



Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

Lecture 9

Frequency response.
Classification of basic automatic systems.

Transfer function

Linear time-invariant SISO system for continuous-time input signal $x(t)$ and output $y(t)$ in a form

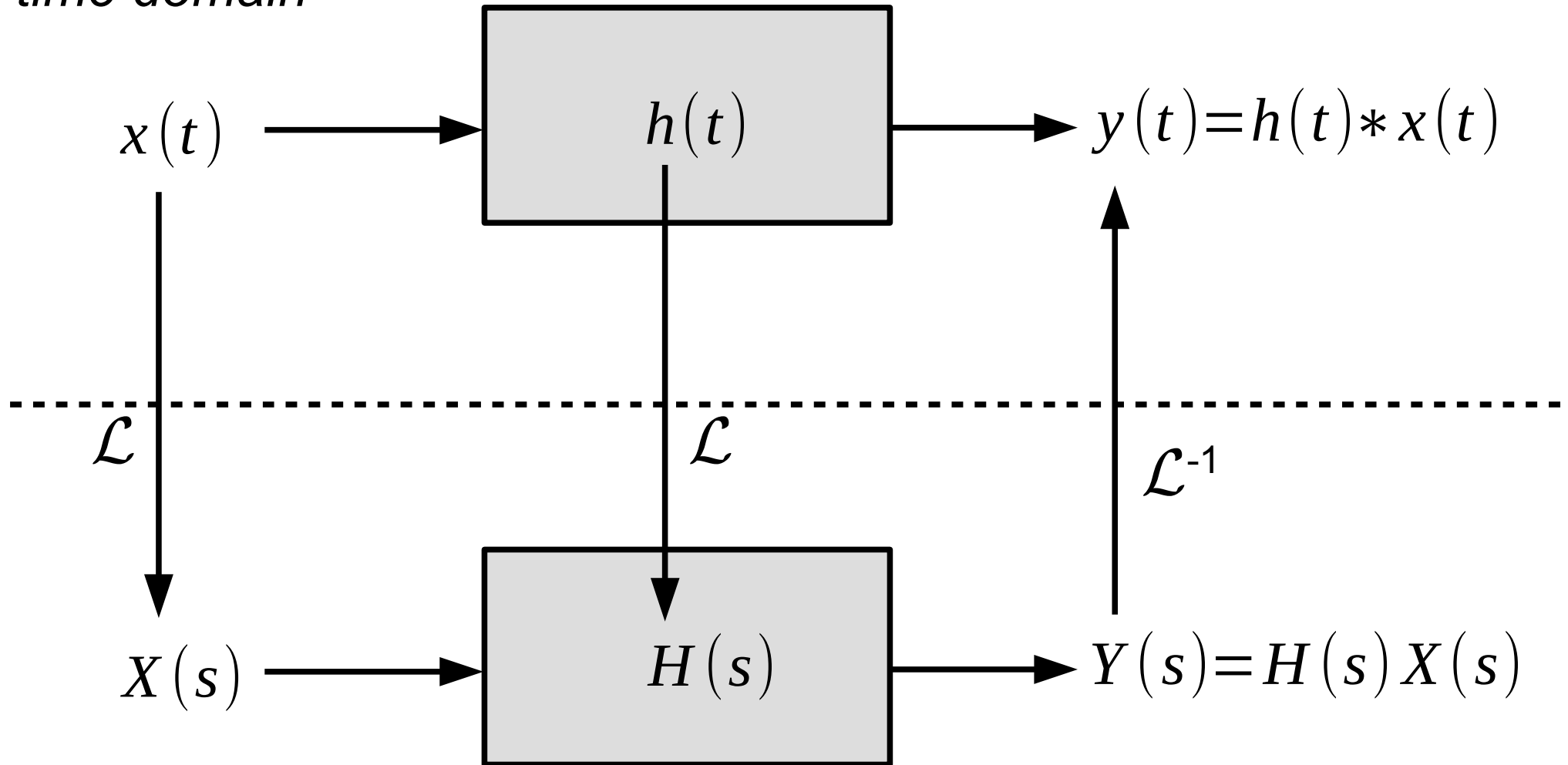
$$\text{Transfer function } H(s) = \frac{Y(s)}{X(s)}$$

$Y(s)$ - Laplace'a transform of an output

$X(s)$ - Laplace'a transform of an input

Input and output

time domain



complex domain

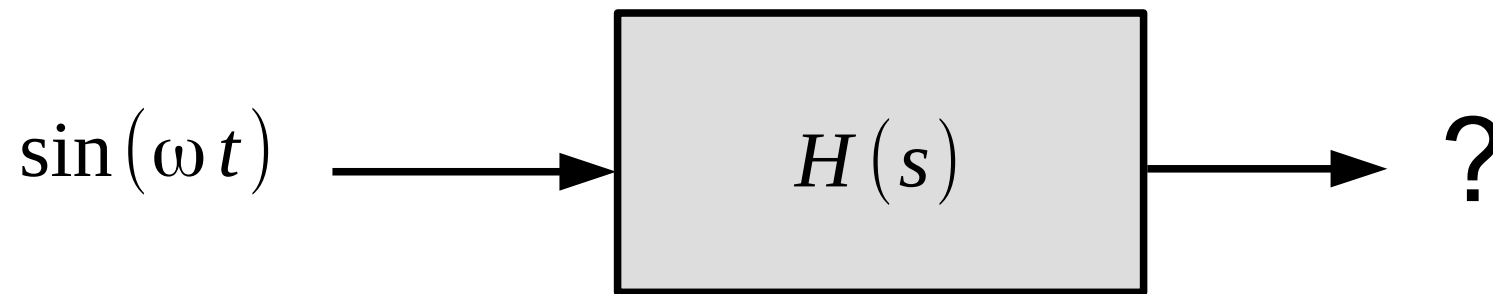
Transfer function & frequency response

Transfer function
(Laplace domain)

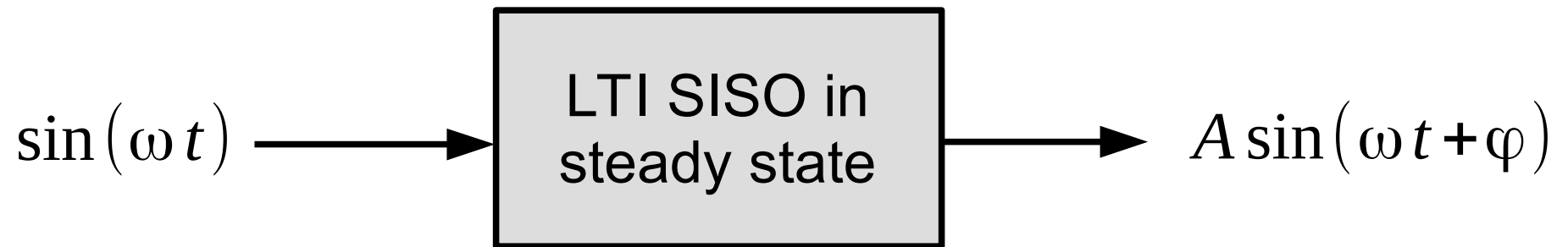
$$H(s)$$

Full system description
(for every possible input)

Transfer function & frequency response



Transfer function & frequency response



Transfer function & frequency response

Transfer function
(Laplace domain)

$$H(s)$$

Full system description
(for every possible input)

↓
 $s = j\omega$

Frequency response
(Fourier domain)

$$H(j\omega)$$

Description of a system in
steady state with harmonic
input

Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A \sin(\omega t + \varphi)$

Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A \sin(\omega t + \varphi)$
harmonic excitation *steady state!!!*

Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A \sin(\omega t + \varphi)$

$$H(s) \xrightarrow{s = j\omega}$$

Transfer function – frequency response

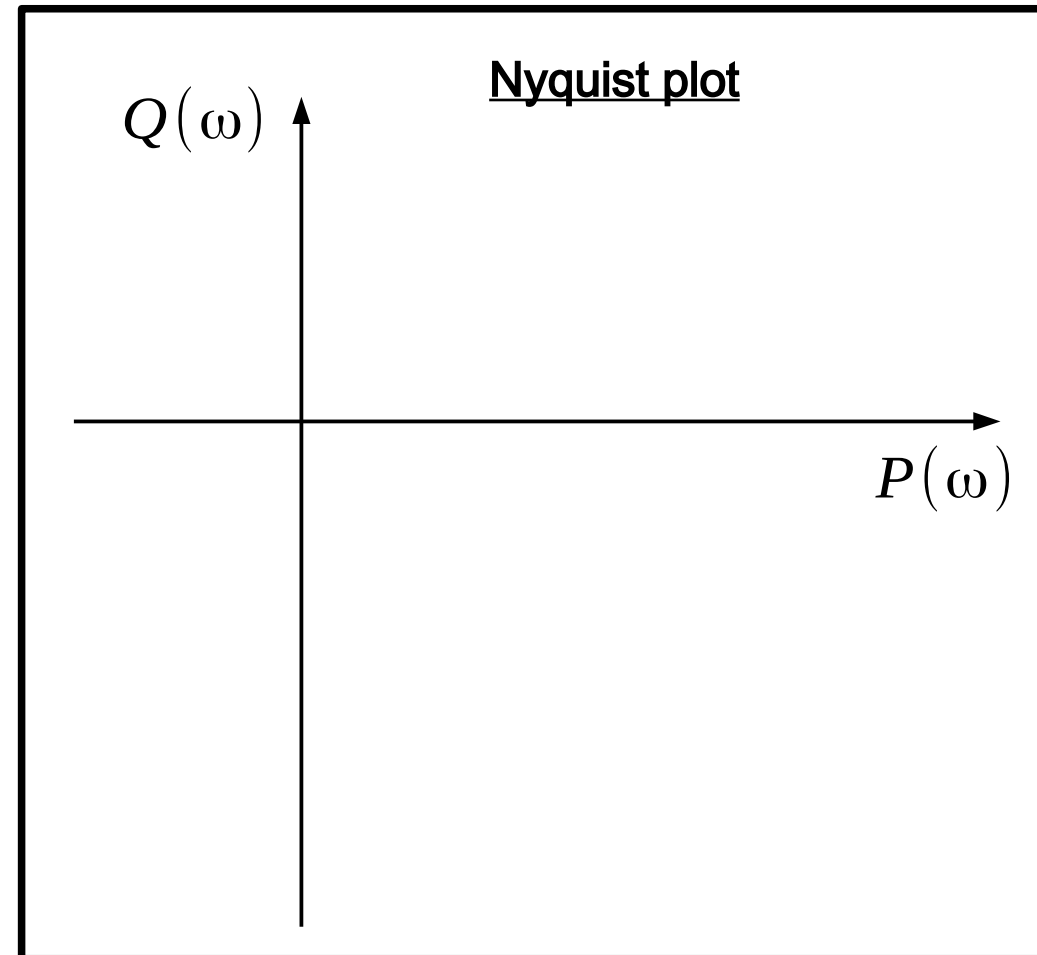
input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A \sin(\omega t + \varphi)$

$$H(s) \xrightarrow{s=j\omega} H(j\omega) = P(\omega) + jQ(\omega)$$

Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A \sin(\omega t + \varphi)$

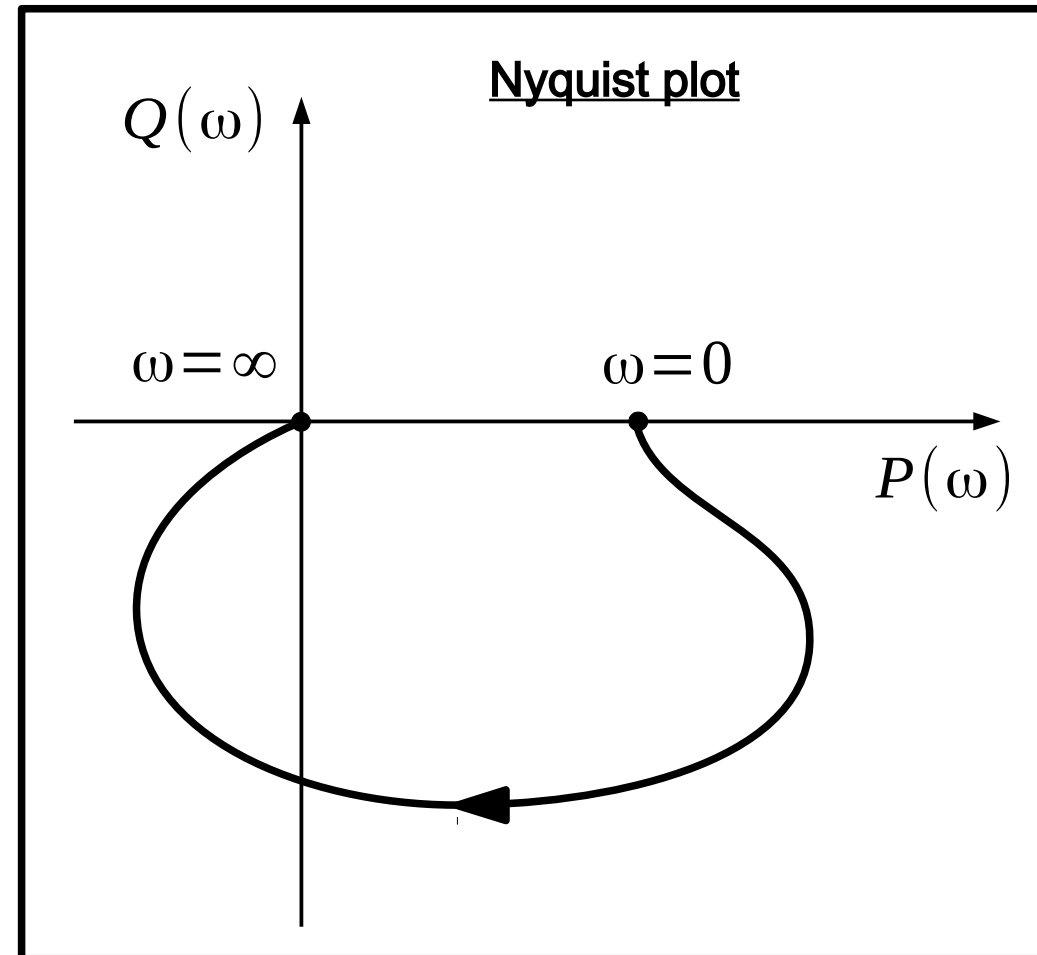
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Transfer function – frequency response

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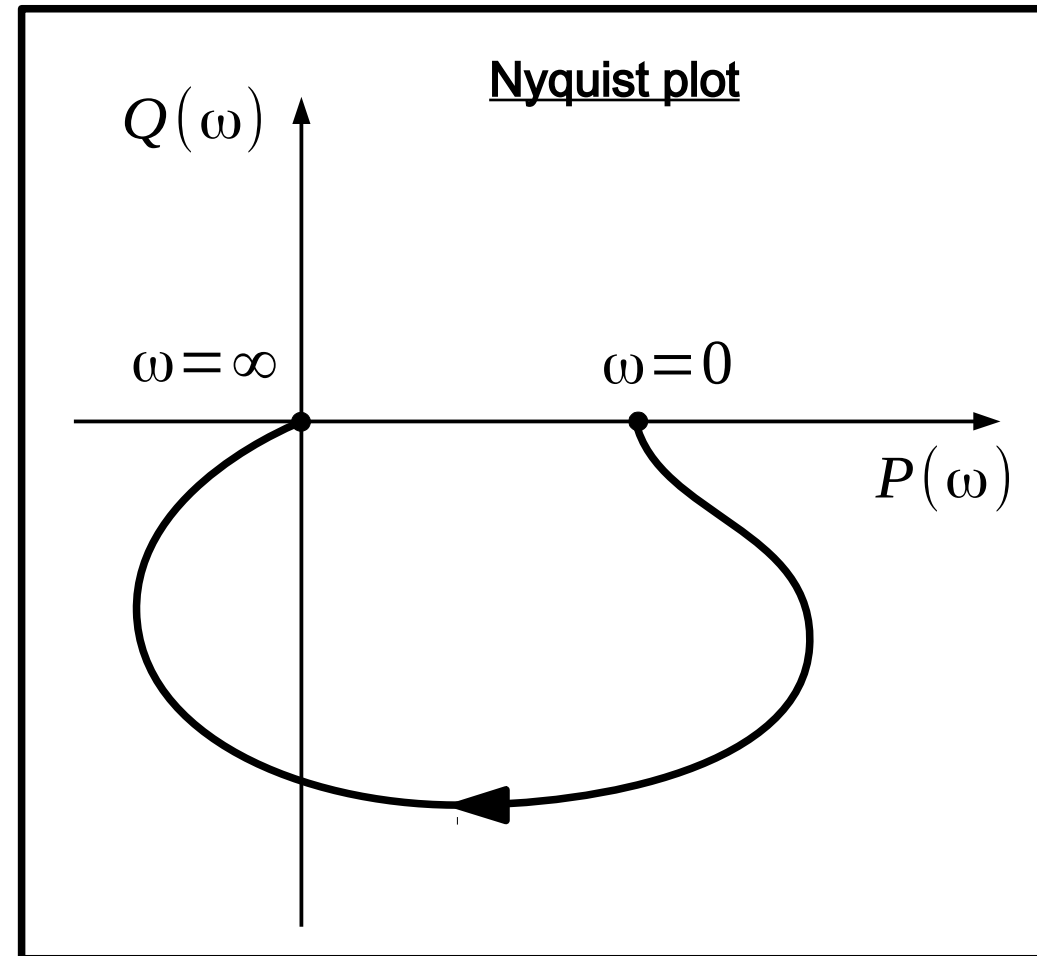


Transfer function – frequency response

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$$H(s) \xrightarrow{s=j\omega} H(j\omega) = P(\omega) + jQ(\omega)$$

$$A(\omega) = |H(j\omega)| = \sqrt{P^2(\omega) + Q^2(\omega)}$$



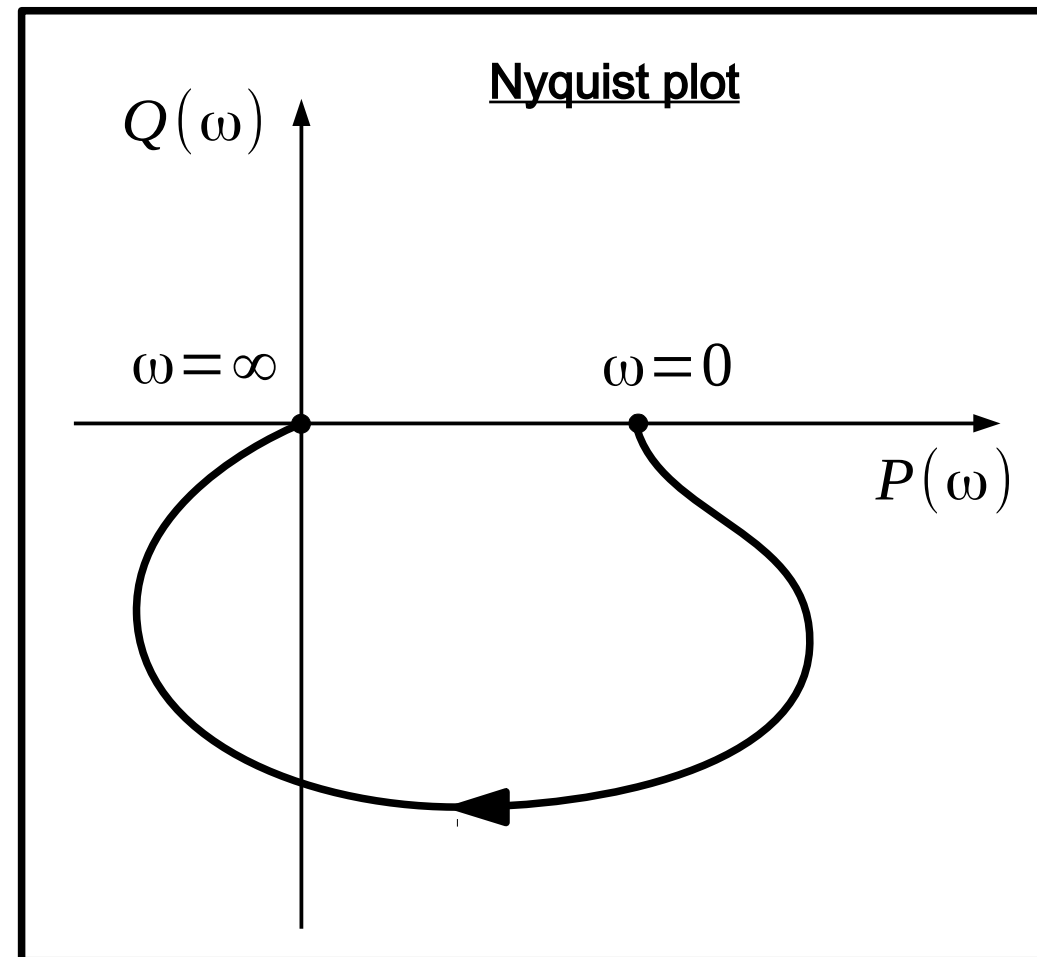
Transfer function – frequency response

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$$\varphi(\omega) = \text{Arg } H(j\omega) = \arctan \frac{Q}{P}$$



