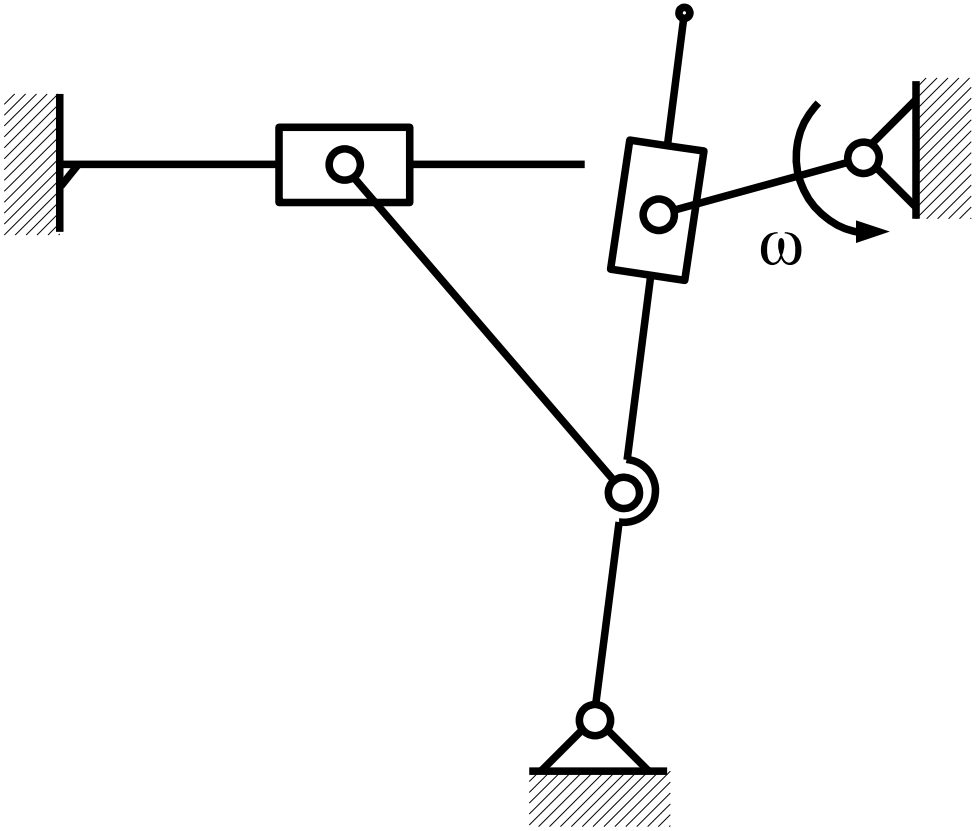
# PAiTM - zima 2014/2015

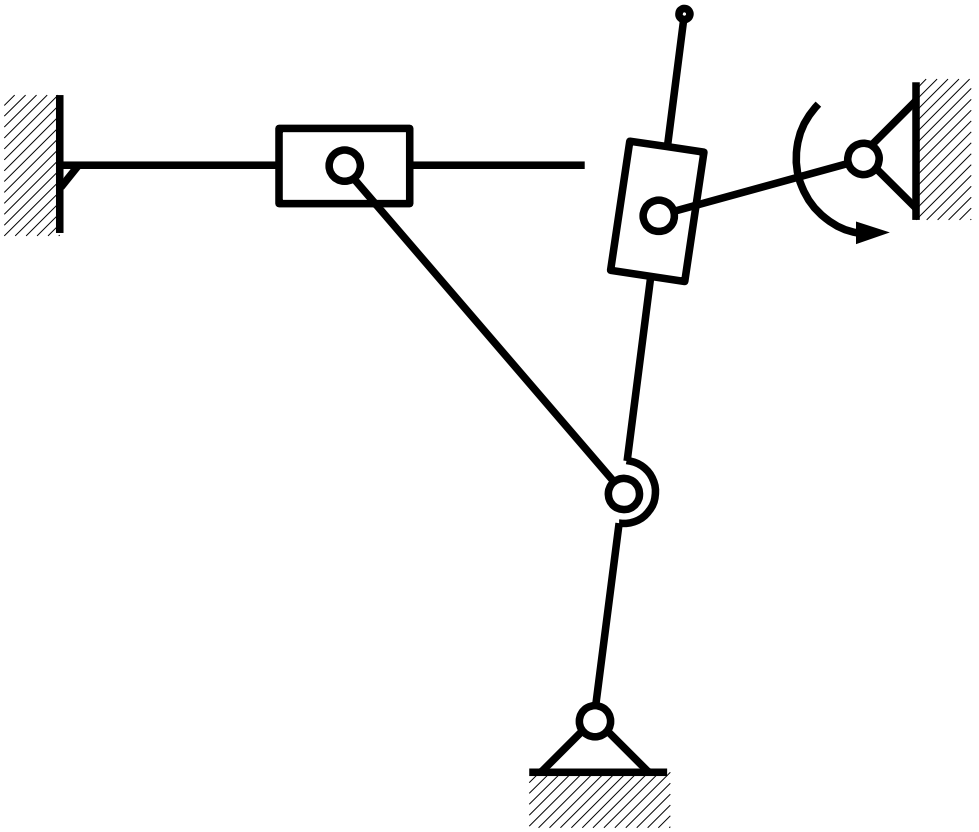
## Wyznaczanie przyspieszeń mechanizmu płaskiego metodą planu przyspieszeń (metoda wykreślna)

Dane: geometria mechanizmu (wymiary elementów, ich położenie i orientacja) oraz stała prędkość kątowa $\omega$ elementu napędowego




Wydział Samochodów i Maszyn Roboczych, Politechnika Warszawska,
Instytut Podstaw Budowy Maszyn, Zakład Mechaniki, mgr inż. Sebastian Korczak

# Jak pracuje ten mechanizm?



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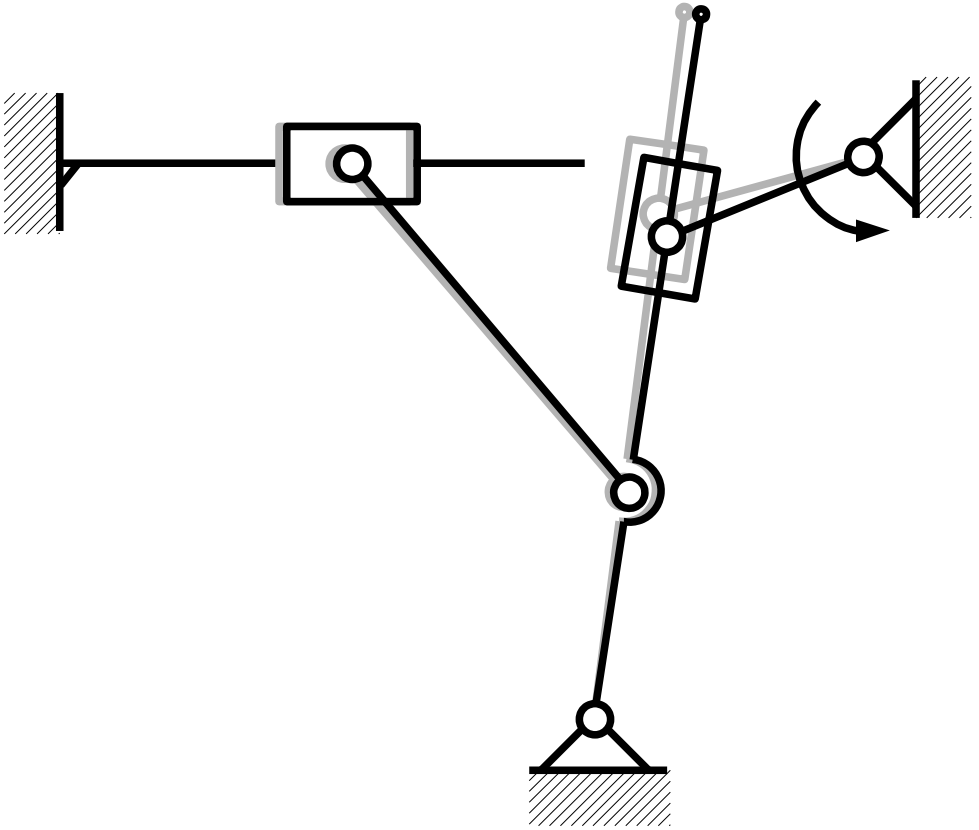
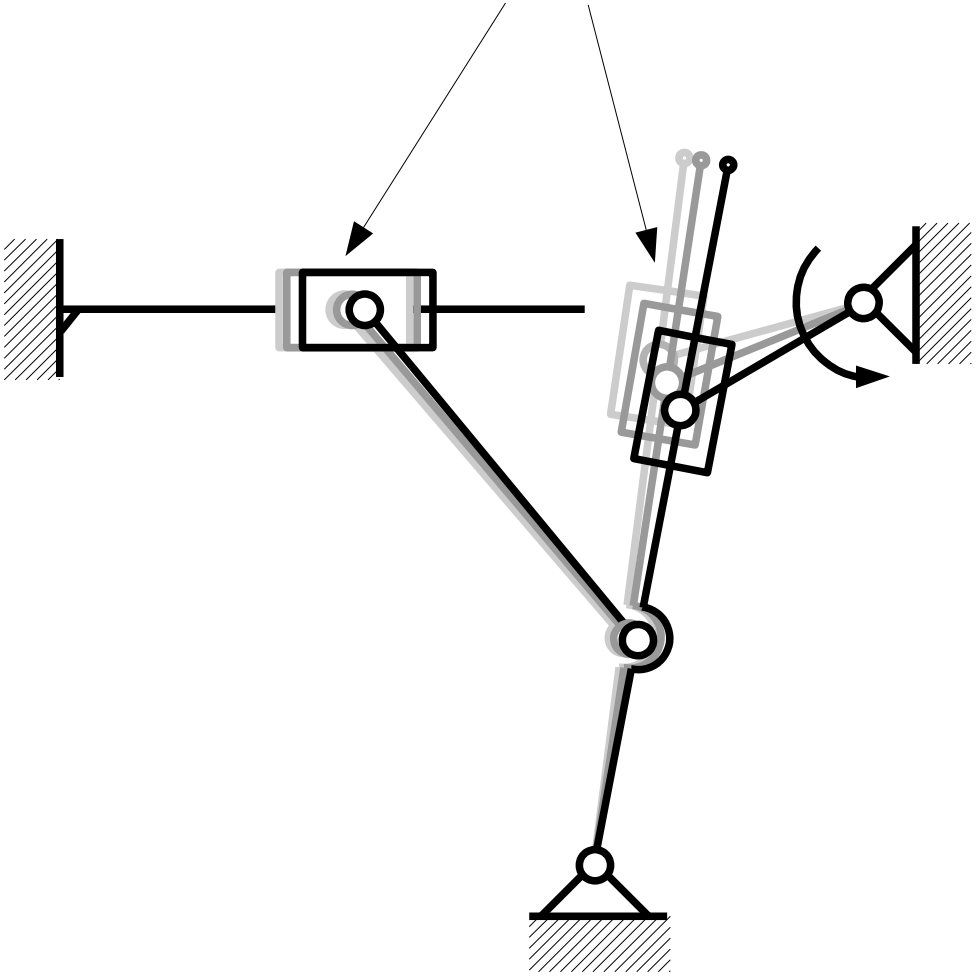
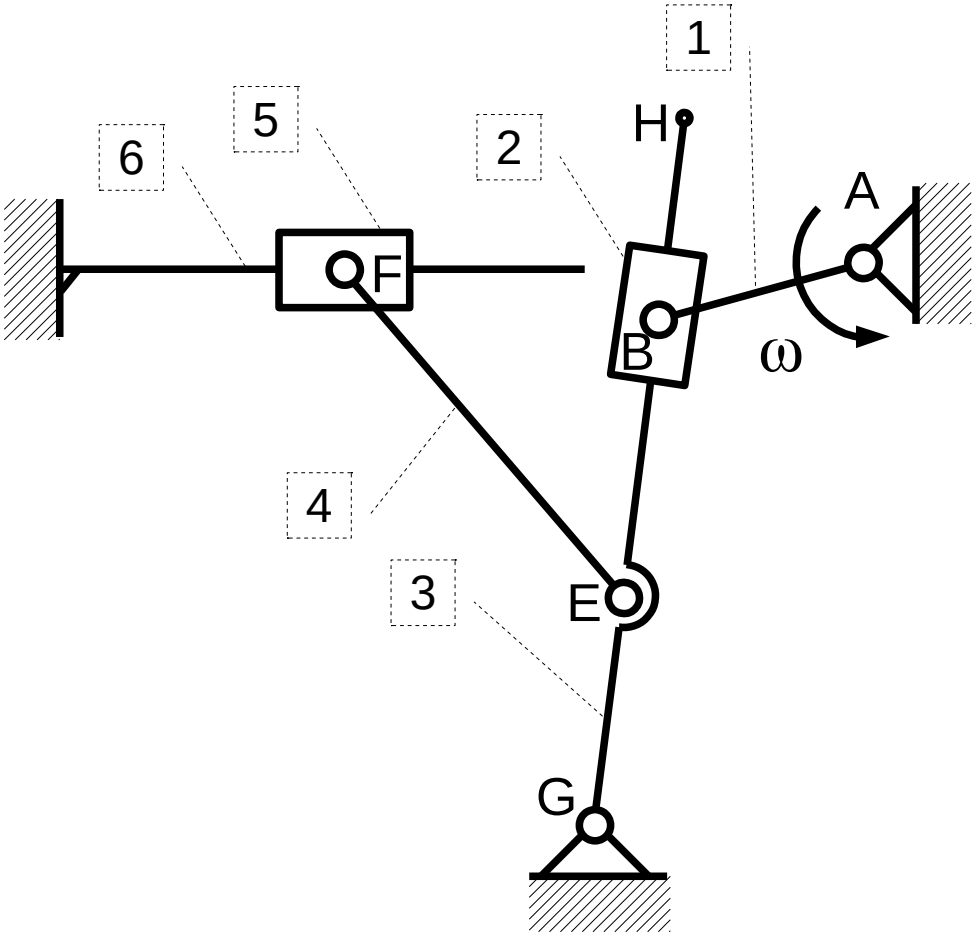


# Jak pracuje ten mechanizm?

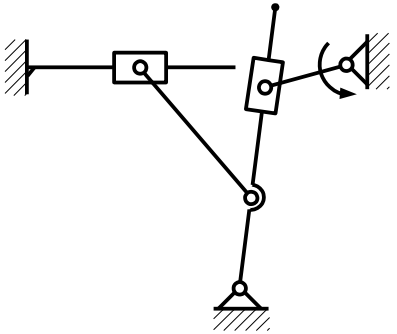
zauważamy ruch względny



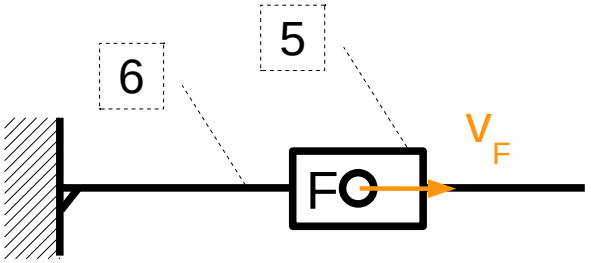
# Wracamy do rozważanego położenia mechanizmu, numerujemy elementy i nazywamy punkty

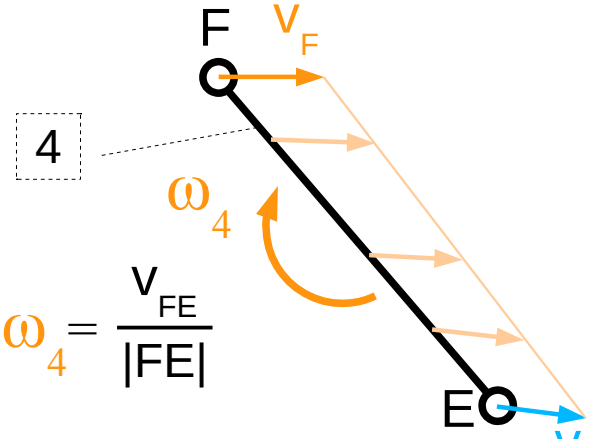


# Przypominamy rozkład prędkości poszczególnych elementów mechanizmu



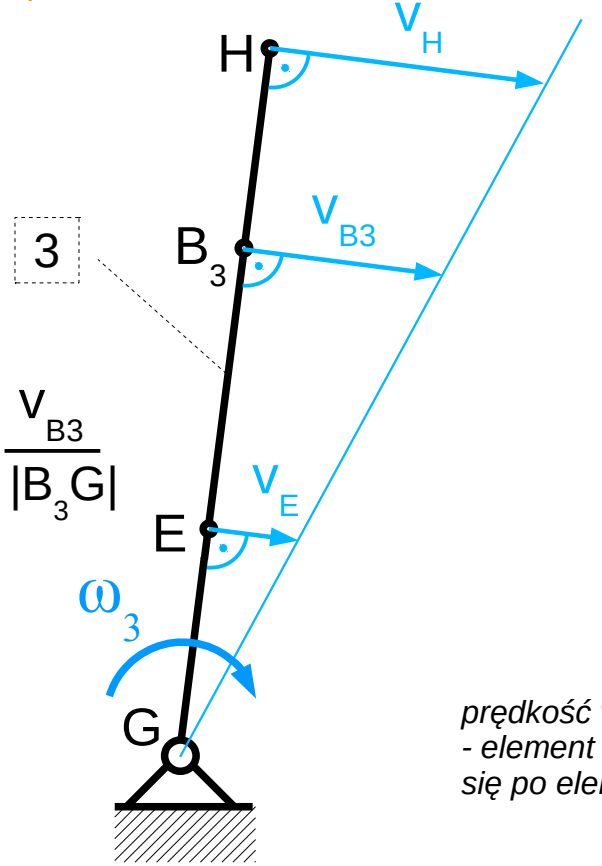
$$\omega > \omega_3 > \omega_4$$



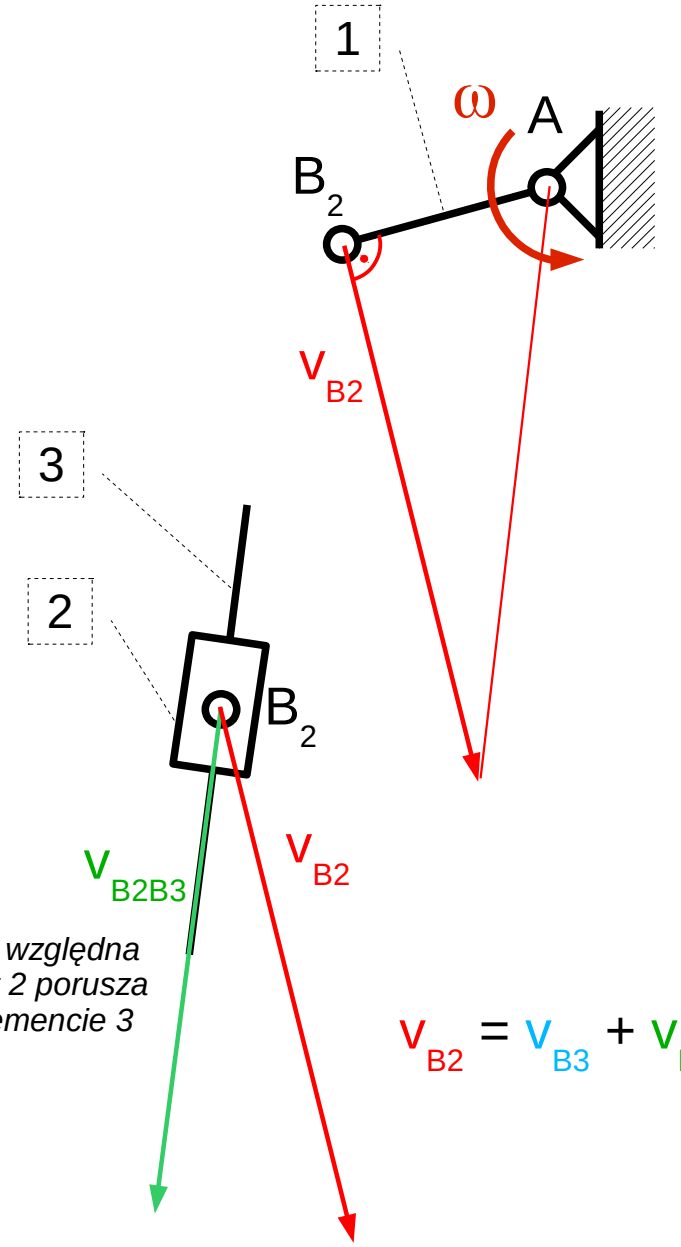

$$\omega_4 = \frac{v_{FE}}{|FE|}$$

$$v_F = v_E + v_{FE}$$




$$\omega_3 = \frac{v_{B3}}{|B_3G|}$$

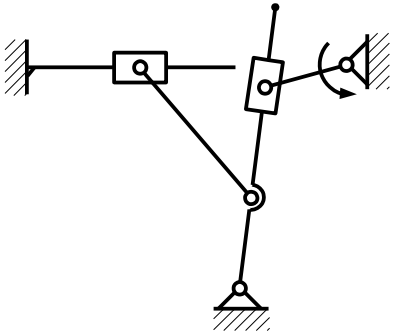
$$v_E = v_{B3}\frac{|EG|}{|B_3G|}$$

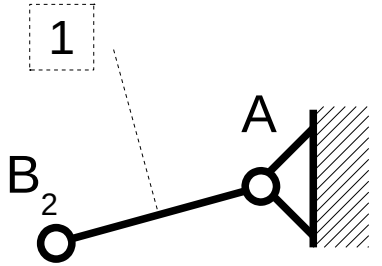



*prędkość względna - element 2 porusza się po elemencie 3*

$$v_{B2} = v_{B3} + v_{B2B3}$$

# Rozpocznijmy analizę przyspieszeń od członu napędowego





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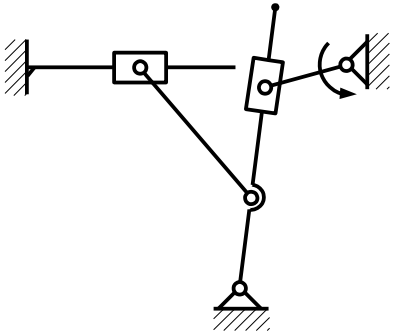
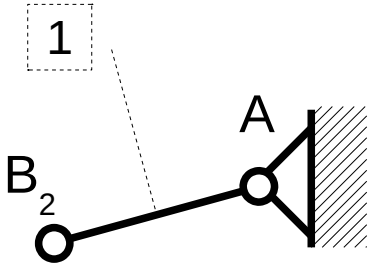


