Faculty of Automotive and Construction Machinery Engineering
WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control

Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

Lecture 9

Frequency response.
Classification of basic automatic systems.

Transfer function

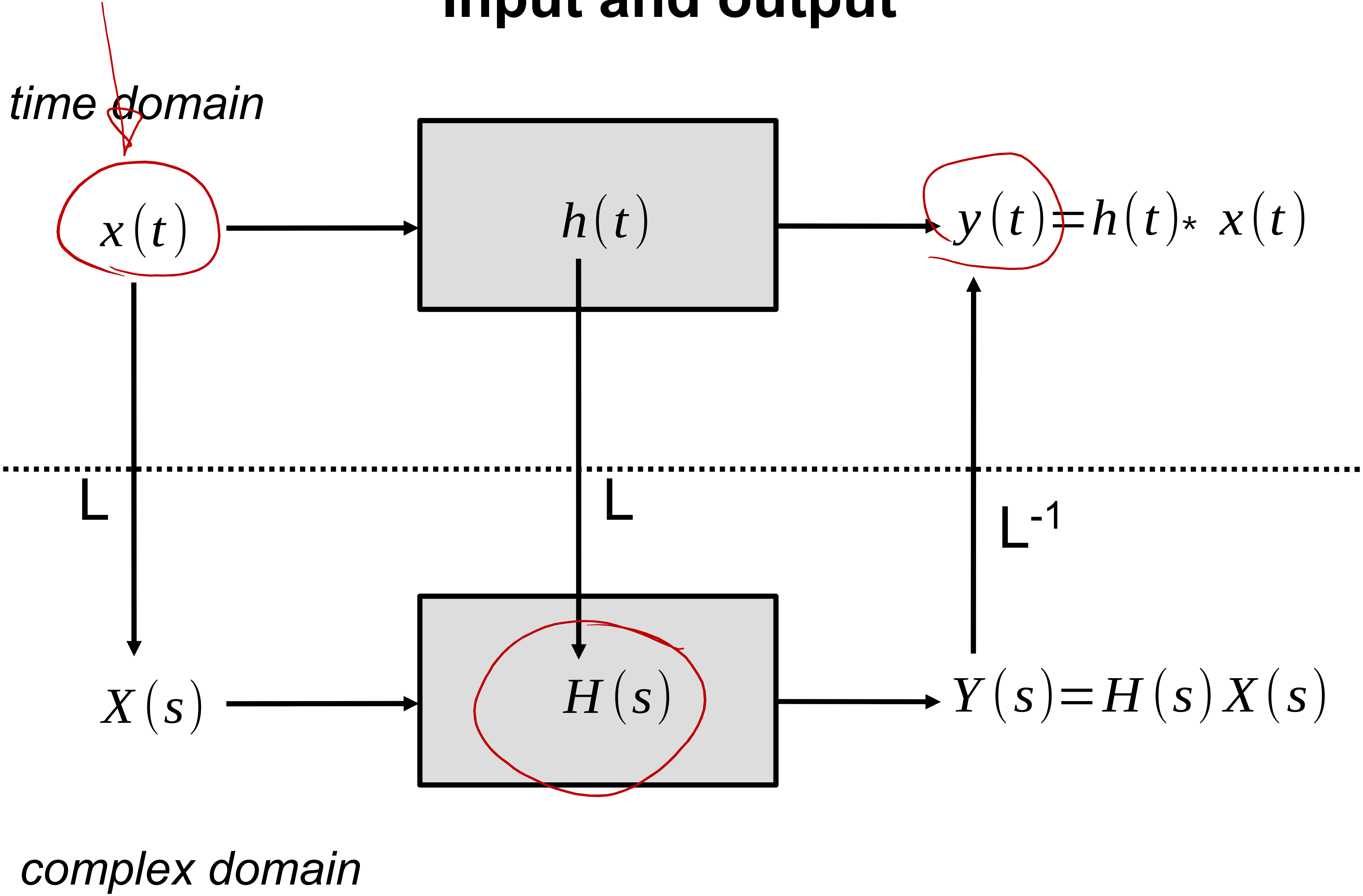
Linear time-invariant SISO system for continous-time input signal *x(t)* and output *y(t)* in a form

Transfer function $$H(s)=\frac{Y(s)}{X(s)}$$

$Y(s)$ - Laplace'a transform of an output

$X(s)$ - Laplace'a transform of an input

Input and output

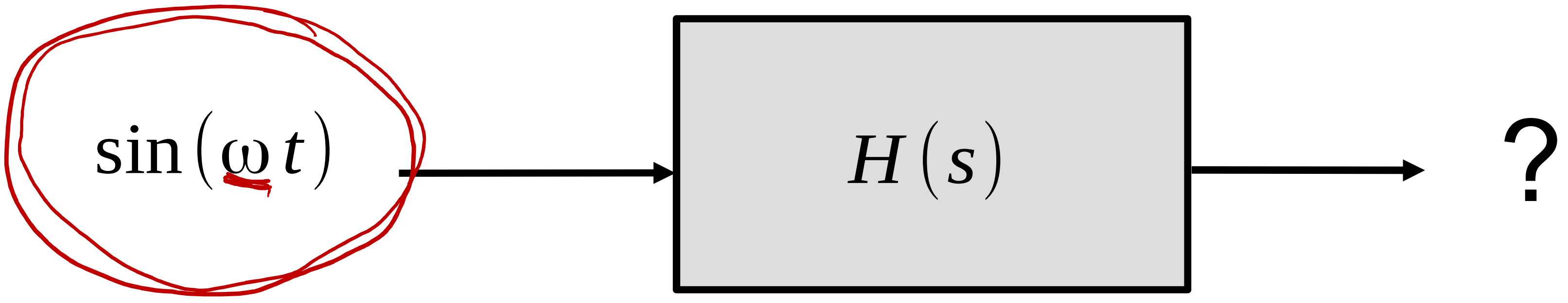


Transfer function & frequency response

Transfer function (Laplace domain) $H(s)$

Full system description
(for every possible input)

Transfer function & frequency response



Transfer function & frequency response

Transfer function & frequency response

Transfer function (Laplace domain)

$H(s)$

Full system description (for every possible input)

$s = j\omega$

Frequency response (Fourier domain)

$H(j\omega)$

Description of a system in <u>steady state with harmonic input</u>

Transfer function – frequency response

input: $x(t)=\sin(\omega t)$ transfer function: $H(s)$ output: $y(t)=A\sin(\omega t+\varphi)$

Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A\sin(\omega t + \varphi)$

harmonic excitation

steady state!!!

Transfer function – frequency response

input: $x(t)=\sin(\omega t)$ transfer function: $H(s)$ output: $y(t)=A\sin(\omega t+\varphi)$

$$H(s)\xrightarrow{s=j\omega} H(j\omega)=P(\omega)+jQ(\omega)$$

Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A\sin(\omega t + \varphi)$

$$H(s) \xrightarrow{s = j\omega} H(j\omega) = P(\omega) + jQ(\omega)$$

Transfer function – frequency response

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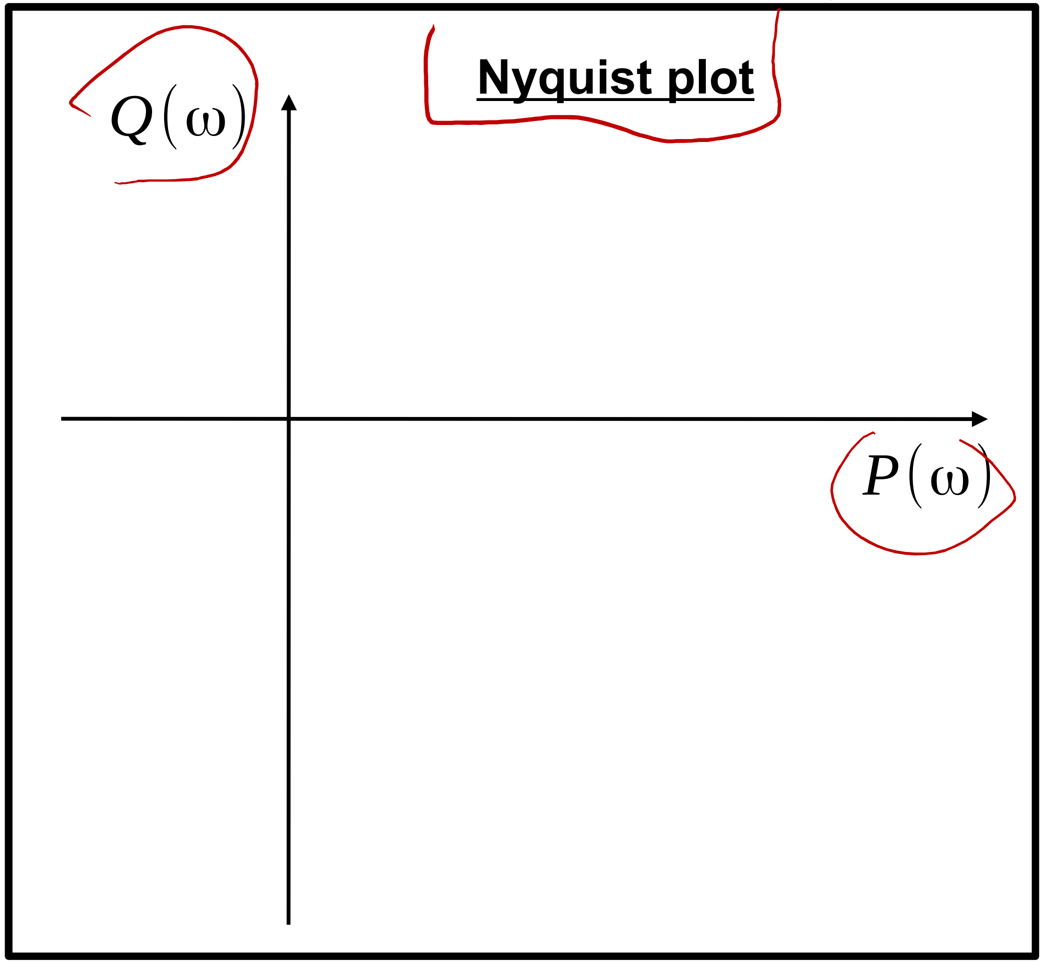
$\omega \in (0;\infty)$



Transfer function – frequency response

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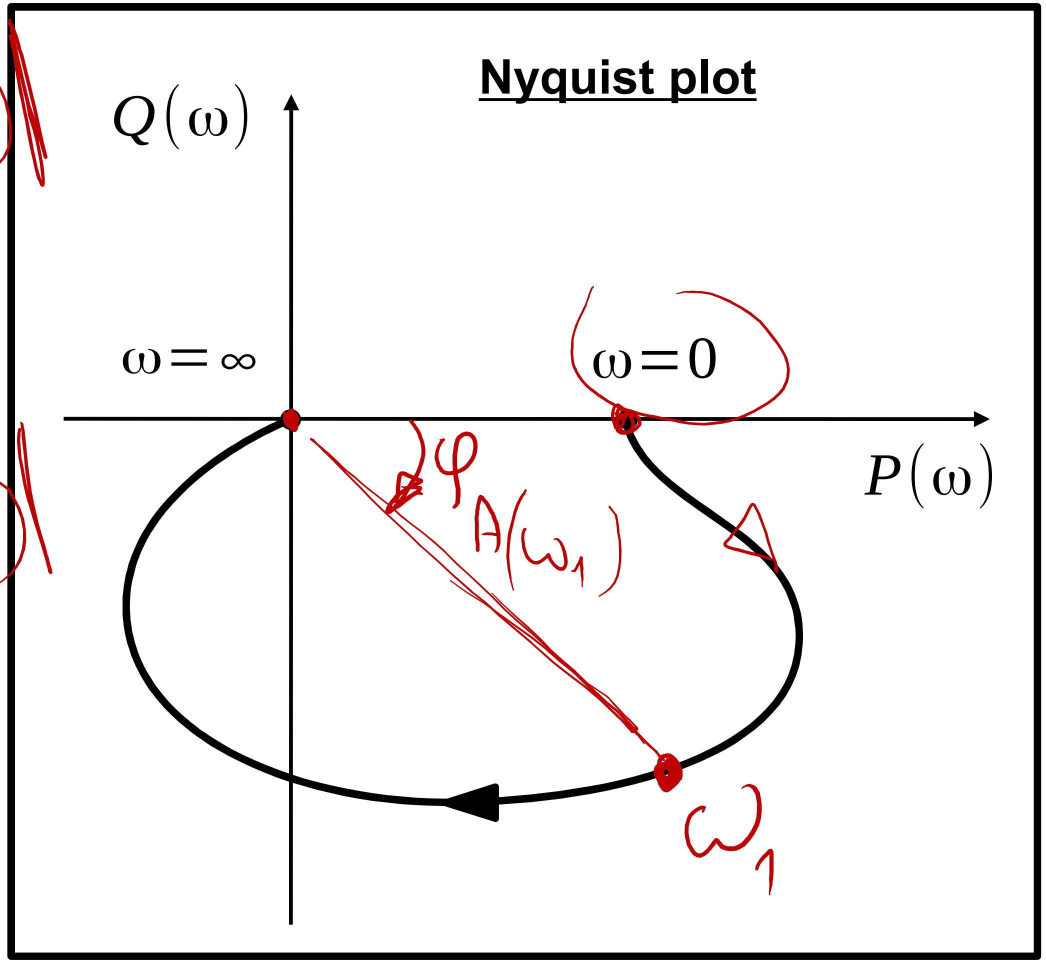
$$H(s)\xrightarrow{s=j\omega}H(j\omega)=P(\omega)+jQ(\omega)$$

$A(\omega)=\sqrt{P^2+Q^2}=|H(j\omega)|$

GAIN

$\varphi(\omega)=\operatorname{atan}\frac{Q}{P}=\operatorname{Arg}[H(j\omega)]$

delay



Transfer function – frequency response

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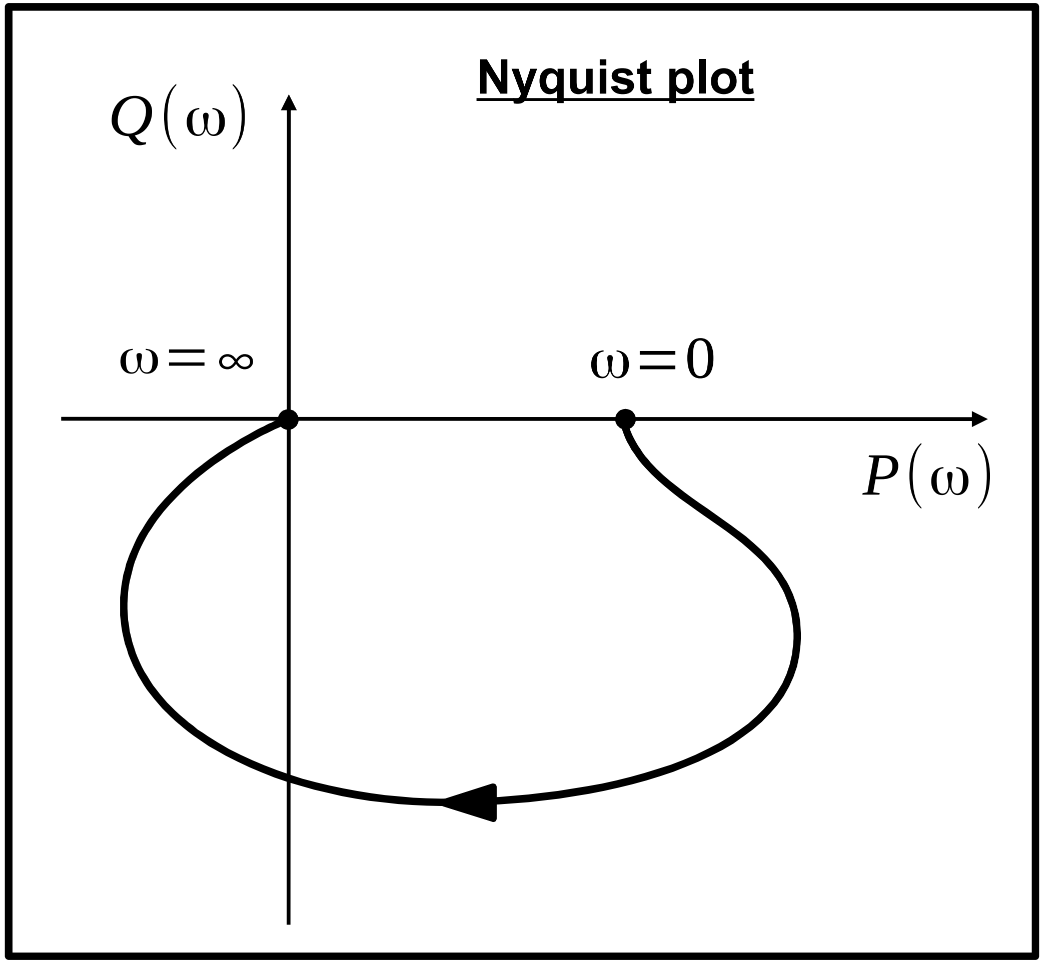
$$A(\omega)=|H(j\omega)|=\sqrt{P^2(\omega)+Q^2(\omega)}$$



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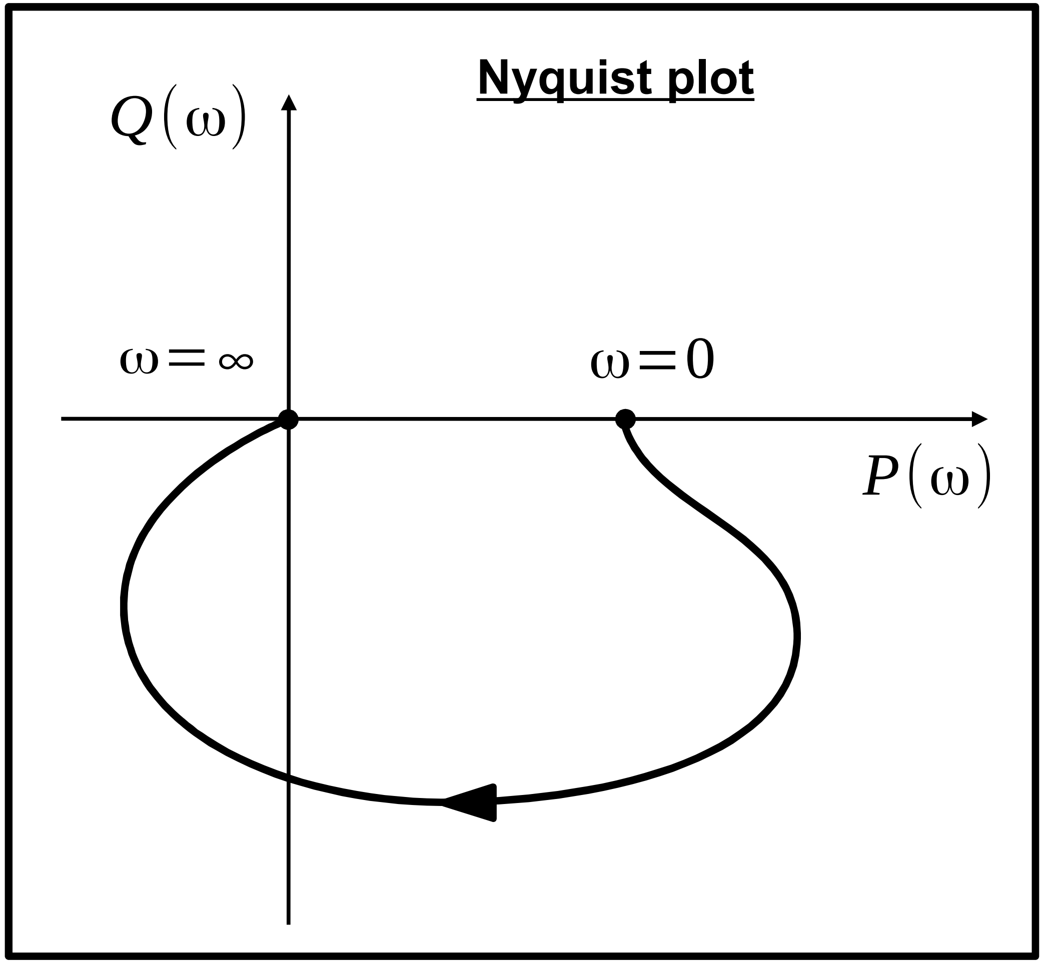

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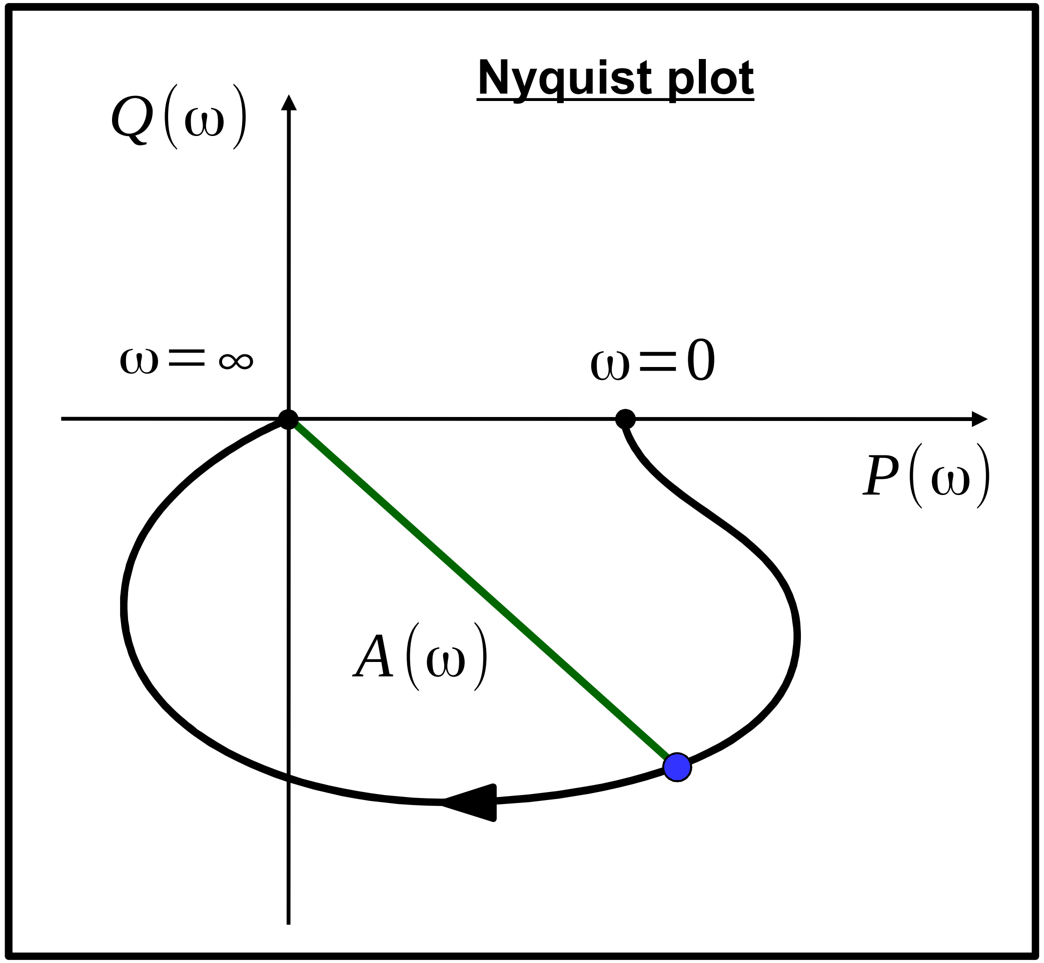