Transfer function – frequency response

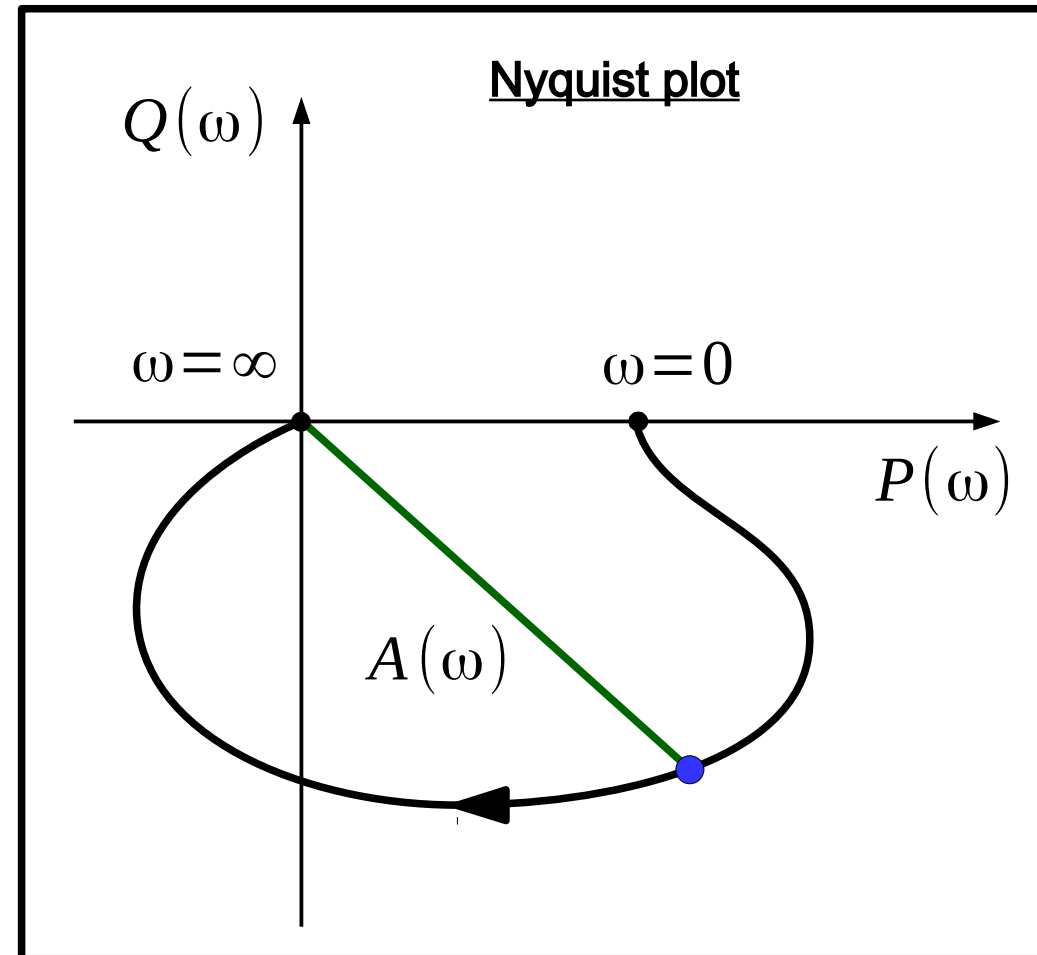
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GAIN

$$\varphi(\omega) = \text{Arg } H(j\omega) = \arctan \frac{Q}{P}$$



Transfer function – frequency response

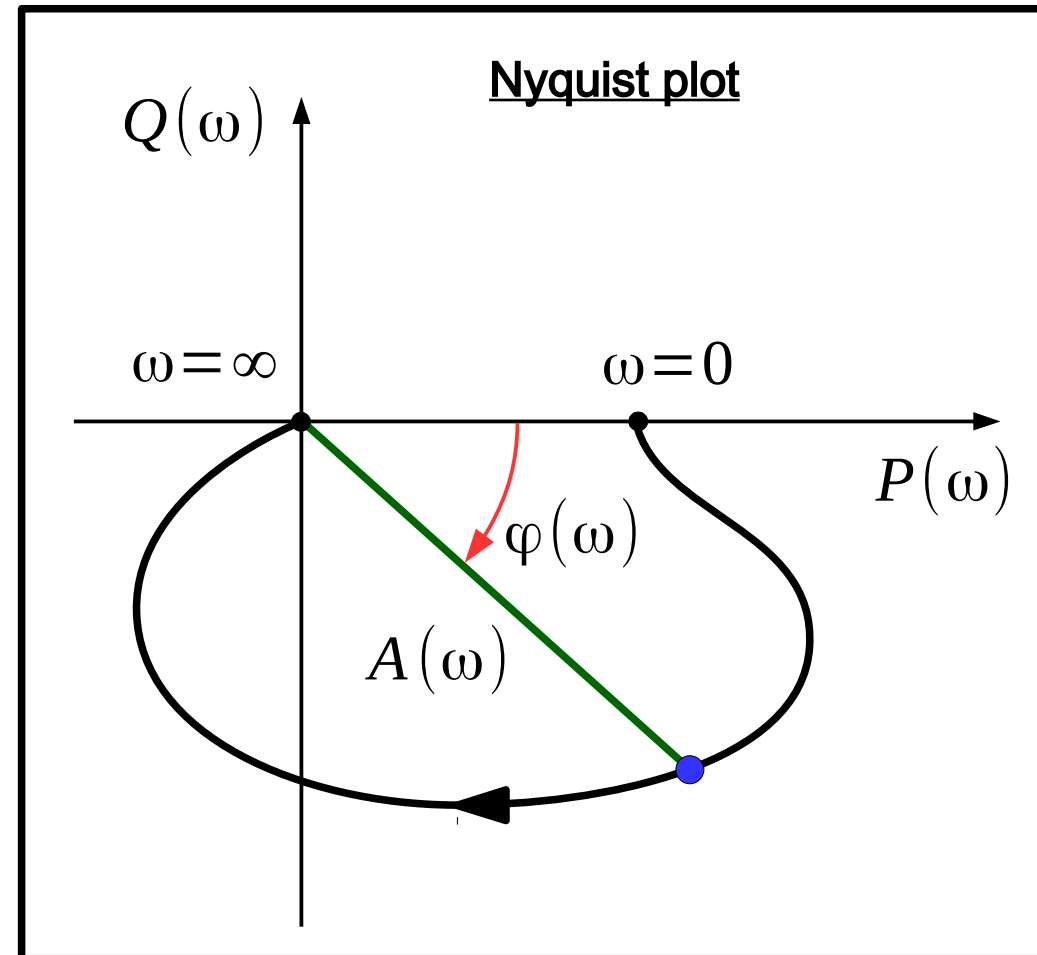
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DELAY



Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A \sin(\omega t + \varphi)$

Bode Plot



Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A \sin(\omega t + \varphi)$

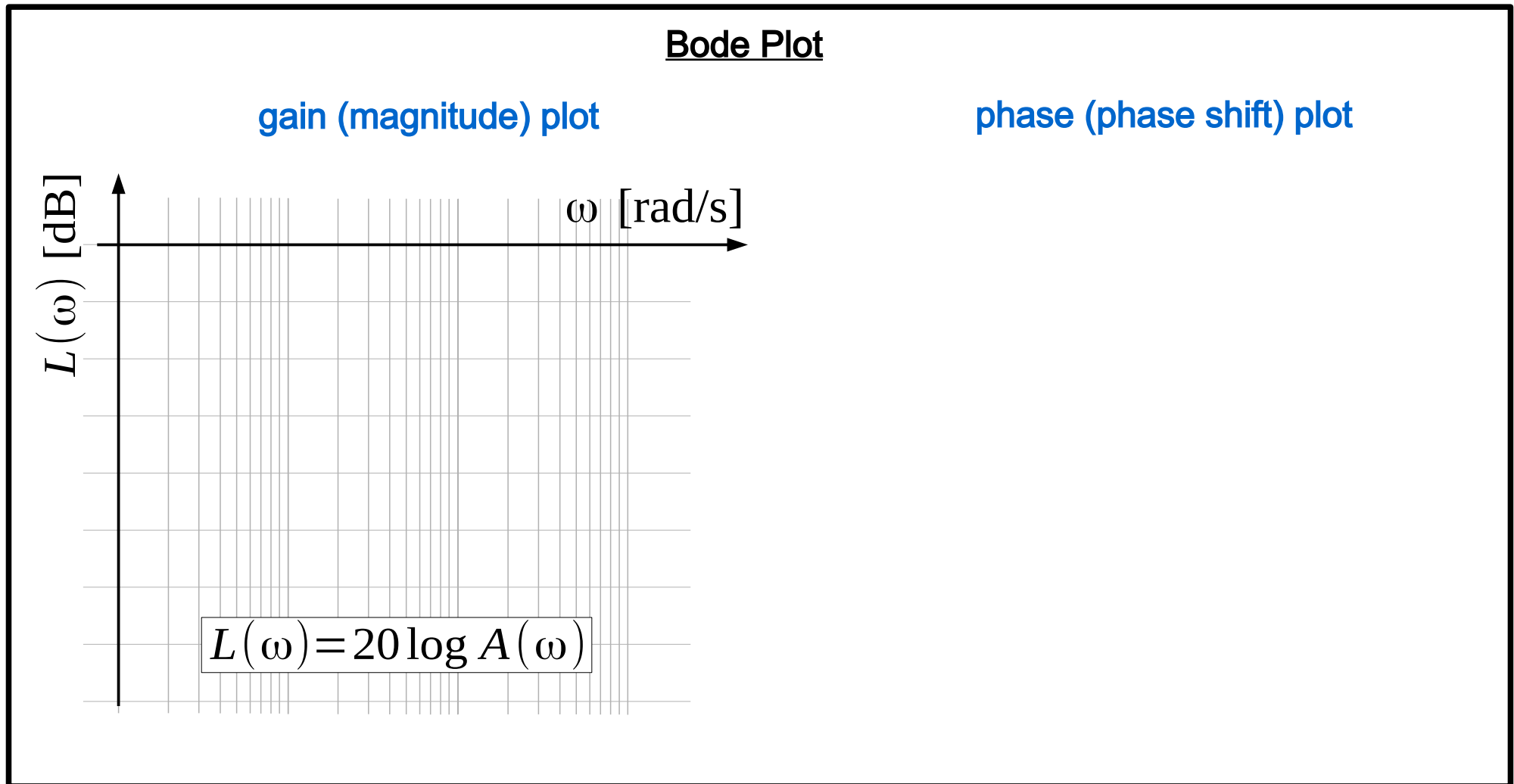
Bode Plot

gain (magnitude) plot

phase (phase shift) plot

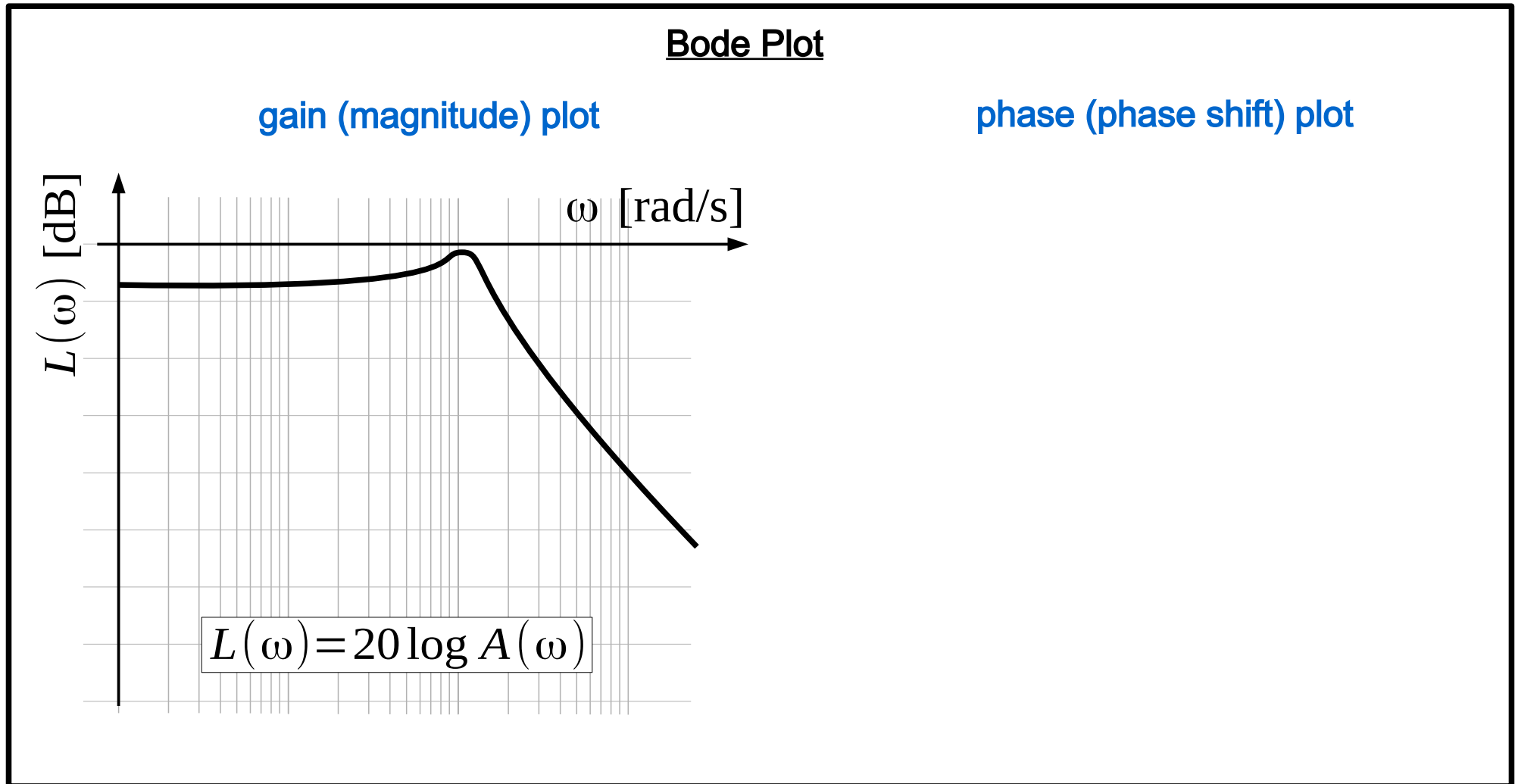
Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A \sin(\omega t + \varphi)$



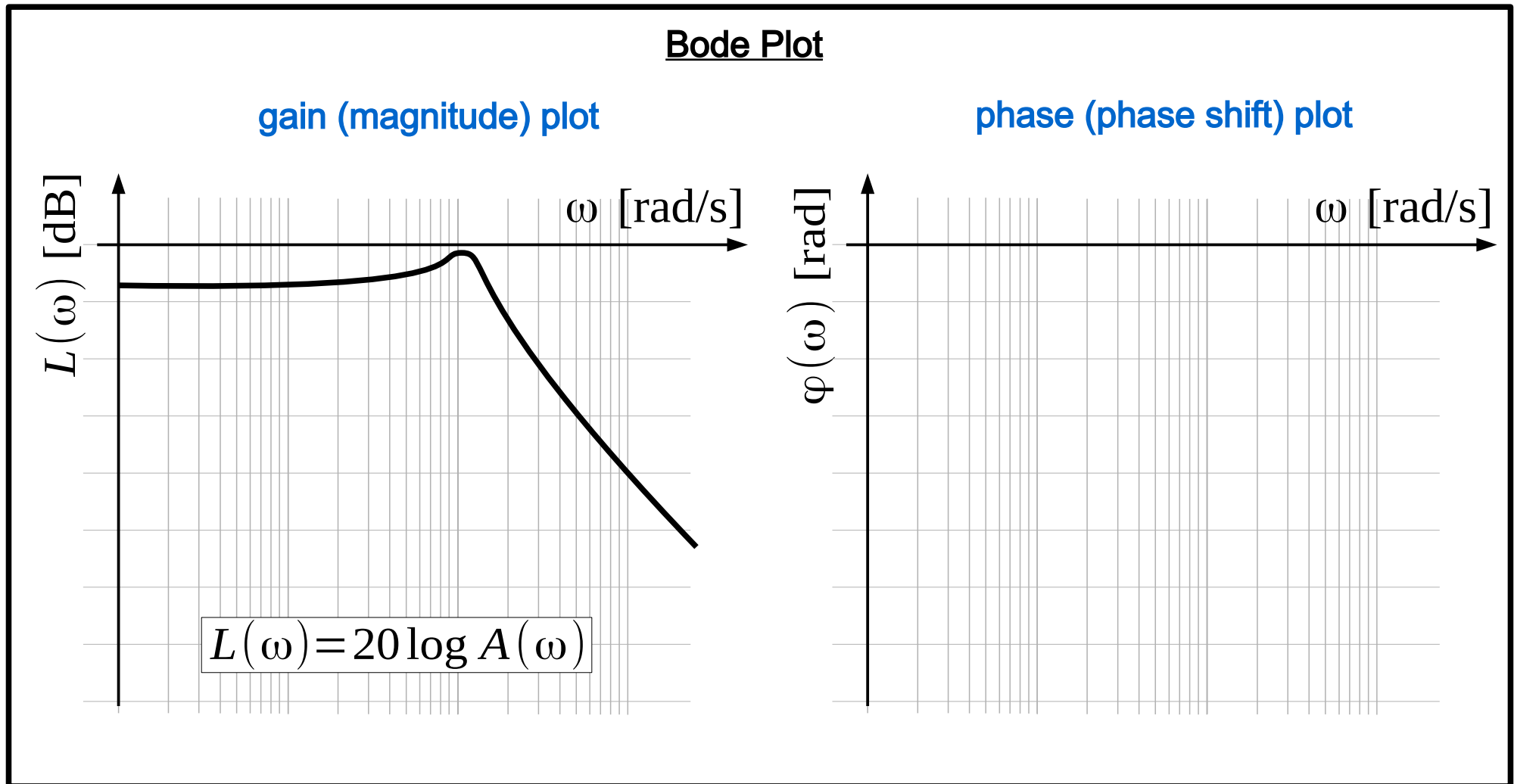
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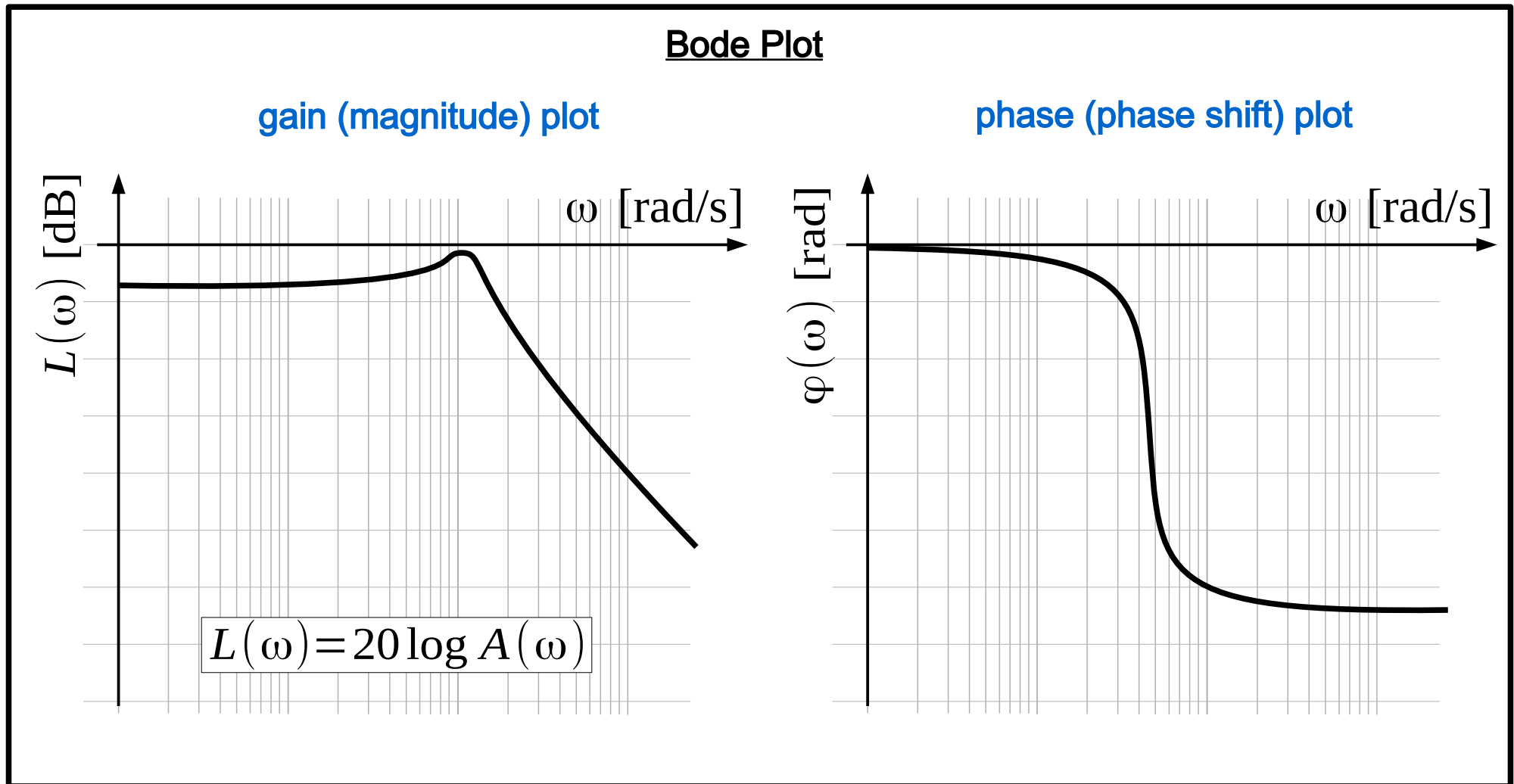
Transfer function – frequency response

