Warsaw University of Technology

The Faculty of Automotive
and Construction Machinery Engineering
Institute of Machine Design Fundamentals
Department of Mechanics
http://www.ipbm.simr.pw.edu.pl/

Theory of Machines and Automatic Control

Winter 2017/2018

Lecturer: Sebastian Korczak, PhD Eng.

Lecture 10

Classification of basic automatic systems with examples.

Classification of basic automatic systems

Element name	Equation	Transfer function
proportional	$y(t)=ku(t)$	k
first order (inertial)	$T\frac{dy(t)}{dt}+y(t)=ku(t)$	$\frac{k}{Ts+1}$
integrator	$y(t)=k\int_0^t u(t)dt$ or $\frac{dy(t)}{dt}=ku(t)$	$\frac{k}{s}$

Classification of basic automatic systems

Element name	Equation	Transfer function
derivative	$y(t)=k\frac{du(t)}{dt}$	ks
derivative with inertia	$T\frac{dy(t)}{dt}+y(t)=k\frac{du(t)}{dt}$	$\frac{ks}{Ts+1}$

Classification of basic automatic systems

Element name	Equation	Transfer function
delay	$y(t)=u(t-\tau)$	$e^{-\tau s}$
second order (oscillator)	$T_1^2 \frac{d^2 y(t)}{dt^2} + T_2 \frac{dy(t)}{dt} + y(t) = ku(t)$	$\frac{k}{T_1^2 s^2 + T_2 s + 1}$

Proportional element

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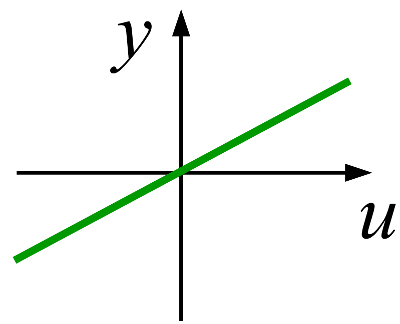
Proportional element

1. Element equation: $y(t) = ku(t)$ $\quad u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y = ku$ $\quad$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



for $k > 0$

Proportional element

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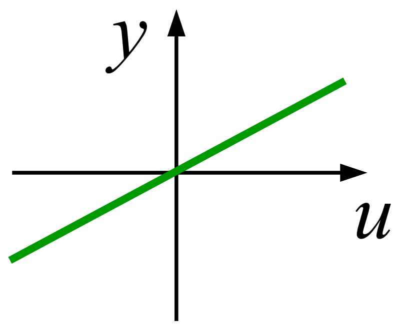

for $k > 0$

3. Transfer function:

Proportional element

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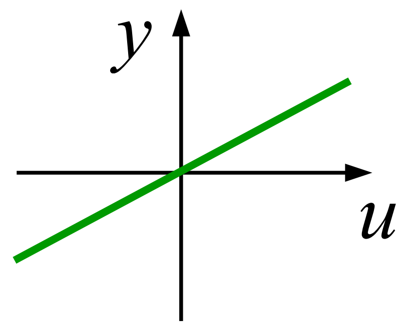

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3. Transfer function: $H(s) = k$

Proportional element

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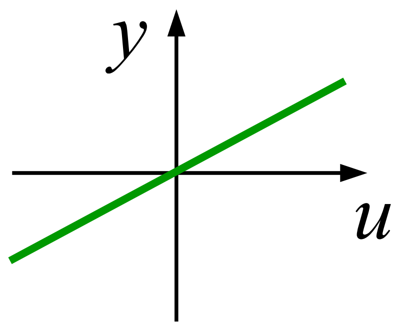

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4. Step response: $\quad$ for $u(t) = u_0 1(t)$

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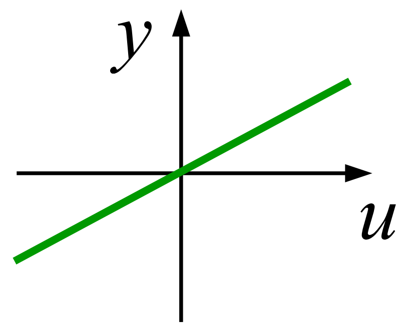

for $k>0$

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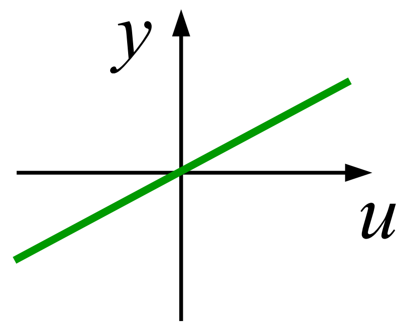

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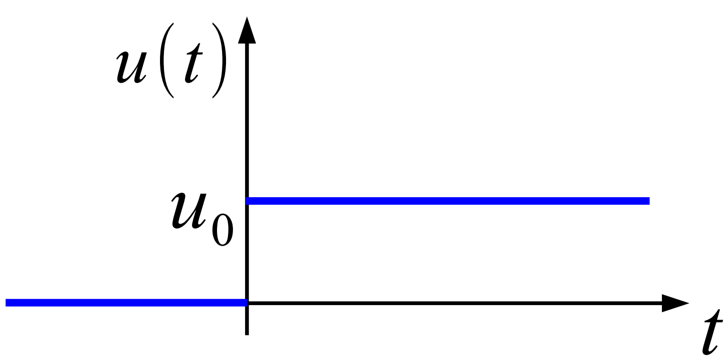

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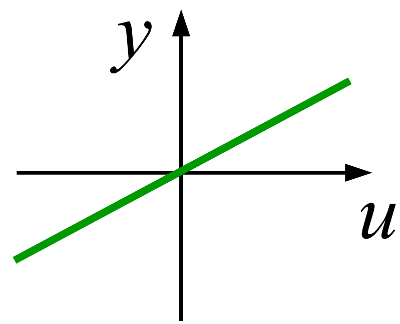

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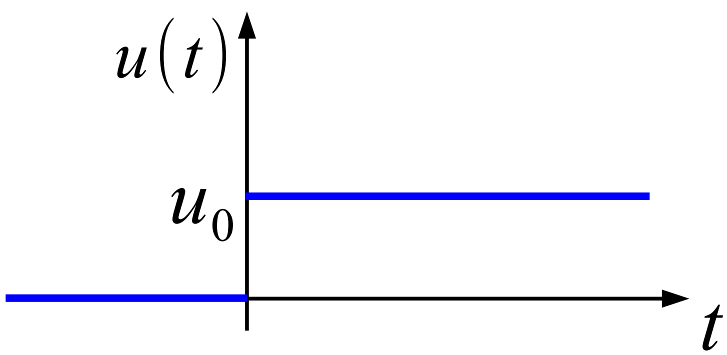

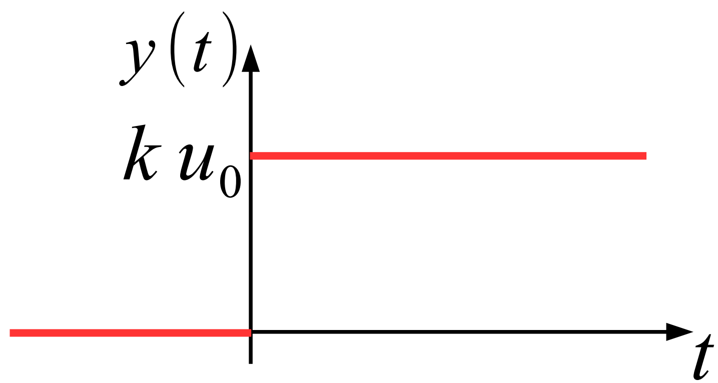

Proportional element

5. Frequency response:

Proportional element

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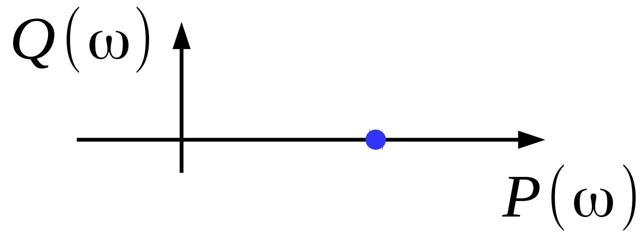

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Proportional element

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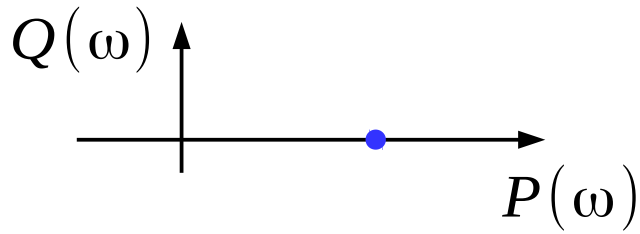

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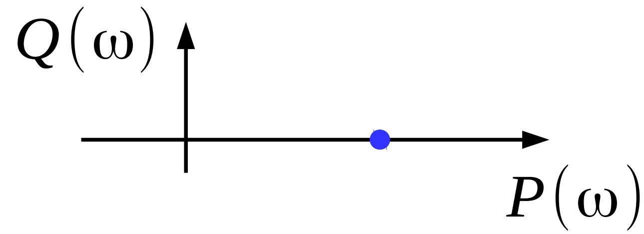

for $k>0$

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k|$

$$L(\omega)=20\log A(\omega)$$

Proportional element

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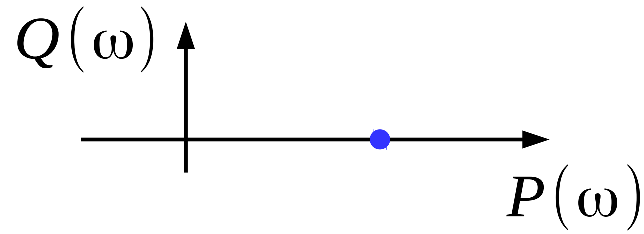

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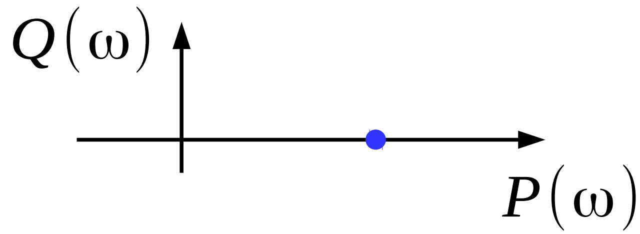

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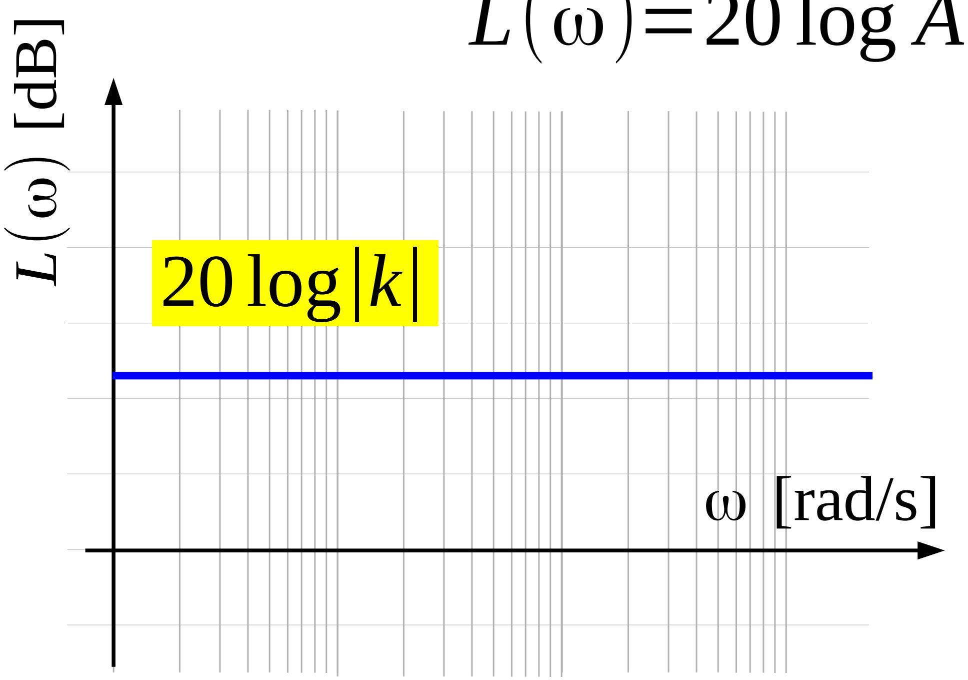

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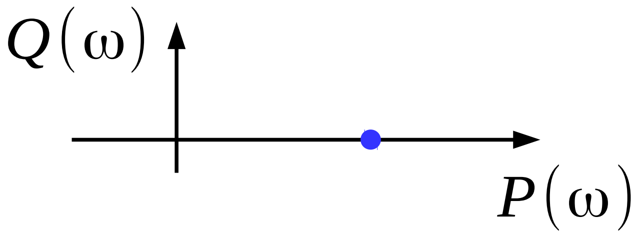

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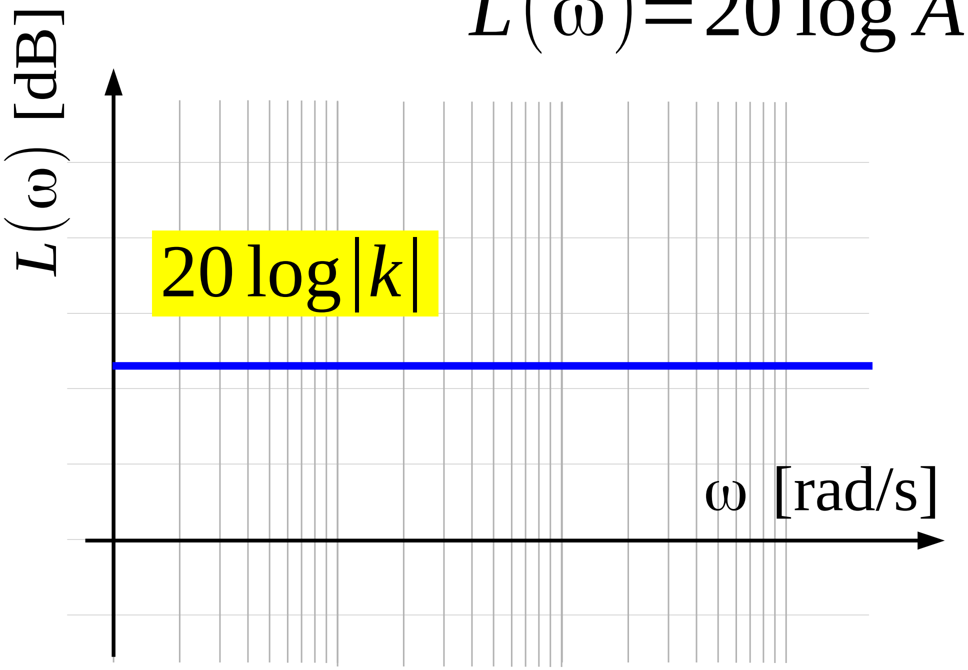
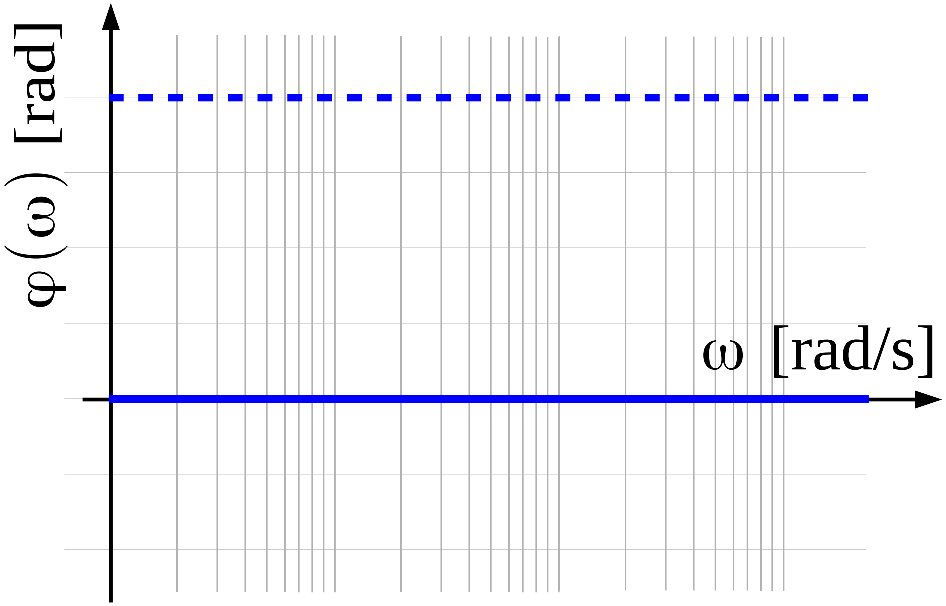
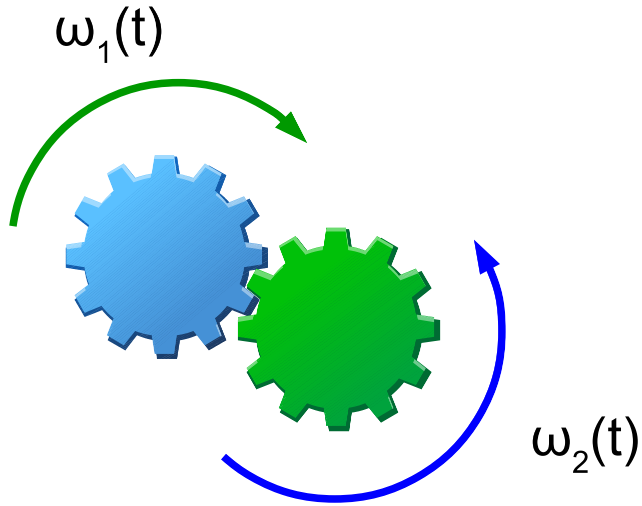




Proportional element

Examples

1

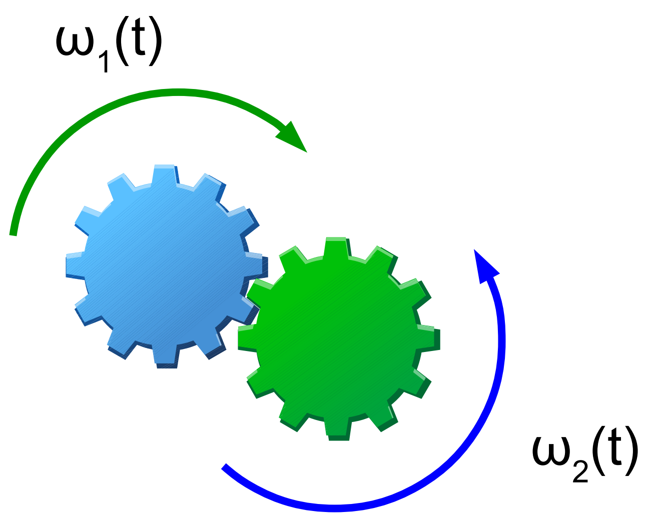


GEARBOX:
input – angular velocity $\omega_1(t)$
output – angular velocity $\omega_2(t)$

Proportional element

Examples

1



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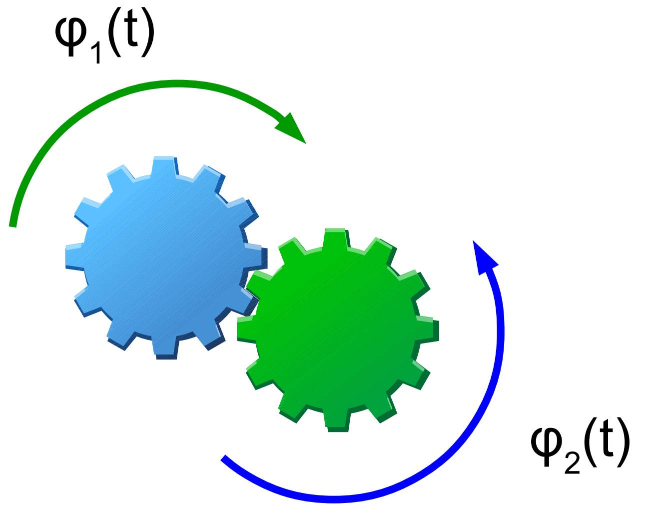
2