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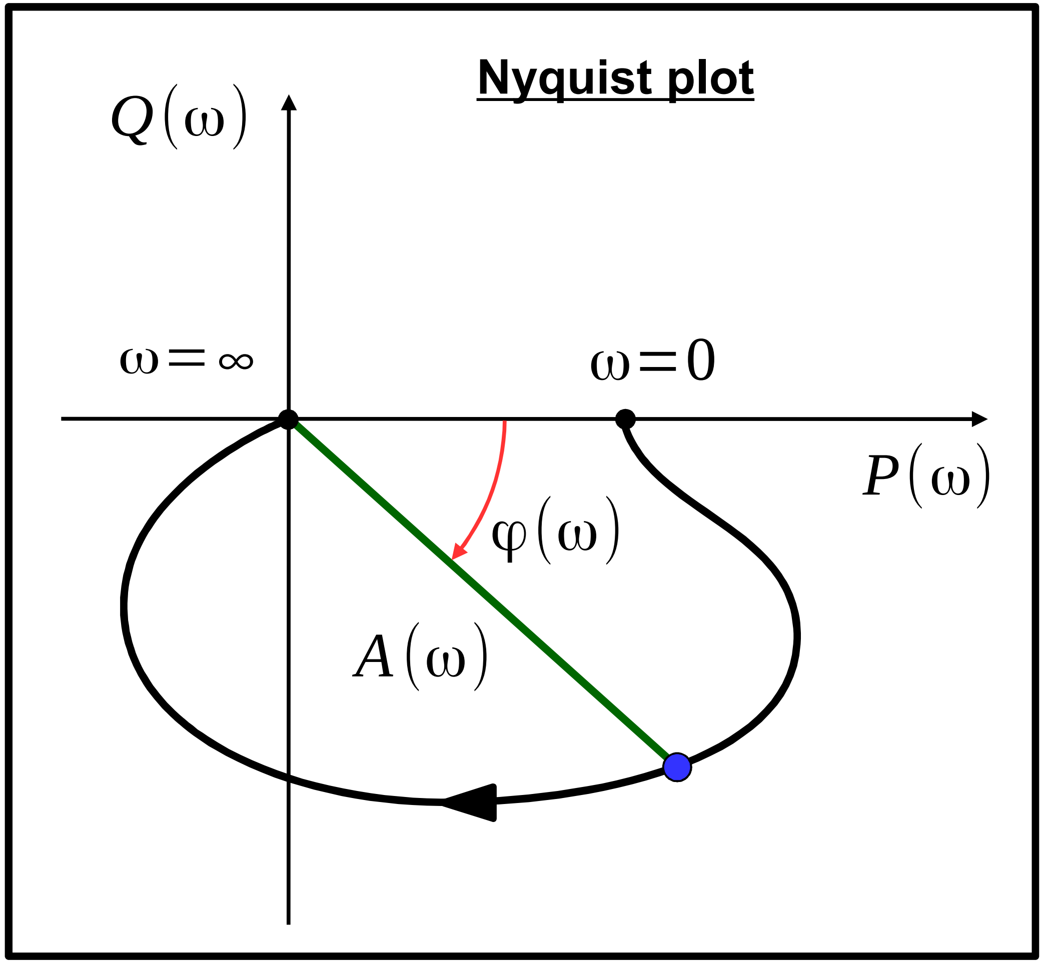
DELAY



Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A\sin(\omega t + \varphi)$

Bode Plot

Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A \sin(\omega t + \varphi)$

Bode Plot

gain (magnitude) plot

delay

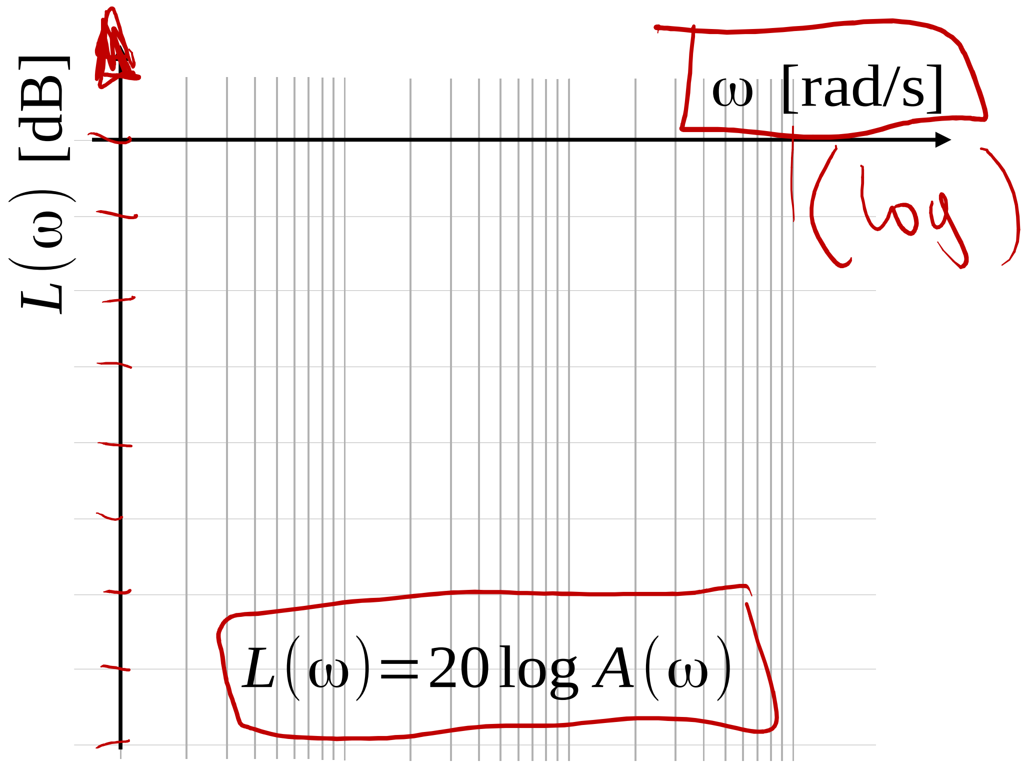
phase (phase shift) plot

Transfer function – frequency response

input: $x(t)=\sin(\omega t)$ transfer function: $H(s)$ output: $y(t)=A\sin(\omega t+\varphi)$

Bode Plot

gain (magnitude) plot



phase (phase shift) plot

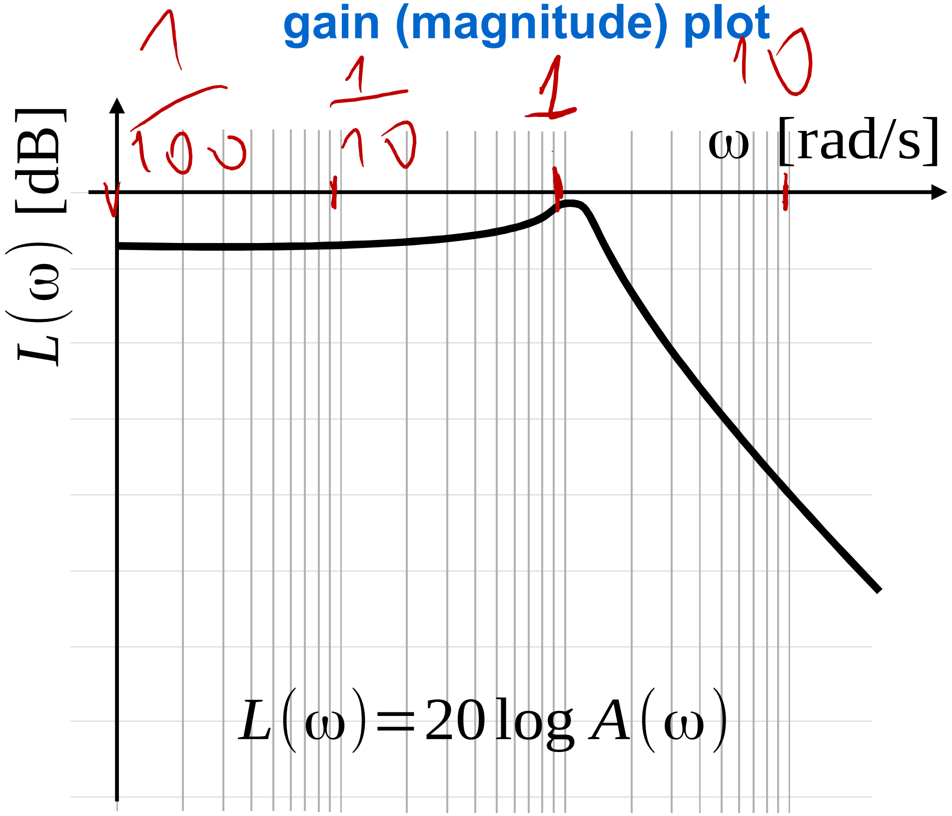
$$L(\omega)=20\cdot\log_{10}A(\omega)\ [dB]$$

Transfer function – frequency response

input: $x(t)=\sin(\omega t)$ transfer function: $H(s)$ output: $y(t)=A\sin(\omega t+\varphi)$

Bode Plot

gain (magnitude) plot



phase (phase shift) plot

Transfer function – frequency response

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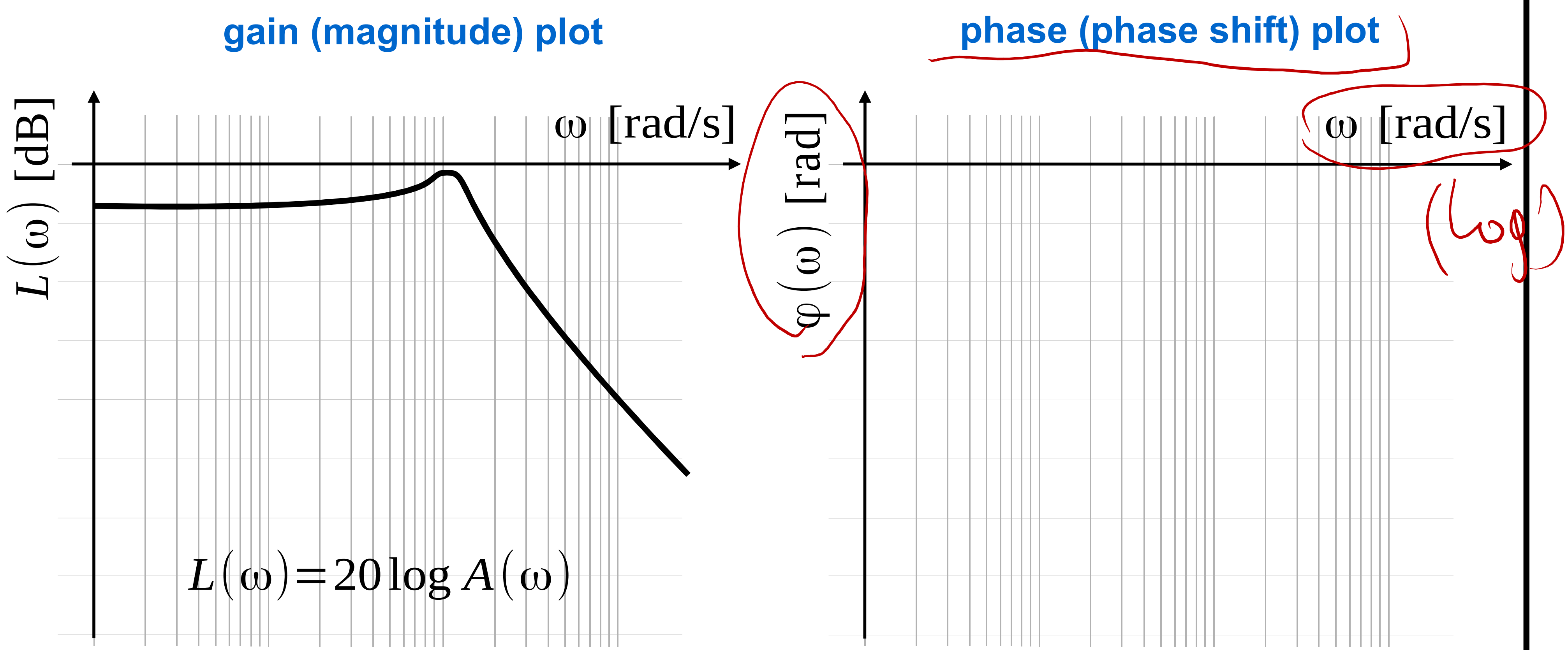
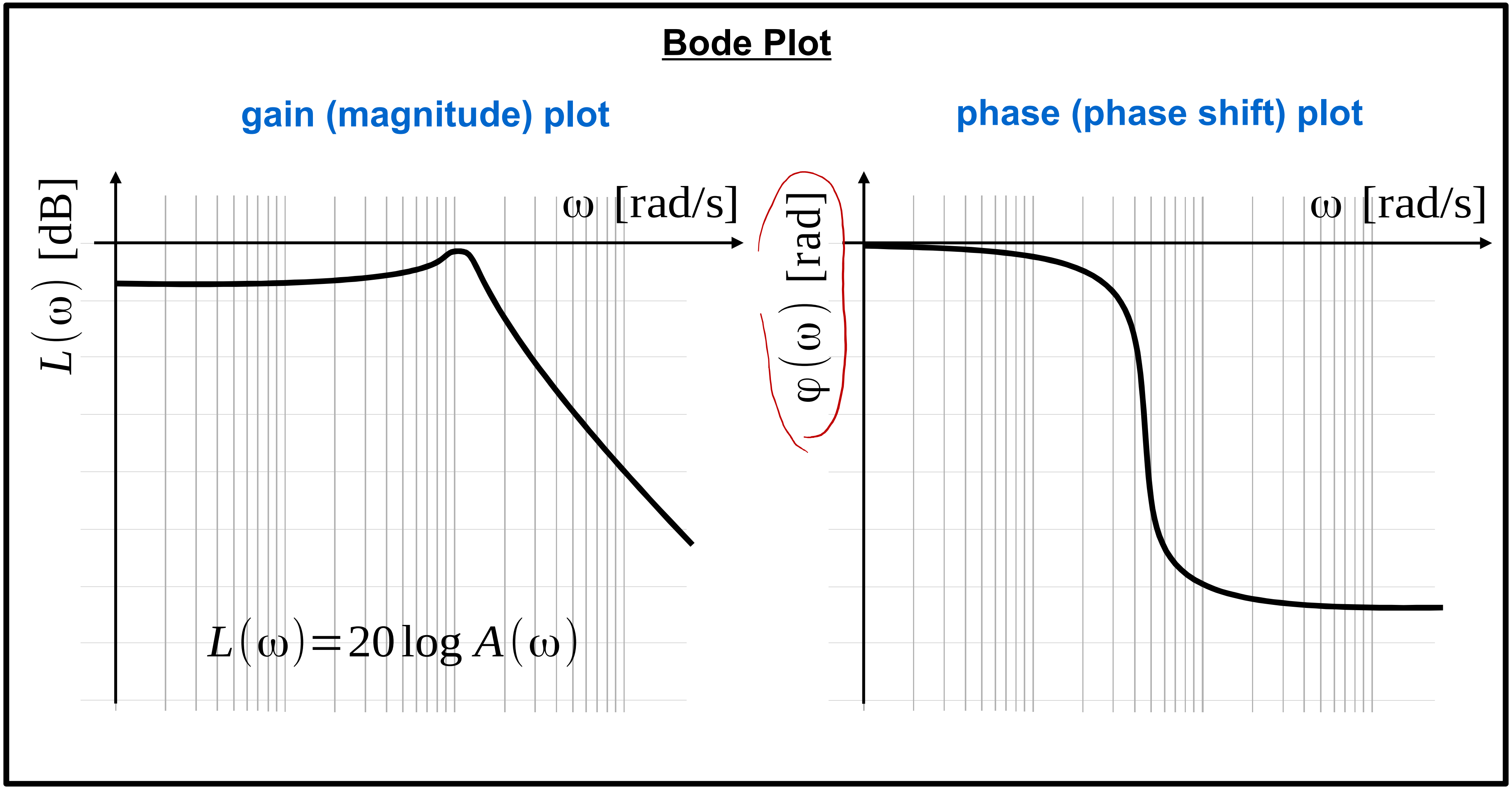
Bode Plot



Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A\sin(\omega t + \varphi)$



Transfer function – frequency response

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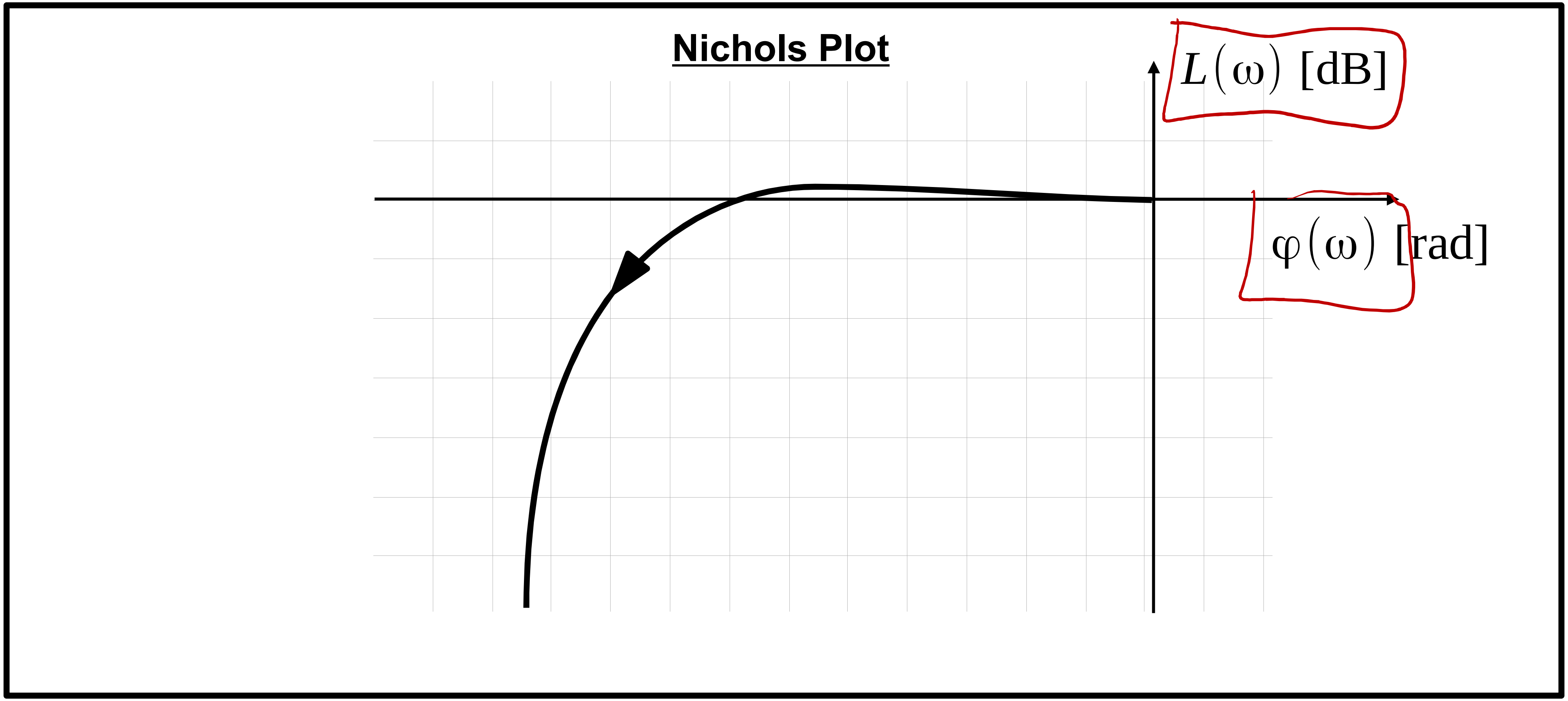


Transfer function – frequency response

A (gain)	20logA [dB]
1000	60
100	40
10	20
1	0
0.1	-20
0.01	-40
0.001	-60

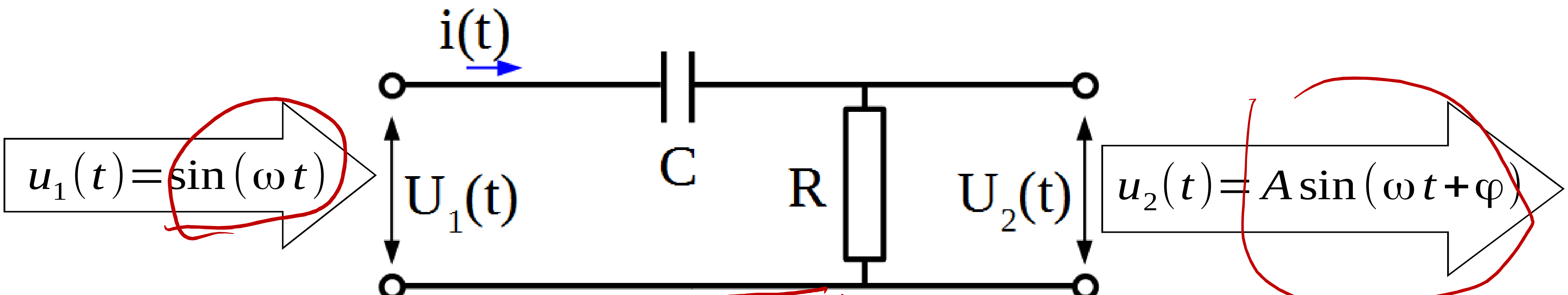
Transfer function – frequency response

Useful for:

- audio components (amplifiers, microphones, loudspeakers, cables)
- wireless components (antennas, amplifiers)
- vibrating systems (suspensions, drivetrains)
- control systems (regulators, objects)

Transfer function – frequency response

RC circuit example



$$H(s)=\frac{Ts}{1+Ts}$$

$$H(j\omega)=\frac{Tj\omega}{1+Tj\omega}=P(\omega)+jQ(\omega)$$

$$\frac{Tj\omega}{(1+Tj\omega)}\cdot\frac{1-Tj\omega}{(1-Tj\omega)}=\frac{Tj\omega(1-Tj\omega)}{1^2-T^2j^2\omega^2}=\frac{Tj\omega-T^2j^2\omega^2}{1+T^2\omega^2}$$

Transfer function – frequency response

RC circuit example

$$H(s)=\frac{Ts}{1+Ts}$$

$$H(j\omega)=\frac{Tj\omega+T^2\omega^2}{1+T^2\omega^2}=\frac{T^2\omega^2}{1+T^2\omega^2}+j\frac{T\omega}{1+T^2\omega^2}$$