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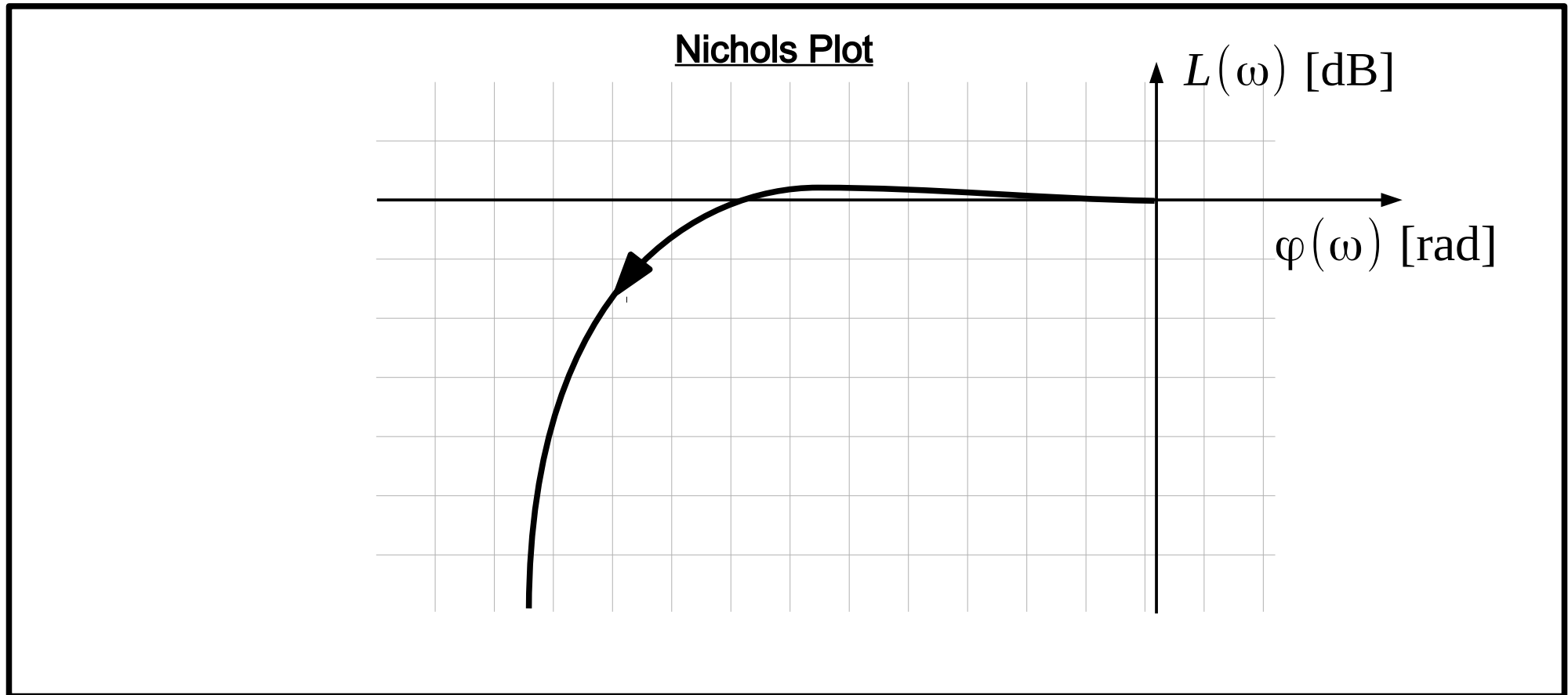
Transfer function – frequency response

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Transfer function – frequency response

input: $x(t) = \sin(\omega t)$ transfer function: $H(s)$ output: $y(t) = A \sin(\omega t + \varphi)$



Transfer function – frequency response

A (gain)	20logA [dB]
1000	60
100	40
10	20
1	0
0.1	-20
0.01	-40
0.001	-60

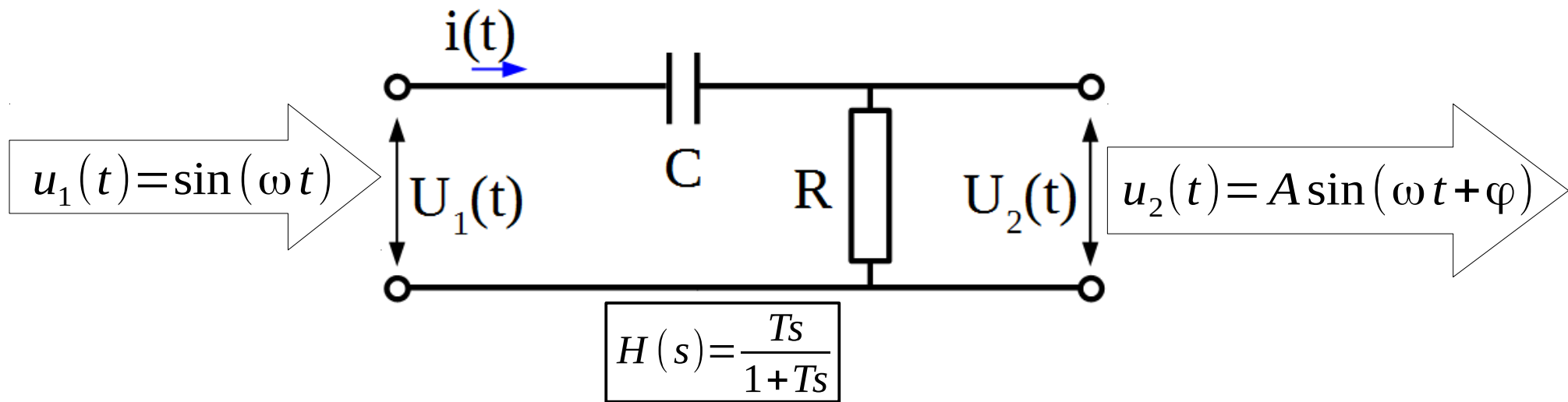
Transfer function – frequency response

Useful for:

- audio components (amplifiers, microphones, loudspeakers, cables)
- wireless components (antennas, amplifiers)
- vibrating systems (suspensions, drivetrains)
- control systems (regulators, objects)

Transfer function – frequency response

RC circuit example



Transfer function – frequency response

RC circuit example

$$H(s) = \frac{Ts}{1+Ts}$$

Transfer function – frequency response

RC circuit example

$$H(s) = \frac{Ts}{1 + Ts}$$

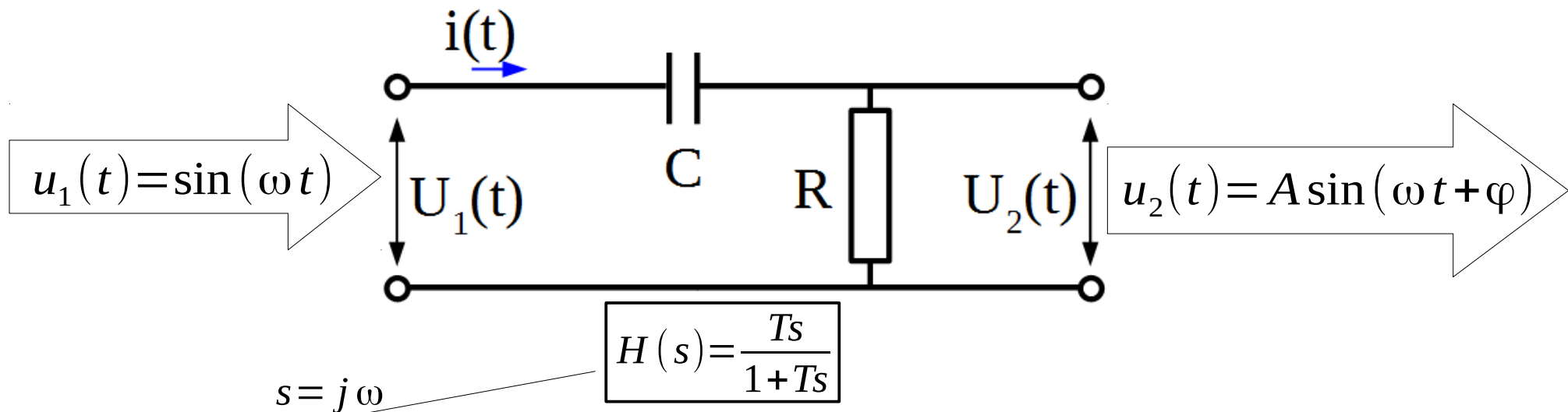
Transfer function – frequency response

RC circuit example

$$H(s) = \frac{Ts}{1 + Ts}$$

Transfer function – frequency response

RC circuit example



$$s = j\omega$$

$$H(s) = \frac{Ts}{1+Ts}$$

$$H(j\omega) = \frac{Tj\omega}{1+Tj\omega} = \frac{Tj\omega}{1+Tj\omega} \cdot \frac{1-Tj\omega}{1-Tj\omega} = \frac{Tj\omega - T^2j^2\omega^2}{1^2 - T^2j^2\omega^2} = \frac{Tj\omega + T^2\omega^2}{1^2 + T^2\omega^2} = \frac{T^2\omega^2}{1^2 + T^2\omega^2} + j \frac{T\omega}{1^2 + T^2\omega^2}$$

$$P(\omega) = \frac{T^2\omega^2}{1+T^2\omega^2} \quad Q(\omega) = \frac{T\omega}{1+T^2\omega^2} \quad A(\omega) = |H(j\omega)| = \sqrt{P^2(\omega) + Q^2(\omega)} = \frac{T\omega}{\sqrt{T^2\omega^2 + 1}}$$

$$L(\omega) = 20 \log A(\omega) = 20 \log \frac{T\omega}{\sqrt{T^2\omega^2 + 1}} = 20 \log T\omega - 20 \log \sqrt{T^2\omega^2 + 1}$$

$$\varphi(\omega) = \arg H(j\omega) = \arctan \frac{Q}{P} = \arctan \left(\frac{1}{T\omega} \right)$$

Transfer function – frequency response

RC circuit example

Nyquist plot

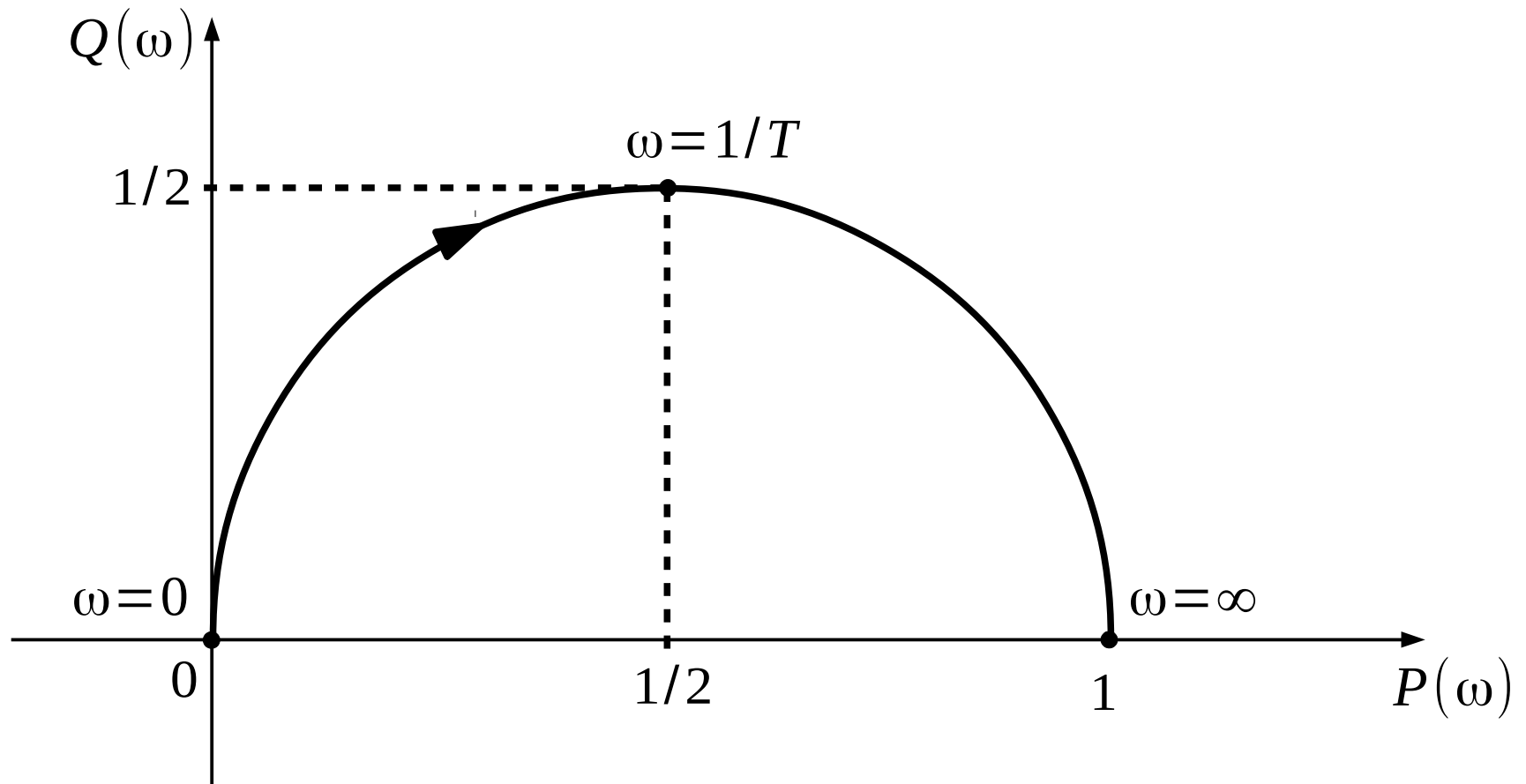
$$P(\omega) = \frac{T^2 \omega^2}{1 + T^2 \omega^2} \quad Q(\omega) = \frac{T \omega}{1 + T^2 \omega^2}$$

Transfer function – frequency response

RC circuit example

Nyquist plot

$$P(\omega) = \frac{T^2 \omega^2}{1 + T^2 \omega^2} \quad Q(\omega) = \frac{T \omega}{1 + T^2 \omega^2}$$



Transfer function – frequency response

RC circuit example

$$A(\omega) = \frac{T \omega}{\sqrt{T^2 \omega^2 + 1}}$$

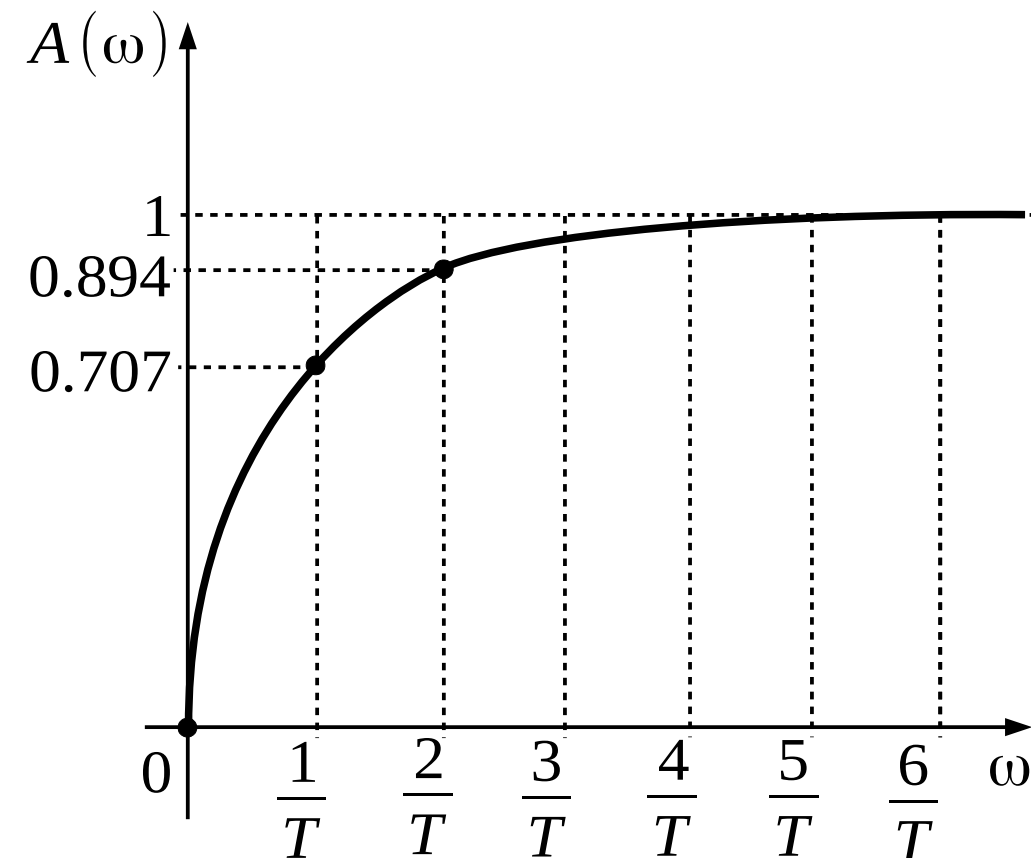
$$\varphi(\omega) = \arctan\left(\frac{1}{T \omega}\right)$$

Transfer function – frequency response

RC circuit example

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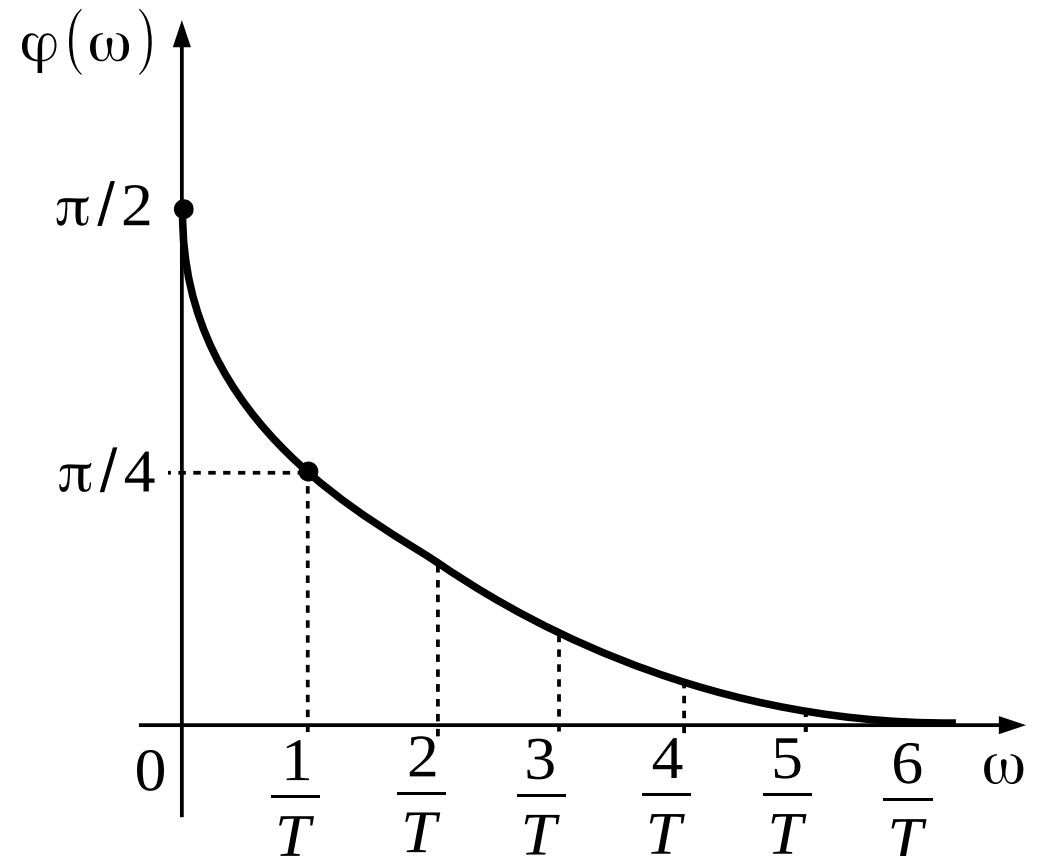
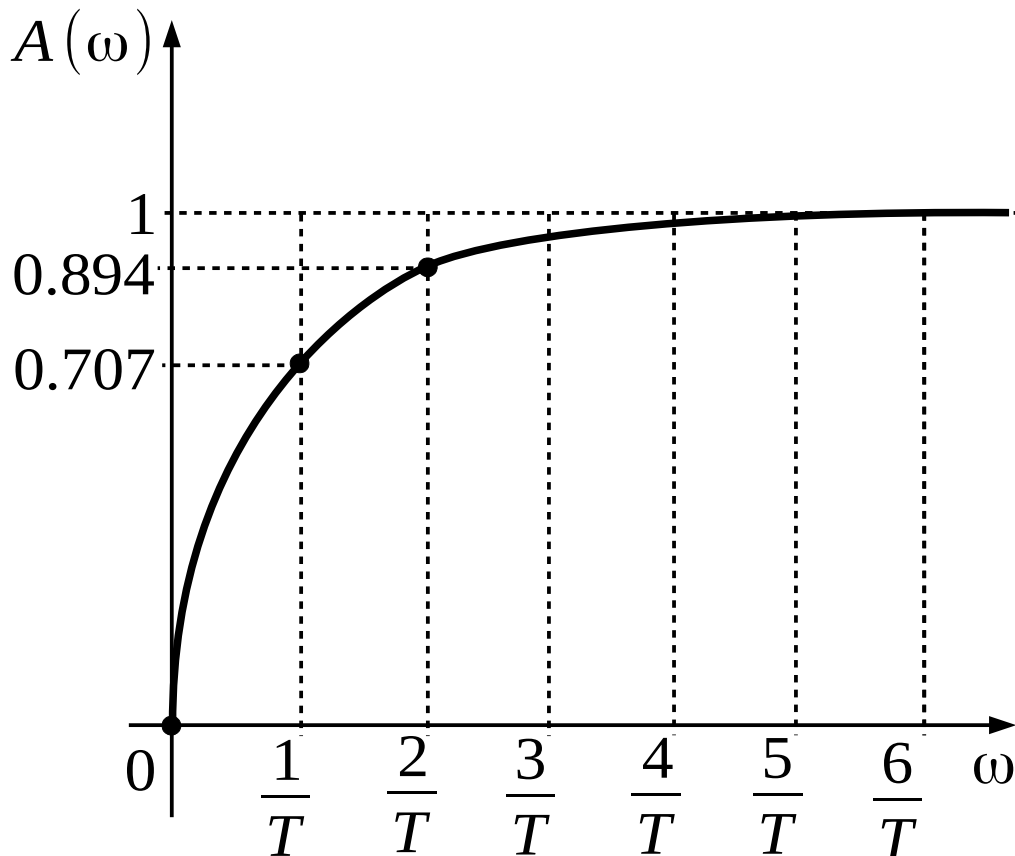