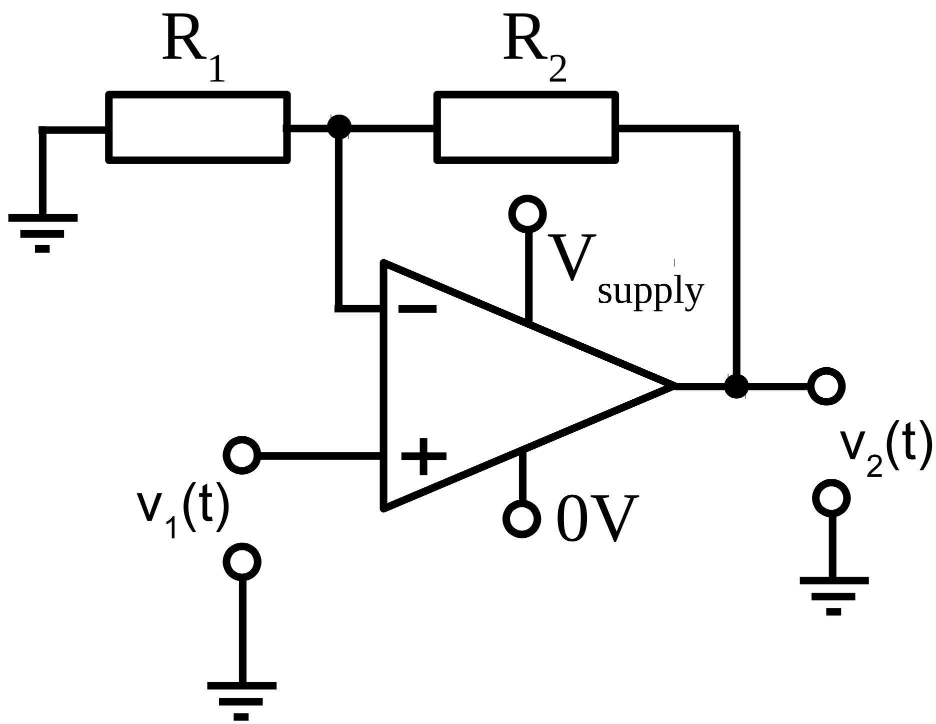


GEARBOX:
input – rotation angle $\varphi_1(t)$
output – rotation angle $\varphi_2(t)$

Proportional element

Examples

3



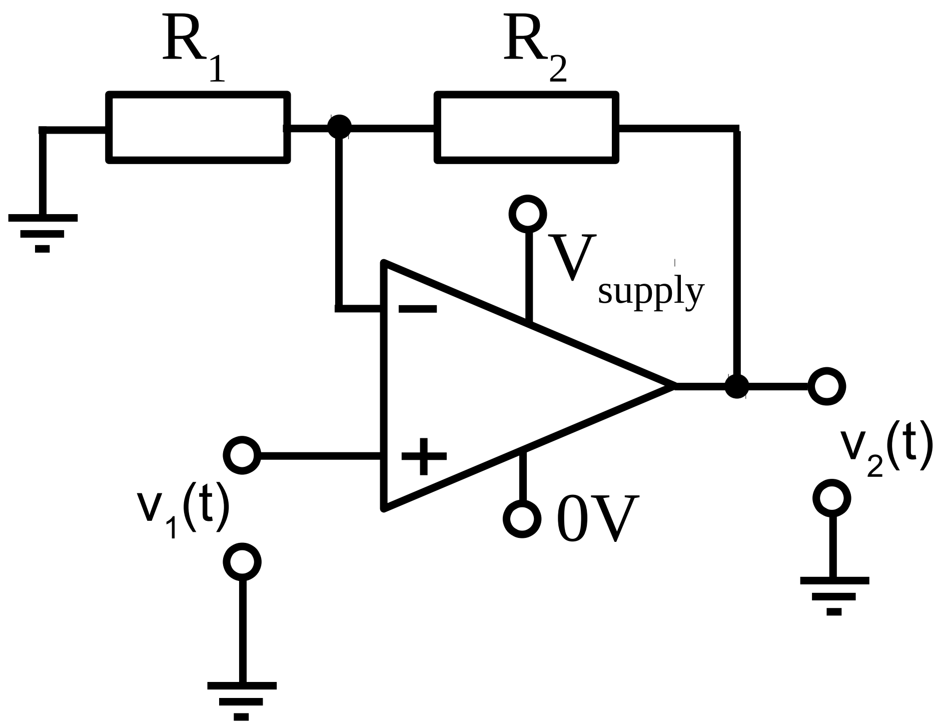
OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = v_1(t)\left(1 + \frac{R_2}{R_1}\right)$$

Proportional element

Examples

3



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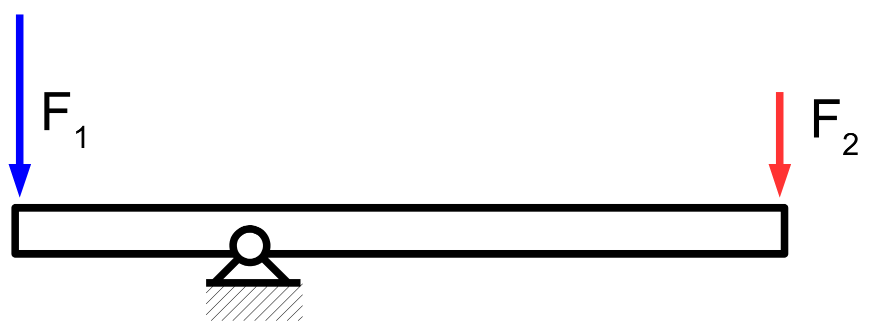
4

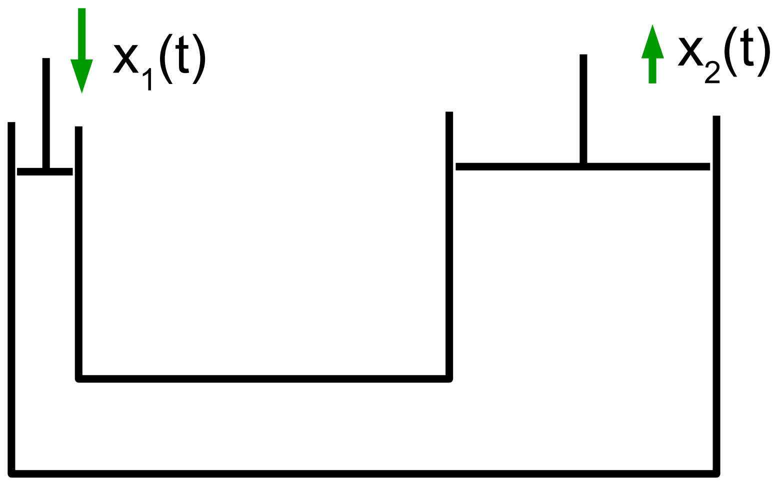


BEAM in steady state:
input – force F_1
output – force F_2

Proportional element

Examples

5

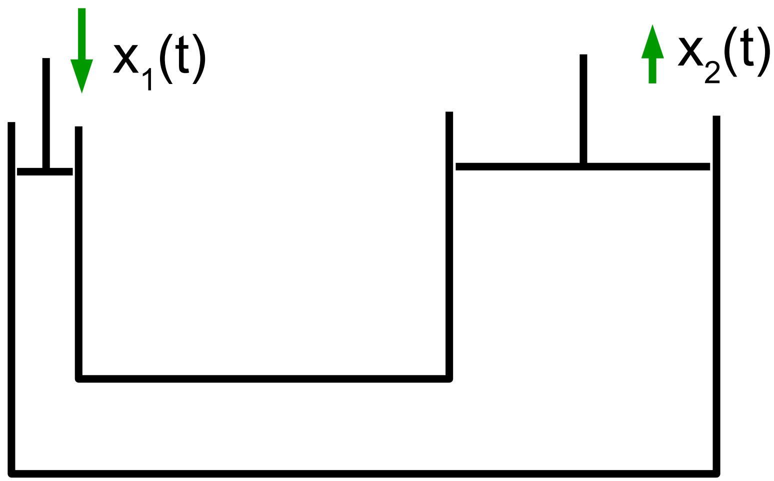


HYDRAULIC LEVER:
input – displacement $x_1(t)$
output – displacement $x_2(t)$

Proportional element

Examples

5



HYDRAULIC LEVER:
input – displacement $x_1(t)$
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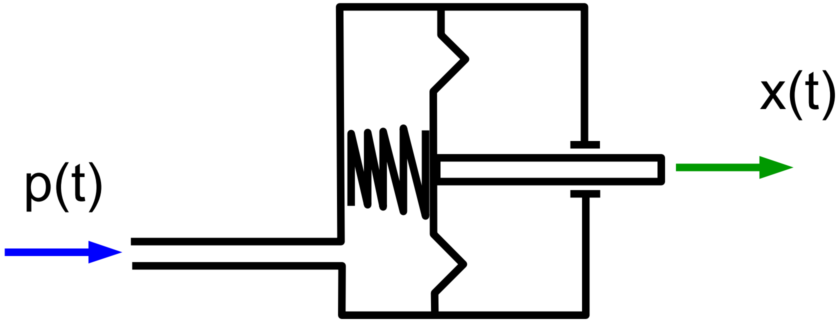
6



PRESSURE ACTUATOR:
input – pressure $p_1(t)$
output – displacement $x(t)$

First-order inertial element

1. Element equation: $T\frac{dy(t)}{dt}+y(t)=ku(t)$

$u(t)$ - input
$y(t)$ - output

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2. Static characteristic (steady state):

for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$

First-order inertial element

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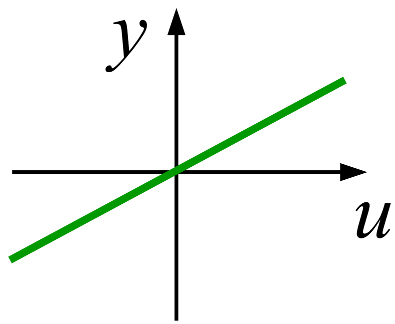

for $k>0$

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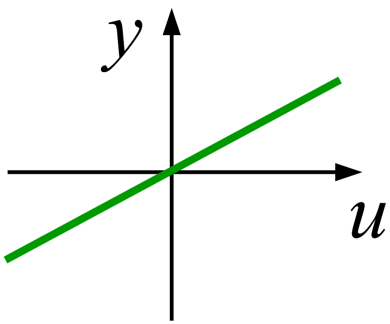

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3. Transfer function:

First-order inertial element

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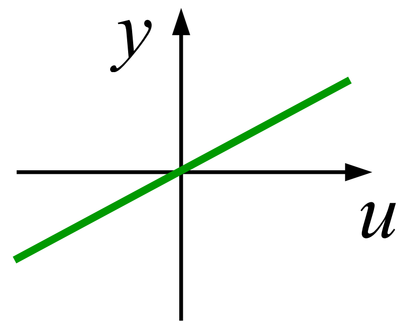

for $k>0$

3. Transfer function: $H(s)=\frac{k}{Ts+1}$

First-order inertial element

4. Step response:

First-order inertial element

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input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0 \frac{1}{s}$

First-order inertial element

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Laplace of output: $Y(s)=H(s)U(s)=\frac{k u_0}{s(Ts+1)}$

First-order inertial element

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input: $u(t) = u_0 1(t)$

Laplace of input: $U(s) = u_0 \frac{1}{s}$

Laplace of output: $Y(s) = H(s)U(s) = \frac{k u_0}{s(Ts+1)}$

output: $y(t) = L^{-1}\{Y(s)\} = k u_0 \left(1 - e^{-t/T}\right)$

First-order inertial element

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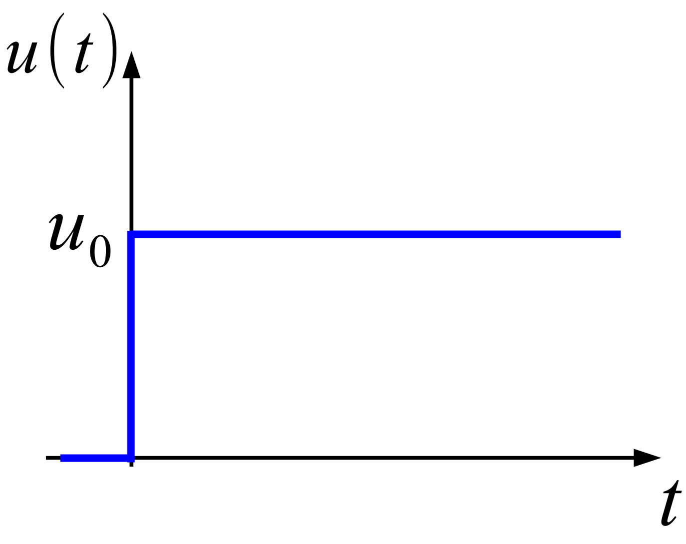

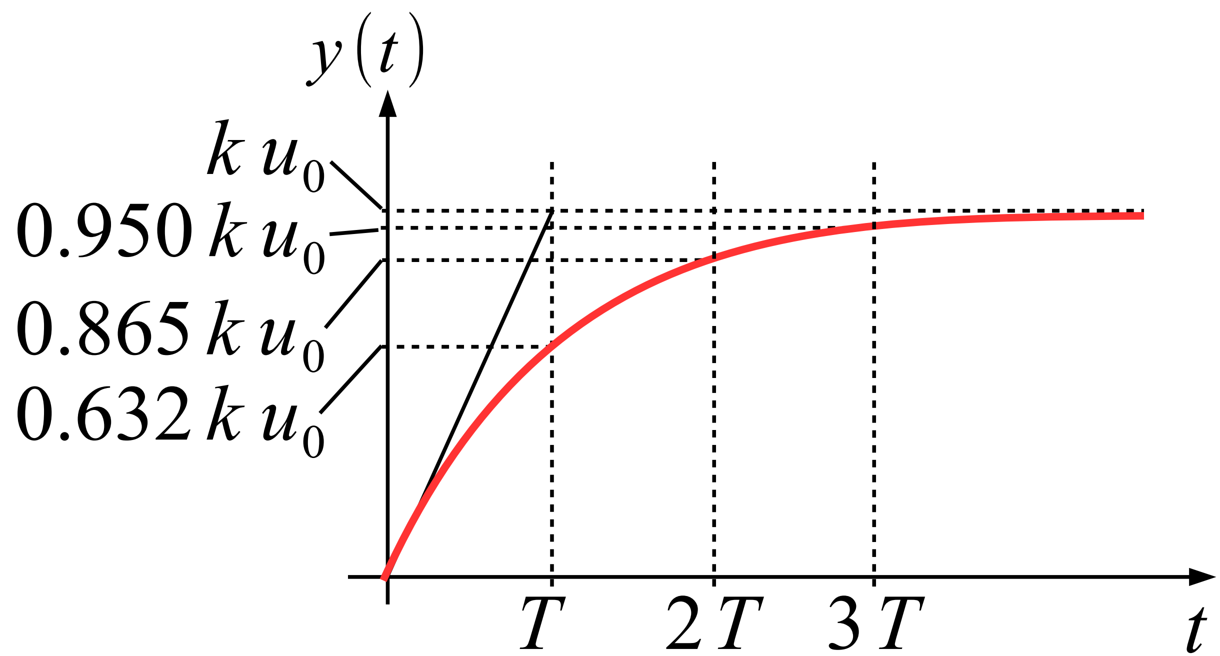

First-order inertial element

5. Frequency response:

First-order inertial element

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$$P(\omega)=\frac{k}{T^2\omega^2+1},\quad Q(\omega)=\frac{-kT\omega}{T^2\omega^2+1}$$

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6. Nyquist plot:

First-order inertial element

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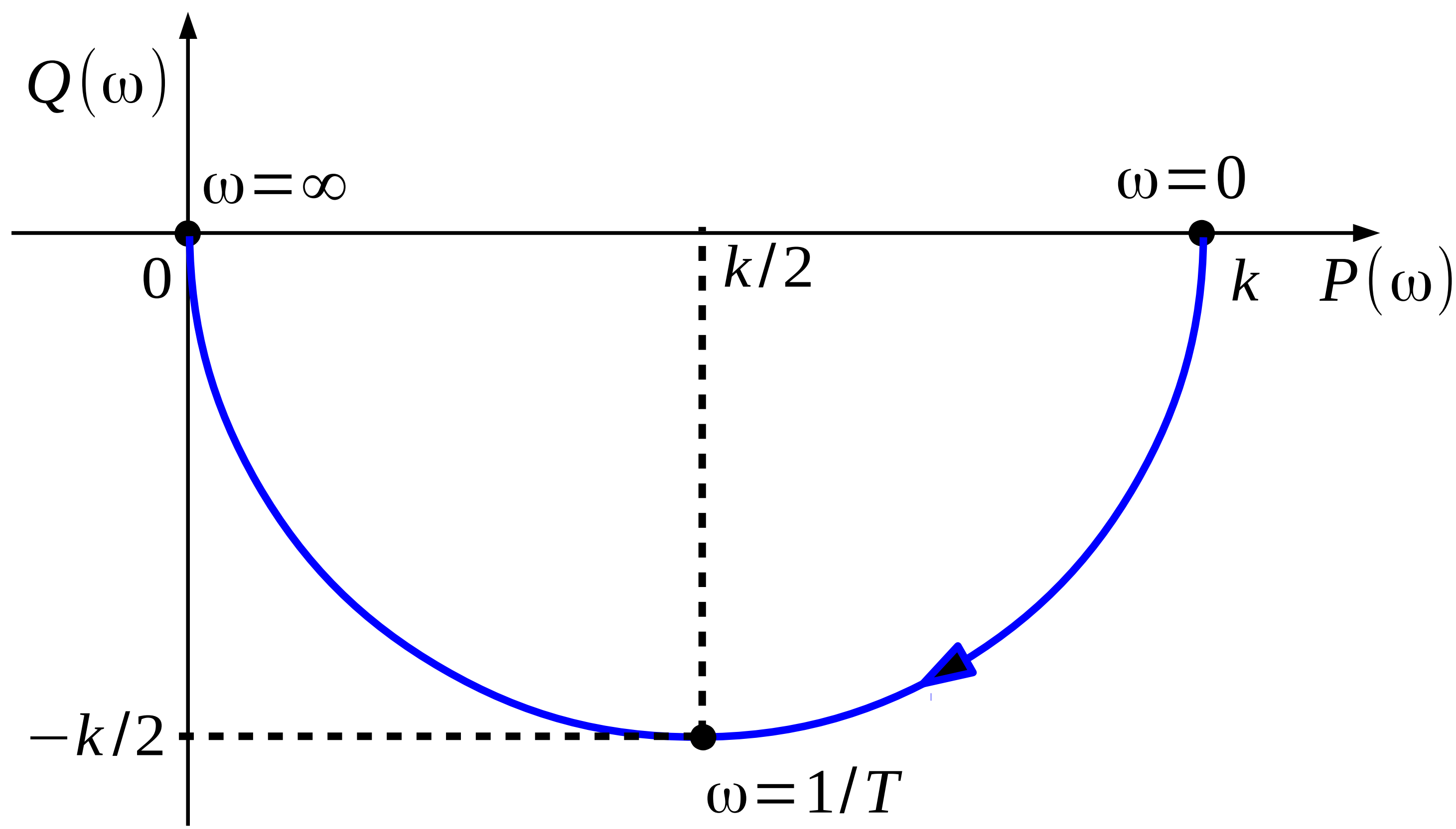
6. Nyquist plot:

for $k>0$



First-order inertial element

7. Bode plot:

First-order inertial element

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First-order inertial element

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$$L(\omega)=20\log A(\omega)=20\log|k|-20\log\sqrt{T^2\omega^2+1}$$