$P(\omega)$ $Q(\omega)$

$$A(\omega)=\sqrt{P^2+Q^2}=\sqrt{\frac{T^4\omega^4+T^2\omega^2}{(1+T^2\omega^2)^2}}=$$

$$=\sqrt{\frac{T^2\omega^2(T^2\omega^2+1)}{(1+T^2\omega^2)^2}}=\frac{T\omega}{\sqrt{1+T^2\omega^2}}$$

$$L(\omega)=20\log A(\omega)=20\log\frac{T\omega}{\sqrt{1+T^2\omega^2}}=20\log T\omega-$$

$$20\log\sqrt{1+T^2\omega^2}$$

$$\varphi(\omega)=\operatorname{atan}\frac{Q}{P}=\operatorname{atan}\left(\frac{1}{T\omega}\right)$$

Transfer function – frequency response

RC circuit example

$$H(s)=\frac{Ts}{1+Ts}$$

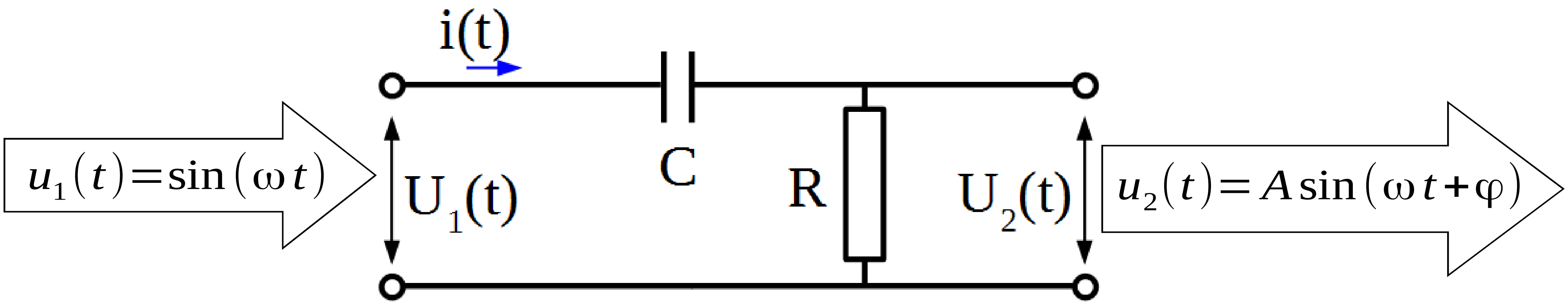
Transfer function – frequency response

RC circuit example

$$H(s) = \frac{Ts}{1+Ts}$$

Transfer function – frequency response

RC circuit example



$$H(s)=\frac{Ts}{1+Ts}$$

$s=j\omega$

$$H(j\omega)=\frac{Tj\omega}{1+Tj\omega}=\frac{Tj\omega}{1+Tj\omega}\cdot\frac{1-Tj\omega}{1-Tj\omega}=\frac{Tj\omega-T^2j^2\omega^2}{1^2-T^2j^2\omega^2}=\frac{Tj\omega+T^2\omega^2}{1^2+T^2\omega^2}=\frac{T^2\omega^2}{1^2+T^2\omega^2}+j\frac{T\omega}{1^2+T^2\omega^2}$$

$$P(\omega)=\frac{T^2\omega^2}{1+T^2\omega^2}\qquad Q(\omega)=\frac{T\omega}{1+T^2\omega^2}\qquad A(\omega)=|H(j\omega)|=\sqrt{P^2(\omega)+Q^2(\omega)}=\frac{T\omega}{\sqrt{T^2\omega^2+1}}$$

$$L(\omega)=20\log A(\omega)=20\log\frac{T\omega}{\sqrt{T^2\omega^2+1}}=20\log T\omega-20\log\sqrt{T^2\omega^2+1}$$

$$\varphi(\omega)=\arg H(j\omega)=\arctan\frac{Q}{P}=\arctan\left(\frac{1}{T\omega}\right)$$

Transfer function – frequency response

RC circuit example

Nyquist plot

$$P(\omega)=\frac{T^2\omega^2}{1+T^2\omega^2} \qquad Q(\omega)=\frac{T\omega}{1+T^2\omega^2}$$

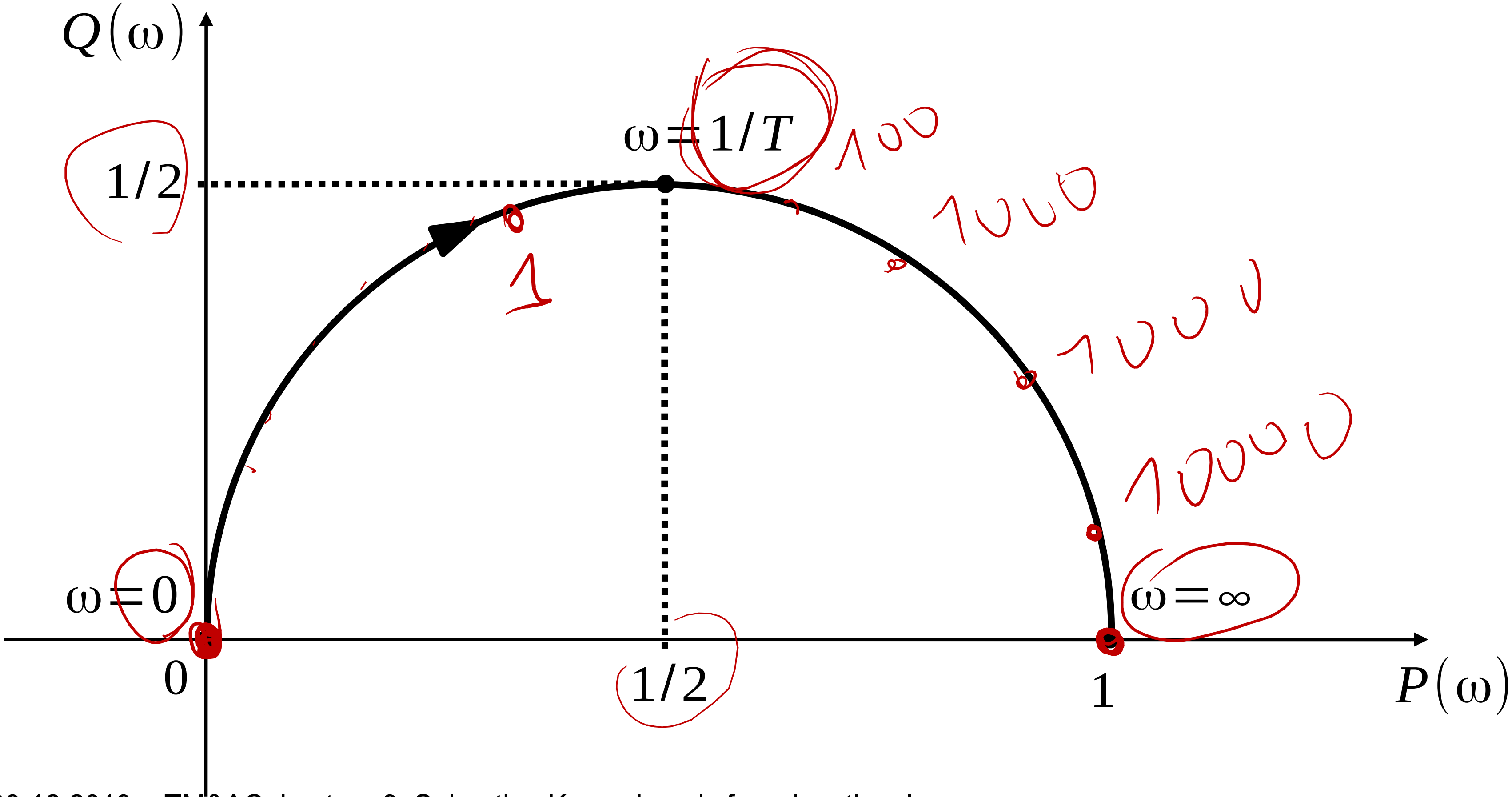
Transfer function – frequency response

RC circuit example

$(P-\frac{1}{2})^2+Q^2=\frac{1}{2}$

Nyquist plot

$$P(\omega)=\frac{T^2\omega^2}{1+T^2\omega^2} \qquad Q(\omega)=\frac{T\omega}{1+T^2\omega^2}$$



Transfer function – frequency response

RC circuit example

$$A(\omega)=\frac{T\omega}{\sqrt{T^2\omega^2+1}}$$

LIN.

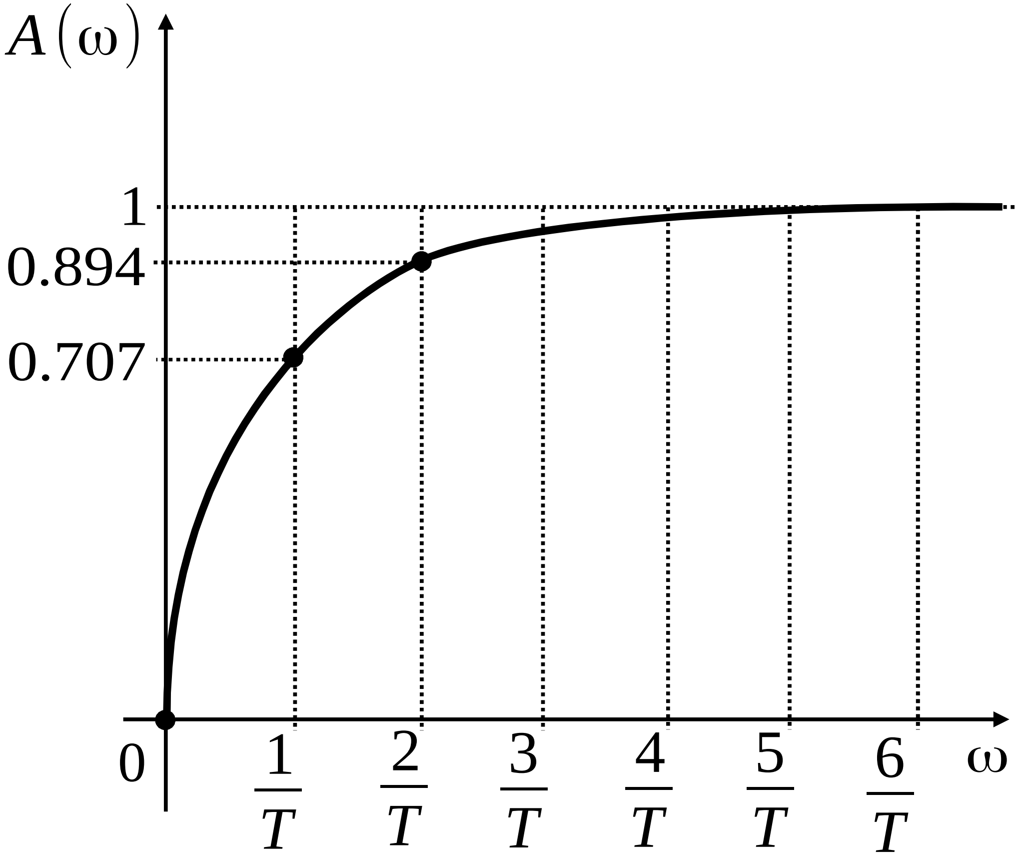
$$\varphi(\omega)=\arctan\left(\frac{1}{T\omega}\right)$$

Transfer function – frequency response

RC circuit example

$$A(\omega)=\frac{T\omega}{\sqrt{T^2\omega^2+1}}$$

$$\varphi(\omega)=\arctan\left(\frac{1}{T\omega}\right)$$

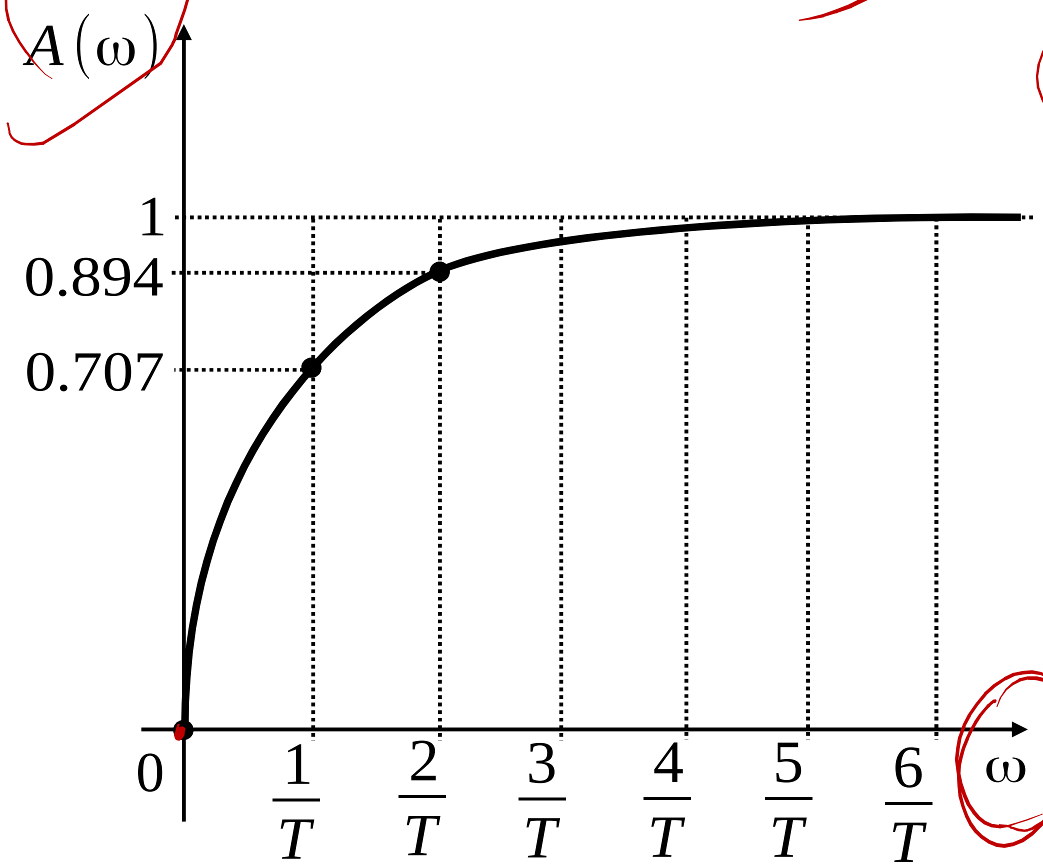


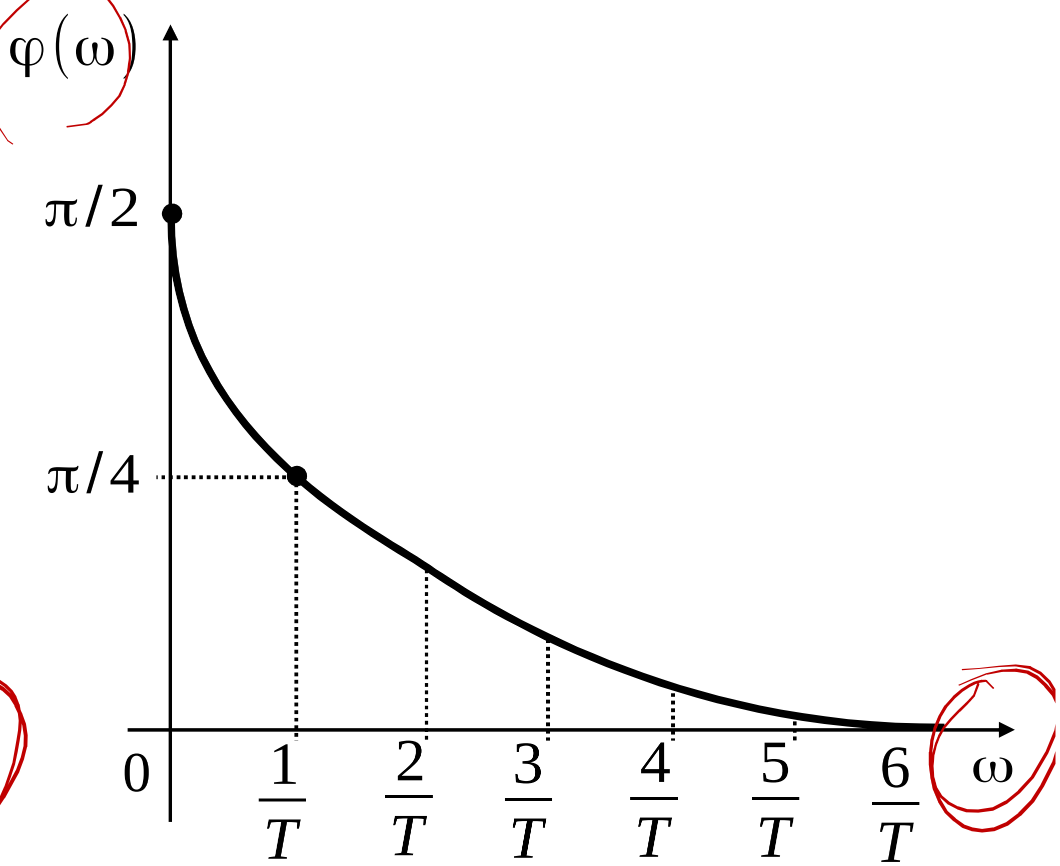
Transfer function – frequency response

RC circuit example

$$A(\omega)=\frac{T\omega}{\sqrt{T^2\omega^2+1}}$$

$$\varphi(\omega)=\arctan\left(\frac{1}{T\omega}\right)$$





Transfer function – frequency response

RC circuit example

$$L(\omega) = 20\log T\omega - 20\log\sqrt{T^2\omega^2+1}$$

$$\varphi(\omega) = \arctan\left(\frac{1}{T\omega}\right)$$

Transfer function – frequency response

RC circuit example

$$L(\omega) = 20\log T\omega - 20\log\sqrt{T^2\omega^2+1}$$

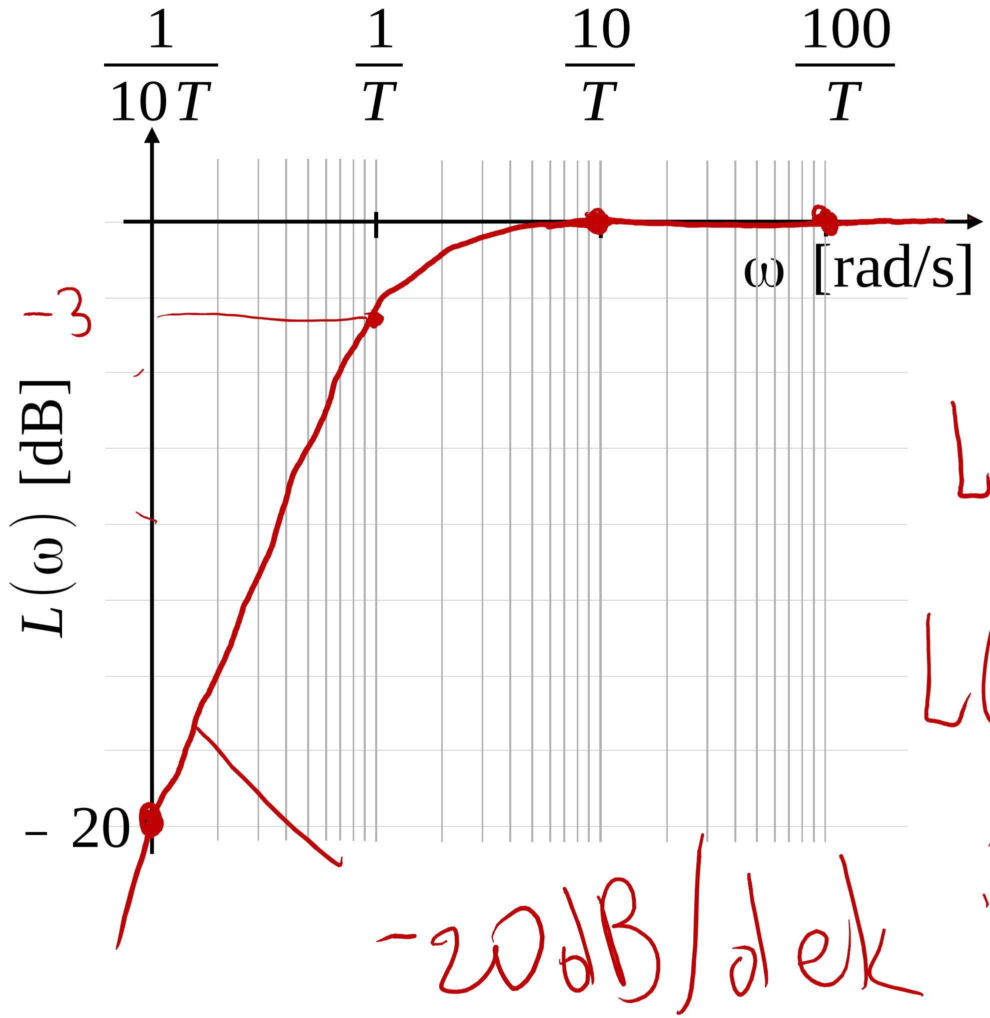
$$L\left(\omega=\frac{1}{T}\right) = 20\log 1 - 20\log\sqrt{2} \approx -3$$

$$L\left(\omega=\frac{10}{T}\right) = 20\log 10 - 20\log\sqrt{101} \approx 0$$

$$L\left(\omega=\frac{100}{T}\right) = 20\log 100 - 20\log\sqrt{10001} = 0$$

$$L\left(\omega=\frac{1}{10T}\right) = 20\log\frac{1}{10} - 20\log\sqrt{\frac{101}{100}} = -20$$

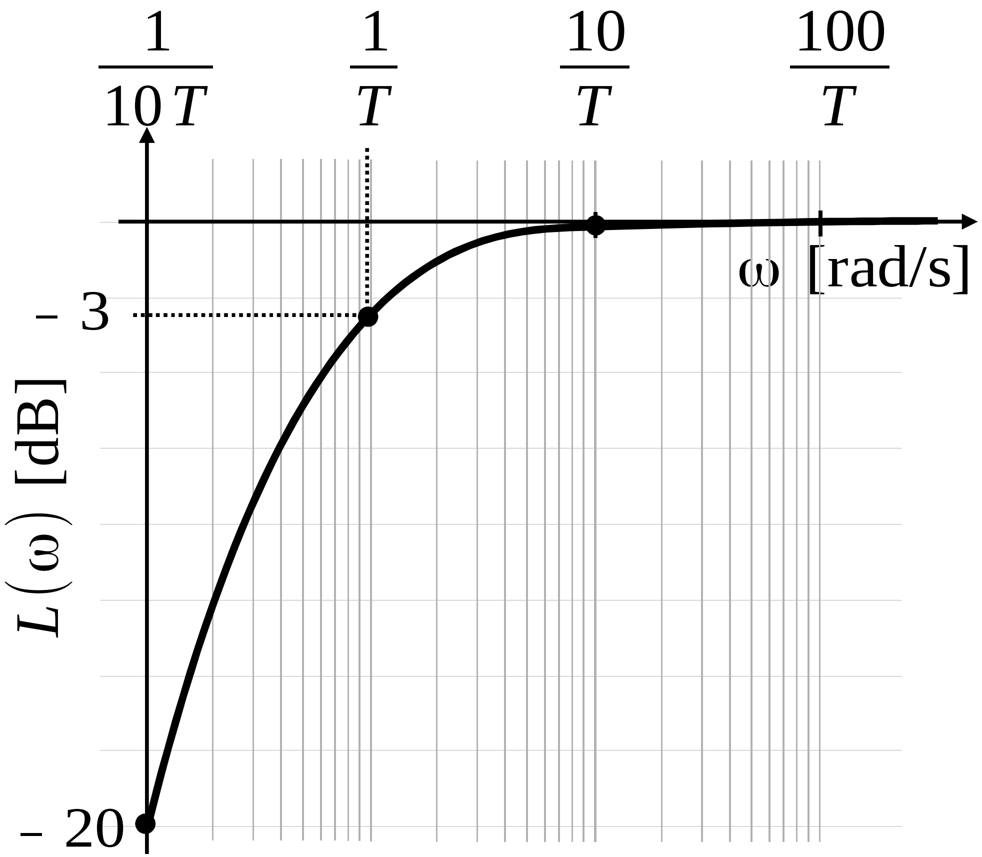
$$L\left(\omega=\frac{1}{100T}\right) = 20\log\frac{1}{100} - 20\log\sqrt{\frac{10001}{10000}} = -40$$