$$p_{B2} = p_A + p^n_{B2A} + p^t_{B2A}$$



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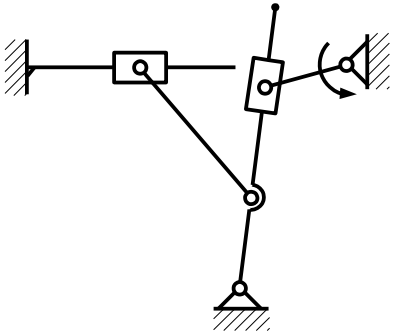
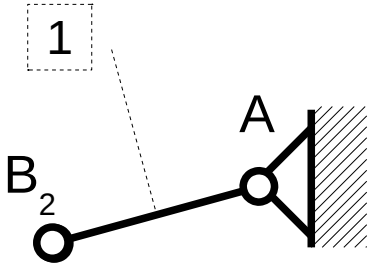


$$p_{B2} = p_A + p^n_{B2A} + p^t_{B2A}$$

$$=0 \quad \| 1$$

$$|p^n_{B2A}| = \omega^2 |B_2A| \qquad |p^t_{B2A}| = \varepsilon |B_2A| = 0$$

$$\varepsilon = \frac{d\omega}{dt} = 0$$

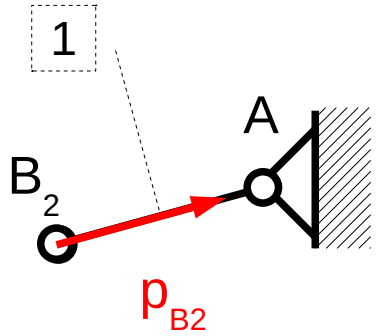


# Rozpocznijmy analizę przyspieszeń od członu napędowego

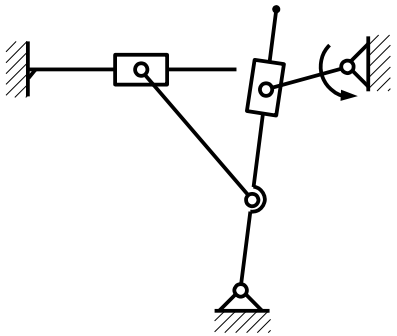
$$p_{B2} = \underline{\underline{p^n_{B2A}}}$$

$$\| 1$$

$$|p^n_{B2A}| = \omega^2 \, |B_2 A|$$

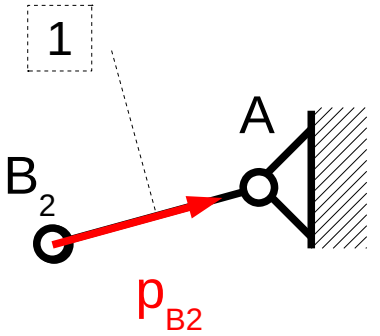


# Rozpatrzmy ruch obrotowy elementu trzeciego

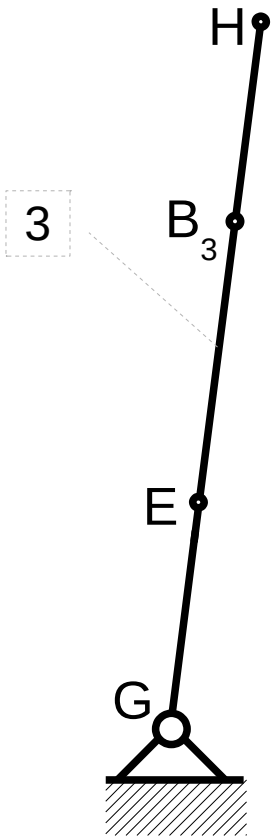


$$p_{B2} = \underline{\underline{p^n_{B2A}}} \;\; \| 1$$

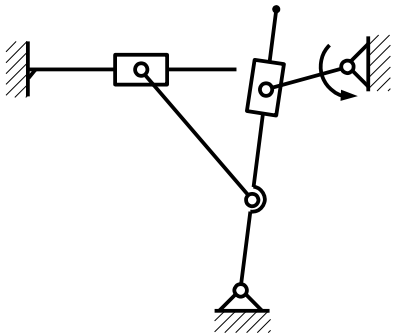
$$|p^n_{B2A}| = \omega^2 \, |B_2 A|$$




$$p_{B3} = p_G + p^n_{B3G} + p^t_{B3G}$$

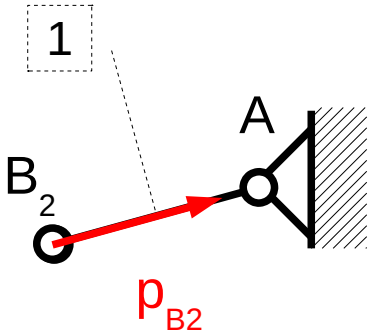


# Rozpatrzmy ruch obrotowy elementu trzeciego



$$p_{B2} = \underline{\underline{p^n_{B2A}}} \quad \underline{\underline{\parallel 1}}$$

$$|p^n_{B2A}| = \omega^2 |B_2 A|$$

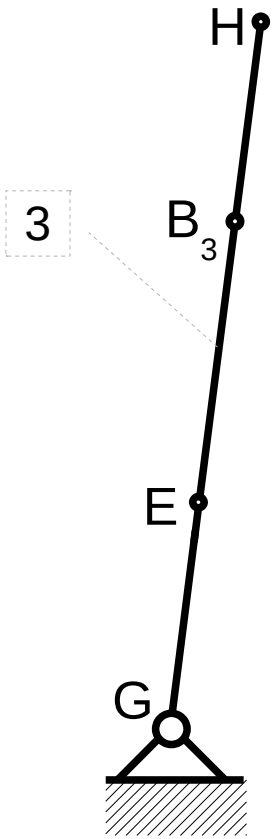



$$p_{B3} = \underline{\underline{p_G}} + \underline{\underline{p^n_{B3G}}} + \underline{p^t_{B3G}}$$

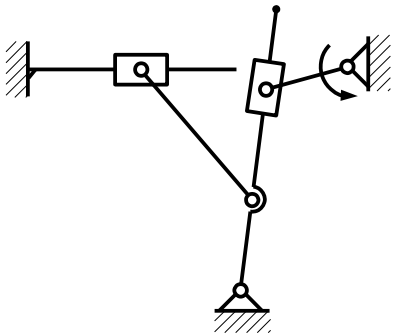
$$\underline{\underline{=0}} \quad \underline{\underline{\parallel 3}} \quad \underline{\perp 3}$$

$$|p^n_{B3G}| = \omega_3^{\,2} |B_3 G|$$

*z planu prędkości*



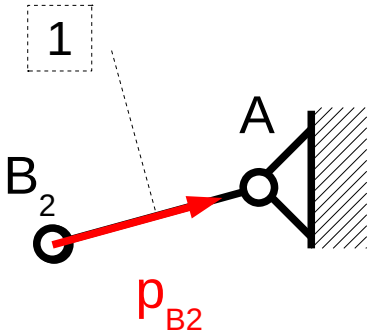
# Rozpatrzmy ruch obrotowy elementu trzeciego



$$p_{B2} = \underline{\underline{p^n_{B2A}}}$$

$$\underline{\underline{\|1}}$$

$$|p^n_{B2A}| = \omega^2 |B_2A|$$

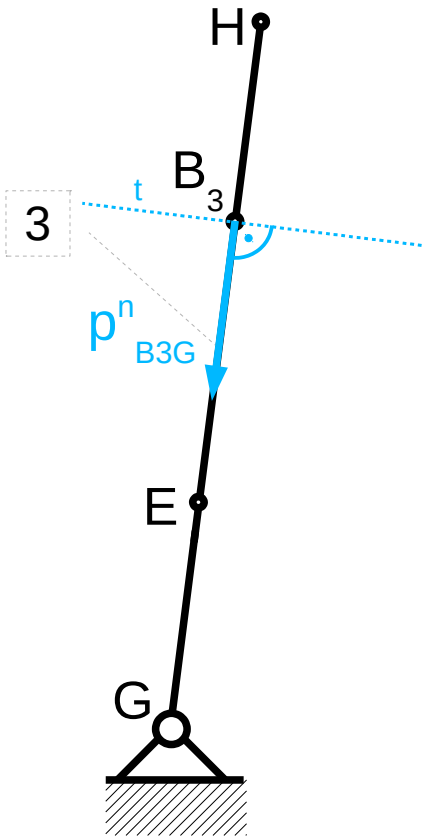



$$p_{B3} = \underline{\underline{p_G}} + \underline{\underline{p^n_{B3G}}} + \underline{p^t_{B3G}}$$

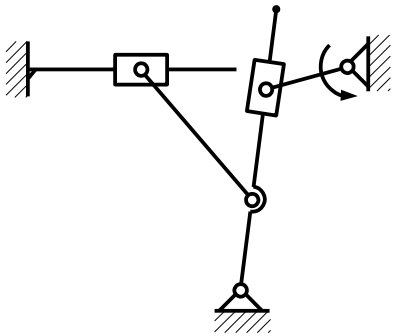
$$\underline{\underline{=0}} \quad \underline{\underline{\|3}} \quad \underline{\perp 3}$$

$$|p^n_{B3G}| = \omega_3^{\,2} |B_3G|$$

*z planu prędkości*



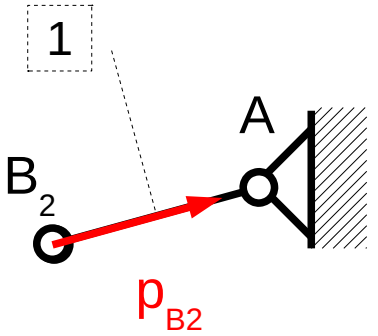
# Rozpatrzmy ruch obrotowy elementu trzeciego



$$p_{B2} = \underline{\underline{p^n_{B2A}}}$$

$$\| 1$$

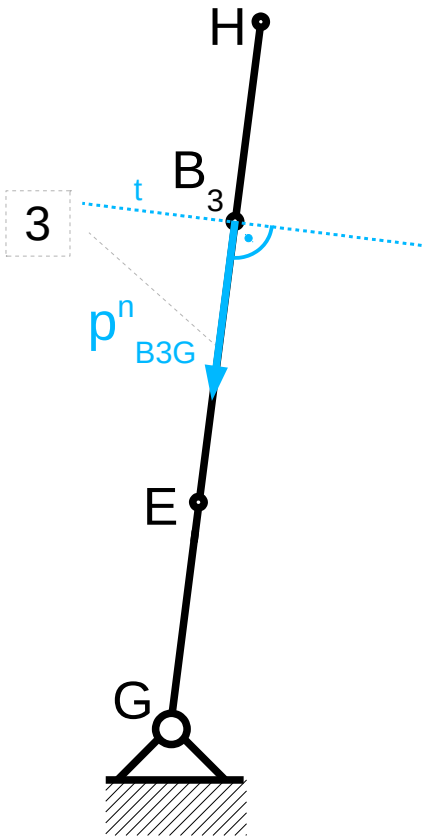
$$|p^n_{B2A}| = \omega^2 |B_2 A|$$



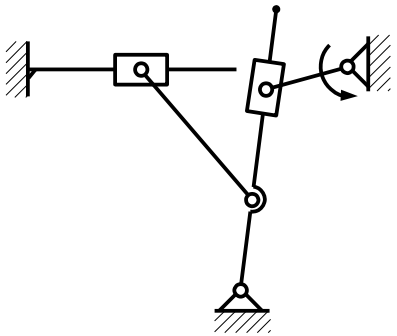

$$p_{B3} = \underline{\underline{p^n_{B3G}}} + \underline{p^t_{B3G}}$$

$$\| 3 \quad \perp 3$$

$$|p^n_{B3G}| = \omega_3^{\,2} |B_3 G|$$



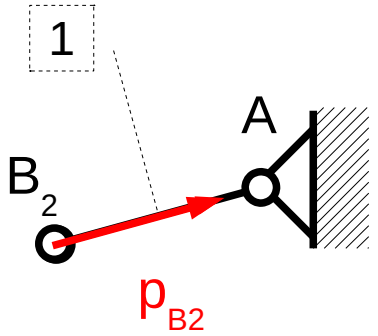
# Rozpatrzmy ruch względny elementów 2 i 3



$$p_{B2} = \underline{\underline{p^n_{B2A}}}$$

$$\underline{\underline{\|1}}$$

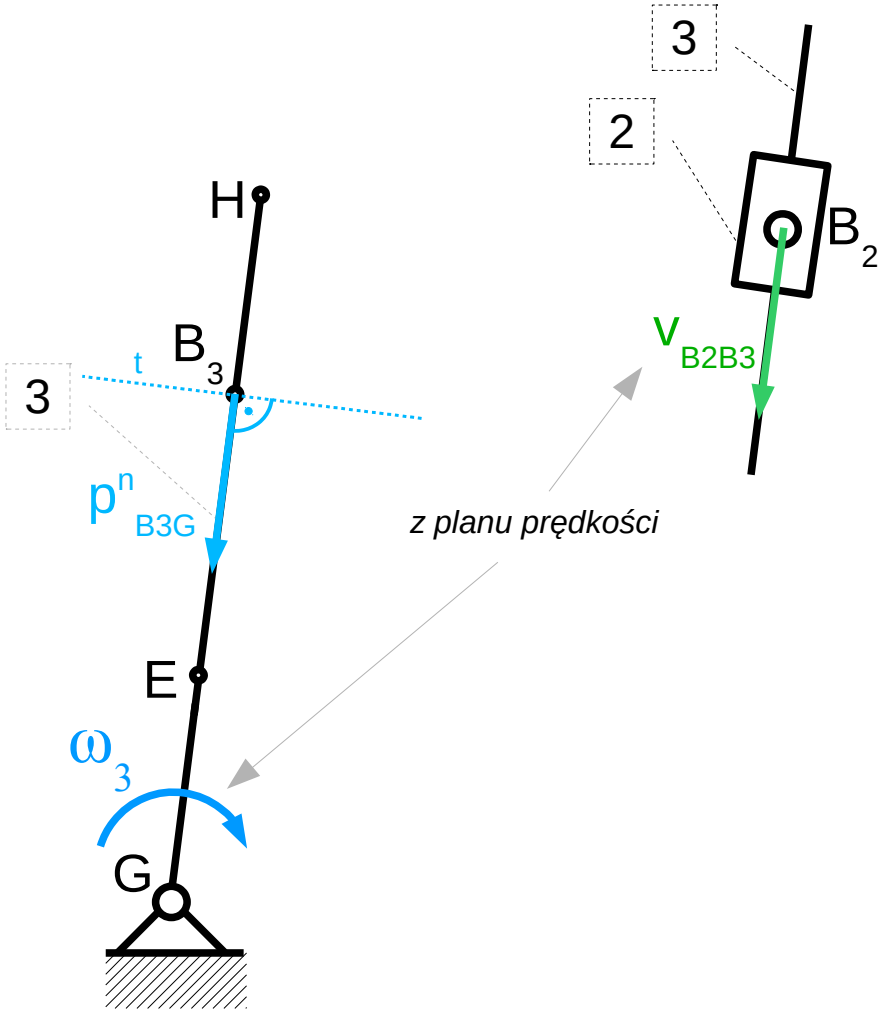
$$|p^n_{B2A}| = \omega^2 |B_2A|$$




$$p_{B3} = \underline{\underline{p^n_{B3G}}} + \underline{p^t_{B3G}}$$

$$\underline{\underline{\|3}} \quad \underline{\perp 3}$$

$$|p^n_{B3G}| = \omega_3^{\,2} |B_3G|$$



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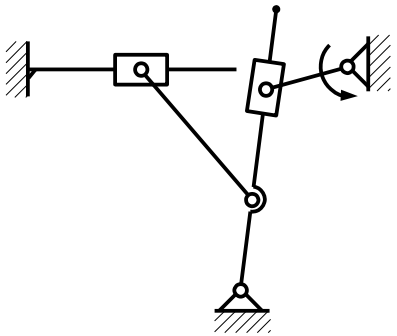
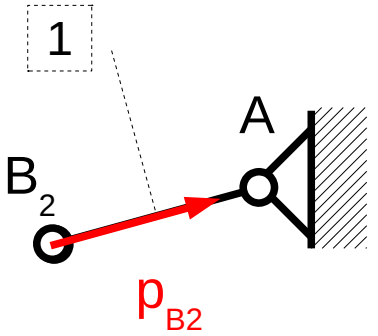


$$p_{B2} = \underline{\underline{p^n_{B2A}}} \quad \| 1$$

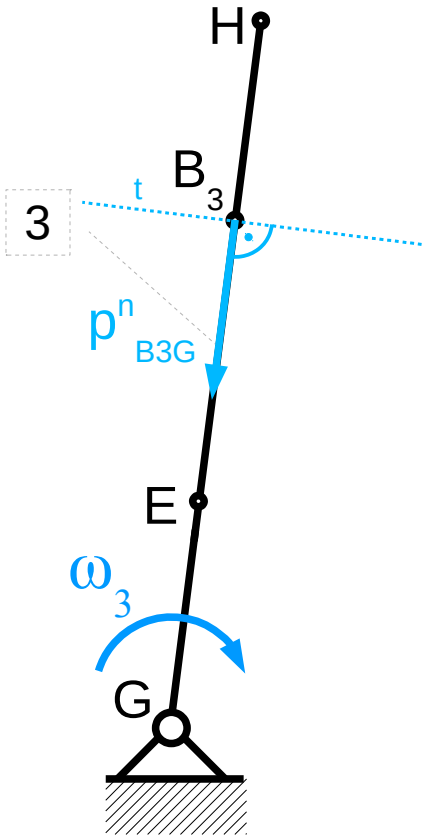
$$|p^n_{B2A}| = \omega^2 |B_2 A|$$

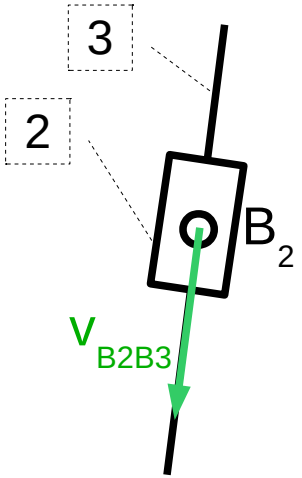



$$p_{B3} = \underline{\underline{p^n_{B3G}}} + \underline{p^t_{B3G}} \quad \| 3 \quad \perp 3$$

$$|p^n_{B3G}| = \omega_3^{\,2} |B_3 G|$$

*RUCH UNOSZENIA: ruch elementu 3*
*RUCH WZGLĘDNY: ruch elementu 2 wzdłuż elementu 3*



