = G_B G_C$$

$$G_A = G_2 G_3$$

$$G_B = \frac{G_1}{1 - G_1 G_A}$$

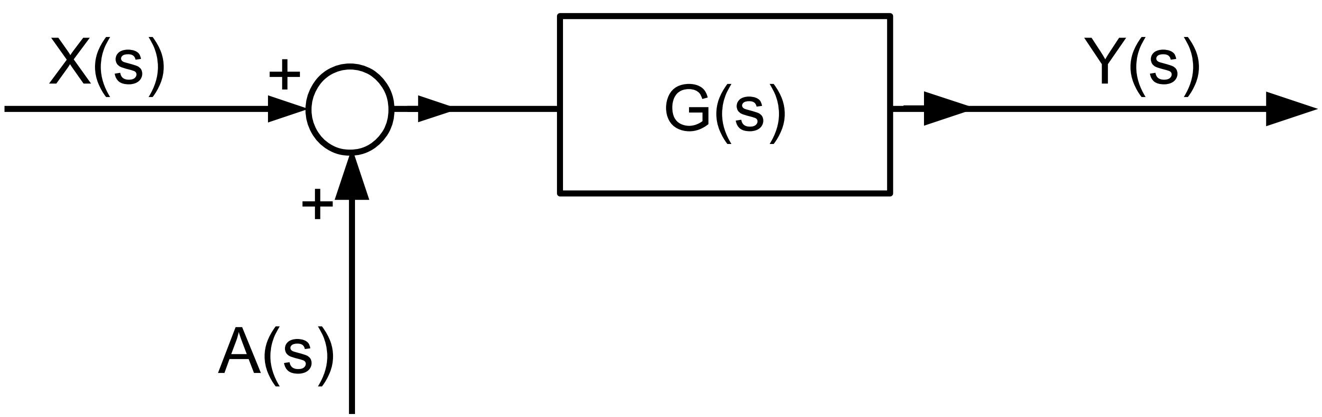
$$G_C = -G_4 + 1$$

EXAMPLE 2

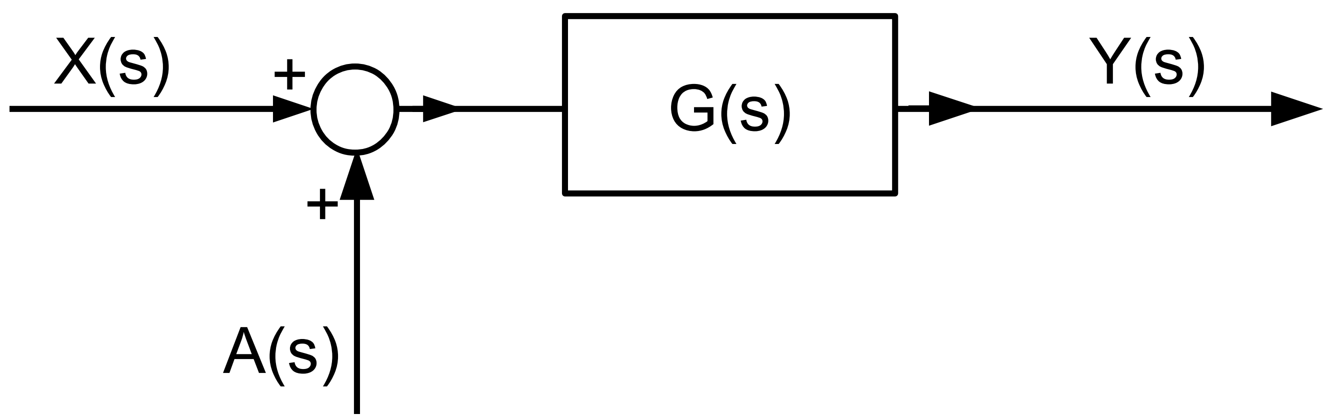


$$G_R = \frac{Y(s)}{X(s)} = G_B G_C$$

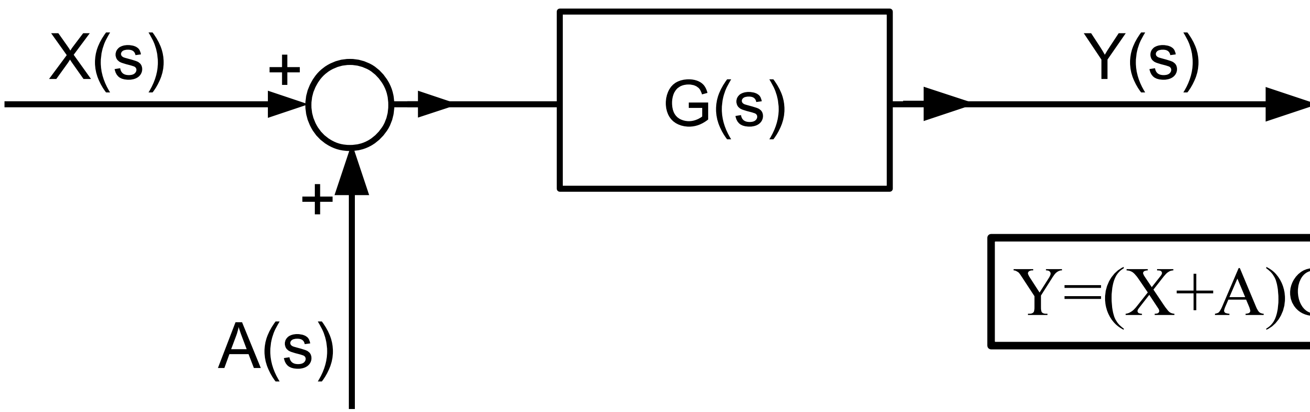
BLOCK DIAGRAM ALGEBRA
order change of sum node and block



BLOCK DIAGRAM ALGEBRA
order change of sum node and block

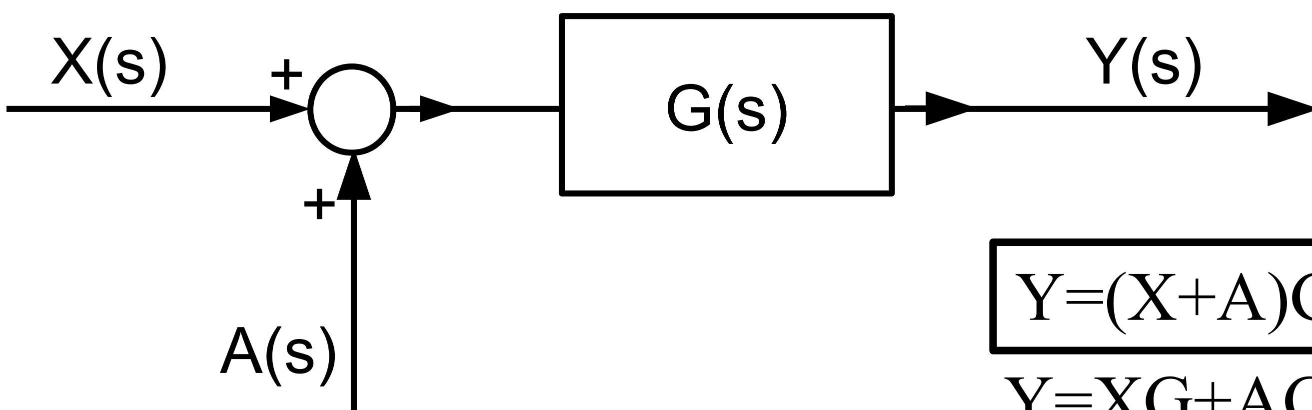


BLOCK DIAGRAM ALGEBRA
order change of sum node and block



Y=(X+A)G

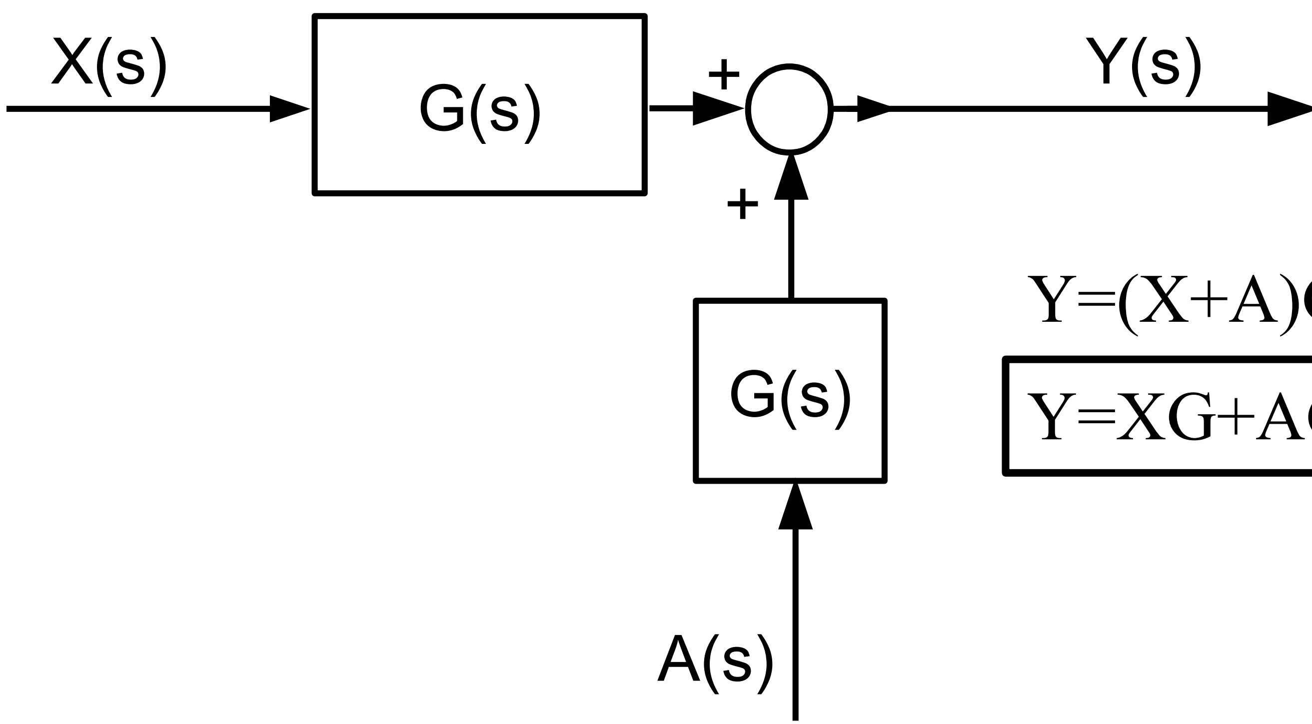
BLOCK DIAGRAM ALGEBRA
order change of sum node and block



$$Y=(X+A)G$$

$$Y=XG+AG$$

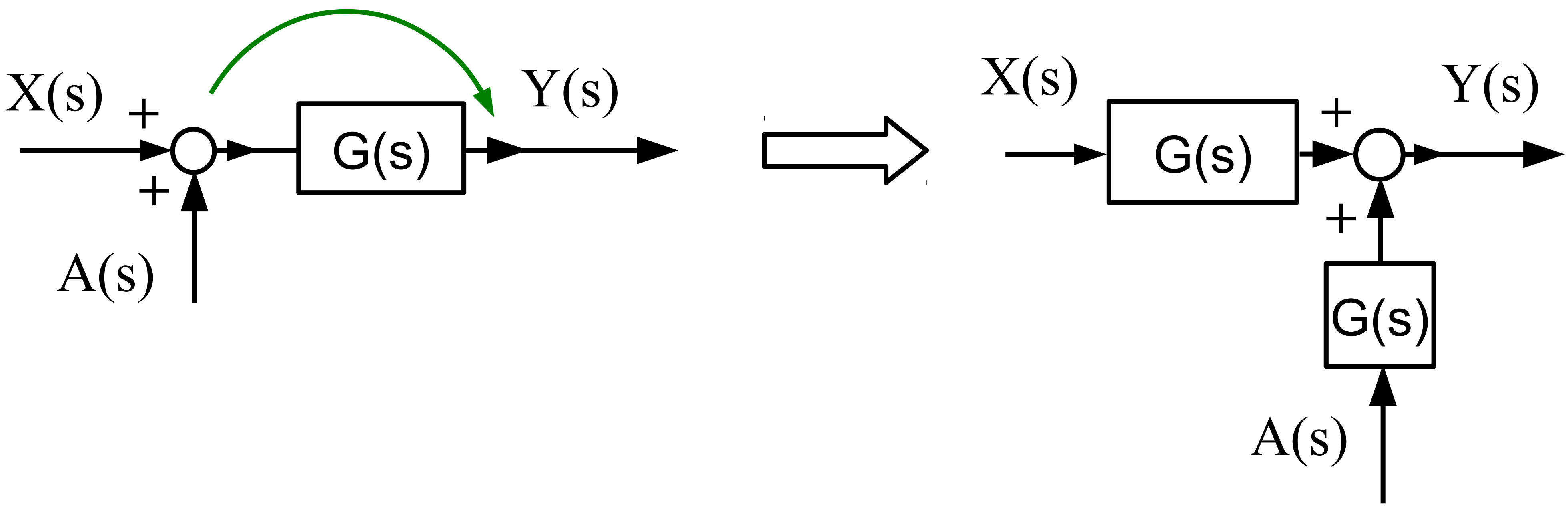
BLOCK DIAGRAM ALGEBRA
order change of sum node and block



$$Y=(X+A)G$$

$$\boxed{Y=XG+AG}$$

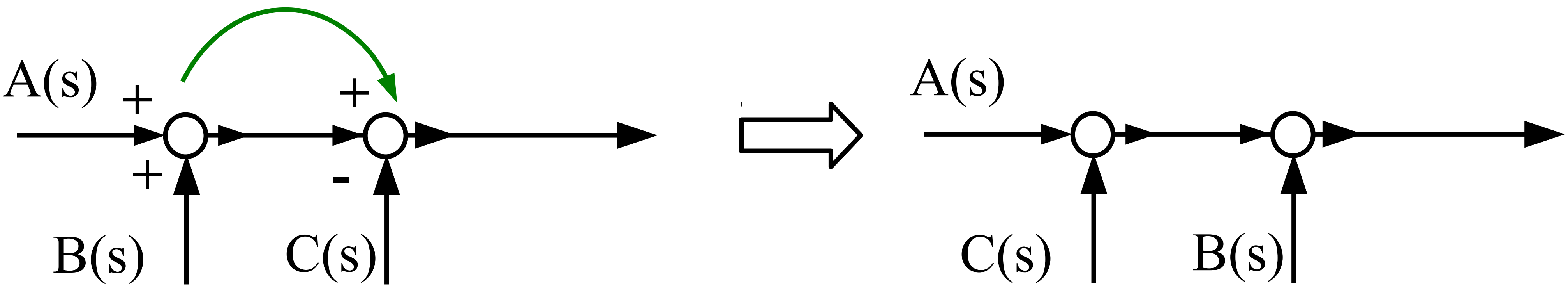
BLOCK DIAGRAM ALGEBRA
order change of sum node and block



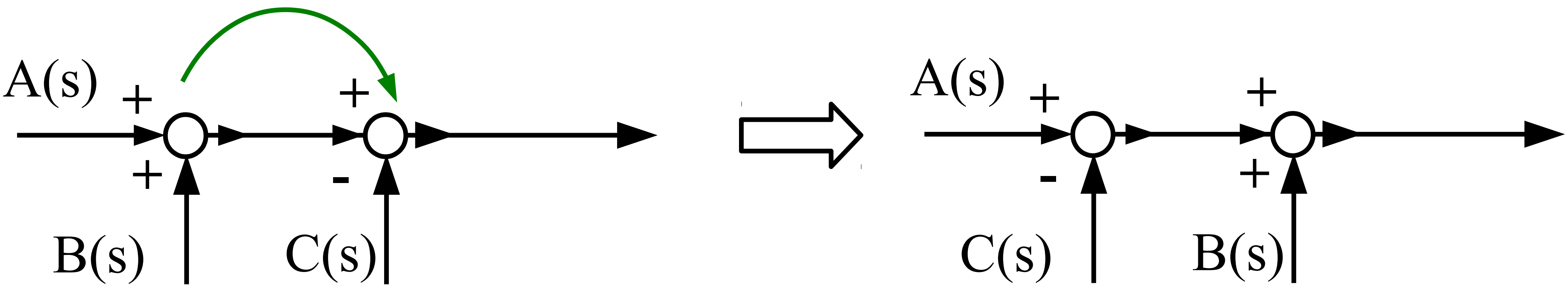
$Y=(X+A)G$

$Y=XG+AG$

BLOCK DIAGRAM ALGEBRA
order change of sum nodes

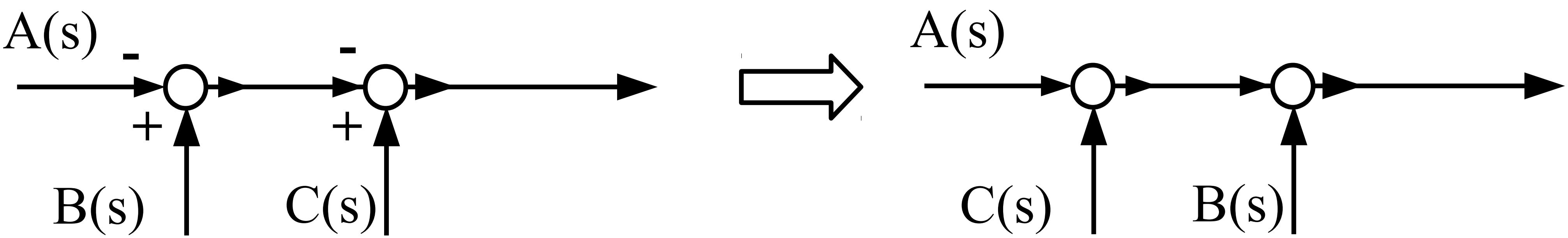


BLOCK DIAGRAM ALGEBRA
order change of sum nodes



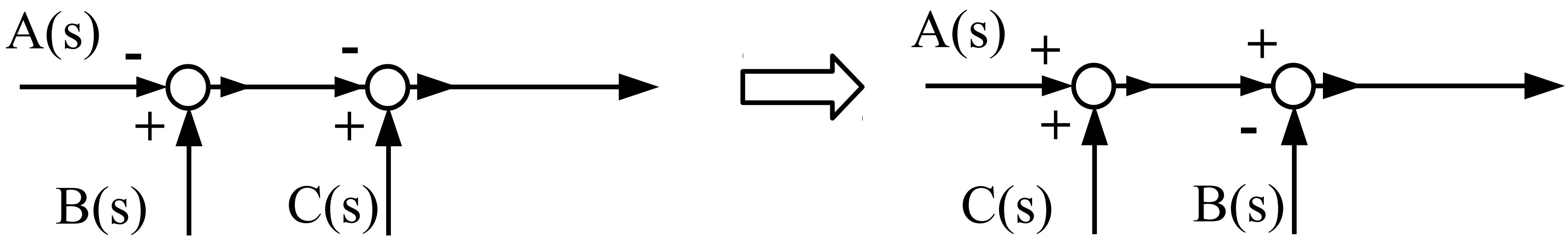
BLOCK DIAGRAM ALGEBRA
order change of sum nodes