Transfer function – frequency response

RC circuit example

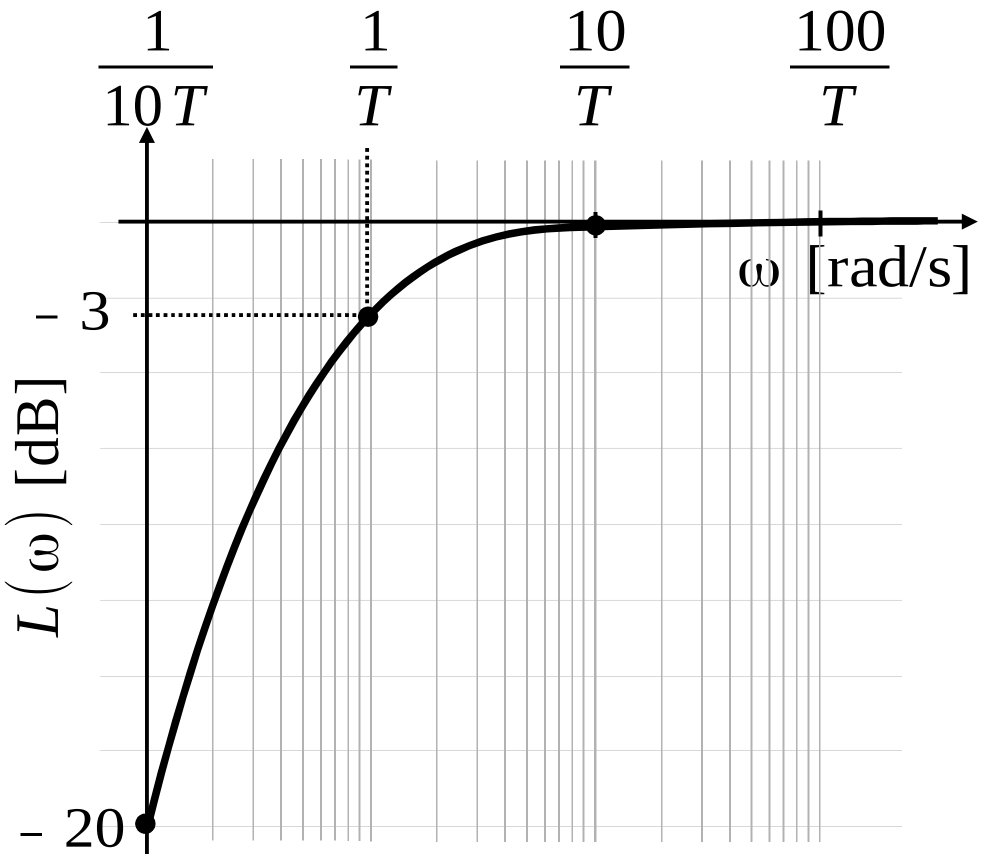
$$L(\omega)=20\log T\omega - 20\log\sqrt{T^2\omega^2+1}$$



Transfer function – frequency response

RC circuit example

$$L(\omega) = 20\log T\omega - 20\log\sqrt{T^2\omega^2+1}$$



$$L\left(\omega=\frac{1}{T}\right)=20\log 1-20\log\sqrt{2}\approx -3$$

$$L\left(\omega=\frac{10}{T}\right)=20\log 10-20\log\sqrt{101}\approx 0$$

$$L\left(\omega=\frac{100}{T}\right)=20\log 100-20\log\sqrt{10001}\approx 0$$

$$L\left(\omega=\frac{1}{10T}\right)=20\log 0{,}1-20\log\sqrt{1{,}01}\approx -20$$

$$L\left(\omega=\frac{1}{100T}\right)=20\log 0{,}01-20\log\sqrt{1{,}0001}\approx -40$$

Transfer function – frequency response

RC circuit example

$$\varphi(\omega = \frac{1}{T}) = \operatorname{atan}(1) = \frac{\pi}{4} = 45^\circ$$

$$\varphi(\omega = \frac{10}{T}) = \operatorname{atan}(\frac{1}{10}) \approx 6^\circ$$

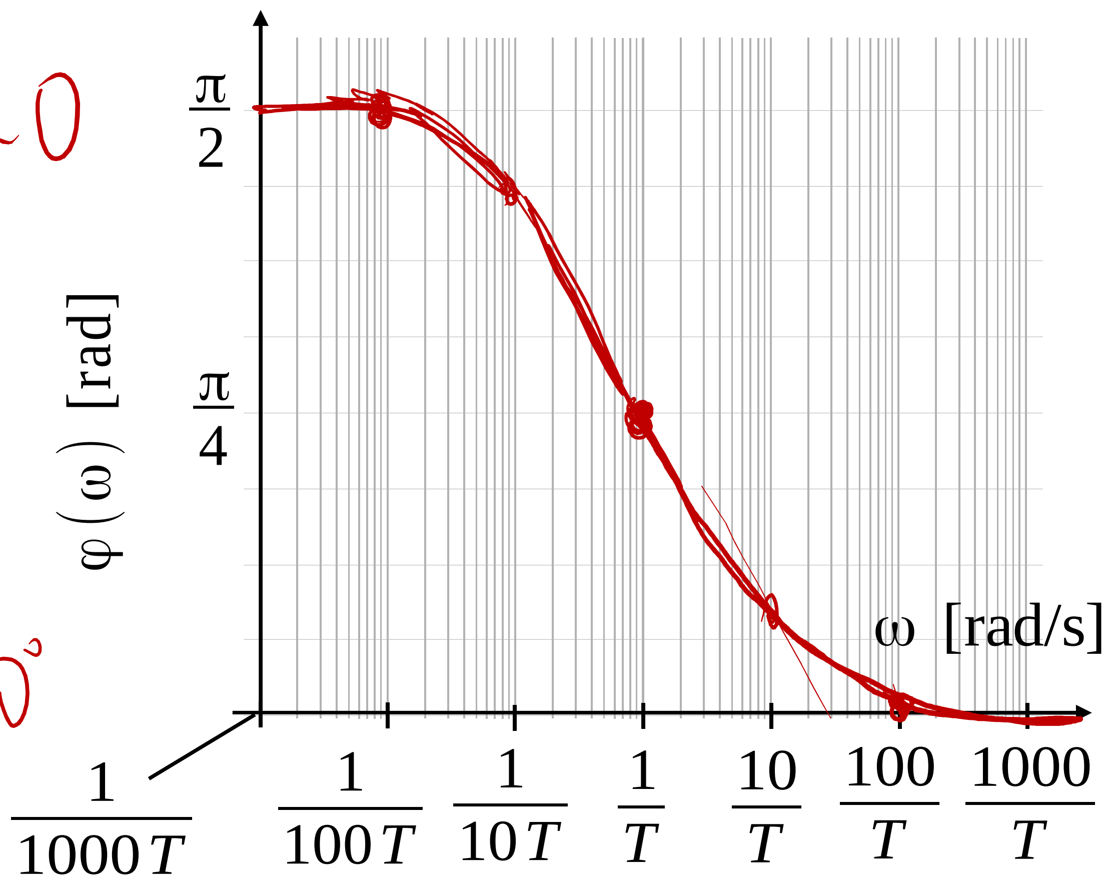
$$\varphi(\omega = \frac{100}{T}) = \operatorname{atan}(\frac{1}{100}) \simeq 0{,}7^\circ \approx 0$$

$$\varphi(\omega = \frac{1000}{T}) = 0$$

$$\varphi(\omega = \frac{1}{10T}) = \operatorname{atan}(10) \simeq 84^\circ$$

$$\varphi(\omega = \frac{1}{100T}) = \operatorname{atan}(100) \simeq 90^\circ$$

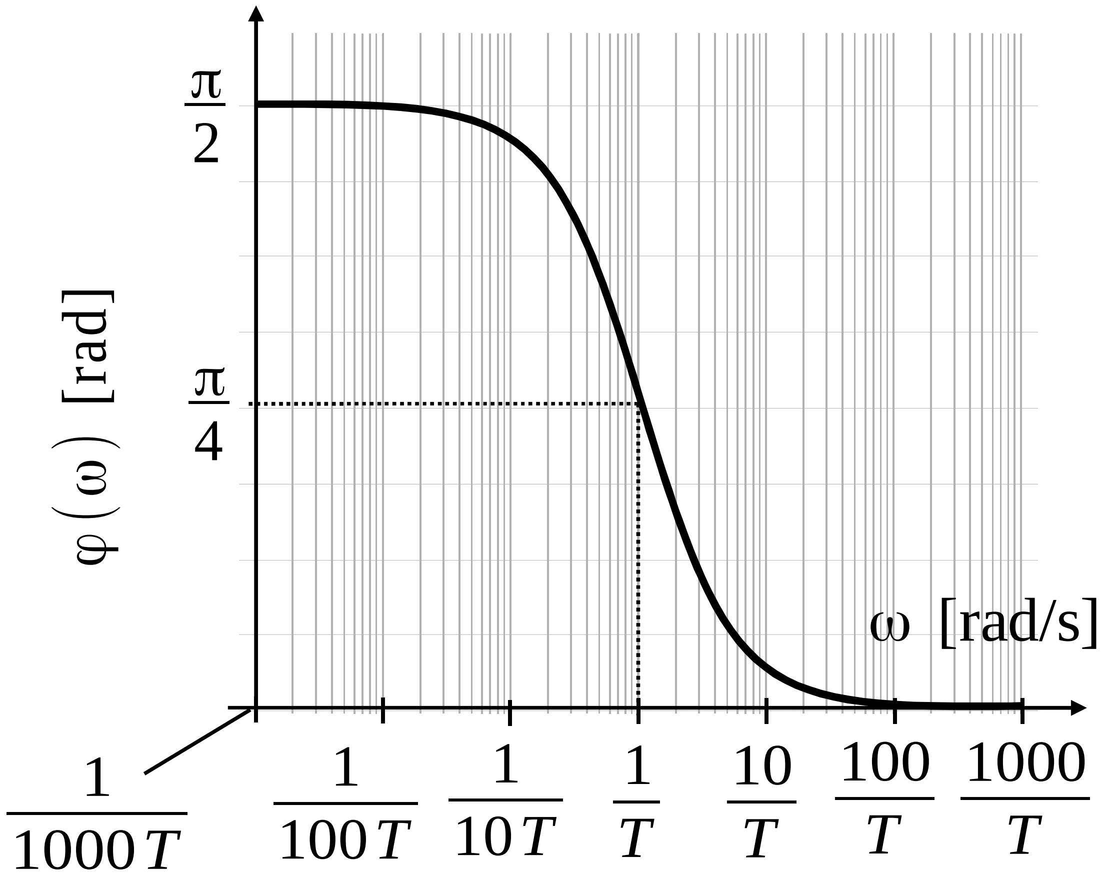
$$\varphi(\omega) = \arctan\left(\frac{1}{T\omega}\right)$$



Transfer function – frequency response

RC circuit example

$$\varphi(\omega) = \arctan\left(\frac{1}{T\omega}\right)$$



Transfer function – frequency response

RC circuit example

$$\varphi(\omega)=\arctan\left(\frac{1}{T\omega}\right)$$

$$\varphi\left(\omega=\frac{1}{T}\right)=\arctan 1=\frac{\pi}{4}=45^{o}$$

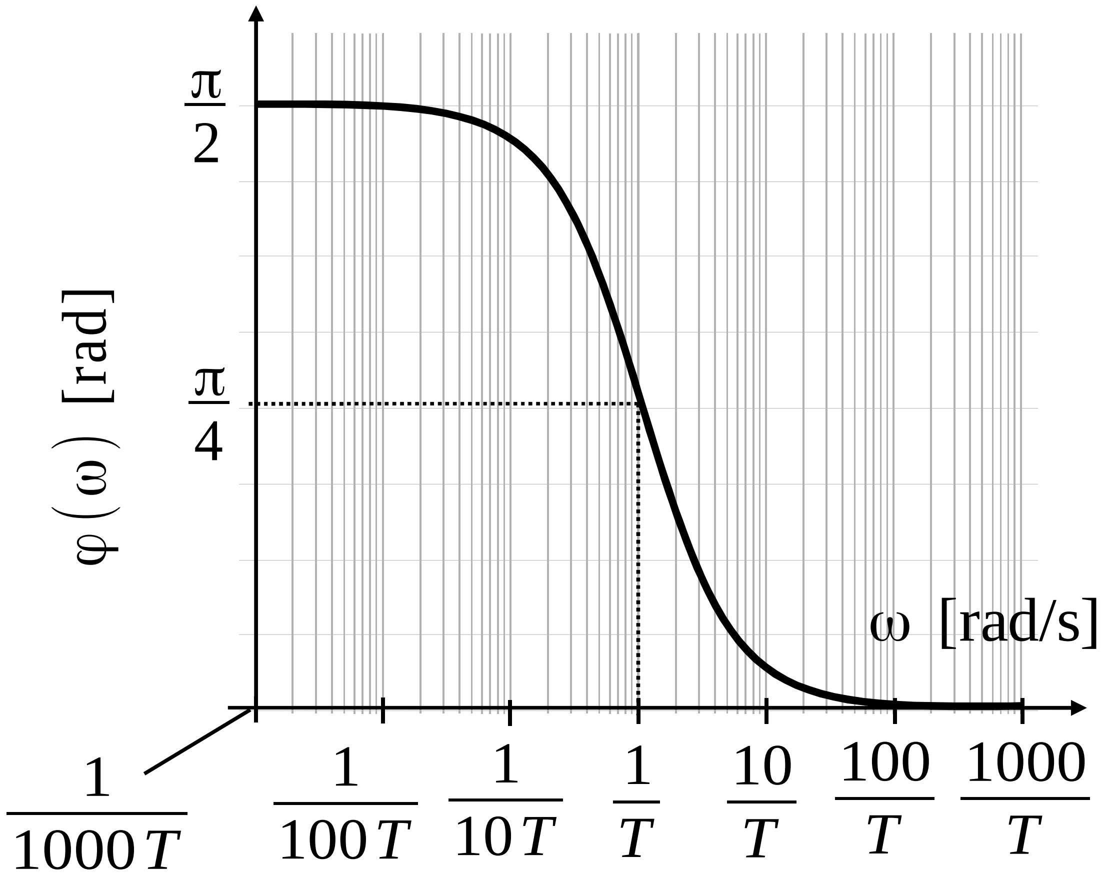
$$\varphi\left(\omega=\frac{10}{T}\right)=\arctan\frac{1}{10}=0{,}1=5{,}71^{o}$$

$$\varphi\left(\omega=\frac{100}{T}\right)=\arctan\frac{1}{100}=0{,}01=0{,}57^{o}$$

$$\varphi\left(\omega=\frac{1000}{T}\right)=\arctan\frac{1}{1000}=0{,}001=0{,}06^{o}$$

$$\varphi\left(\omega=\frac{1}{10T}\right)=\arctan 10=1{,}47=84{,}29^{o}$$

$$\varphi\left(\omega=\frac{1}{100T}\right)=\arctan 100=1{,}56=89{,}43^{o}$$

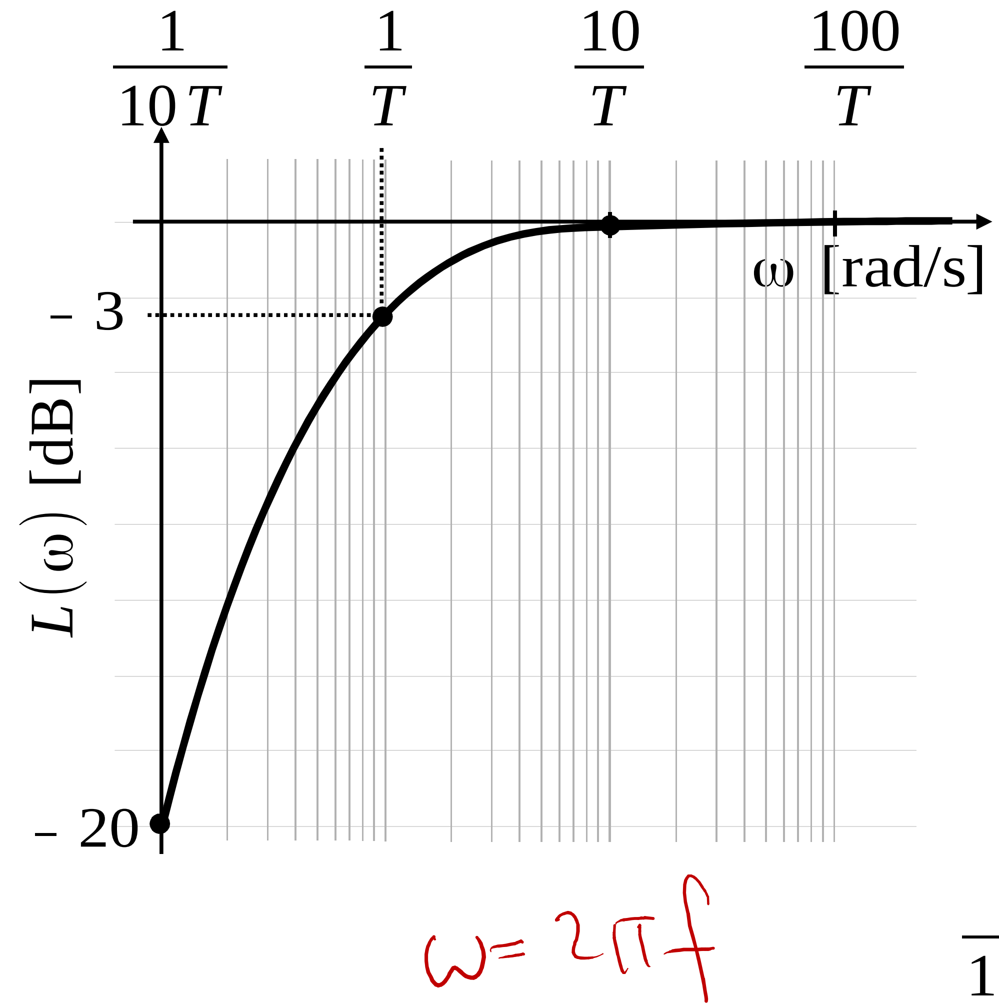


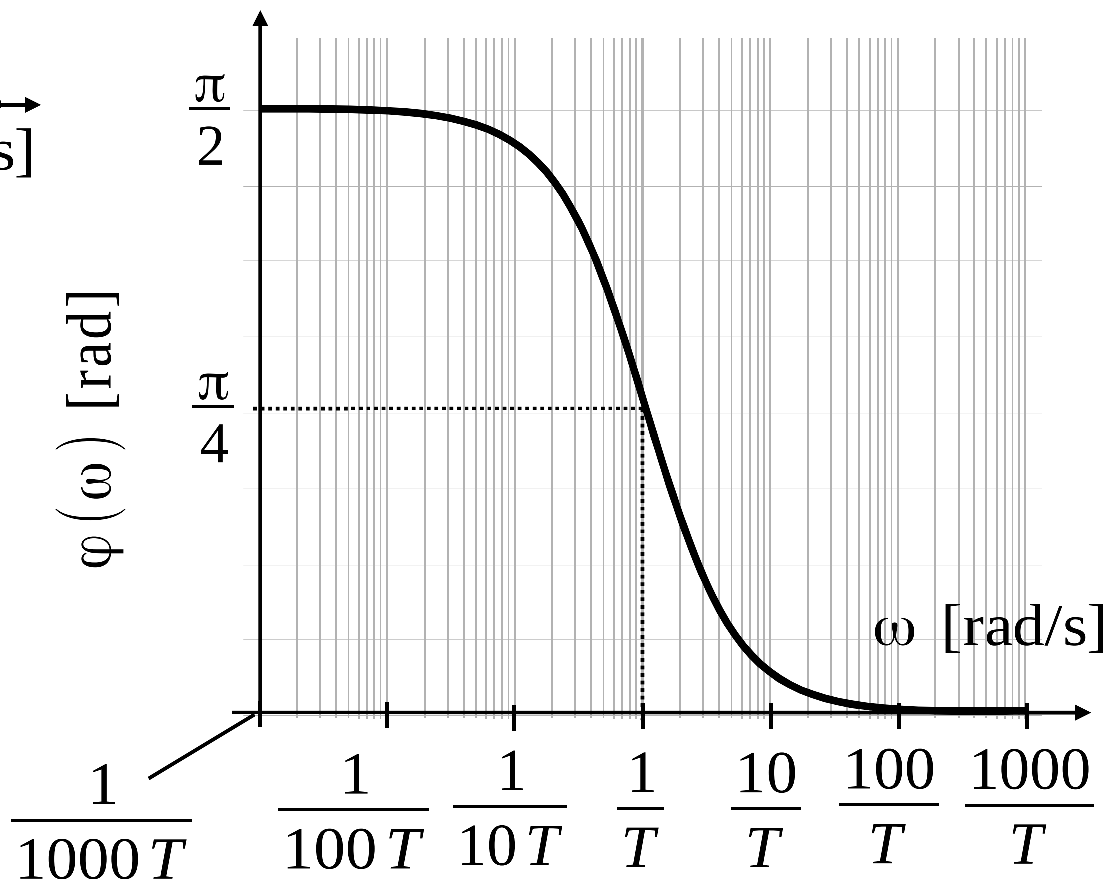
Transfer function – frequency response

RC circuit example

$$L(\omega) = 20\log T\omega - 20\log\sqrt{T^2\omega^2+1} \qquad \varphi(\omega) = \arctan\left(\frac{1}{T\omega}\right)$$