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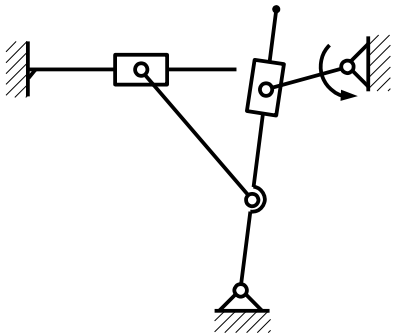
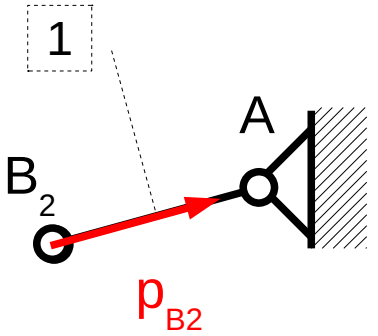


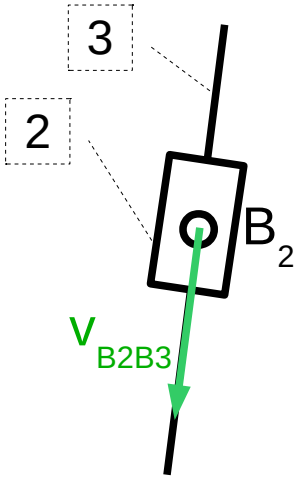
$$p_{B2} = \underline{\underline{p^n_{B2A}}} \quad \| 1$$

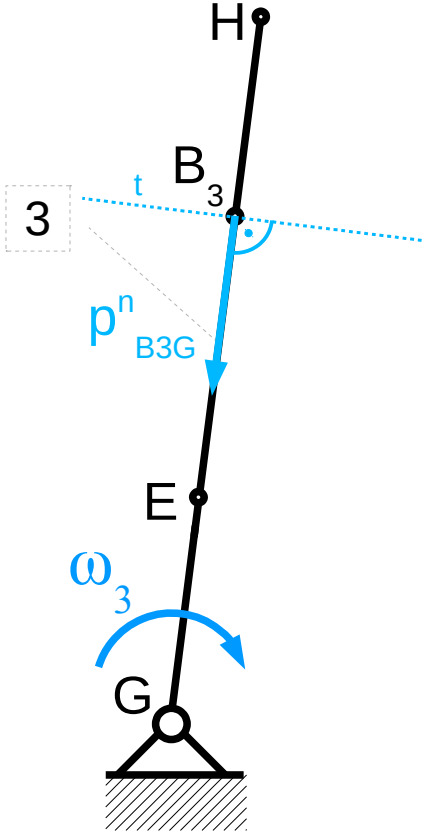
$$|p^n_{B2A}| = \omega^2 |B_2 A|$$




$$p_{B3} = \underline{\underline{p^n_{B3G}}} + \underline{p^t_{B3G}} \quad \| 3 \quad \perp 3$$

$$|p^n_{B3G}| = \omega_3^{\,2} |B_3 G|$$



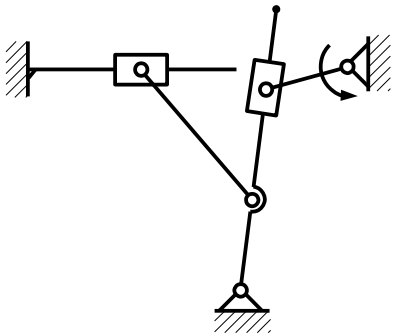




*RUCH UNOSZENIA: ruch elementu 3*
*RUCH WZGLĘDNY: ruch elementu 2 wzdłuż elementu 3*

*RÓWNANIE RUCHU WZGLĘDNEGO:*

$$p_{B2} = p^u_{B3} + p^w + p^c$$

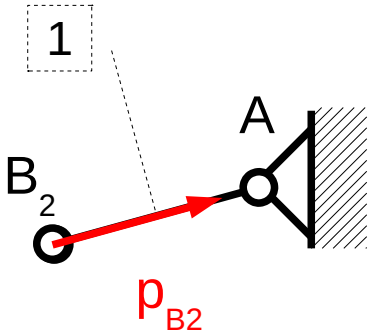
# Rozpatrzmy ruch względny elementów 2 i 3



$$p_{B2} = \underline{\underline{p^n_{B2A}}}$$

$$\| 1$$

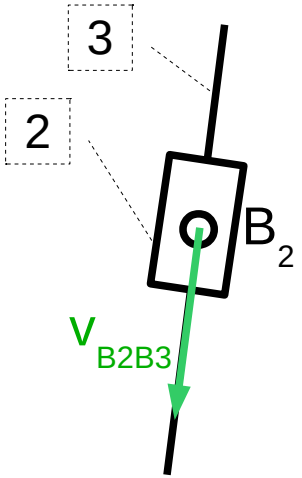
$$|p^n_{B2A}| = \omega^2 |B_2 A|$$

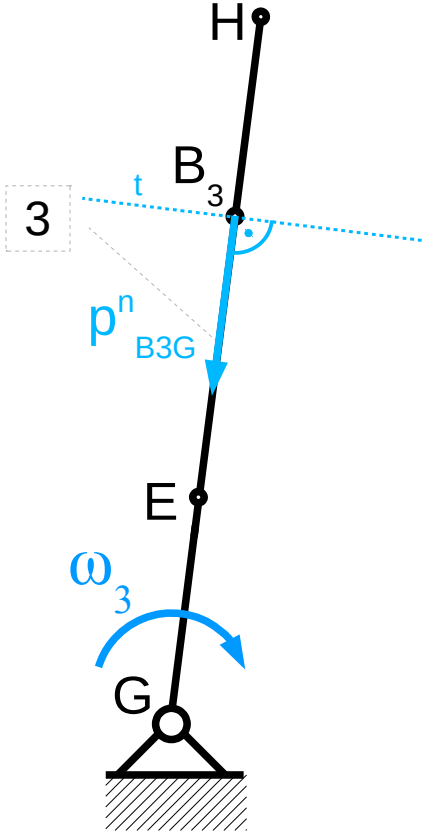



$$p_{B3} = \underline{\underline{p^n_{B3G}}} + \underline{p^t_{B3G}}$$

$$\| 3 \quad \perp 3$$

$$|p^n_{B3G}| = \omega_3^{\,2} |B_3 G|$$

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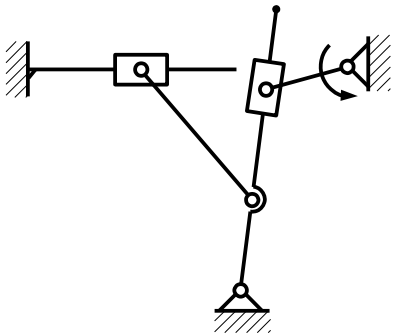
*RUCH WZGLĘDNY: ruch elementu 2 wzdłuż elementu 3*

*RÓWNANIE RUCHU WZGLĘDNEGO:*

$$p_{B2} = p^u_{B3} + p^w + p^c$$

$$p^n_{B2A} = p^n_{B3G} + p^t_{B3G} + p^w_{B2B3} + p^c$$

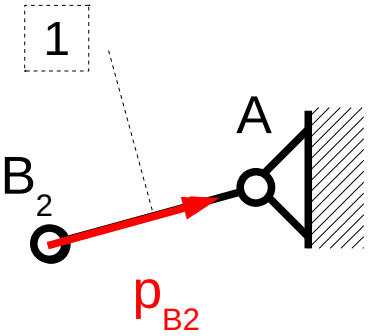
# Rozpatrzmy ruch względny elementów 2 i 3



$$p_{B2} = \underline{\underline{p^n_{B2A}}}$$

$$||1$$

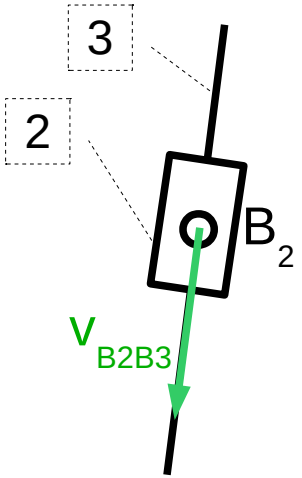
$$|p^n_{B2A}| = \omega^2 |B_2A|$$

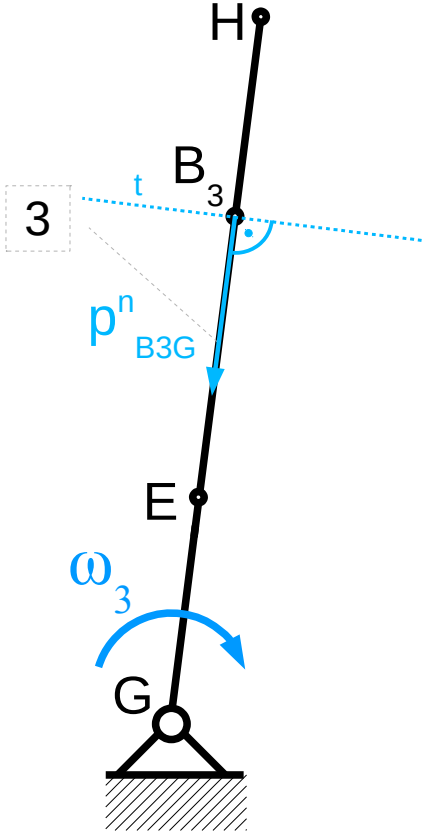



$$p_{B3} = \underline{\underline{p^n_{B3G}}} + \underline{p^t_{B3G}}$$

$$||3 \quad \perp 3$$

$$|p^n_{B3G}| = \omega_3^{\,2} |B_3G|$$




*RUCH UNOSZENIA: ruch elementu 3*
*RUCH WZGLĘDNY: ruch elementu 2 wzdłuż elementu 3*

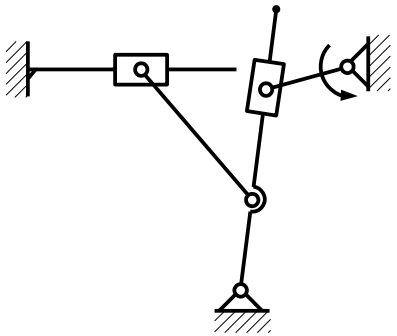
*RÓWNANIE RUCHU WZGLĘDNEGO:*

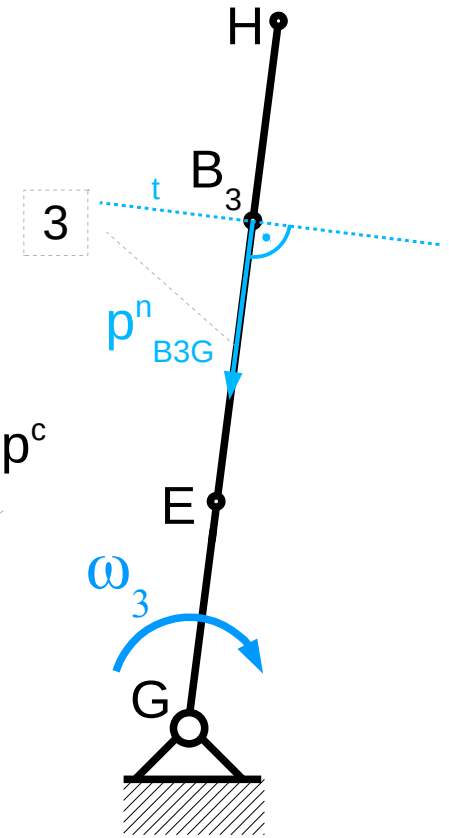
$$p_{B2} = p^u_{B3} + p^w + p^c$$

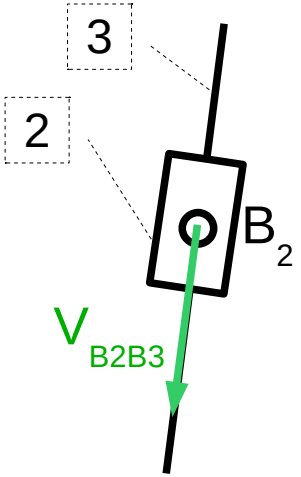
$$\underline{\underline{p^n_{B2A}}} = \underline{\underline{p^n_{B3G}}} + \underline{p^t_{B3G}} + \underline{p^w_{B2B3}} + p^c$$

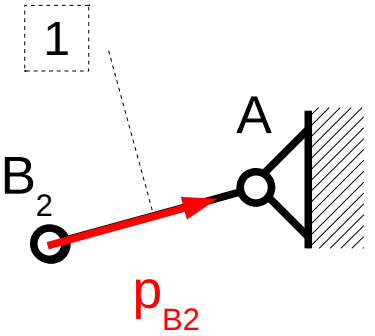
$$||1 \quad ||3 \quad \perp 3 \quad ||3$$

# Przyspieszenie coriolisa





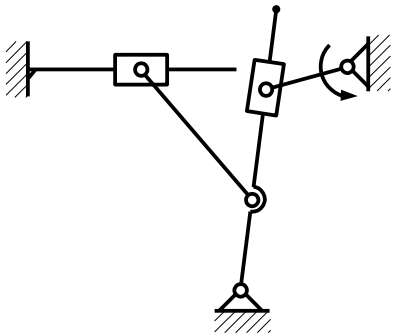

$$p_{B2} = p^{u}_{B3} + p^{w} + p^{c}$$

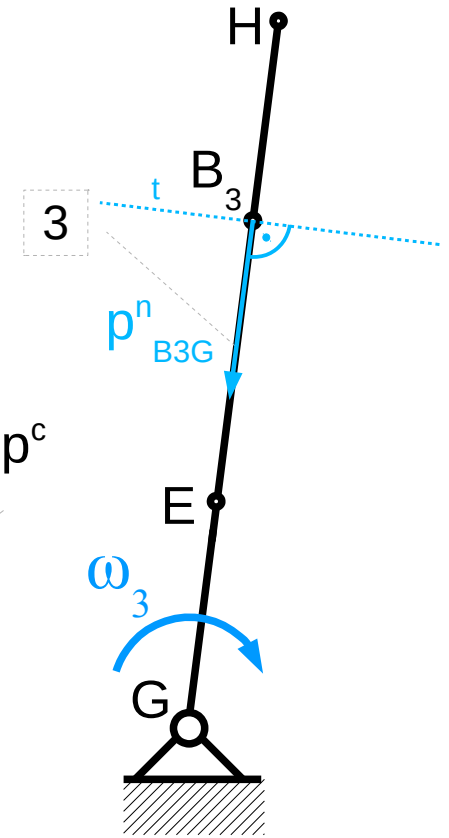
$$\underline{\underline{p^{n}_{B2A}}} = \underline{\underline{p^{n}_{B3G}}} + \underline{p^{t}_{B3G}} + \underline{p^{w}_{B2B3}} + p^{c}$$

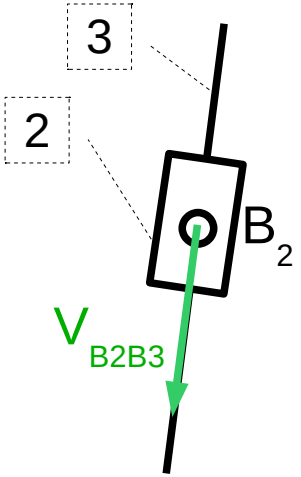
$$\underline{\underline{||1}} \qquad \underline{\underline{||3}} \qquad \underline{\perp 3} \qquad \underline{||3}$$

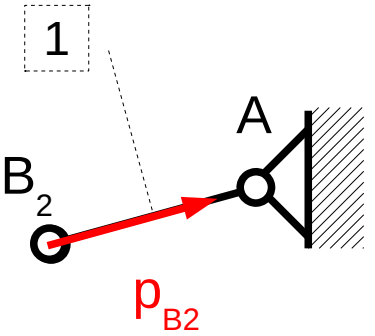
$$p^{c} = 2\omega_{3} \times V_{B2B3}$$

# Przyspieszenie coriolisa






$$p_{B2} = p^u_{B3} + p^w + p^c$$

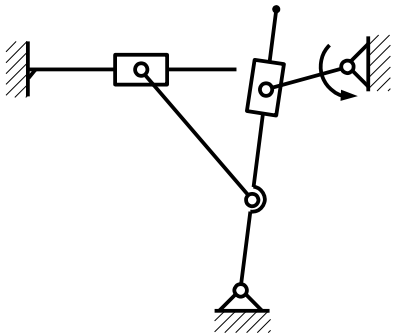
$$\underline{\underline{p^n_{B2A}}} = \underline{\underline{p^n_{B3G}}} + \underline{p^t_{B3G}} + \underline{p^w_{B2B3}} + p^c$$

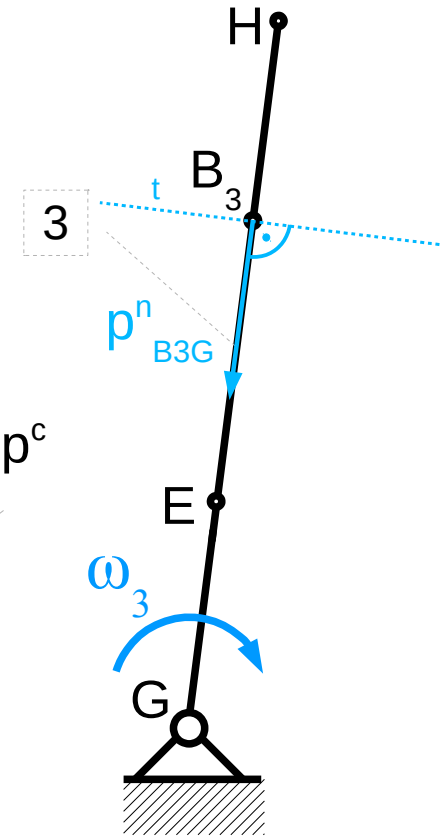
$$\underline{\underline{||1}} \qquad \underline{\underline{||3}} \qquad \underline{\perp 3} \qquad \underline{||3}$$

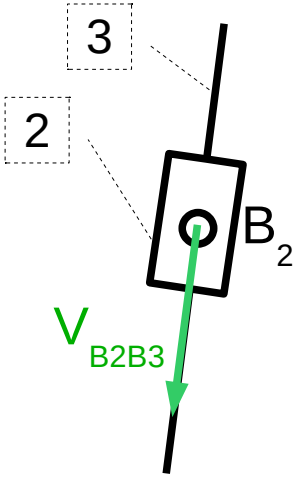
$$p^c = 2\omega_3 \times V_{B2B3}$$

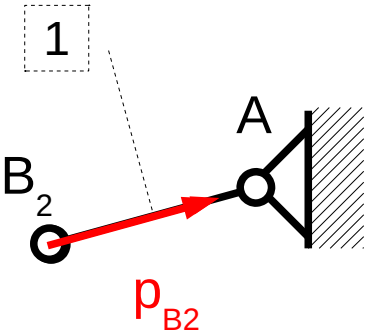
$$|p^c| = 2\omega_3 \, |V_{B2B3}| \sin(\sphericalangle(\omega_3, V_{B2B3}))$$

# Przyspieszenie coriolisa





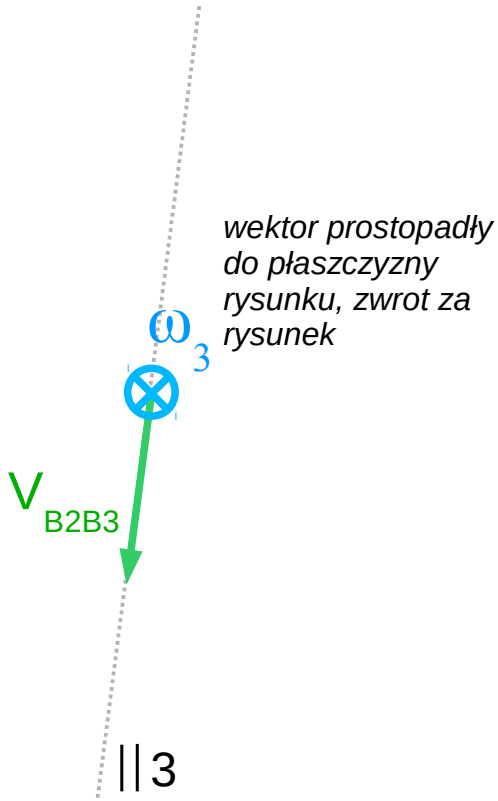

$$p_{B2} = p^{u}_{B3} + p^{w} + p^{c}$$

$$\underline{\underline{p^{n}_{B2A}}} = \underline{\underline{p^{n}_{B3G}}} + \underline{p^{t}_{B3G}} + \underline{p^{w}_{B2B3}} + p^{c}$$