Transfer function – frequency response

RC circuit example

$$A(\omega) = \frac{T\omega}{\sqrt{T^2\omega^2 + 1}}$$

$$\varphi(\omega) = \arctan\left(\frac{1}{T\omega}\right)$$



Transfer function – frequency response

RC circuit example

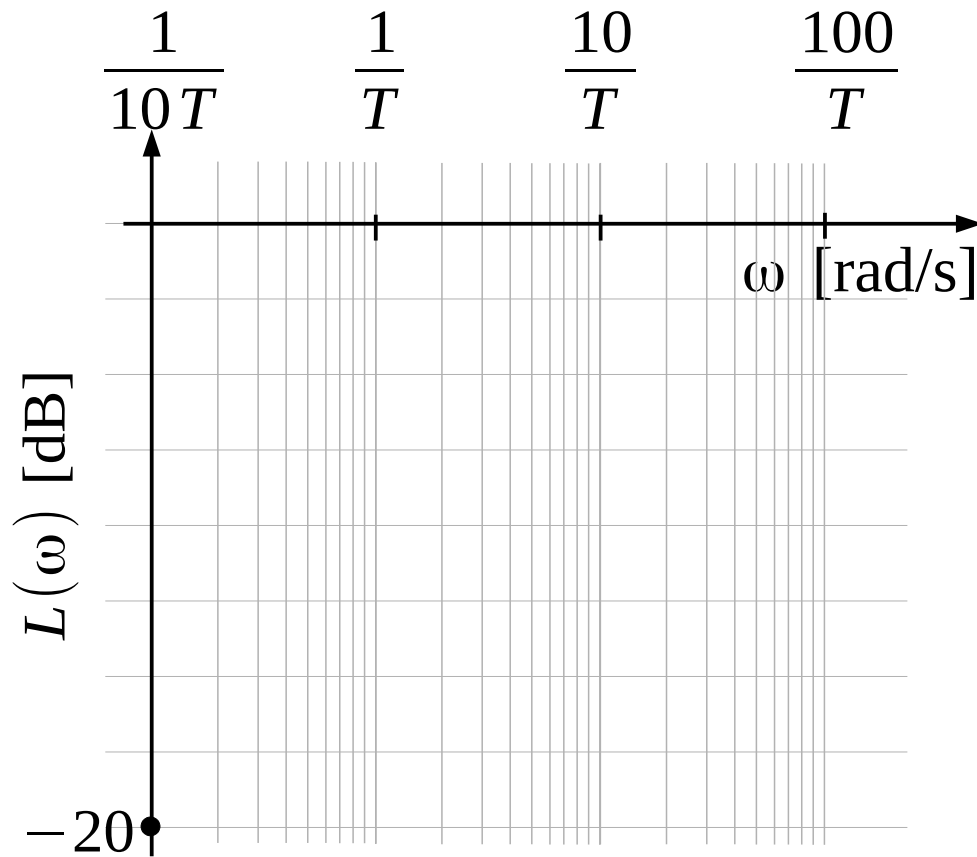
$$L(\omega) = 20 \log T \omega - 20 \log \sqrt{T^2 \omega^2 + 1}$$

$$\varphi(\omega) = \arctan\left(\frac{1}{T \omega}\right)$$

Transfer function – frequency response

RC circuit example

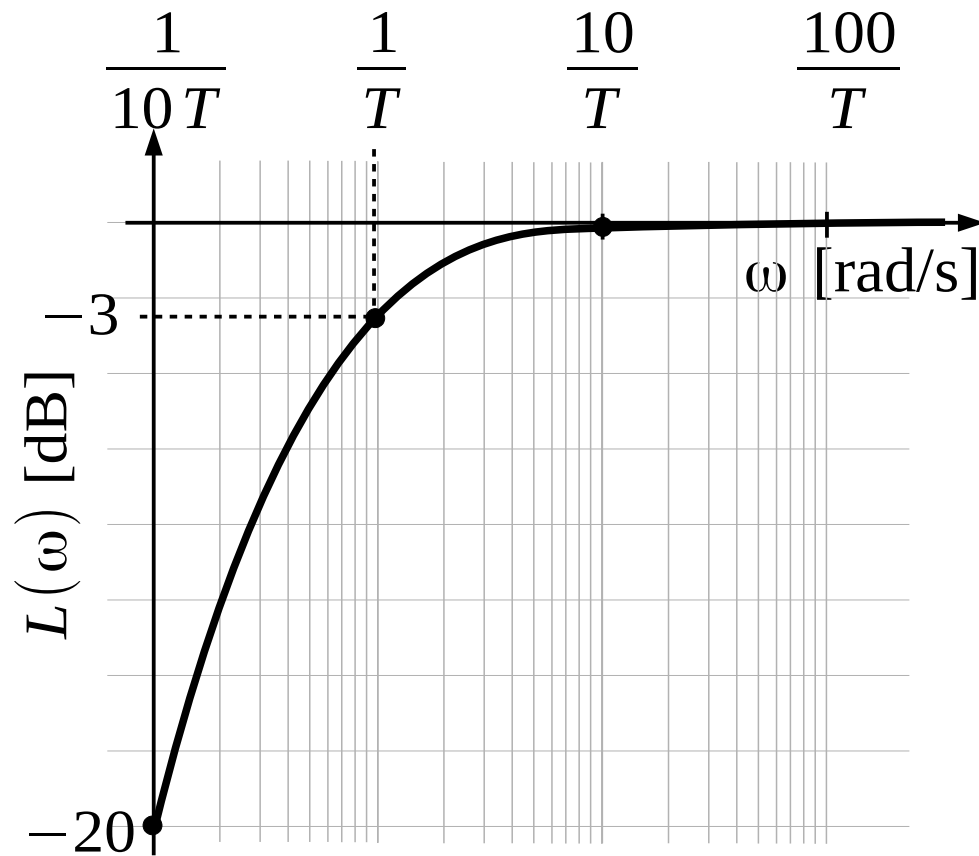
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Transfer function – frequency response

RC circuit example

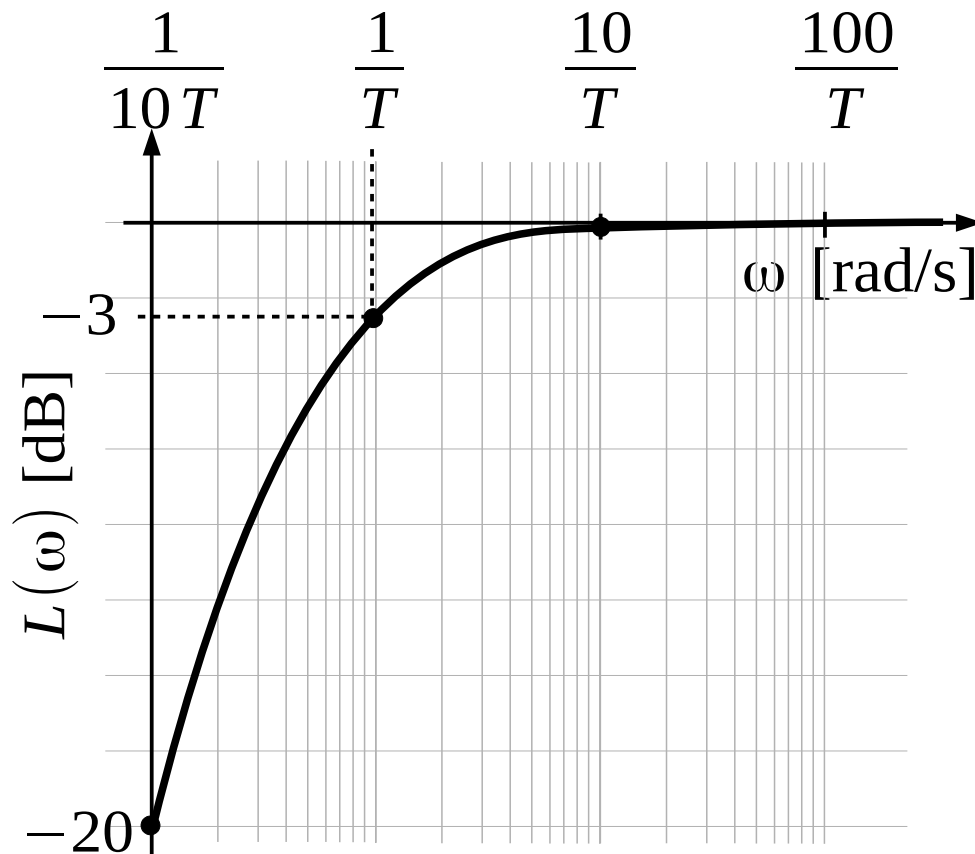
$$L(\omega) = 20 \log T \omega - 20 \log \sqrt{T^2 \omega^2 + 1}$$



Transfer function – frequency response

RC circuit example

$$L(\omega) = 20 \log T \omega - 20 \log \sqrt{T^2 \omega^2 + 1}$$



$$L\left(\omega = \frac{1}{T}\right) = 20 \log 1 - 20 \log \sqrt{2} \approx -3$$

$$L\left(\omega = \frac{10}{T}\right) = 20 \log 10 - 20 \log \sqrt{101} \approx 0$$

$$L\left(\omega = \frac{100}{T}\right) = 20 \log 100 - 20 \log \sqrt{10001} \approx 0$$

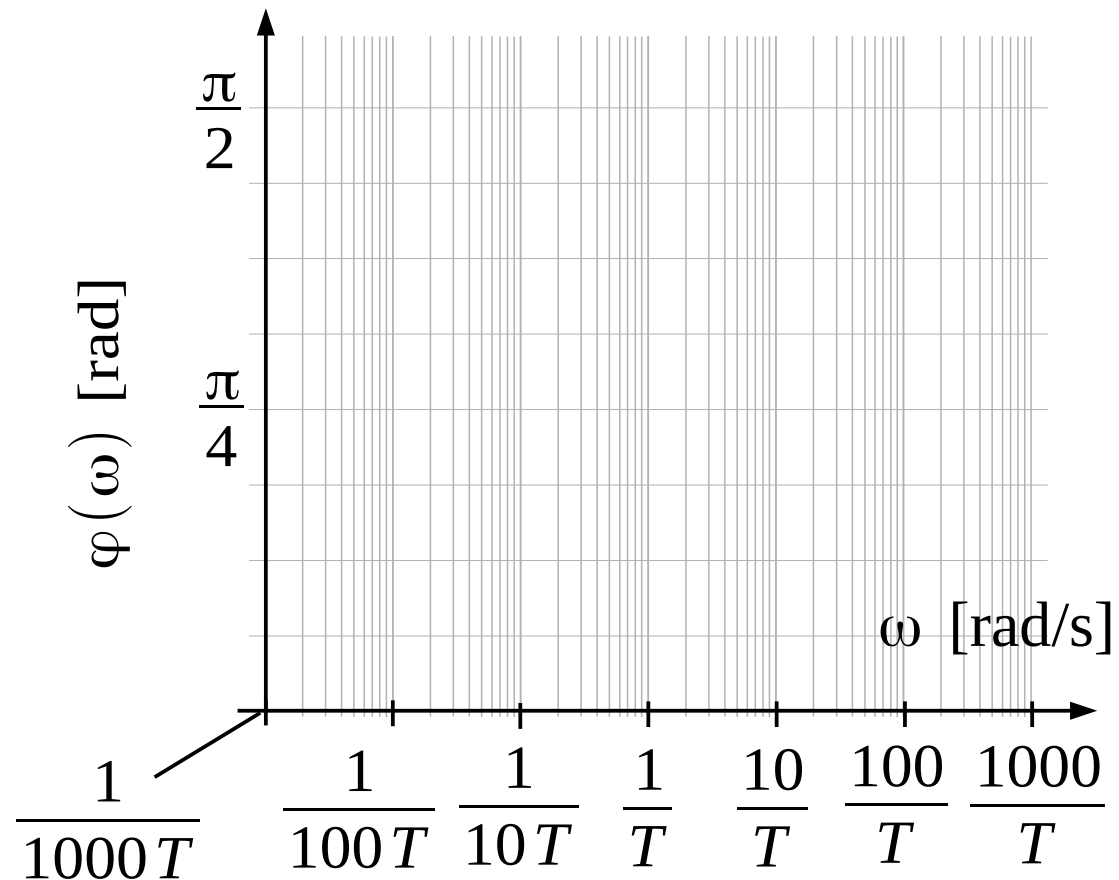
$$L\left(\omega = \frac{1}{10T}\right) = 20 \log 0,1 - 20 \log \sqrt{1,01} \approx -20$$

$$L\left(\omega = \frac{1}{100T}\right) = 20 \log 0,01 - 20 \log \sqrt{1,0001} \approx -40$$

Transfer function – frequency response

RC circuit example

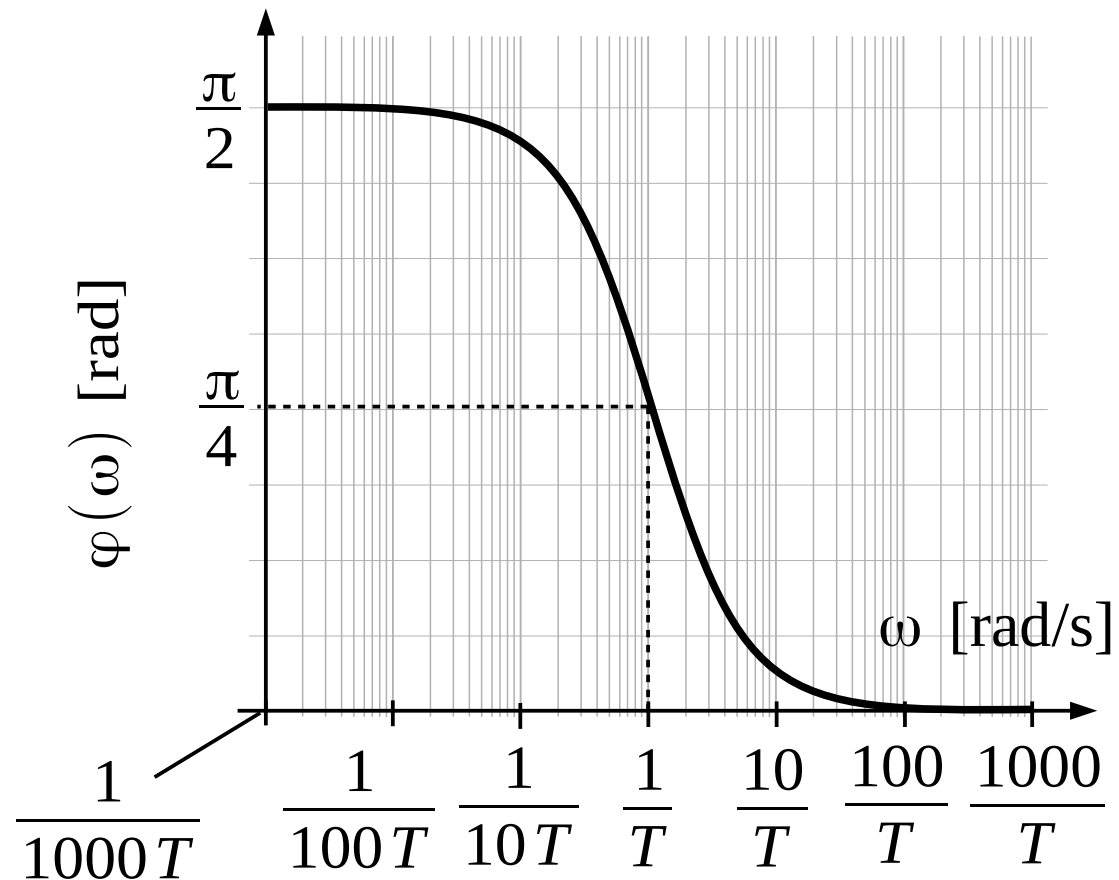
$$\varphi(\omega) = \arctan\left(\frac{1}{T\omega}\right)$$



Transfer function – frequency response

RC circuit example

$$\varphi(\omega) = \arctan\left(\frac{1}{T\omega}\right)$$



Transfer function – frequency response

RC circuit example

$$\varphi(\omega) = \arctan\left(\frac{1}{T\omega}\right)$$

$$\varphi\left(\omega = \frac{1}{T}\right) = \arctan 1 = \frac{\pi}{4} = 45^\circ$$

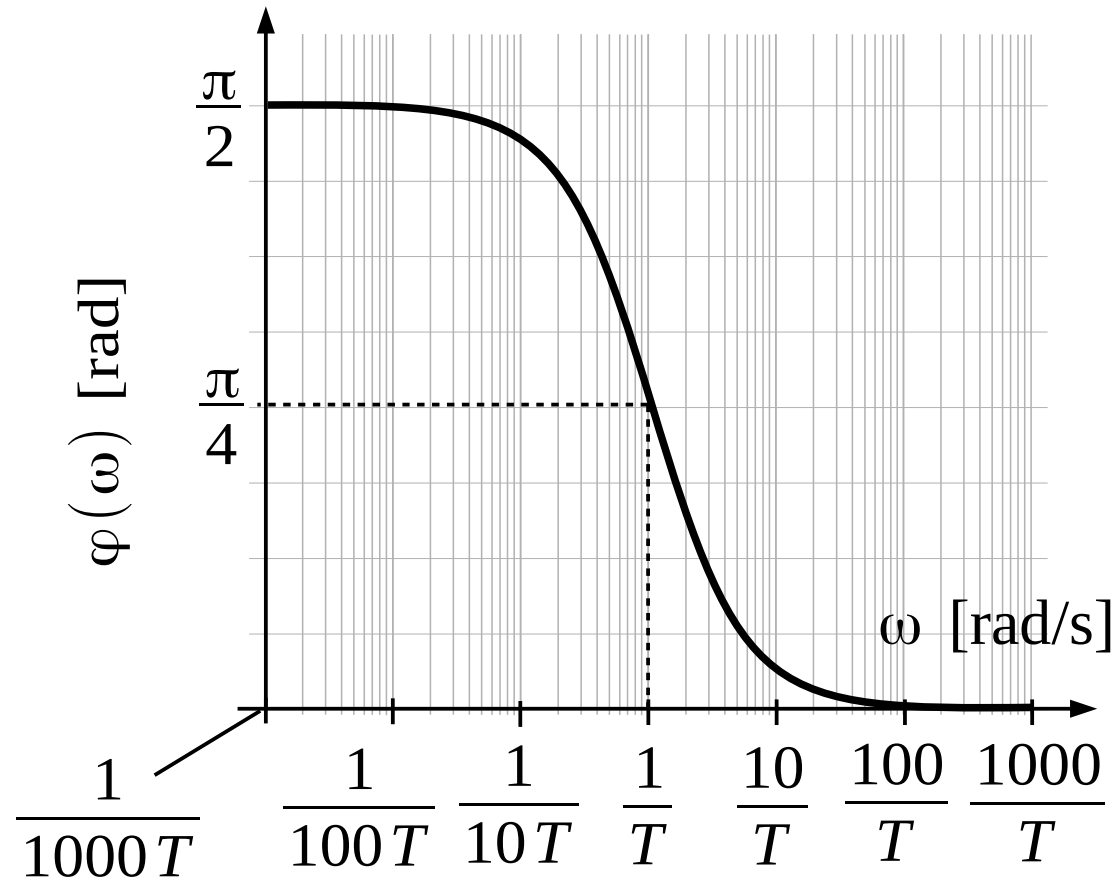
$$\varphi\left(\omega = \frac{10}{T}\right) = \arctan \frac{1}{10} = 0,1 = 5,71^\circ$$

$$\varphi\left(\omega = \frac{100}{T}\right) = \arctan \frac{1}{100} = 0,01 = 0,57^\circ$$

$$\varphi\left(\omega = \frac{1000}{T}\right) = \arctan \frac{1}{1000} = 0,001 = 0,06^\circ$$

$$\varphi\left(\omega = \frac{1}{10T}\right) = \arctan 10 = 1,47 = 84,29^\circ$$

$$\varphi\left(\omega = \frac{1}{100T}\right) = \arctan 100 = 1,56 = 89,43^\circ$$