Example 2 – attention to signs

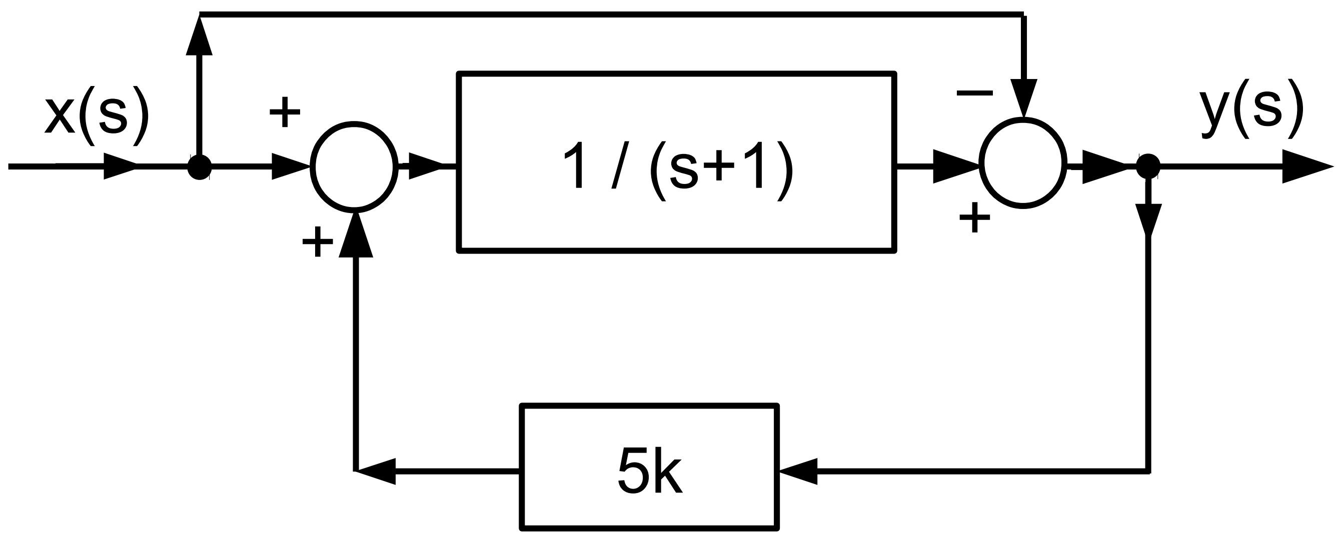


BLOCK DIAGRAM ALGEBRA
order change of sum nodes

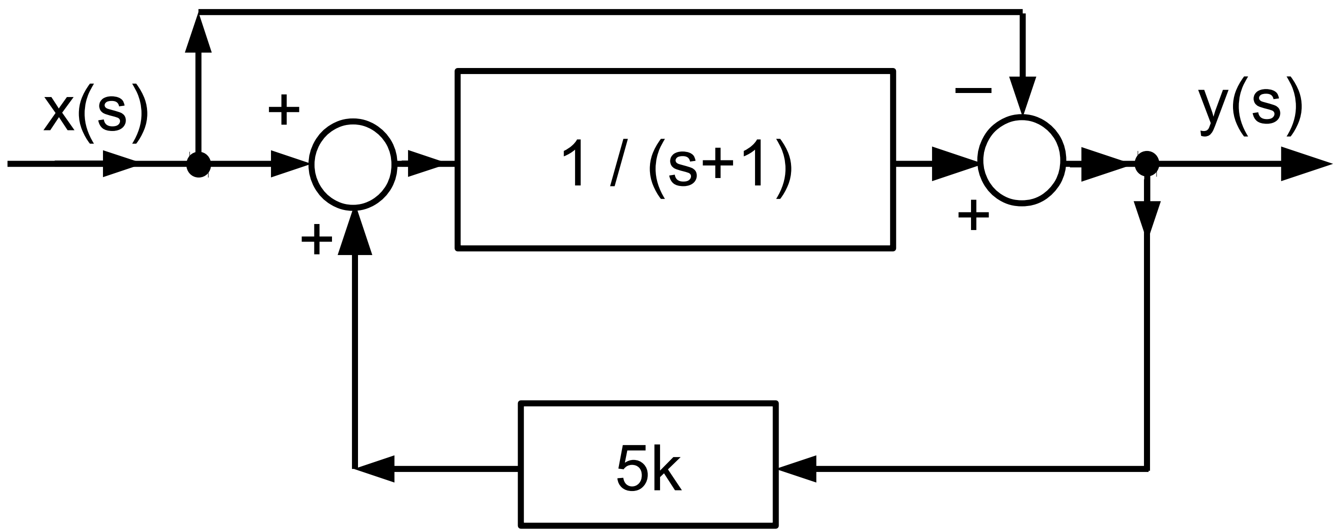
Example 2 – attention to signs

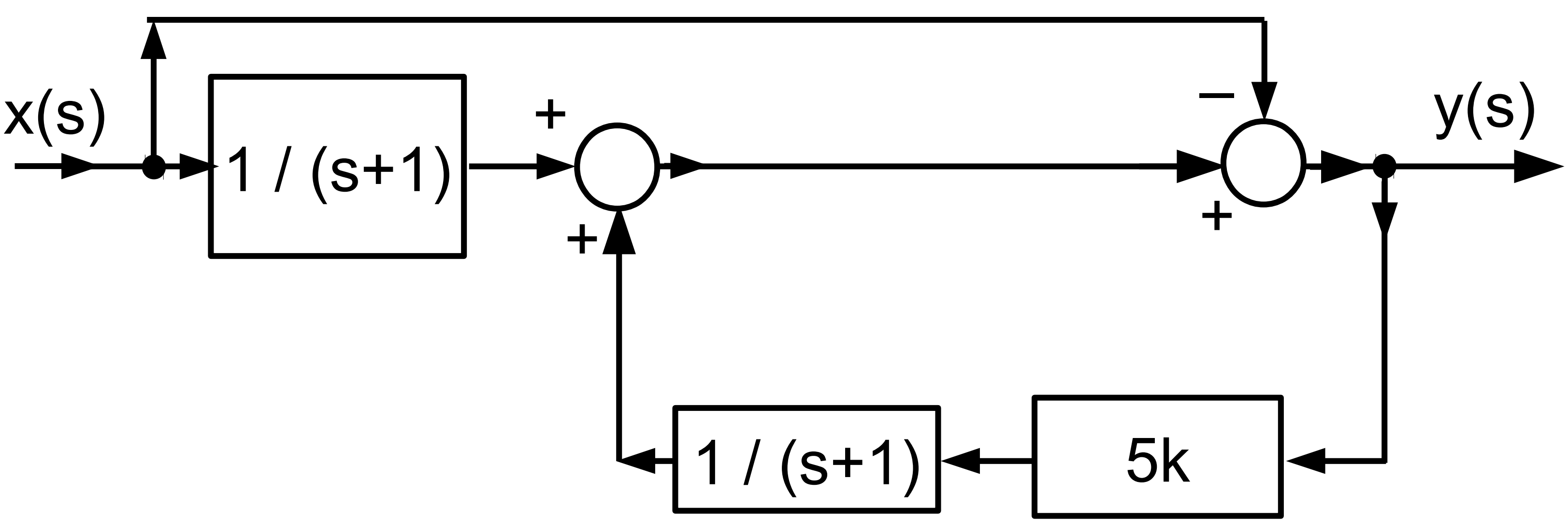


EXAMPLE 3

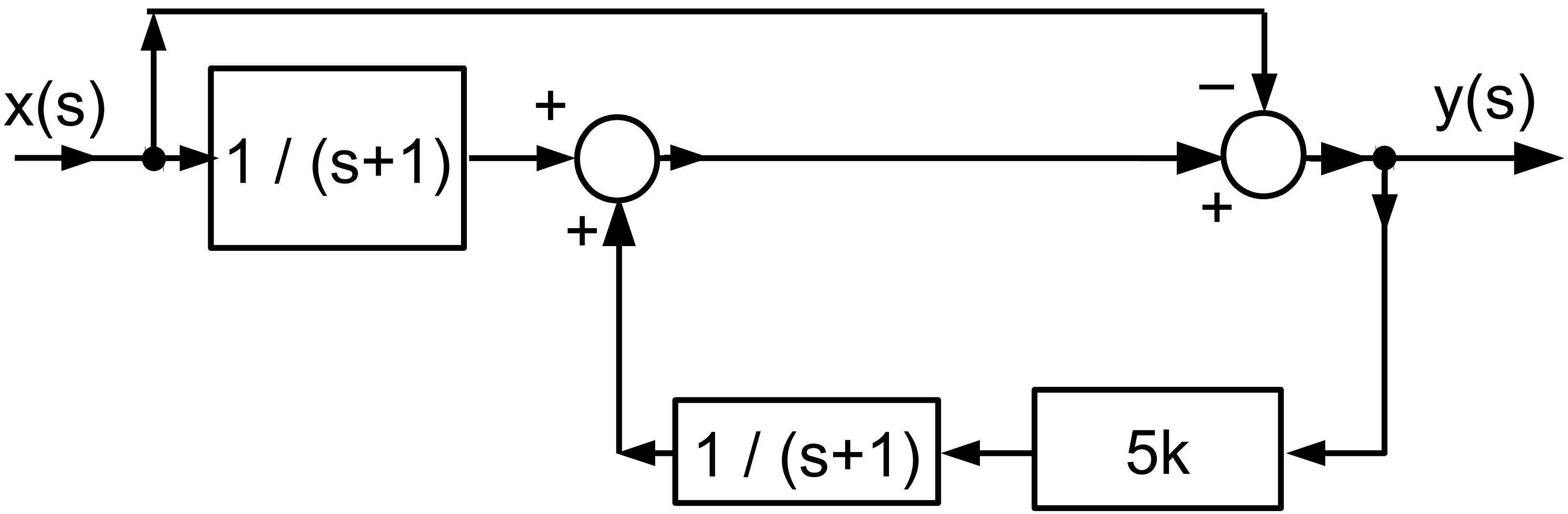


EXAMPLE 3

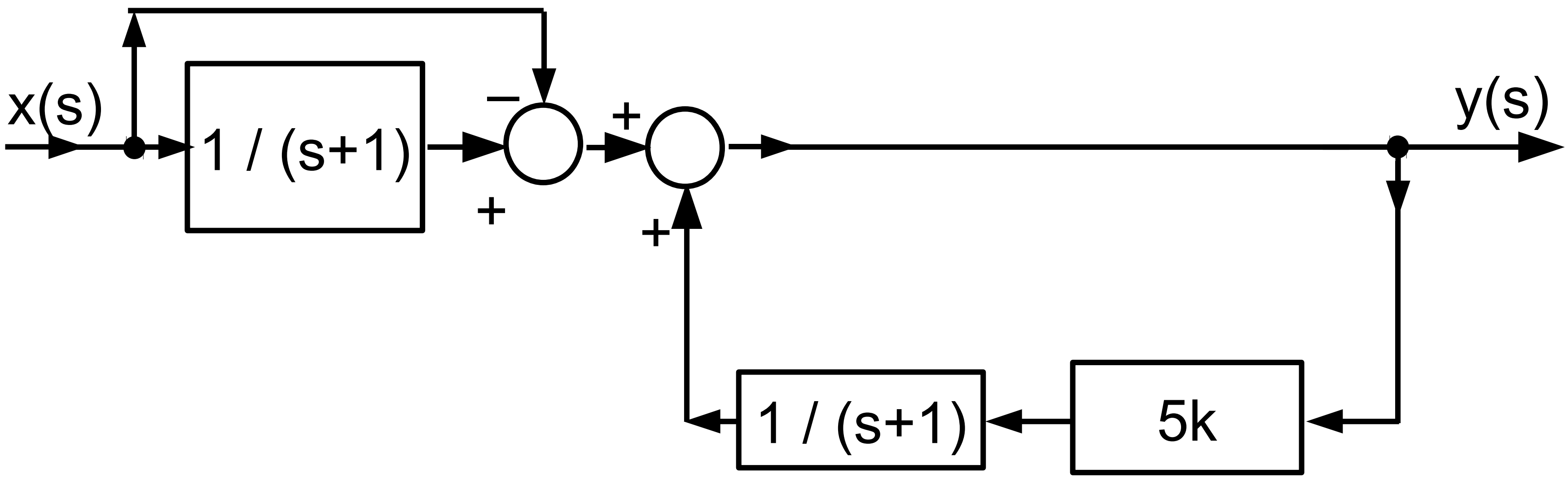




EXAMPLE 3

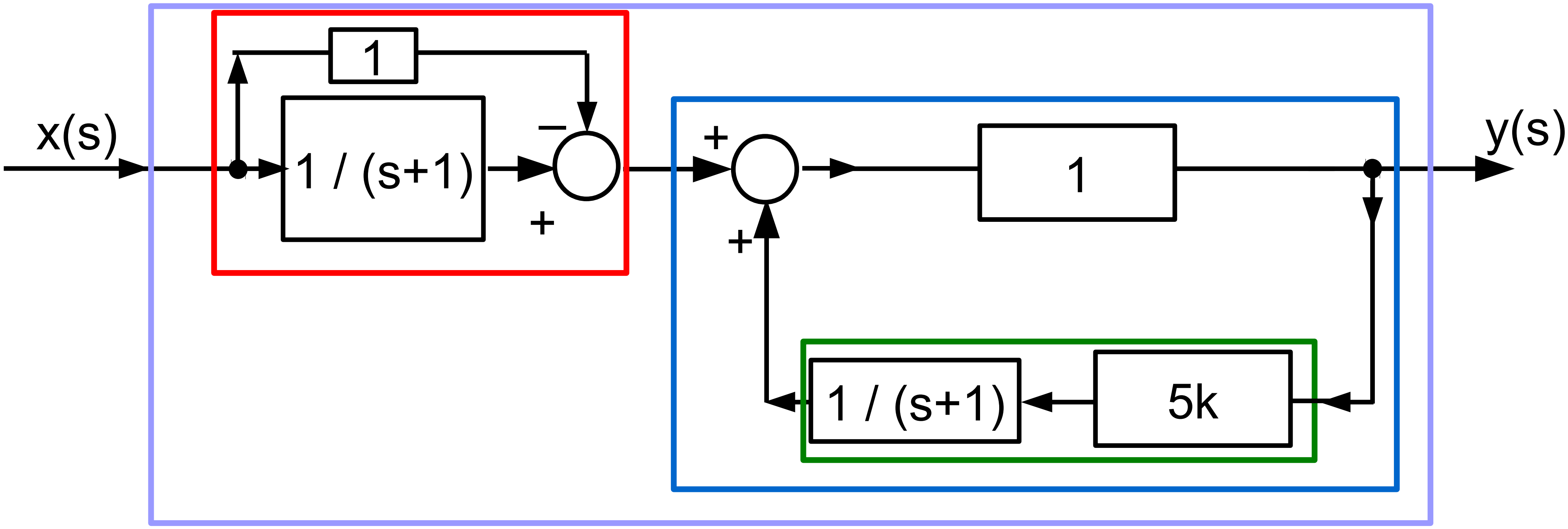


EXAMPLE 3

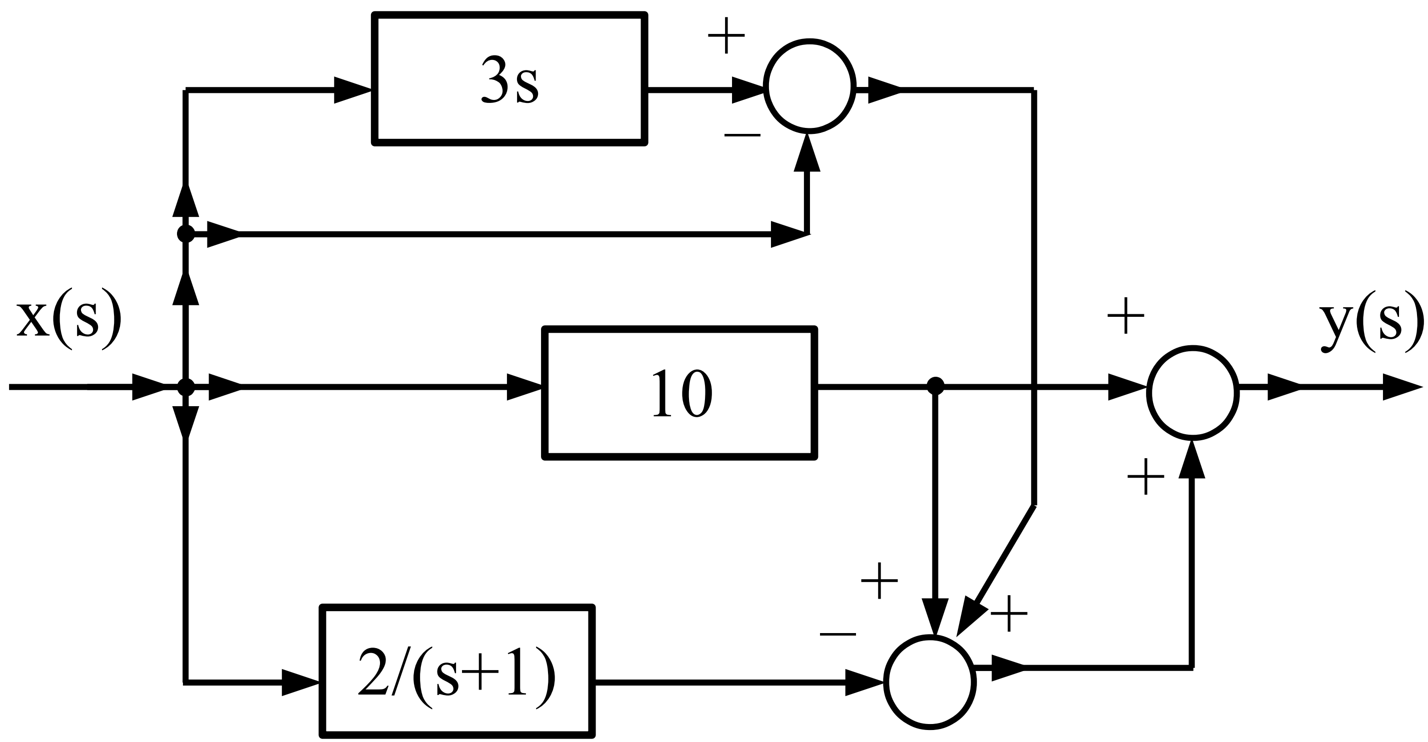


be careful with signs when changing sum points order

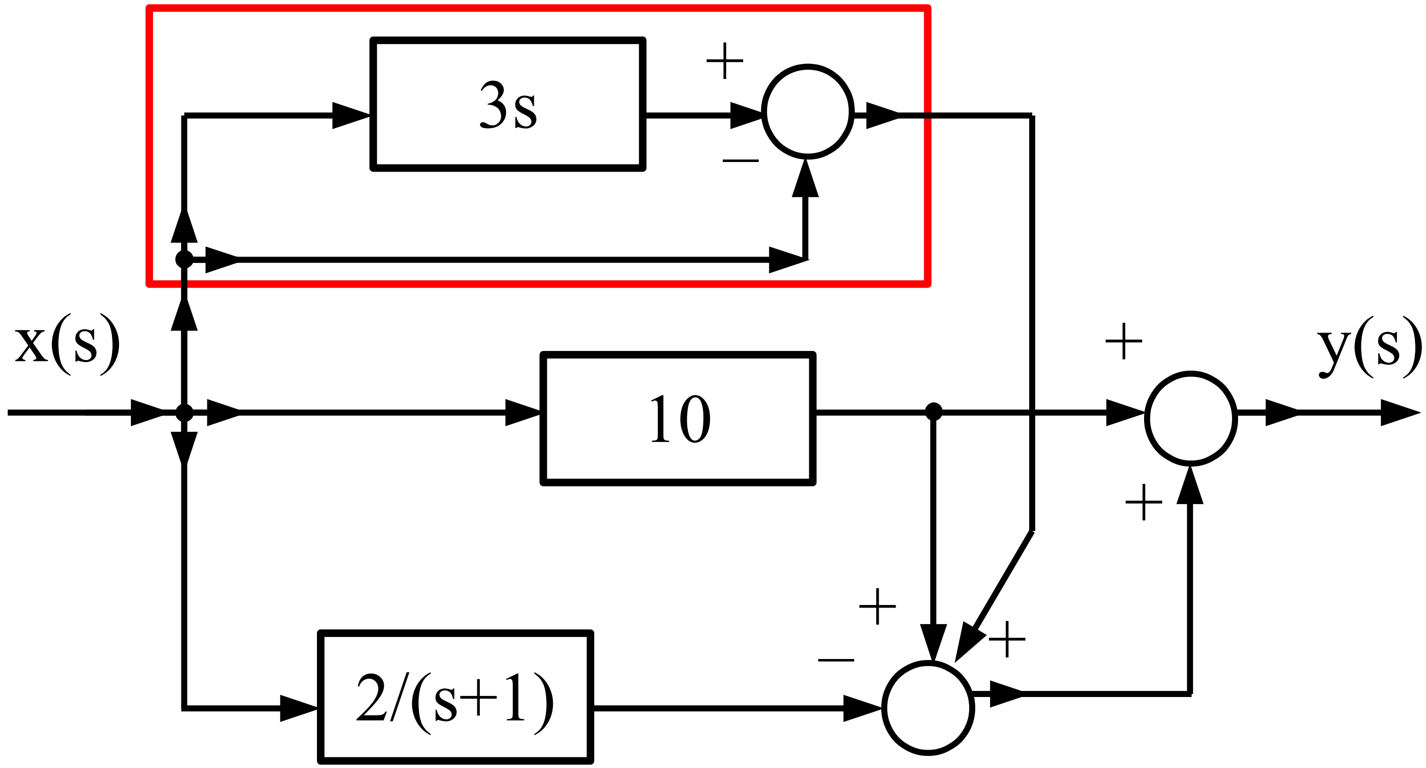
EXAMPLE 3



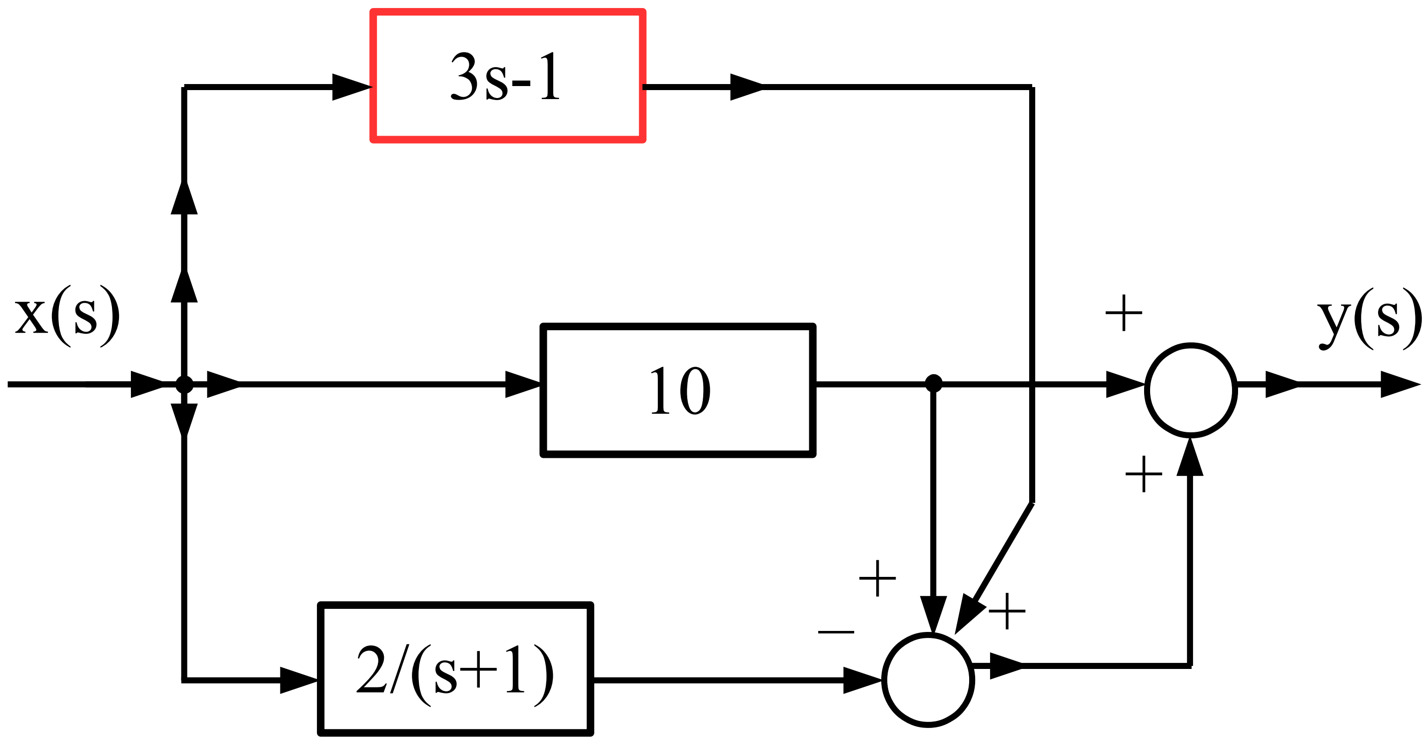
EXAMPLE 4



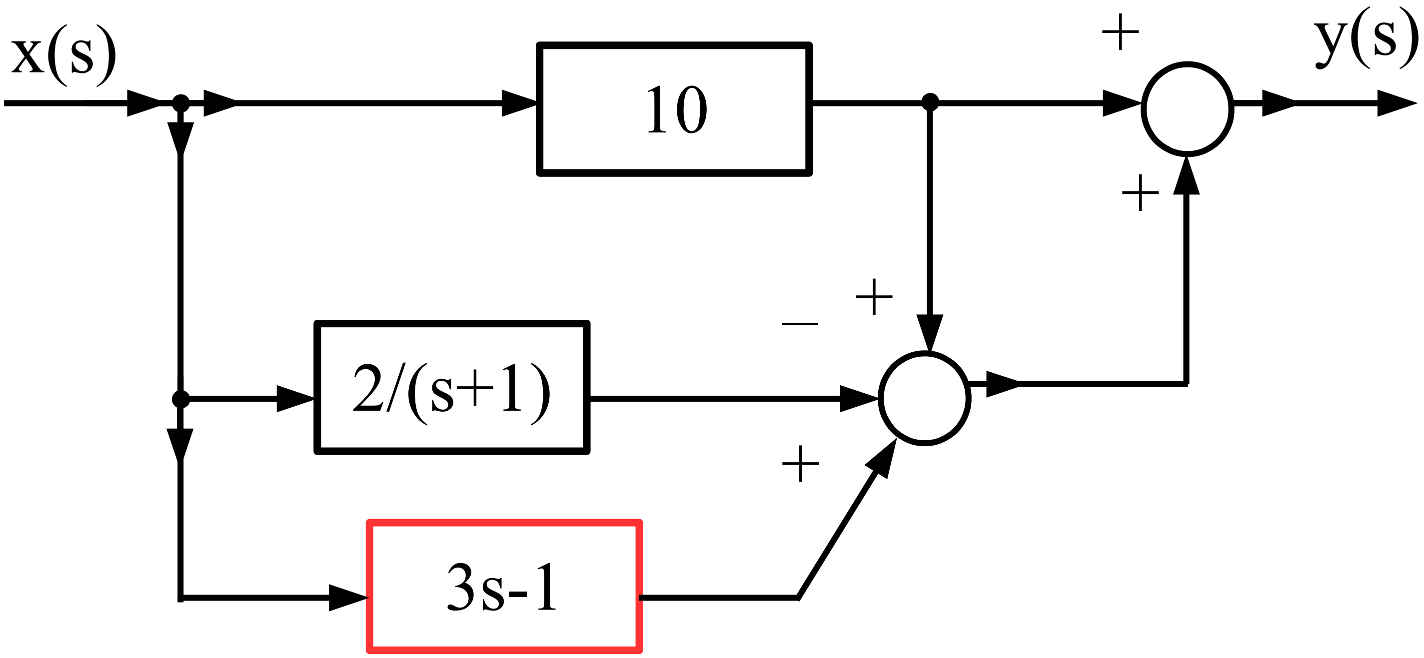
EXAMPLE 4



EXAMPLE 4



EXAMPLE 4



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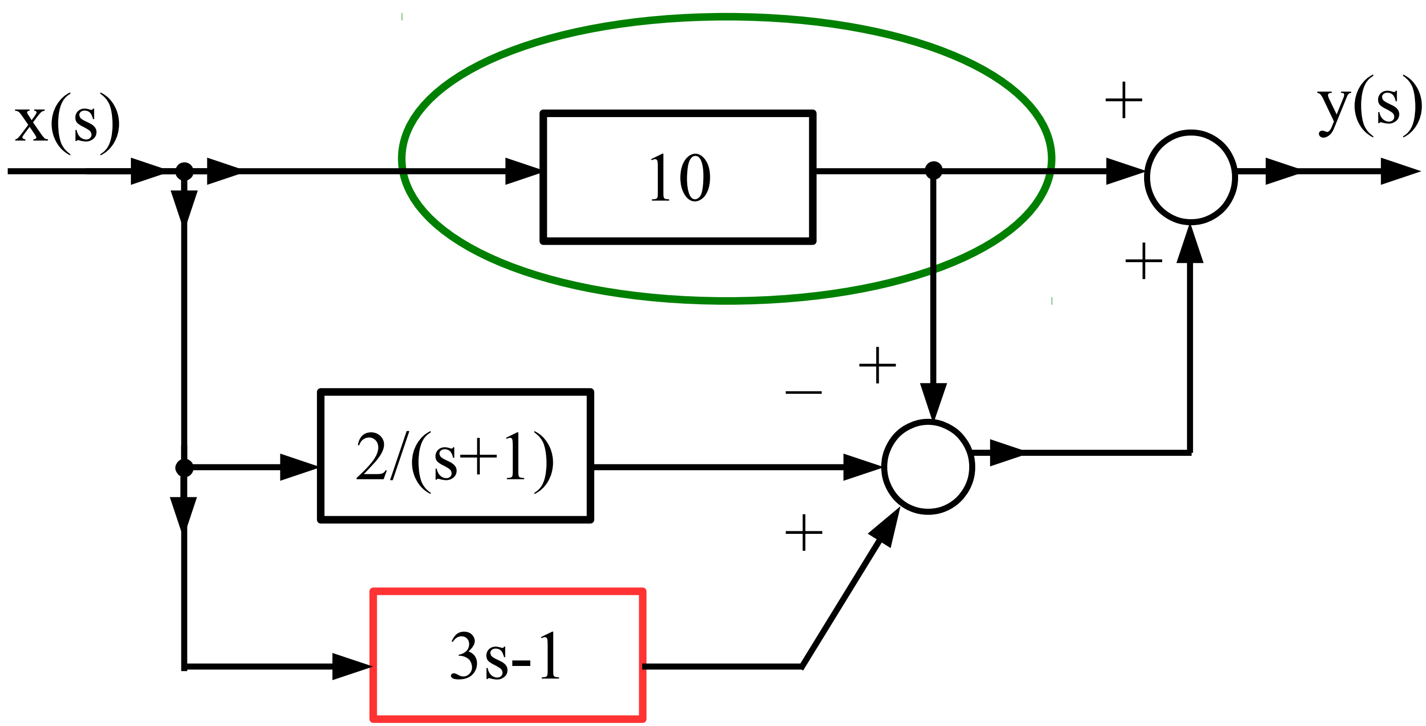
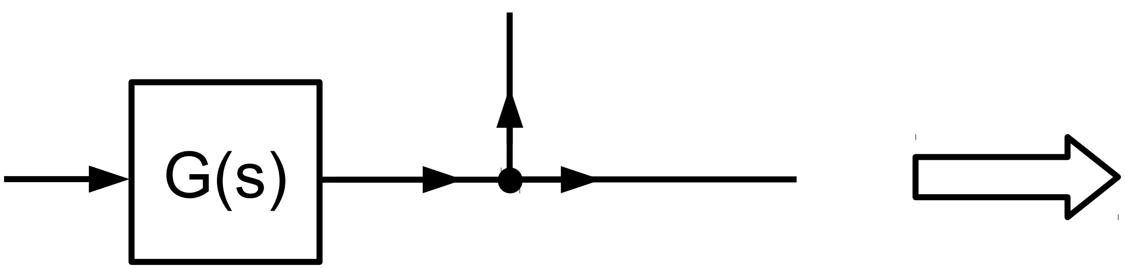