Bode plot

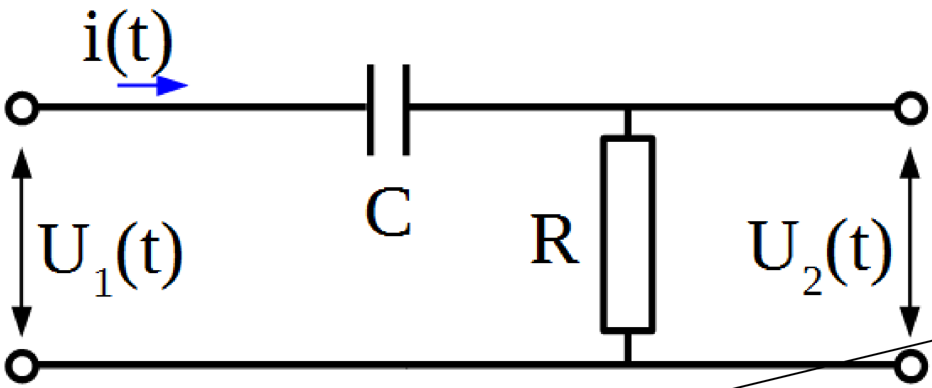




Transfer function – frequency response

RC circuit example

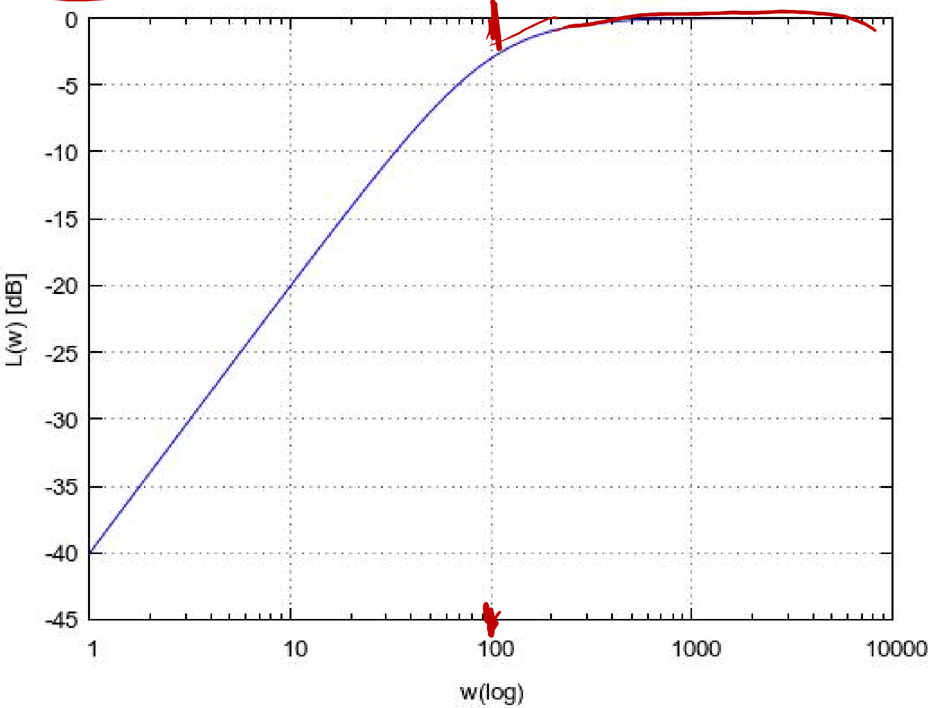
$$u_1(t) = \sin(\omega t)$$

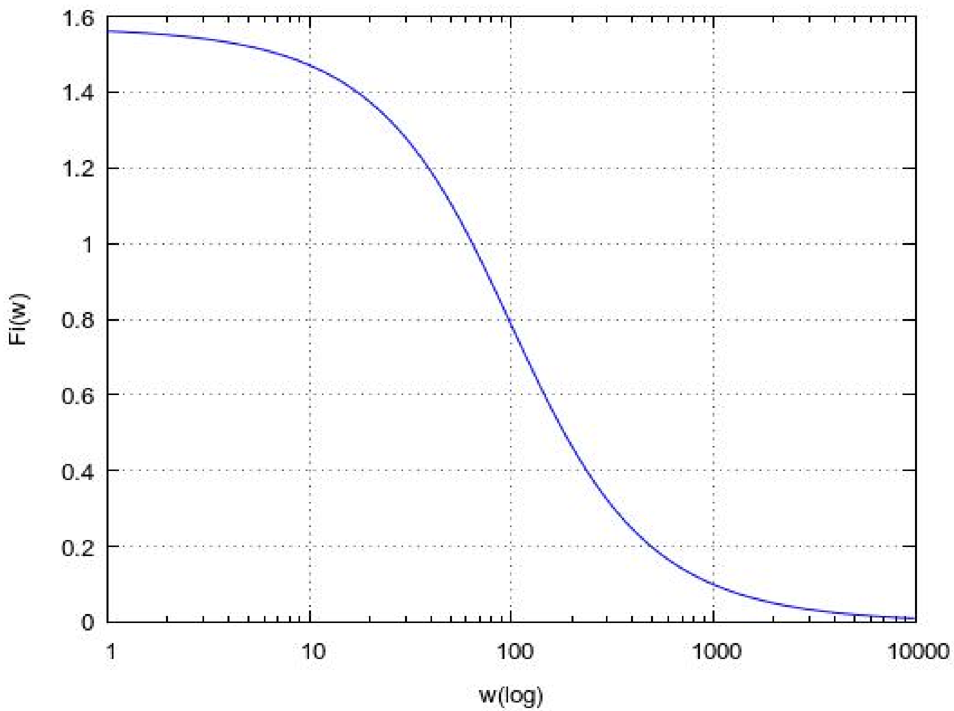


$$u_2(t) = A\sin(\omega t + \varphi)$$

Example:

$R = 1\,k\Omega,\ C = 10\,\mu F$

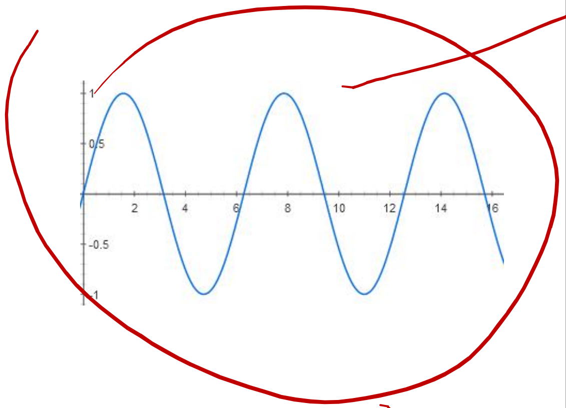


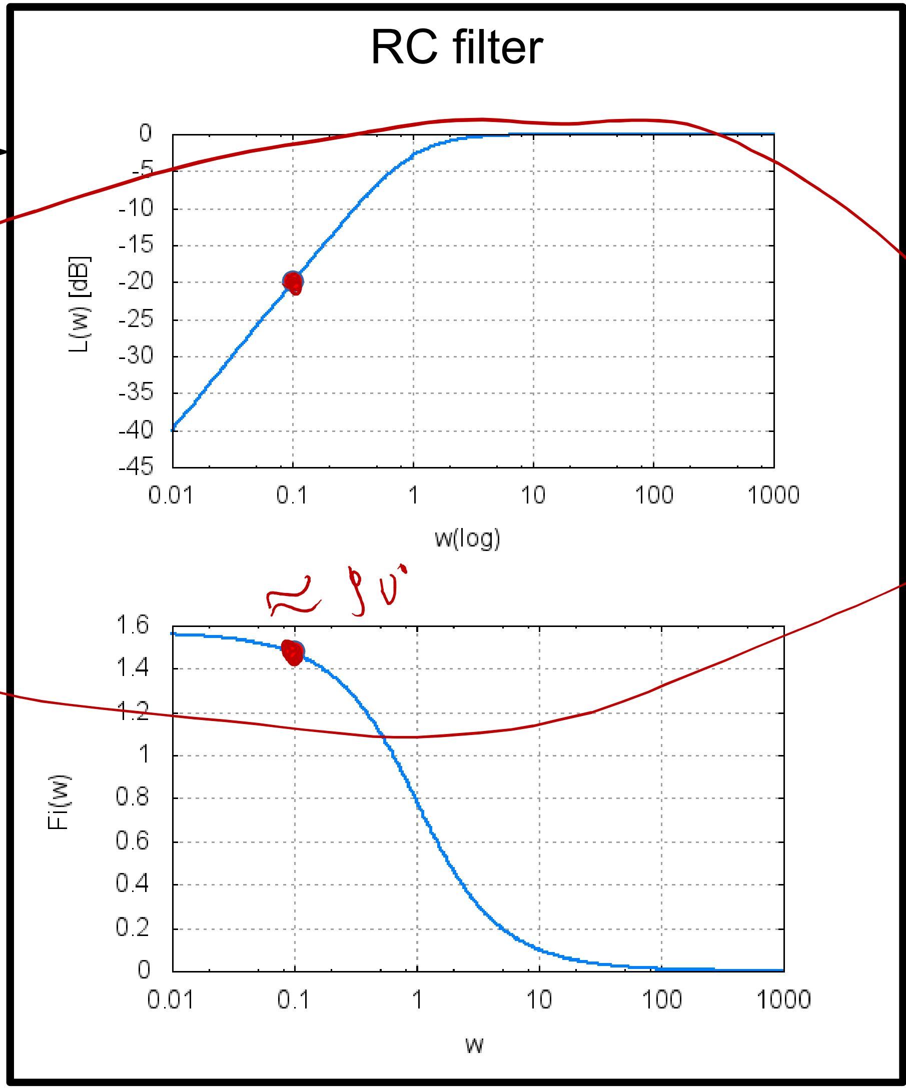


Transfer function – frequency response

RC circuit example

$$u_1(t)=\sin(\omega t)$$



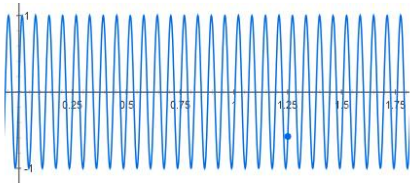


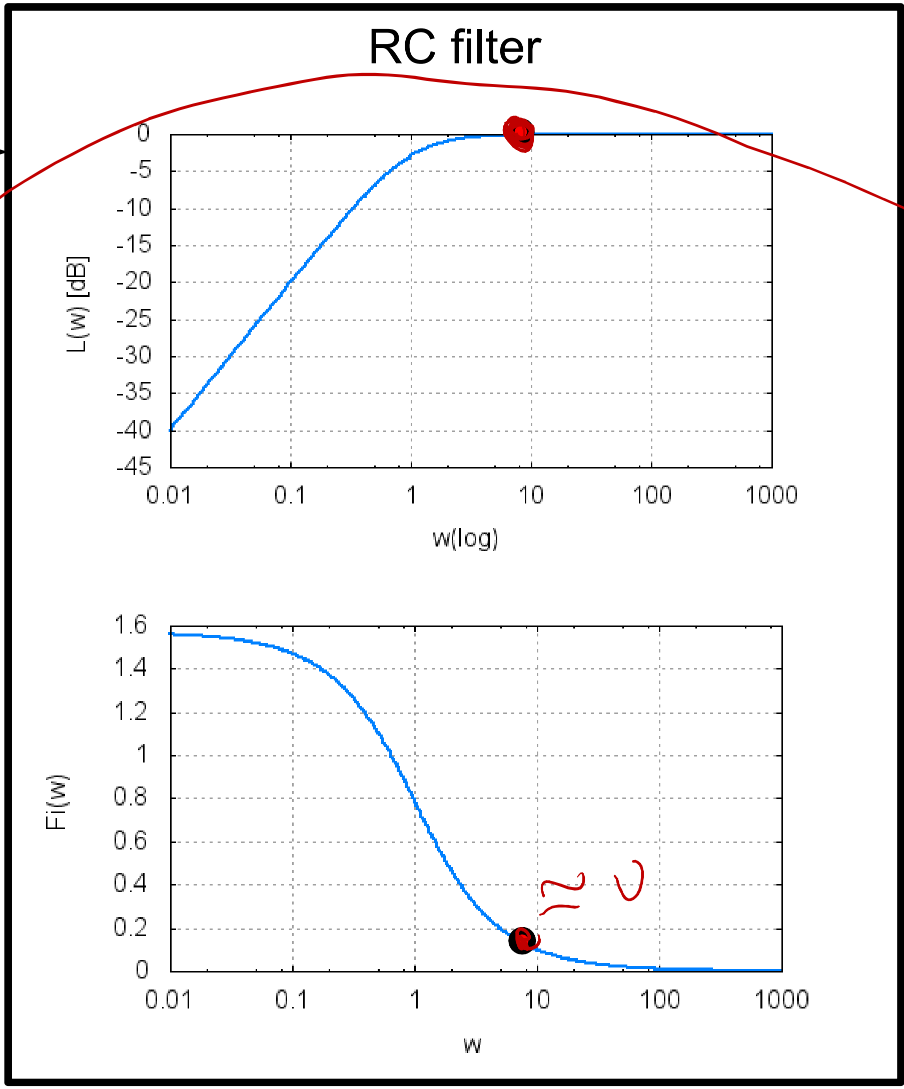
$$u_2(t)=A\sin(\omega t+\varphi)$$

Transfer function – frequency response

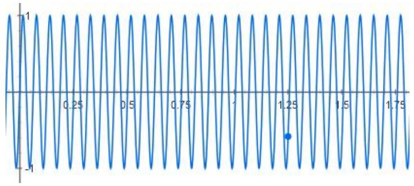
RC circuit example

$$u_1(t) = \sin \omega t$$



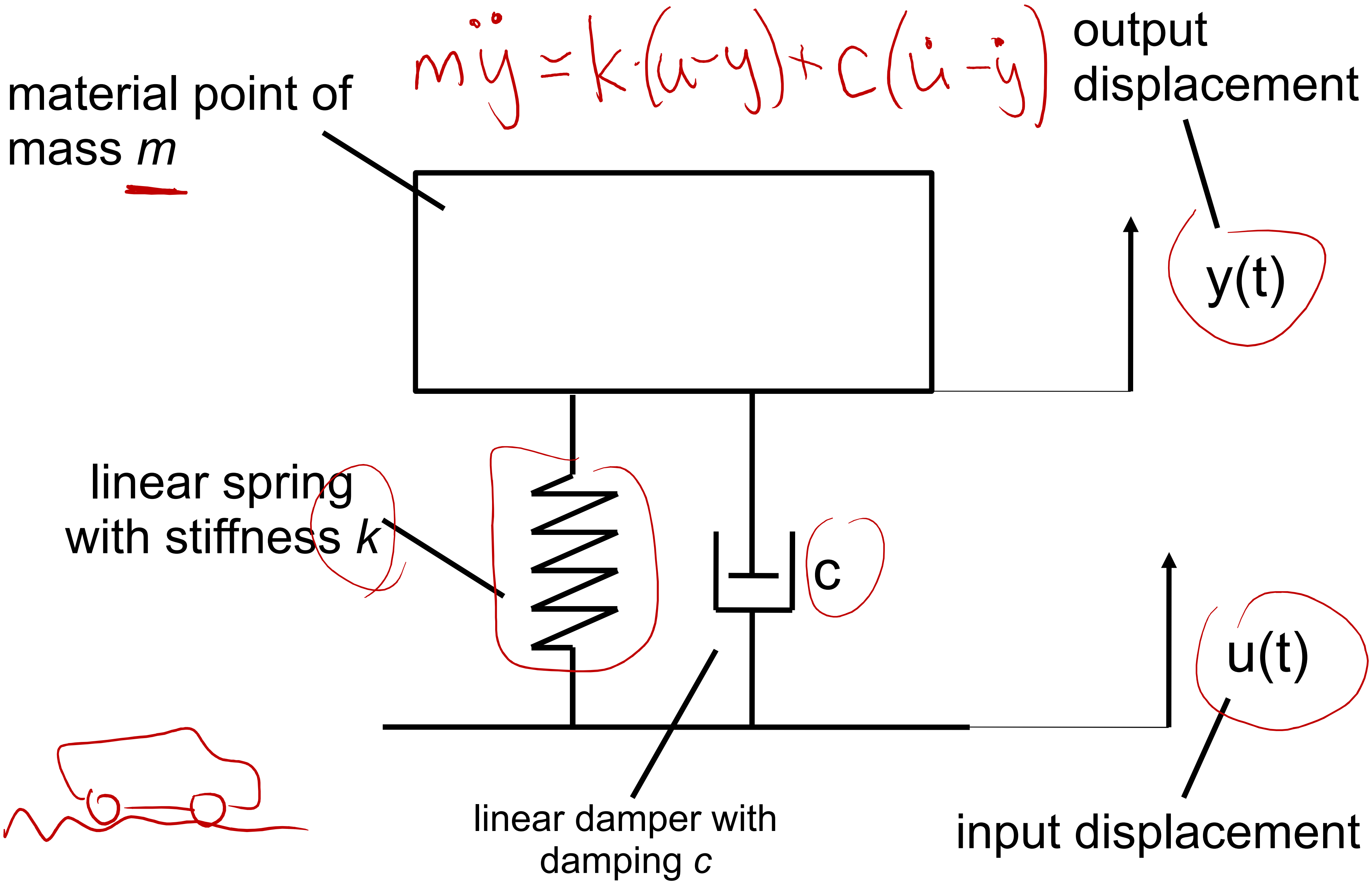


$$u_2(t) = A \sin(\omega t + \phi)$$



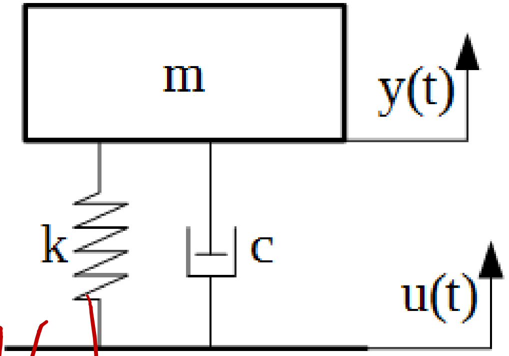
Transfer function – frequency response

Example 2 - vibrating system



Transfer function – frequency response

Example 2 - vibrating system



$$m\ddot{y} + c\dot{y} + ky = k u + c\dot{u}$$

$$ms^2 Y(s) + c\,s\,Y(s) + kY(s) = kU(s) + csU(s)$$

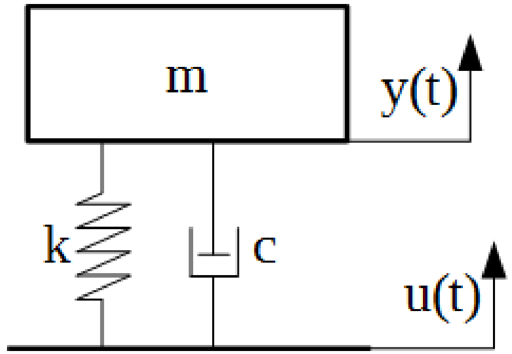
$$H(s) = \frac{Y(s)}{U(s)} = \frac{k + cs}{ms^2 + cs + k}$$

$$H(j\omega) = \frac{k + cj\omega}{-m\omega^2 + cj\omega + k} = \frac{k + jc\omega}{(k - m\omega^2) + jc\omega} \cdot \frac{(k+jc\omega)((k-m\omega^2)-jc\omega)}{(k-m\omega^2) - jc\omega}$$

$$= \frac{(k+jc\omega)(k-m\omega^2) - (k+jc\omega)jc\omega}{(k-m\omega^2)^2 + c^2\omega^2} = P(\omega) + jQ(\omega)$$

Transfer function – frequency response

Example 2 - vibrating system



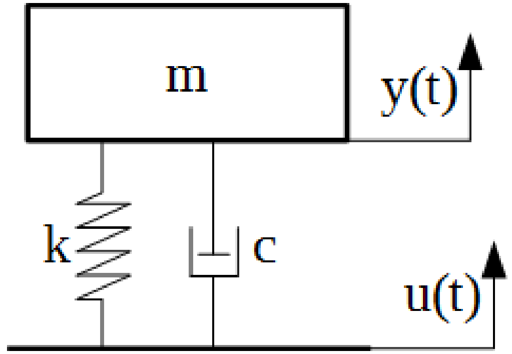
$$m\,\ddot{y}(t)+c\,\dot{y}(t)+k\,y(t)=c\,\dot{u}(t)+k\,u(t)$$

$$H(s)=\frac{c\,s+k}{m\,s^2+c\,s+k}$$

$$P(\omega)=\frac{k^2+c^2\omega^2-km\omega^2}{\left(k-m\omega^2\right)^2+c^2\omega^2}\ ,\ Q(\omega)=\frac{-\,cm\omega^3}{\left(k-m\omega^2\right)^2+c^2\omega^2}$$

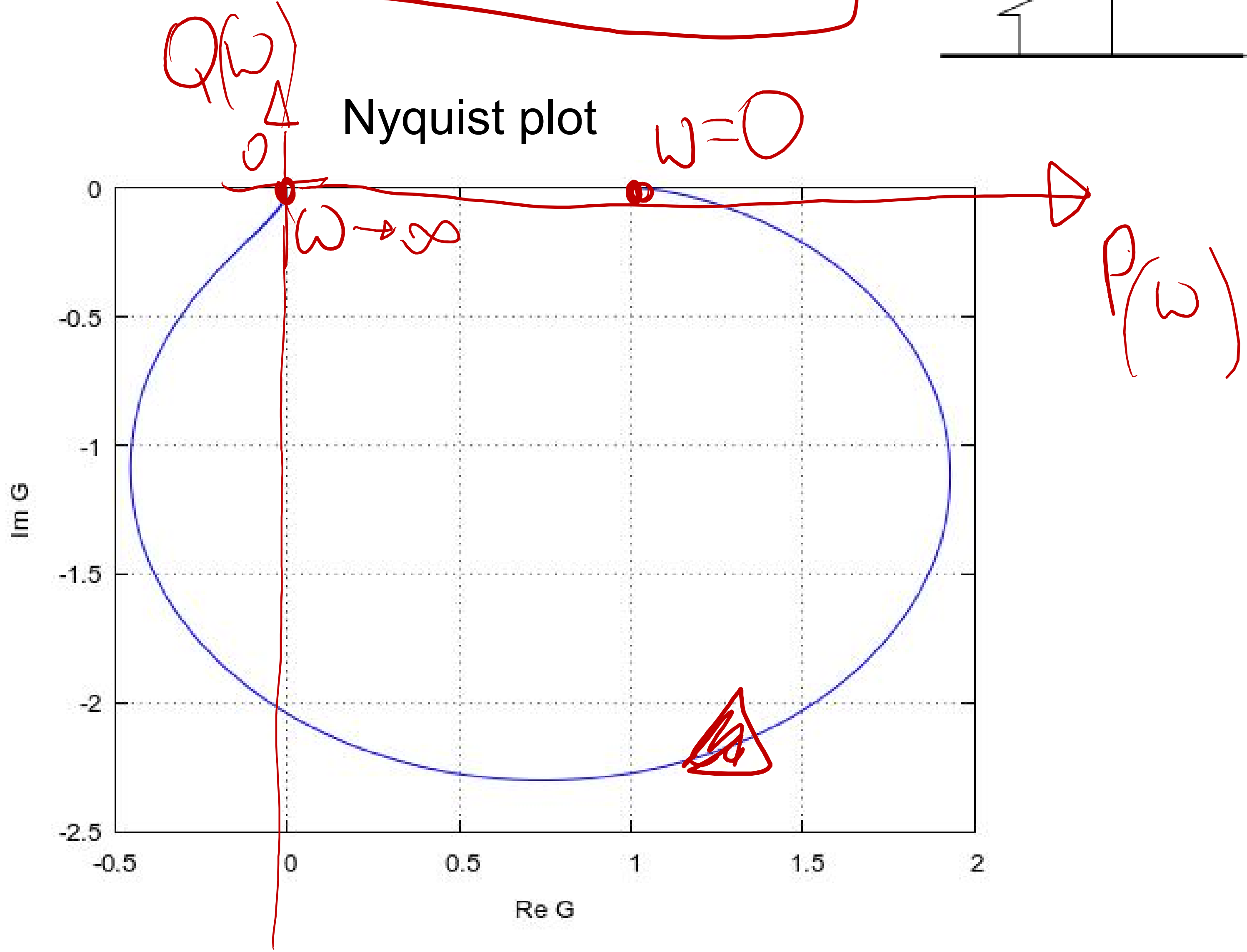
Transfer function – frequency response

Example 2 - vibrating system



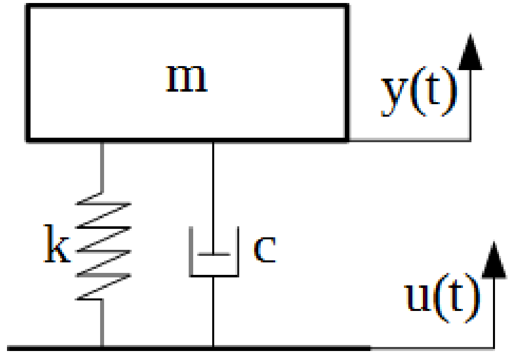
Plots for: $m = 300\,\text{kg},\ c = 800\,\frac{\text{Ns}}{\text{m}},\ k = 11000\,\frac{\text{N}}{\text{m}}$