First-order inertial element

7. Bode plot: $A(\omega) = \sqrt{P^2 + Q^2} = |k| / \sqrt{T^2 \omega^2 + 1}$

$$L(\omega) = 20 \log A(\omega) = 20 \log |k| - 20 \log \sqrt{T^2 \omega^2 + 1}$$

$$\varphi(\omega) = \arctan \frac{Q}{P} = \arctan(-T\omega)$$

First-order inertial element

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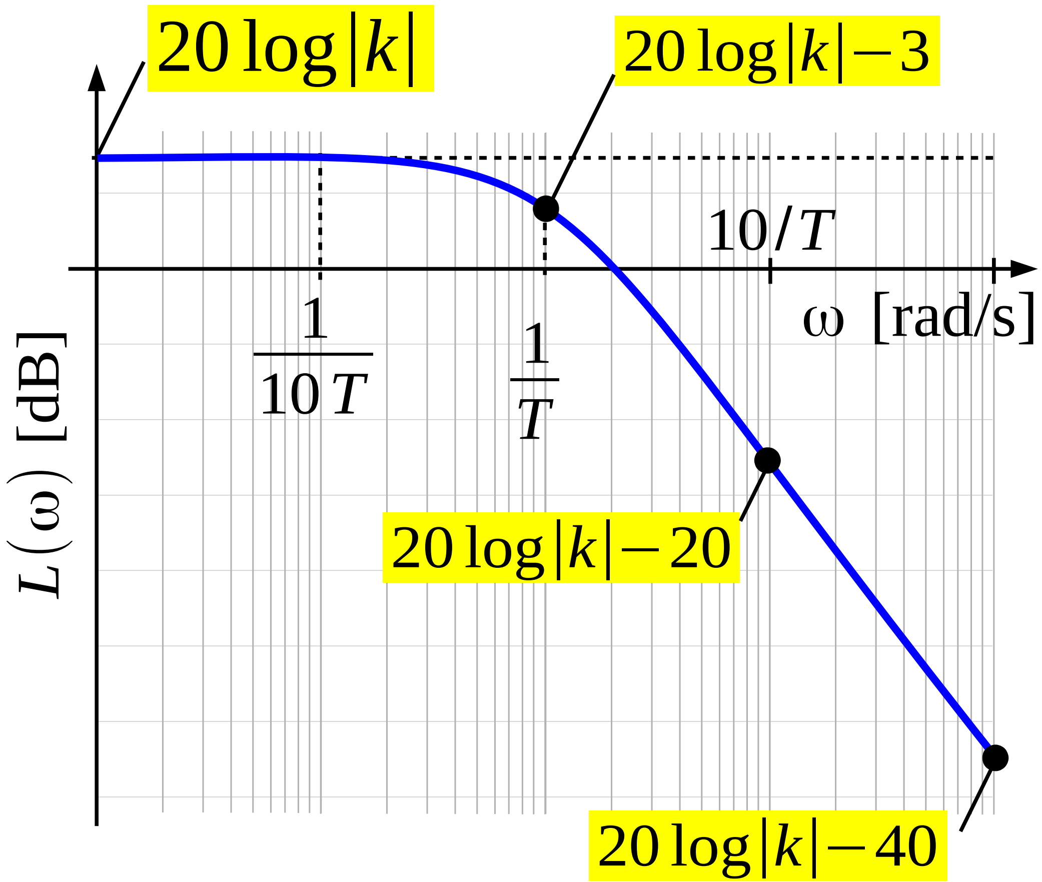

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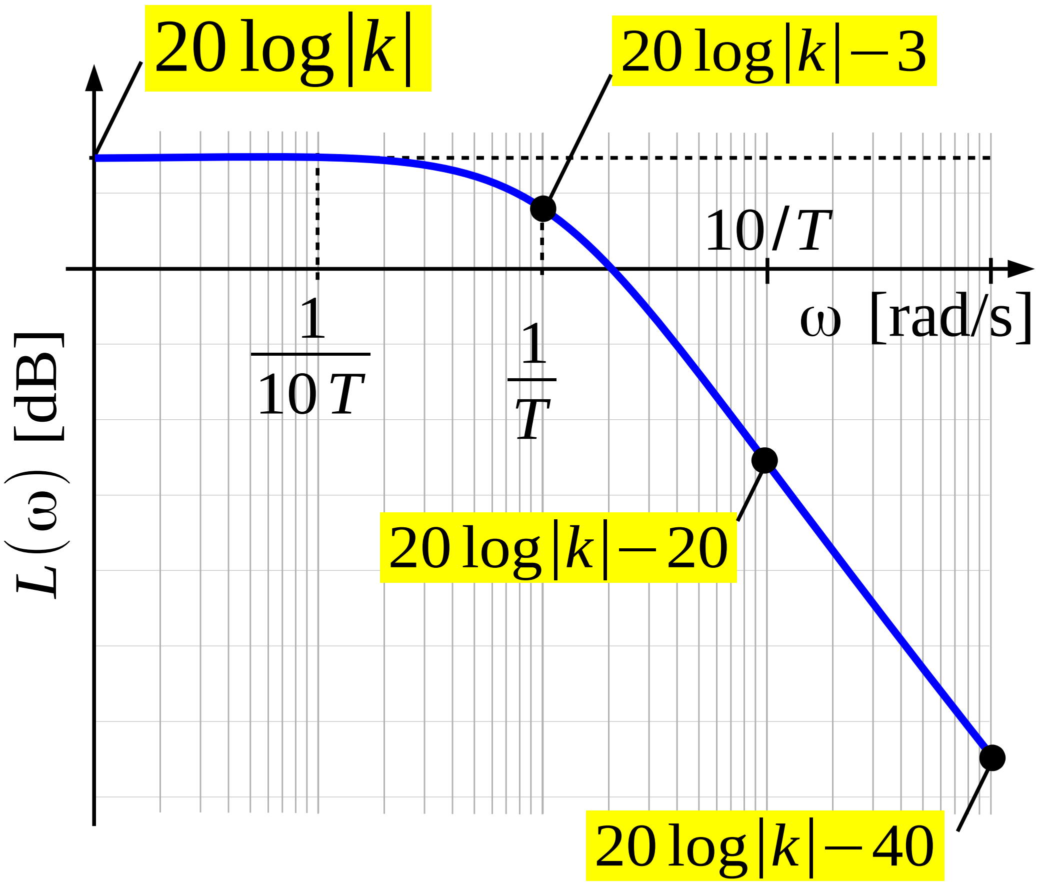
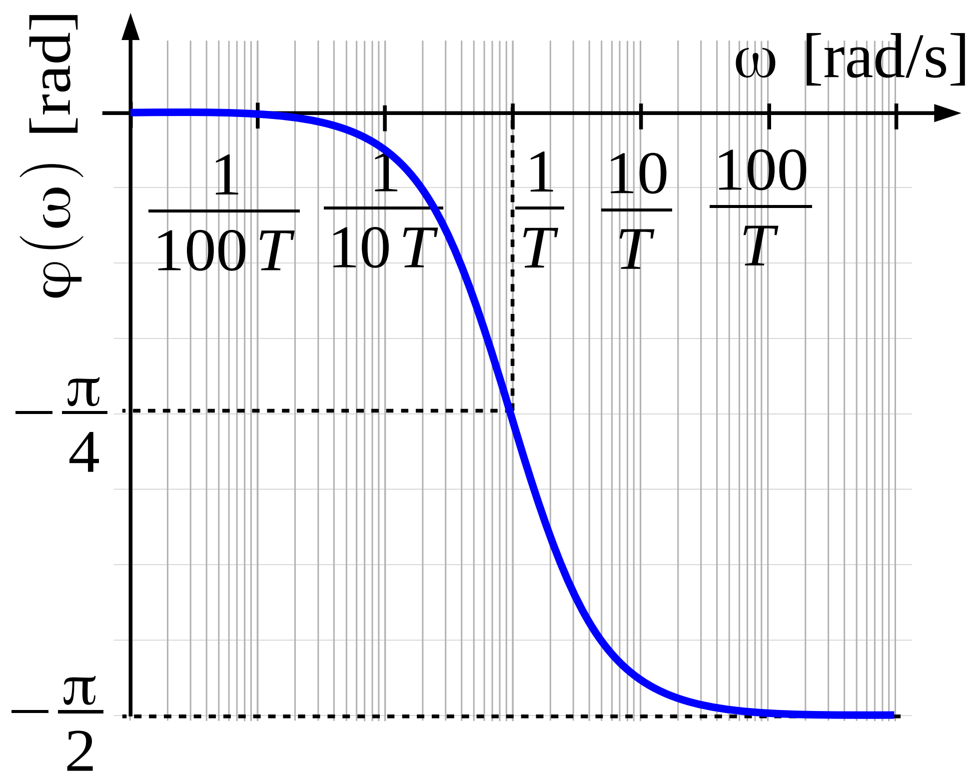
$$\varphi(\omega)=\arctan\frac{Q}{P}=\arctan(-T\omega) \qquad \text{for } k>0$$

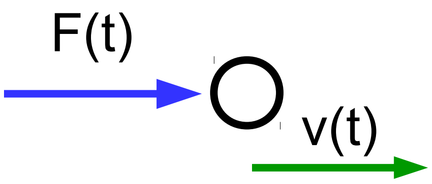




First-order inertial element

Examples

1



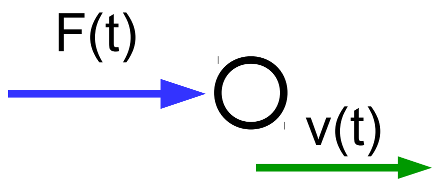
LINEAR MOTION OF A MATERIAL POINT WITH LINEAR DAMPING:
input – force F(t)
output – velocity v(t)

example: car is driving on a flat surface with air resistance proportional
To its velocity, described using machine equation of motion, with assumption
of constant reduced mass.

First-order inertial element

Examples

1



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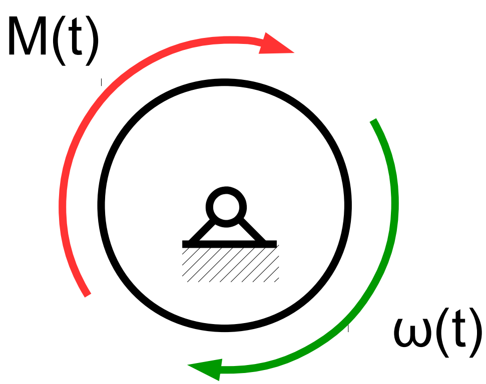
2

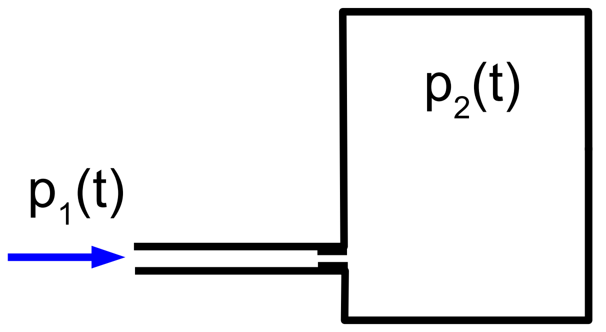


ANGULAR MOTION OF A RIGID BODY WITH LINEAR DAMPING:
input – torque M(t)
output – angular velocity ω(t)

First-order inertial element

Examples

3

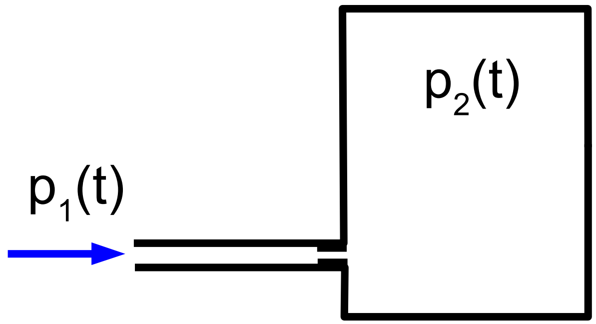


AIR CONTAINER:
input – pressure $p_1(t)$
output – pressure $p_2(t)$

First-order inertial element

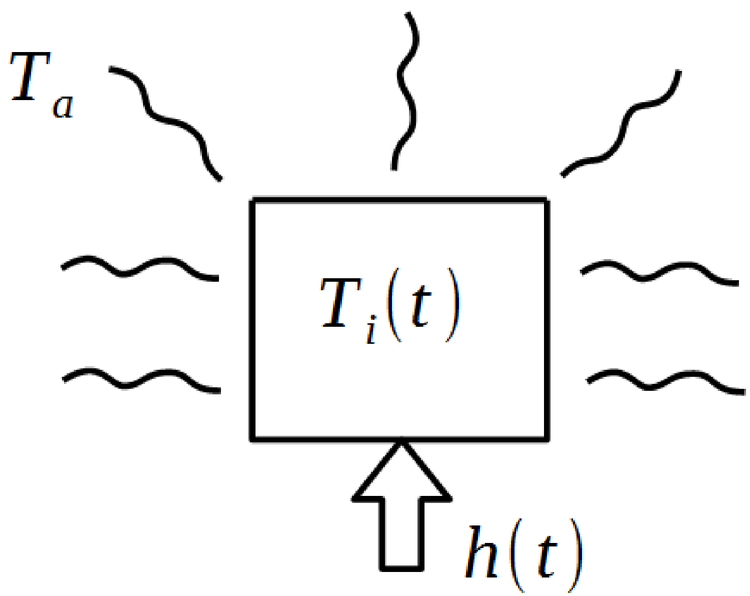
Examples

3



AIR CONTAINER:
input – pressure $p_1(t)$
output – pressure $p_2(t)$

4



HEATED OBJECT WITH SMALL INERTIA:
input – heater power h(t)
output – object temperature $T_i(t)$

Integrator

1. Element equation: $$\frac{dy(t)}{dt} = k\,u(t)$$

$u(t)$ - input
$y(t)$ - output

Integrator

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---

2. Static characteristic (steady state):

for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$

Integrator

1. Element equation: $$\frac{dy(t)}{dt} = k\,u(t)$$

$u(t)$ - input
$y(t)$ - output

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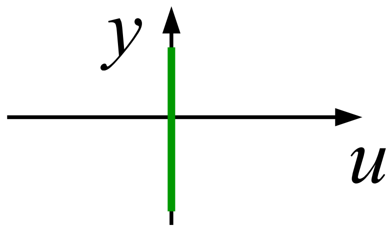

Integrator

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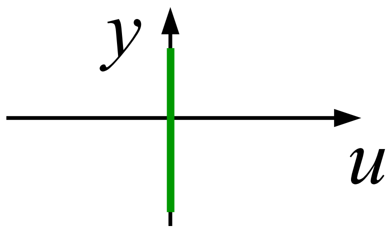

3. Transfer function:

Integrator

1. Element equation: $$\frac{dy(t)}{dt}=k\,u(t)$$

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$y(t)$ - output

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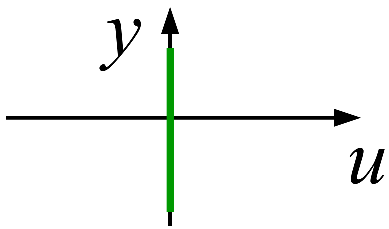

3. Transfer function: $$H(s)=\frac{k}{s}$$

Integrator

4. Step response:

Integrator

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input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0 \frac{1}{s}$

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Integrator

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output: $y(t)=L^{-1}\{Y(s)\}=k u_0 t$

Integrator

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Laplace of input: $U(s)=u_0 \frac{1}{s}$

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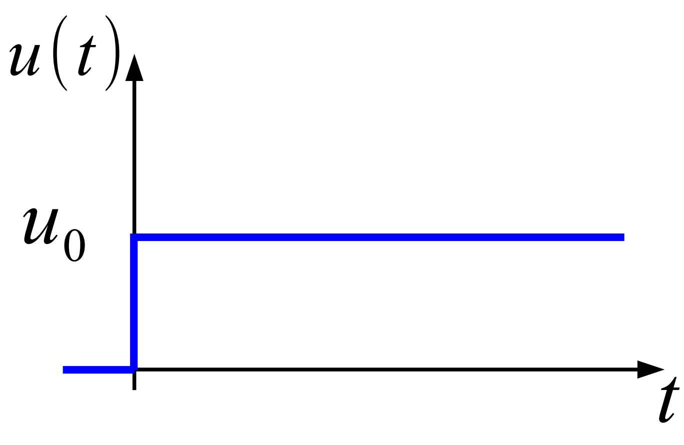
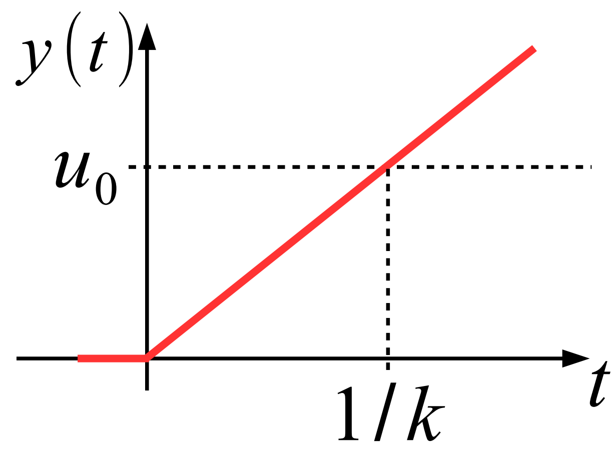
for $k>0$





Integrator

5. Frequency response:

Integrator

5. Frequency response: $H(j\omega)=\frac{k}{j\omega}$

Integrator

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$$P(\omega)=0,\ \ Q(\omega)=-\frac{k}{\omega}$$

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6. Nyquist plot:

Integrator

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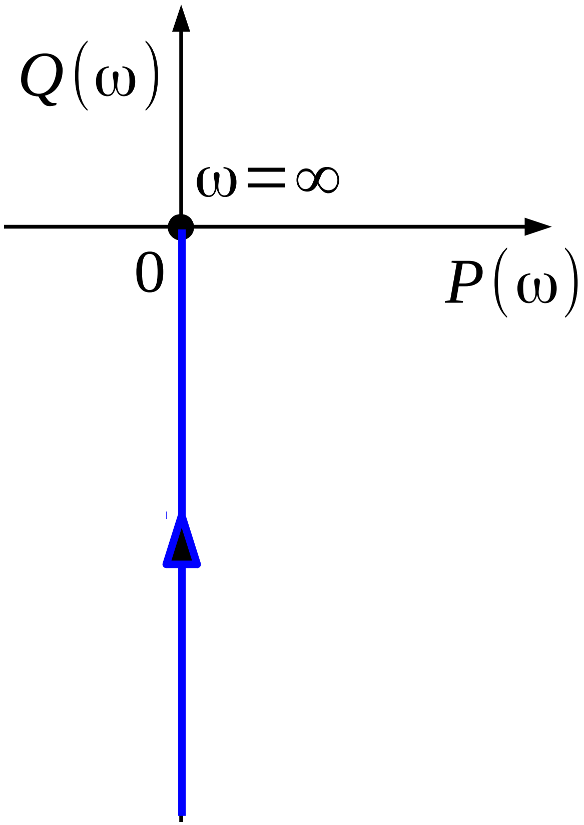
for $k>0$