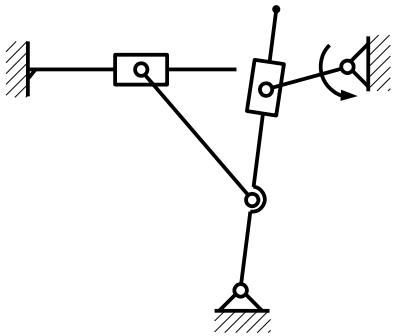
$$\underline{\underline{||1}} \qquad \underline{\underline{||3}} \qquad \underline{\perp 3} \qquad \underline{||3}$$

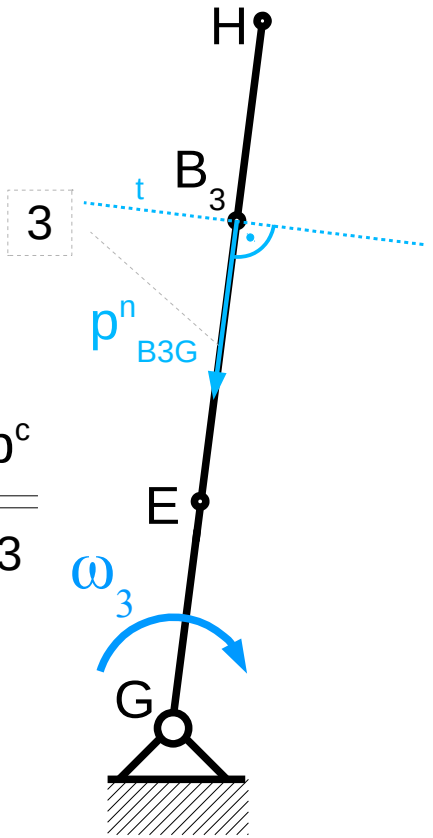
$$p^{c} = 2\omega_3 \times V_{B2B3}$$

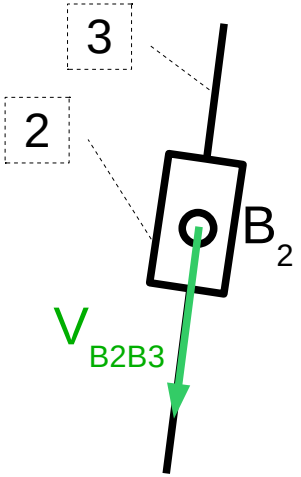
$$|p^{c}| = 2\omega_3 \, |V_{B2B3}| \sin(\sphericalangle(\omega_3, V_{B2B3}))$$

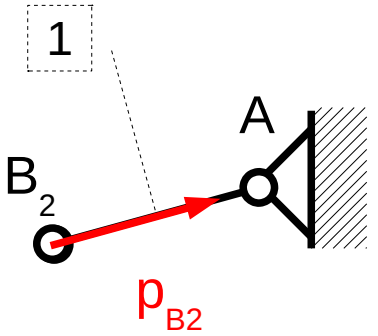


# Przyspieszenie coriolisa





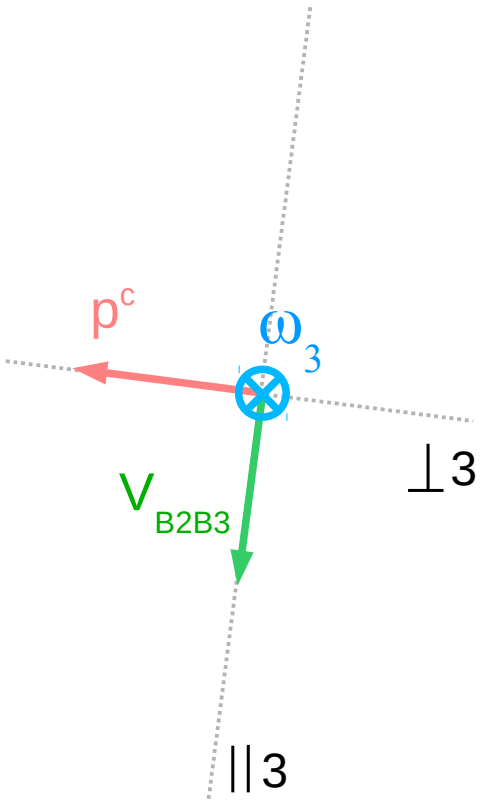

$$p_{B2} = p^{u}_{B3} + p^{w} + p^{c}$$

$$\underset{\|1}{\underline{\underline{p^{n}_{B2A}}}} = \underset{\|3}{\underline{\underline{p^{n}_{B3G}}}} + \underset{\perp 3}{\underline{p^{t}_{B3G}}} + \underset{\|3}{\underline{p^{w}_{B2B3}}} + \underset{\perp 3}{\underline{\underline{p^{c}}}}$$

$$p^{c} = 2\omega_3 \times V_{B2B3}$$

$$|p^{c}| = 2|\omega_3|\,|V_{B2B3}|\sin(\sphericalangle(\omega_3, V_{B2B3})) = 2|\omega_3|\,|V_{B2B3}|$$

*kąt prosty*



# Plan przyspieszeń

$$\underset{\parallel 1}{\overline{\overline{p^{n}_{B2A}}}} = \underset{\parallel 3}{\overline{\overline{p^{n}_{B3G}}}} + \underset{\perp 3}{\overline{p^{t}_{B3G}}} + \underset{\parallel 3}{\overline{p^{w}_{B2B3}}} + \underset{\perp 3}{\overline{\overline{p^{c}}}}$$

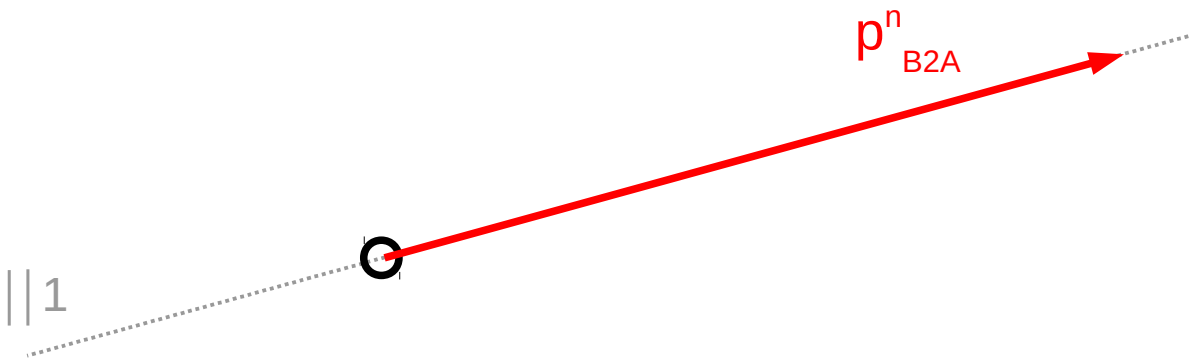
$$\underset{\parallel 1}{\overline{\overline{p^{n}_{B2A}}}} - \underset{\perp 3}{\overline{\overline{p^{c}}}} - \underset{\parallel 3}{\overline{p^{w}_{B2B3}}} = \underset{\parallel 3}{\overline{\overline{p^{n}_{B3G}}}} + \underset{\perp 3}{\overline{p^{t}_{B3G}}}$$

# Plan przyspieszeń

$$\underset{\|\,1}{\overline{\overline{p^{n}_{B2A}}}} - \underset{\perp\,3}{\overline{\overline{p^{c}}}} - \underset{\|\,3}{\overline{p^{w}_{B2B3}}} = \underset{\|\,3}{\overline{\overline{p^{n}_{B3G}}}} + \underset{\perp\,3}{\overline{p^{t}_{B3G}}}$$

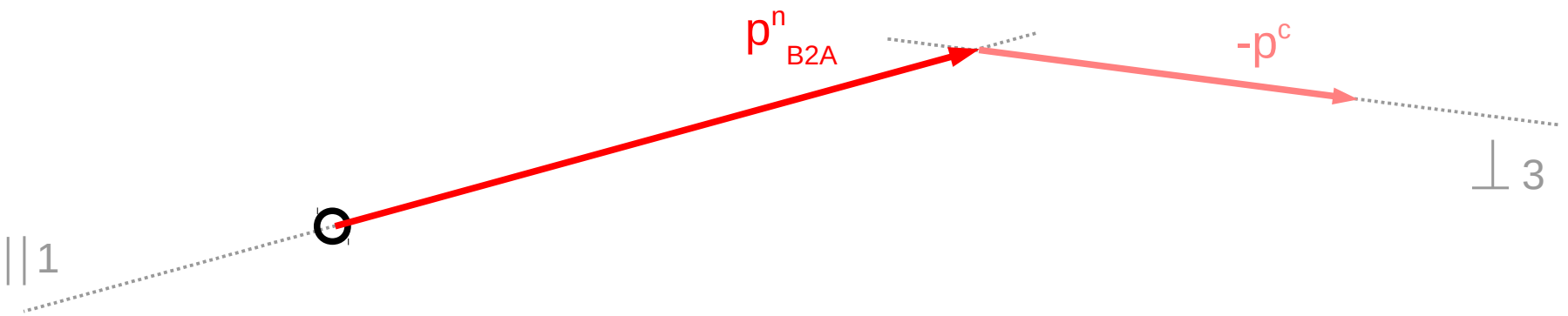
# Plan przyspieszeń

$$\underset{\parallel 1}{\underline{\underline{p^{n}_{B2A}}}} - \underset{\perp 3}{\underline{\underline{p^{c}}}} - \underset{\parallel 3}{\underline{p^{w}_{B2B3}}} = \underset{\parallel 3}{\underline{\underline{p^{n}_{B3G}}}} + \underset{\perp 3}{\underline{p^{t}_{B3G}}}$$



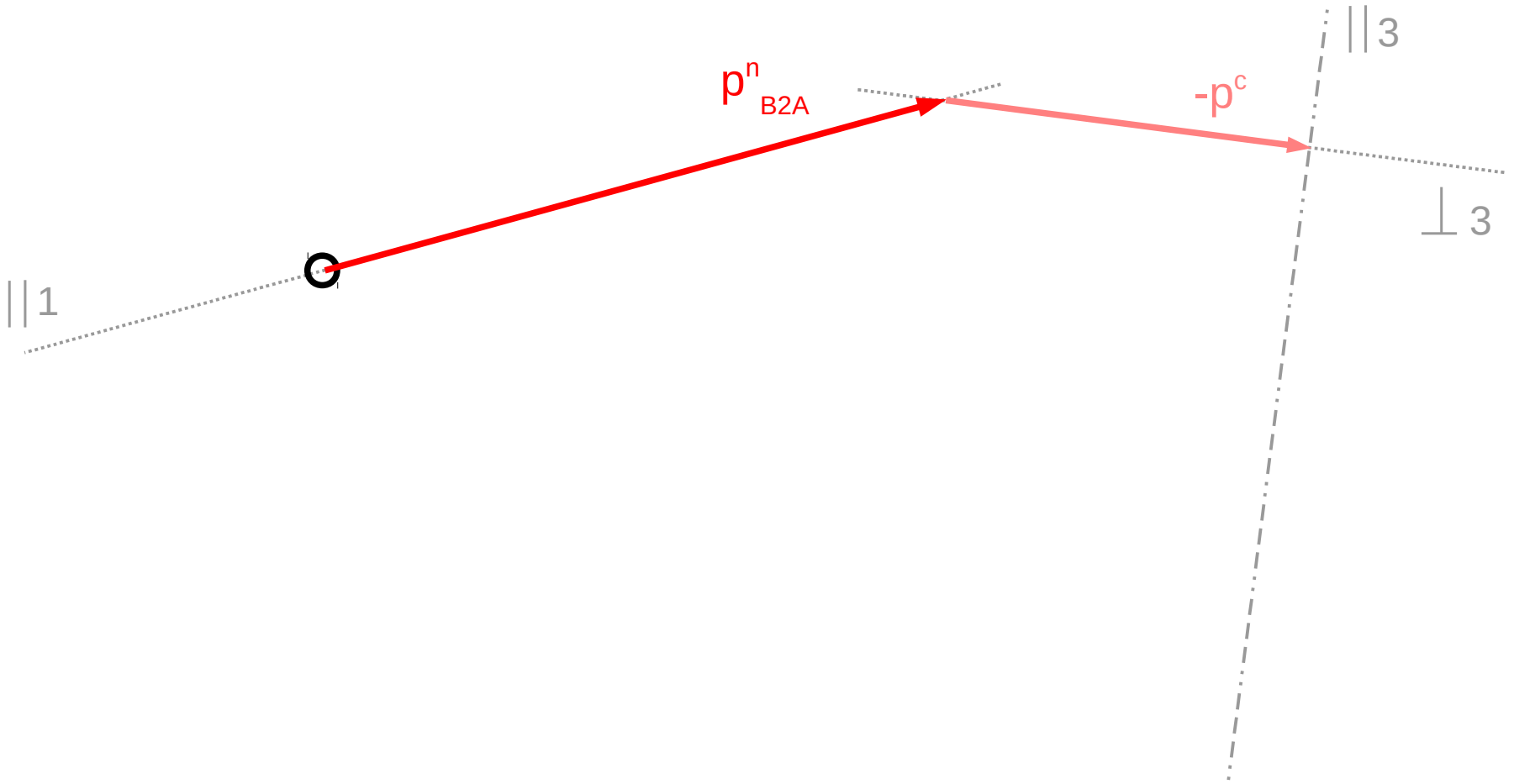
# Plan przyspieszeń

$$\underset{\parallel 1}{\underline{\underline{p^n_{B2A}}}} - \underset{\perp 3}{\underline{\underline{p^c}}} - \underset{\parallel 3}{\underline{p^w_{B2B3}}} = \underset{\parallel 3}{\underline{\underline{p^n_{B3G}}}} + \underset{\perp 3}{\underline{p^t_{B3G}}}$$



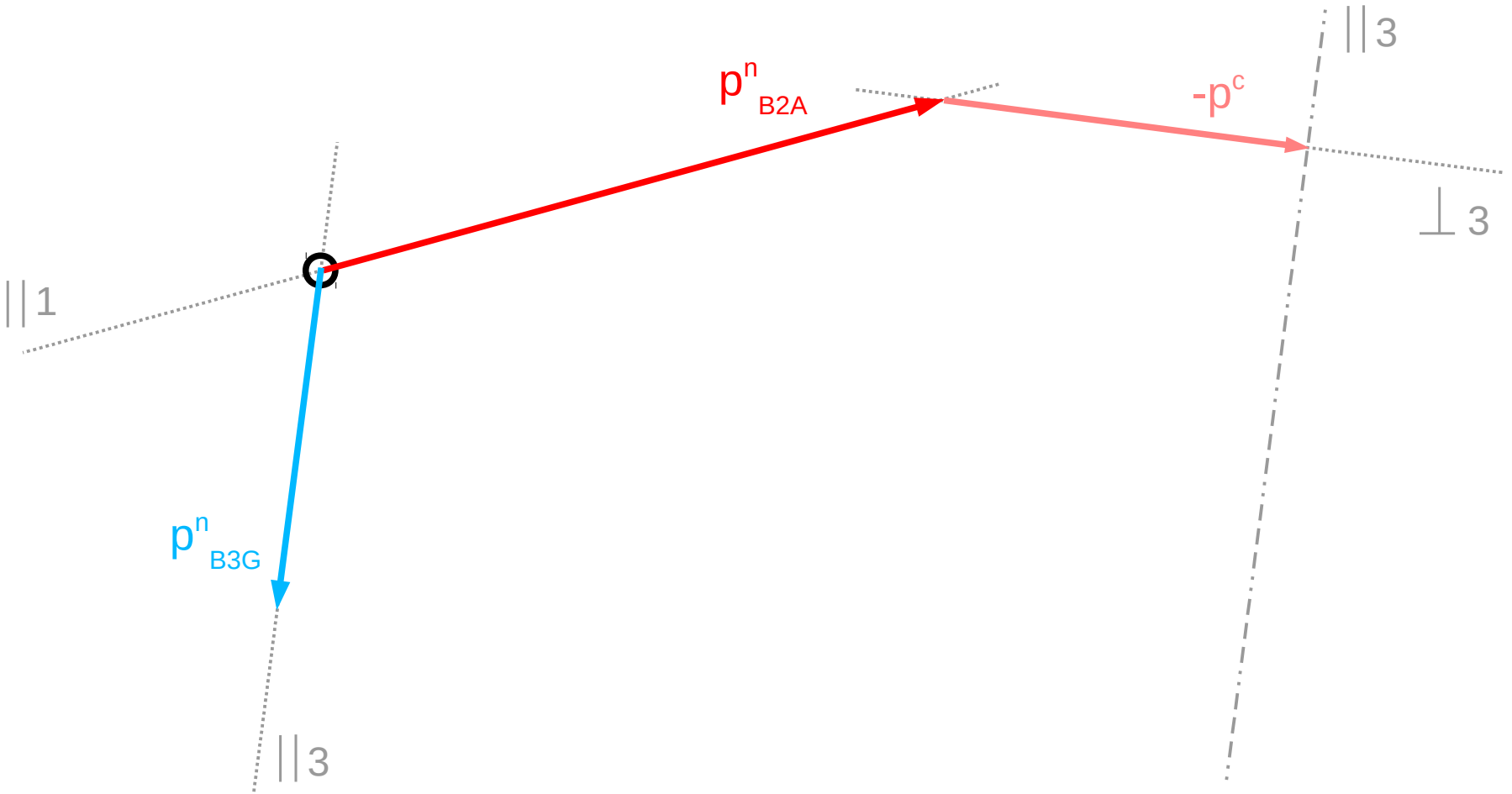
# Plan przyspieszeń

$$\underset{\parallel 1}{\overset{=}{p^{n}_{B2A}}} - \underset{\perp 3}{\overset{=}{p^{c}}} - \underset{\parallel 3}{\overline{p^{w}_{B2B3}}} = \underset{\parallel 3}{\overset{=}{p^{n}_{B3G}}} + \underset{\perp 3}{\overline{p^{t}_{B3G}}}$$