BLOCK DIAGRAM ALGEBRA
order change of block and information node



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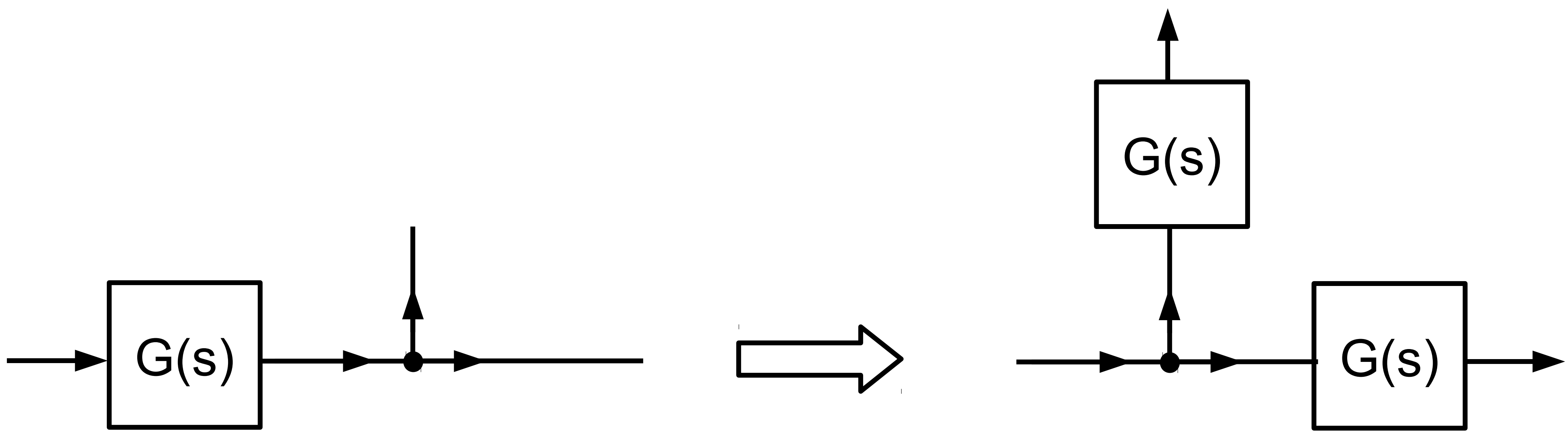

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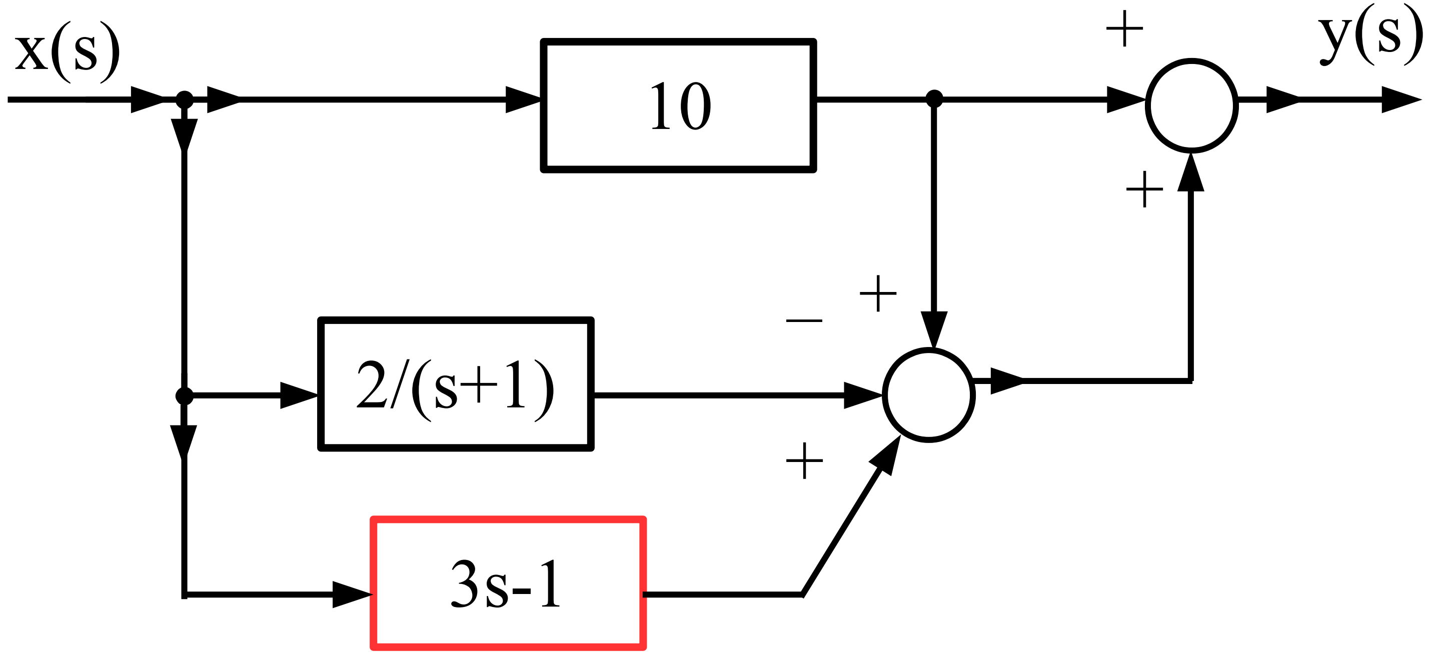

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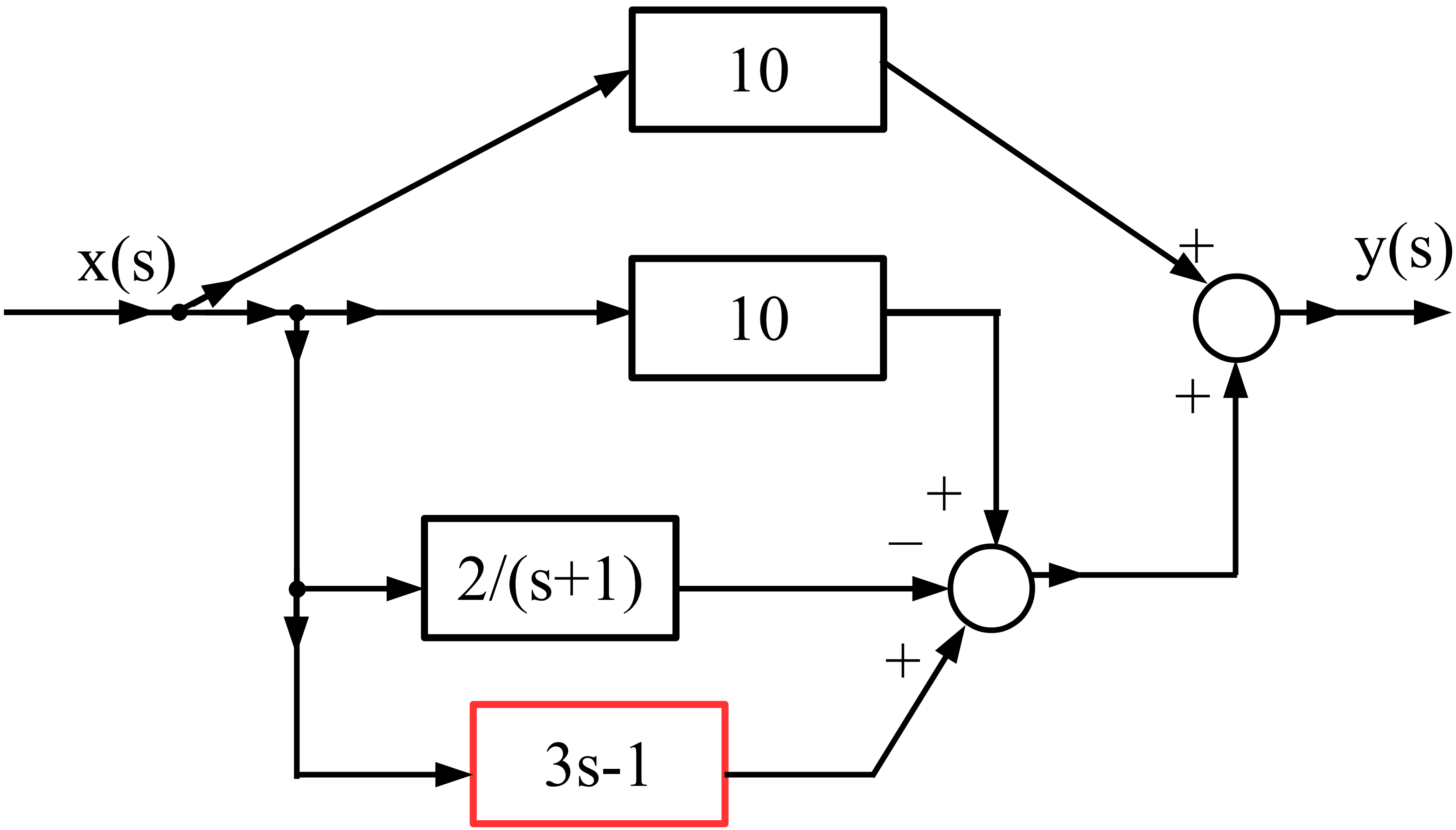

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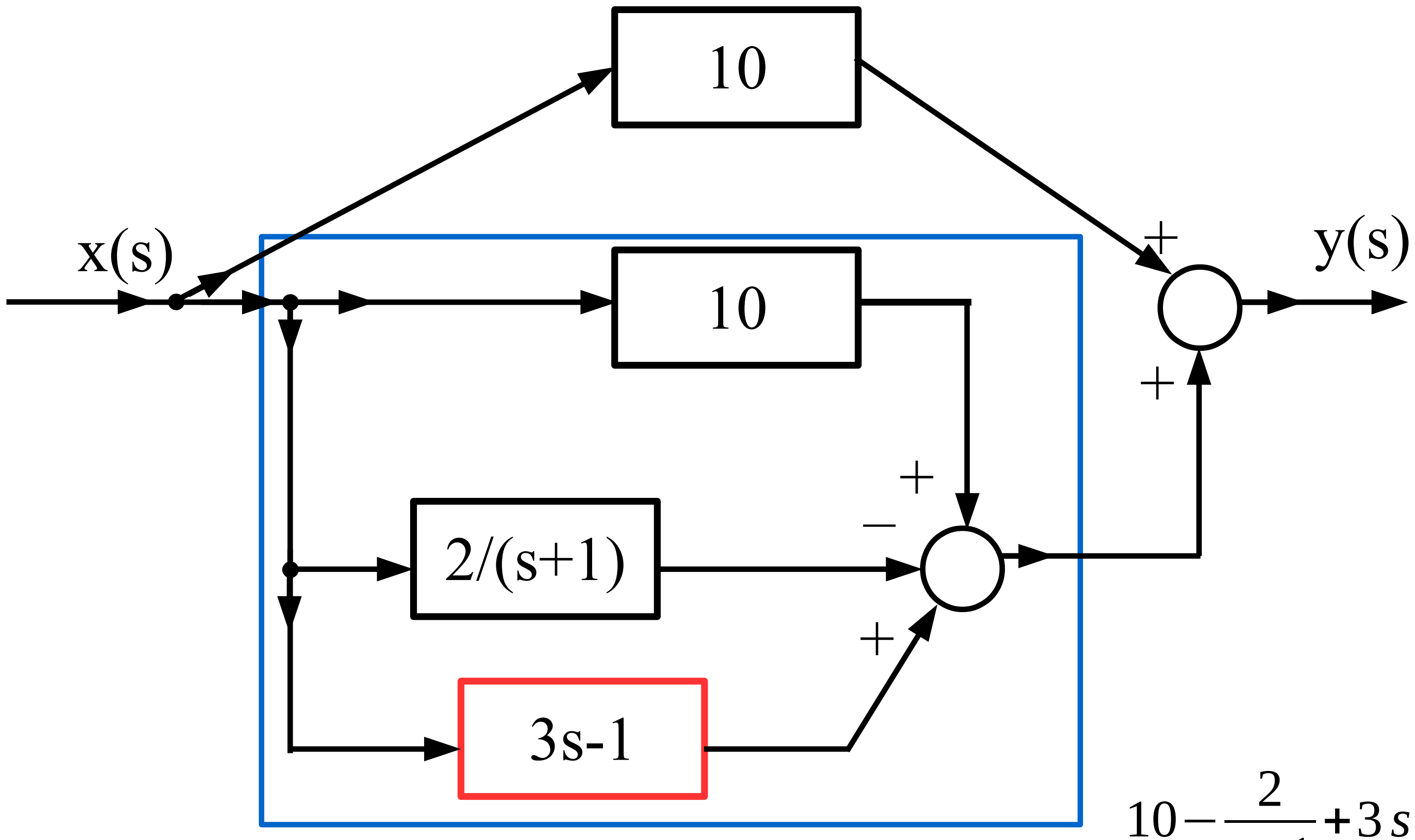
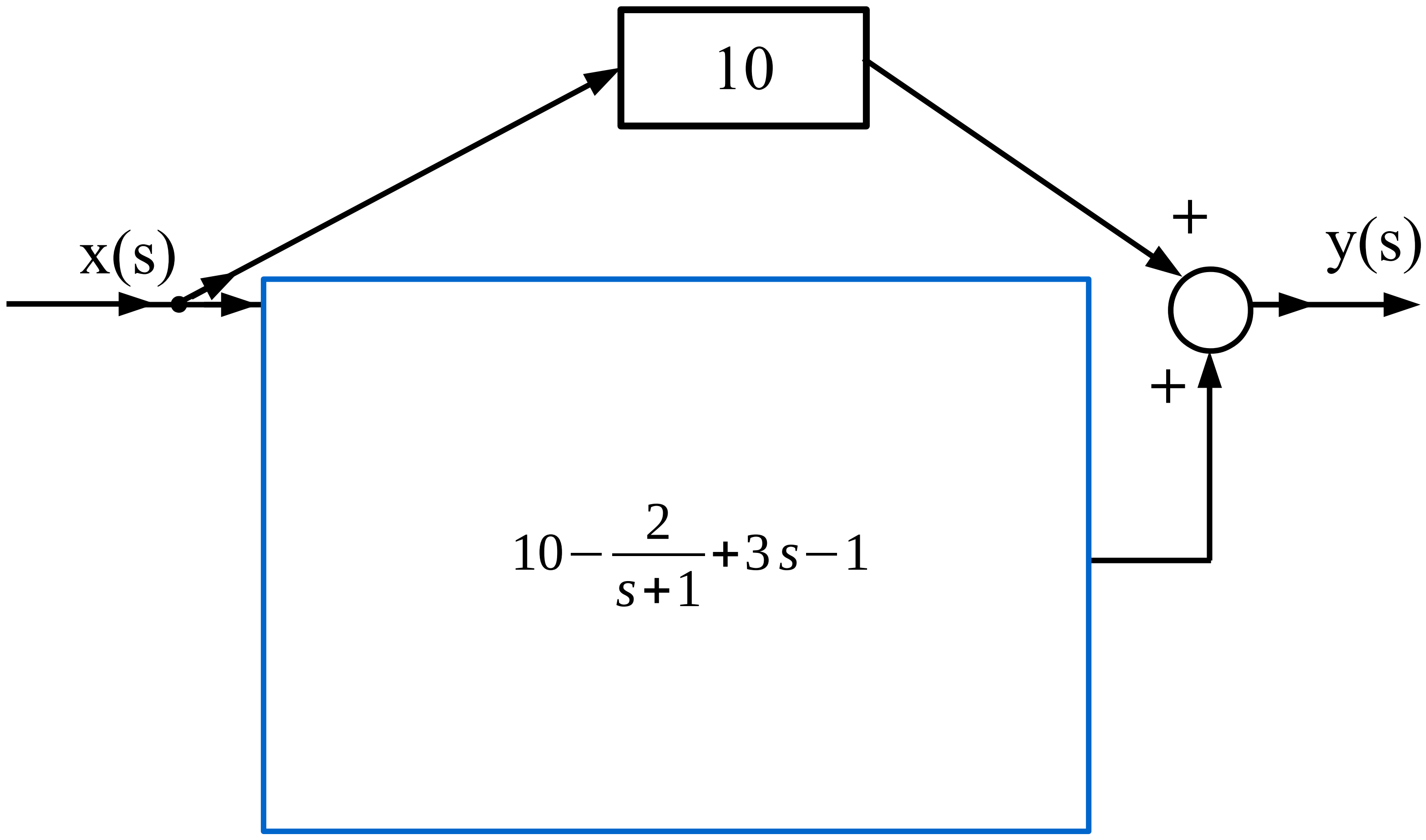


$$10-\frac{2}{s+1}+3s-1$$

EXAMPLE 4



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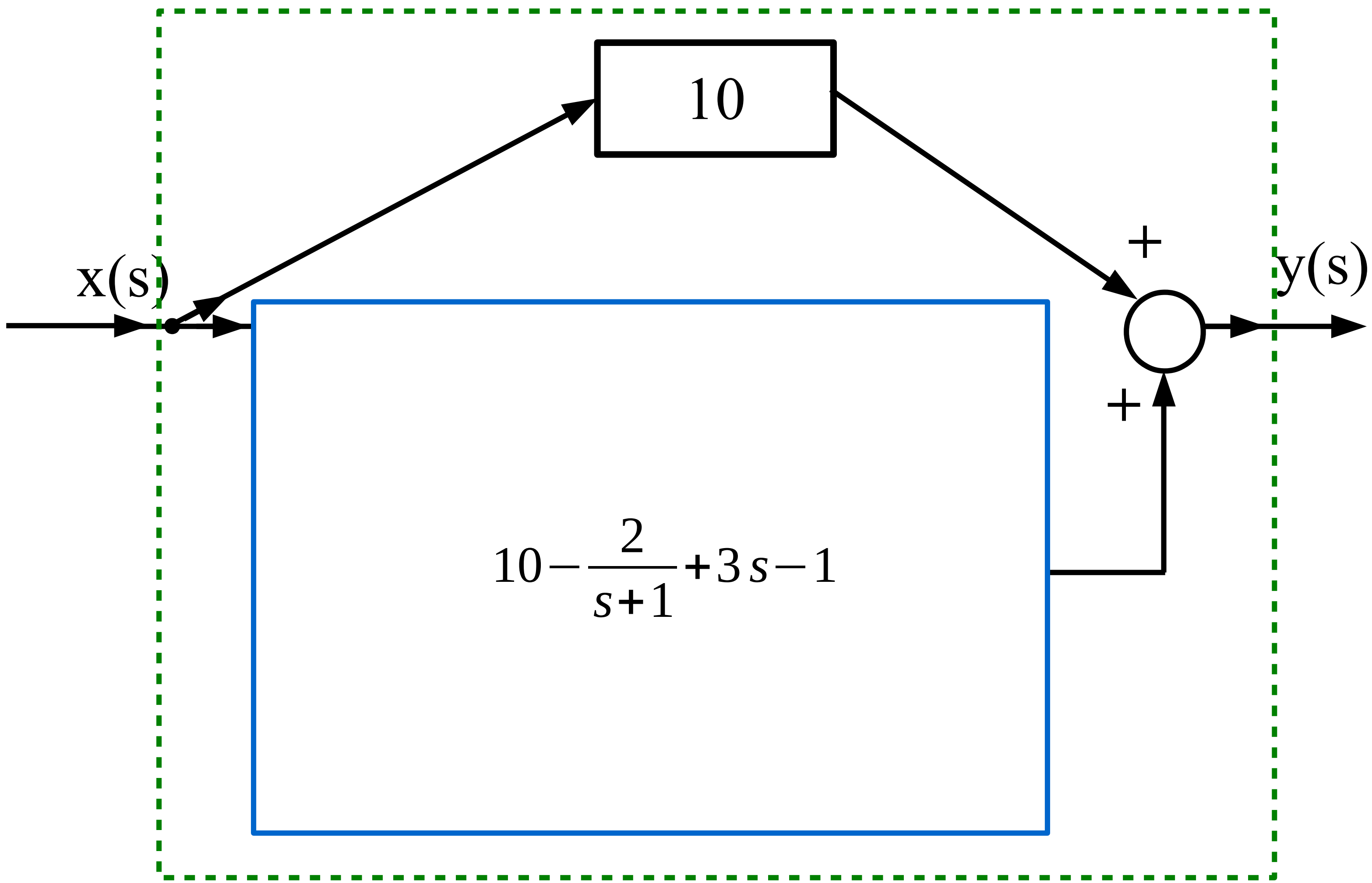

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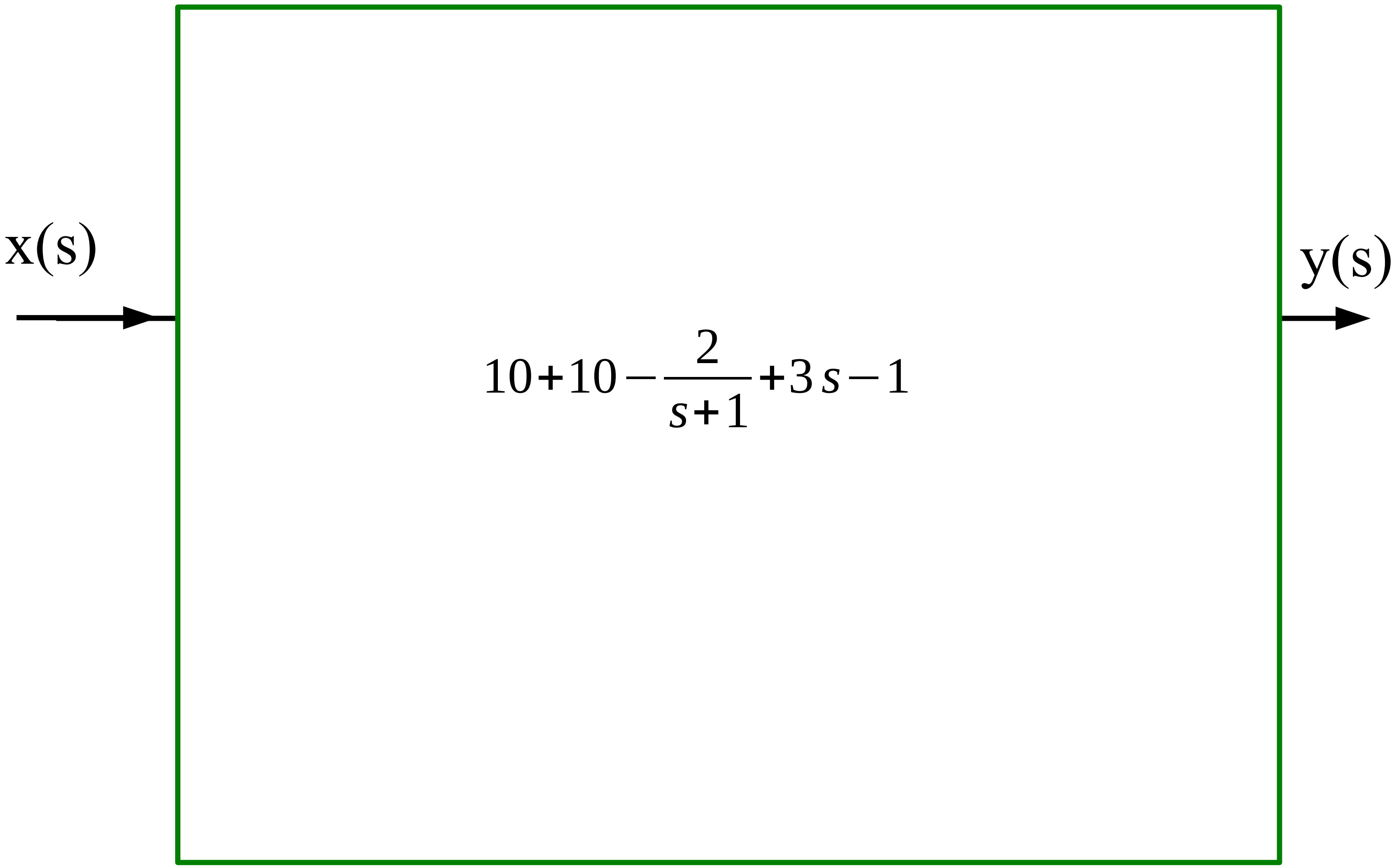