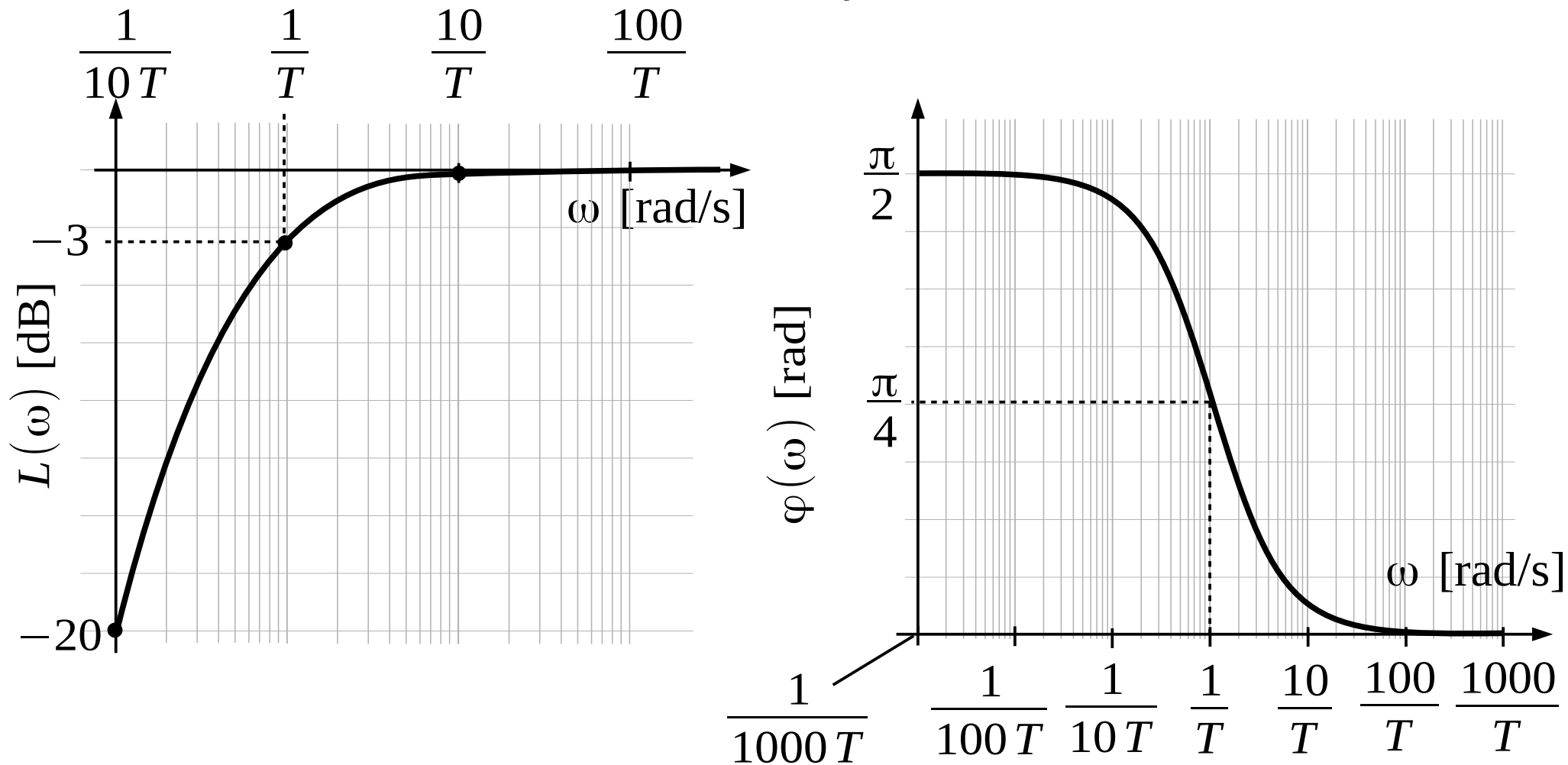
Transfer function – frequency response

RC circuit example

$$L(\omega) = 20 \log T \omega - 20 \log \sqrt{T^2 \omega^2 + 1}$$

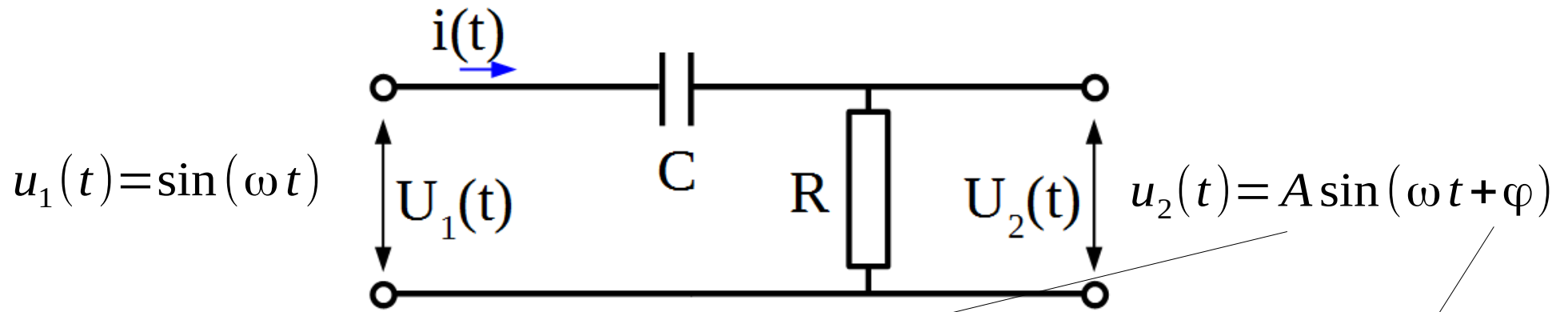
$$\varphi(\omega) = \arctan\left(\frac{1}{T \omega}\right)$$

Bode plot



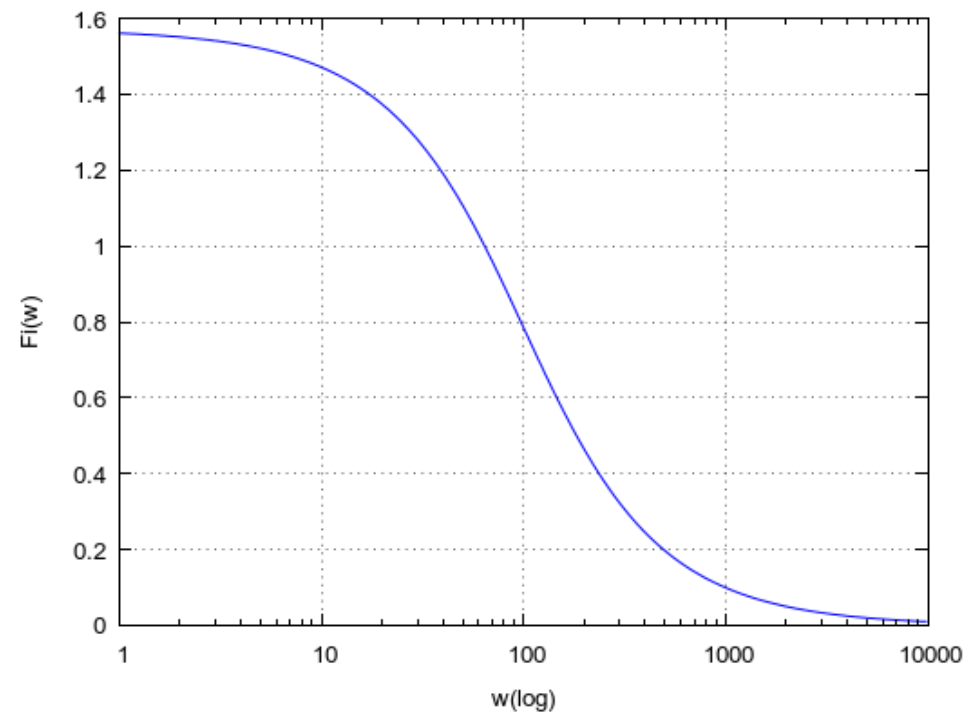
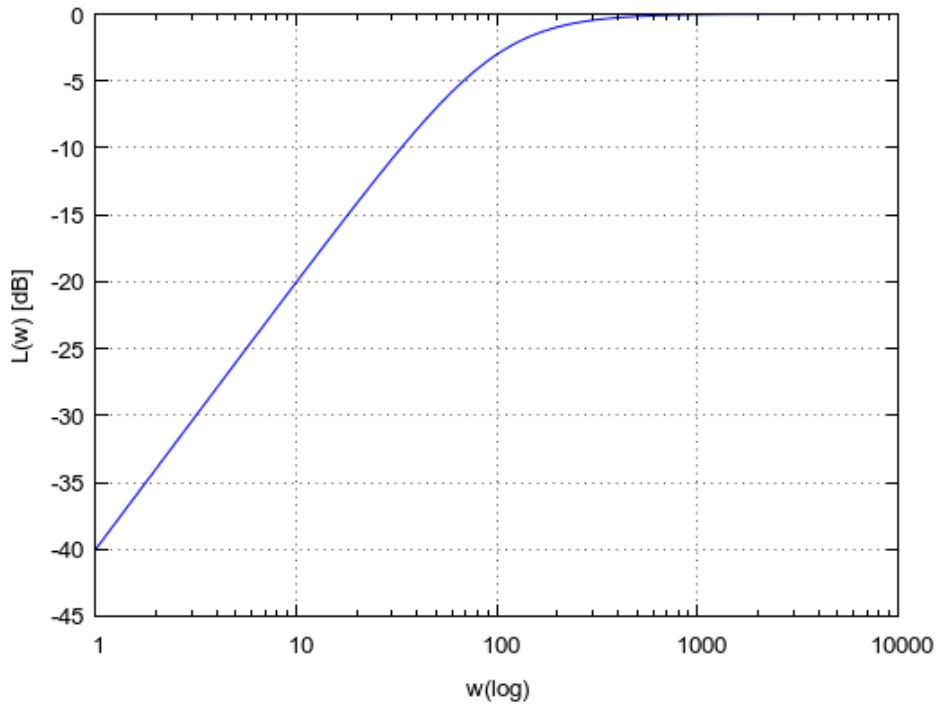
Transfer function – frequency response

RC circuit example



Example:

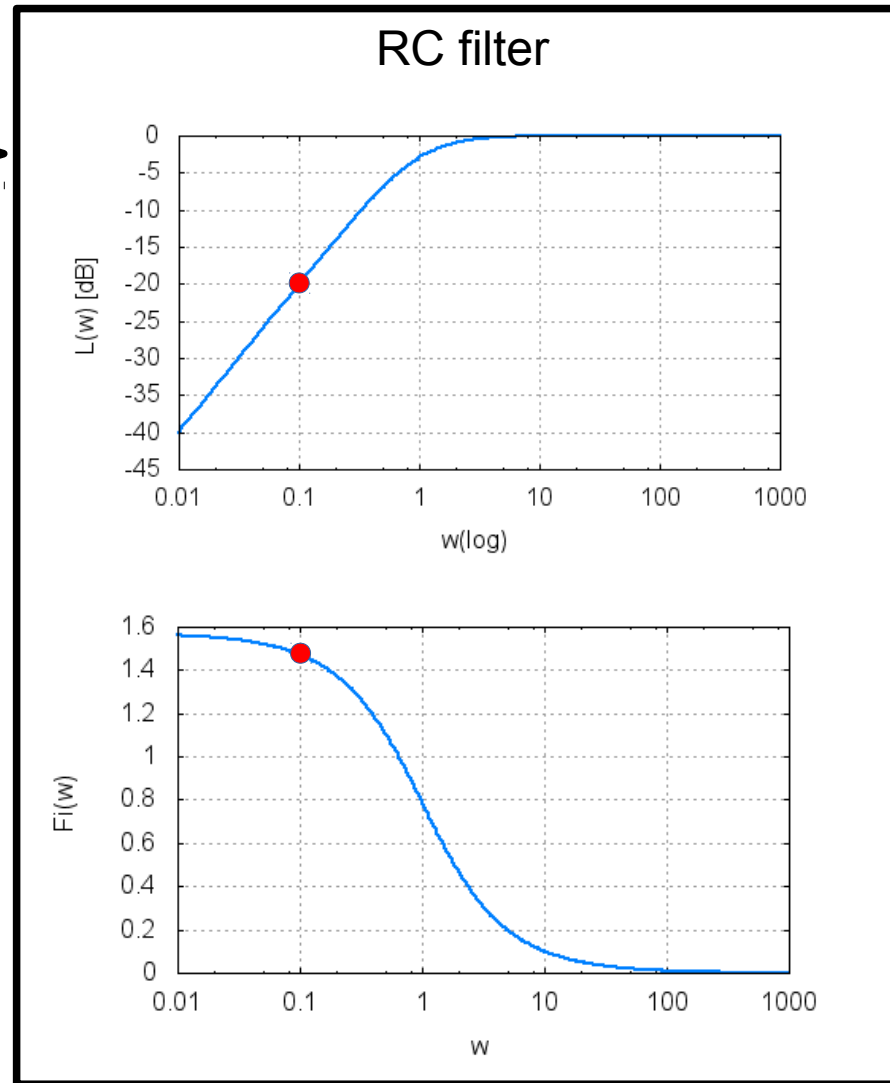
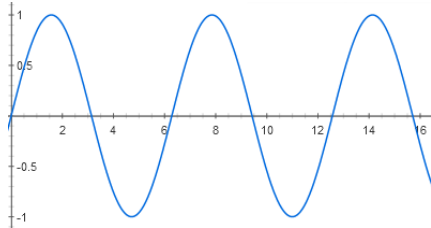
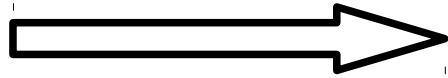
$$R = 1 \text{ k}\Omega, C = 10 \mu\text{F}$$



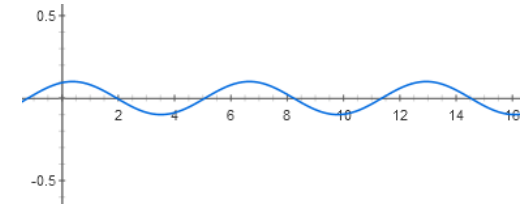
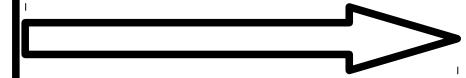
Transfer function – frequency response

RC circuit example

$$u_1(t) = \sin(\omega t)$$

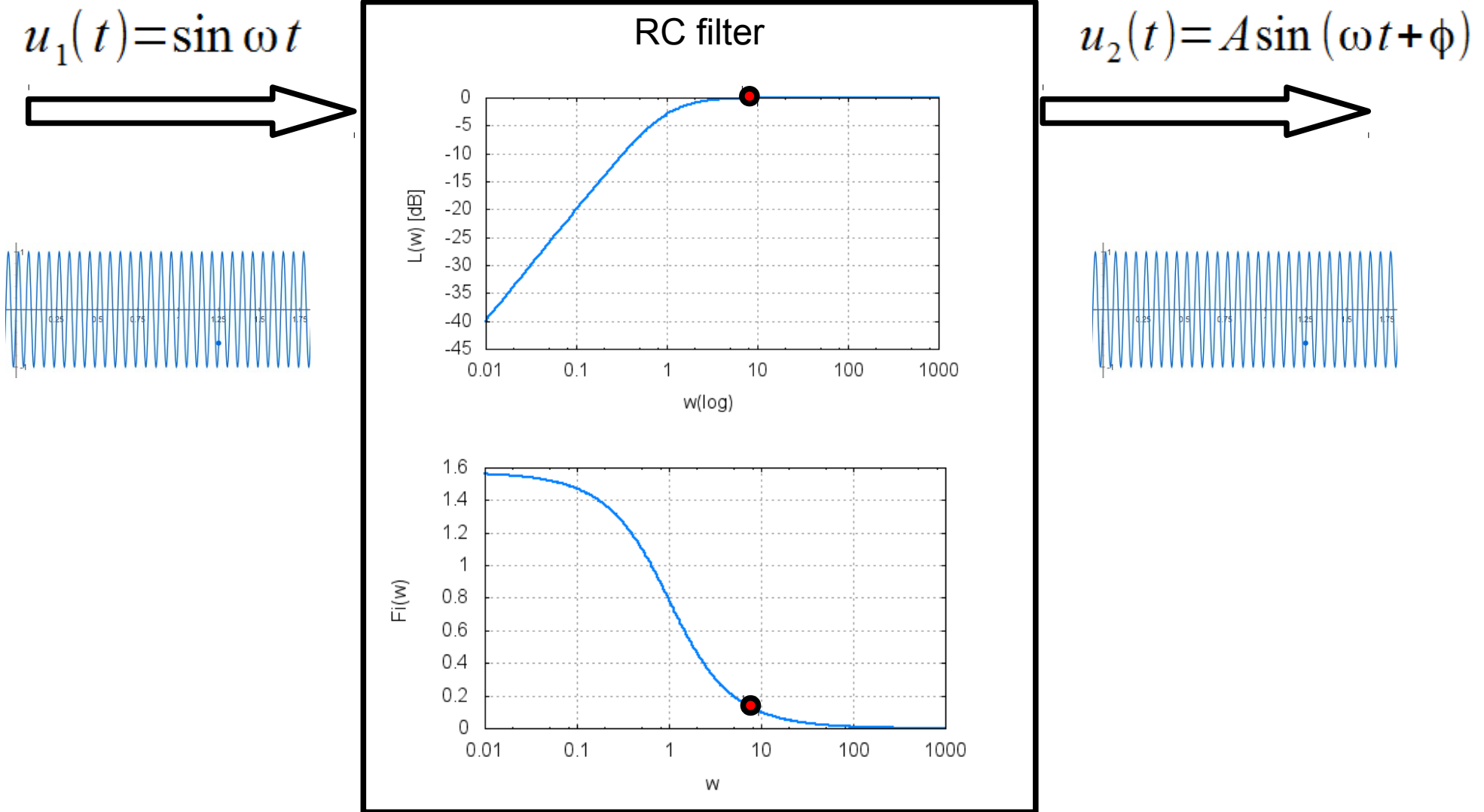


$$u_2(t) = A \sin(\omega t + \varphi)$$



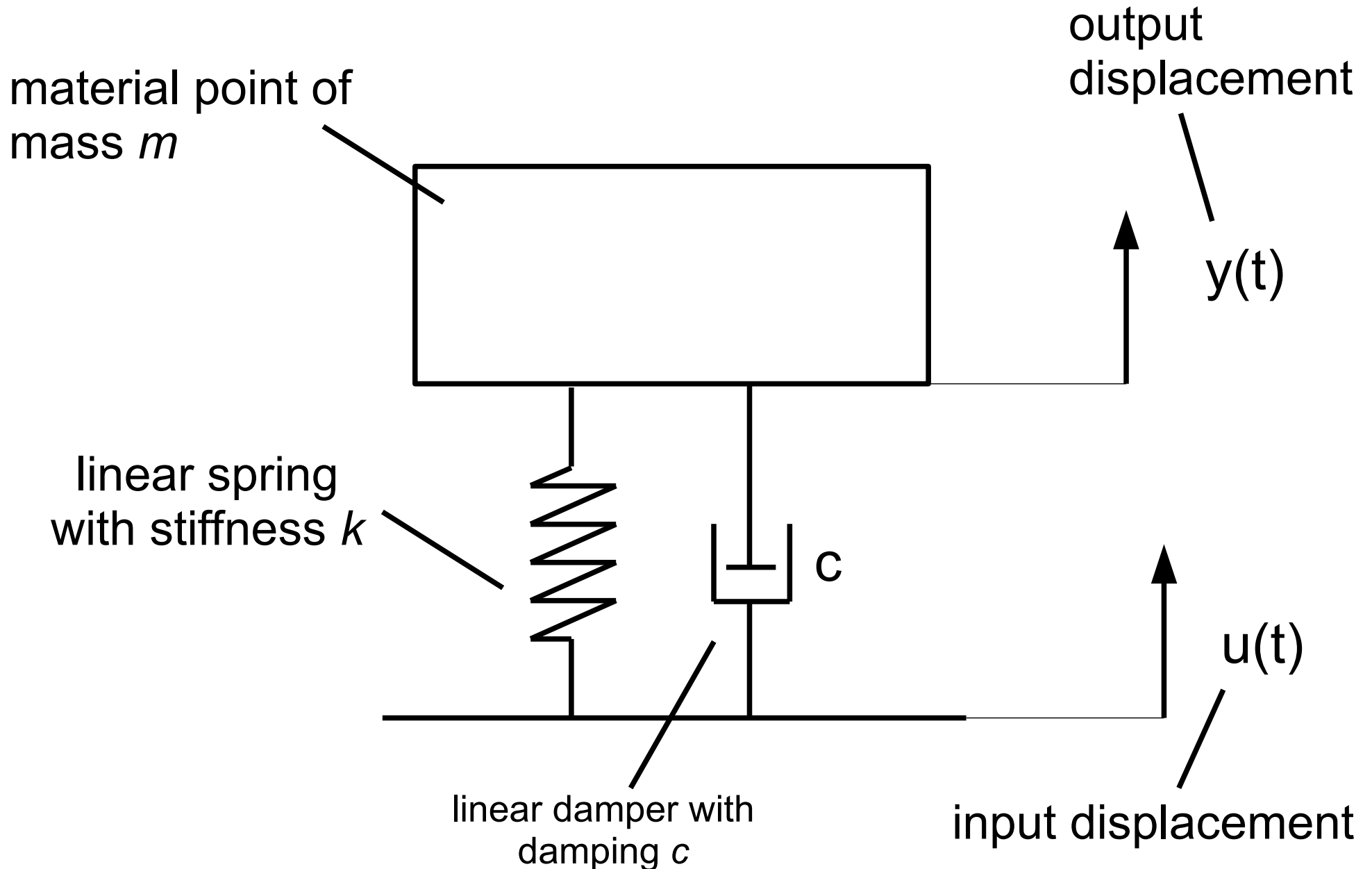
Transfer function – frequency response

RC circuit example



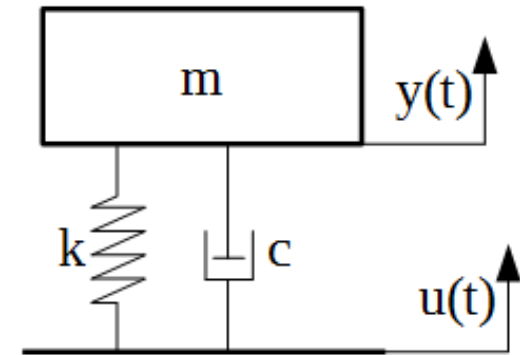
Transfer function – frequency response

Example 2 - vibrating system



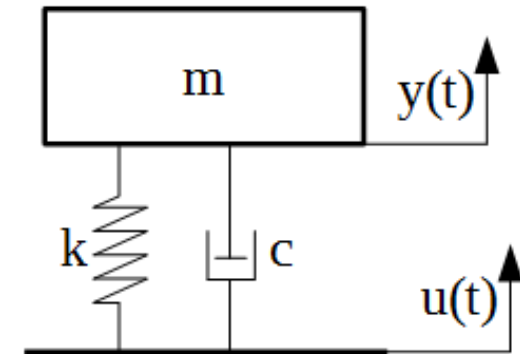
Transfer function – frequency response

Example 2 - vibrating system



Transfer function – frequency response

Example 2 - vibrating system



$$m \ddot{y}(t) + c \dot{y}(t) + k y(t) = c \dot{u}(t) + k u(t)$$

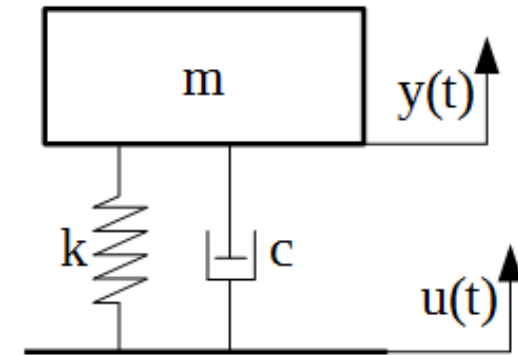
$$H(s) = \frac{cs + k}{ms^2 + cs + k}$$

$$P(\omega) = \frac{k^2 + c^2 \omega^2 - km\omega^2}{(k - m\omega^2)^2 + c^2 \omega^2}, \quad Q(\omega) = \frac{-cm\omega^3}{(k - m\omega^2)^2 + c^2 \omega^2}$$