Nyquist plot

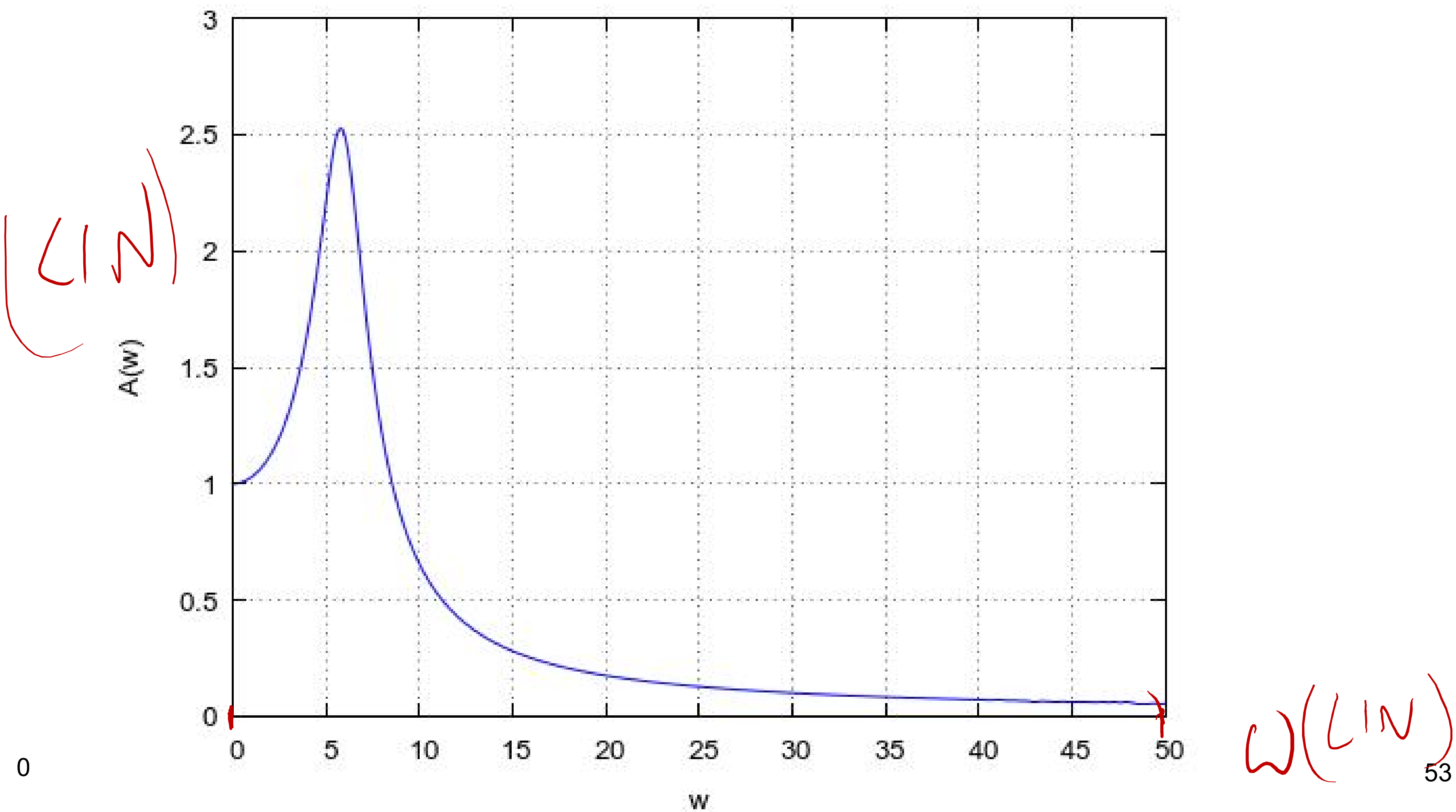


Transfer function – frequency response

Example 2 - vibrating system



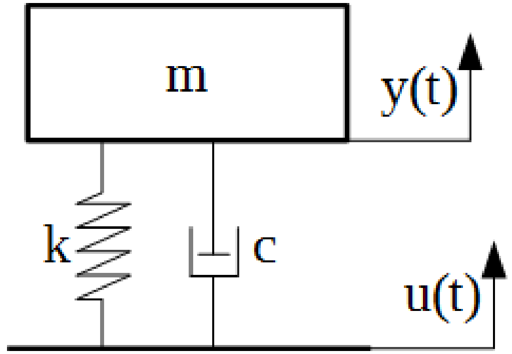
gain plot



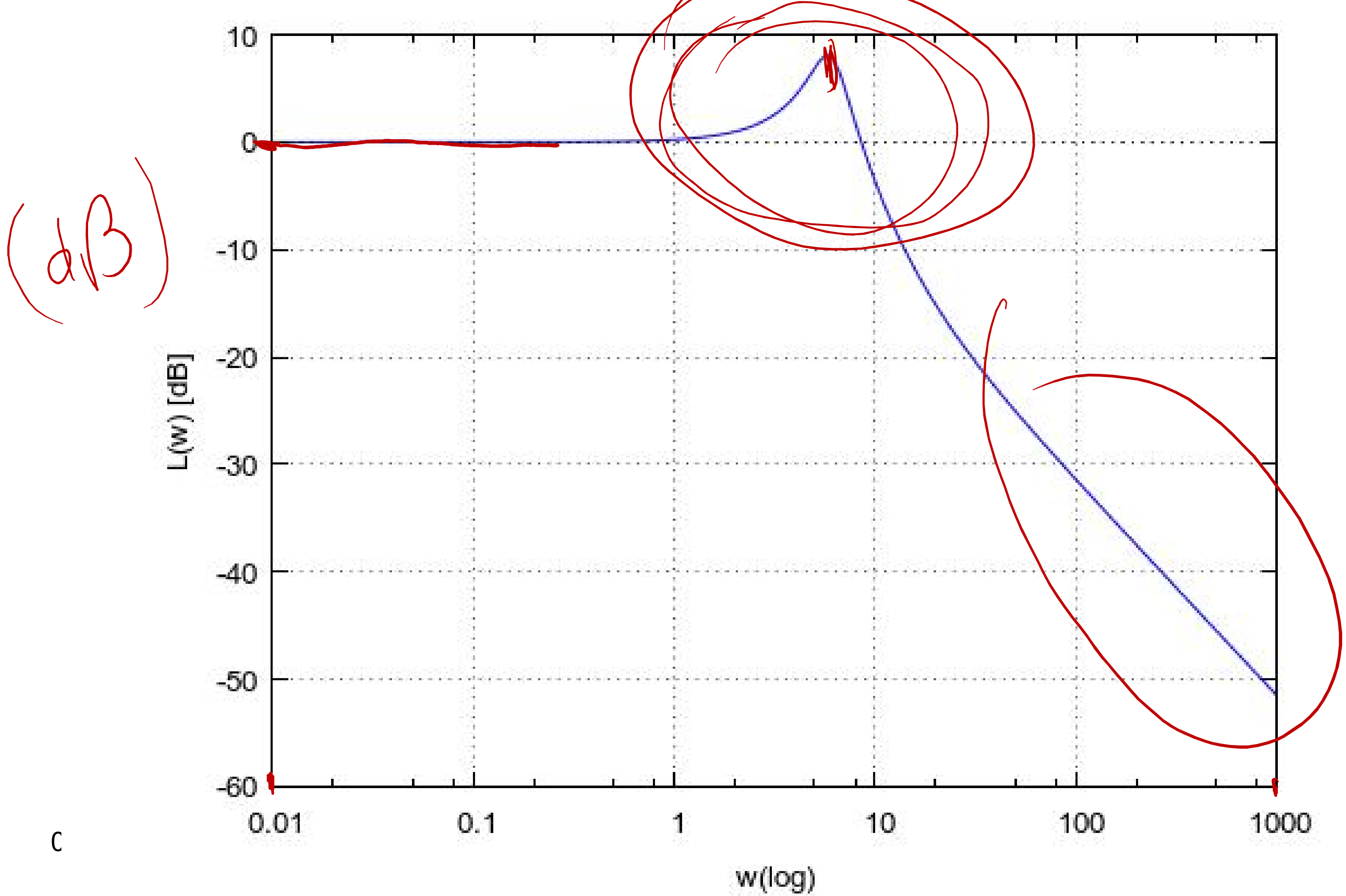
0

Transfer function – frequency response

Example 2 - vibrating system

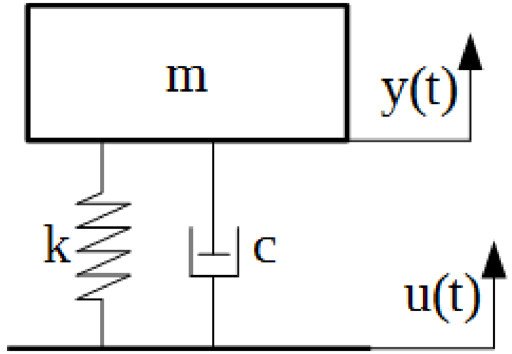


gain plot

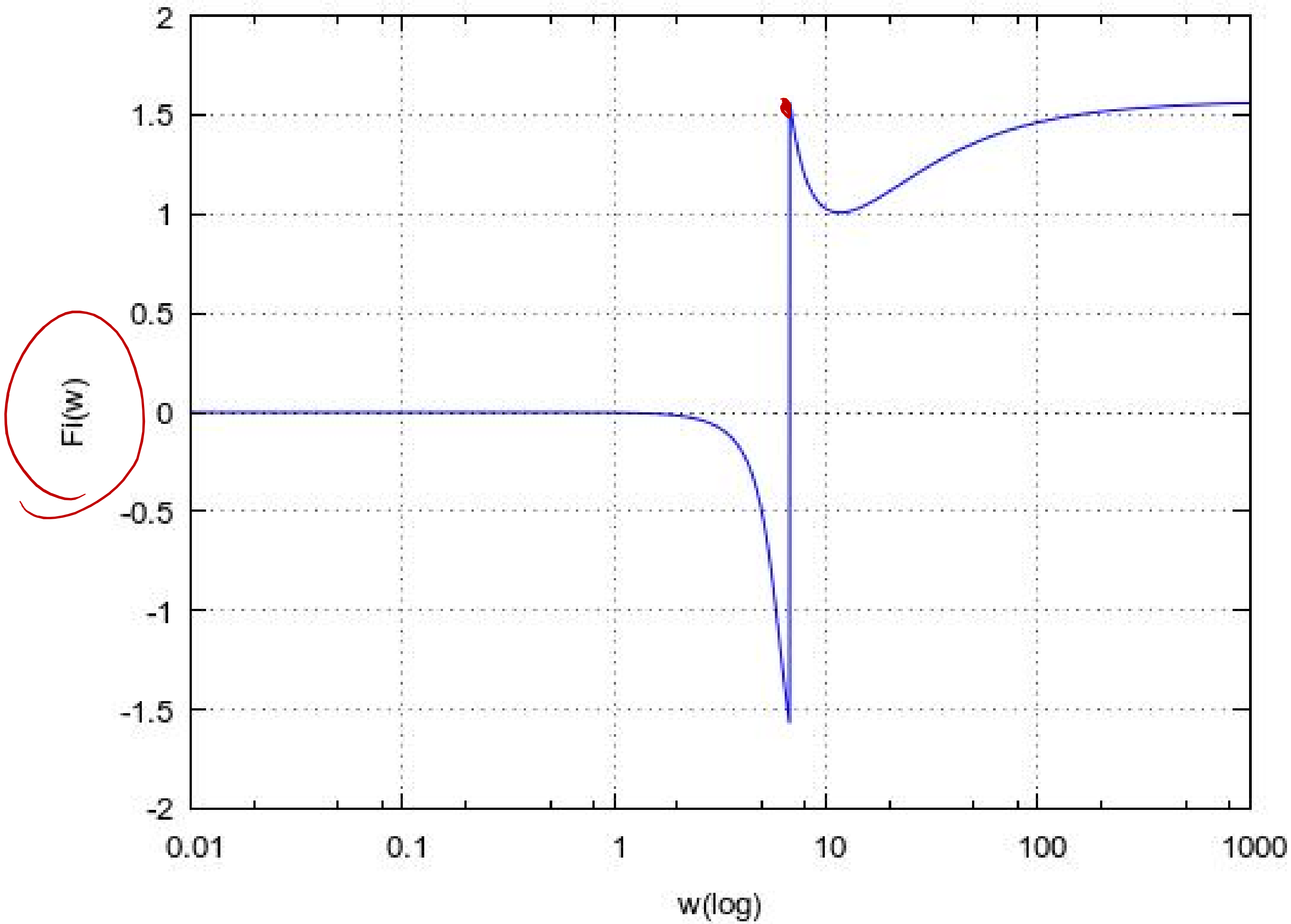


Transfer function – frequency response

Example 2 - vibrating system

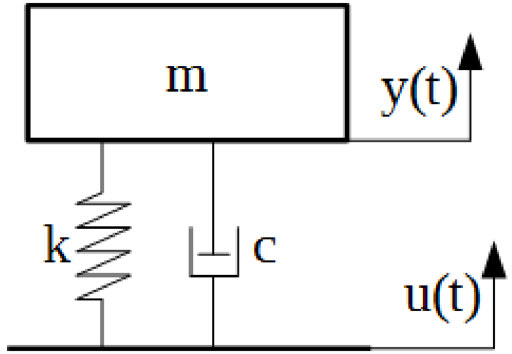


phase plot with „atan"

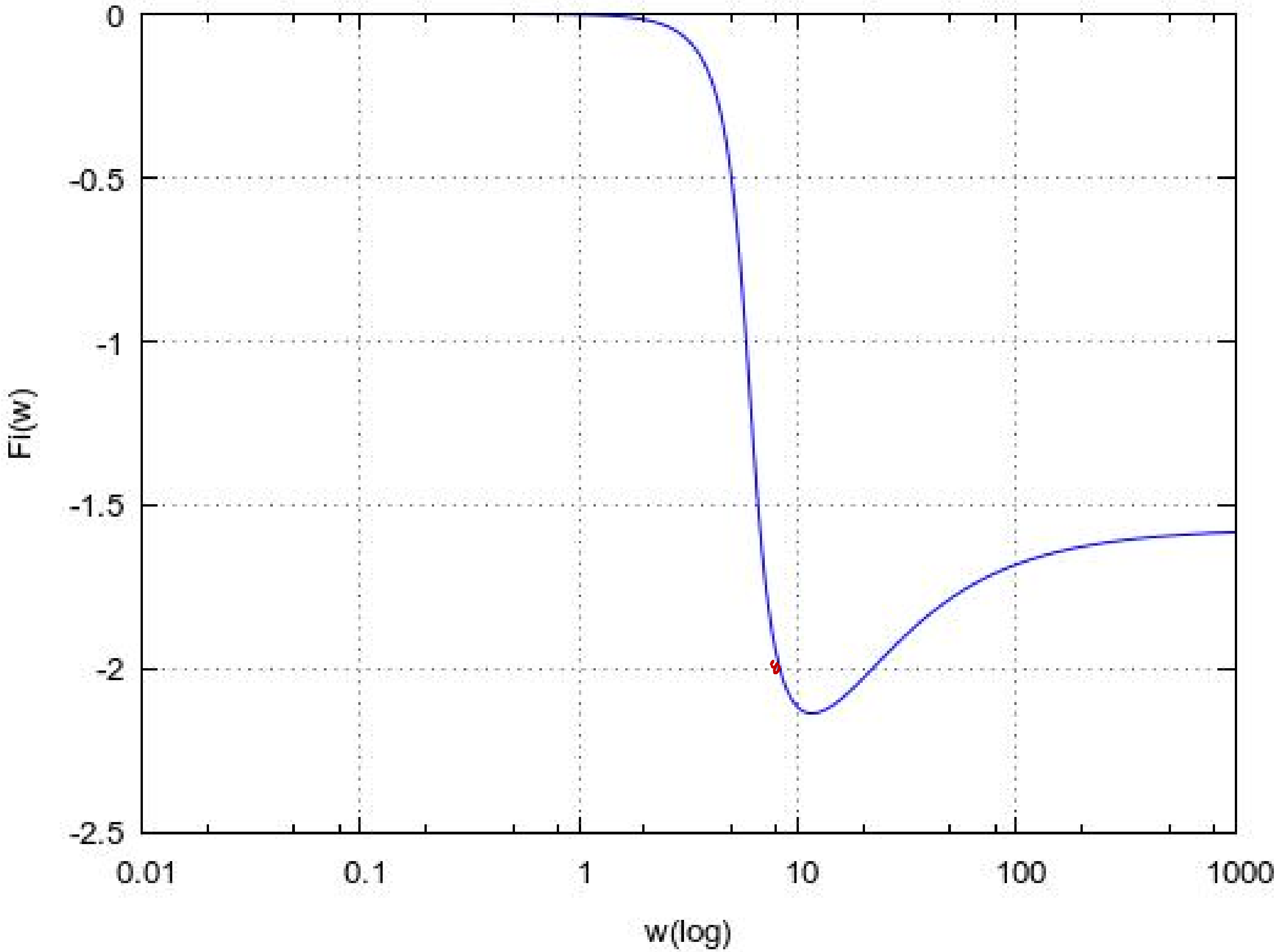


Transfer function – frequency response

Example 2 - vibrating system

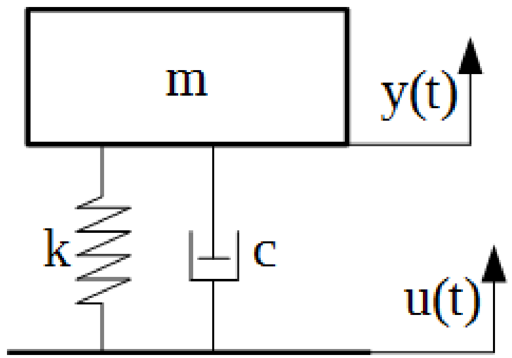


phase plot with „atan2”

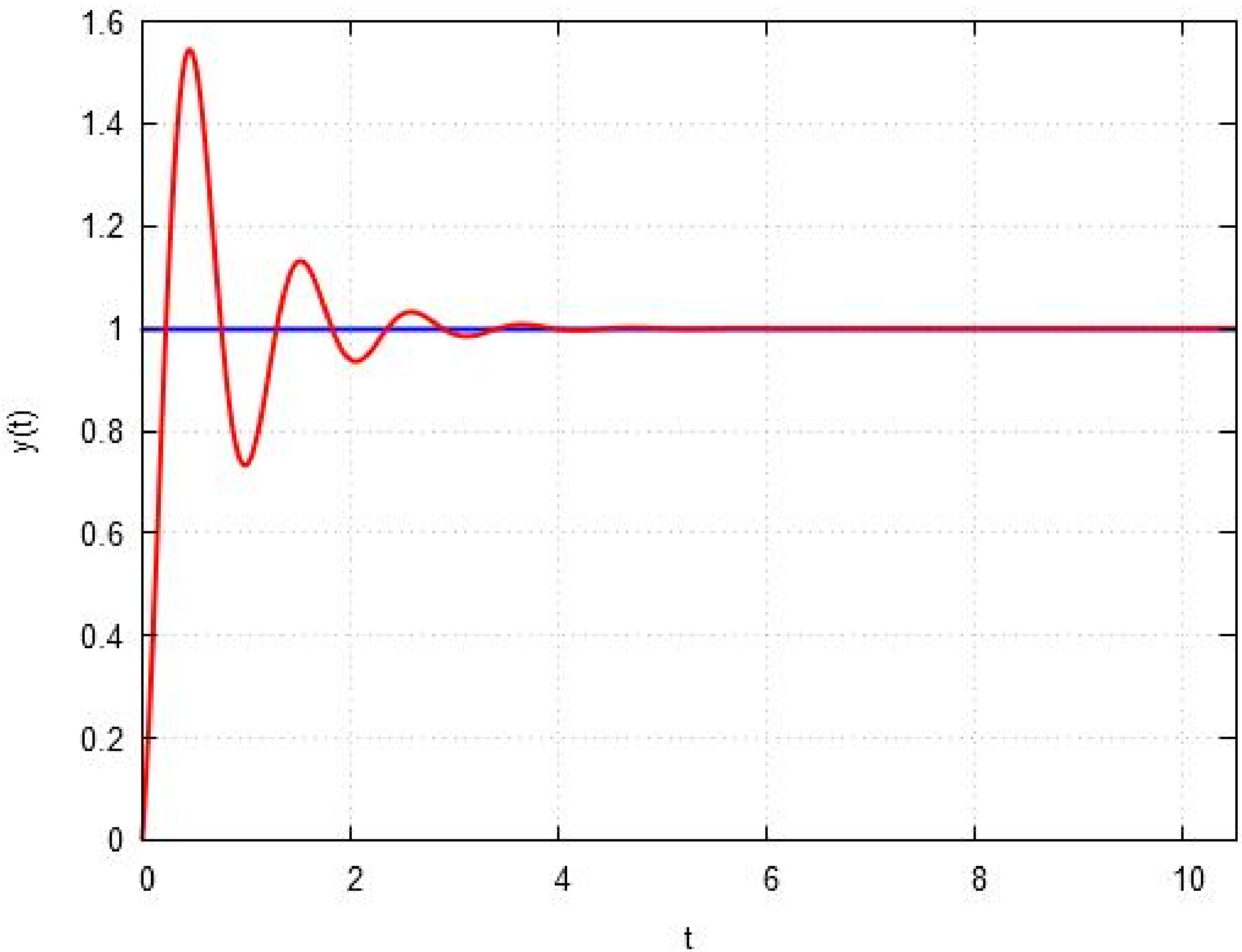


Transfer function – frequency response

Example 2 - vibrating system



Step response



Classification of basic automatic systems

Element name	Equation	Transfer function
proportional	$y(t)=ku(t)$	k
first order (inertial)	$T\frac{dy(t)}{dt}+y(t)=ku(t)$	$\frac{k}{Ts+1}$
integrator	$y(t)=k\int_0^t u(t)dt$ or $\frac{dy(t)}{dt}=ku(t)$	$\frac{k}{s}$

Classification of basic automatic systems

Element name	Equation	Transfer function
derivative	$y(t) = k\frac{du(t)}{dt}$	ks
derivative with inertia	$T\frac{dy(t)}{dt} + y(t) = k\frac{du(t)}{dt}$	$\frac{ks}{Ts+1}$

$k \in \mathbb{R}$

$k \in \mathbb{R}$, $T \in \mathbb{R}_+$

Classification of basic automatic systems

Element name	Equation	Transfer function
delay	$y(t)=u(t-\tau)$	$e^{-\tau s}$
second order (oscillator)	$T_1^2\frac{d^2y(t)}{dt^2}+T_2\frac{dy(t)}{dt}+y(t)=ku(t)$	$\frac{k}{T_1^2 s^2+T_2 s+1}$

$\tau \in \mathbb{R}_+$ [s]

$k \in \mathbb{R}$; $T_1, T_2 \in \mathbb{R}_+$

Proportional element

1. Element equation: $y(t)=ku(t)$ $u(t)$ - input, $y(t)$ - output

$$\mathcal{L} \rightarrow Y(s) = k \cdot U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = k$$

$$H(j\omega) = k \; ; \quad P(\omega) = k \; ; \quad Q(\omega) = 0$$

$$A = \sqrt{P^2 + Q^2} = |k| \rightarrow L(\omega)[dB] = 20\log|k|$$

$$\varphi = \operatorname{atan}\frac{Q}{P} = \operatorname{atan}(0) = 0$$

Proportional element

1. Element equation: $y(t)=ku(t)$ $u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$

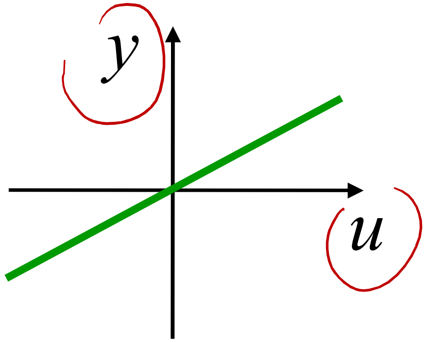
input = const.