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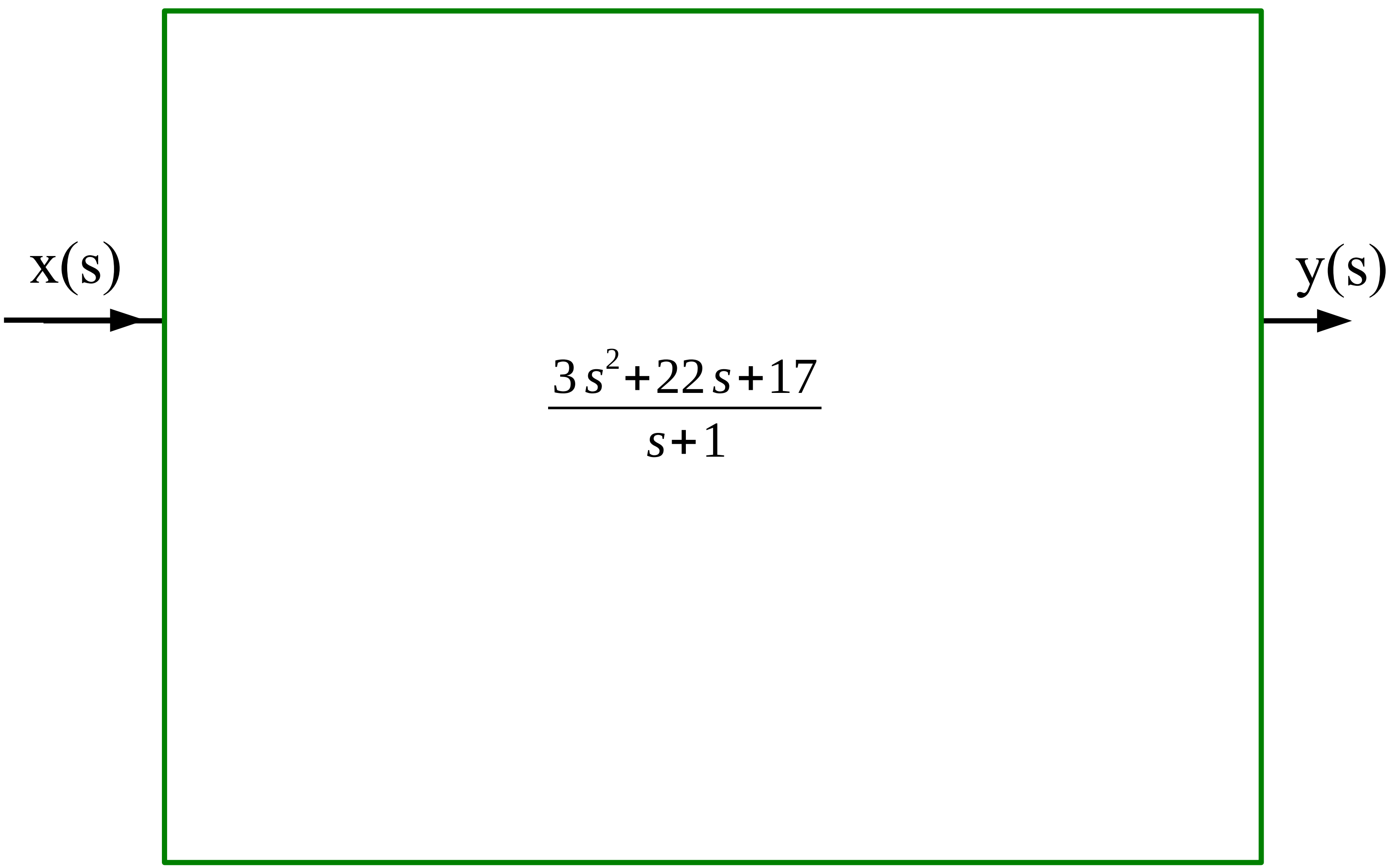
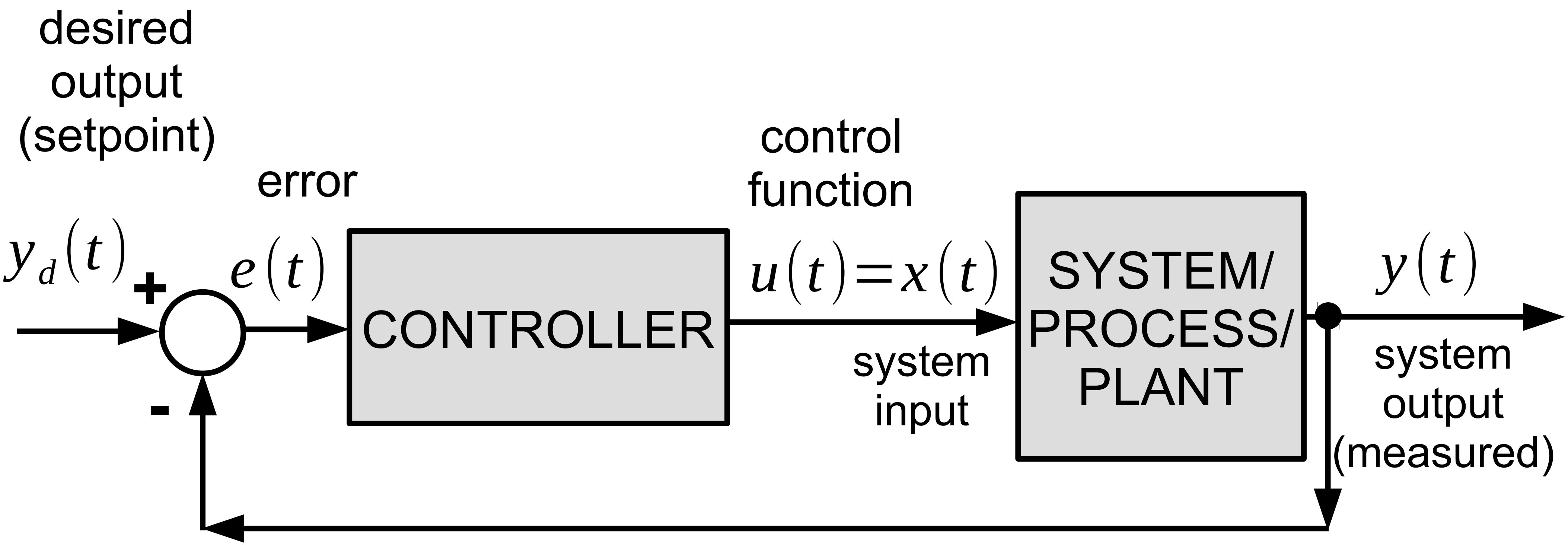


Controllers

Closed loop control



Closed loop control

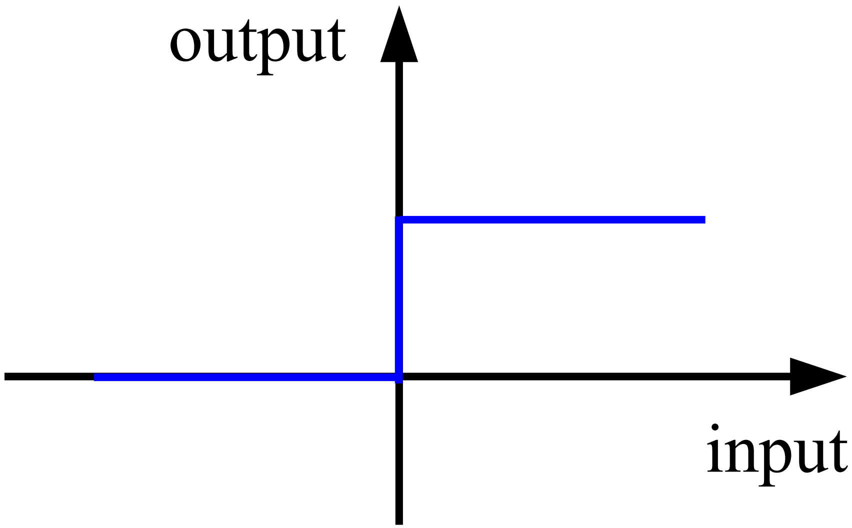
Types of controllers

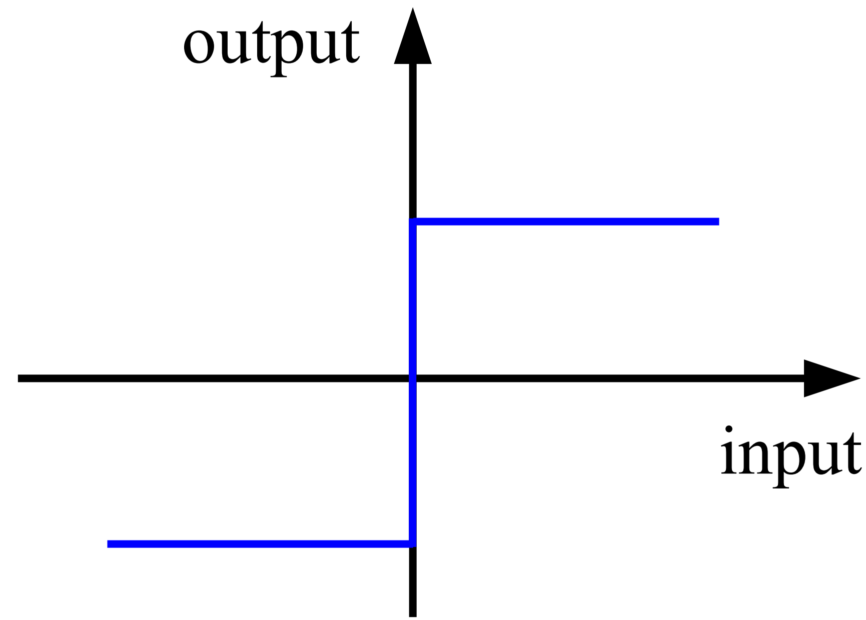
- ON/OFF
- three state
- Proportional (P)
- Integrator (I)
- Differentiator (D)
- Proportional-inegral-derivative (PID)

Types of controllers

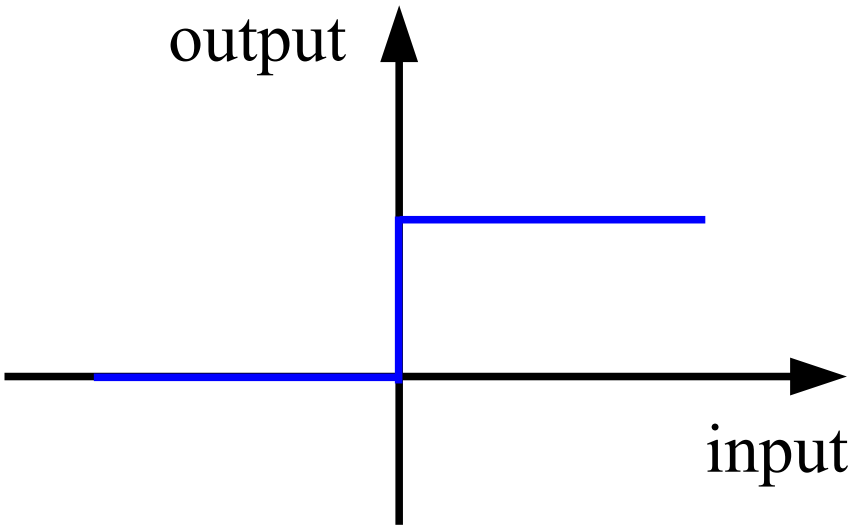
Controller	Transfer function
P	k_p
I	$\frac{1}{T_i s}$
D (ideal)	$T_d s$
D (real, with inertia)	$\frac{T_d s}{T s+1}$
PI	$k_p\left(1+\frac{1}{T_i s}\right)$
PD	$k_p(1+T_d s)$
PID (standard form)	$k_p\left(1+\frac{1}{T_i s}+T_d s\right)$
PID (ideal form)	$K_p+K_i\frac{1}{s}+K_d s$

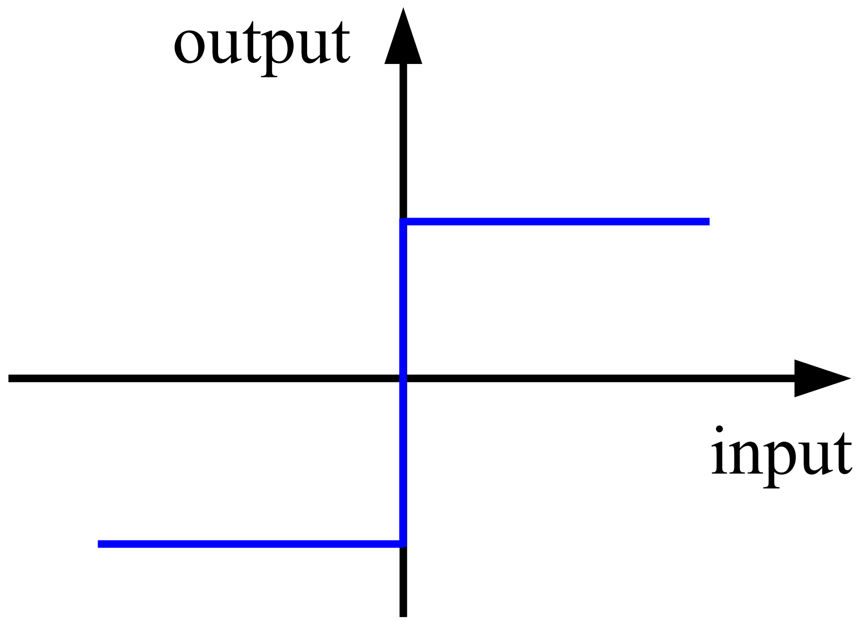
RELAY / ON-OFF / TWO STATE / BANG-BANG CONTROLLER



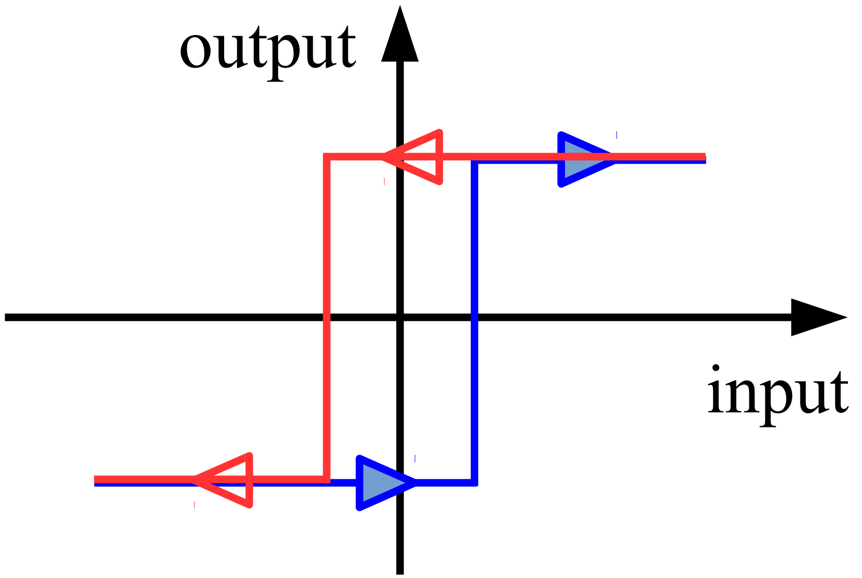


RELAY / ON-OFF / TWO STATE / BANG-BANG CONTROLLER

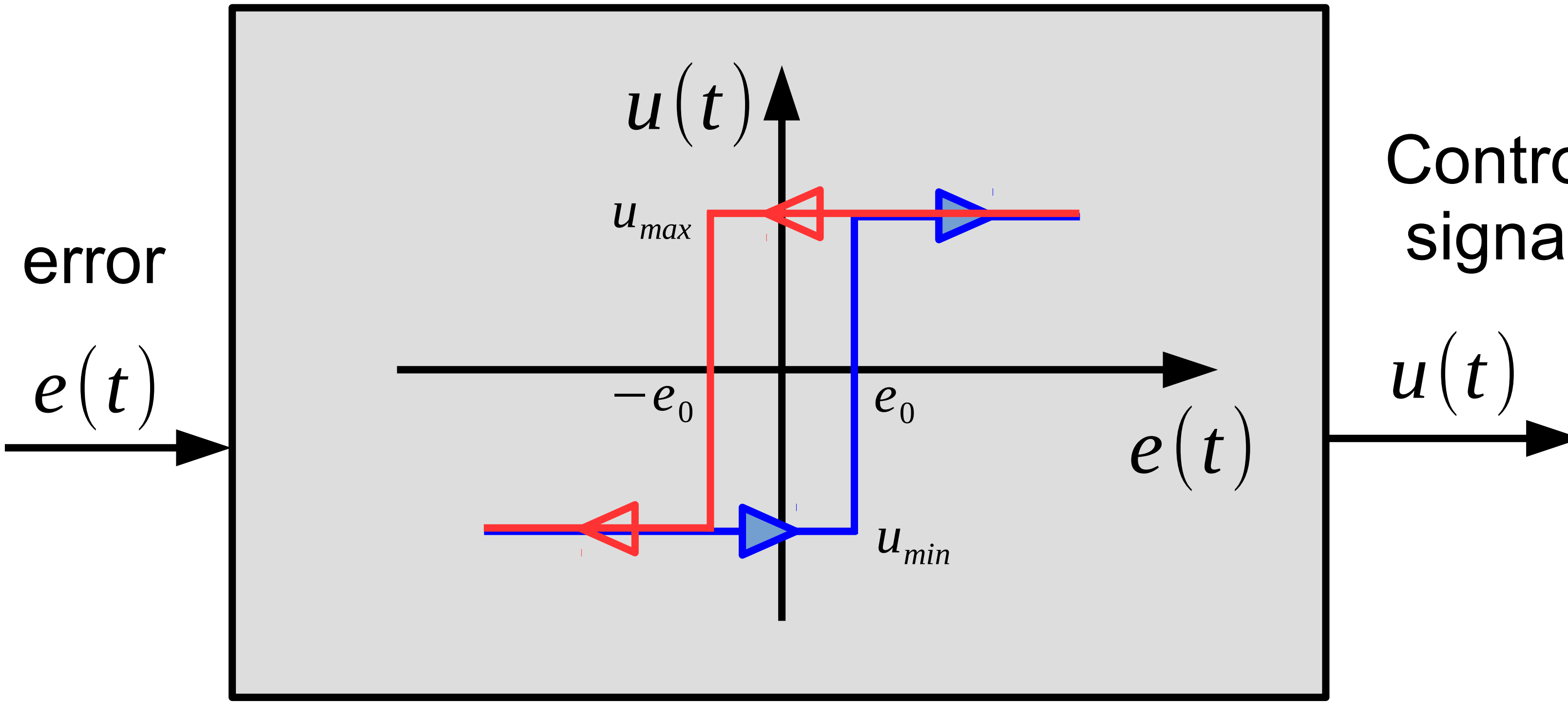




real
(with hysteresis)



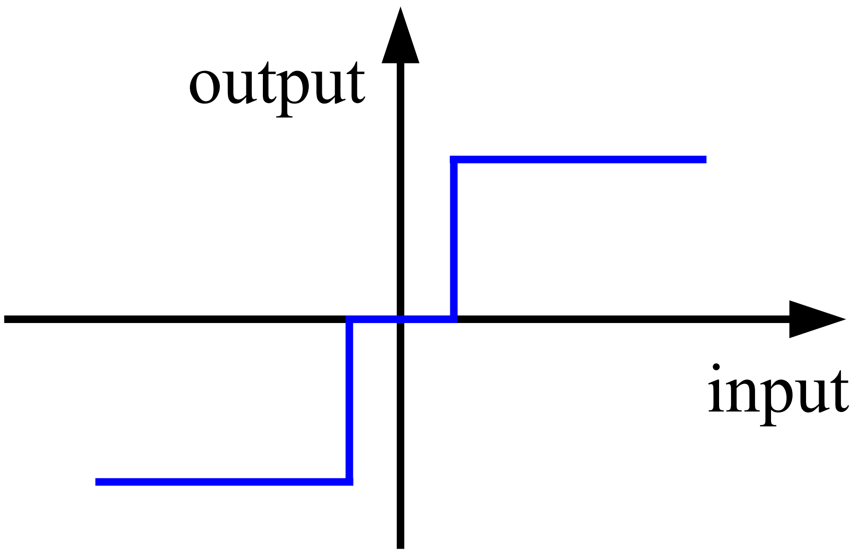
RELAY / ON-OFF / TWO STATE / BANG-BANG CONTROLLER



$$u(t)=\begin{cases} u_{max}, \text{ if } e>e_0 \\ u_{min}, \text{ if } e<-e_0 \\ \text{no change, in other situations} \end{cases}$$

e_o - mechanical or programmed hysteresis

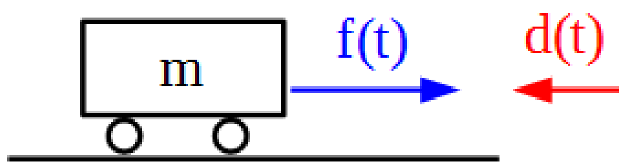
THREE STATE CONTROLLER



EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

Car on a flat surface
m – mass,
f(t) – driving force,
d(t)=c*v(t) – air resistance,
v(t) – velocity



$$m\frac{dv(t)}{dt}=f(t)-d(t)$$

$$H(s)=\frac{V(s)}{F(s)}=\frac{1}{ms+c}$$

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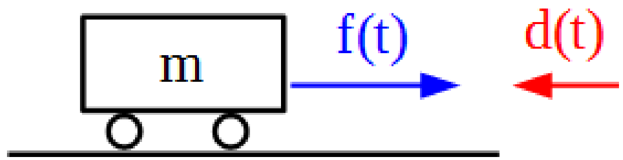