Integrator

7. Bode plot:

Integrator

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|\frac{k}{\omega}|$

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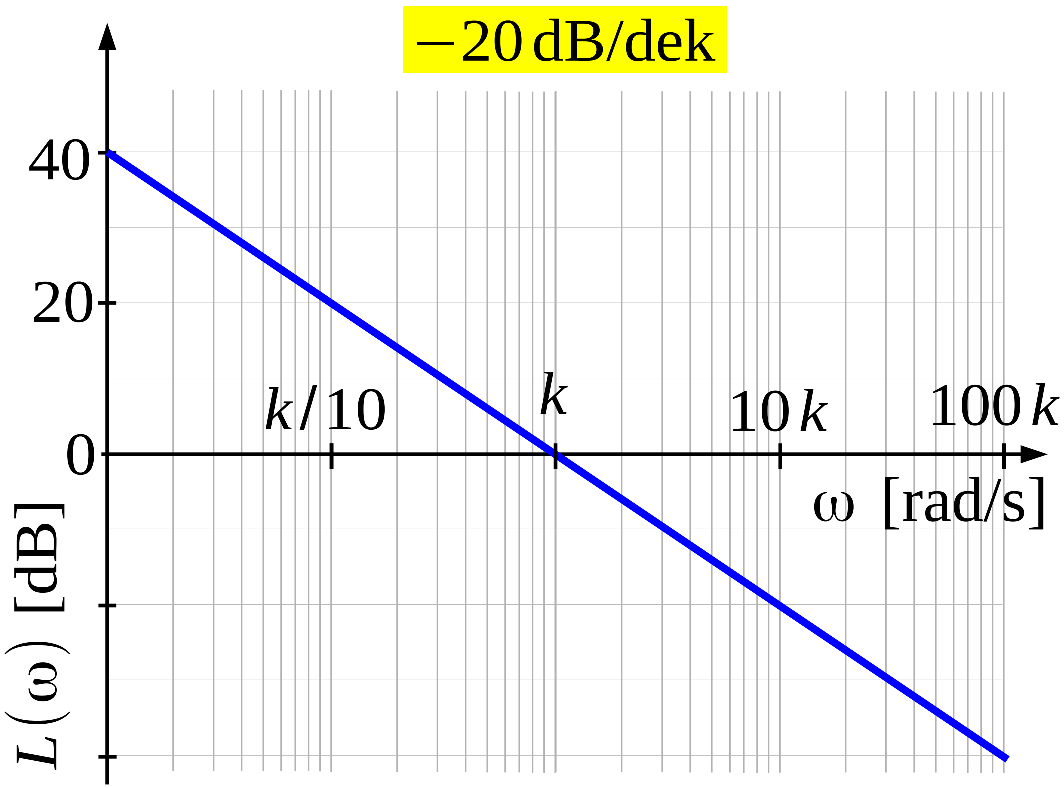
$$L(\omega)=20\log A(\omega)=20\log|\frac{k}{\omega}| \qquad \varphi(\omega)=\arctan\frac{Q}{P}=\arctan(-\infty)$$

Integrator

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=\left|\frac{k}{\omega}\right|$ for $k>0$

$$L(\omega)=20\log A(\omega)=20\log\left|\frac{k}{\omega}\right| \qquad \varphi(\omega)=\arctan\frac{Q}{P}=\arctan(-\infty)$$



Integrator

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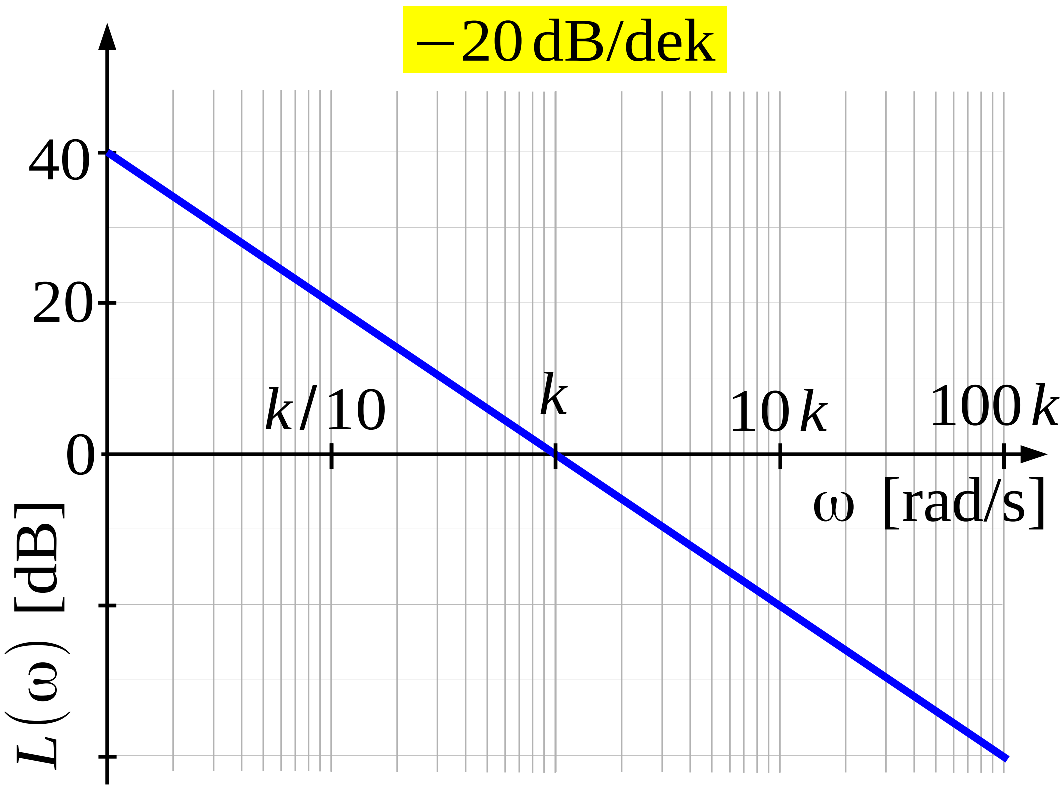
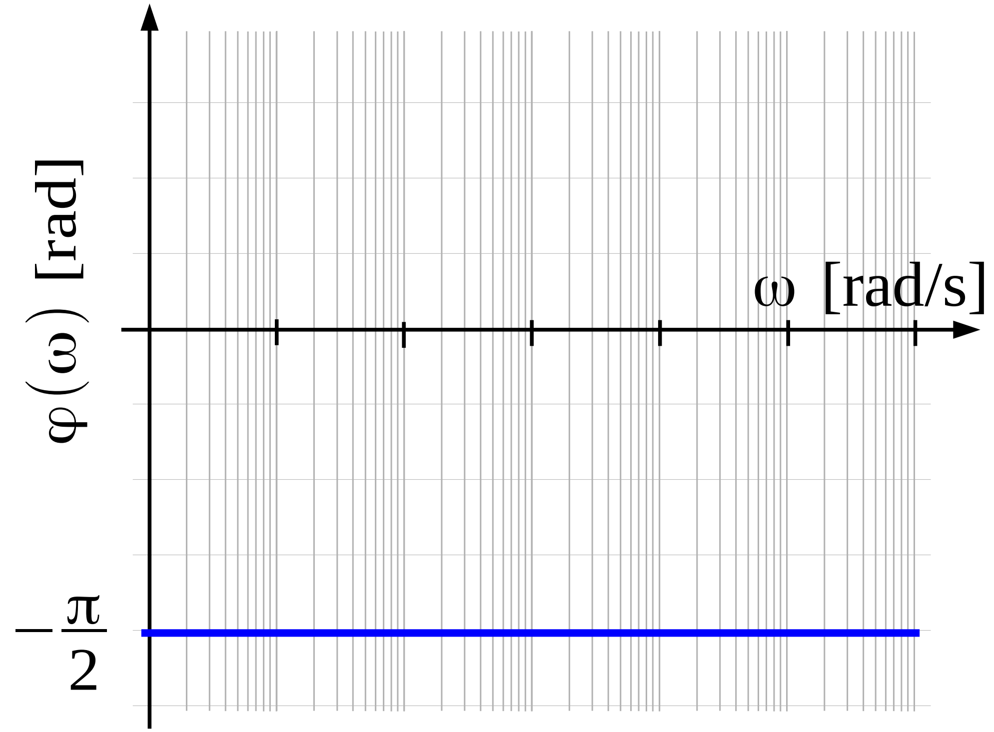
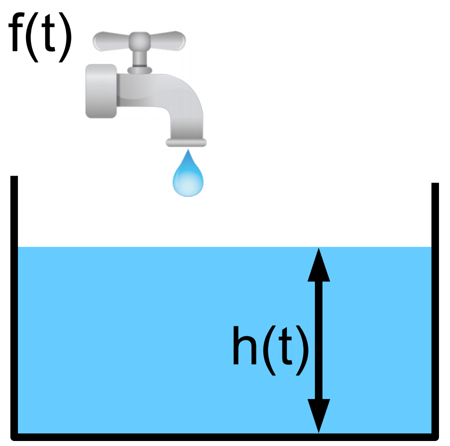




Integrator

Examples

1

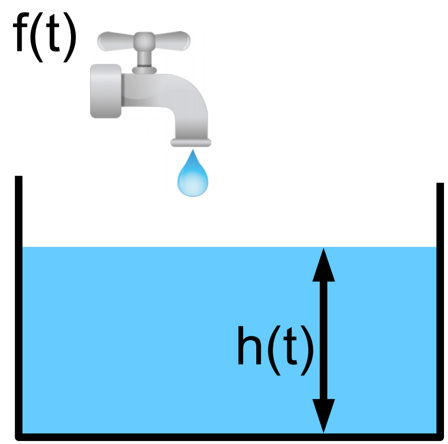


PRISM LIQUID TANK:
input – liquid inflow f(t)
output – liquid level h(t)

Integrator

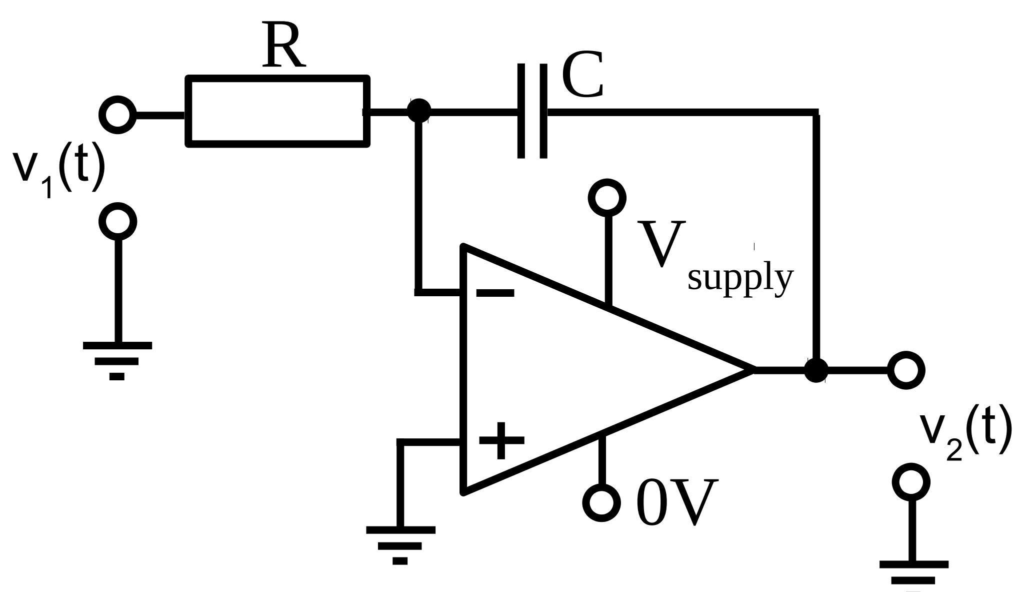
Examples

1



PRISM LIQUID TANK:
input – liquid inflow f(t)
output – liquid level h(t)

2



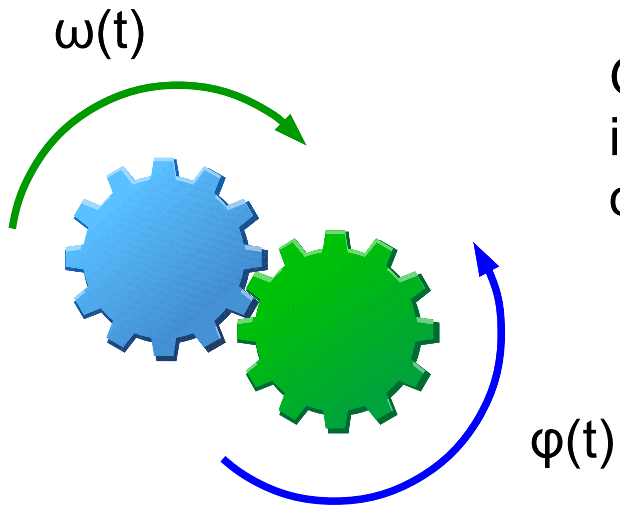
OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = \frac{1}{RC} \int_0^t v_1(t)\, dt$$

Integrator

Examples

3

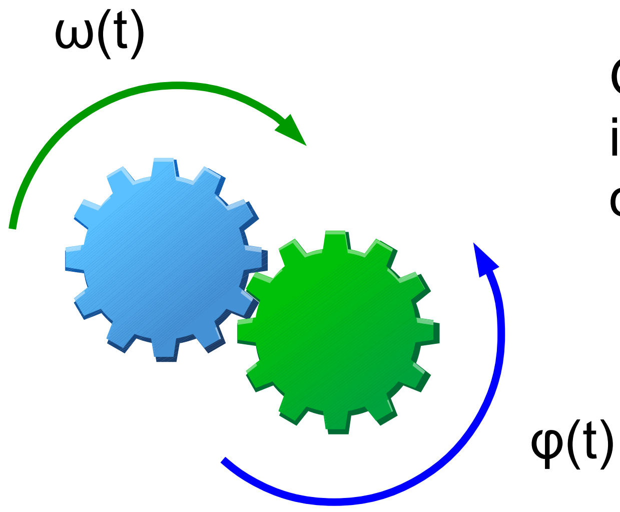


GEARBOX:
input – angular velocity ω(t)
output – rotation angle φ(t)

Integrator

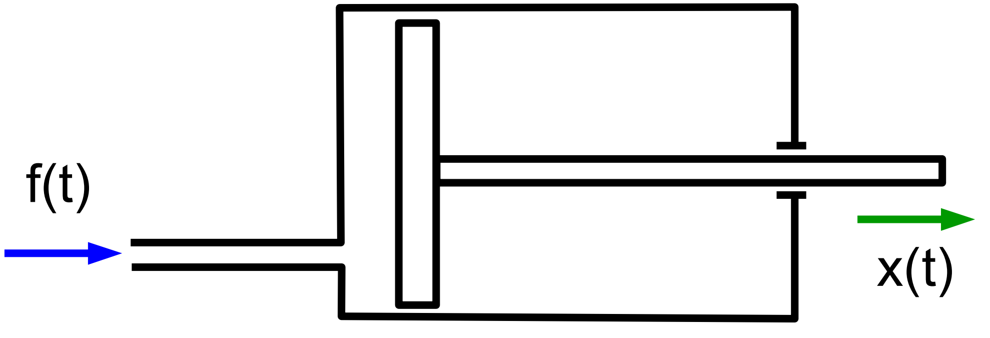
Examples

3



GEARBOX:
input – angular velocity ω(t)
output – rotation angle φ(t)

4



HYDRAULIC CYLINDER:
input – volume inflow f(t)
output – displacement x(t)

Differentiator

1. Element equation: $y(t)=k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

Differentiator

1. Element equation: $y(t) = k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

---

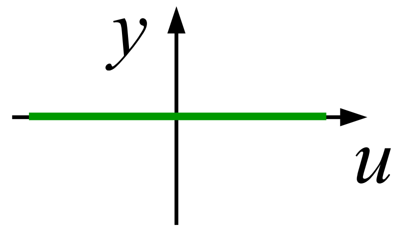
2. Static characteristic (steady state): for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$

Differentiator

1. Element equation: $y(t)=k\dfrac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $y=0$ for $\dfrac{dy}{dt}=0\wedge\dfrac{du}{dt}=0$



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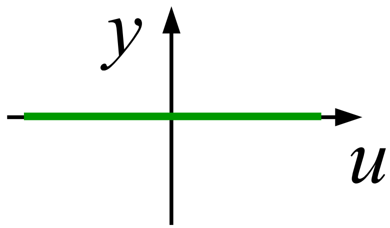

3. Transfer function:

Differentiator

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$u(t)$ - input
$y(t)$ - output

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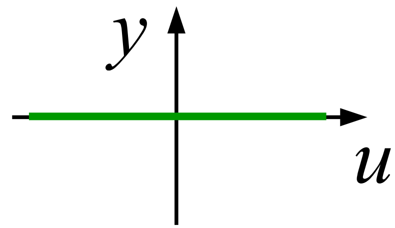

3. Transfer function: $H(s) = k\,s$

Differentiator

4. Step response:

Differentiator

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input: $u(t) = u_0 1(t)$

Laplace of input: $U(s) = u_0 \frac{1}{s}$

Differentiator

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Laplace of input: $U(s) = u_0 \frac{1}{s}$

Laplace of output: $Y(s) = H(s)\, U(s) = k\, u_0$

Differentiator

4. Step response:

input: $u(t) = u_0 1(t)$

Laplace of input: $U(s) = u_0 \frac{1}{s}$

Laplace of output: $Y(s) = H(s)U(s) = k u_0$

output: $y(t) = L^{-1}\{Y(s)\} = k u_0 \delta(t)$

Differentiator

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Laplace of input: $U(s) = u_0 \frac{1}{s}$

Laplace of output: $Y(s) = H(s)U(s) = k\,u_0$

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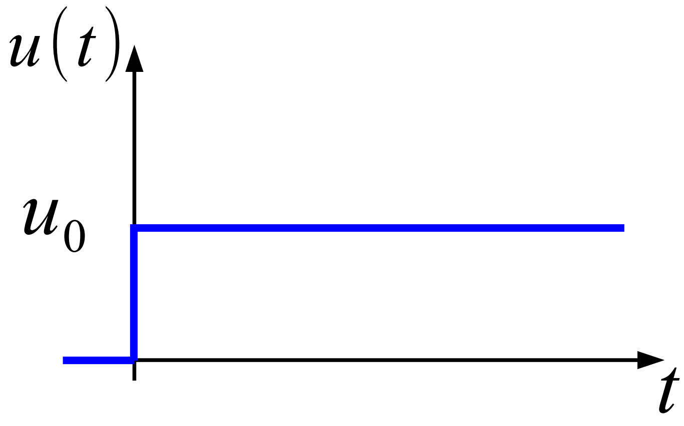

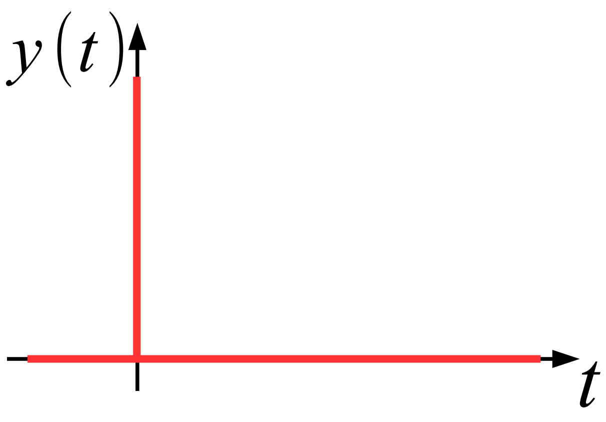

Differentiator

5. Frequency response:

Differentiator

5. Frequency response: $H(j\omega)=jk\omega$

$$P(\omega)=0,\ \ Q(\omega)=k\omega$$

Differentiator

5. Frequency response: $H(j\omega) = jk\omega$

$$P(\omega) = 0, \quad Q(\omega) = k\omega$$

6. Nyquist plot:

Differentiator

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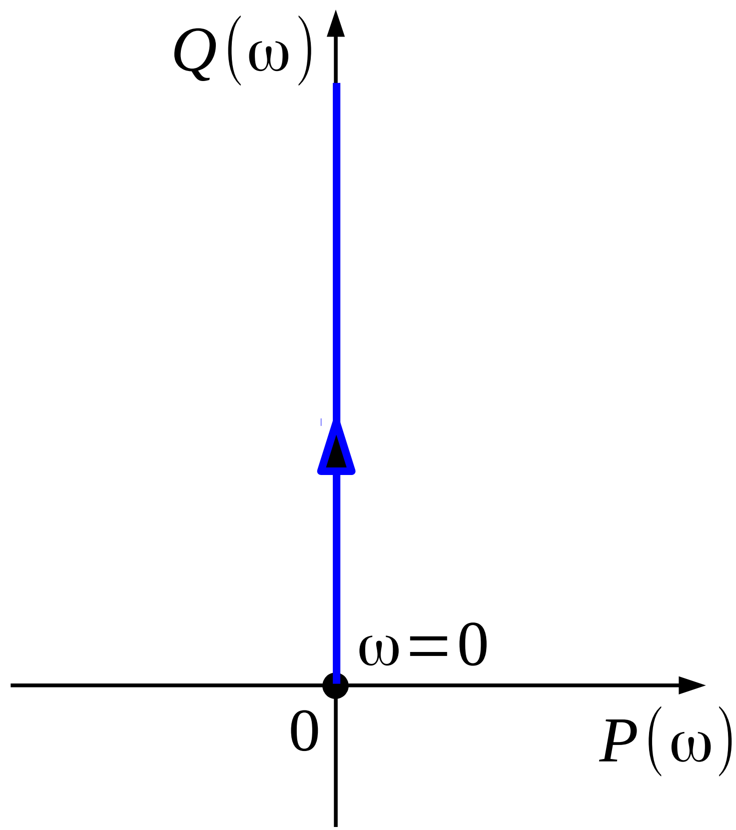
$$P(\omega)=0,\;\; Q(\omega)=k\omega$$