$y = k \cdot u$

Proportional element

1. Element equation: $y(t)=ku(t)$ $\quad u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y=ku$ $\quad$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$

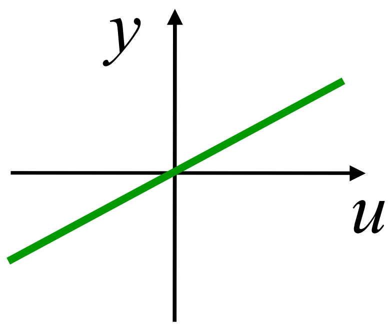


for $k>0$

Proportional element

1. Element equation: $y(t)=ku(t)$ $\quad u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y=ku$ $\quad$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$



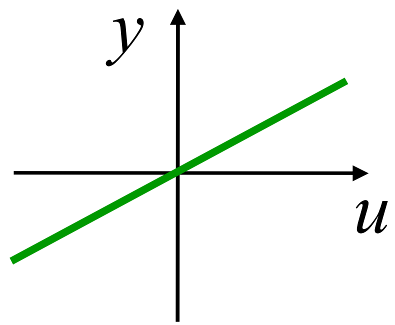
for $k>0$

3. Transfer function:

Proportional element

1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



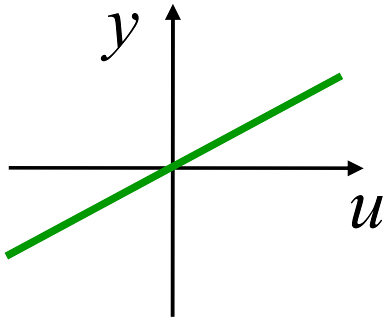
for $k > 0$

3. Transfer function: $H(s) = k$

Proportional element

1. Element equation: $y(t)=ku(t)$ $u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y=ku$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$



for $k>0$

3. Transfer function: $H(s)=k$

4. Step response: for $u(t)=u_0 1(t)$

$U(s)=u_0 \frac{1}{s}$

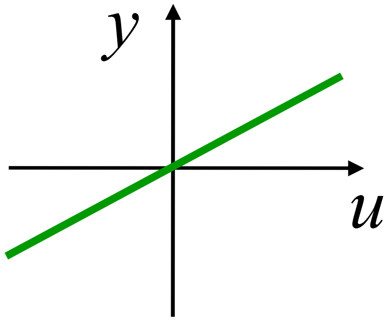
$Y(s)=H(s)\cdot U(s)=k\cdot u_0 \frac{1}{s}$

$y(t)=\mathcal{L}^{-1}\{k u_0 \frac{1}{s}\}=k u_0 1(t)$

Proportional element

1. Element equation: $y(t)=ku(t)$ $u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y=ku$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$



for $k>0$

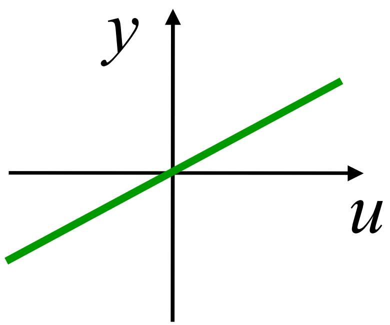
3. Transfer function: $H(s)=k$

4. Step response: $y(t)=k\,u_0\,1(t)$ for $u(t)=u_0\,1(t)$

Proportional element

1. Element equation: $y(t)=ku(t)$ $u(t)$ - input, $y(t)$ - output

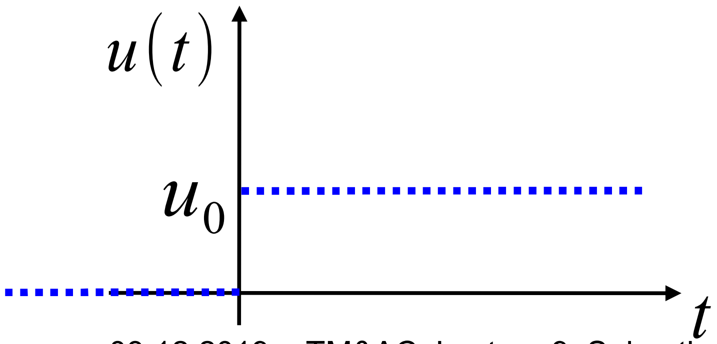
2. Static characteristic (steady state): $y=ku$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$



for $k>0$

3. Transfer function: $H(s)=k$

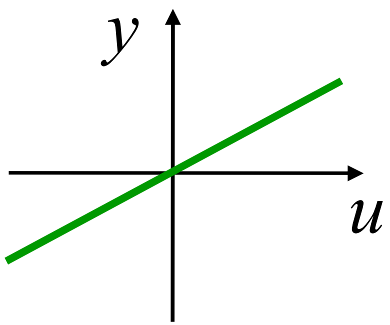
4. Step response: $y(t)=k\,u_0\,1(t)$ for $u(t)=u_0\,1(t)$



Proportional element

1. Element equation: $y(t)=ku(t)$ $u(t)$ - input, $y(t)$ - output

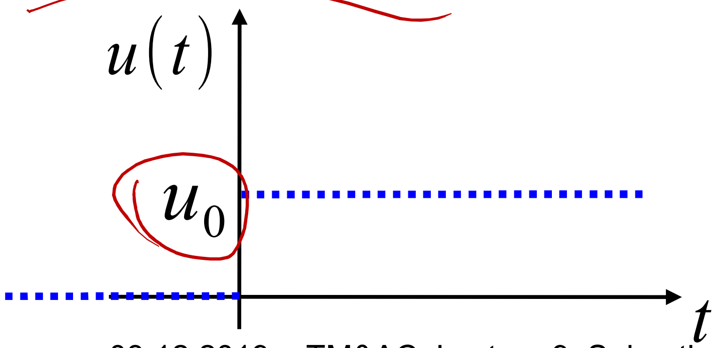
2. Static characteristic (steady state): $y=ku$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$

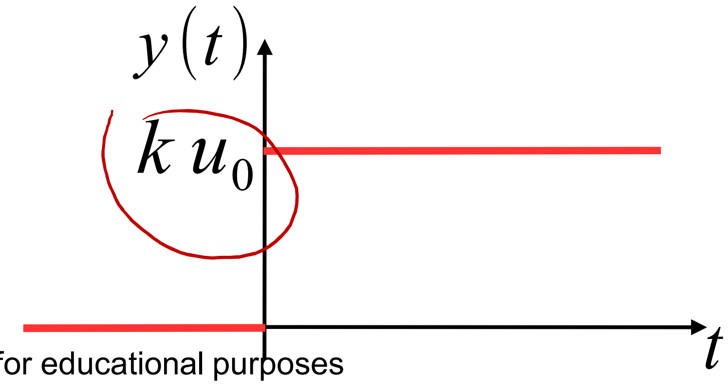


for $k>0$

3. Transfer function: $H(s)=k$

4. Step response: $y(t)=k\,u_0 1(t)$ for $u(t)=u_0 1(t)$





Proportional element

5. Frequency response:

Proportional element

5. Frequency response: $H(j\omega)=k$

Proportional element

5. Frequency response: $H(j\omega)=k$ $P(\omega)=k,\ Q(\omega)=0$

Proportional element

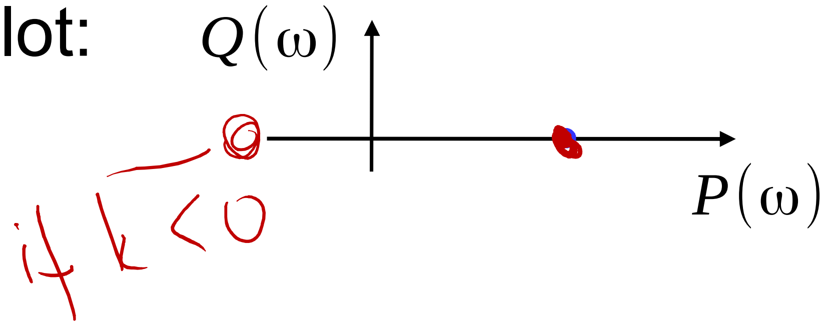
5. Frequency response: $H(j\omega)=k \qquad P(\omega)=k,\ Q(\omega)=0$

6. Nyquist plot:

Proportional element

5. Frequency response: $H(j\omega)=k$ $P(\omega)=k,\ Q(\omega)=0$

6. Nyquist plot:

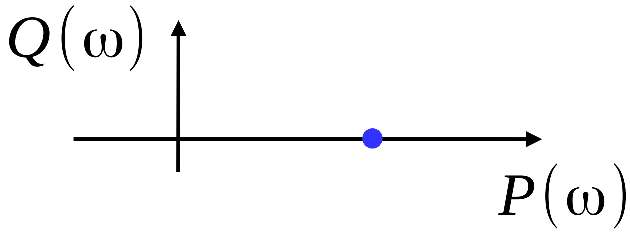


for $k>0$

Proportional element

5. Frequency response: $H(j\omega)=k$ $\qquad P(\omega)=k,\ Q(\omega)=0$

6. Nyquist plot:



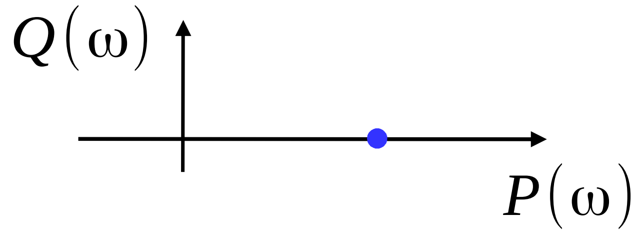
for $k>0$

7. Bode plot:

Proportional element

5. Frequency response: $H(j\omega)=k \qquad P(\omega)=k,\ Q(\omega)=0$

6. Nyquist plot:



for $k>0$

7. Bode plot:

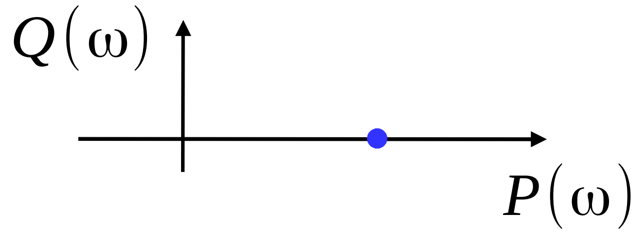
$$A(\omega)=\sqrt{P^2+Q^2}=|k|$$

$$L(\omega)=20\log A(\omega)$$

Proportional element

5. Frequency response: $H(j\omega)=k$ $\quad P(\omega)=k,\ Q(\omega)=0$

6. Nyquist plot:



for $k>0$

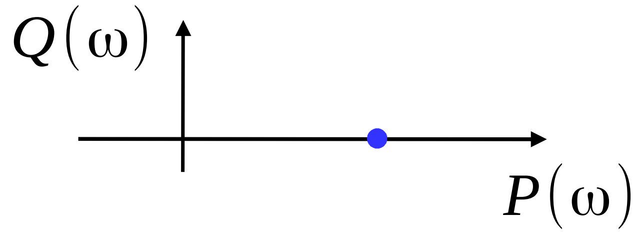
7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k|$ $\quad \varphi(\omega)=\arctan\frac{Q}{P}=\begin{cases}0, \text{ dla } k\geq 0\\ \pi, \text{ dla } k<0\end{cases}$

$L(\omega)=20\log A(\omega)$

Proportional element

5. Frequency response: $H(j\omega)=k$ $\quad P(\omega)=k,\ Q(\omega)=0$

6. Nyquist plot:



for $k>0$

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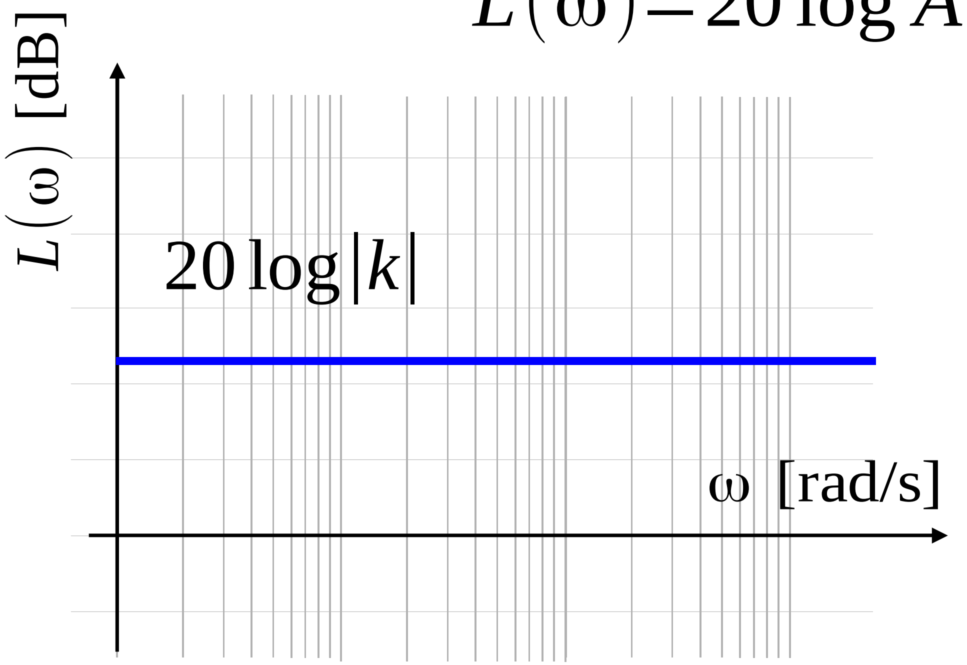
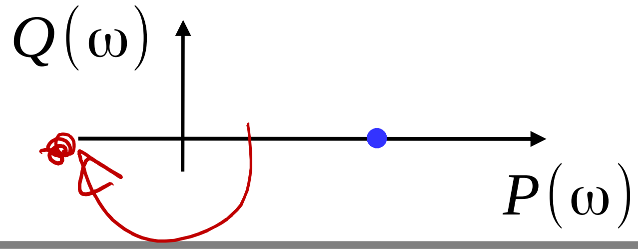
$$L(\omega)=20\log A(\omega)$$



Proportional element

5. Frequency response: $H(j\omega)=k$ $\quad P(\omega)=k,\ Q(\omega)=0$

6. Nyquist plot:



for $k>0$

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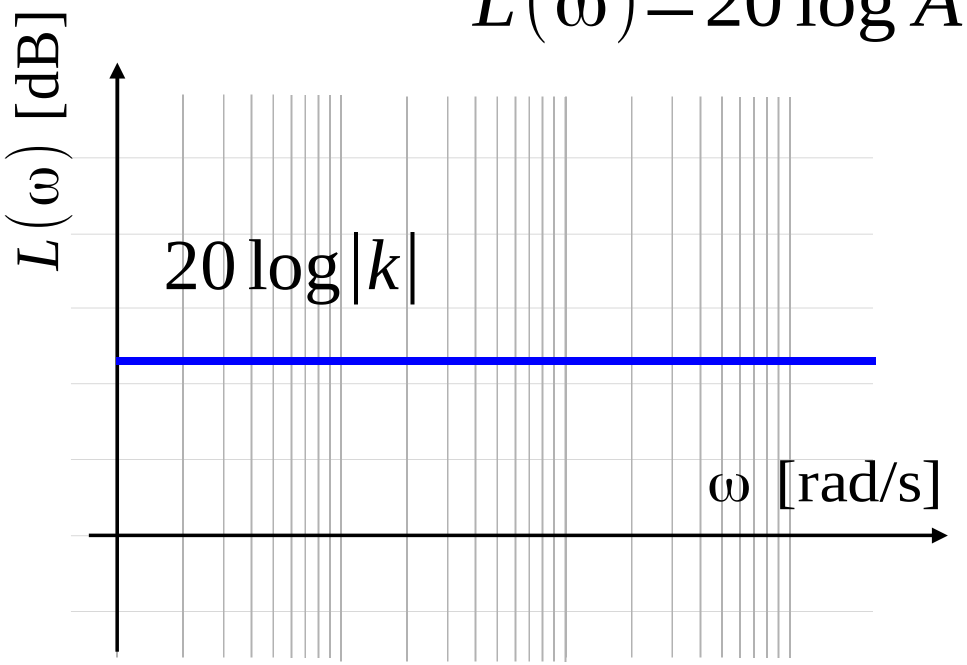
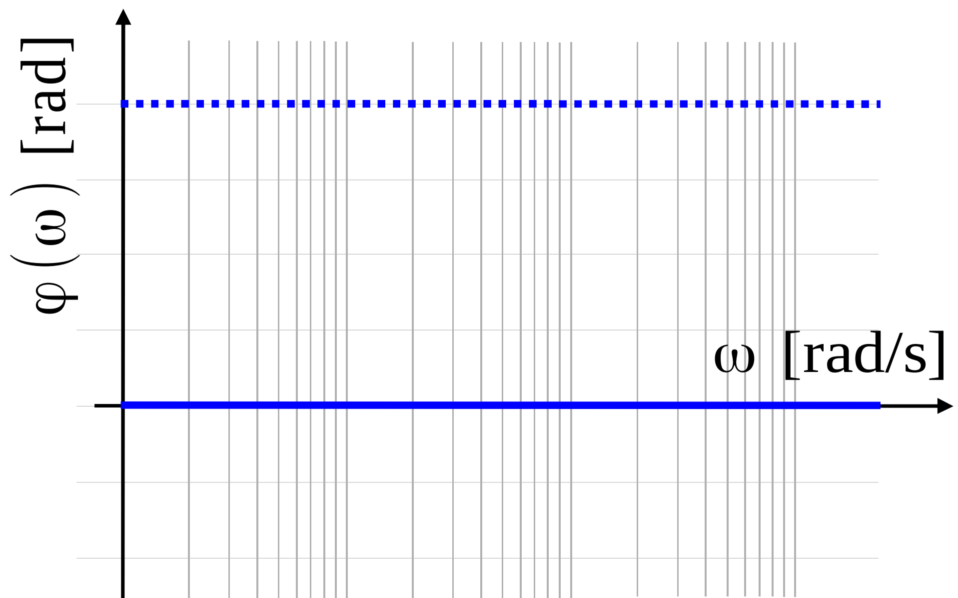
$L(\omega)=20\log A(\omega)$

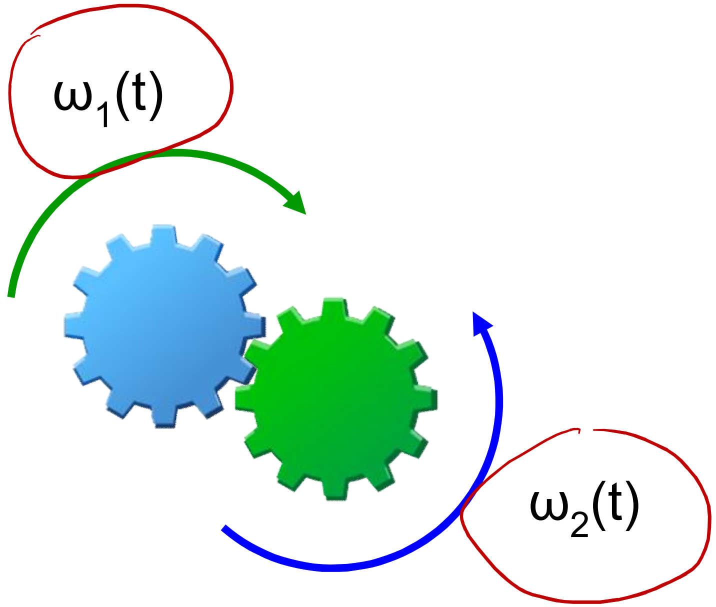




Proportional element

Examples

1

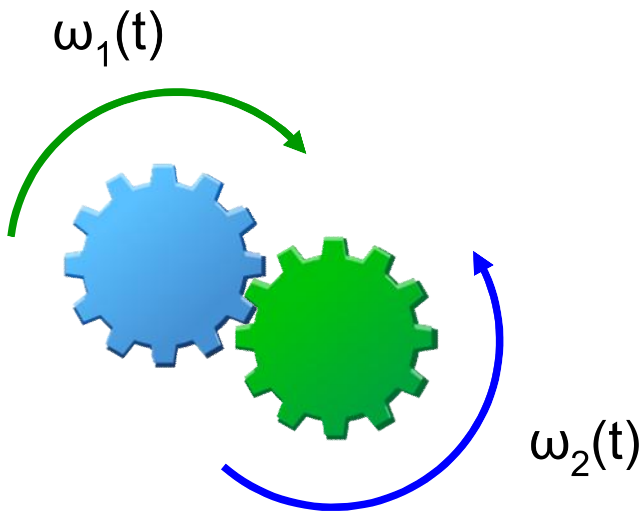


GEARBOX:
input – angular velocity $\omega_1(t)$
output – angular velocity $\omega_2(t)$