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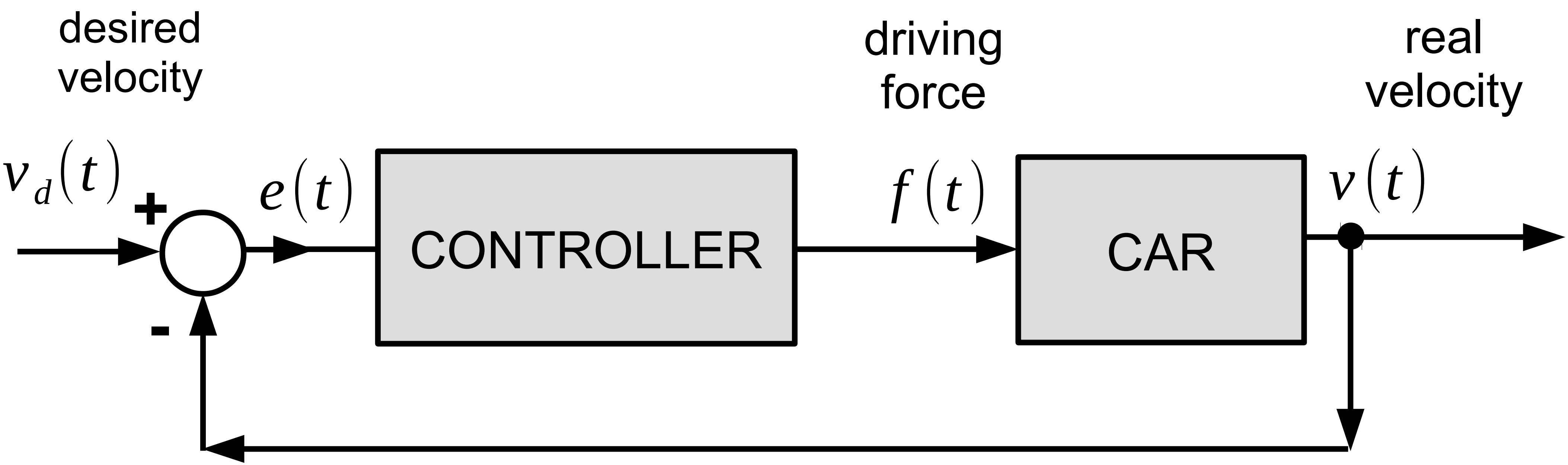

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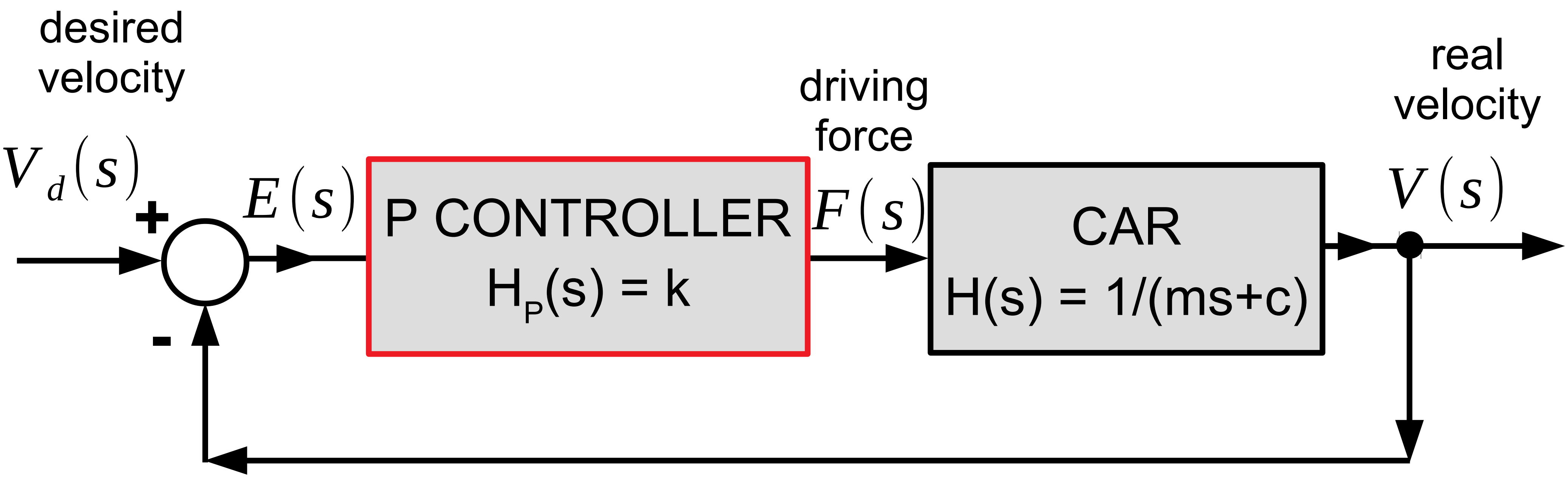

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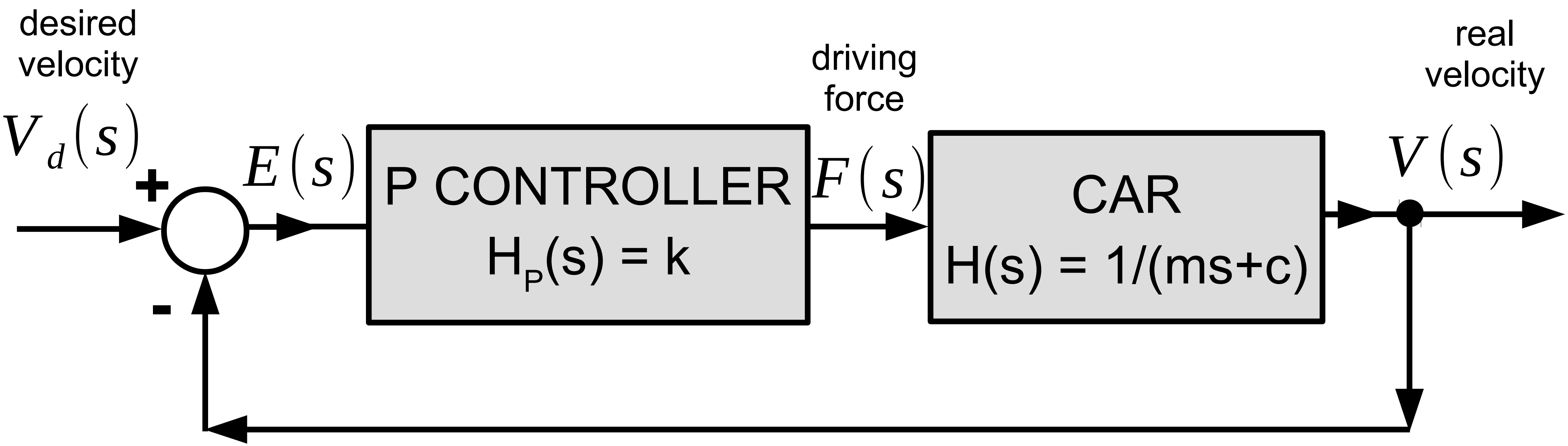
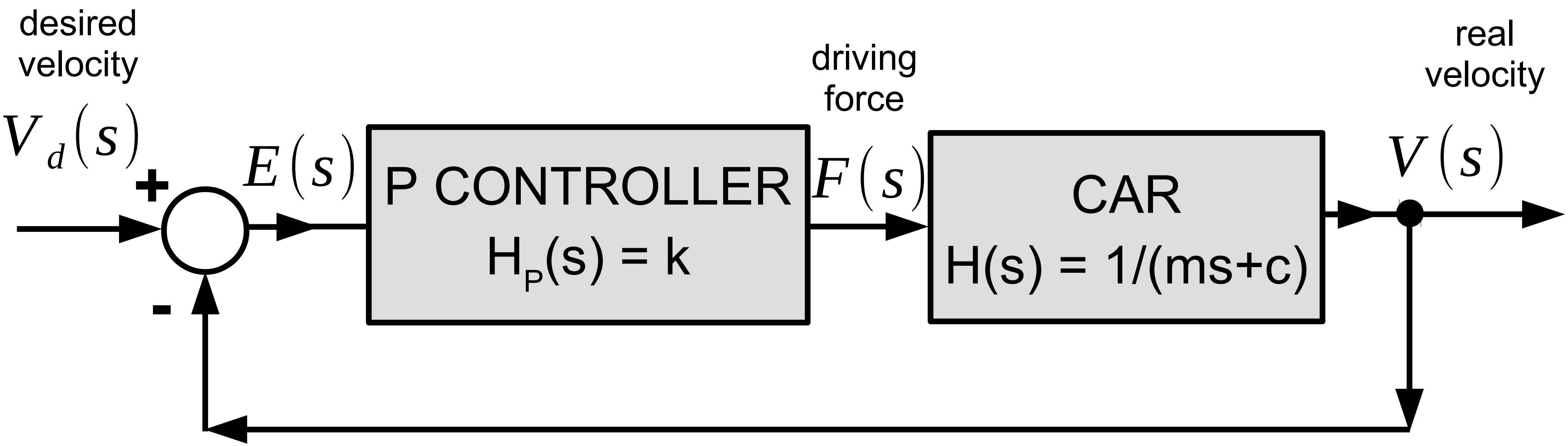


$$H_R(s) = \frac{H_P(s)H(s)}{1 + H_P(s)H(s)}$$

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)



$$H_R(s)=\frac{H_P(s)H(s)}{1+H_P(s)H(s)}$$

$$H_R(s)=\frac{k}{ms+c+k}$$

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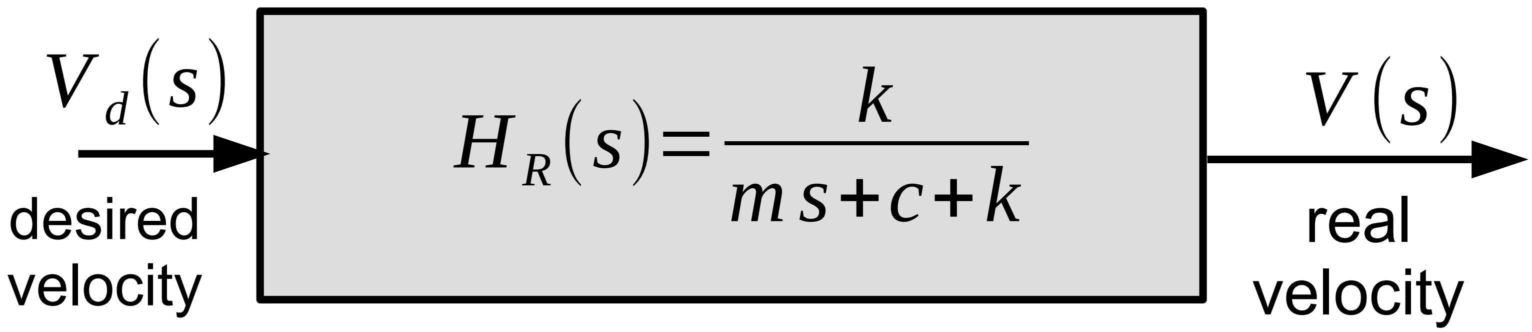
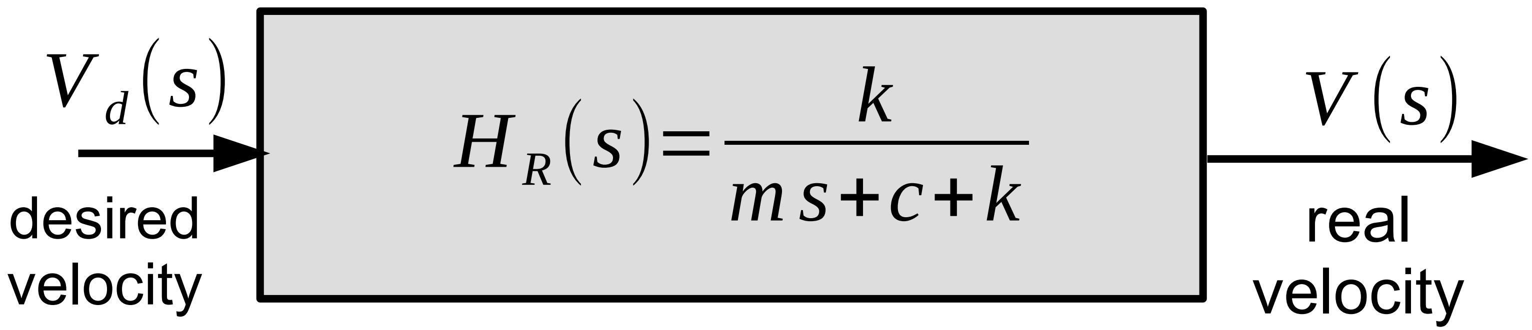
Speed control (cruise control, autocruise, tempomat)



Input function: $v_d(t)=v_0 1(t)$

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)



Input function: $v_d(t)=v_0 1(t)$ Laplace of input: $V_d(s)=v_0\frac{1}{s}$

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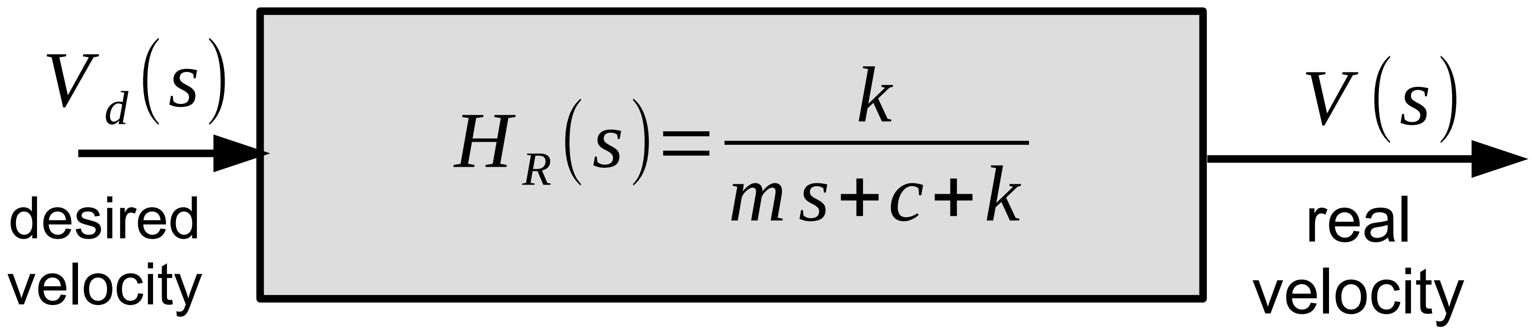
Speed control (cruise control, autocruise, tempomat)



Input function: $v_d(t)=v_0 1(t)$ Laplace of input: $V_d(s)=v_0\frac{1}{s}$

Laplace of output:

$$V(s)=V_d(s)H_R(s)$$

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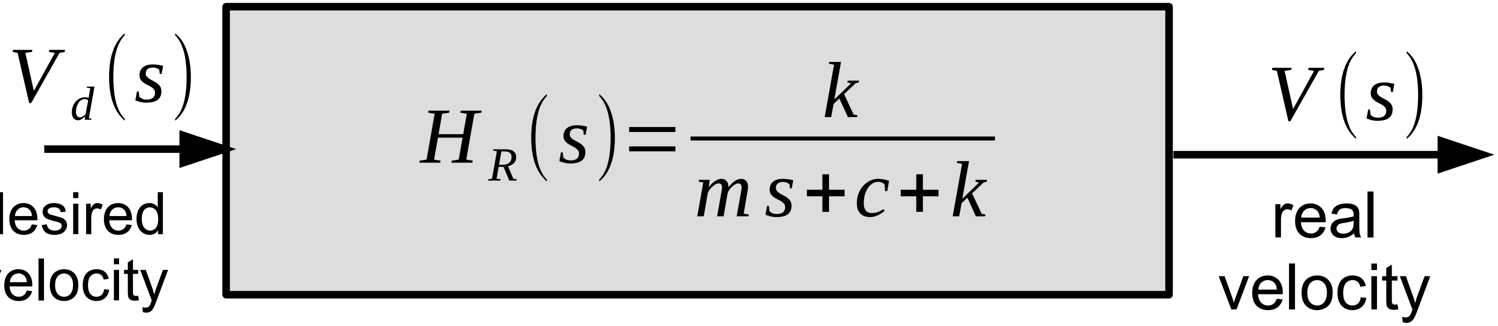

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Laplace of output:

$$V(s)=V_d(s)H_R(s)=\frac{v_0 k}{s(ms+c+k)}$$

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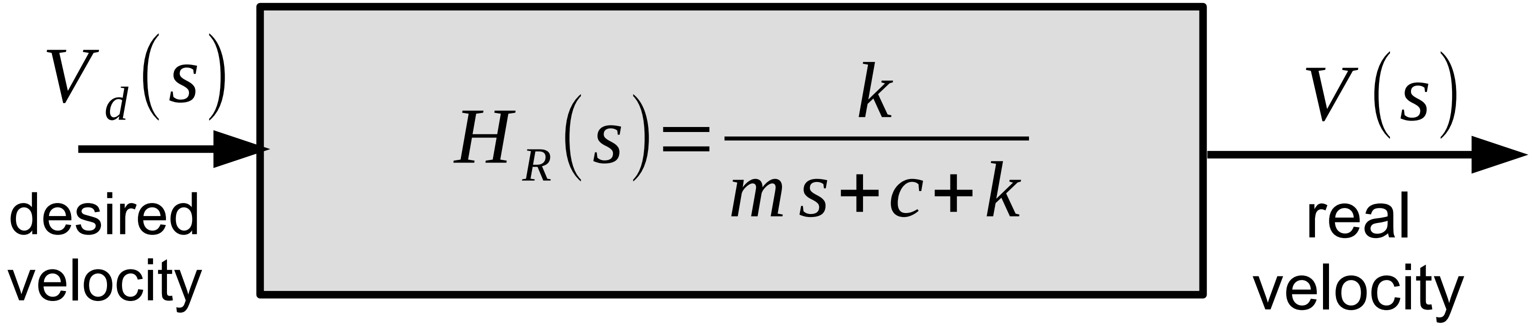

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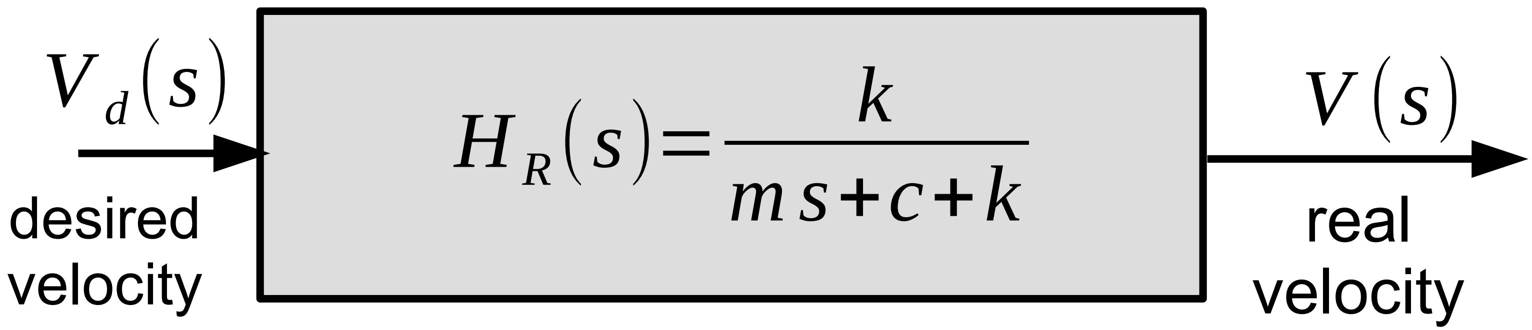
Laplace of output:

$$V(s)=V_d(s)H_R(s)=\frac{v_0 k}{s(ms+c+k)}=\frac{v_0 k}{c+k}\frac{\frac{c+k}{m}}{s\left(s+\frac{c+k}{m}\right)}$$

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)



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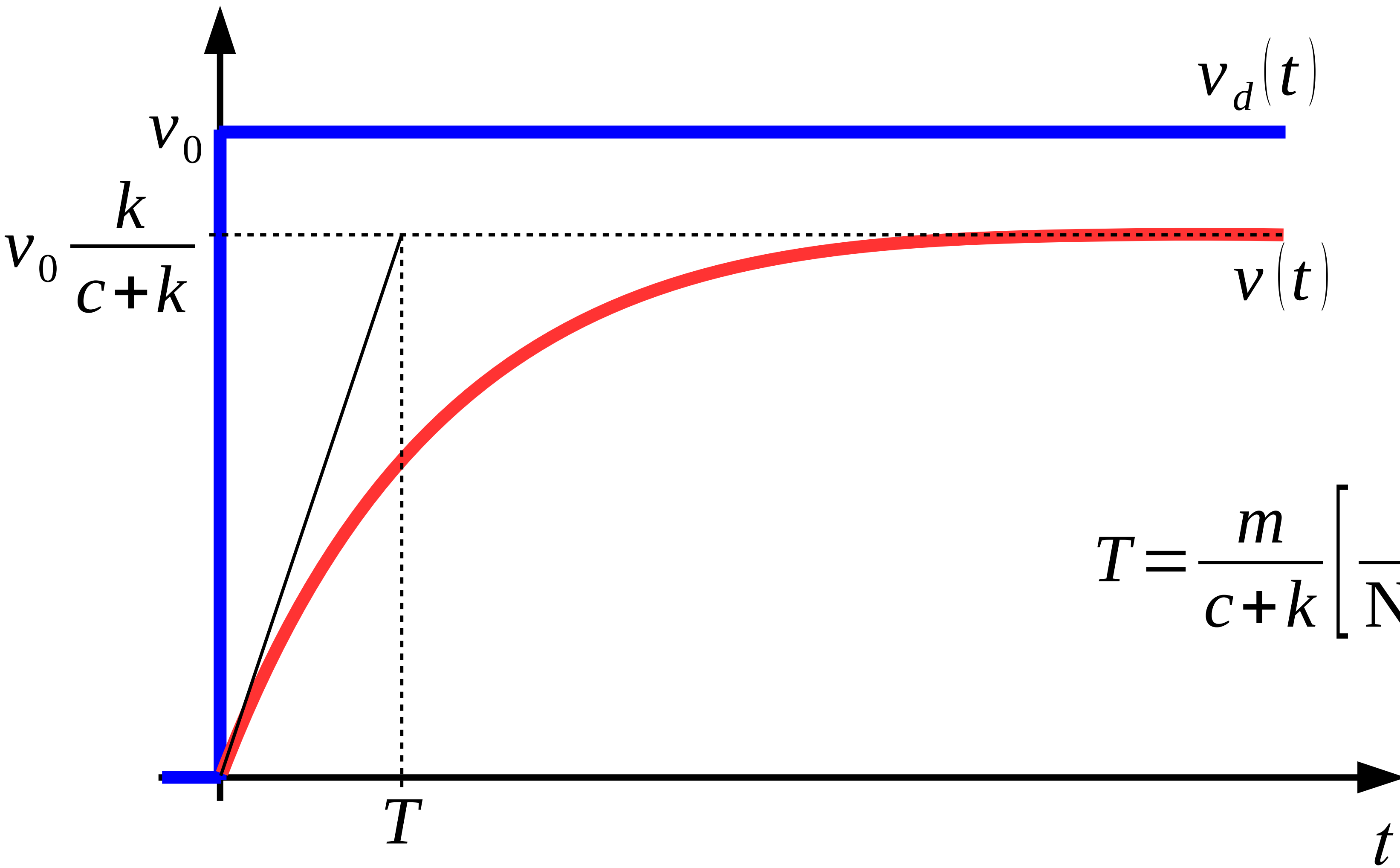
Output:

$$v(t)=\frac{v_0 k}{c+k}\left(1-\exp\left(-\frac{c+k}{m}t\right)\right)$$

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

$$v(t) = v_0 \frac{k}{c+k}\left(1 - \exp\left(-\frac{c+k}{m} t\right)\right)$$



$$T = \frac{m}{c+k}\left[\frac{\text{kg}}{\text{Ns/m}} = \text{s}\right]$$

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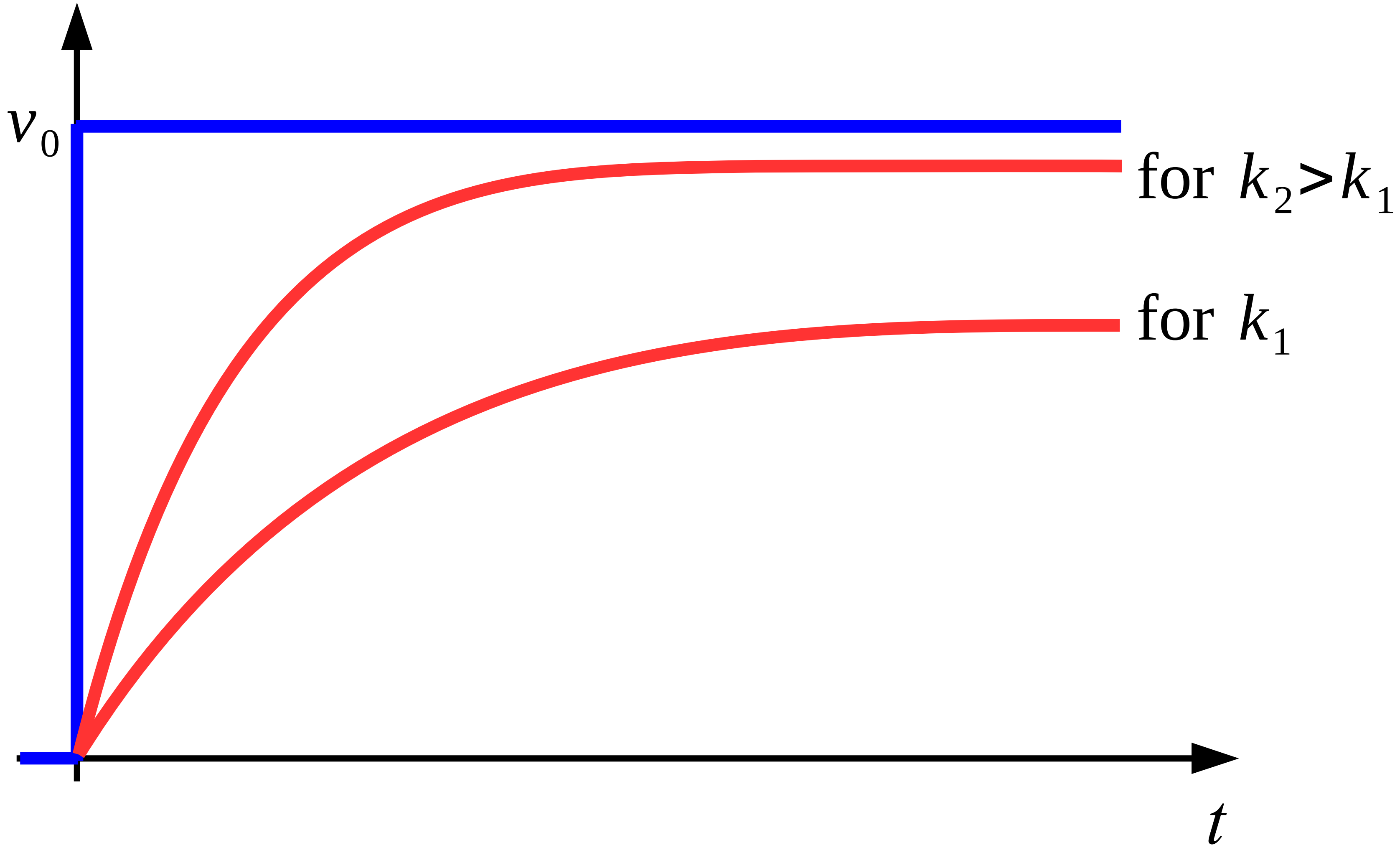