6. Nyquist plot:

for $k>0$



Differentiator

7. Bode plot:

Differentiator

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k\,\omega|$

Differentiator

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$$L(\omega)=20\log A(\omega)=20\log|k\,\omega|$$

Differentiator

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$$L(\omega)=20\log A(\omega)=20\log|k\,\omega|$$

$$\varphi(\omega)=\arctan\frac{Q}{P}=\arctan(\infty)$$

Differentiator

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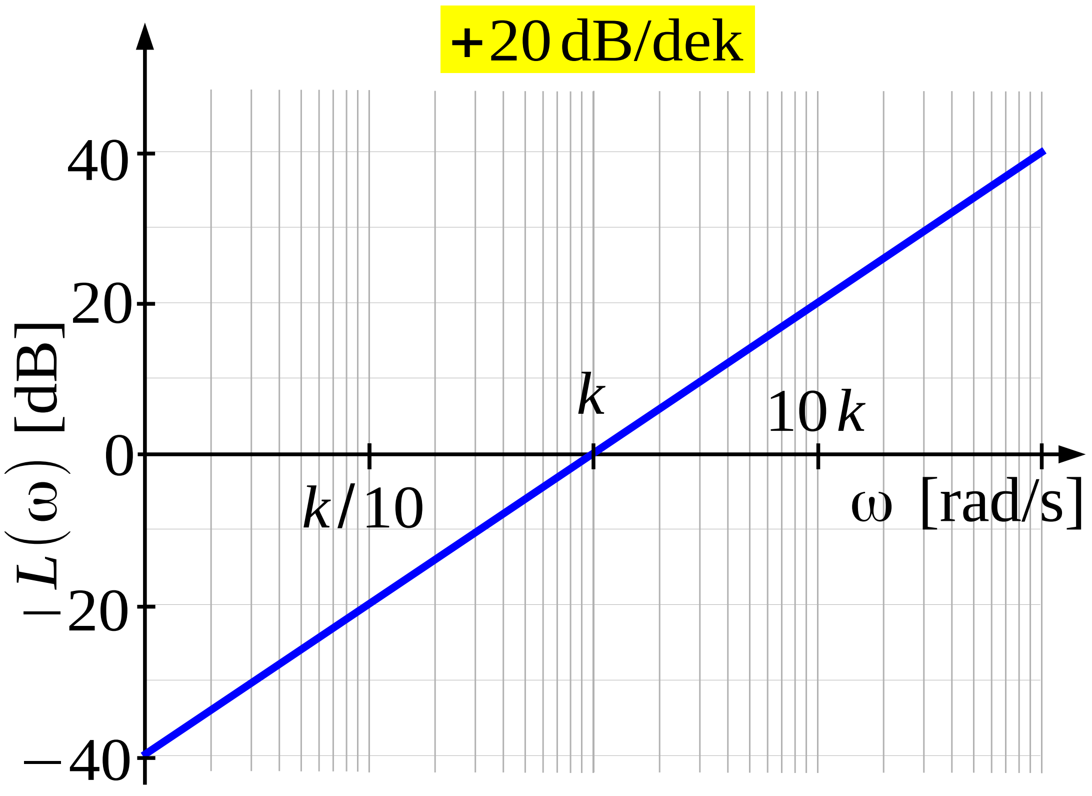

Differentiator

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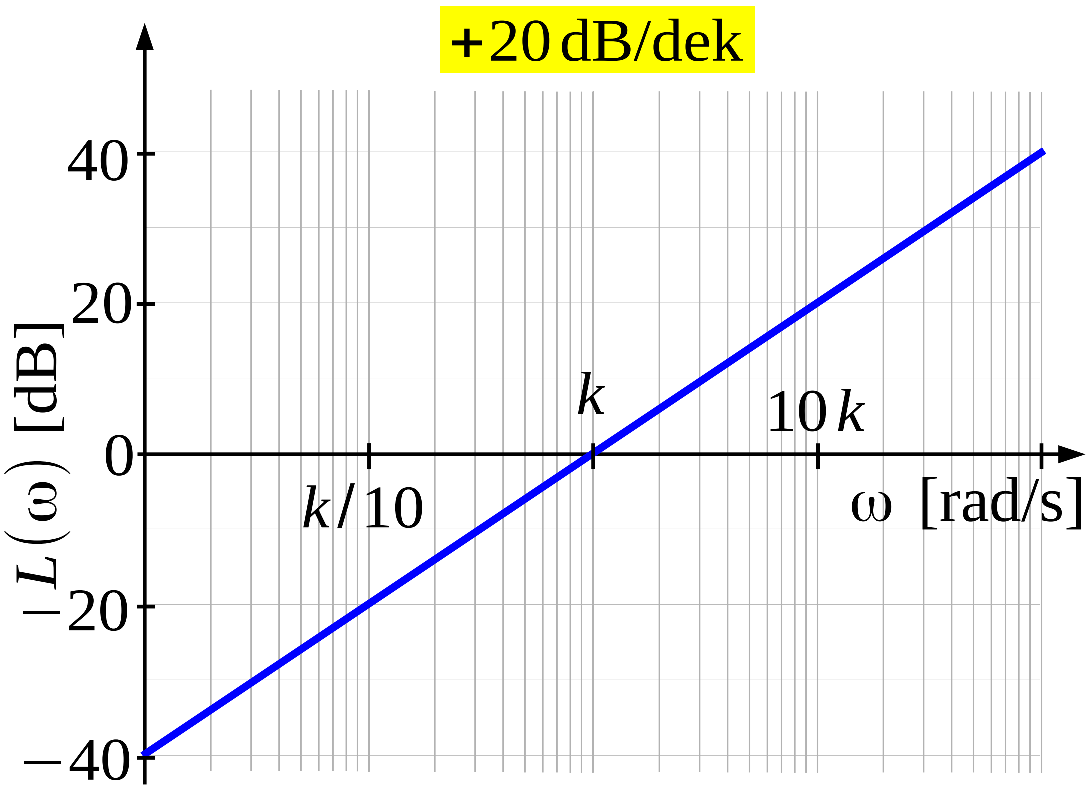
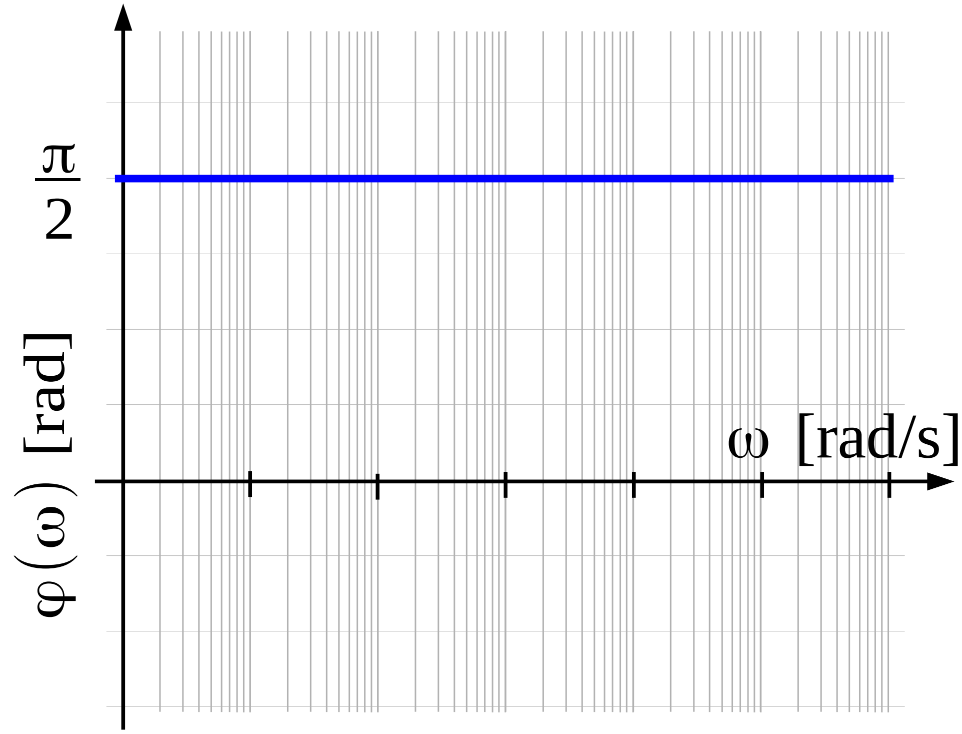


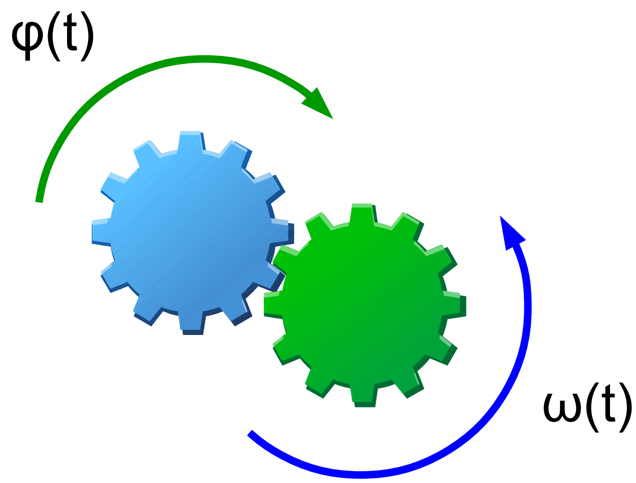
for $k>0$



Differentiator

Examples

1

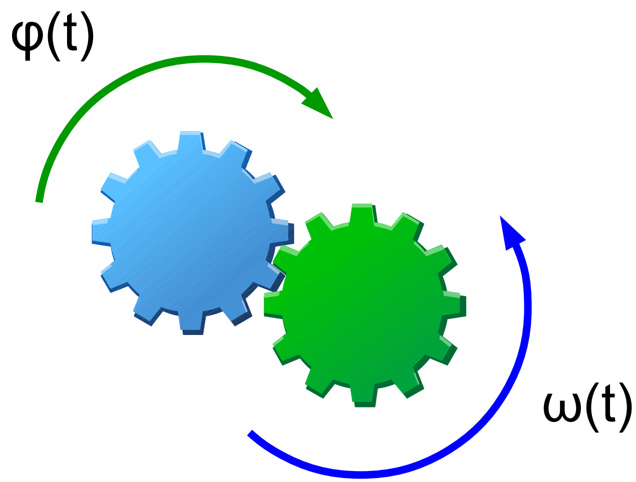


GEARBOX:
input – rotation angle $\varphi(t)$
output – angular velocity $\omega(t)$

Differentiator

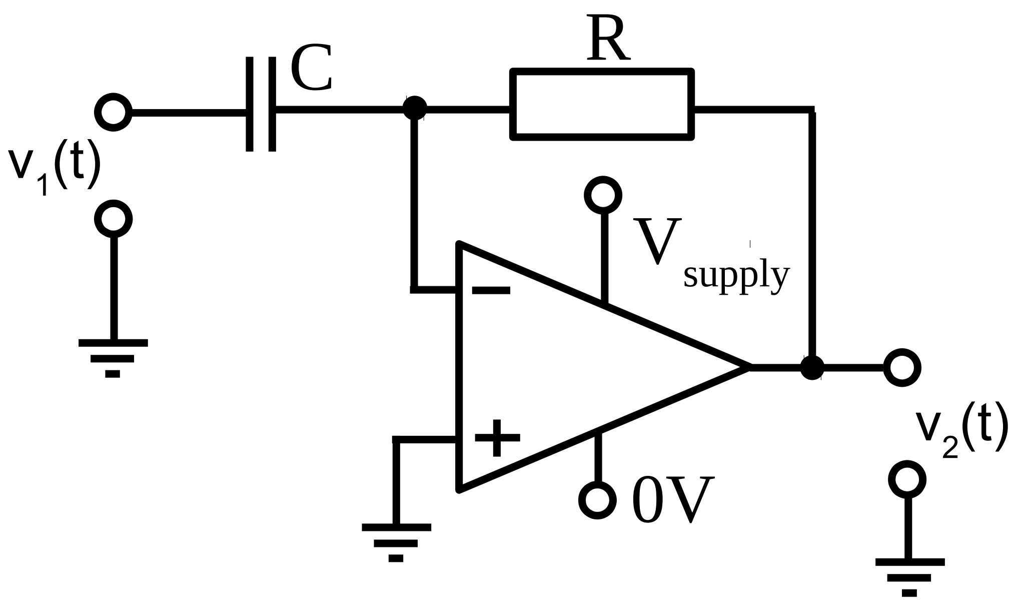
Examples

1



GEARBOX:
input – rotation angle φ(t)
output – angular velocity ω(t)

2



OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = -RC\frac{dv_1(t)}{dt}$$

Real differentiator (derivative+1st order)

1. Element equation: $T\frac{dy(t)}{dt}+y(t)=k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

Real differentiator (derivative+1st order)

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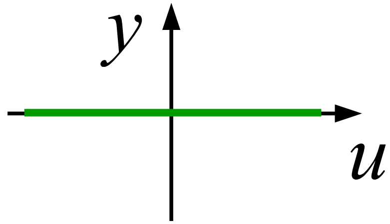

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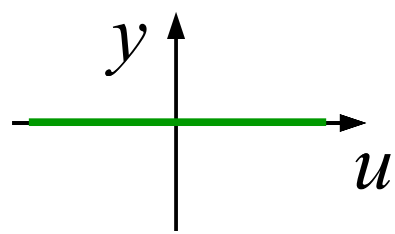

3. Transfer function:

Real differentiator (derivative+1st order)

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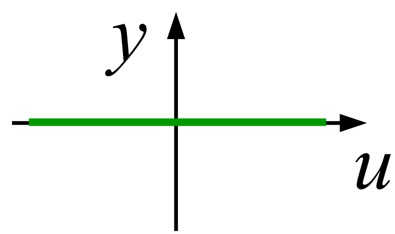

3. Transfer function: $H(s)=\frac{k\,s}{Ts+1}$

Real differentiator (derivative+1st order)

4. Step response:

Real differentiator (derivative+1st order)

4. Step response:

input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0 \frac{1}{s}$

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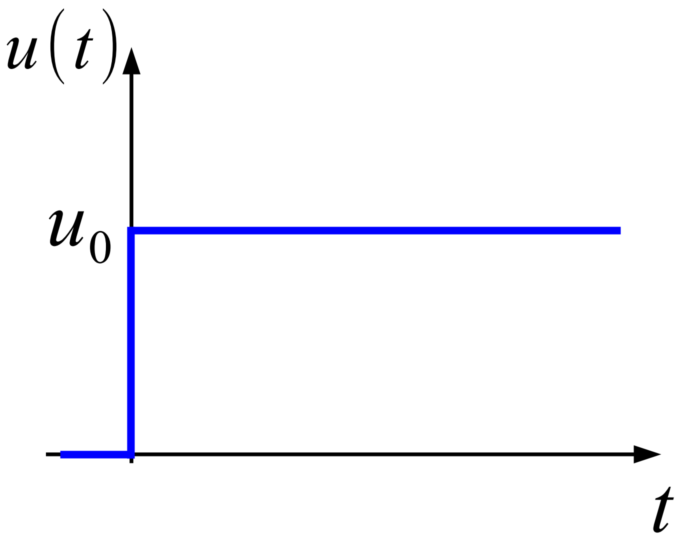
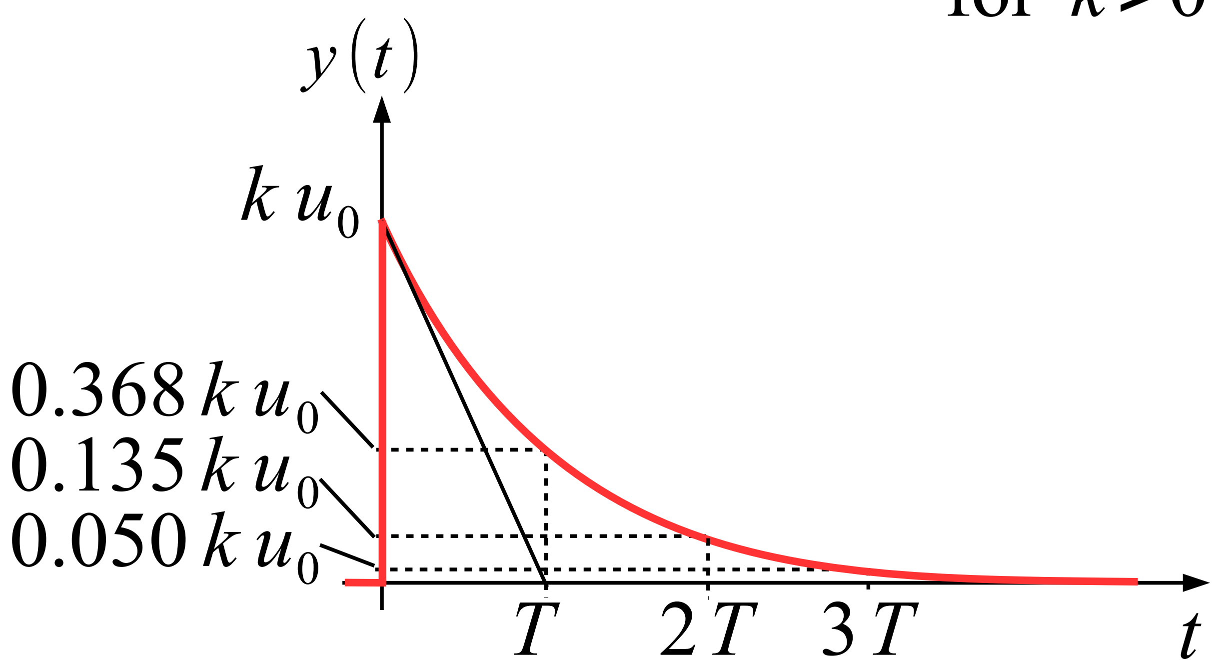
output: $y(t) = L^{-1}\{Y(s)\} = k u_0 e^{-t/T}$

for $k > 0$





Real differentiator (derivative+1st order)

5. Frequency response:

Real differentiator (derivative+1st order)

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Real differentiator (derivative+1st order)