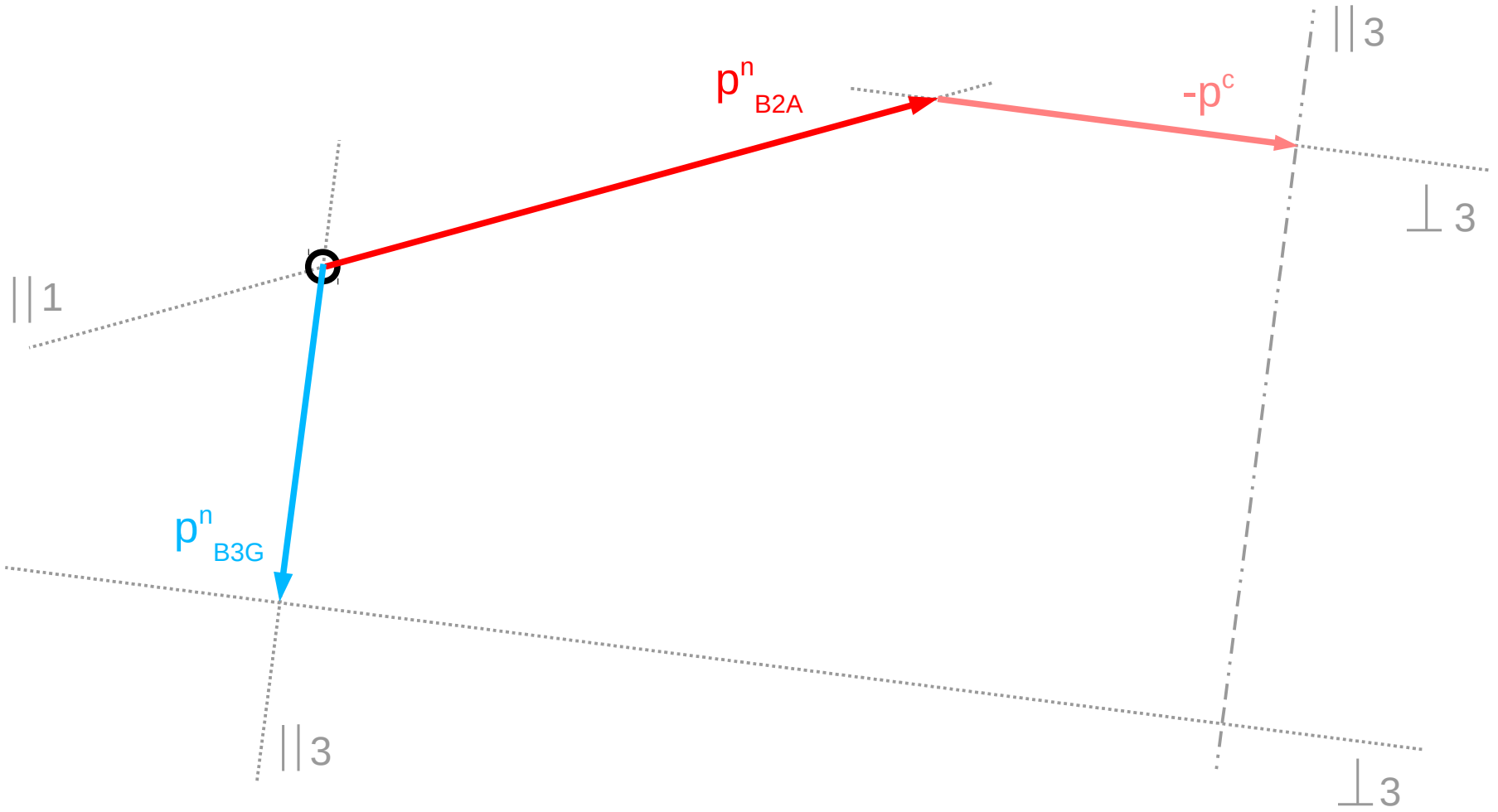
# Plan przyspieszeń

$$\underset{\parallel 1}{\overline{\overline{p^{n}_{B2A}}}} - \underset{\perp 3}{\overline{\overline{p^{c}}}} - \underset{\parallel 3}{\overline{p^{w}_{B2B3}}} = \underset{\parallel 3}{\overline{\overline{p^{n}_{B3G}}}} + \underset{\perp 3}{\overline{p^{t}_{B3G}}}$$



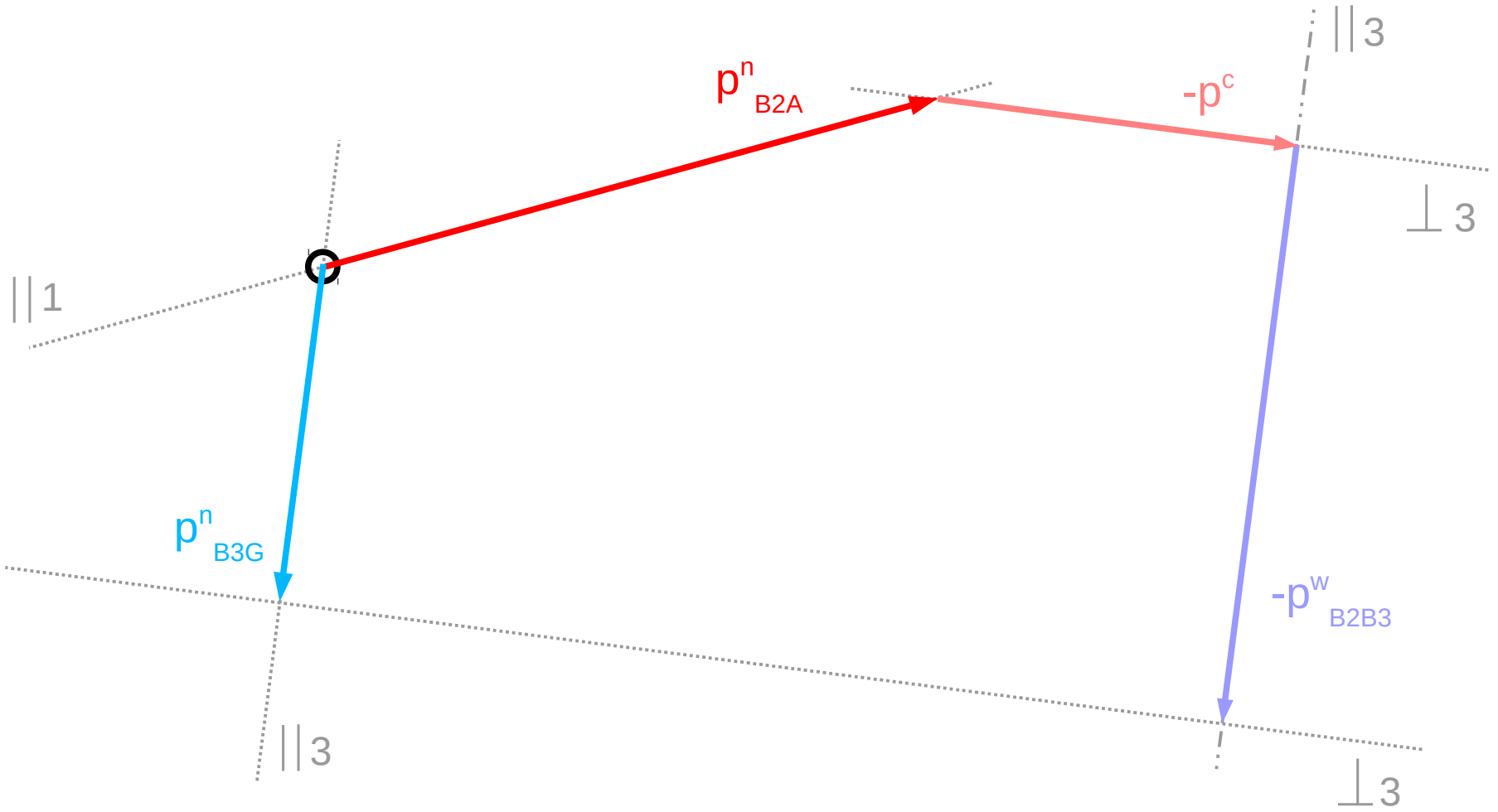
# Plan przyspieszeń

$$\underset{\parallel 1}{\overline{\overline{p^{n}_{B2A}}}} - \underset{\perp 3}{\overline{\overline{p^{c}}}} - \underset{\parallel 3}{\overline{p^{w}_{B2B3}}} = \underset{\parallel 3}{\overline{\overline{p^{n}_{B3G}}}} + \underset{\perp 3}{\overline{p^{t}_{B3G}}}$$



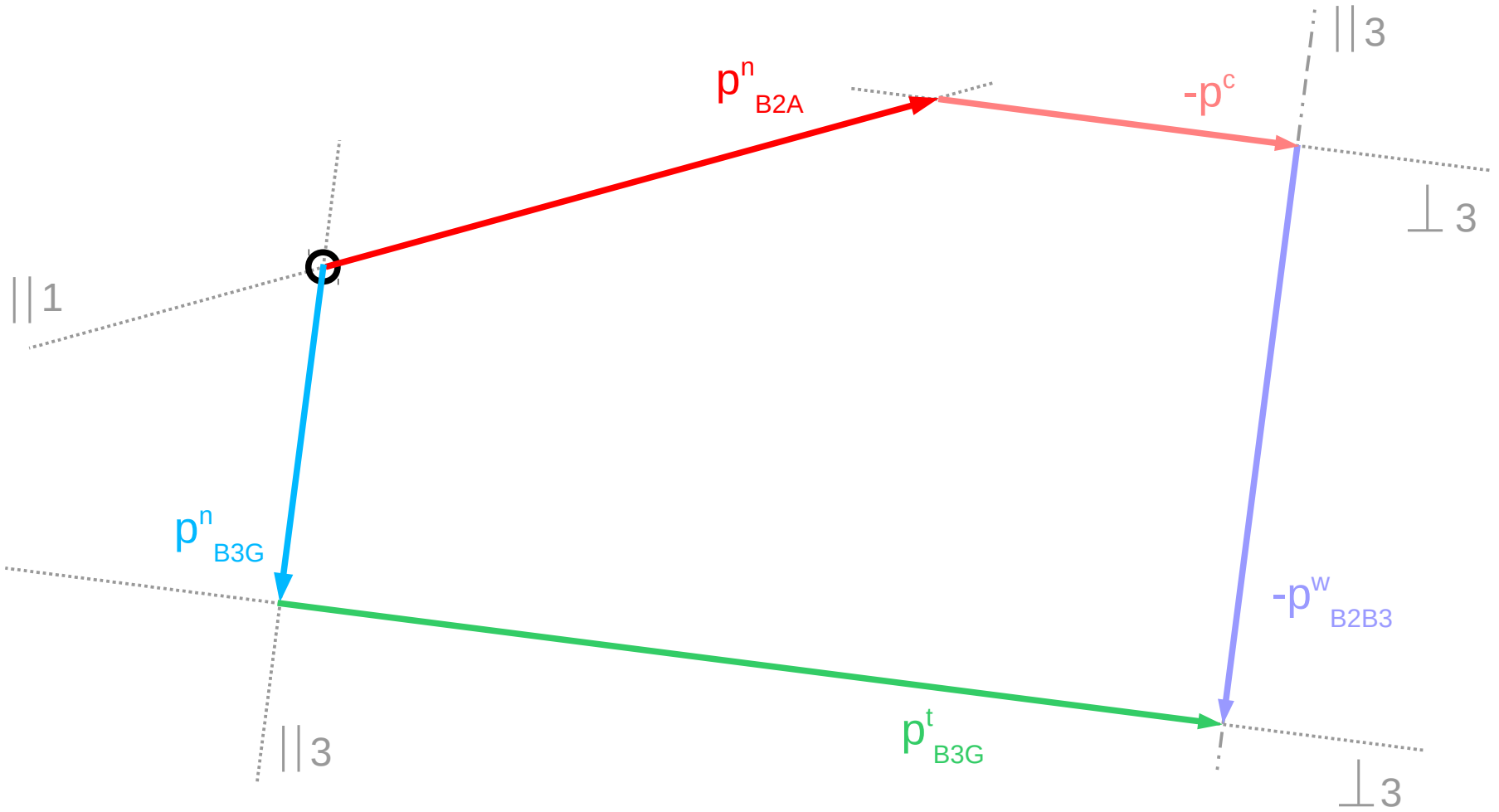
# Plan przyspieszeń

$$\underset{\parallel 1}{\overrightarrow{p^{n}_{B2A}}} - \underset{\perp 3}{\overrightarrow{p^{c}}} - \underset{\parallel 3}{\overrightarrow{p^{w}_{B2B3}}} = \underset{\parallel 3}{\overrightarrow{p^{n}_{B3G}}} + \underset{\perp 3}{\overrightarrow{p^{t}_{B3G}}}$$



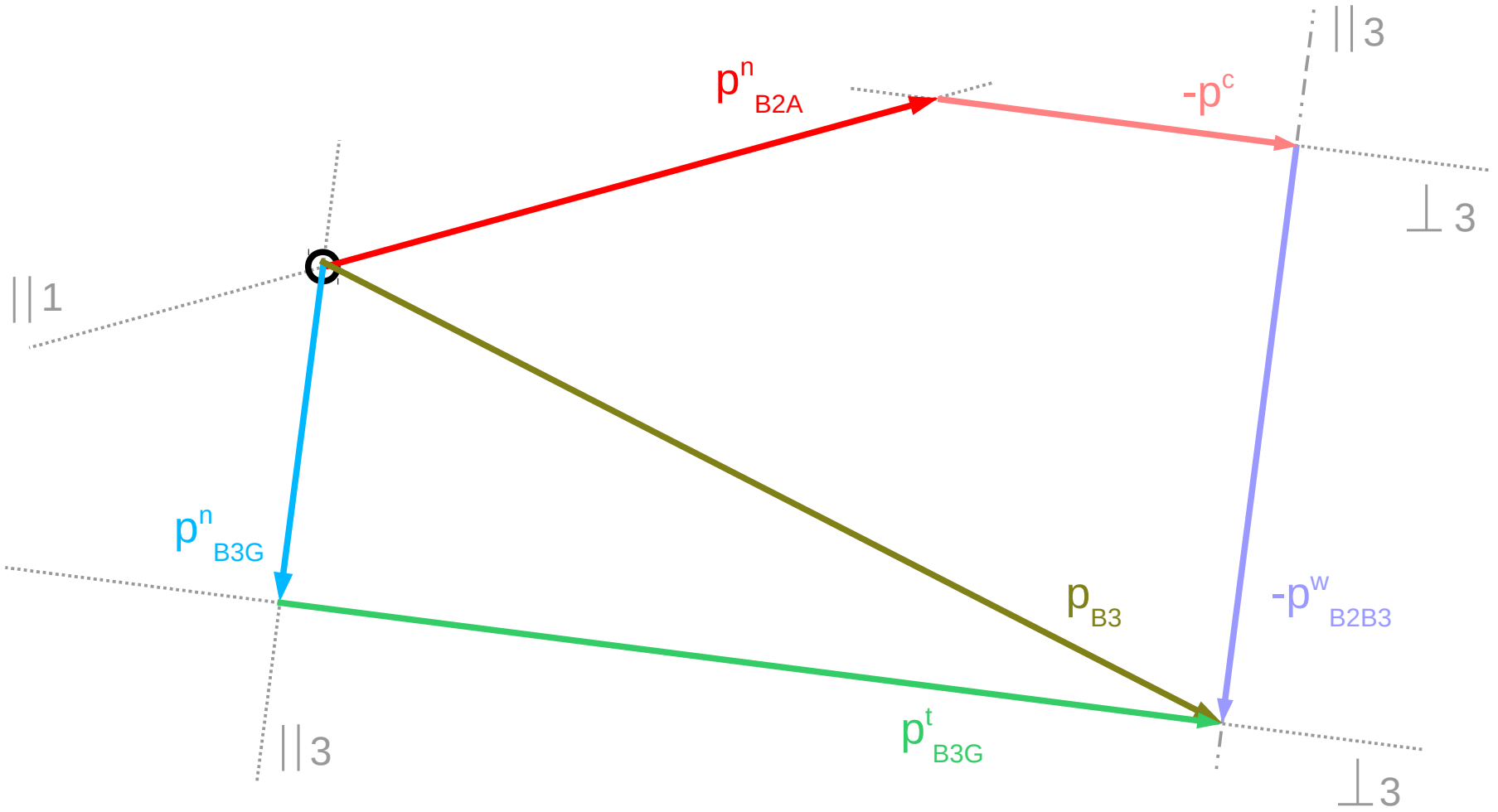
# Plan przyspieszeń

$$\underset{\parallel 1}{p^{n}_{B2A}} - \underset{\perp 3}{p^{c}} - \underset{\parallel 3}{p^{w}_{B2B3}} = \underset{\parallel 3}{p^{n}_{B3G}} + \underset{\perp 3}{p^{t}_{B3G}}$$

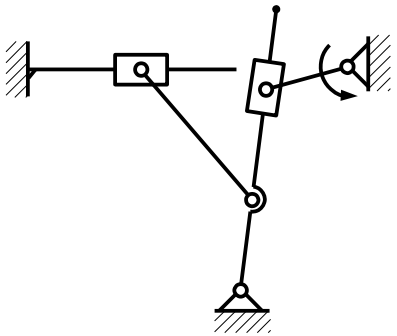


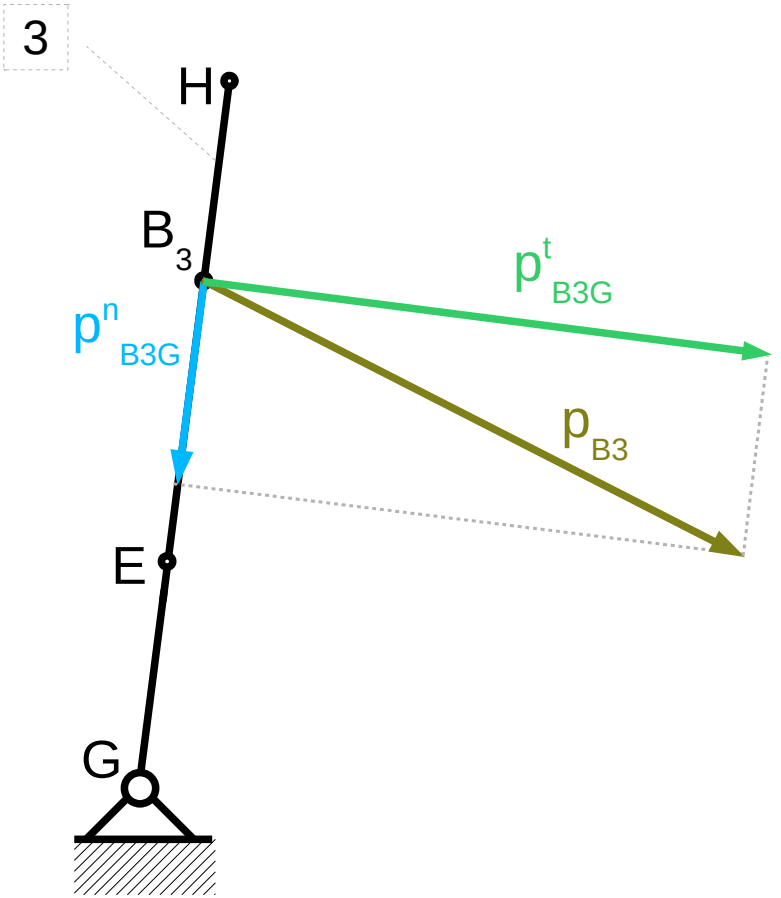
# Plan przyspieszeń

$$\underset{\parallel 1}{p^{n}_{B2A}} - \underset{\perp 3}{p^{c}} - \underset{\parallel 3}{p^{w}_{B2B3}} = \underset{\parallel 3}{p^{n}_{B3G}} + \underset{\perp 3}{p^{t}_{B3G}}$$