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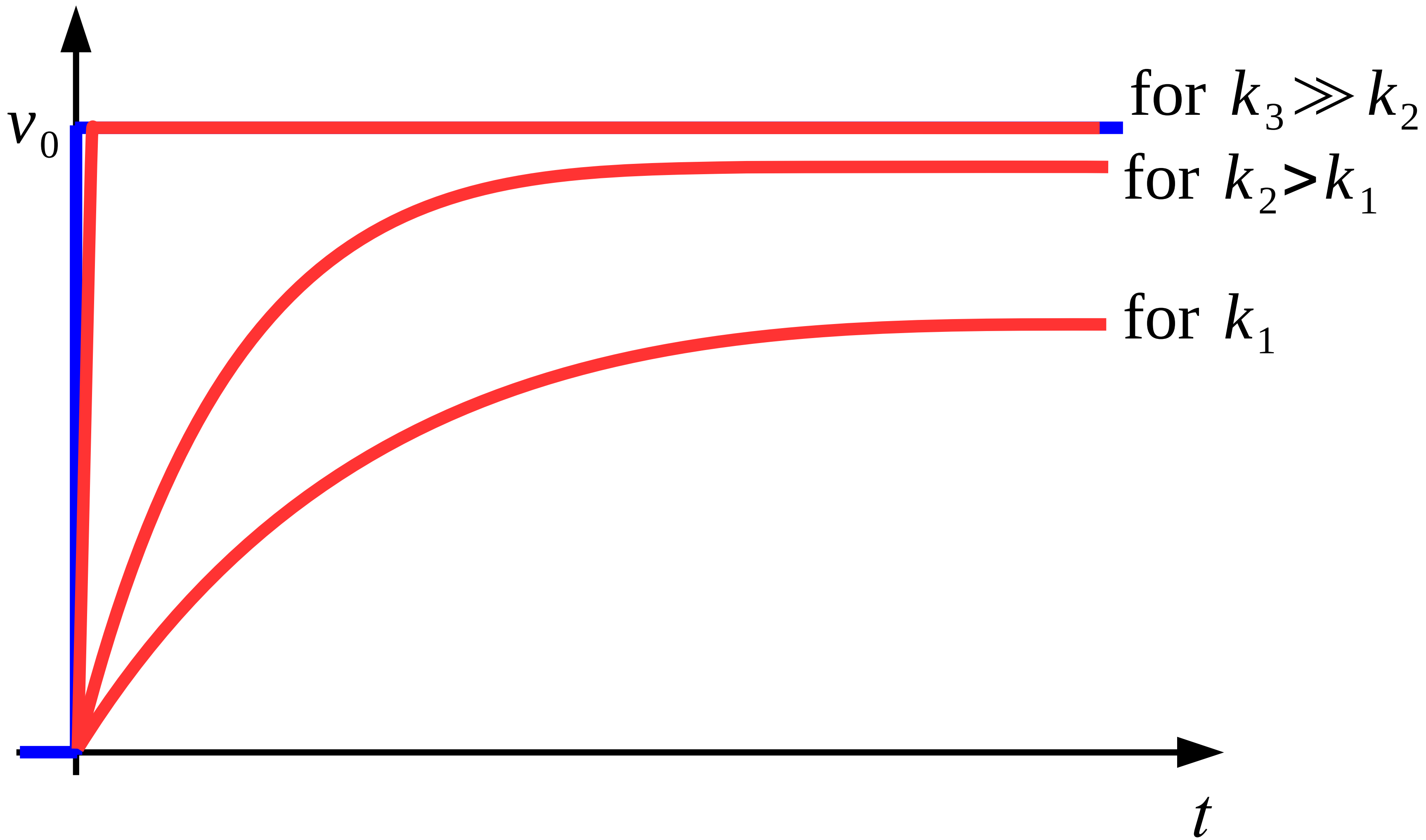

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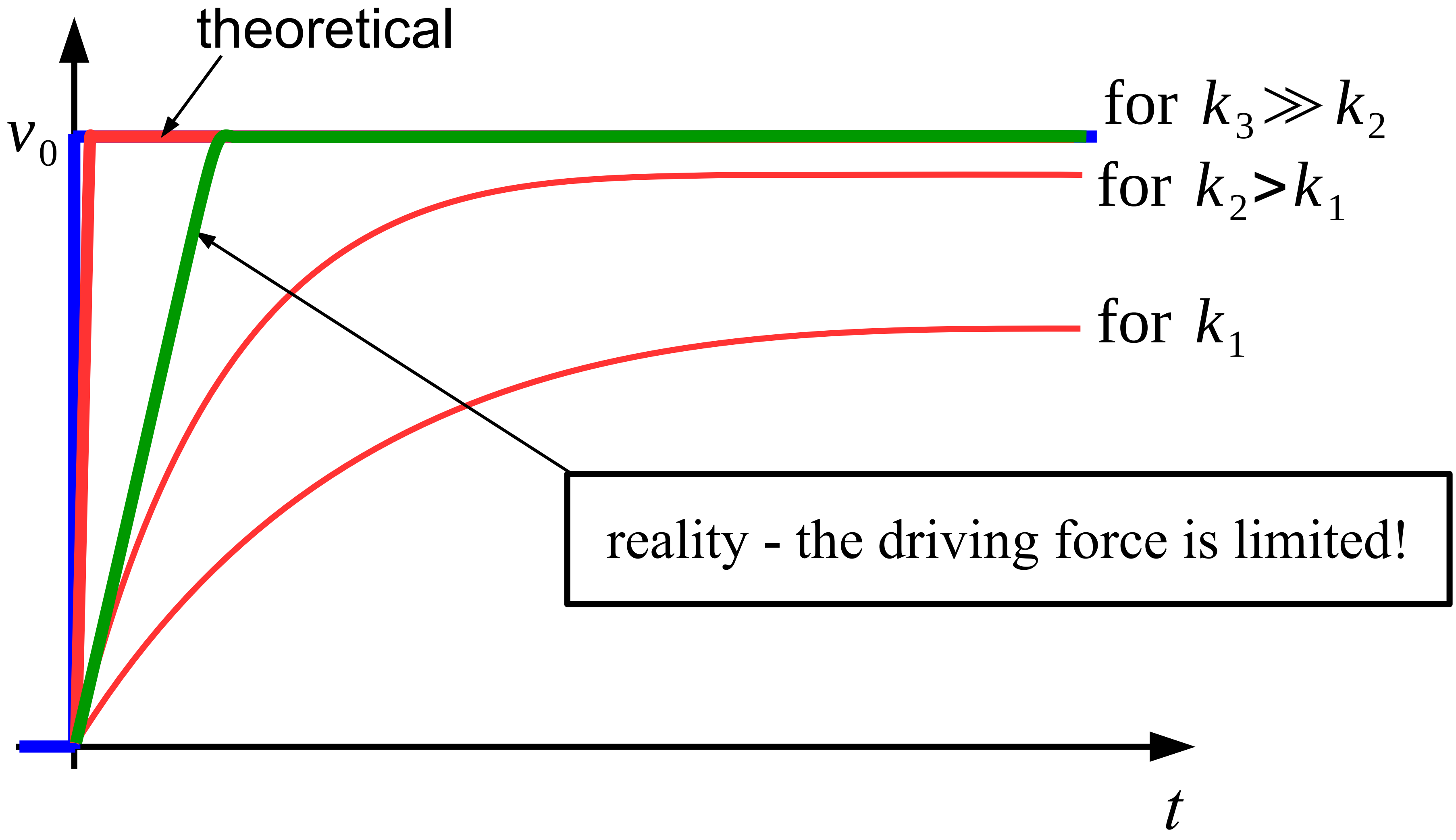
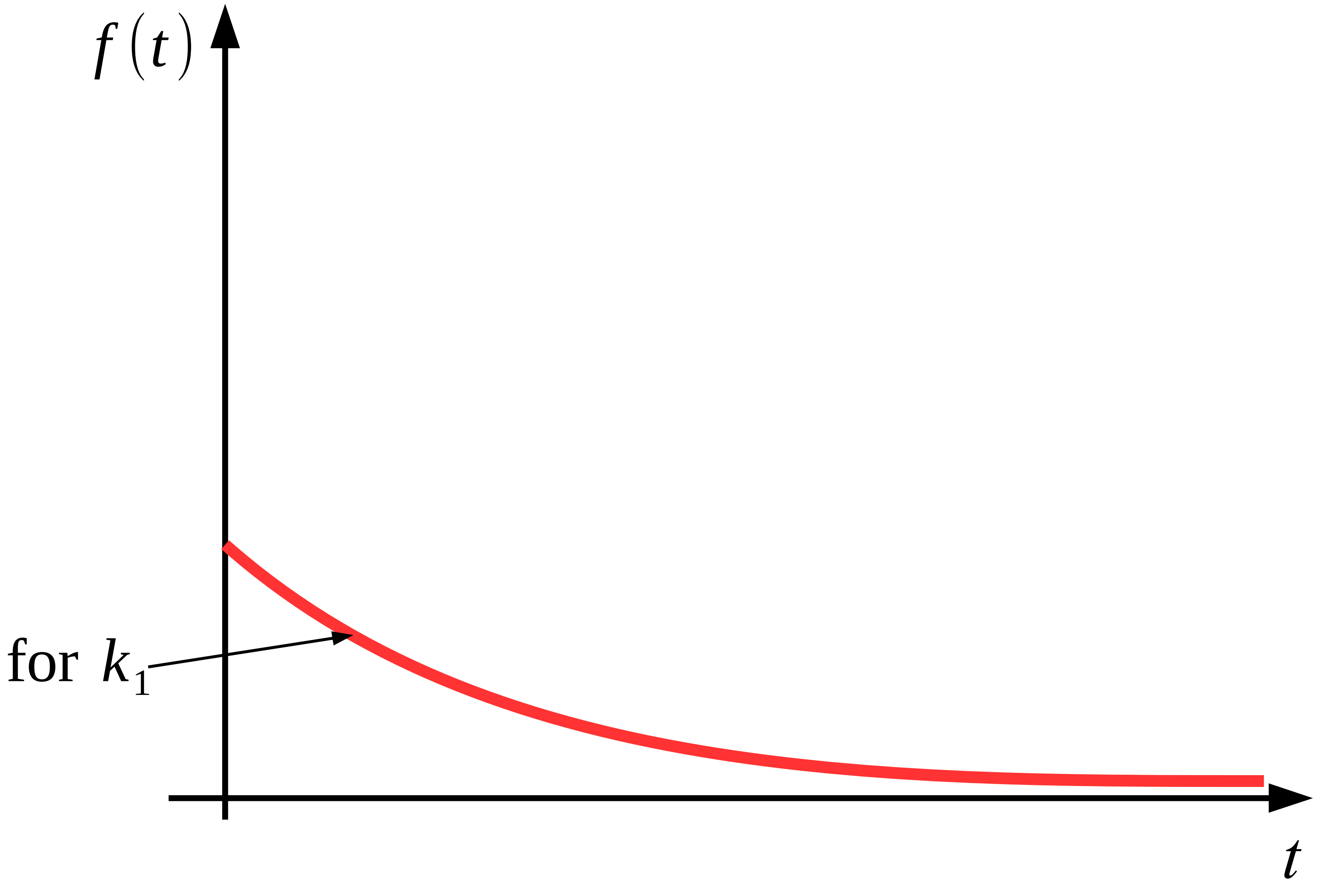


EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

driving force



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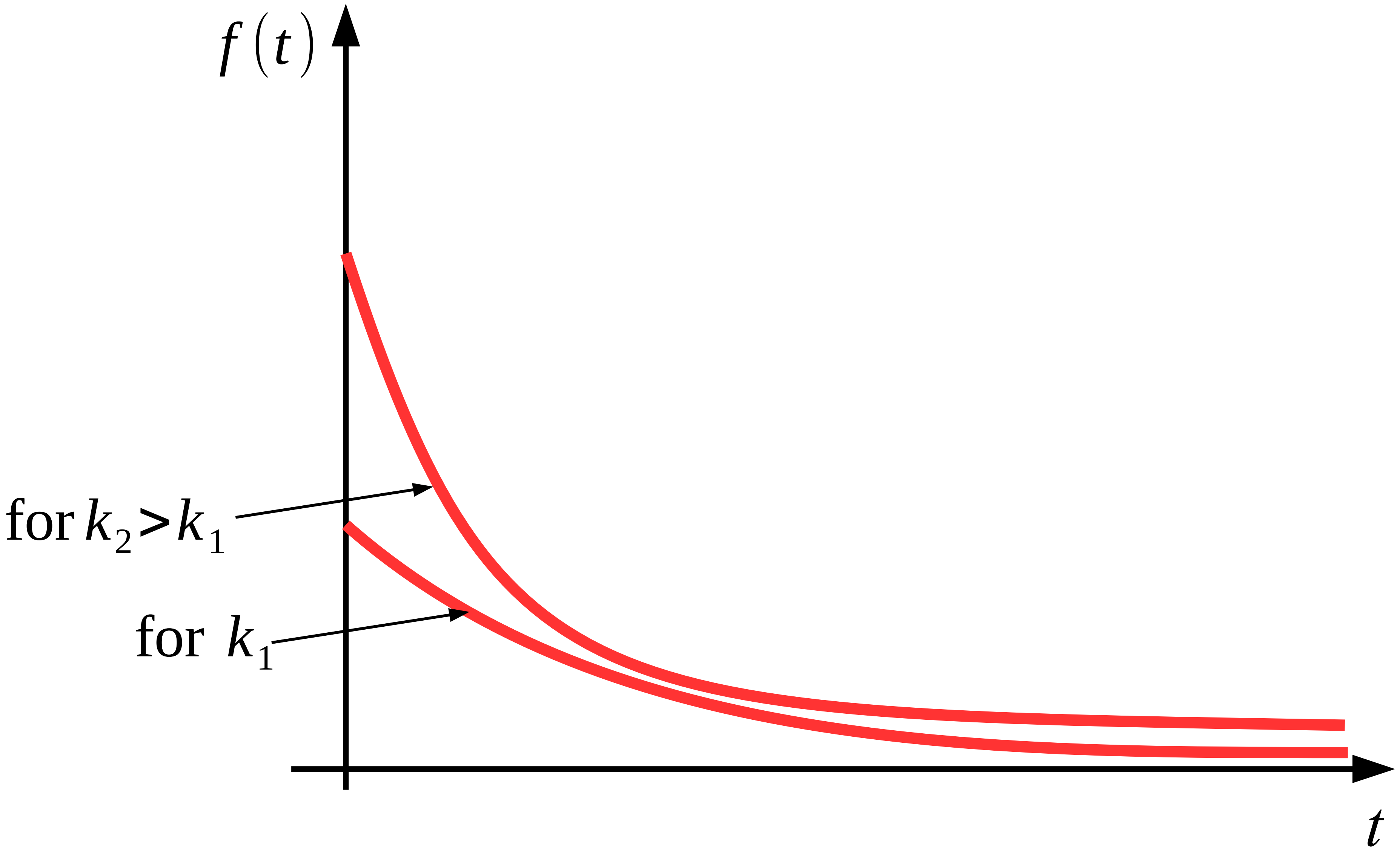

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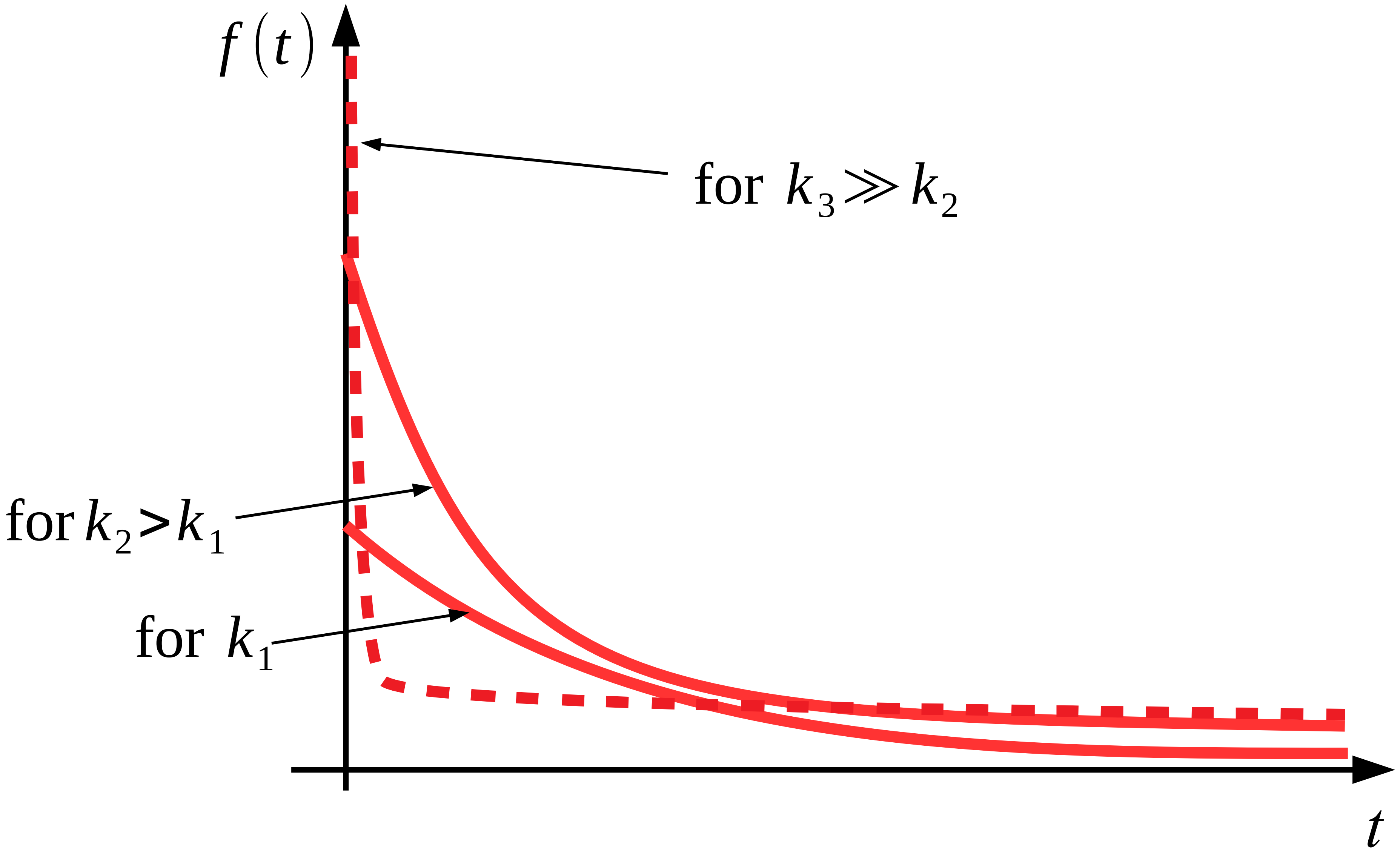

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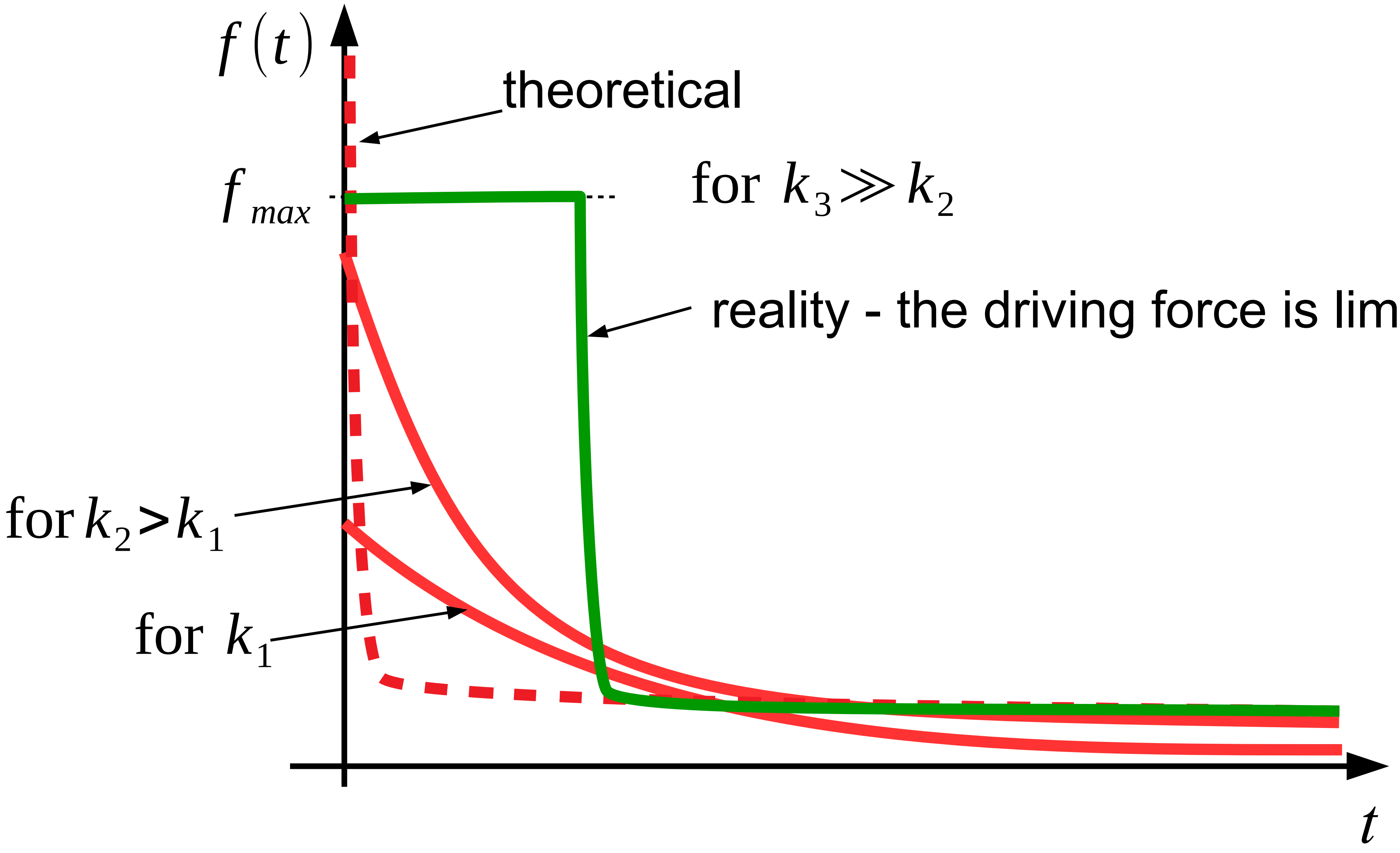