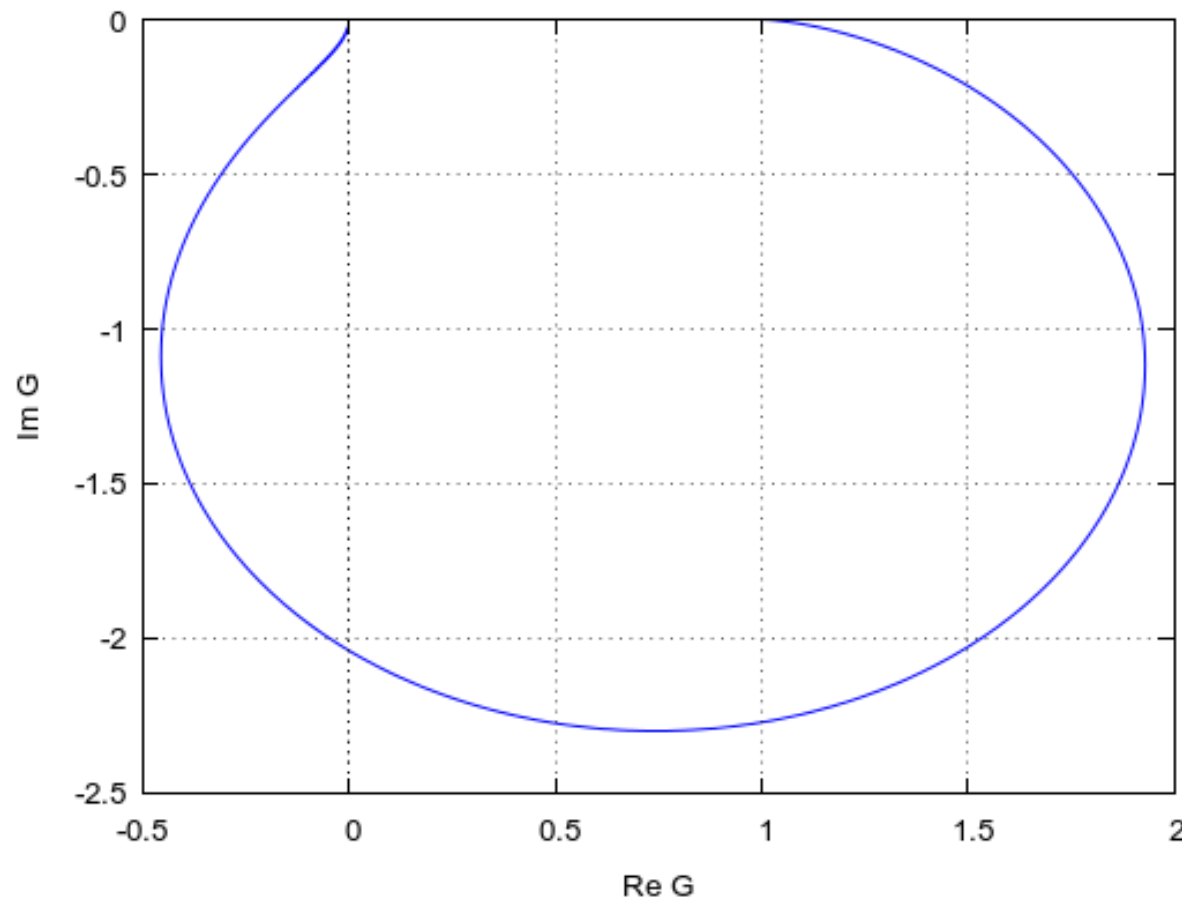
Transfer function – frequency response

Example 2 - vibrating system

Plots for: $m = 300 \text{ kg}$, $c = 800 \frac{\text{Ns}}{\text{m}}$, $k = 11000 \frac{\text{N}}{\text{m}}$

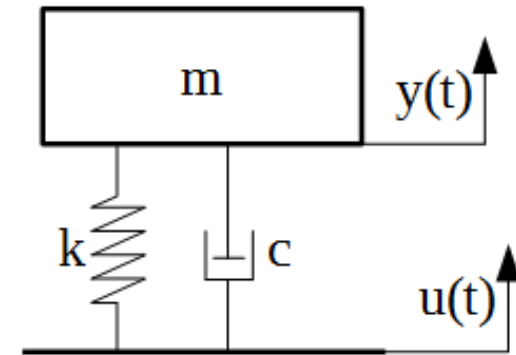


Nyquist plot

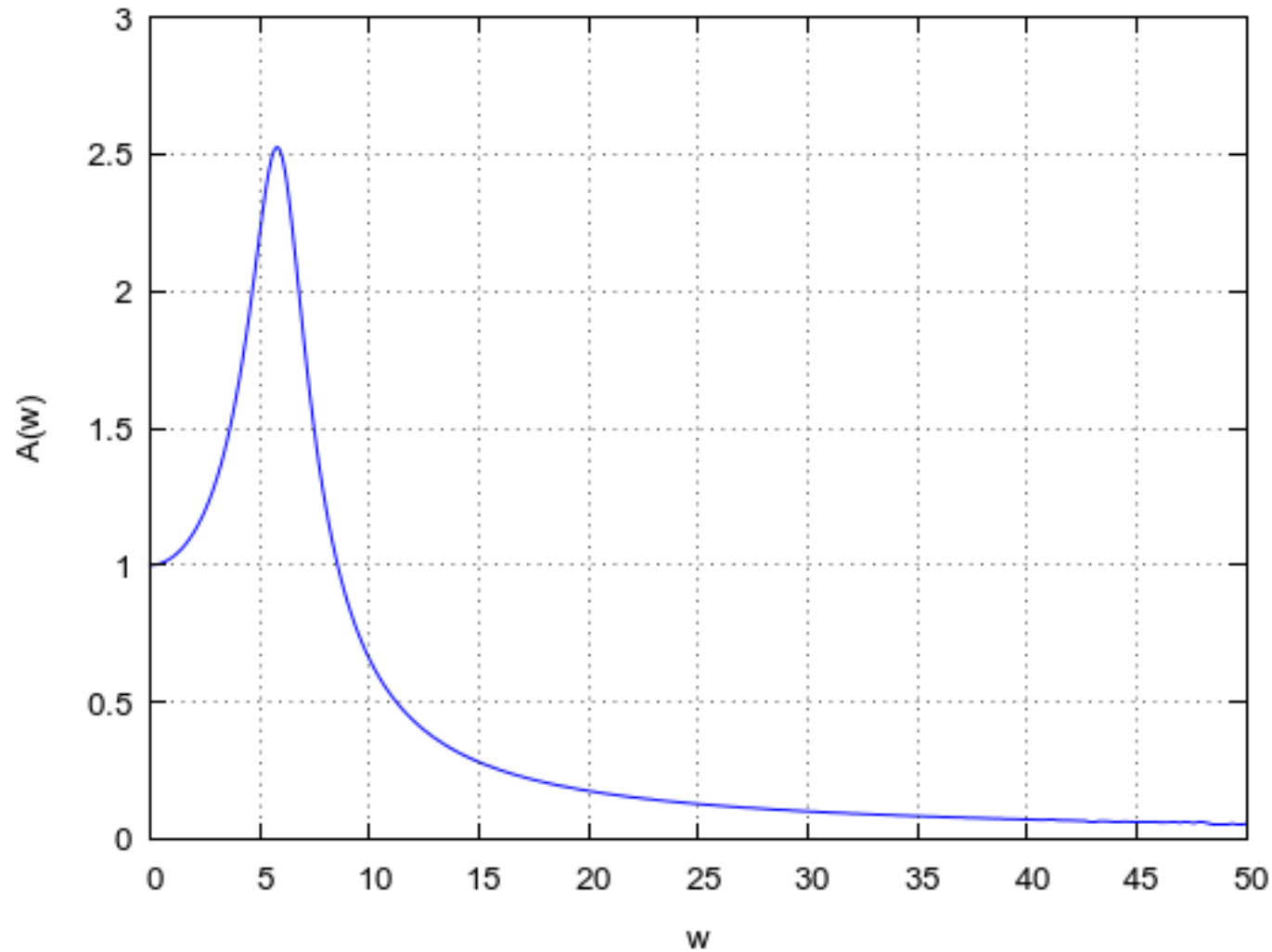


Transfer function – frequency response

Example 2 - vibrating system

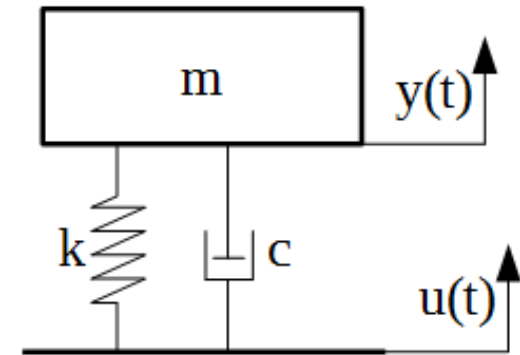


gain plot

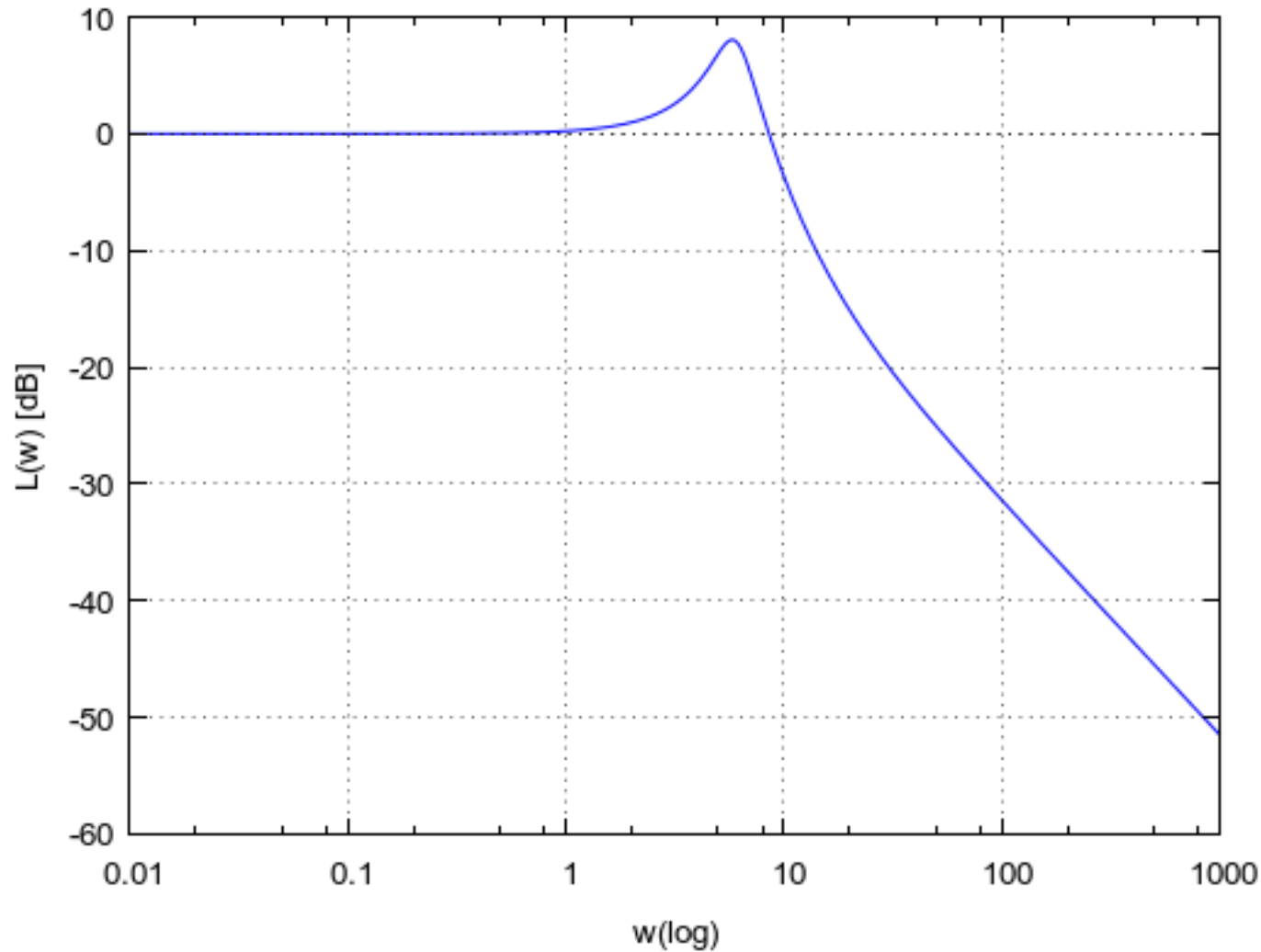


Transfer function – frequency response

Example 2 - vibrating system

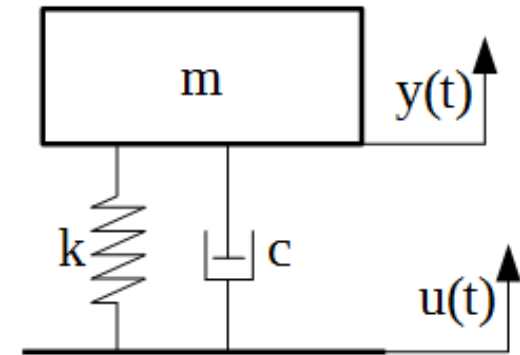


gain plot

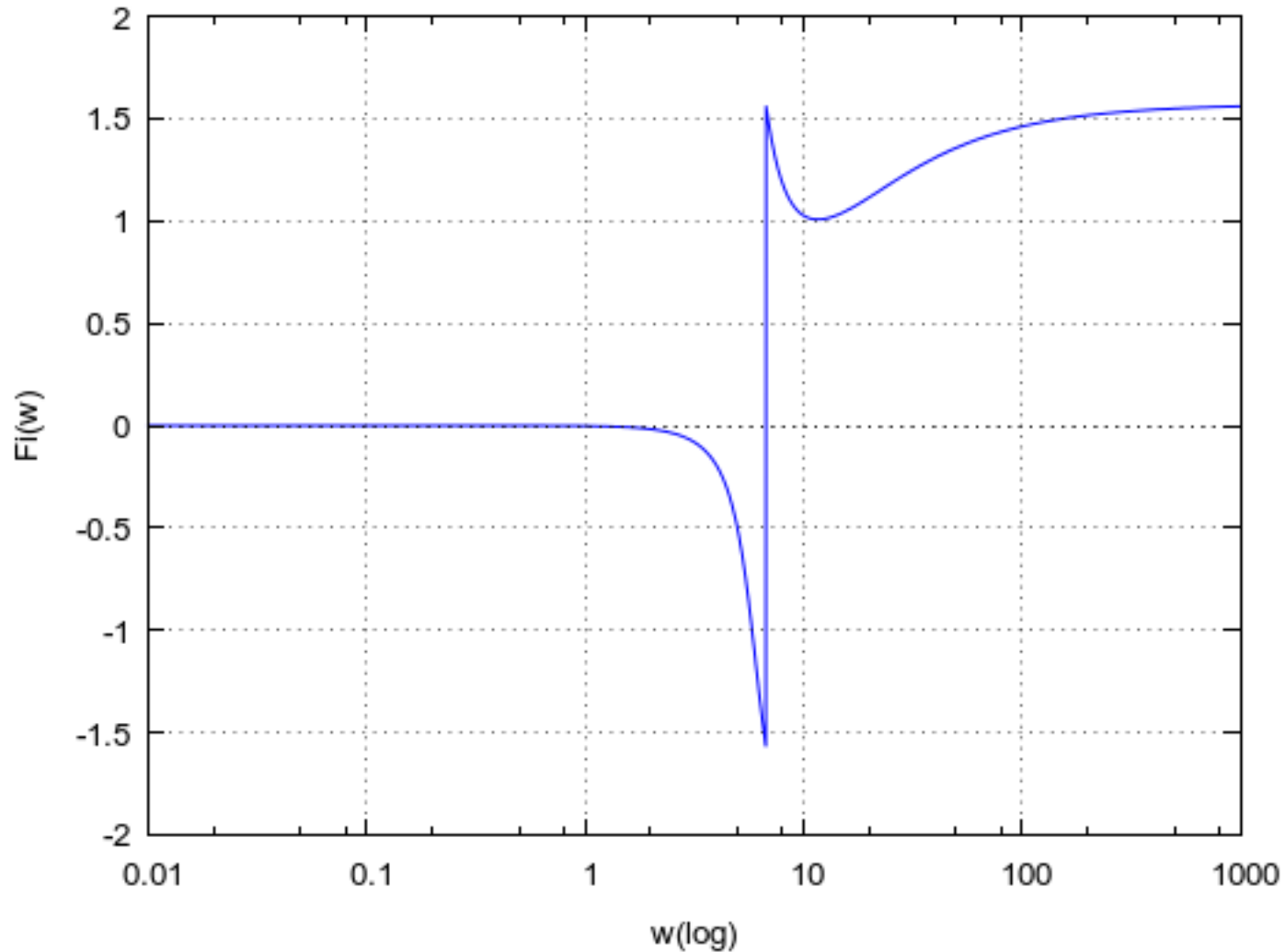


Transfer function – frequency response

Example 2 - vibrating system

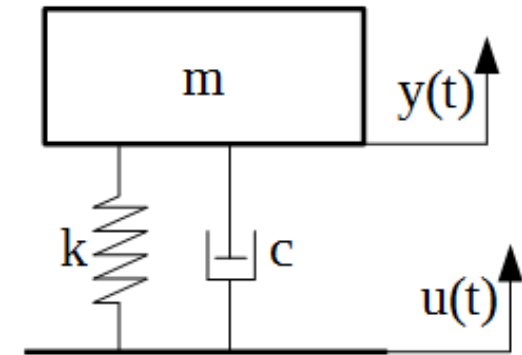


phase plot with „atan”

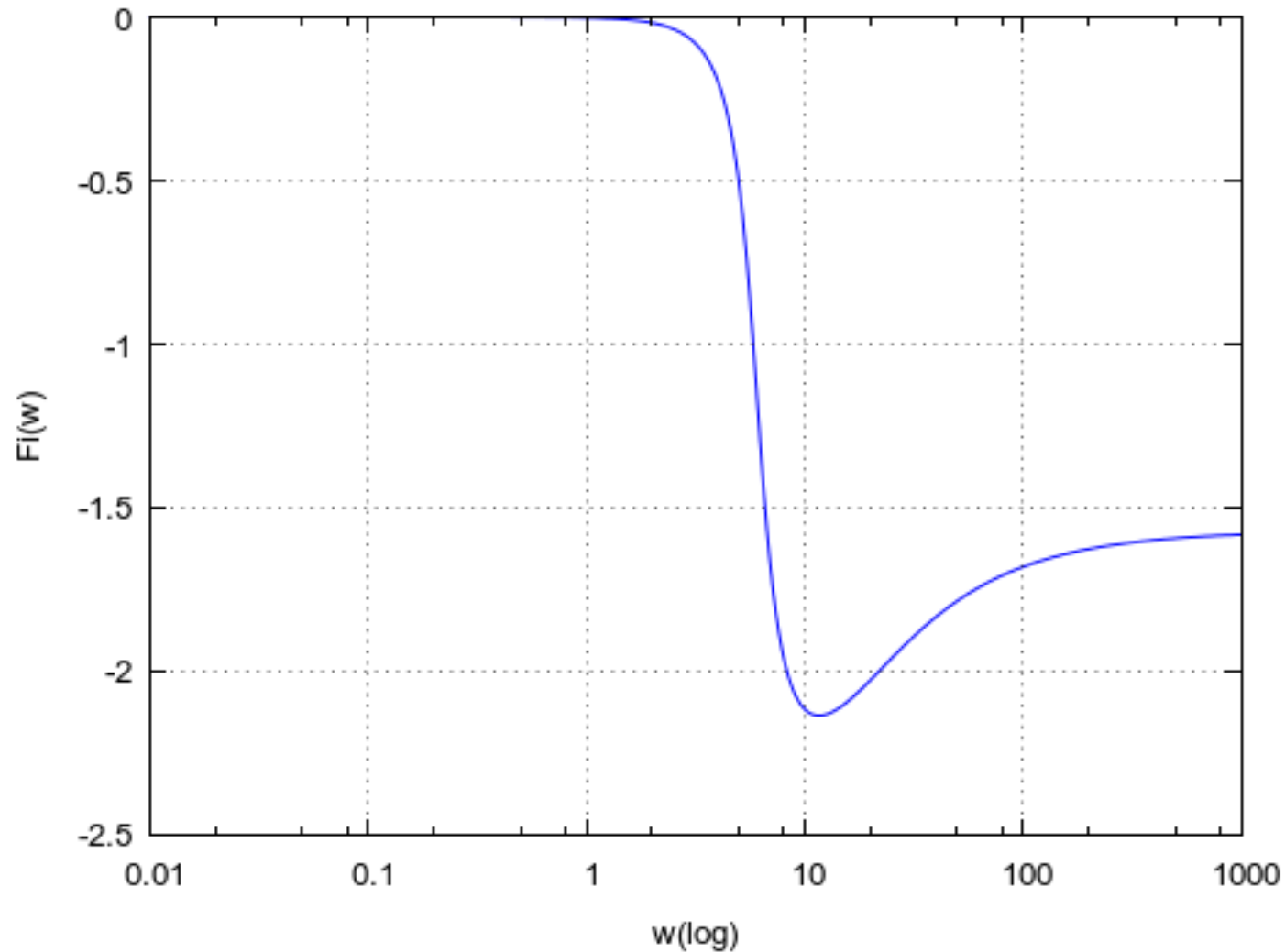


Transfer function – frequency response

Example 2 - vibrating system

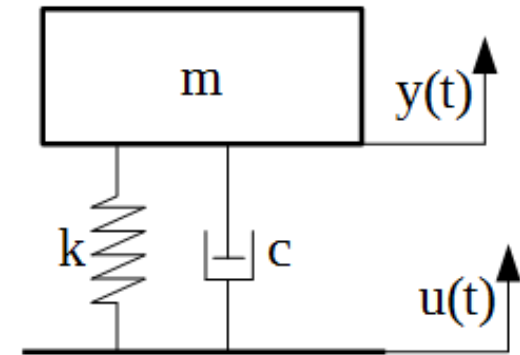


phase plot with „atan2”