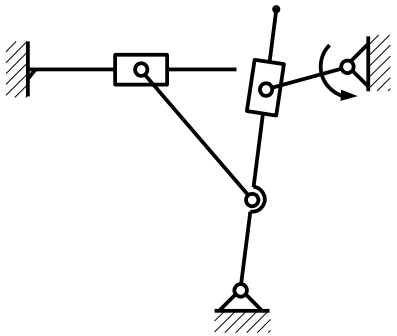


# Przyspieszenia dla elementu 3

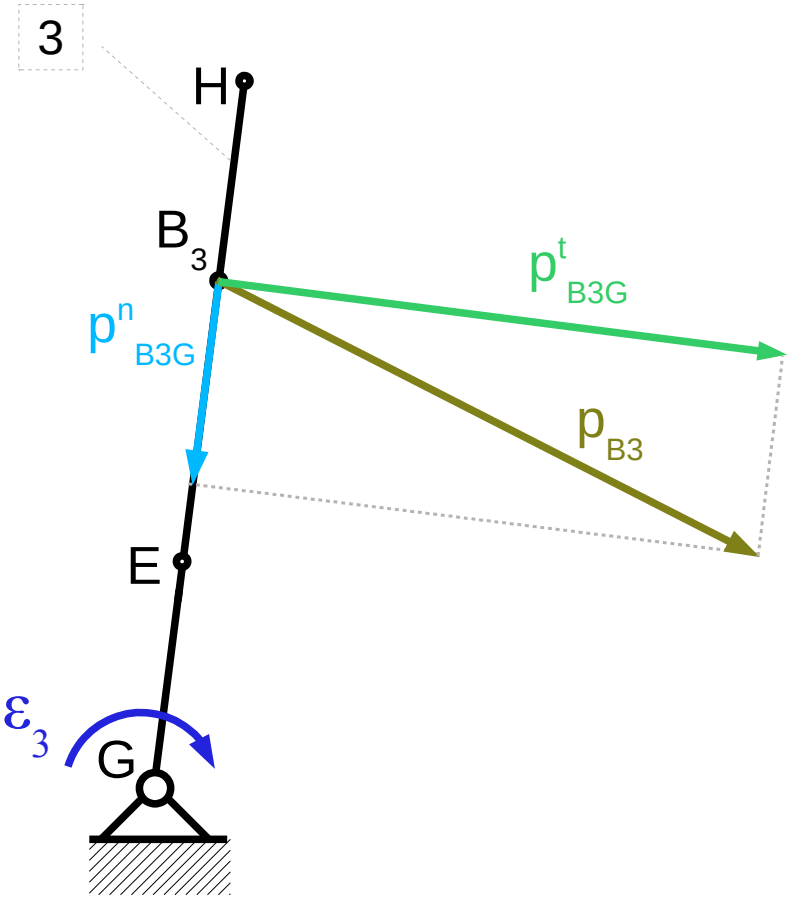




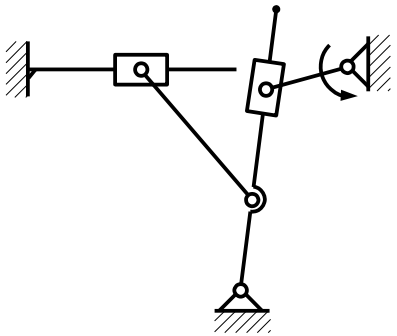
# Przyspieszenia dla elementu 3



$$\varepsilon_3 = \frac{|p^t_{B3G}|}{|B_3G|}$$



# Przyspieszenia dla punktu E

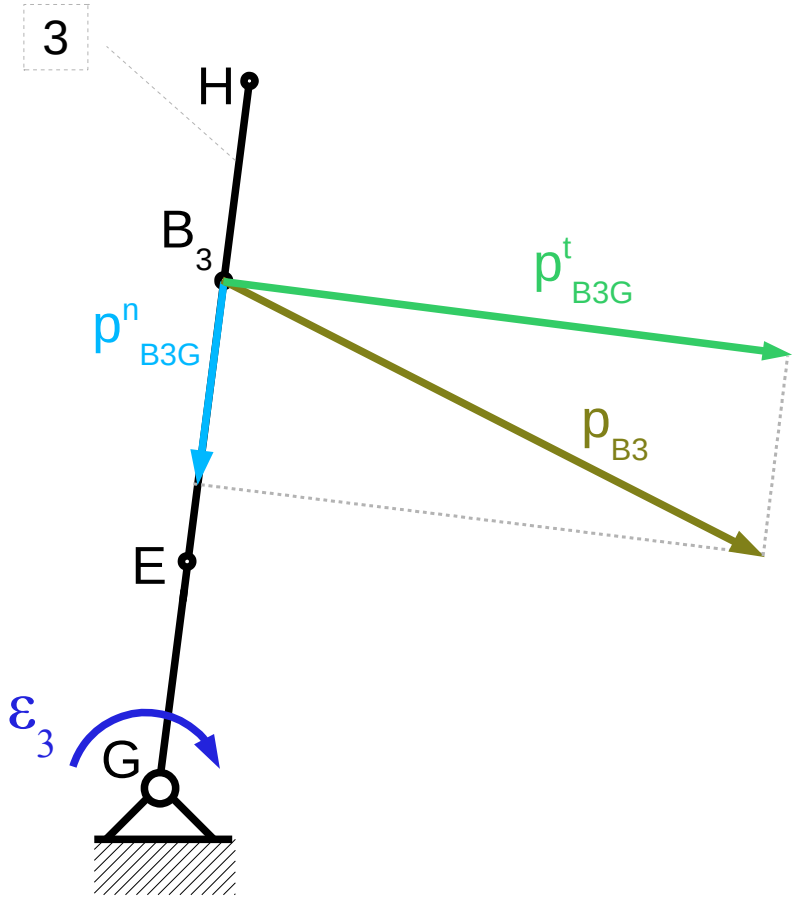


$$\varepsilon_3 = \frac{|p^t_{B3G}|}{|B_3G|}$$

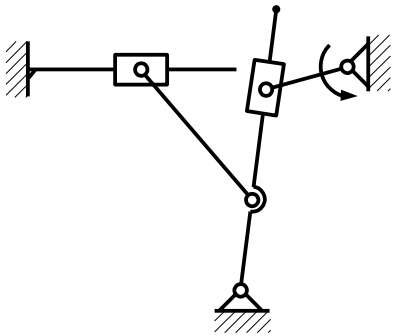
$$p_E = p_G + p^n_{EG} + p^t_{EG}$$

$$|p^n_{EG}| = \omega_3^{\,2}\,|EG|$$

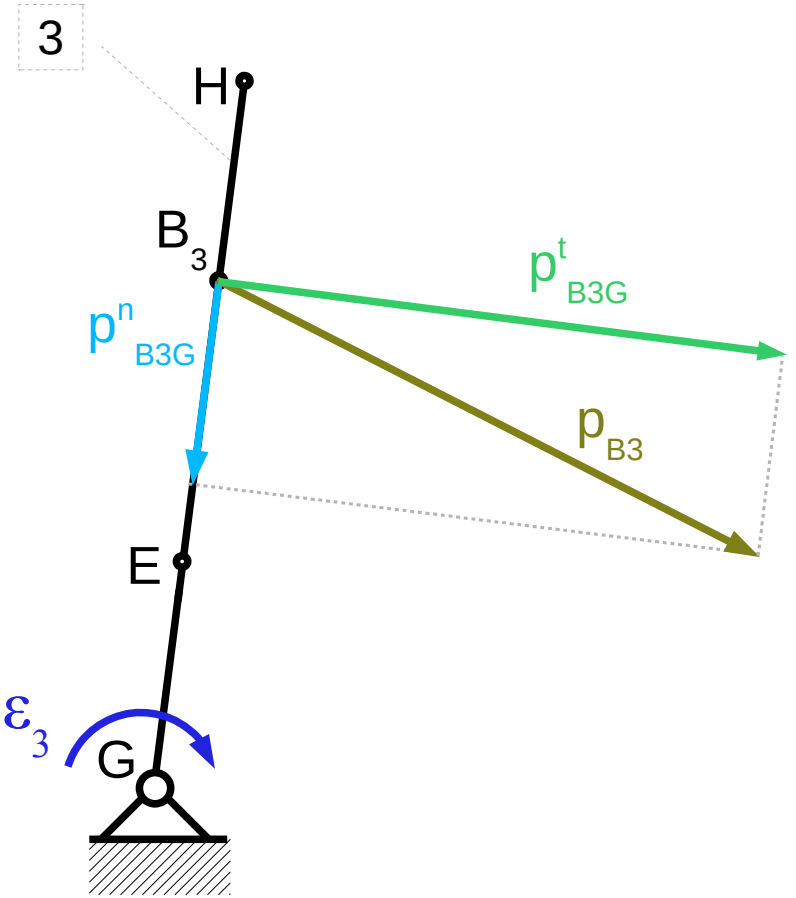
$$|p^t_{EG}| = \varepsilon_3\,|EG|$$



# Przyspieszenia dla punktu E



$$\varepsilon_3 = \frac{|p^t_{B3G}|}{|B_3G|}$$




$$p_E = p_G + p^n_{EG} + p^t_{EG}$$

$$|p^n_{EG}| = \omega_3^{\,2}\,|EG|$$

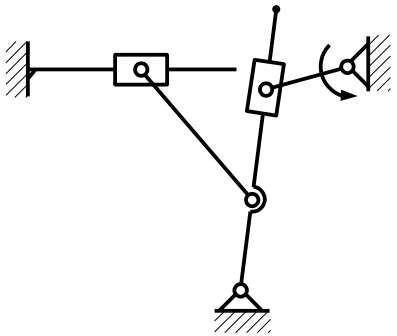
$$|p^t_{EG}| = \varepsilon_3\,|EG|$$

*podstawiamy zależności*

$$|p^n_{B3G}| = \omega_3^{\,2}\,|B_3G|$$

$$|p^t_{B3G}| = \varepsilon_3\,|B_3G|$$

# Przyspieszenia dla punktu E



$$\varepsilon_3 = \frac{|p^t_{B3G}|}{|B_3G|}$$

$$p_E = p_G + p^n_{EG} + p^t_{EG}$$

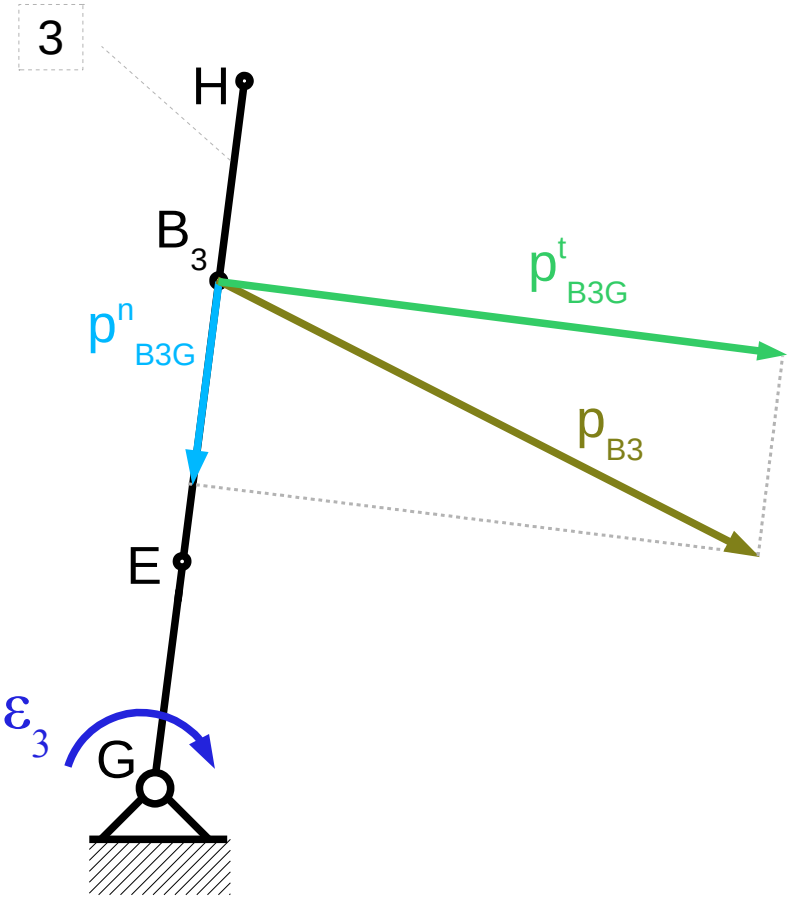
$$|p^n_{EG}| = \omega_3^{\,2}\,|EG| = |p^n_{B3G}|\,\frac{|EG|}{|B_3G|}$$

$$|p^t_{EG}| = \varepsilon_3\,|EG| = |p^t_{B3G}|\,\frac{|EG|}{|B_3G|}$$

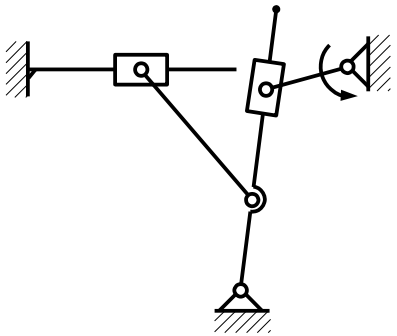
*podstawiamy zależności*

$$|p^n_{B3G}| = \omega_3^{\,2}\,|B_3G|$$

$$|p^t_{B3G}| = \varepsilon_3\,|B_3G|$$



# Przyspieszenia dla punktu E



$$\varepsilon_3 = \frac{|p^t_{B3G}|}{|B_3G|}$$

$$p_E = p_G + p^n_{EG} + p^t_{EG}$$

$$|p^n_{EG}| = \omega_3^{\,2}\,|EG| = |p^n_{B3G}|\,\frac{|EG|}{|B_3G|}$$

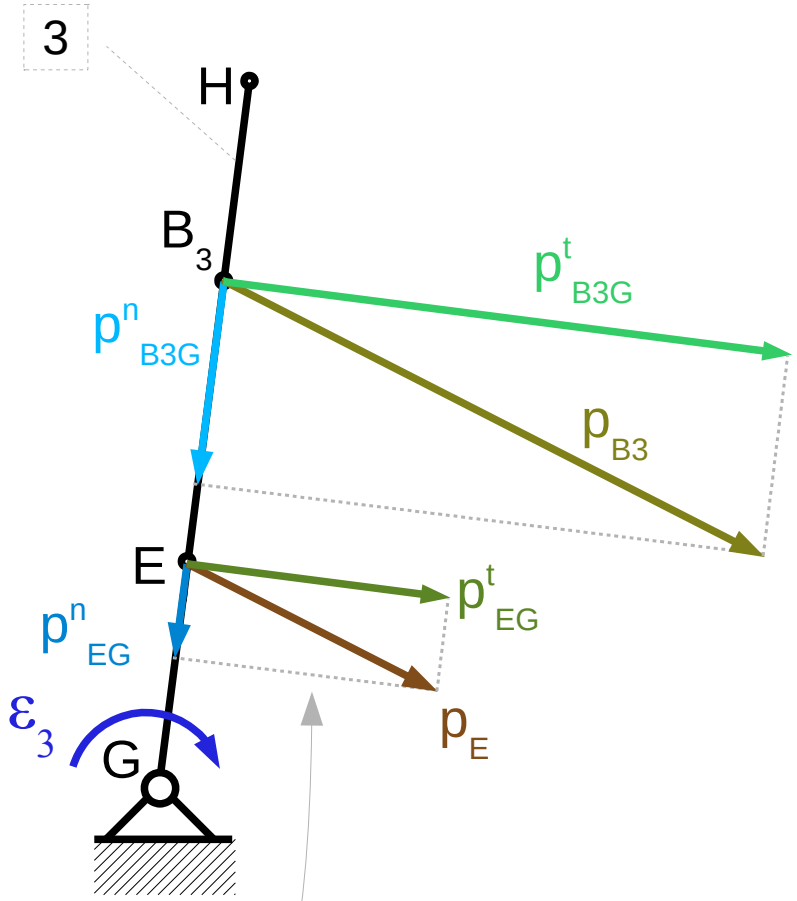
$$|p^t_{EG}| = \varepsilon_3\,|EG| = |p^t_{B3G}|\,\frac{|EG|}{|B_3G|}$$

*podstawiamy zależności*

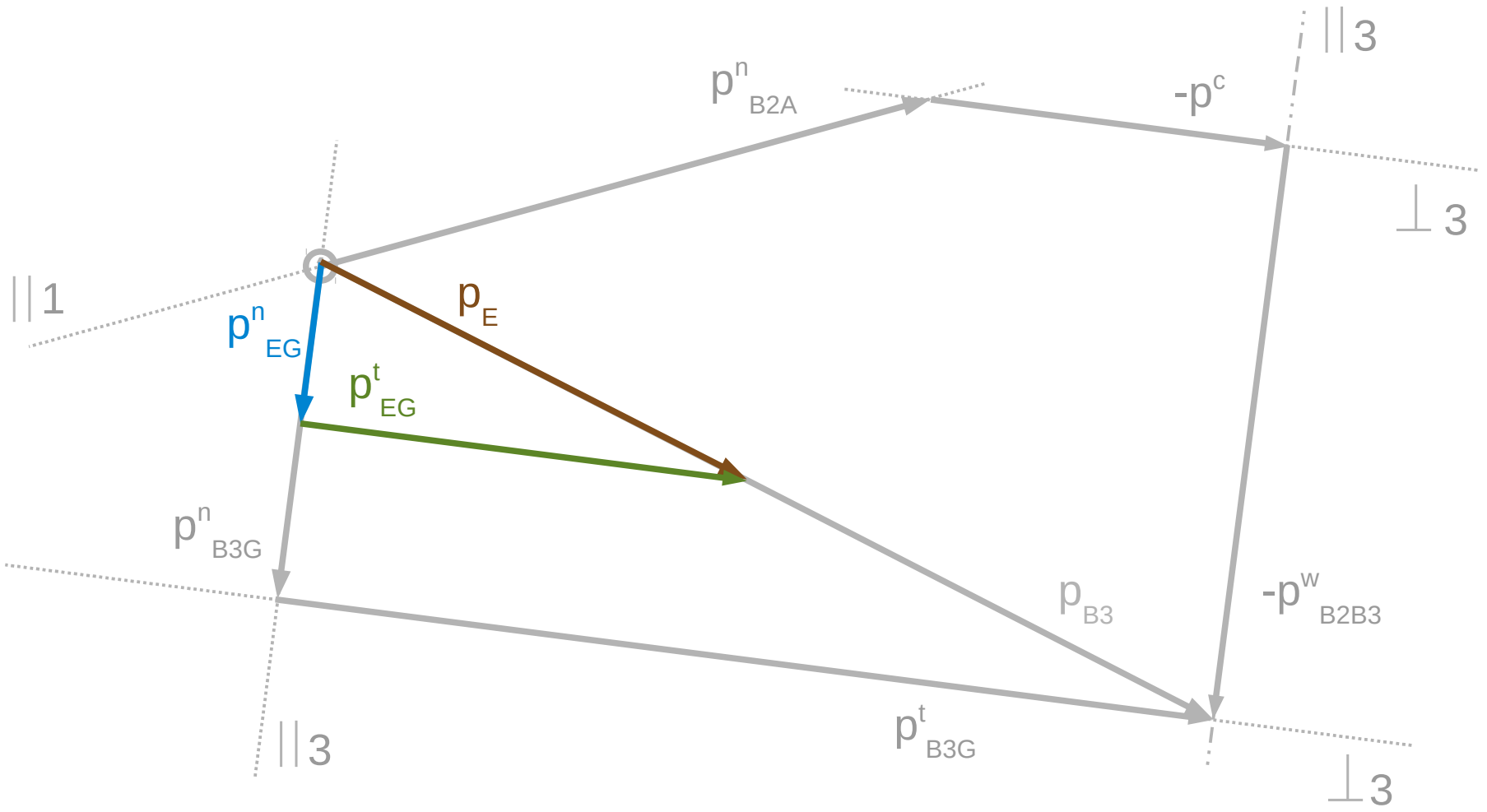
$$|p^n_{B3G}| = \omega_3^{\,2}\,|B_3G|$$

$$|p^t_{B3G}| = \varepsilon_3\,|B_3G|$$

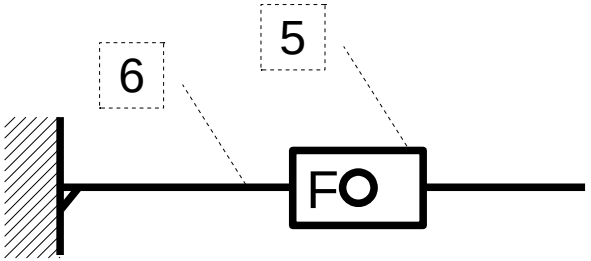
*i mamy proporcjonalnie mniejsze przyspieszenia*

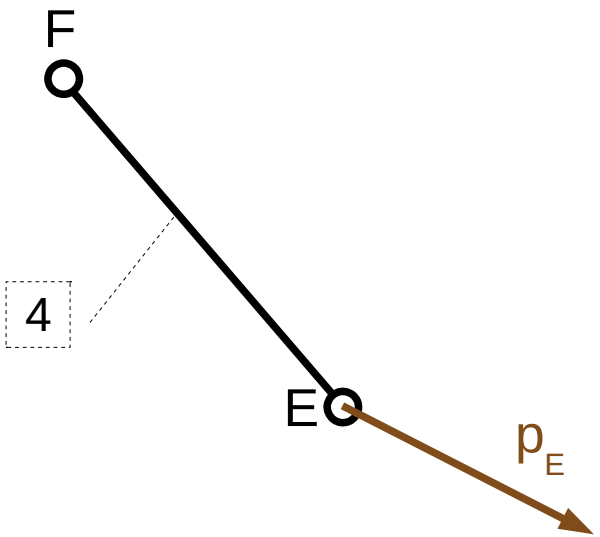


# Plan przyspieszeń



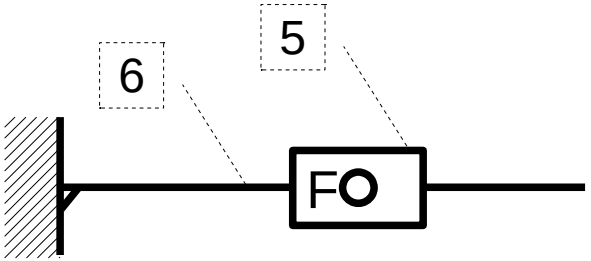
# Przyspieszenia punktów elementu 4

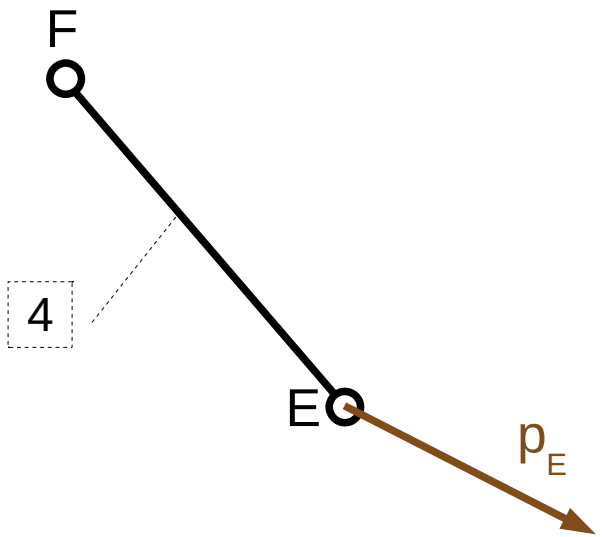


# Przyspieszenia punktów elementu 4

$$p_F = p_E + p^n_{FE} + p^t_{FE}$$



# Przyspieszenia punktów elementu 4

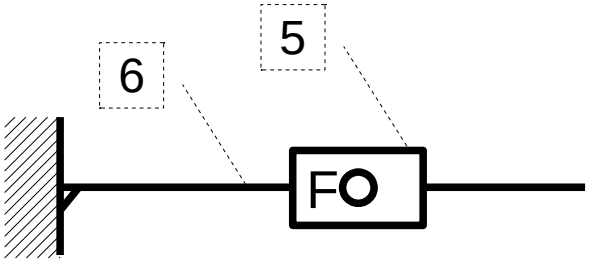
$$\underline{\underline{p_F}} = \underline{\underline{p_E}} + \underline{\underline{p^n_{FE}}} + \underline{p^t_{FE}}$$