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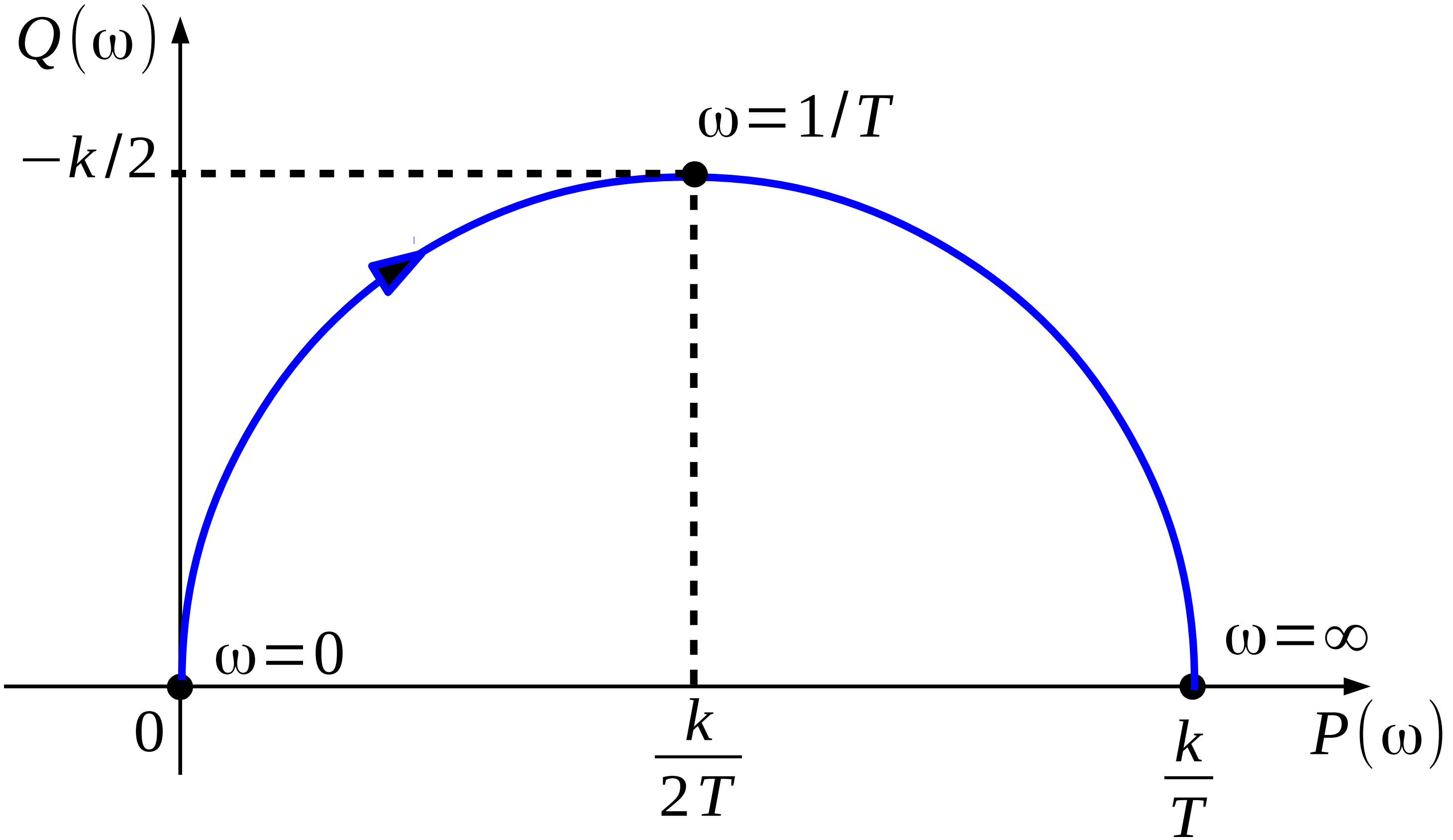
6. Nyquist plot:

for $k>0$



Real differentiator (derivative+1st order)

7. Bode plot:

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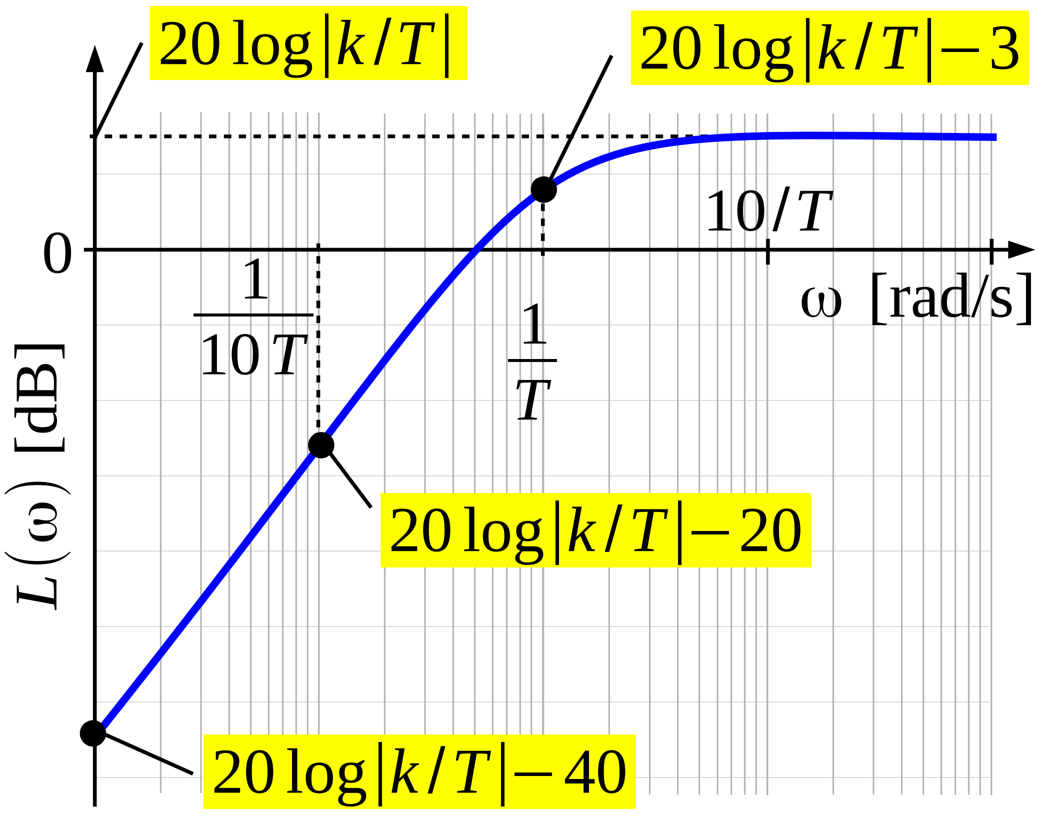

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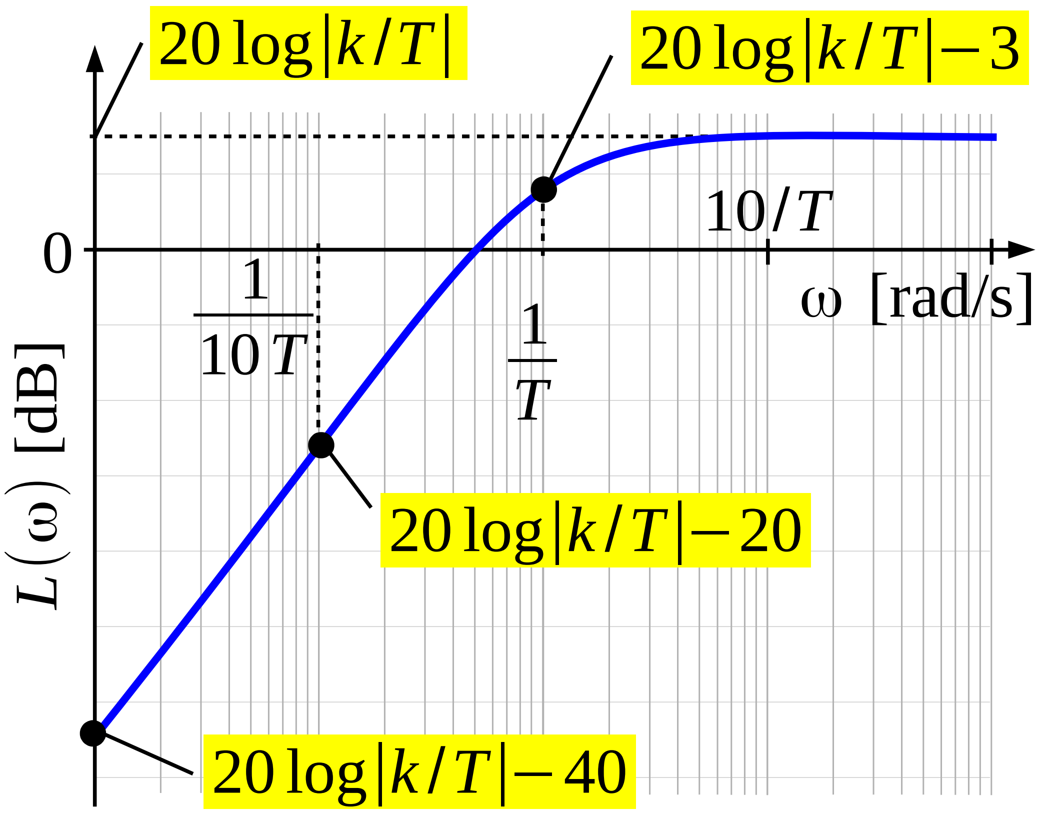
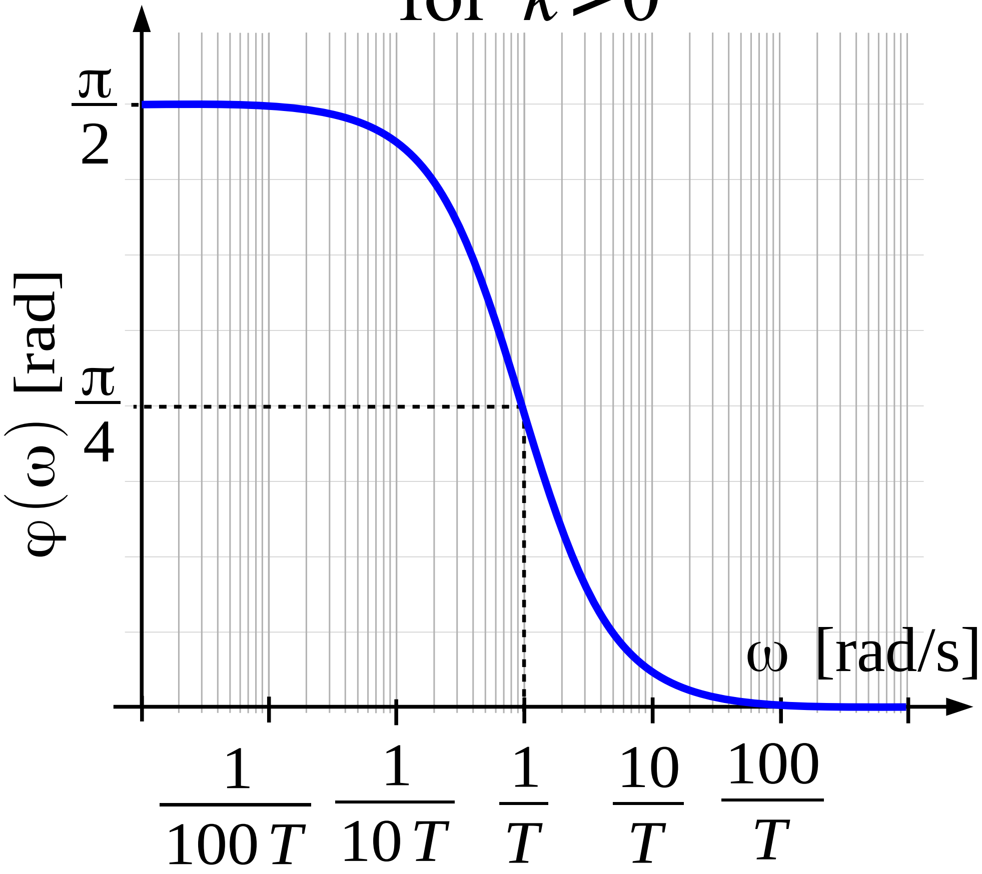


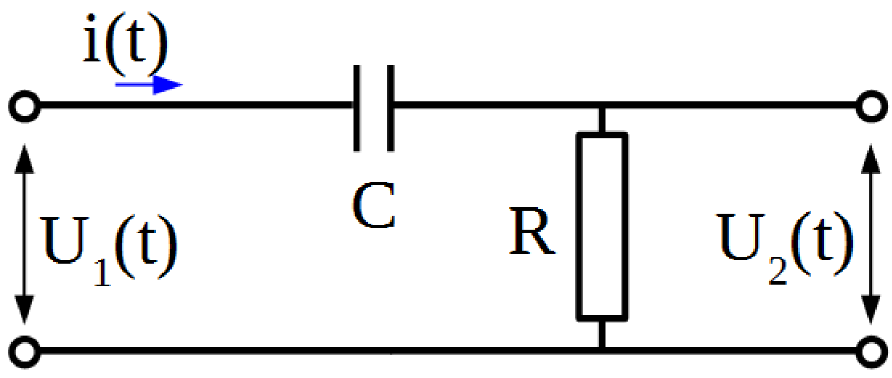
for $k>0$



Real differentiator (derivative+1st order)

Examples

1



RC CIRCUIT:
input – voltage $u_1(t)$
output – voltage $u_2(t)$

Delay

$u(t)$ - input
$y(t)$ - output

1. Element equation: $y(t)=u(t-\tau)$

Delay

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---

2. Static characteristic (steady state): for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$

Delay

1. Element equation: $y(t)=u(t-\tau)$

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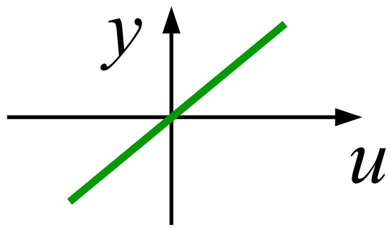
$y(t)$ - output

2. Static characteristic (steady state): $y=u$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$



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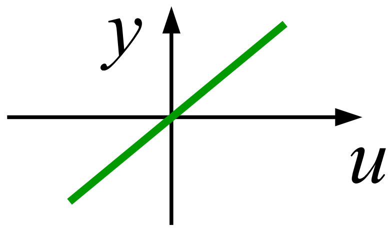

3. Transfer function:

Delay

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$y(t)$ - output

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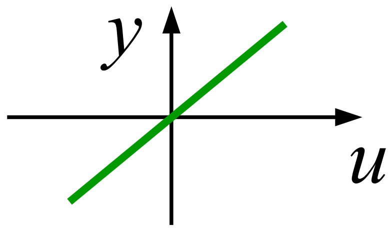

3. Transfer function: $H(s) = e^{-\tau s}$

Delay

4. Step response:

Delay

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input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0 \frac{1}{s}$

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Laplace of input: $U(s) = u_0 \frac{1}{s}$

Laplace of output: $Y(s) = H(s) U(s) = \frac{u_0}{s} e^{-\tau s}$

Delay

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input: $u(t)=u_0 1(t)$

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Laplace of output: $Y(s)=H(s)U(s)=\frac{u_0}{s}e^{-\tau s}$

output: $y(t)=L^{-1}\{Y(s)\}=u_0 1(t-\tau)$

Delay

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input: $u(t) = u_0 1(t)$

Laplace of input: $U(s) = u_0 \frac{1}{s}$

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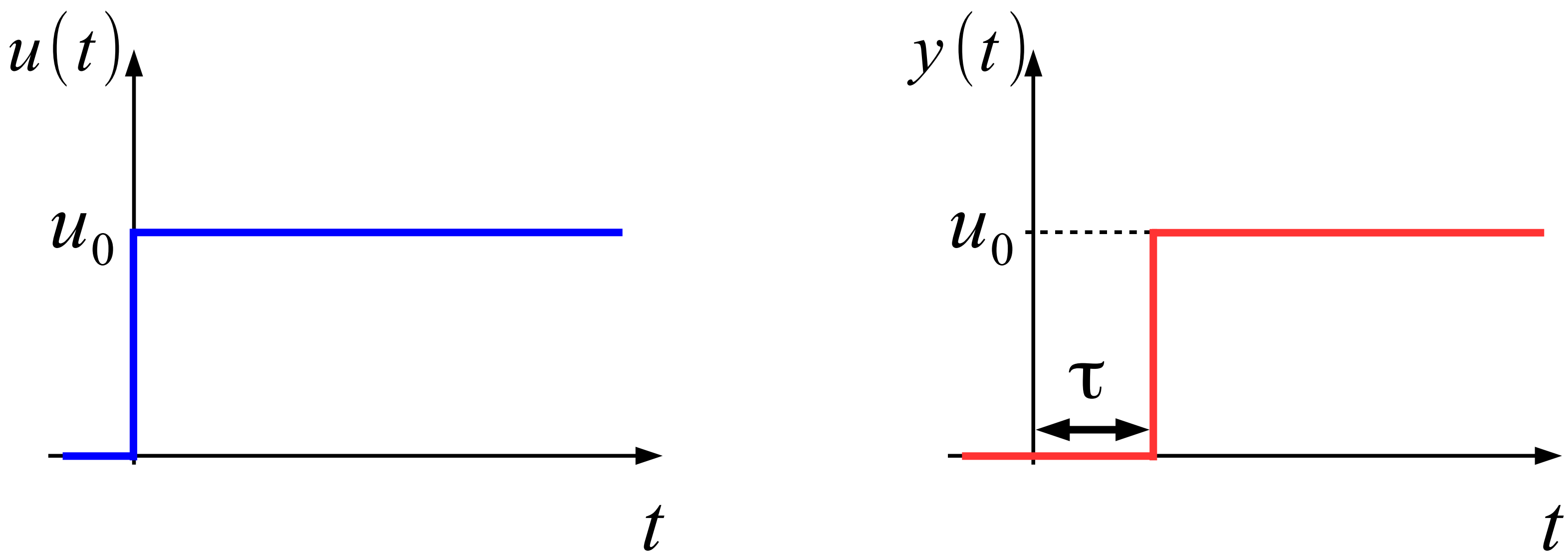

Delay

5. Frequency response:

Delay

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$$P(\omega)=\cos(\tau\omega),\;\; Q(\omega)=-\sin(\tau\omega)$$

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Delay

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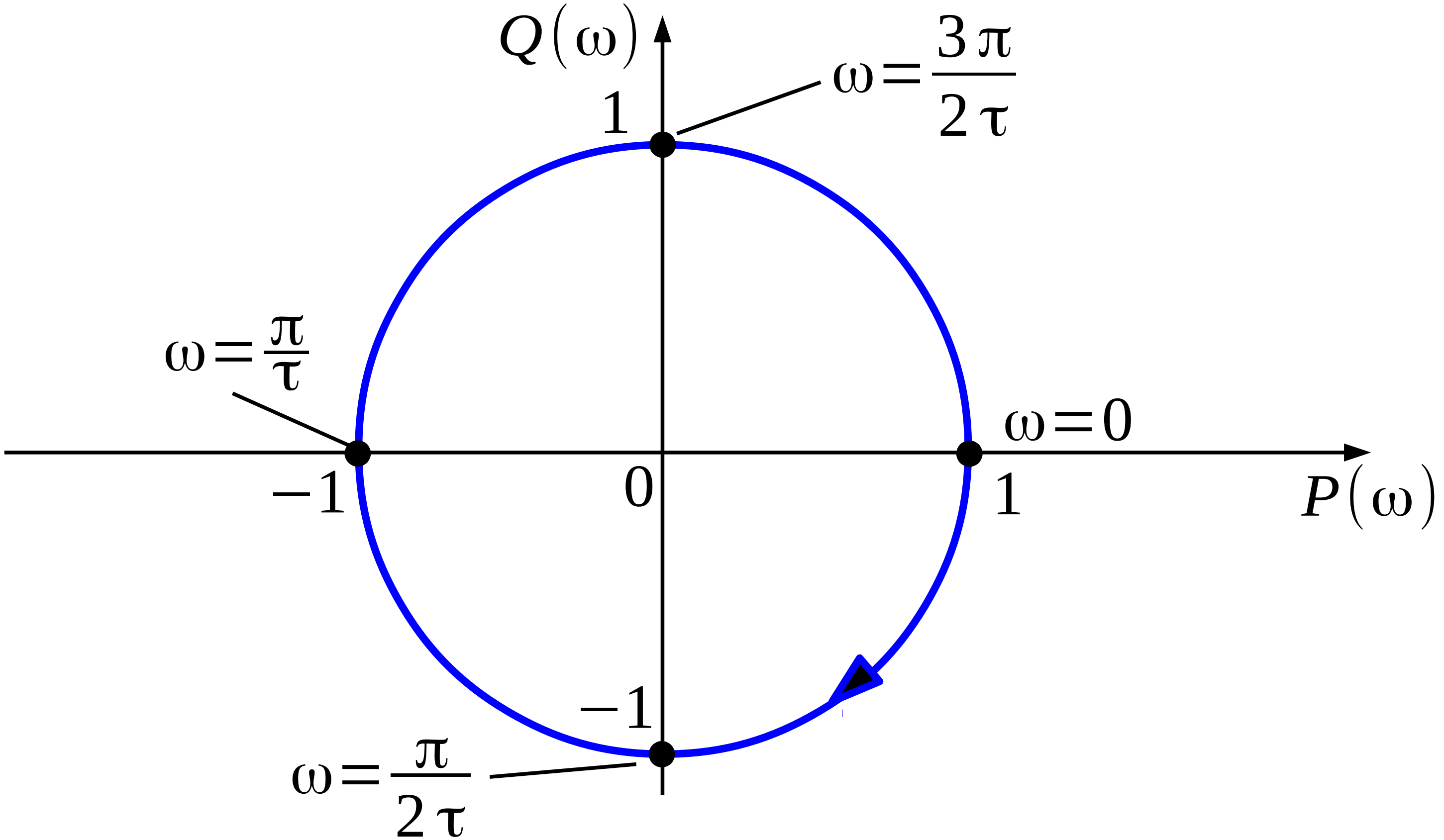