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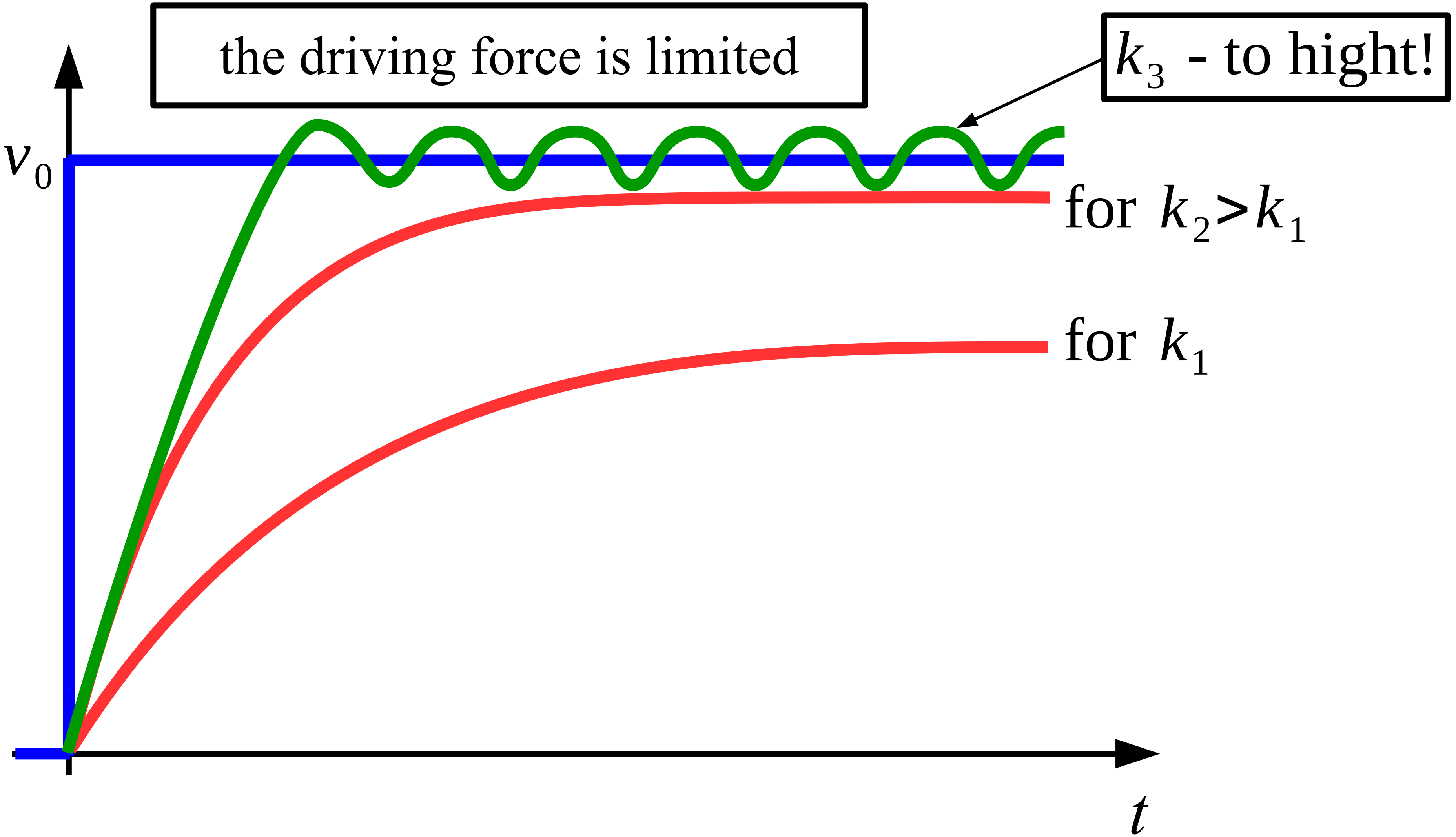

NOTE

Control signal limitation
=
System is nonlinear
=
Linear model (transfer function) is not
Valid in all situations

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

$$H_R(s)=\frac{k}{ms+c+k},\ H(j\omega)=\frac{k}{mj\omega+c+k}$$

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Speed control (cruise control, autocruise, tempomat)

$$H_R(s)=\frac{k}{ms+c+k},\quad H(j\omega)=\frac{k}{mj\omega+c+k}$$

$$P(\omega)=\frac{k(c+k)}{m^2\omega^2+(c+k)^2},\quad Q(\omega)=\frac{-km\omega}{m^2\omega^2+(c+k)^2}$$

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Speed control (cruise control, autocruise, tempomat)

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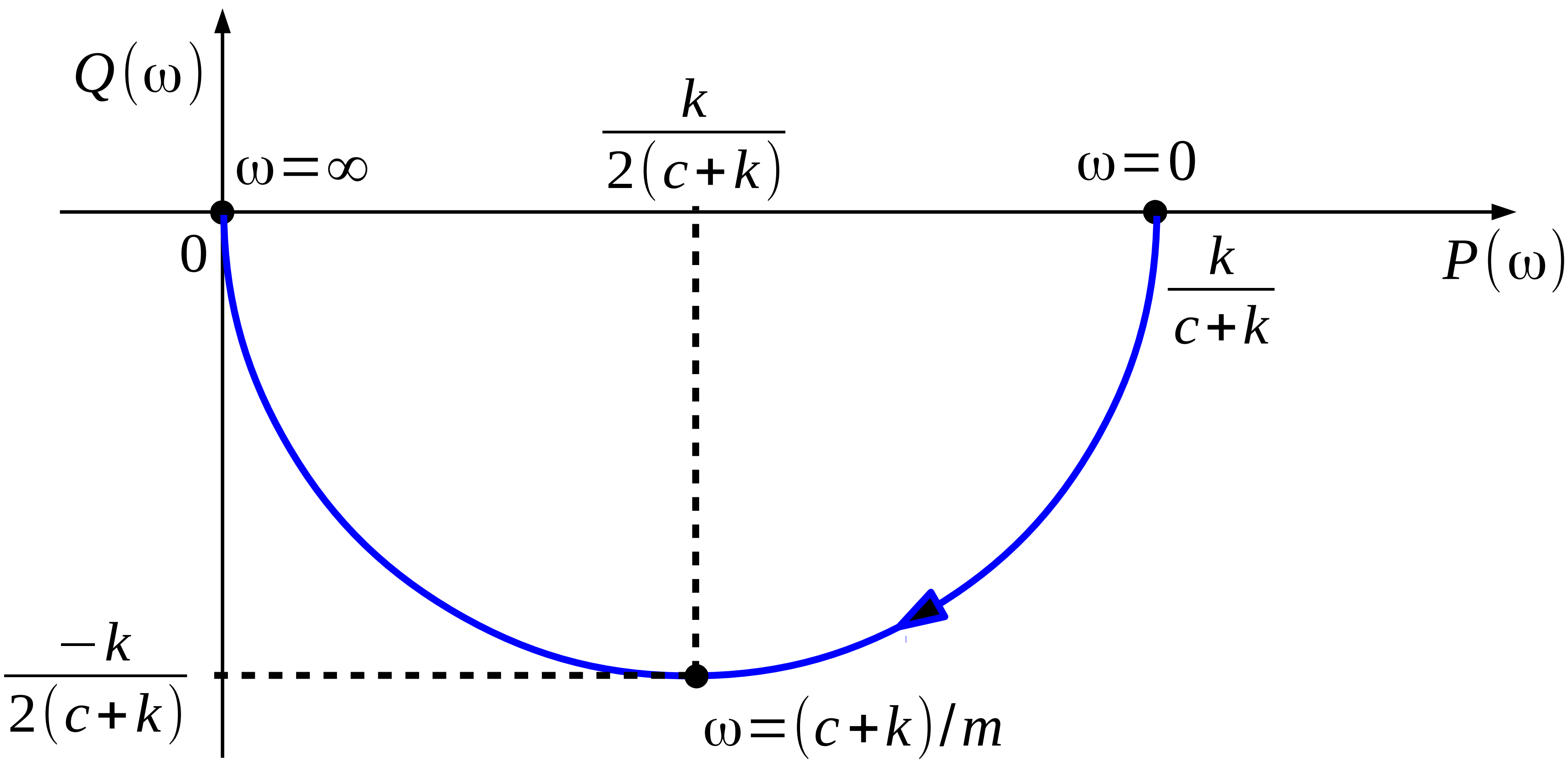
for $k>0$



EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

$$A(\omega) = \sqrt{P^2 + Q^2} = |k| / \sqrt{m^2 \omega^2 + c + k}$$

$$L(\omega) = 20 \log A(\omega) = 20 \log |k| - 20 \log \sqrt{m^2 \omega^2 + (c+k)^2}$$

$$\varphi(\omega) = \arctan \frac{Q}{P} = \arctan \left(-\frac{m \omega}{c+k} \right)$$

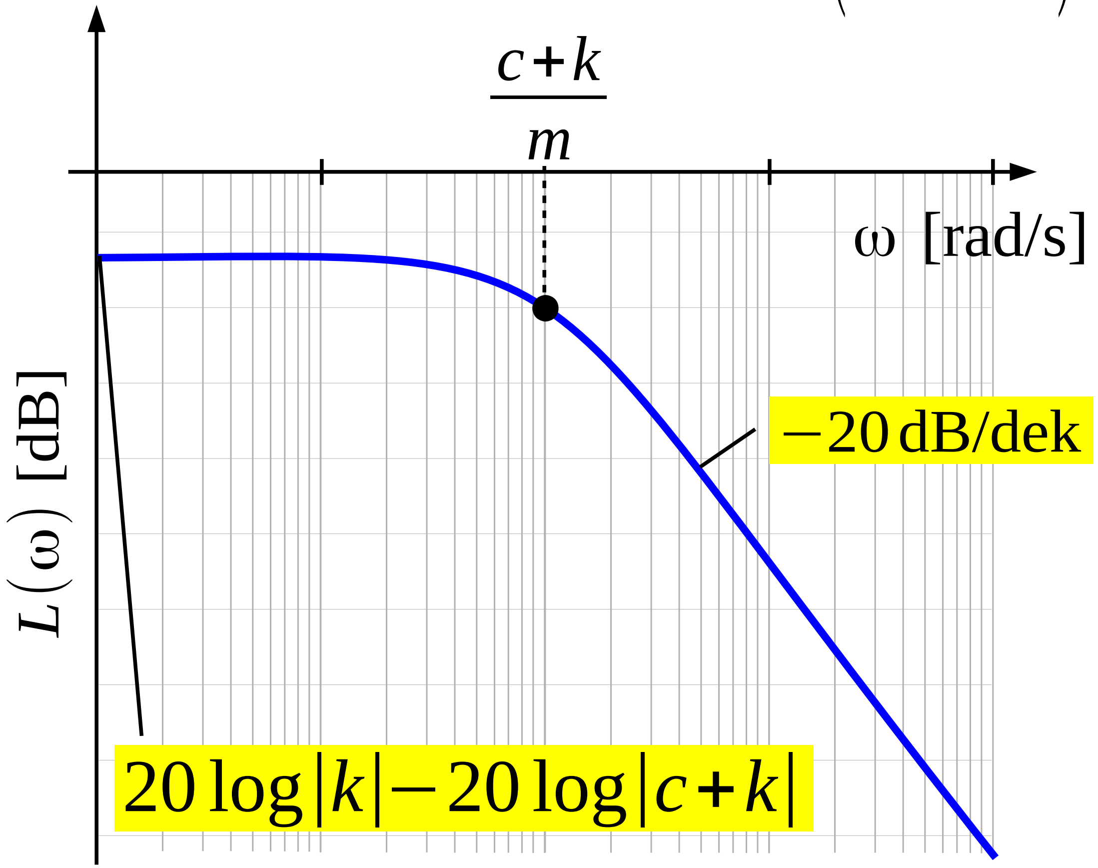
EXAMPLE 1

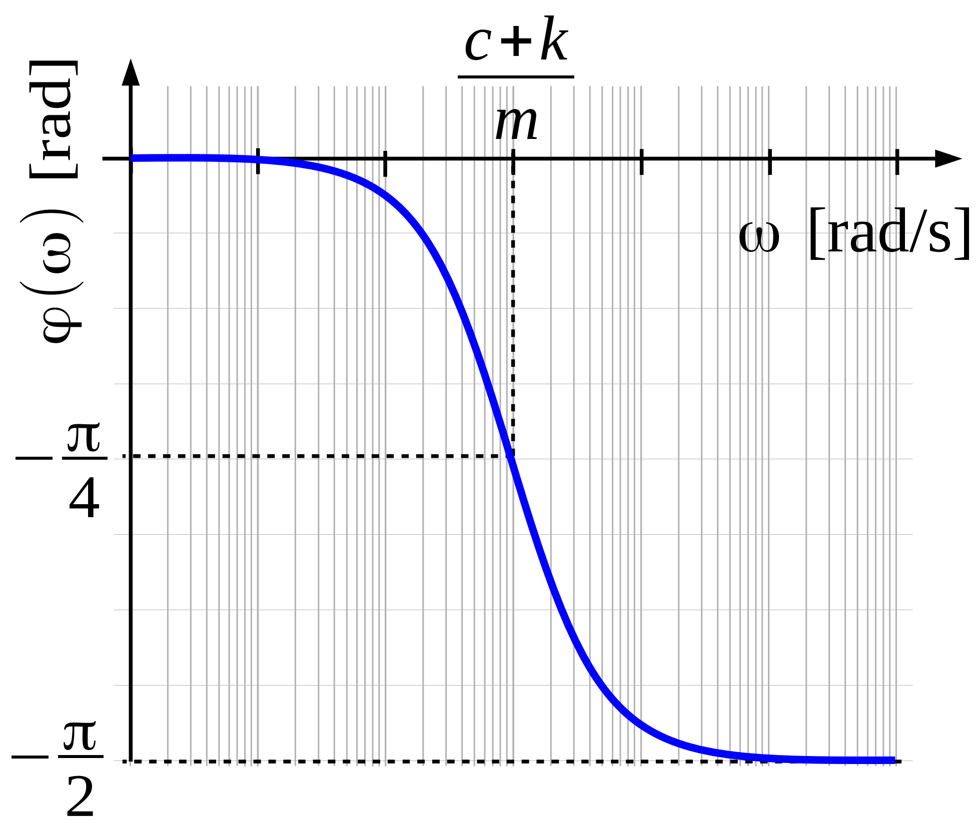
Speed control (cruise control, autocruise, tempomat)

$$A(\omega)=\sqrt{P^2+Q^2}=|k|/\sqrt{m^2\omega^2+c+k}$$

$$L(\omega)=20\log A(\omega)=20\log|k|-20\log\sqrt{m^2\omega^2+(c+k)^2}$$

$$\varphi(\omega)=\arctan\frac{Q}{P}=\arctan\left(-\frac{m\omega}{c+k}\right)$$





EXAMPLE 1

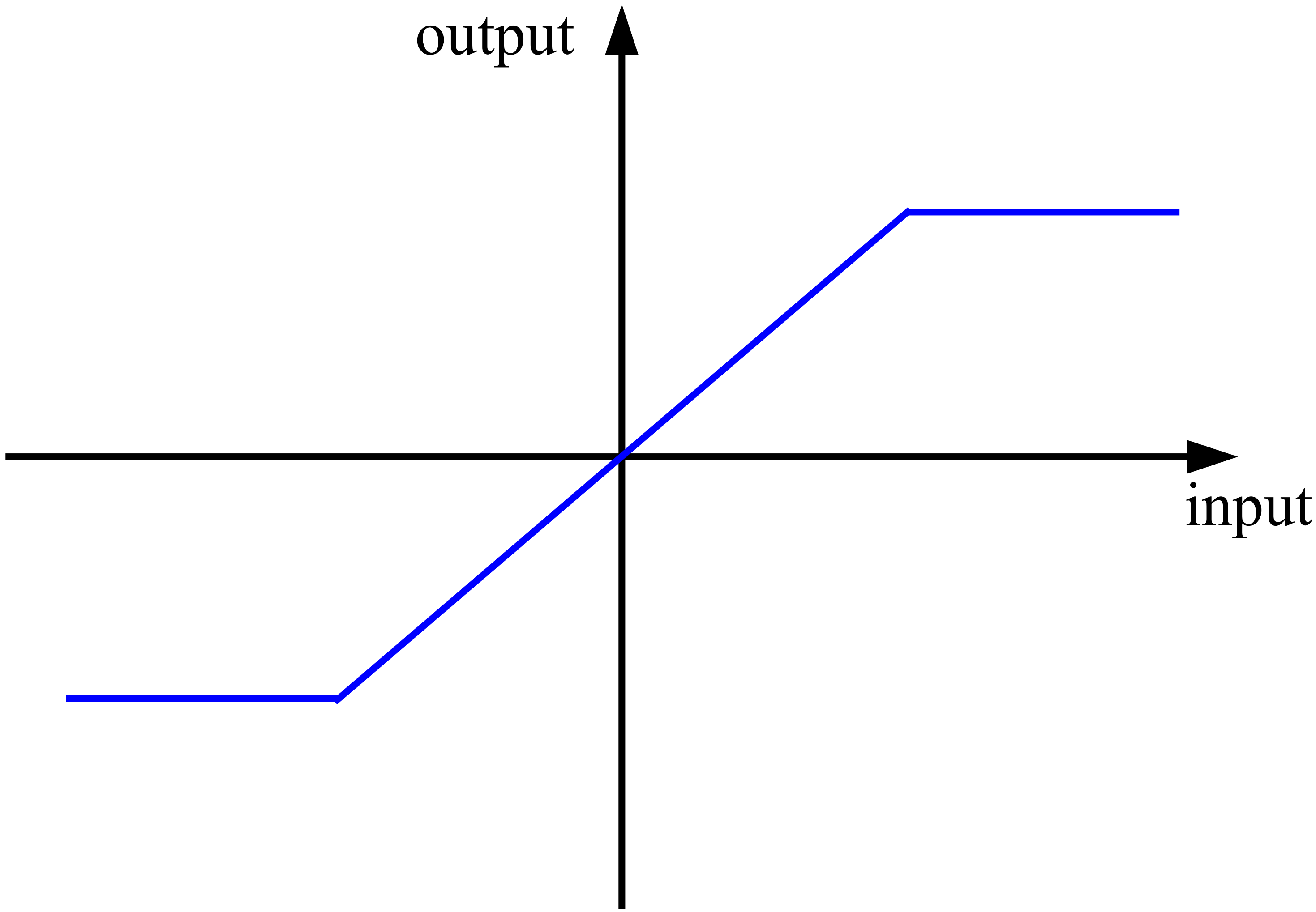
Speed control (cruise control, autocruise, tempomat)

Conclusions for "P" controller + first order system

- constant error in steady state
- P gain increasing = rise time decreasing + error decreasing
- control signal limitations = rise time limitations
- control signal limitations = nonlinear system

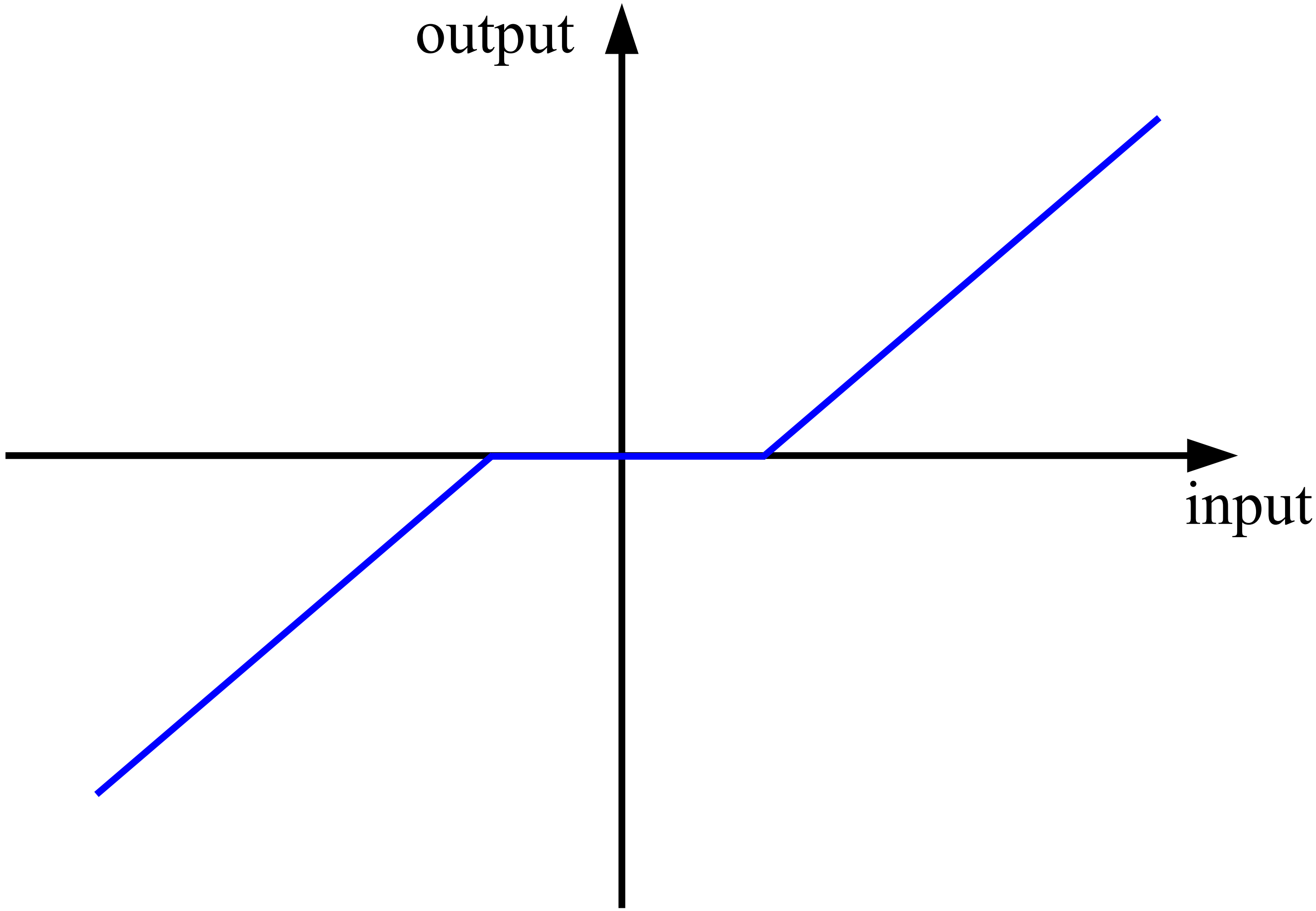
NONLINEARITIES

Symmetric hard limiting <u>saturation</u>



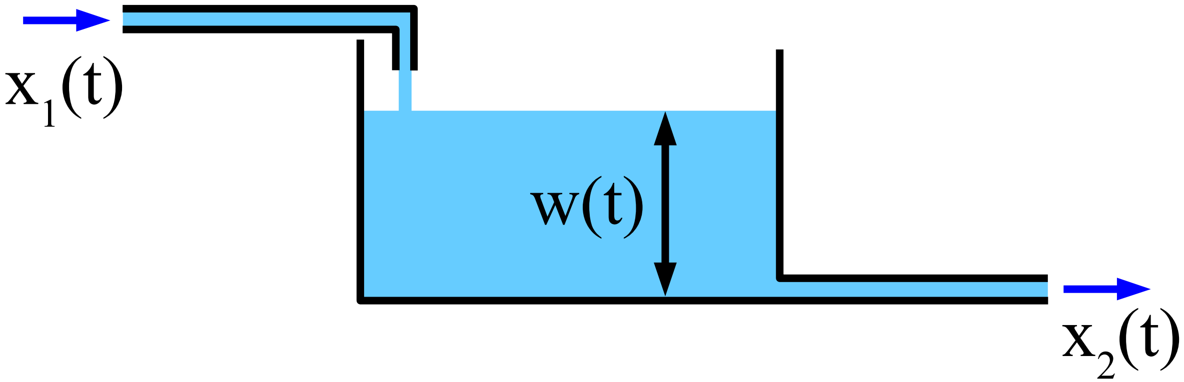
NONLINEARITIES

Dead zone



EXAMPLE 2

Water level control



$x_1(t)[m^3/s]$ - inflow of a liquid (controlled)