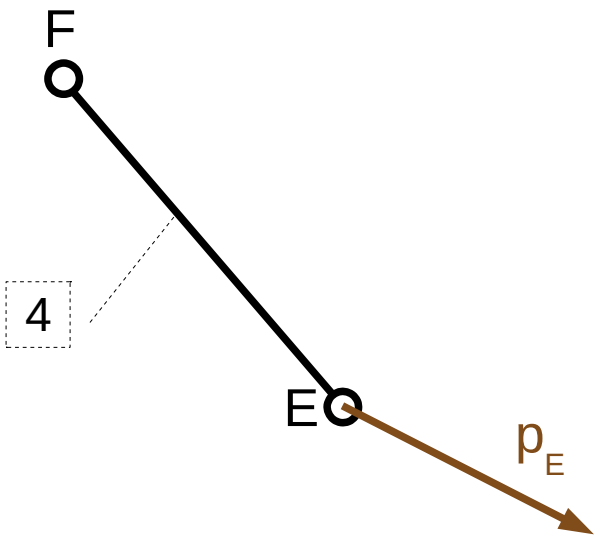
$$\underline{\underline{p_F}} = \underline{\underline{p_E}} + \underline{\underline{p^n_{FE}}} + \underline{p^t_{FE}}$$

$$\underline{\underline{p^n_{FE}}} \;\|\; 4 \qquad \underline{p^t_{FE}} \perp 4$$

$$|p^n_{FE}| = \omega_4^{\,2}\,|FE|$$

*z planu prędkości*



# Przyspieszenia punktów elementu 4

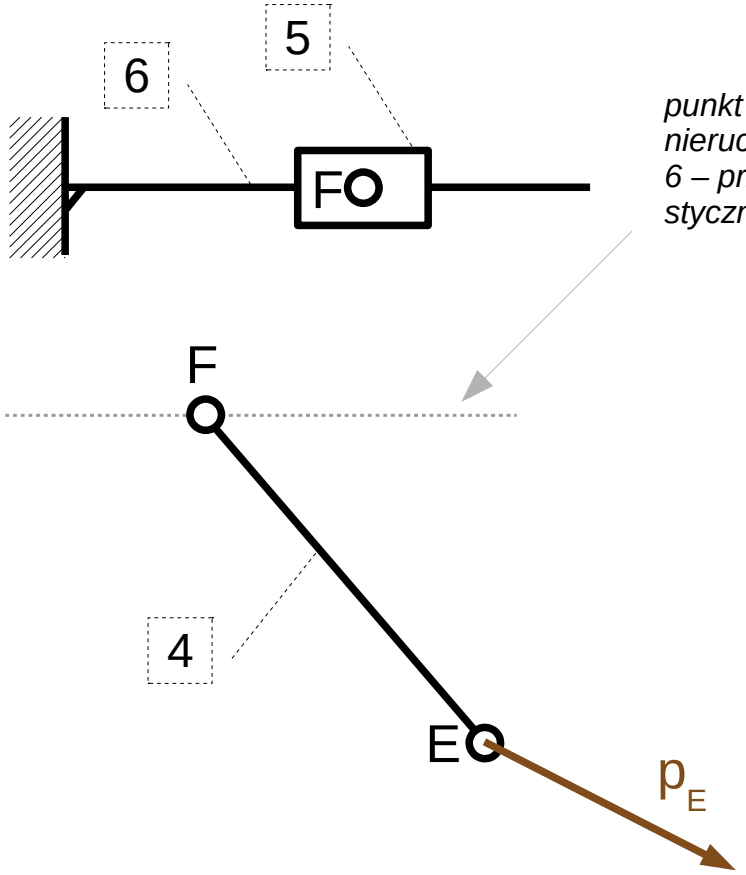
$$\underline{p_F} = \underline{\underline{p_E}} + \underline{\underline{p^n_{FE}}} + \underline{p^t_{FE}}$$

$$\underline{\;}\,6 \qquad \underline{\underline{\;}}\,4 \qquad \perp 4$$

$$|p^n_{FE}| = \omega_4^{\,2}\,|FE|$$

*z planu prędkości*

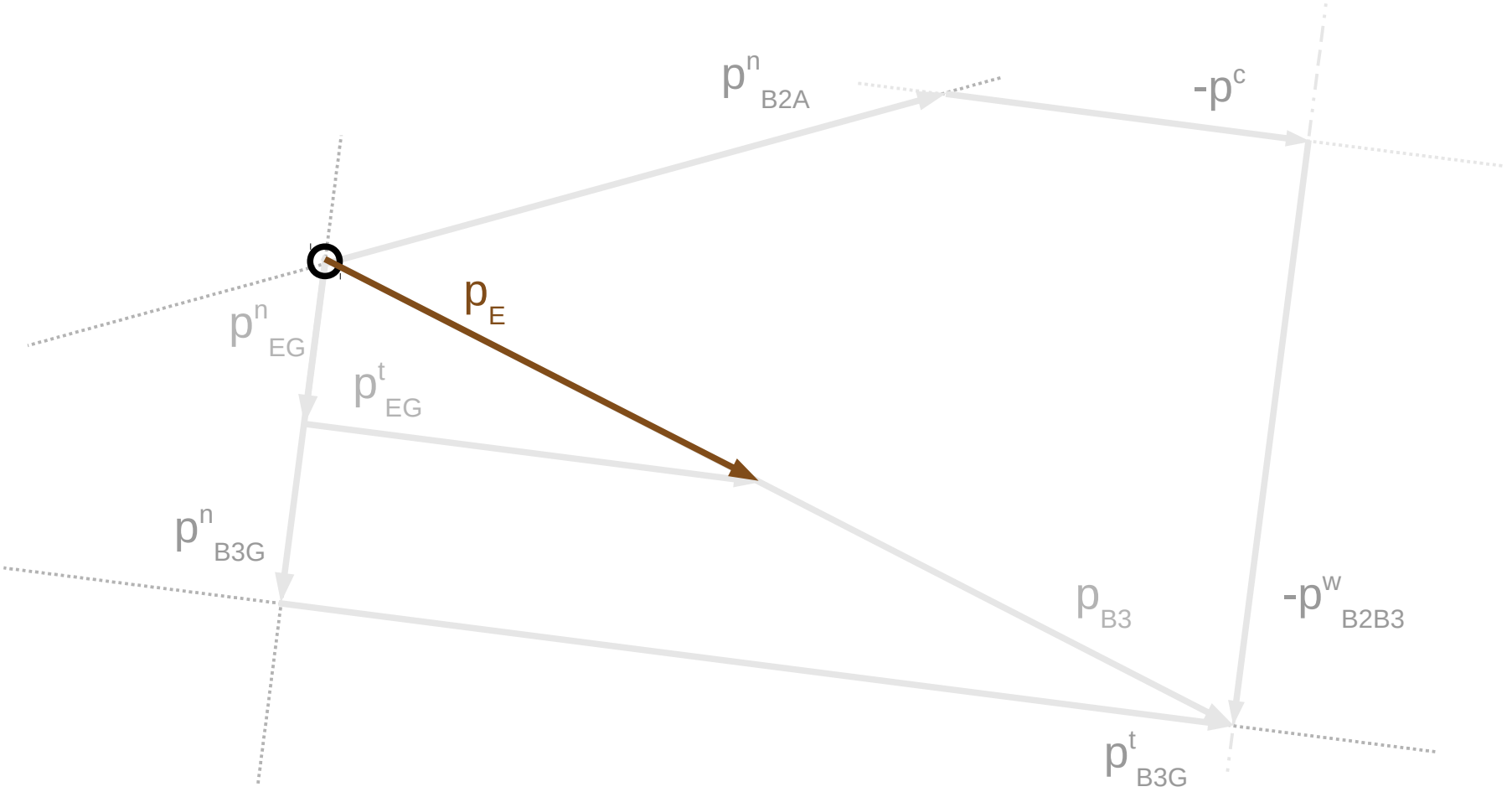
*punkt F porusza się po nieruchomym elemencie 6 – przyspieszenie jest styczne do toru ruchu*



# Plan przyspieszeń

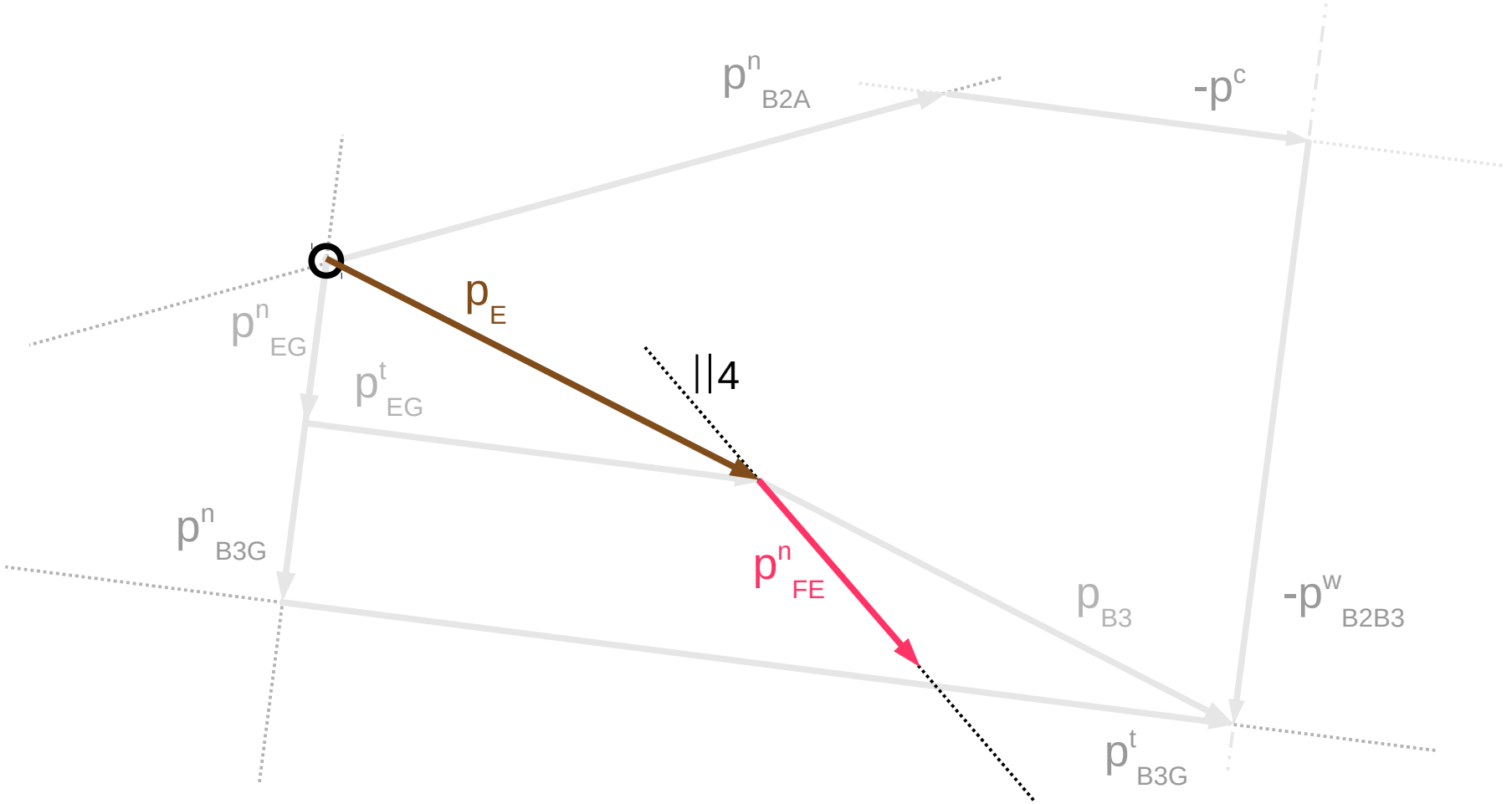
$$\underline{p_F} = \underline{\underline{p_E}} + \underline{\underline{p^n_{FE}}} + \underline{p^t_{FE}}$$

$$\underline{p_F} \parallel 6 \qquad \underline{\underline{p_E}} \qquad \underline{\underline{p^n_{FE}}} \parallel 4 \qquad \underline{p^t_{FE}} \perp 4$$



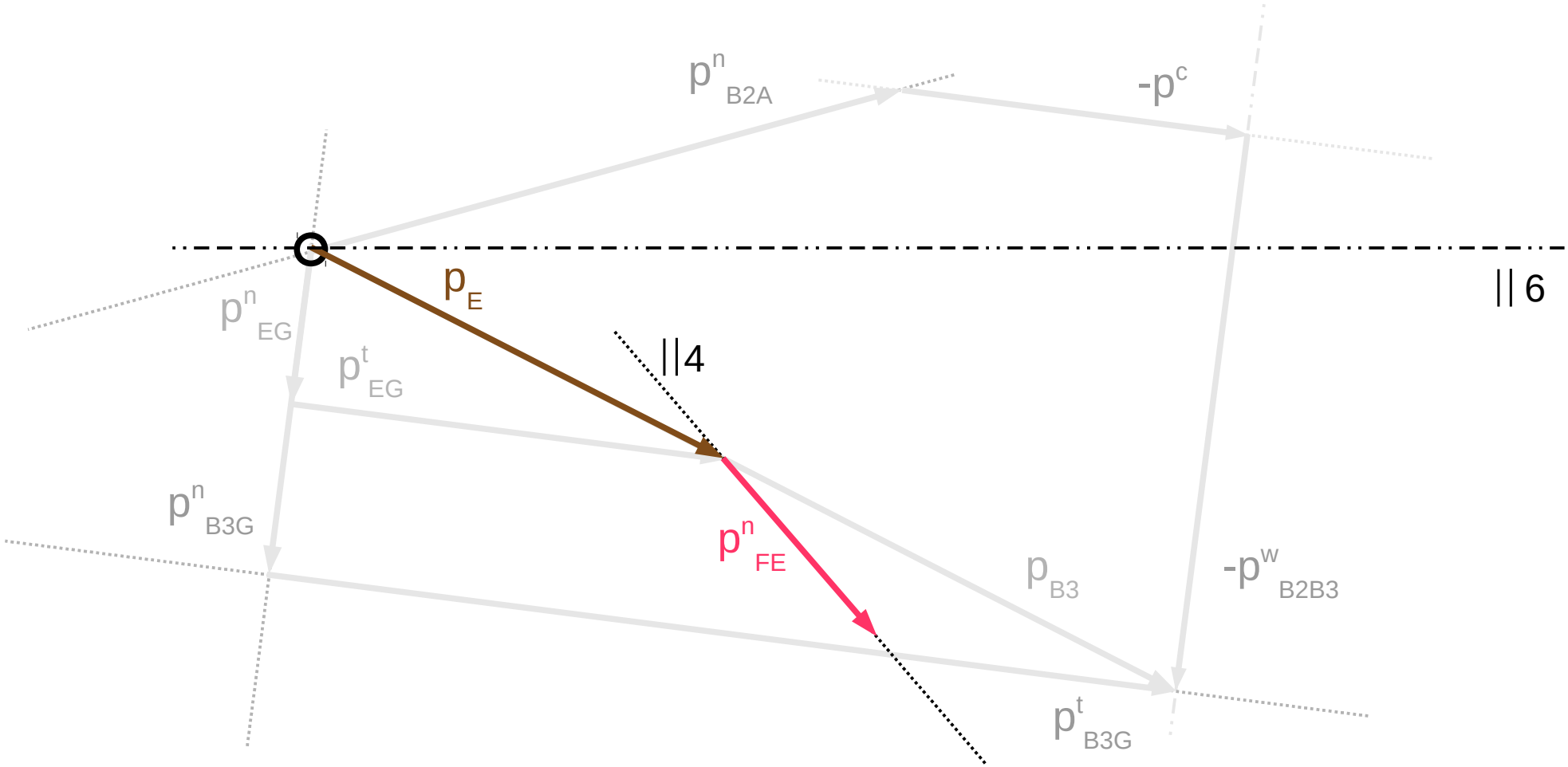
# Plan przyspieszeń

$$\underline{p_F} \;||\,6 = \underline{\underline{p_E}} \;||\,4 + \underline{\underline{p^n_{FE}}} \;||\,4 + \underline{p^t_{FE}} \;\perp 4$$



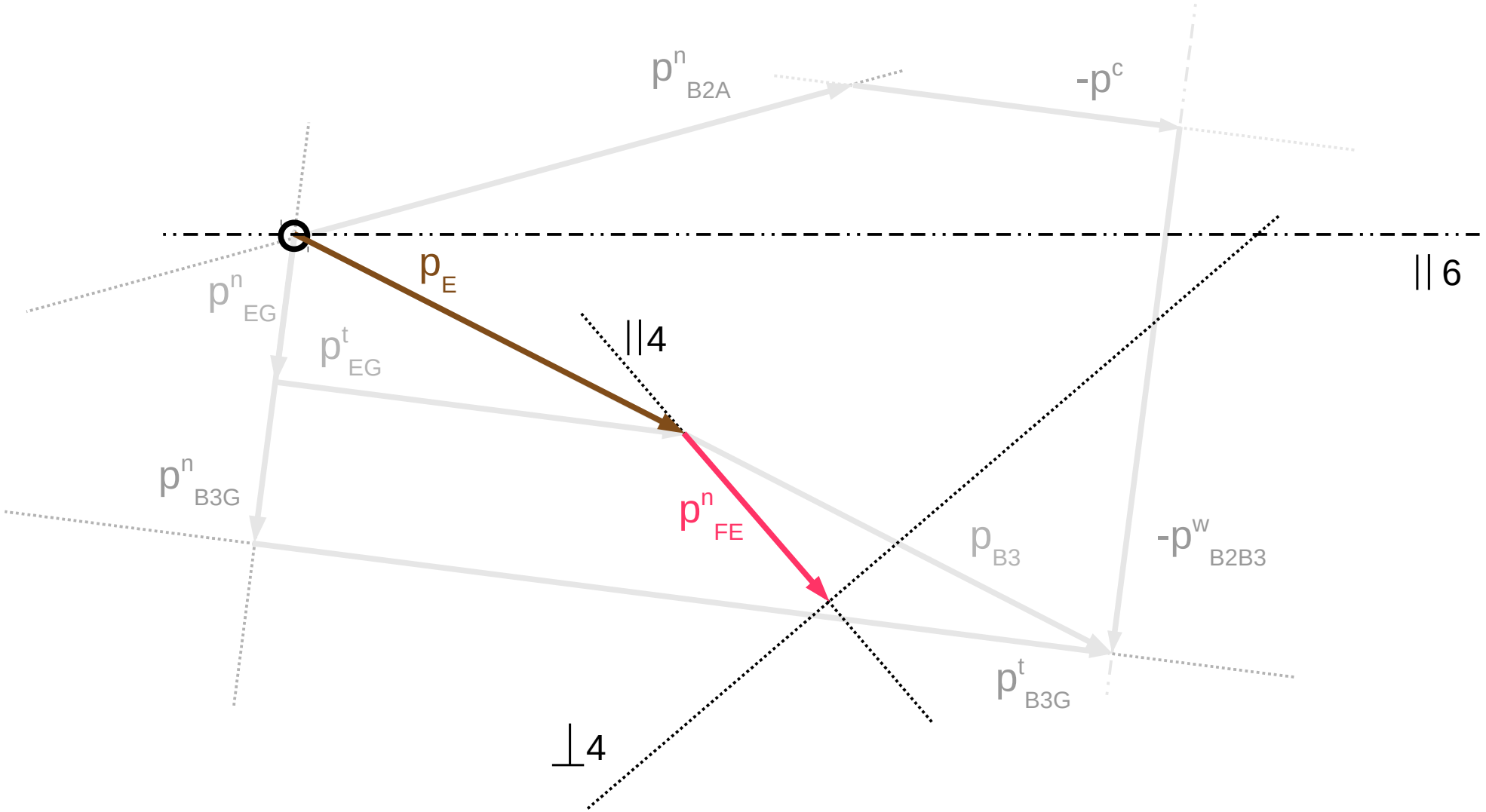
# Plan przyspieszeń

$$\underline{p_F} \;||\,6 = \underline{\underline{p_E}} + \underline{\underline{p^n_{FE}}} \;||\,4 + \underline{p^t_{FE}} \;\perp 4$$