Delay

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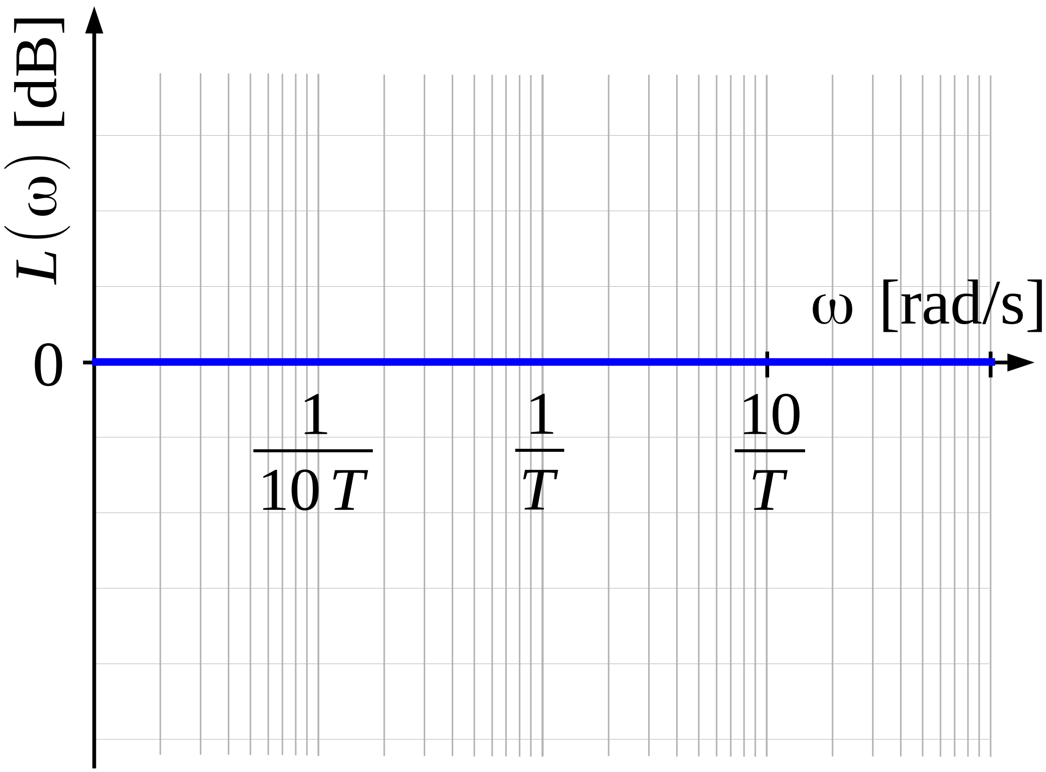

Delay

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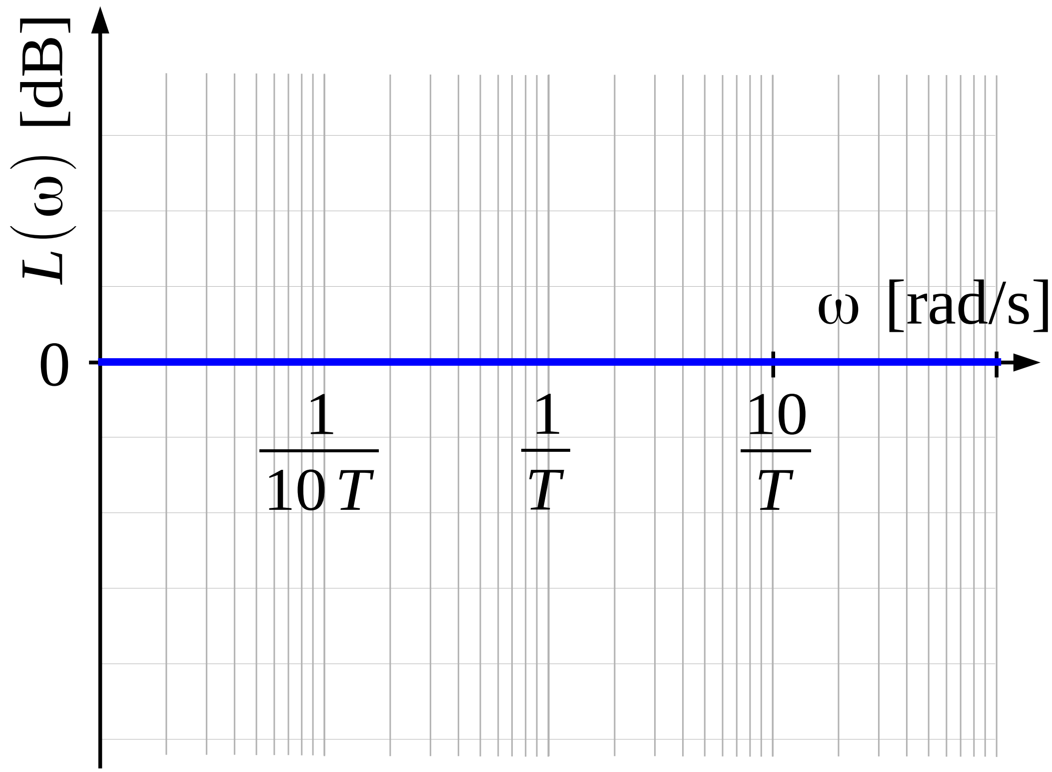
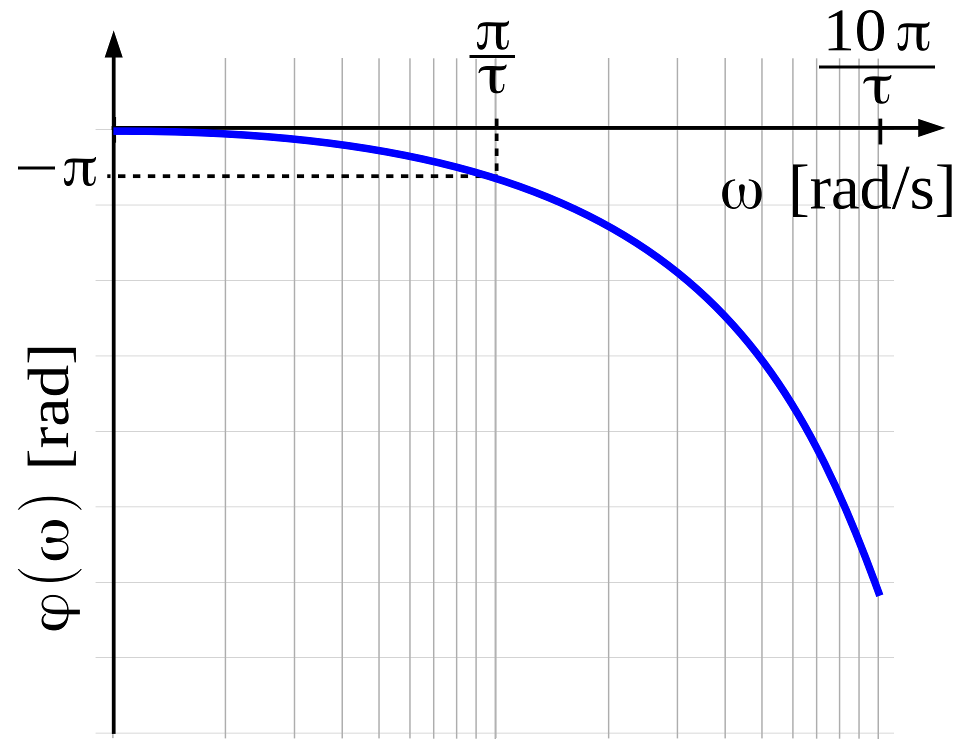
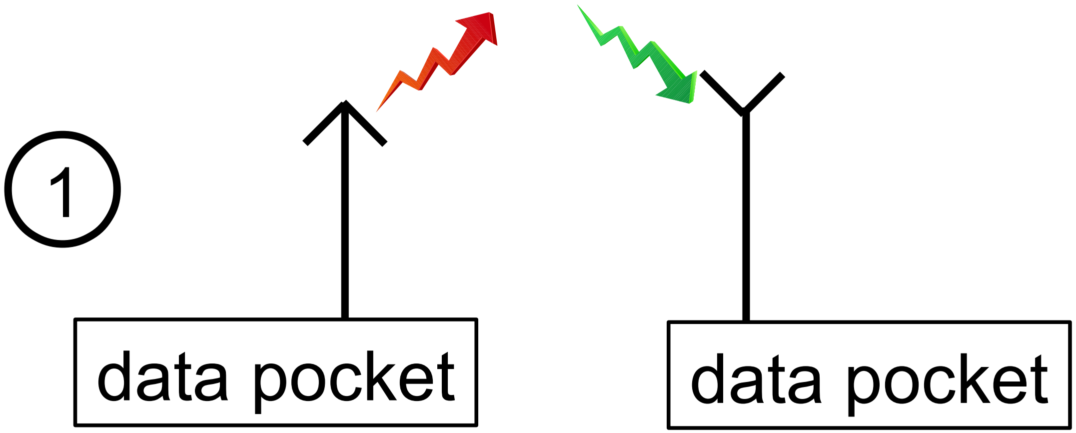




Delay

Examples



WIRELESS TRANSMISSION:
input – sent data
output – received data

Second-order inertial element

1. Element equation: $T_1^2 \frac{d^2 y(t)}{dt^2} + T_2 \frac{dy(t)}{dt} + y(t) = k\,u(t)$

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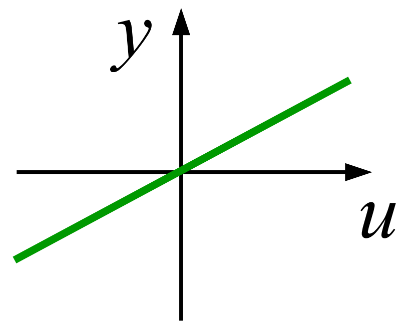

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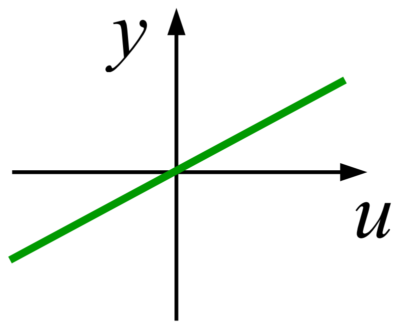

3. Transfer function:

Second-order inertial element

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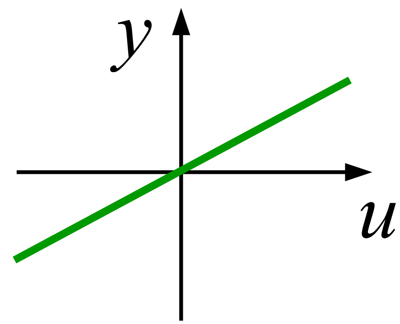

3. Transfer function: $H(s) = \frac{k}{T_1^2 s^2 + T_2 s + 1}$

Second-order inertial element

4. Step response:

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input: $u(t) = u_0 1(t)$

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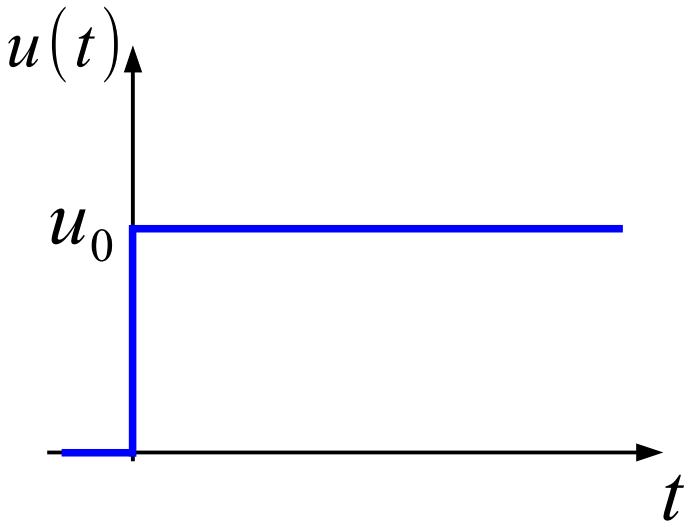
output: $y(t)=L^{-1}\{Y(s)\}=$

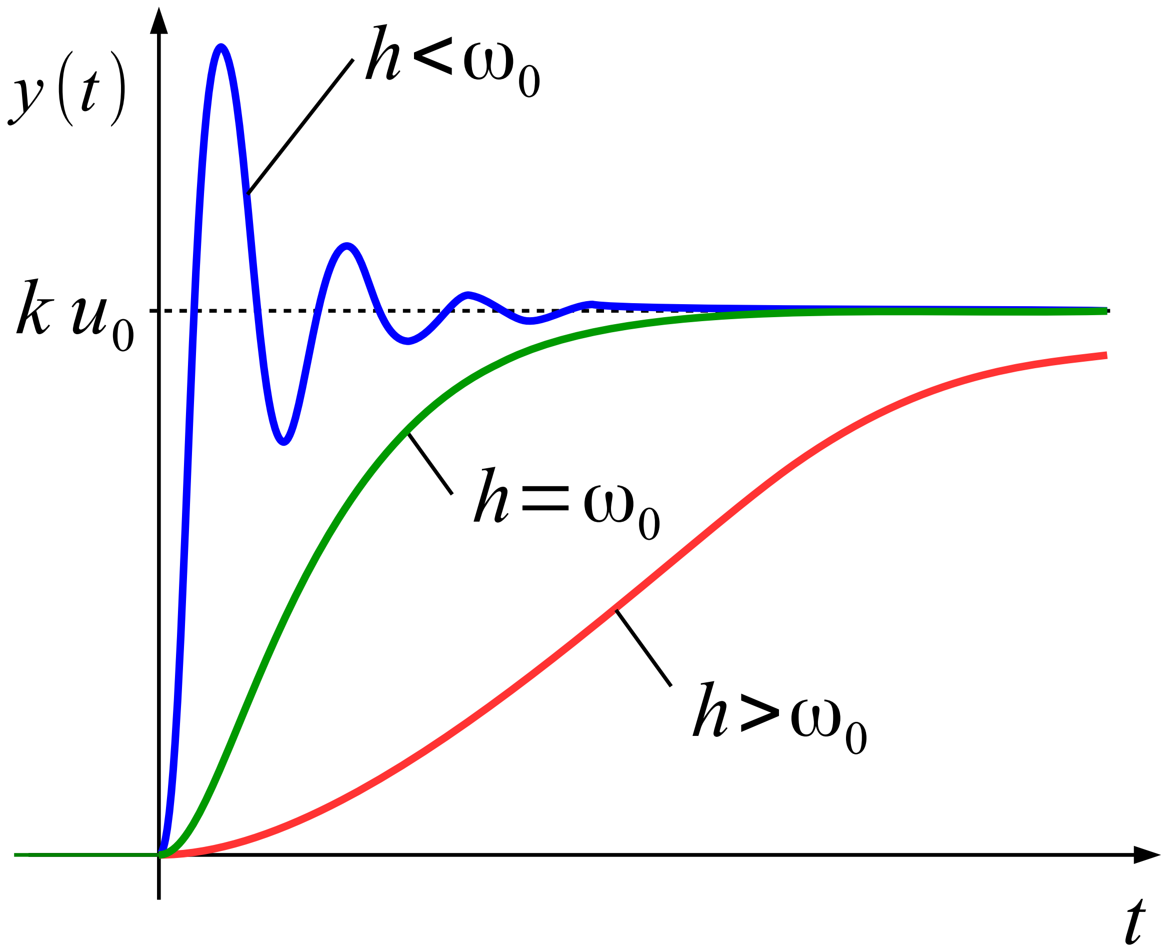
$$=\begin{cases} \frac{k u_0}{T_1^2}\left(1-e^{-ht}\left(\cos\omega t+\frac{h}{\omega}\sin\omega t\right)\right), \text{ for } h\leq\omega_0 \\ \frac{k u_0}{T_1^2}\left(1+e^{-ht}\left(\left(\frac{h+w}{2w}-1\right)e^{-wt}-\frac{h+w}{2w}e^{wt}\right)\right), \text{ for } h\geq\omega_0 \end{cases}$$

where: $h=\frac{T_2}{2T_1^2},\ \omega_0=\frac{1}{T_1},\ \omega=\sqrt{\omega_0^2-h^2},\ w=\sqrt{h^2-\omega_0^2}$

Second-order inertial element

4. Step response:





Second-order inertial element

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$$P(\omega)=\frac{k\left(1-T_1^2\omega^2\right)}{\left(1-T_1^2\omega^2\right)^2+T_2^2\omega^2},\quad Q(\omega)=\frac{-k\,T_2\,\omega}{\left(1-T_1^2\omega^2\right)^2+T_2^2\omega^2}$$

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6. Nyquist plot:

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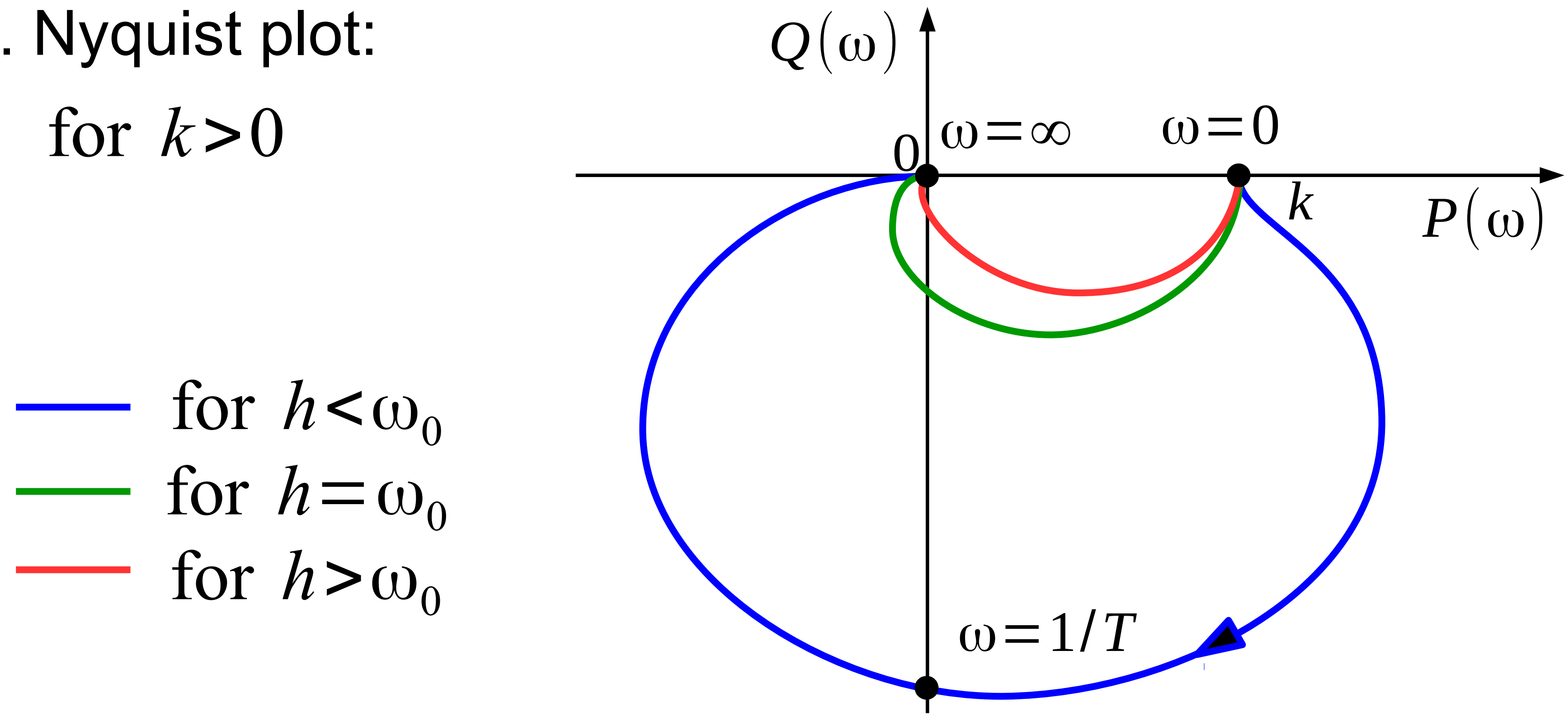
for $k>0$



Second-order inertial element

7. Bode plot:

Second-order inertial element

7. Bode plot: $A(\omega) = \sqrt{P^2 + Q^2}$

$$L(\omega) = 20 \log A(\omega)$$

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Second-order inertial element

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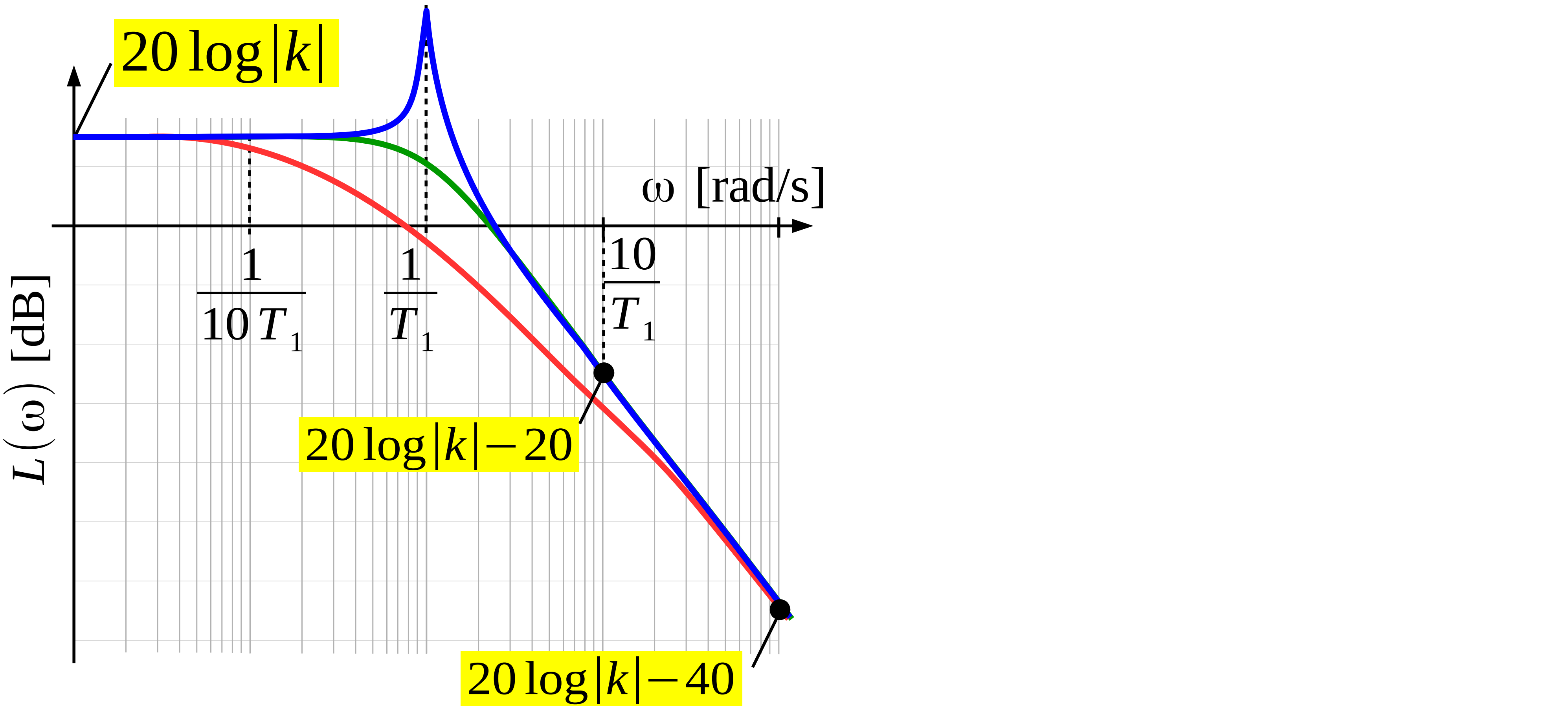

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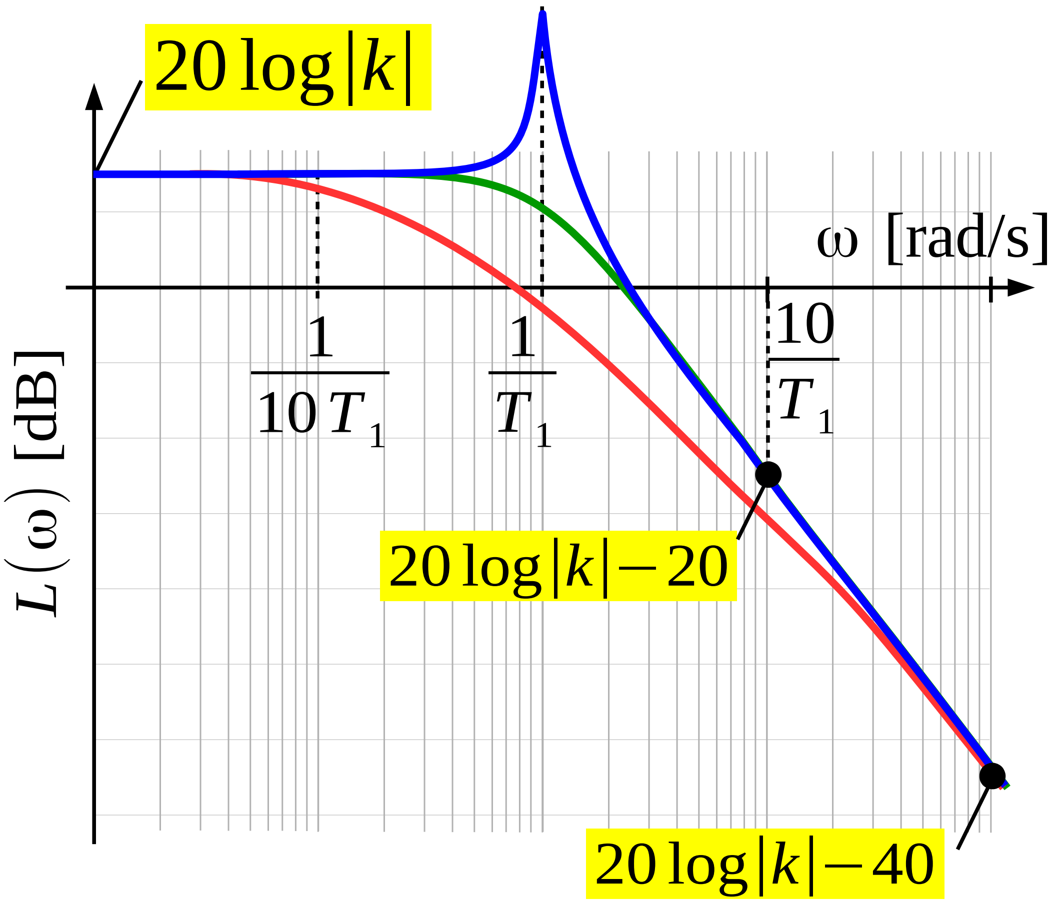
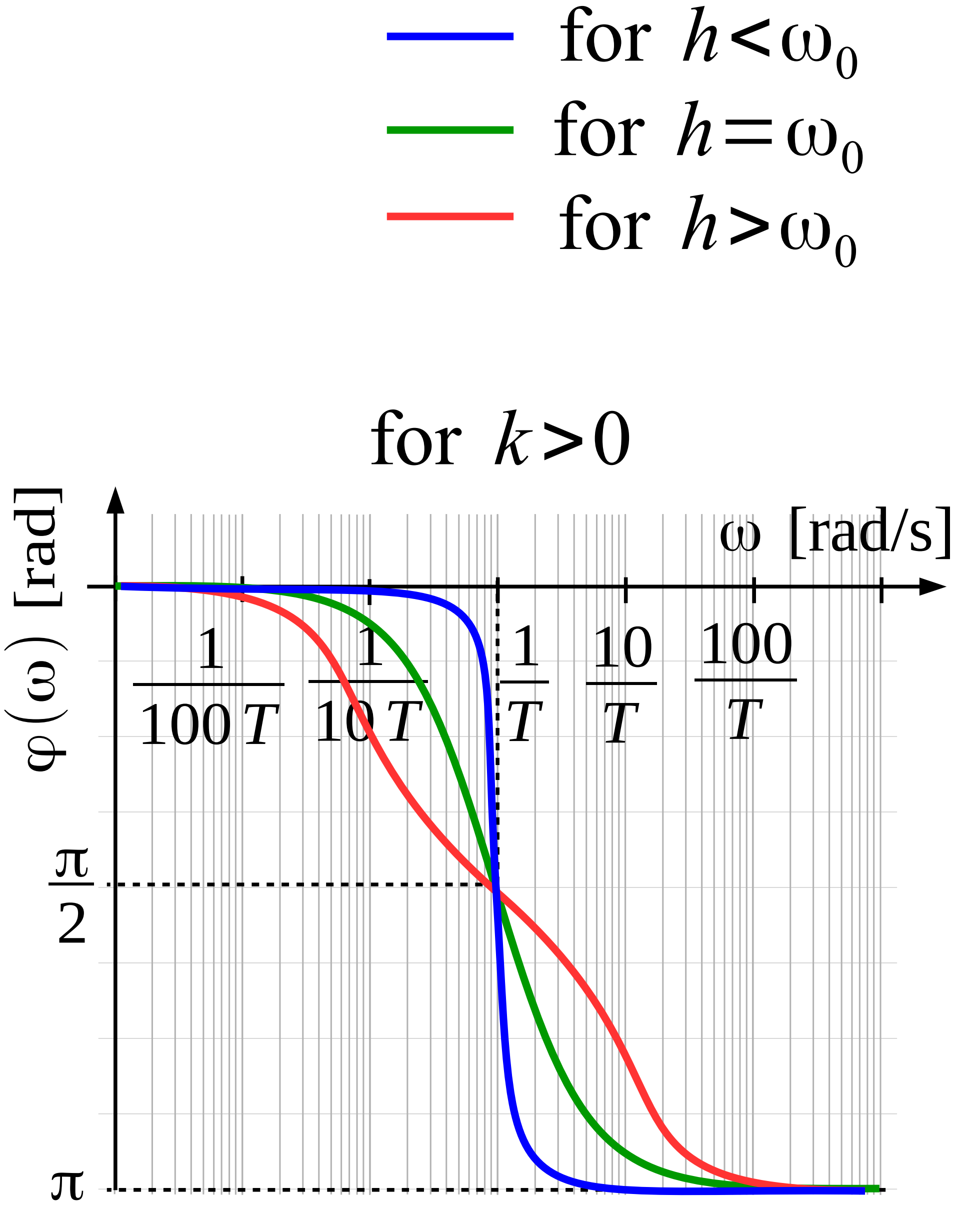


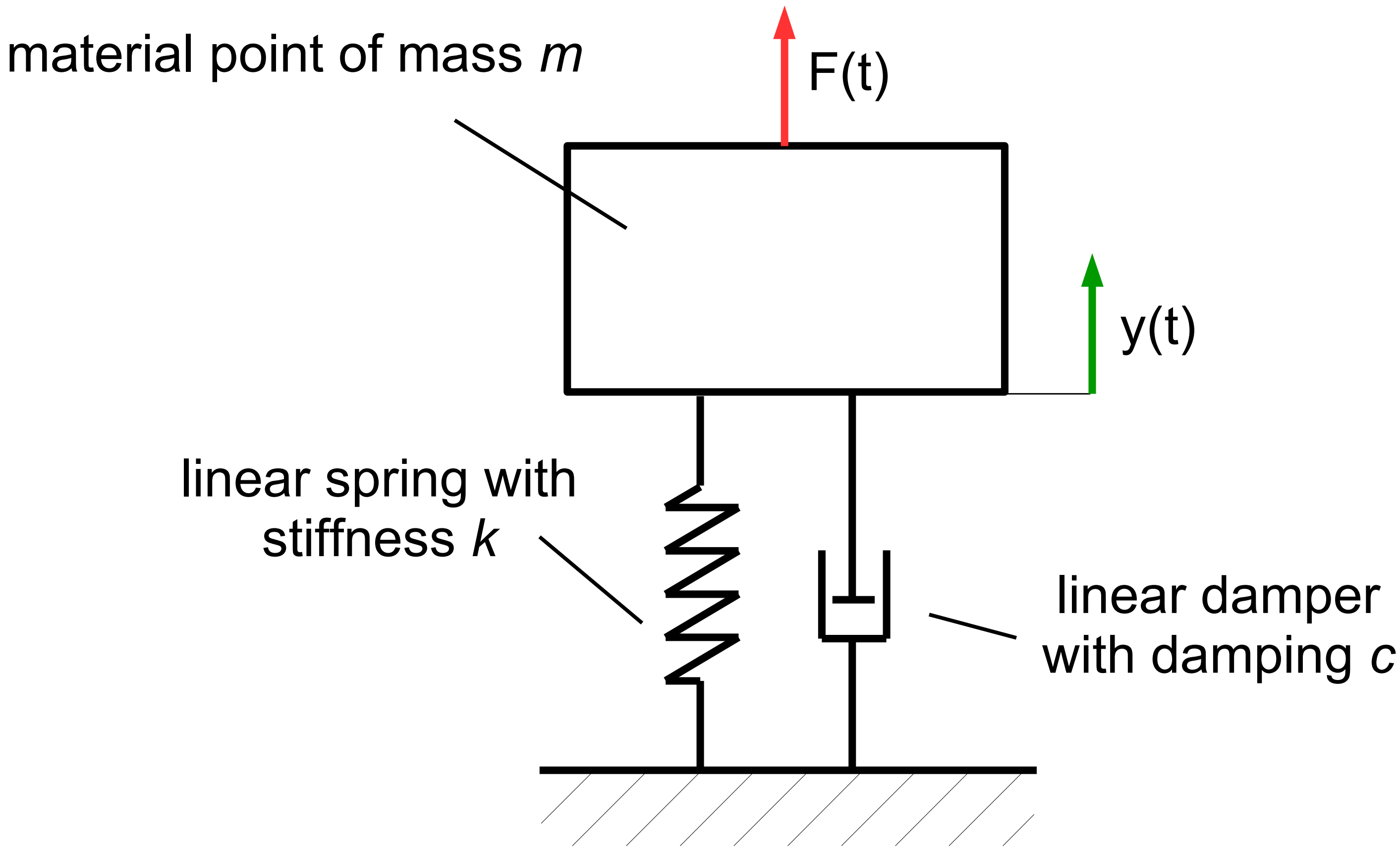


Second-order inertial element

Examples

1

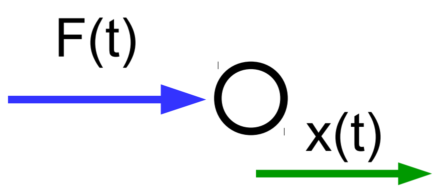
VIBRATING SYSTEM:
input – force F(t)
output – displacement y(t)



Second-order inertial element

Examples

2



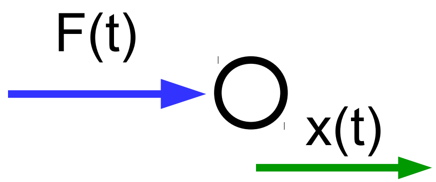
LINEAR MOTION OF A MATERIAL POINT WITH LINEAR DAMPING:
input – force F(t)
output – displacement x(t)

example: car driving on a flat surface with air resistance proportional to velocity, described using machine equation of motion, with assumption of constant reduced mass.

Second-order inertial element

Examples

2



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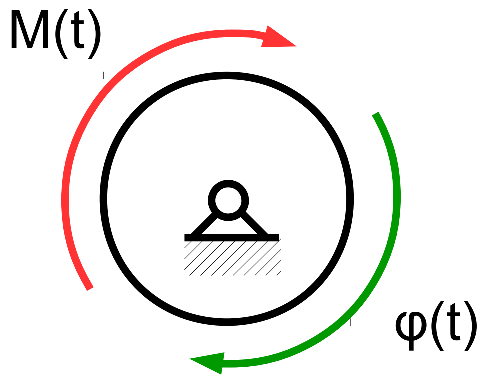
3

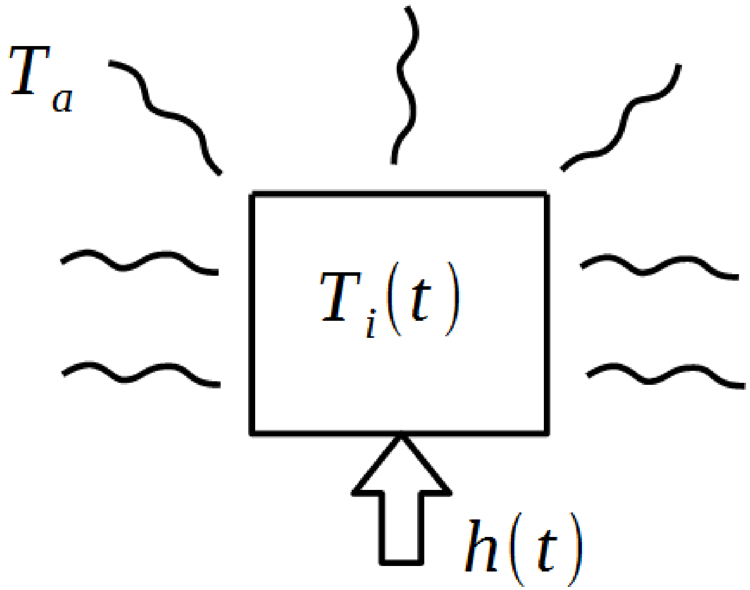


ANGULAR MOTION OF A RIGID BODY WITH LINEAR DAMPING:
input – torque M(t)
output – angle φ(t)

Second-order inertial element

Examples

4



HEATED OBJECT WITH HIGH INERTIA:
input – heater power h(t)
output – object temperature $T_i(t)$

Classification of basic automatic systems

Element name	Transfer function
proportional	k
first order (inertial)	$\frac{k}{Ts+1}$
integrator	$\frac{k}{s}$
differentiator	ks
differentiator with inertia	$\frac{ks}{Ts+1}$
delay	$e^{-\tau s}$
second order (oscillator)	$\frac{k}{T_1^2 s^2 + T_2 s + 1}$