Proportional element

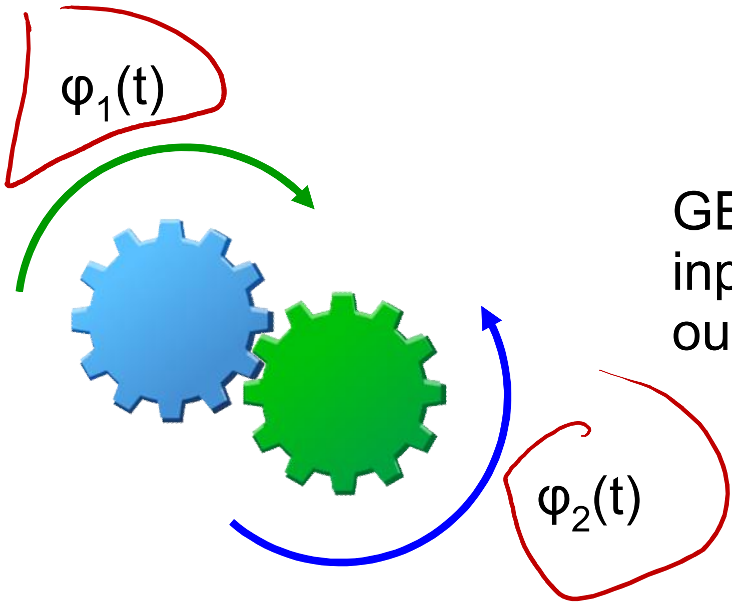
Examples

1



GEARBOX:
input – angular velocity $\omega_1(t)$
output – angular velocity $\omega_2(t)$

2

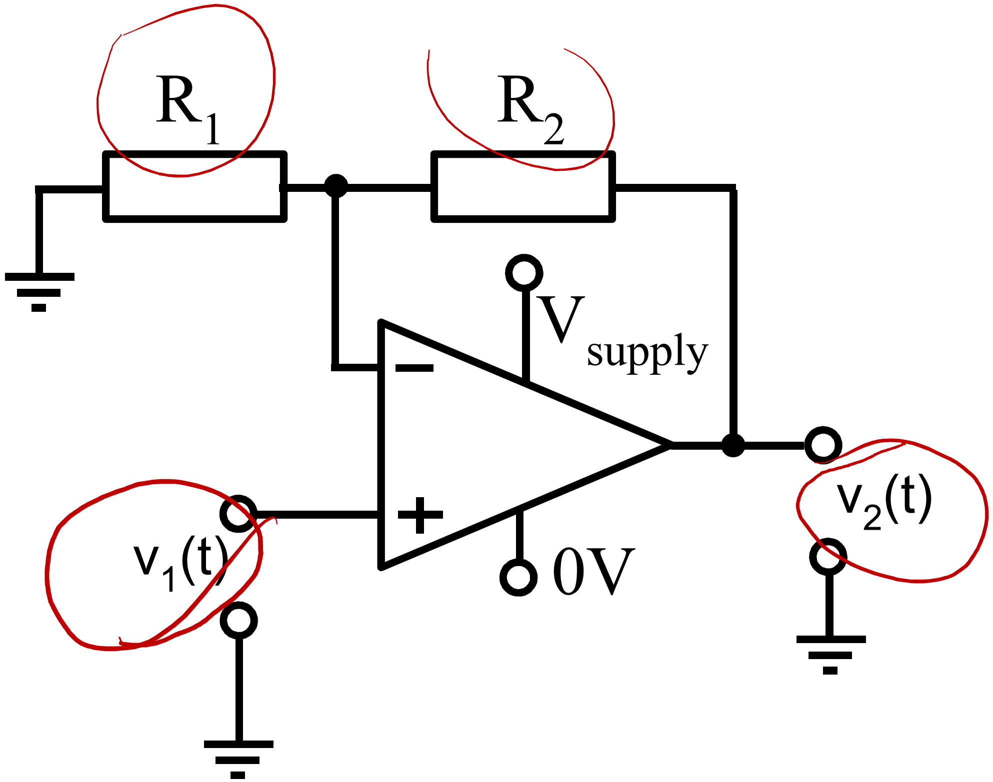


GEARBOX:
input – rotation angle $\varphi_1(t)$
output – rotation angle $\varphi_2(t)$

Proportional element

Examples

3



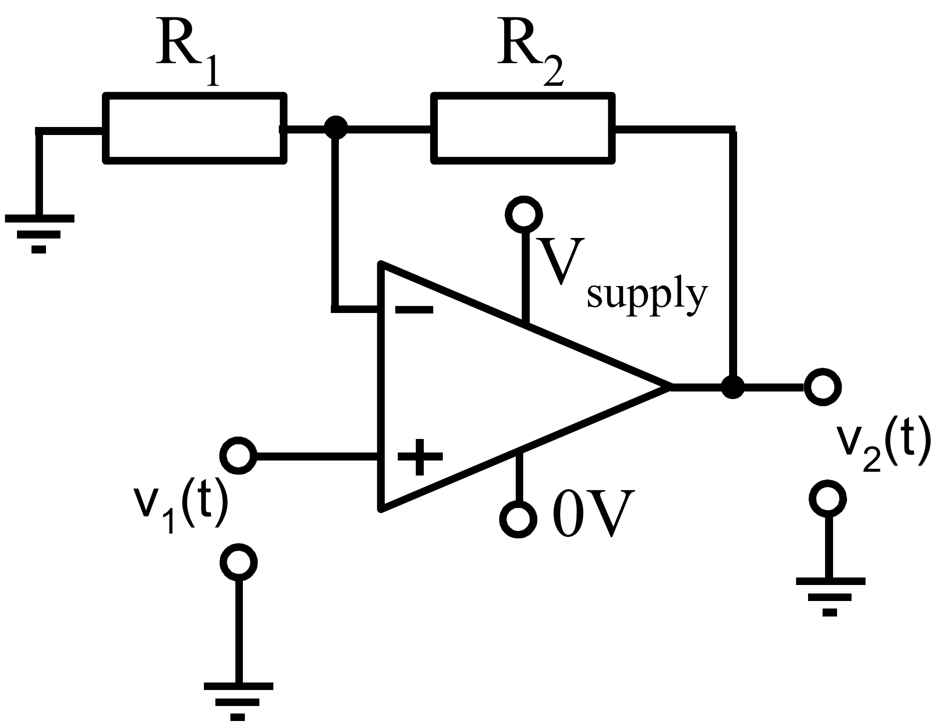
OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = v_1(t)\left(1 + \frac{R_2}{R_1}\right)$$

Proportional element

Examples

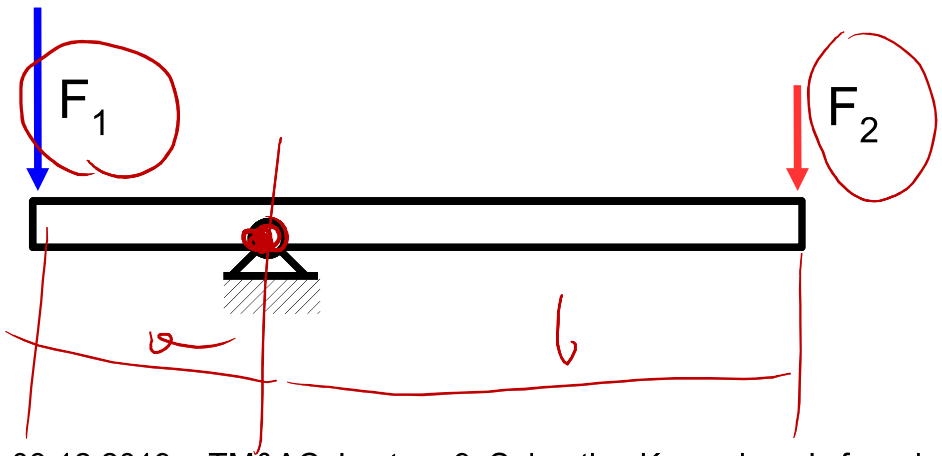
3



OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = v_1(t)\left(1 + \frac{R_2}{R_1}\right)$$

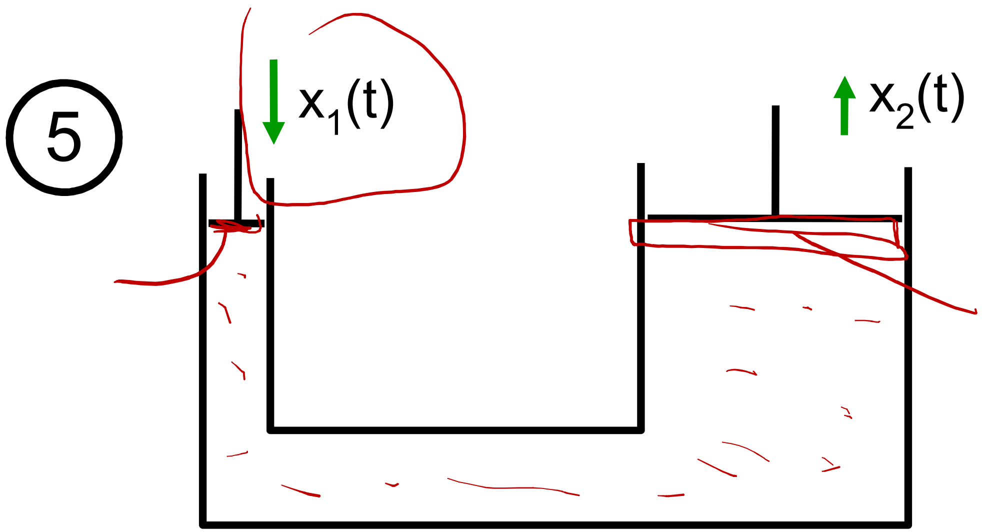
4



BEAM in steady state:
input – force F_1
output – force F_2

Proportional element

Examples

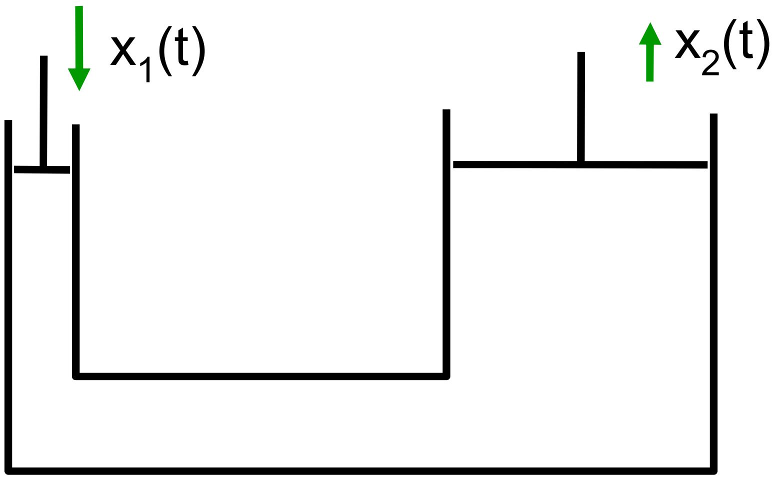


HYDRAULIC LEVER:
input – displacement $x_1(t)$
output – displacement $x_2(t)$

Proportional element

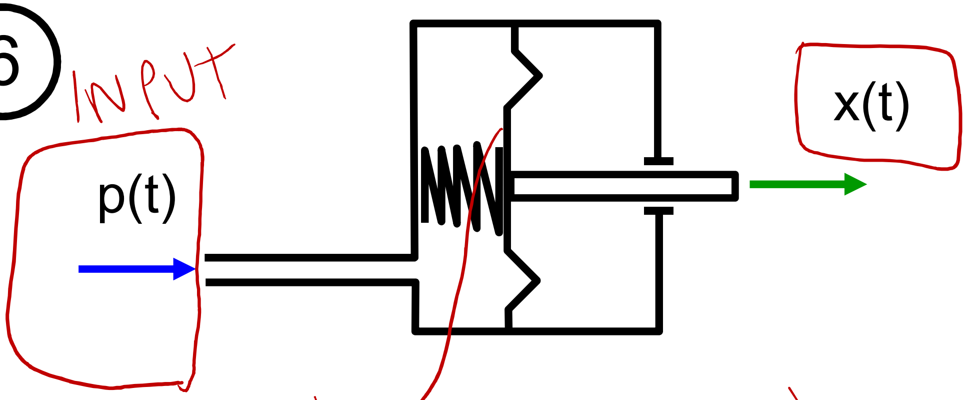
Examples

5



HYDRAULIC LEVER:
input – displacement $x_1(t)$
output – displacement $x_2(t)$

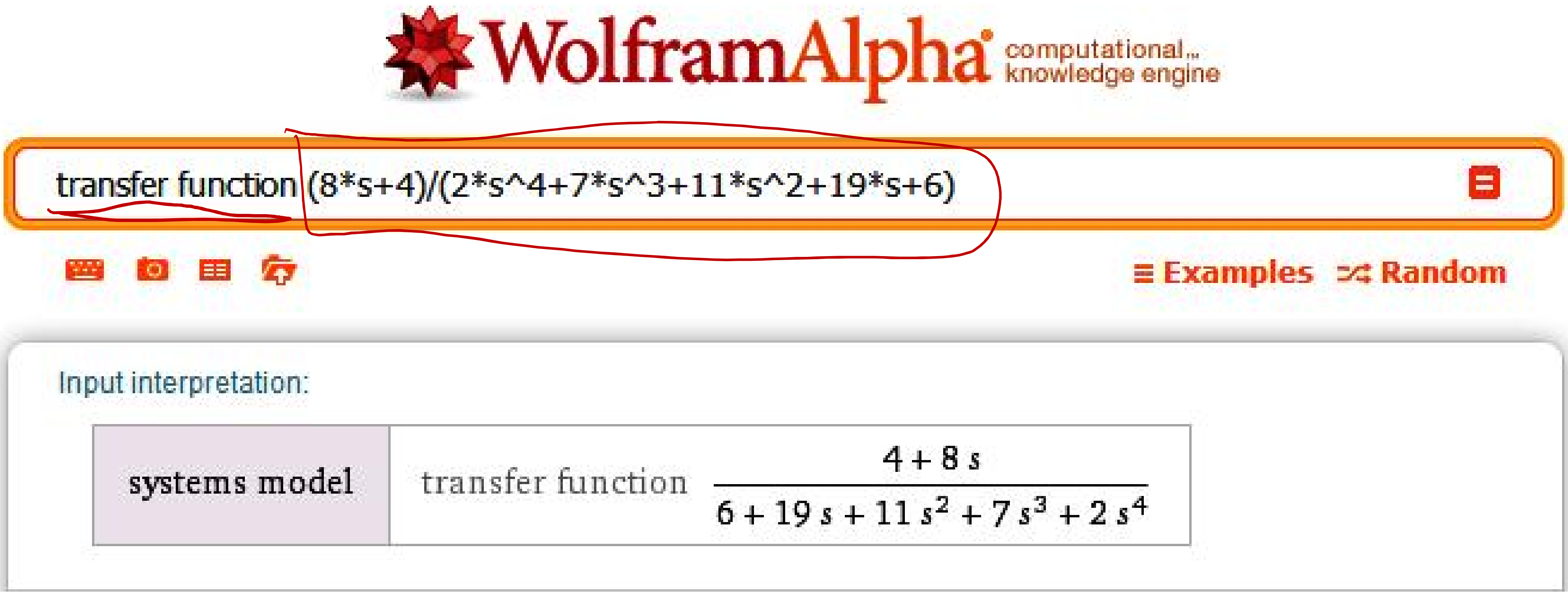
6



$p(t) \cdot A = k \cdot x(t)$

PRESSURE ACTUATOR:
input – pressure $p_1(t)$
output – displacement x(t)

WolframAlpha



WolframAlpha

WolframAlpha computational... knowledge engine

partial fraction decomposition s/(s^3+4*s^2+5*s+2)

Examples Random

Assuming "s" is a variable | Use as a unit instead

Input:

partial fractions	$\frac{s}{s^3+4s^2+5s+2}$

Result:

Step-by-step solution

$$\frac{s}{s^3+4s^2+5s+2} = -\frac{2}{s+2} + \frac{2}{s+1} - \frac{1}{(s+1)^2}$$

WolframAlpha

WolframAlpha computational... knowledge engine

inverse laplace transform s/(s^3+ 4*s^2+5*s+2)

≡ Examples ⤮ Random

Assuming "s" is a variable | Use as a unit instead

Input:

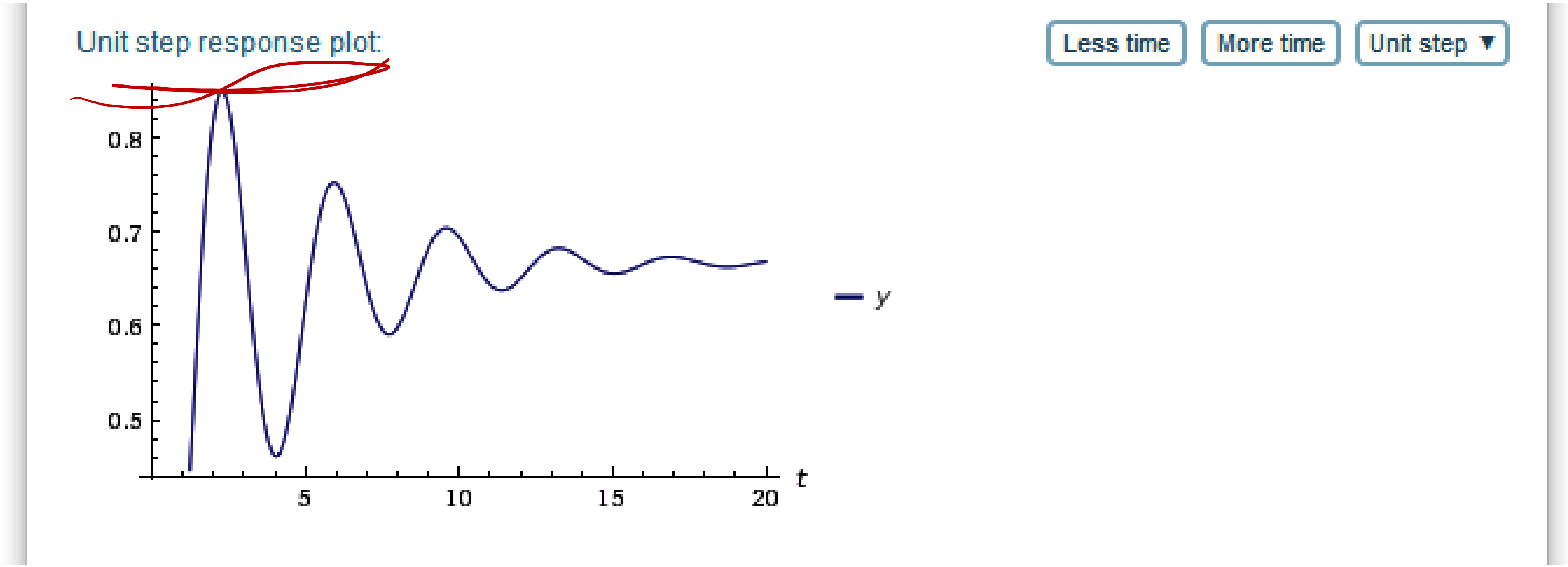
$$\mathcal{L}_s^{-1}\left[\frac{s}{s^3+4s^2+5s+2}\right](t)$$

$\mathcal{L}_s^{-1}[f(s)](t)$ is the inverse Laplace transform of $f(s)$ with real variable t

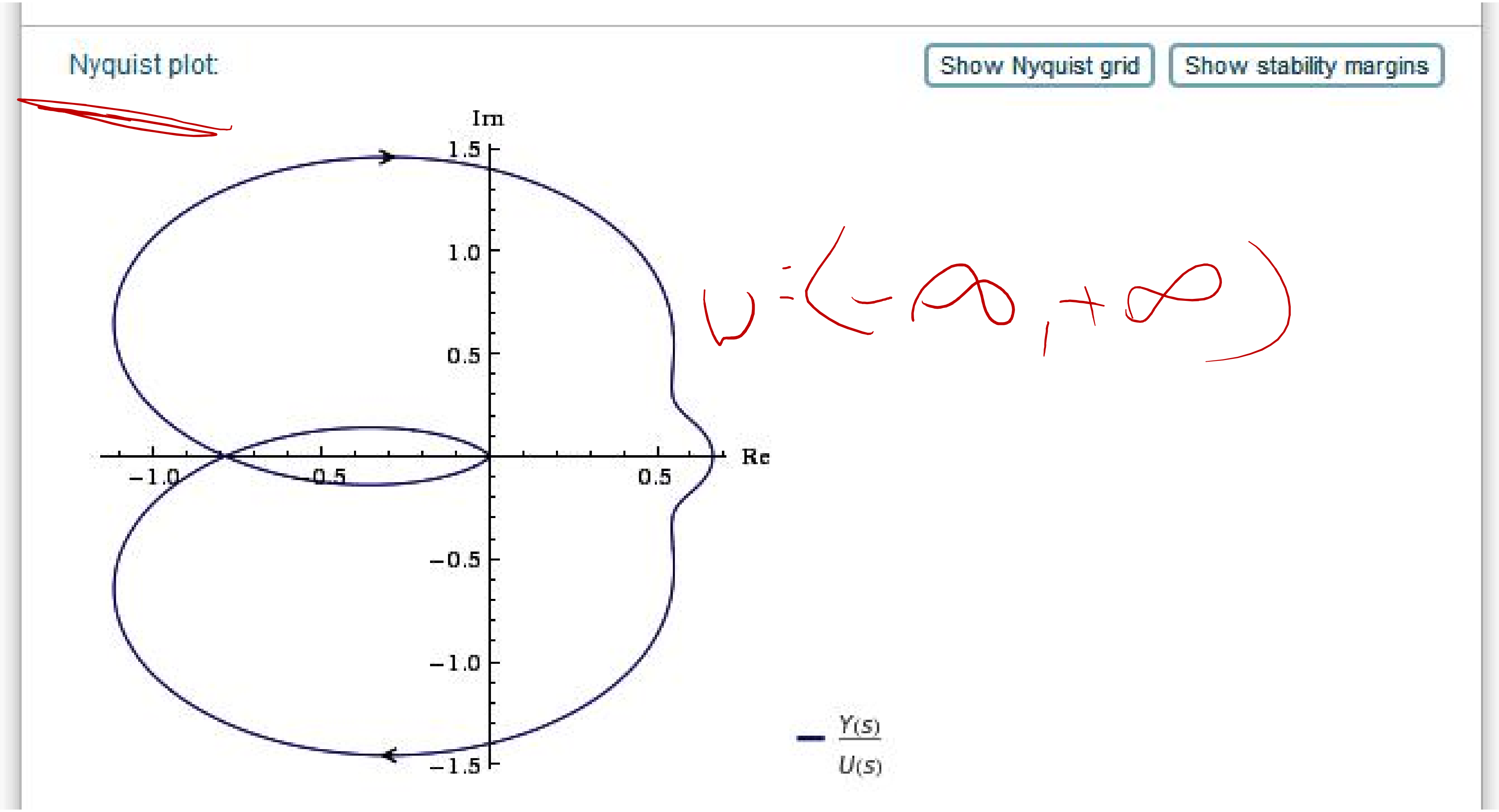
Result:

$$-e^{-2t}\left(e^t\, t-2\, e^t+2\right)$$

WolframAlpha



WolframAlpha



WolframAlpha