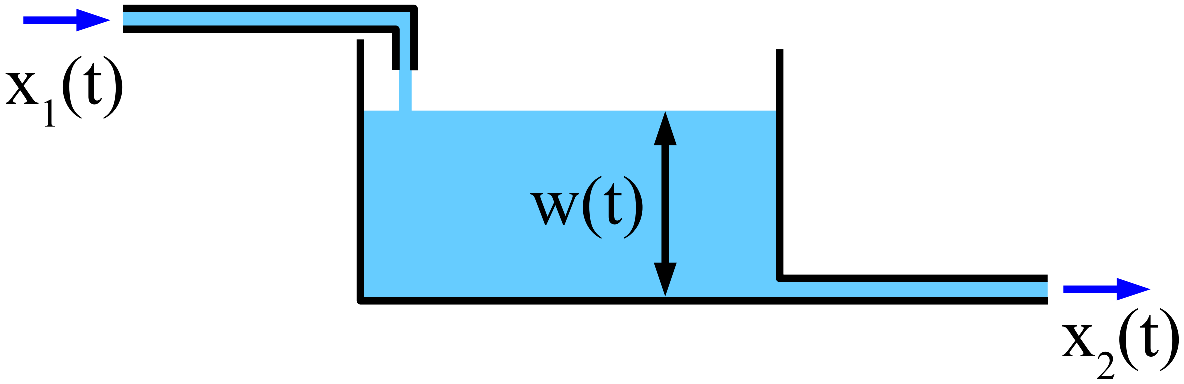
$x_2(t)[m^3/s]$ - outflow of a liquid (not controlled)

$w(t)[m]$ - level of a liquid in a tank

$A[m^2]$ - constant surface area

EXAMPLE 2

Water level control



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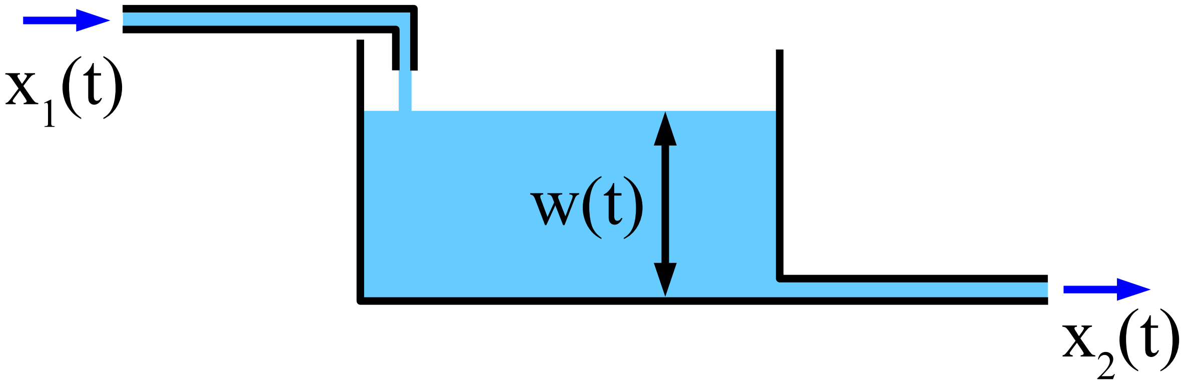
$A[m^2]$ - constant surface area

$$\frac{dv(t)}{dt} = x_1(t) - x_2(t)$$

EXAMPLE 2

Water level control



$x_1(t)[m^3/s]$ - inflow of a liquid (controlled)

$x_2(t)[m^3/s]$ - outflow of a liquid (not controlled)

$w(t)[m]$ - level of a liquid in a tank

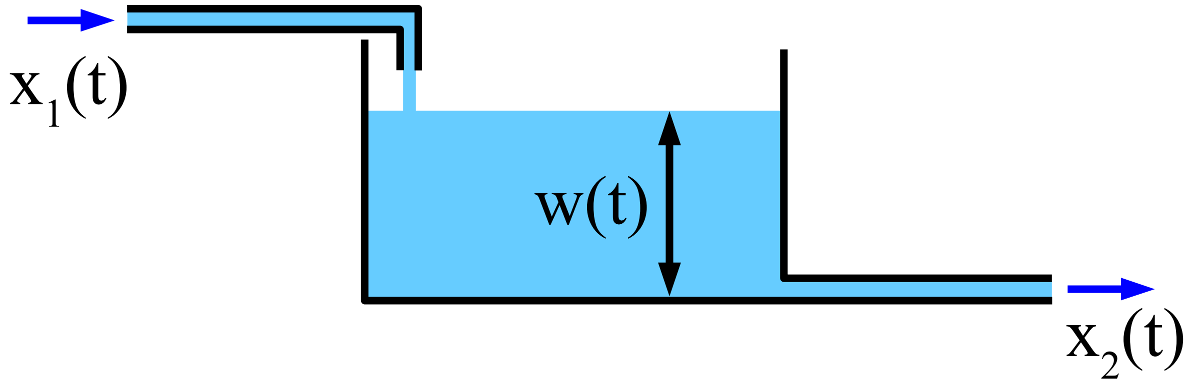
$A[m^2]$ - constant surface area

$$\frac{dv(t)}{dt}=x_1(t)-x_2(t)$$

$$A\frac{dw(t)}{dt}=x_1(t)-x_2(t)$$

EXAMPLE 2

Water level control



$x_1(t)[m^3/s]$ - inflow of a liquid (controlled)

$x_2(t)[m^3/s]$ - outflow of a liquid (not controlled)

$w(t)[m]$ - level of a liquid in a tank

$A[m^2]$ - constant surface area

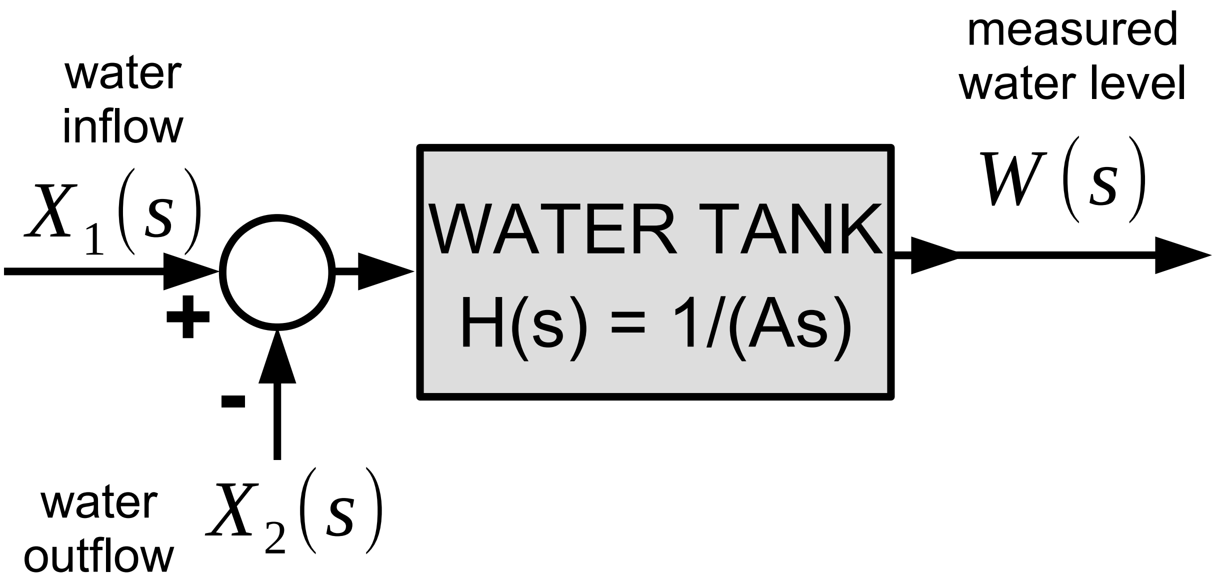
$$\frac{dv(t)}{dt} = x_1(t) - x_2(t)$$

$$A\frac{dw(t)}{dt} = x_1(t) - x_2(t)$$

$$H(s) = \frac{W(s)}{X_1(s) - X_2(s)} = \frac{1}{A\,s}$$

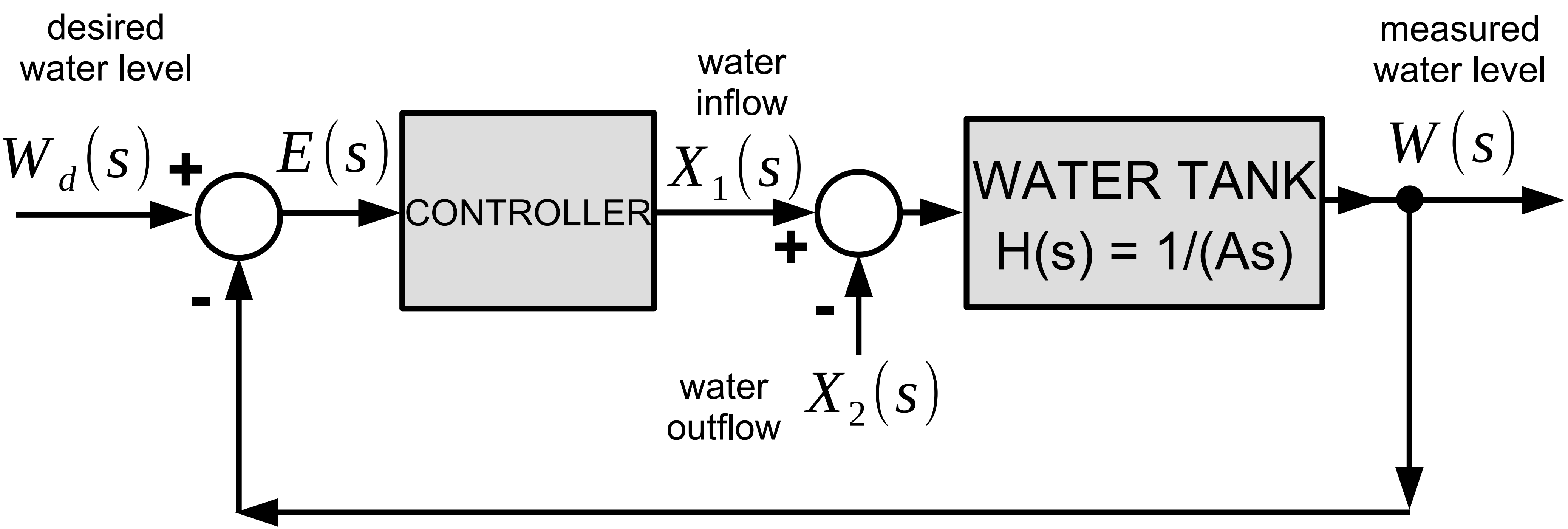
EXAMPLE 2

Water level control



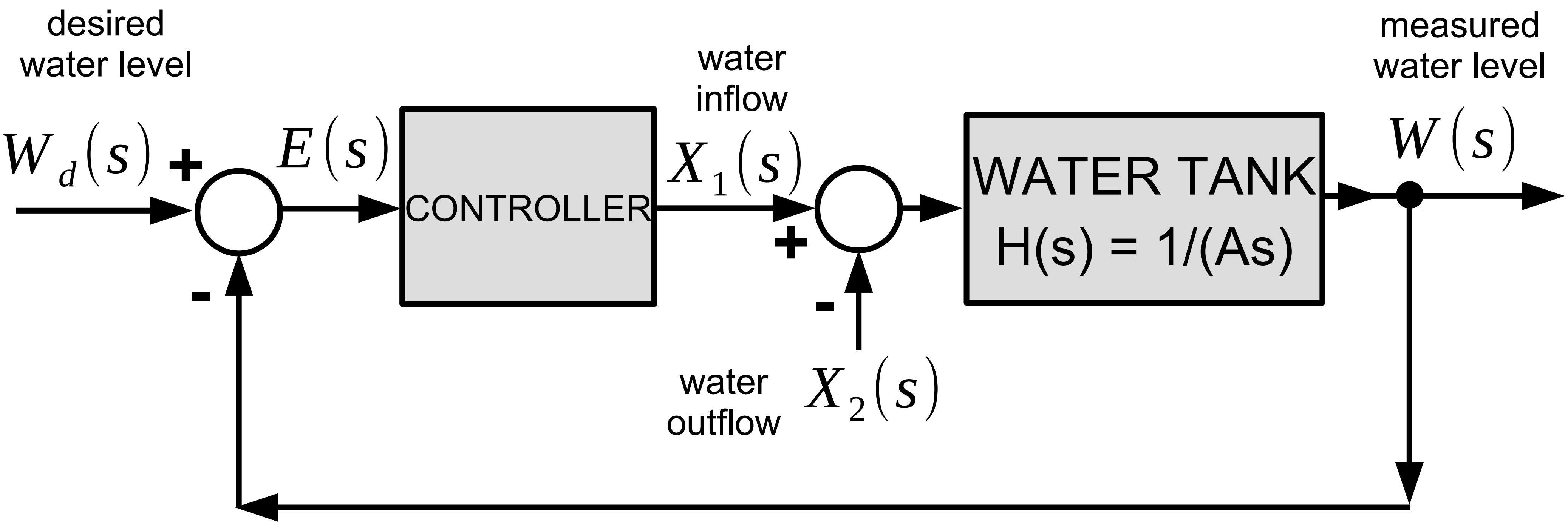
EXAMPLE 2

Water level control



EXAMPLE 2

Water level control



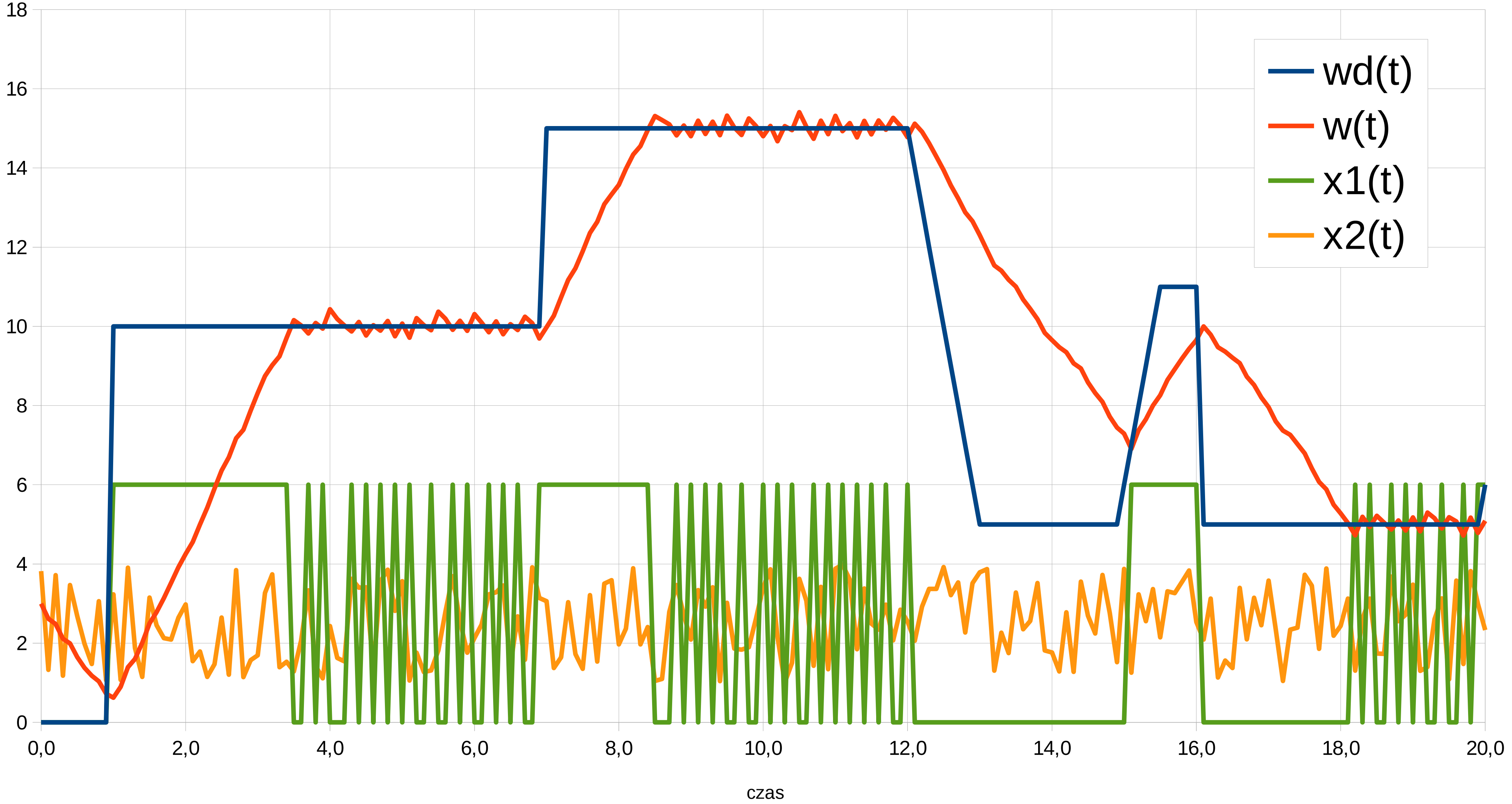
<u>Proposed controllers</u>:

- ideal on/off controller
- on/off controller with hysteresis
- proportional controller

EXAMPLE 2

Water level control

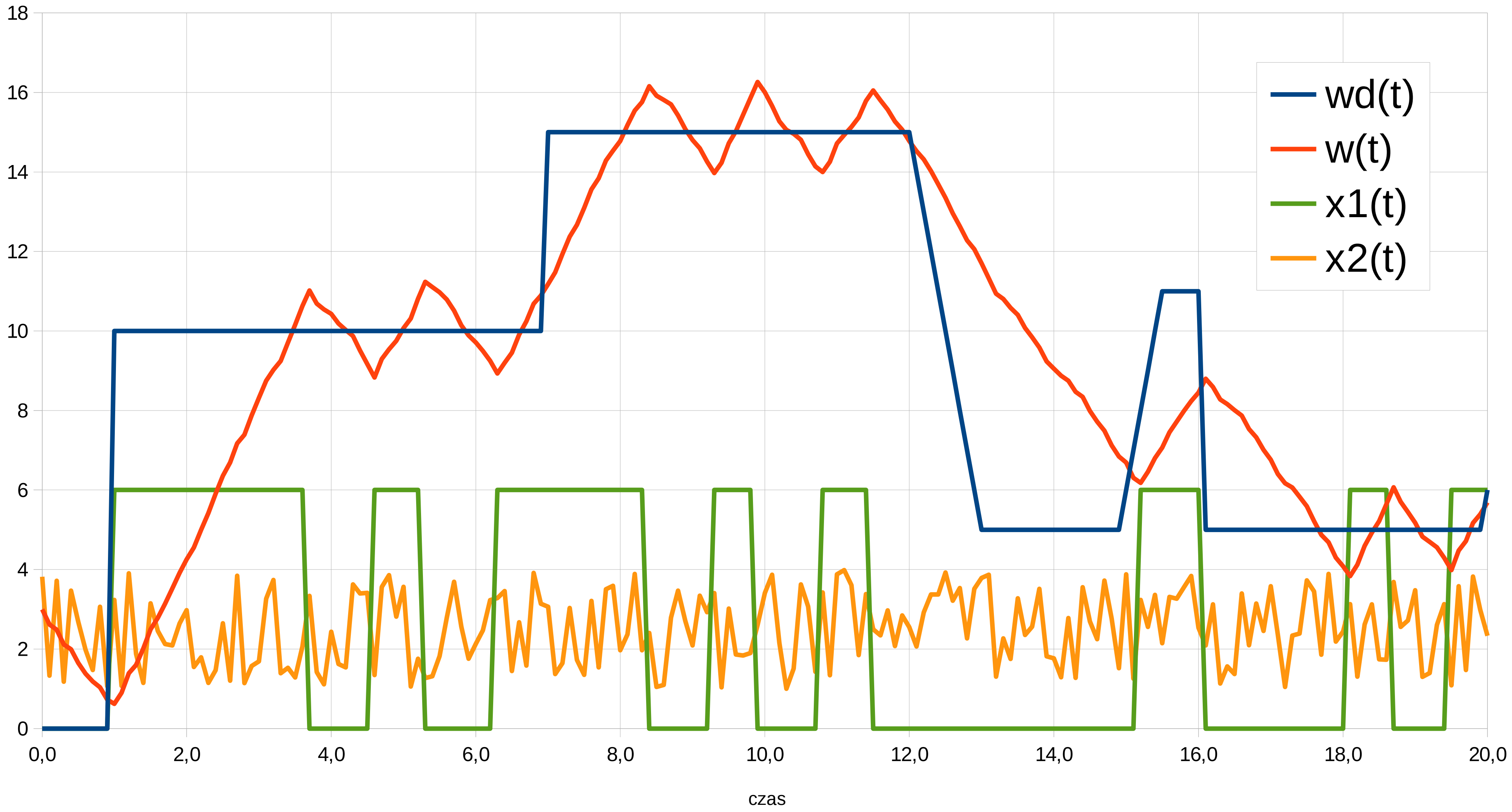
ideal on/off controller



EXAMPLE 2

Water level control

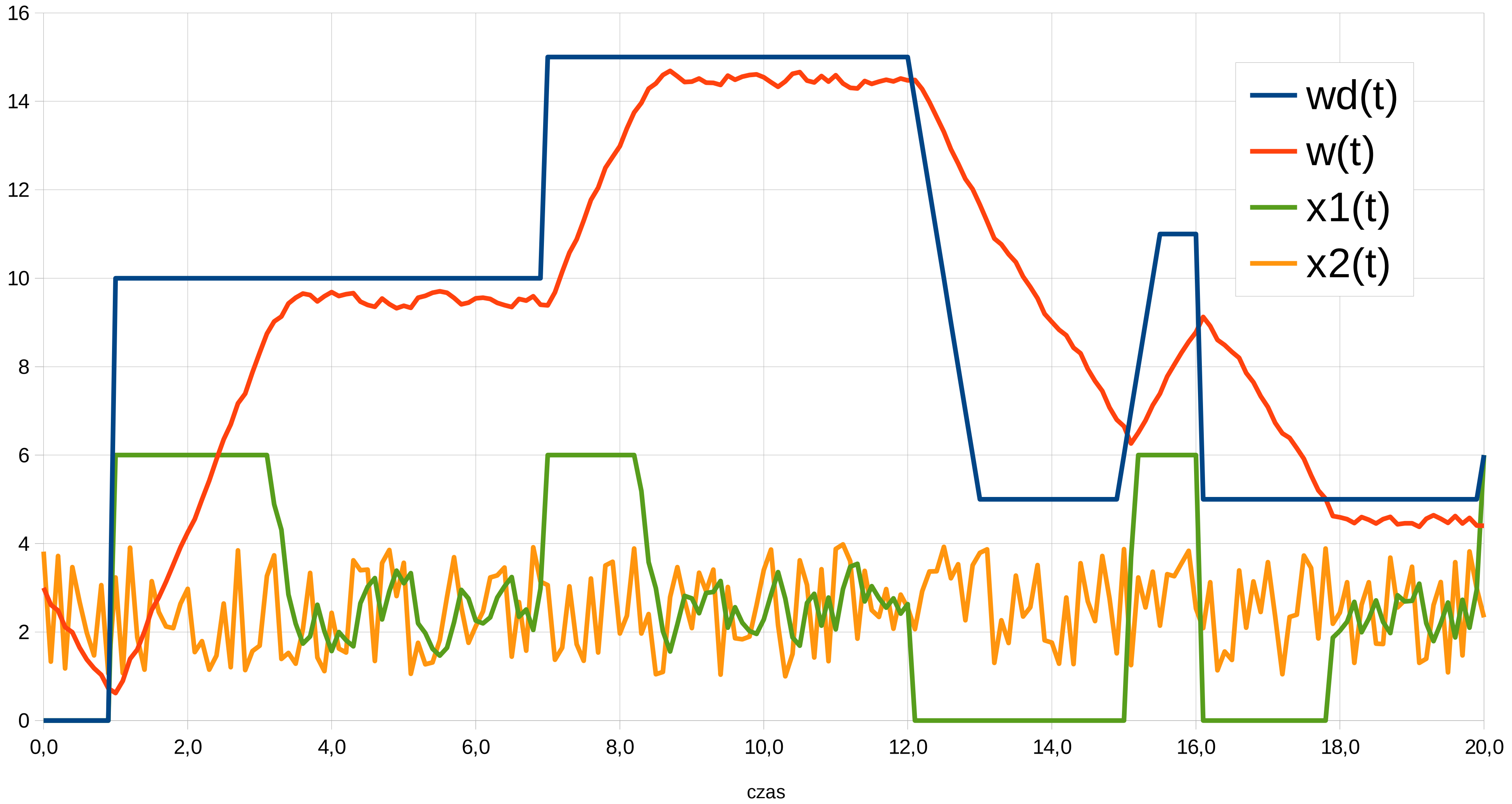
on/off controller with hysteresis



EXAMPLE 2

Water level control

proportional controller (small k_P)



EXAMPLE 2

Water level control

proportional controller (high k_P)