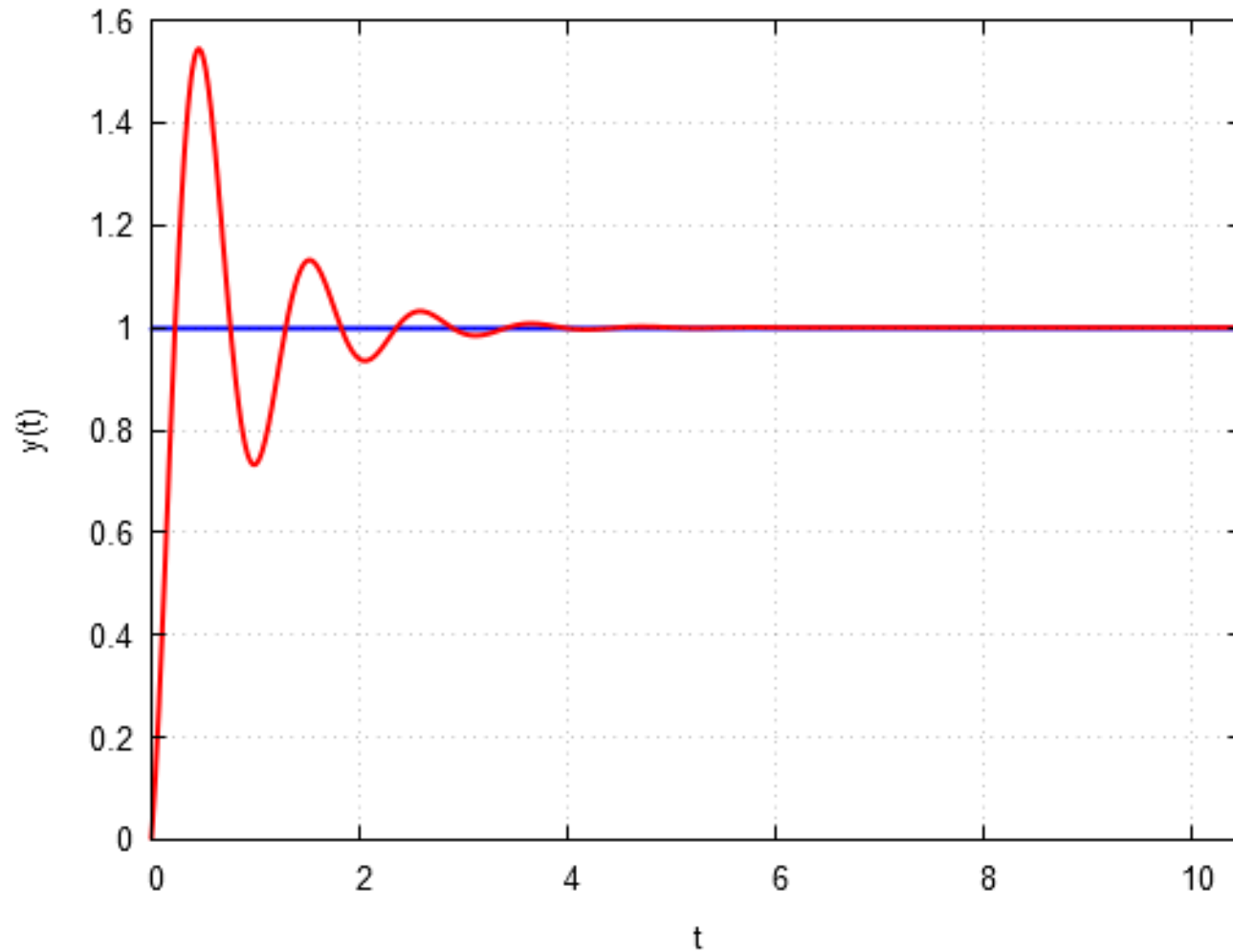


Transfer function – frequency response

Example 2 - vibrating system



Step response



Classification of basic automatic systems

Element name	Equation	Transfer function
proportional	$y(t) = ku(t)$	k
first order (inertial)	$T \frac{dy(t)}{dt} + y(t) = ku(t)$	$\frac{k}{Ts + 1}$
integrator	$y(t) = k \int_0^t u(t) dt$ <p style="text-align: center;">or</p> $\frac{dy(t)}{dt} = ku(t)$	$\frac{k}{s}$

Classification of basic automatic systems

Element name	Equation	Transfer function
derivative	$y(t) = k \frac{du(t)}{dt}$	ks
derivative with inertia	$T \frac{dy(t)}{dt} + y(t) = k \frac{du(t)}{dt}$	$\frac{ks}{Ts + 1}$

Classification of basic automatic systems

Element name	Equation	Transfer function
delay	$y(t) = u(t - \tau)$	$e^{-\tau s}$
second order (oscillator)	$T_1^2 \frac{d^2 y(t)}{dt^2} + T_2 \frac{dy(t)}{dt} + y(t) = ku(t)$	$\frac{k}{T_1^2 s^2 + T_2 s + 1}$

Proportional element

1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

Proportional element

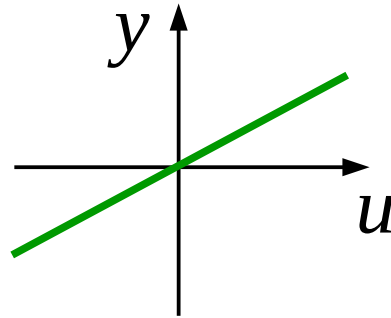
1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$

Proportional element

1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$

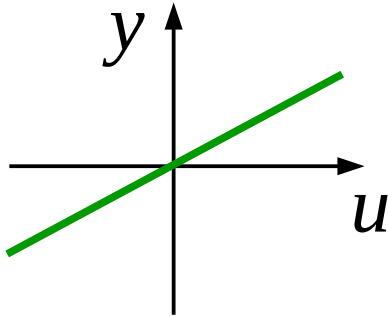


for $k > 0$

Proportional element

1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



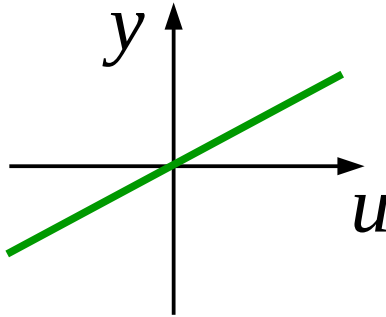
for $k > 0$

3. Transfer function:

Proportional element

1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



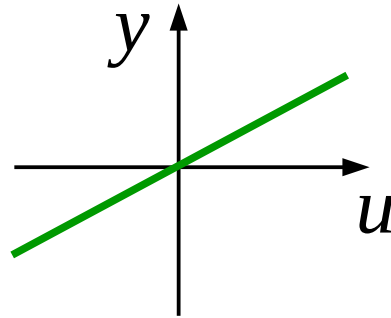
for $k > 0$

3. Transfer function: $H(s) = k$

Proportional element

1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



for $k > 0$

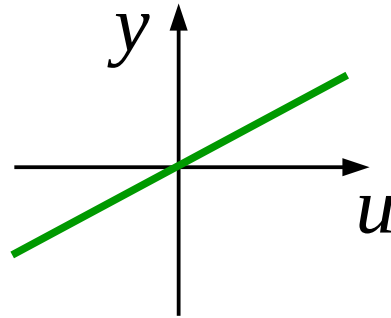
3. Transfer function: $H(s) = k$

4. Step response: for $u(t) = u_0 1(t)$

Proportional element

1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



for $k > 0$

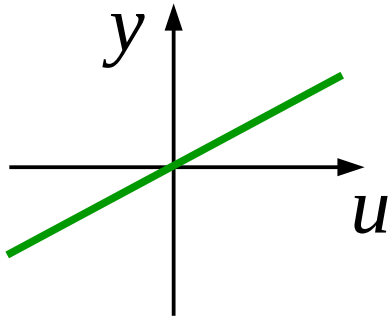
3. Transfer function: $H(s) = k$

4. Step response: $y(t) = k u_0 1(t)$ for $u(t) = u_0 1(t)$

Proportional element

1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

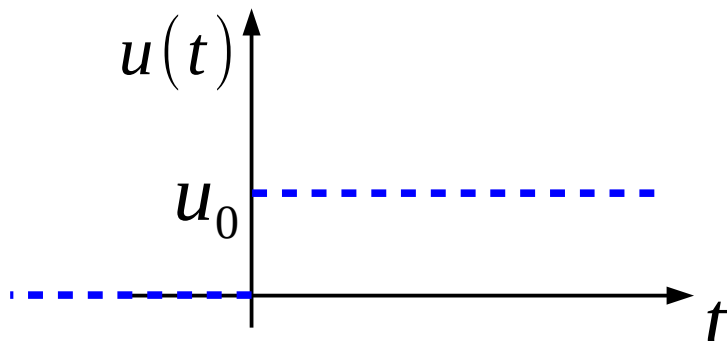
2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



for $k > 0$

3. Transfer function: $H(s) = k$

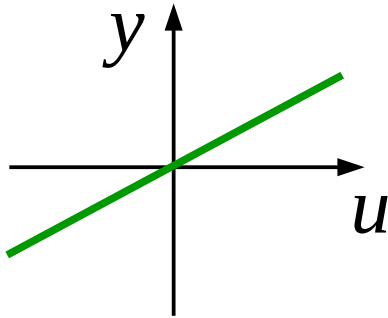
4. Step response: $y(t) = k u_0 1(t)$ for $u(t) = u_0 1(t)$



Proportional element

1. Element equation: $y(t) = ku(t)$ $u(t)$ - input, $y(t)$ - output

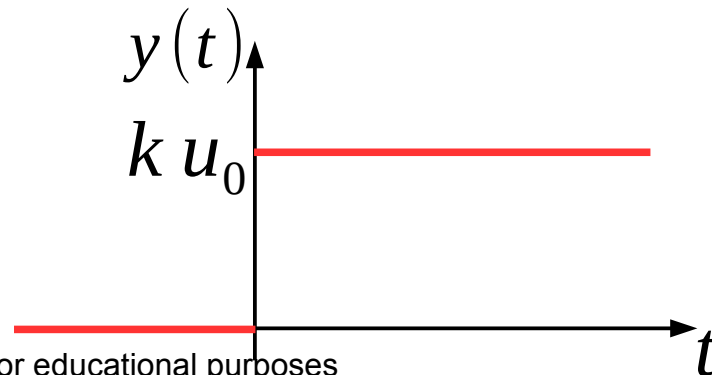
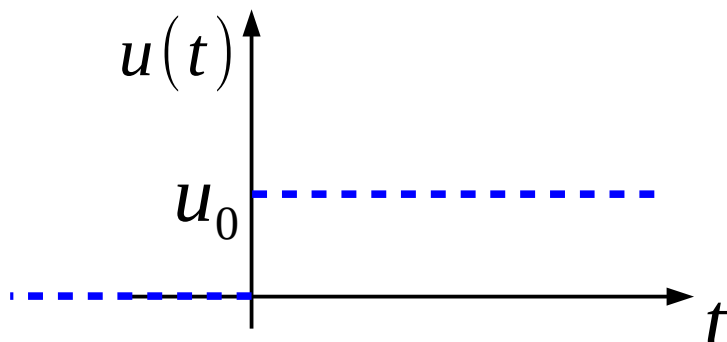
2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



for $k > 0$

3. Transfer function: $H(s) = k$

4. Step response: $y(t) = k u_0 1(t)$ for $u(t) = u_0 1(t)$



Proportional element

5. Frequency response:

Proportional element

5. Frequency response: $H(j\omega) = k$

Proportional element

5. Frequency response: $H(j\omega) = k$ $P(\omega) = k$, $Q(\omega) = 0$

Proportional element

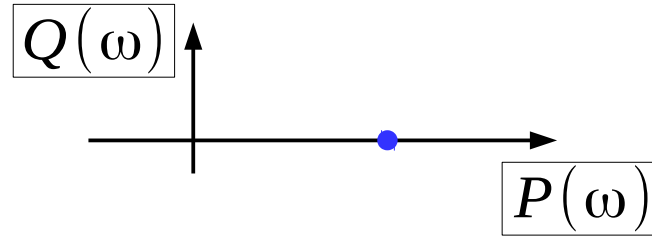
5. Frequency response: $H(j\omega) = k$ $P(\omega) = k$, $Q(\omega) = 0$

6. Nyquist plot:

Proportional element

5. Frequency response: $H(j\omega) = k$ $P(\omega) = k$, $Q(\omega) = 0$

6. Nyquist plot:

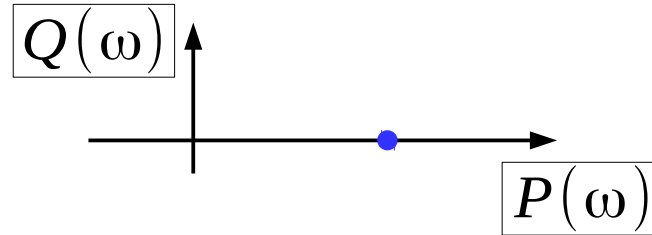


for $k > 0$

Proportional element

5. Frequency response: $H(j\omega) = k$ $P(\omega) = k$, $Q(\omega) = 0$

6. Nyquist plot:

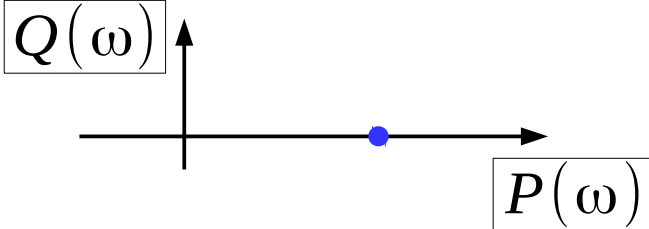


for $k > 0$

7. Bode plot:

Proportional element

5. Frequency response: $H(j\omega) = k$ $P(\omega) = k$, $Q(\omega) = 0$

6. Nyquist plot:  for $k > 0$

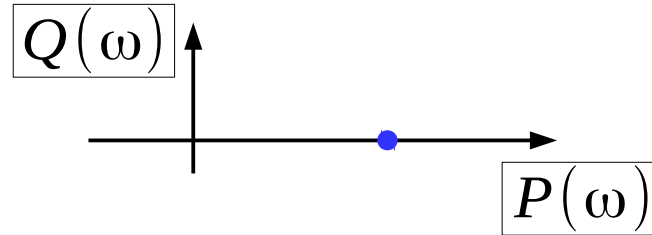
The Nyquist plot shows a horizontal axis labeled $P(\omega)$ and a vertical axis labeled $Q(\omega)$. A blue dot is located on the positive $P(\omega)$ axis, representing the frequency response for $k > 0$.

7. Bode plot: $A(\omega) = \sqrt{P^2 + Q^2} = |k|$
 $L(\omega) = 20 \log A(\omega)$

Proportional element

5. Frequency response: $H(j\omega) = k$ $P(\omega) = k$, $Q(\omega) = 0$

6. Nyquist plot:



for $k > 0$

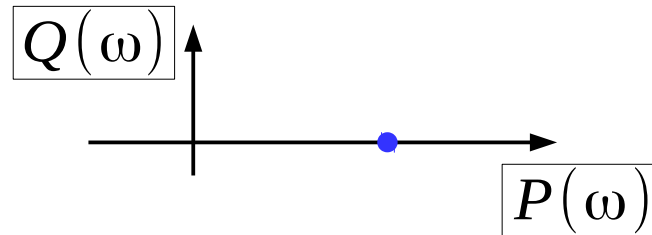
7. Bode plot: $A(\omega) = \sqrt{P^2 + Q^2} = |k|$
 $L(\omega) = 20 \log A(\omega)$

$$\varphi(\omega) = \arctan \frac{Q}{P} = \begin{cases} 0, & \text{dla } k \geq 0 \\ \pi, & \text{dla } k < 0 \end{cases}$$

Proportional element

5. Frequency response: $H(j\omega) = k$ $P(\omega) = k$, $Q(\omega) = 0$

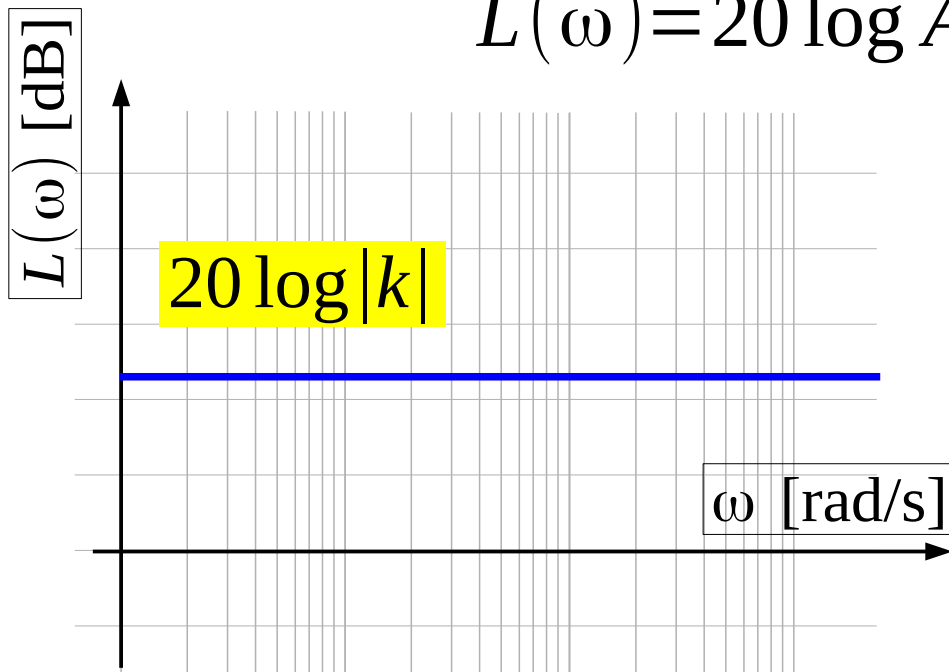
6. Nyquist plot:



for $k > 0$

7. Bode plot: $A(\omega) = \sqrt{P^2 + Q^2} = |k|$
 $L(\omega) = 20 \log A(\omega)$

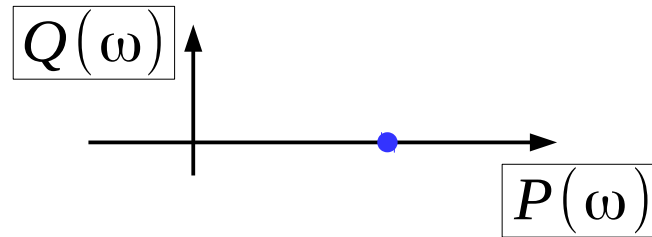
$$\varphi(\omega) = \arctan \frac{Q}{P} = \begin{cases} 0, & \text{dla } k \geq 0 \\ \pi, & \text{dla } k < 0 \end{cases}$$



Proportional element

5. Frequency response: $H(j\omega) = k$ $P(\omega) = k, Q(\omega) = 0$

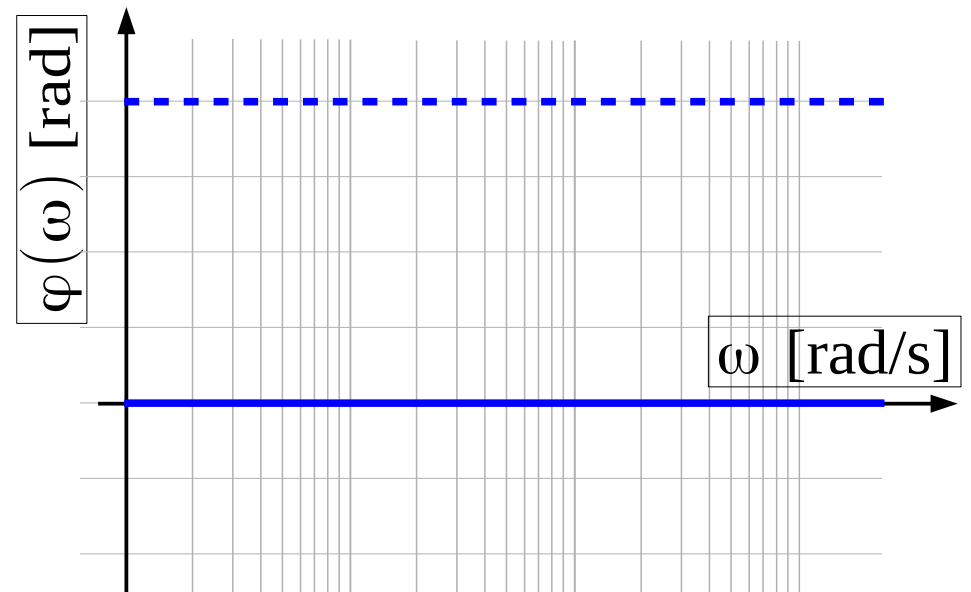
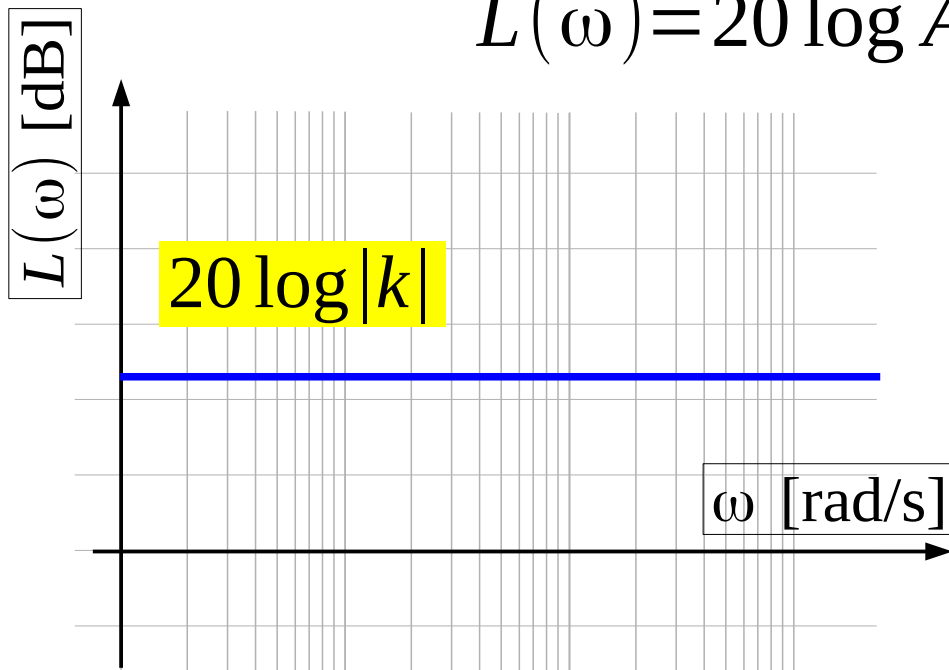
6. Nyquist plot:



for $k > 0$

7. Bode plot: $A(\omega) = \sqrt{P^2 + Q^2} = |k|$
 $L(\omega) = 20 \log A(\omega)$

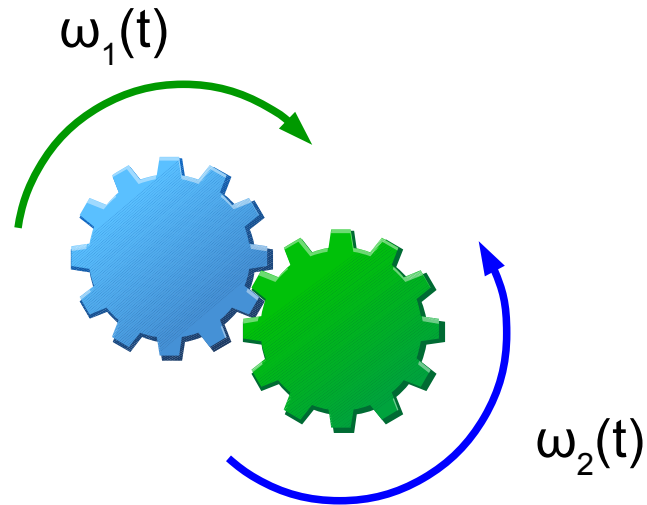
$$\varphi(\omega) = \arctan \frac{Q}{P} = \begin{cases} 0, & \text{dla } k \geq 0 \\ \pi, & \text{dla } k < 0 \end{cases}$$