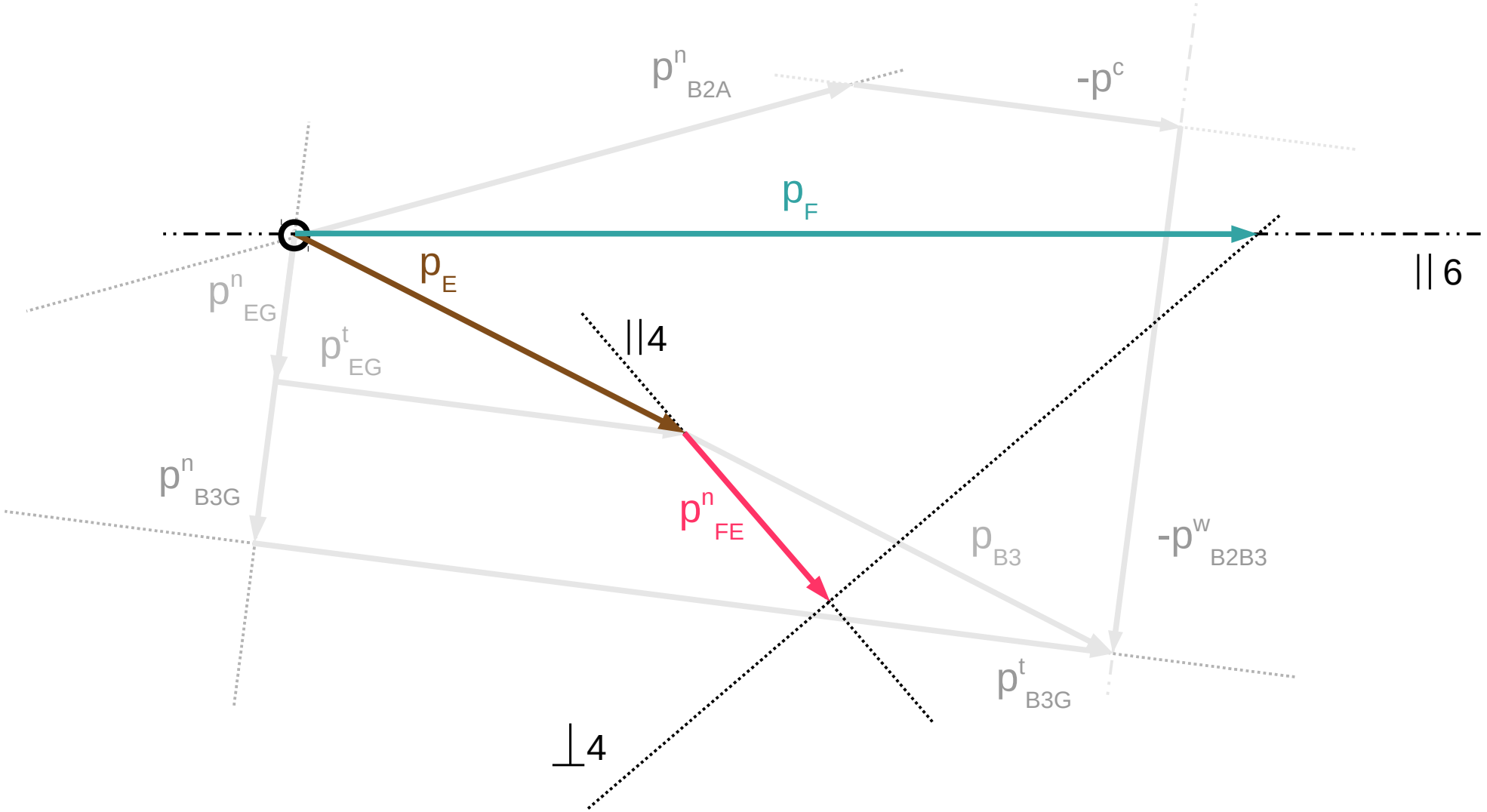
# Plan przyspieszeń

$$\underline{p_F} \;||\,6 = \underline{\underline{p_E}} + \underline{\underline{p^n_{FE}}} \;||\,4 + \underline{p^t_{FE}} \;\perp 4$$



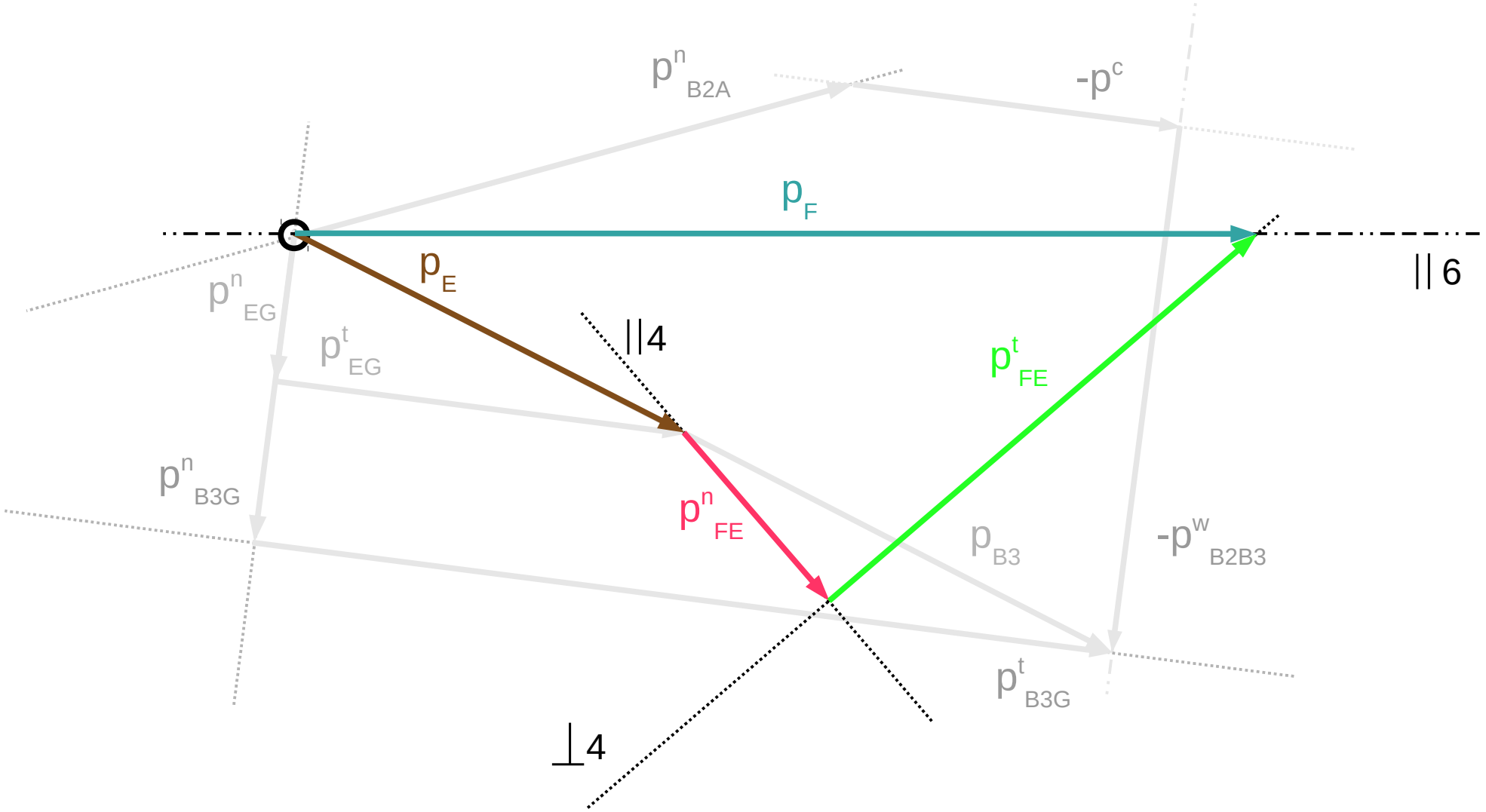
# Plan przyspieszeń

$$\underline{p_F}\ (\parallel 6) = \underline{\underline{p_E}} + \underline{\underline{p^n_{FE}}}\ (\parallel 4) + \underline{p^t_{FE}}\ (\perp 4)$$

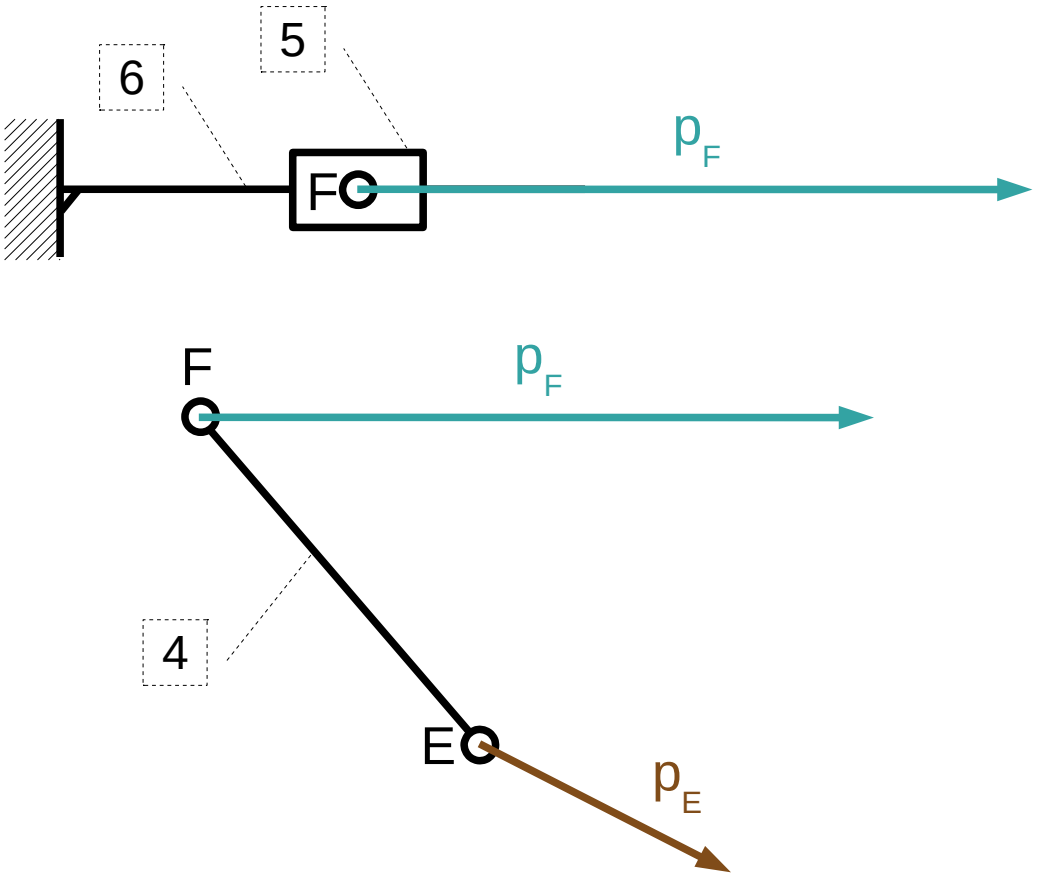


# Plan przyspieszeń

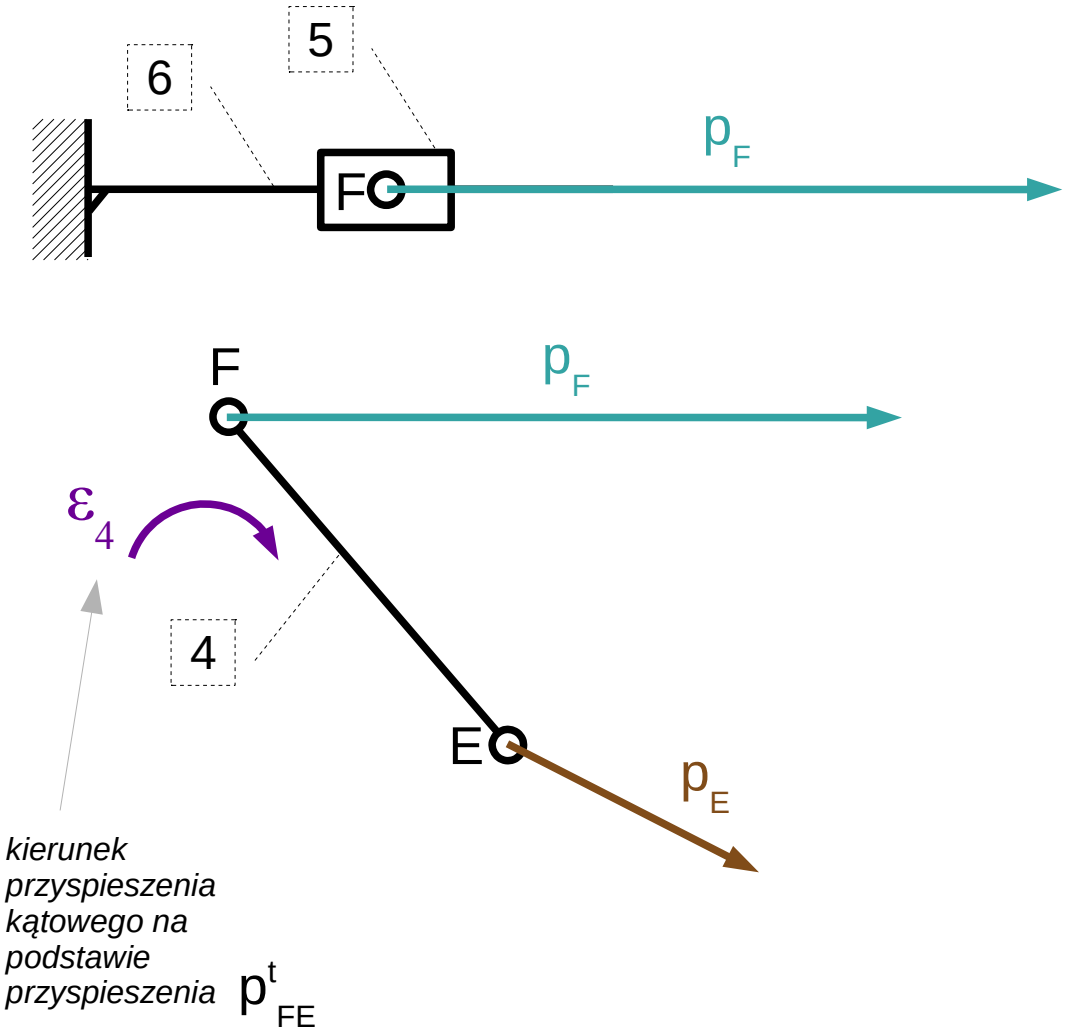
$$\underline{p_F} \;(\parallel 6) = \underline{\underline{p_E}} + \underline{\underline{p^n_{FE}}} \;(\parallel 4) + \underline{p^t_{FE}} \;(\perp 4)$$



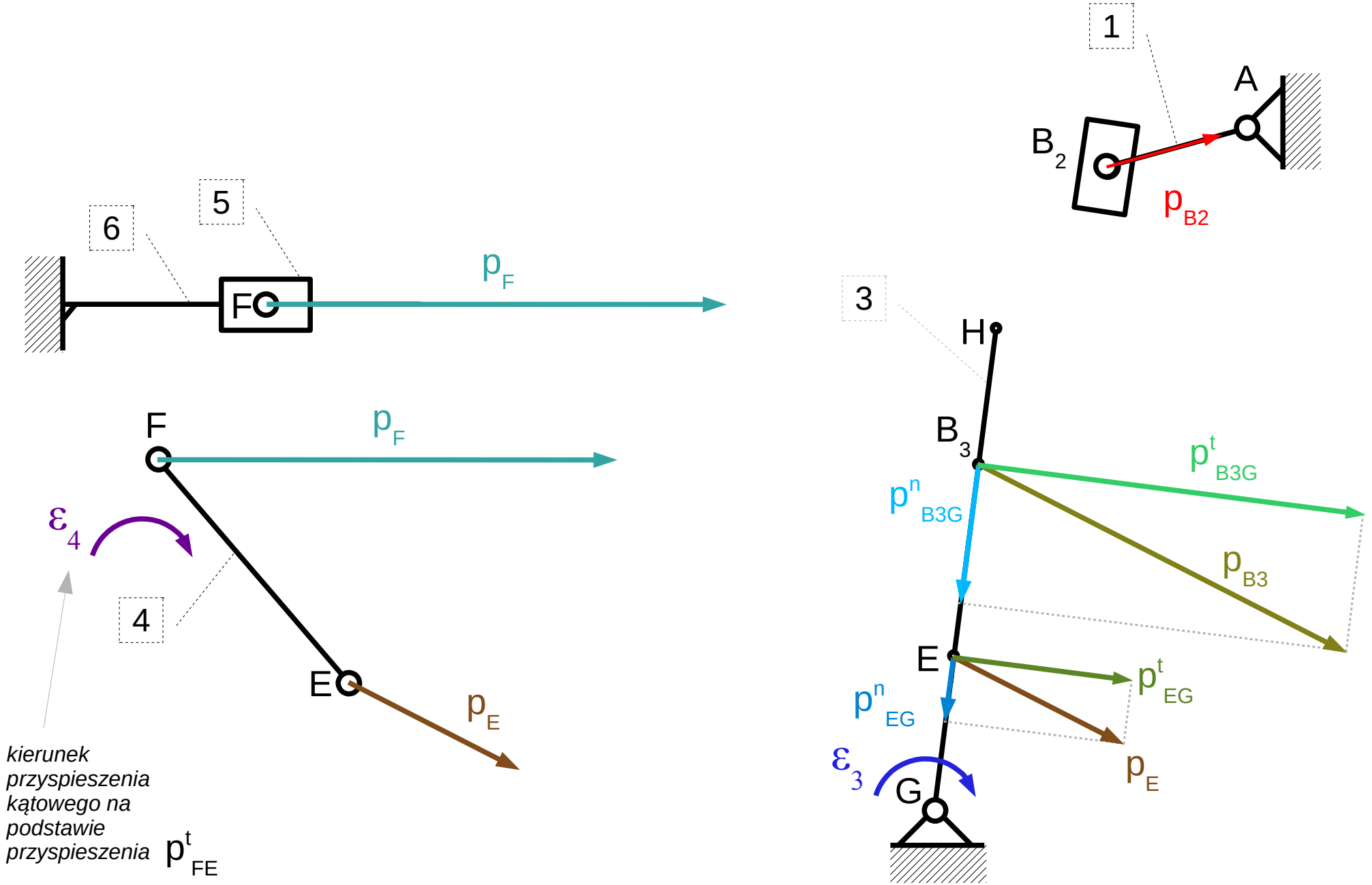
# Przyspieszenia punktów elementu 4



# Przyspieszenia punktów elementu 4



# Przyspieszenia w całym mechanizmie



# Wybrane przyspieszenia w mechanizmie