Proportional element

Examples

1



GEARBOX:

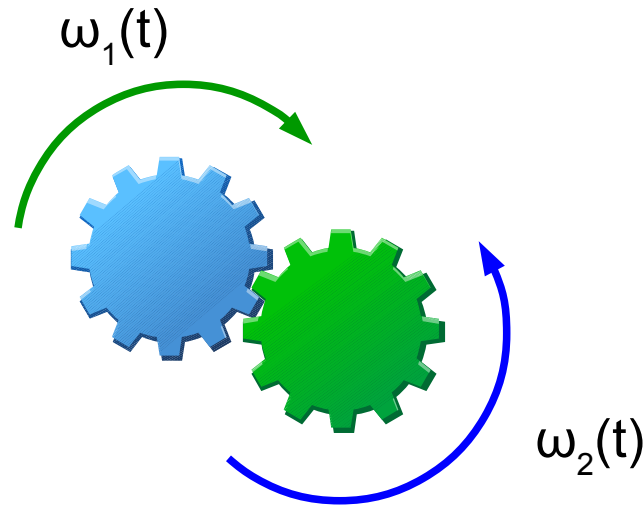
input – angular velocity $\omega_1(t)$

output – angular velocity $\omega_2(t)$

Proportional element

Examples

1

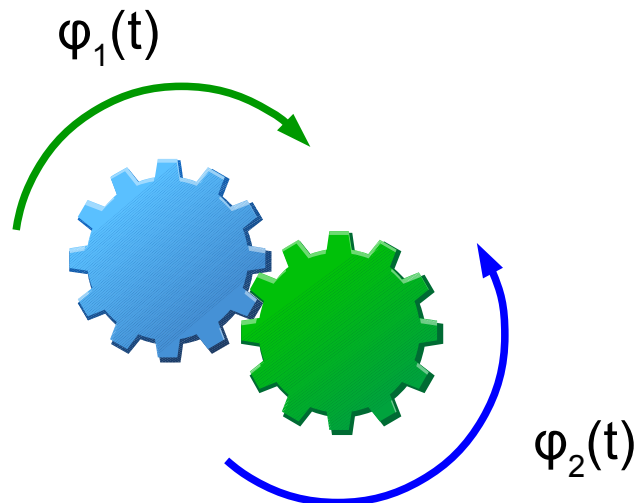


GEARBOX:

input – angular velocity $\omega_1(t)$

output – angular velocity $\omega_2(t)$

2



GEARBOX:

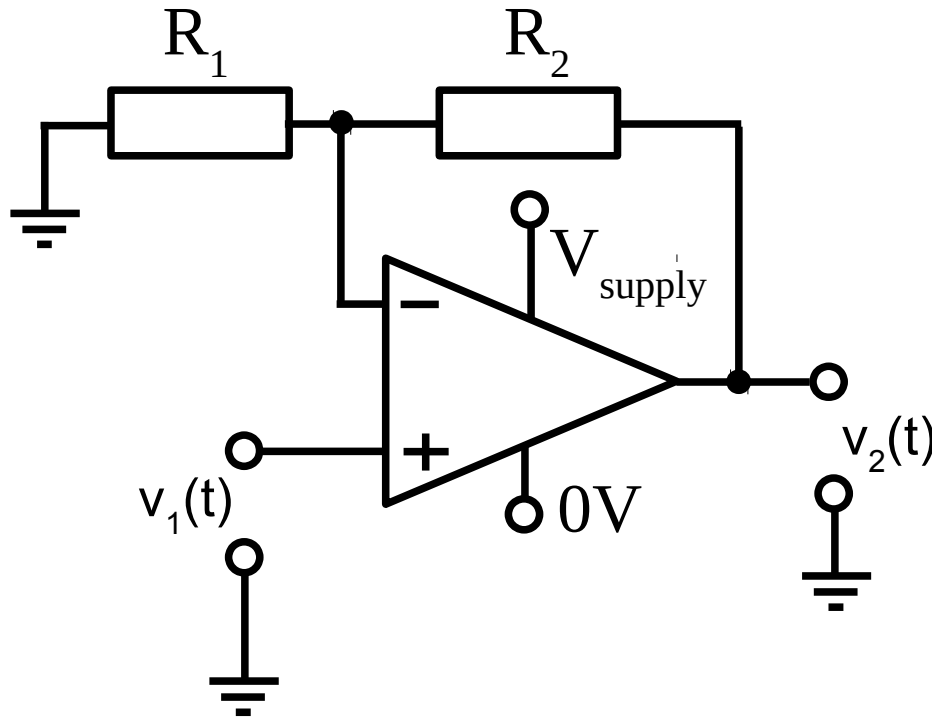
input – rotation angle $\varphi_1(t)$

output – rotation angle $\varphi_2(t)$

Proportional element

Examples

3



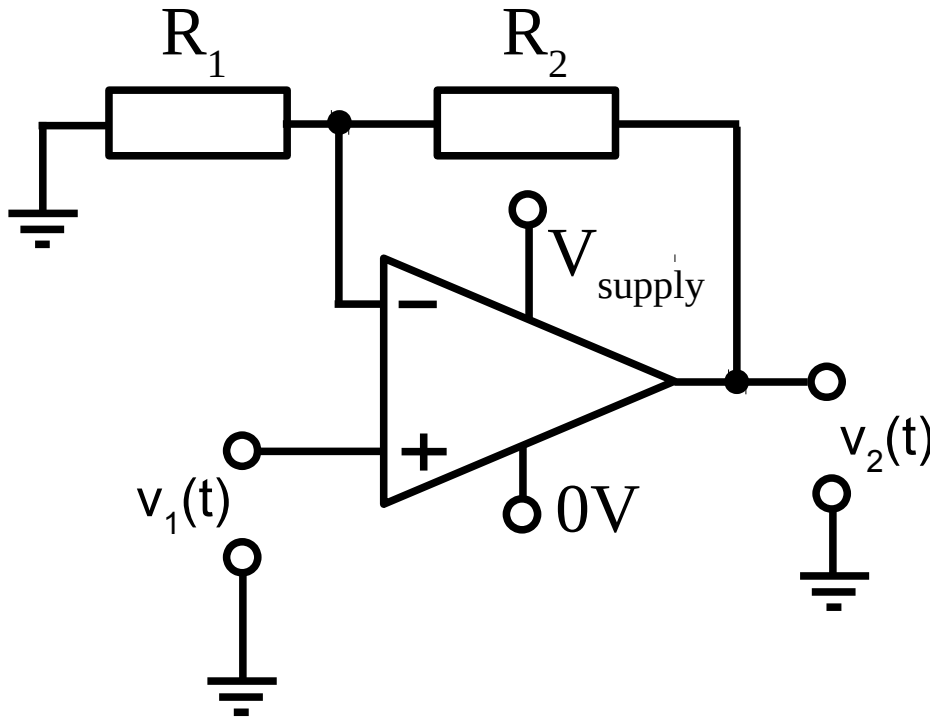
OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = v_1(t) \left(1 + \frac{R_2}{R_1} \right)$$

Proportional element

Examples

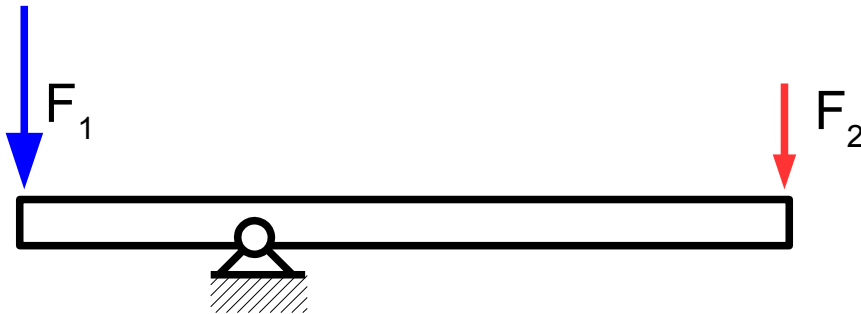
3



OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = v_1(t) \left(1 + \frac{R_2}{R_1} \right)$$

4

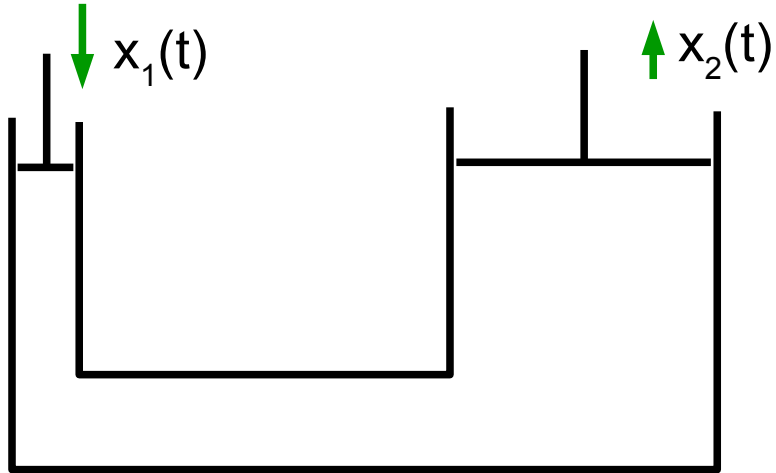


BEAM in steady state:
input – force F_1
output – force F_2

Proportional element

Examples

5



HYDRAULIC LEVER:

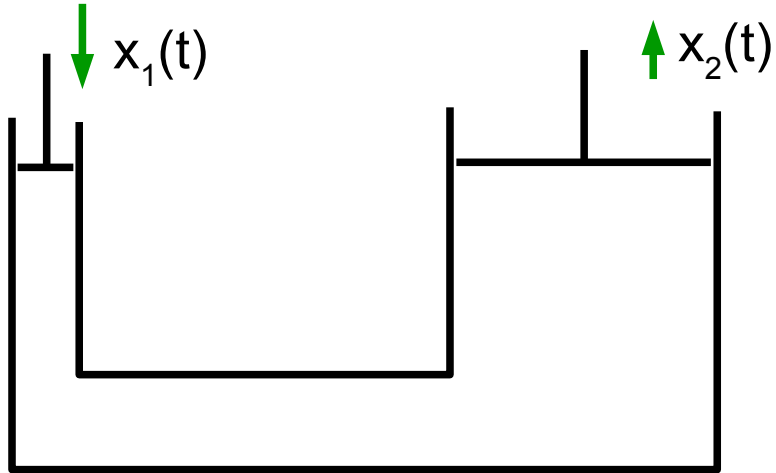
input – displacement $x_1(t)$

output – displacement $x_2(t)$

Proportional element

Examples

5

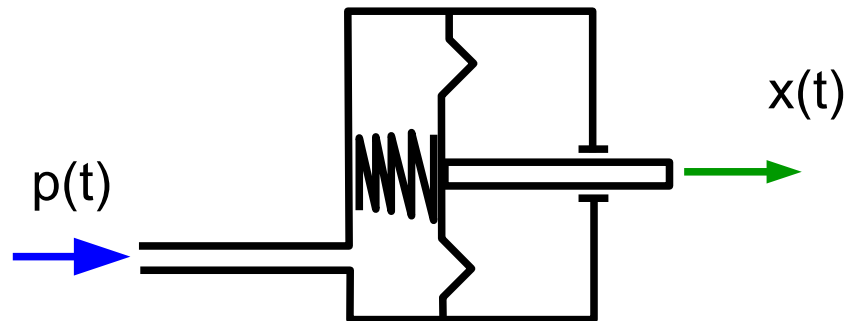


HYDRAULIC LEVER:

input – displacement $x_1(t)$

output – displacement $x_2(t)$

6



PRESSURE ACTUATOR:

input – pressure $p_1(t)$

output – displacement $x(t)$

WolframAlpha



transfer function $(8*s+4)/(2*s^4+7*s^3+11*s^2+19*s+6)$



[Examples](#) [Random](#)

Input interpretation:

systems model

transfer function
$$\frac{4 + 8 s}{6 + 19 s + 11 s^2 + 7 s^3 + 2 s^4}$$

WolframAlpha



partial fraction decomposition $s/(s^3+4s^2+5s+2)$



Examples Random

Assuming "s" is a variable | Use as a unit instead

Input:

partial fractions

$$\frac{s}{s^3 + 4s^2 + 5s + 2}$$

Result:

Step-by-step solution

$$\frac{s}{s^3 + 4s^2 + 5s + 2} = -\frac{2}{s + 2} + \frac{2}{s + 1} - \frac{1}{(s + 1)^2}$$

WolframAlpha



inverse laplace transform $s/(s^3 + 4s^2 + 5s + 2)$



Examples Random

Assuming "s" is a variable | Use as a [unit](#) instead

Input:

$$\mathcal{L}_s^{-1}\left[\frac{s}{s^3 + 4s^2 + 5s + 2}\right](t)$$

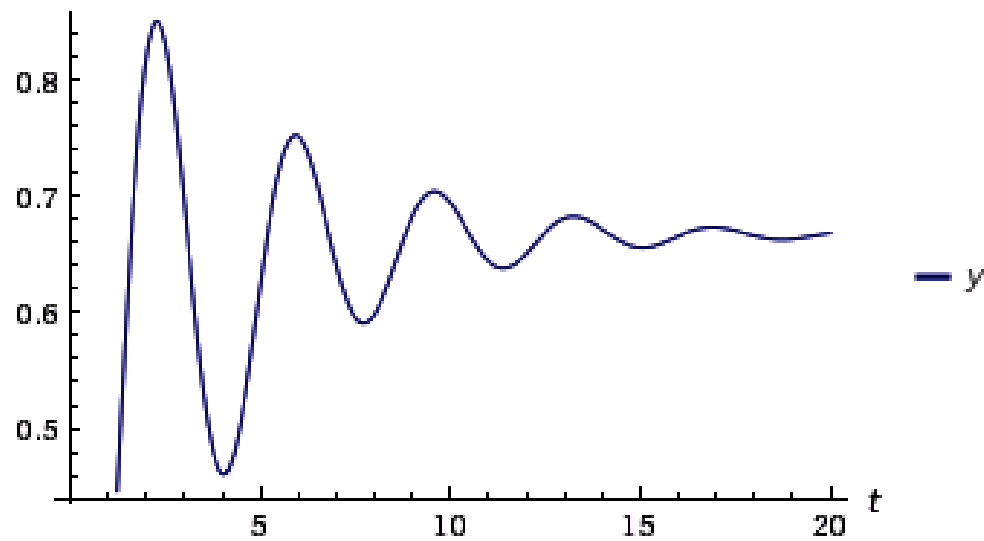
$\mathcal{L}_s^{-1}[f(s)](t)$ is the inverse Laplace transform of $f(s)$ with real variable t

Result:

$$-e^{-2t} (e^t t - 2e^t + 2)$$

WolframAlpha

Unit step response plot:



Less time

More time

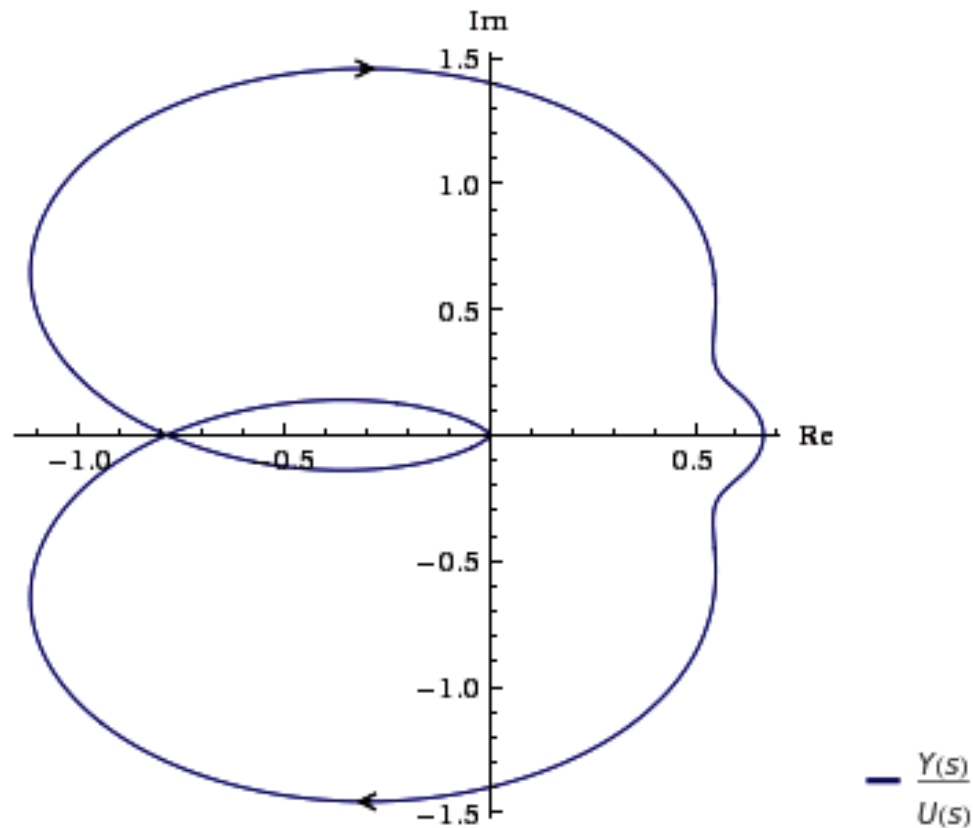
Unit step ▼

WolframAlpha

Nyquist plot:

Show Nyquist grid

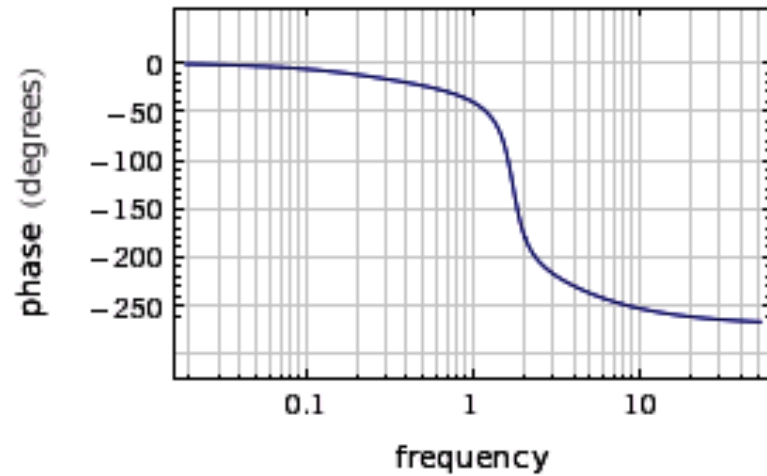
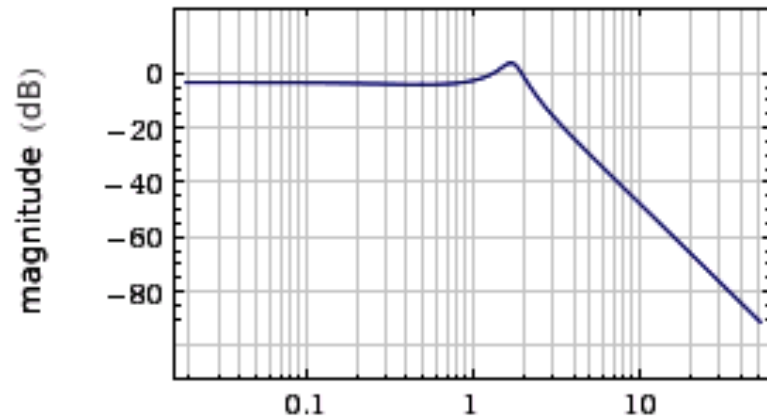
Show stability margins



WolframAlpha

Bode plot:

Show stability margins



— $\frac{Y(s)}{U(s)}$