Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control

Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

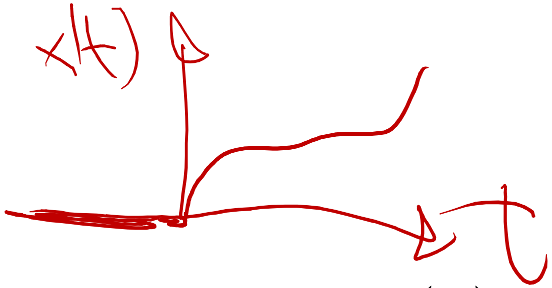
Lecture 8

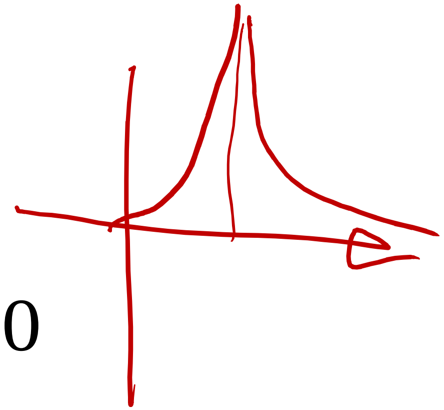
Laplace transform.

Transfer function.

Inputs and outputs in time domain.

Laplace transform





Assumption: $x(t)$ - signal such that for $t<0$ $\quad x(t)=0$

$$\mathcal{L}\{x(t)\} \overset{def}{=} \boxed{\int_0^\infty x(t)\, e^{-st}\, dt}, \text{ where } s \in \mathbb{C}$$

complex!

$$\boxed{x(t)} \longrightarrow \boxed{X(s)}$$

time domain

complex domain

$$s = \underbrace{\sigma}_{Re} + j\underbrace{\omega}_{Im}$$

$$j = \sqrt{-1}$$

Laplace transform

Assumption: $x(t)$ - signal such that for $t<0 \quad x(t)=0$

Laplace transform of x(t): $X(s)=L\{x(t)\}=\int_0^\infty x(t)e^{-st}dt$

Laplace transform

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where: $s\in\mathbb{C}$, $s=\sigma+j\omega$, $j=\sqrt{-1}$

Laplace transform

<u>Assumption</u>: $x(t)$ - signal such that for $t<0 \quad x(t)=0$

<u>Laplace transform of x(t)</u>: $$X(s)=L\{x(t)\}=\int_0^{\infty} x(t)e^{-st}dt$$

where: $s \in \mathbb{C}, \quad s=\sigma+j\omega, \quad j=\sqrt{-1}$

<u>Inverse Laplace transform of x(t)</u>: $$x(t)=L^{-1}\{X(s)\}=\frac{1}{2\pi j}\lim_{\omega\to\infty}\int_{\gamma-j\omega}^{\gamma+j\omega} X(s)e^{st}ds$$

$$\mathcal{L}^{-1}\{X(s)\} \longrightarrow x(t)$$

Laplace transform

Assumption: $x(t)$ - signal such that for $t<0 \quad x(t)=0$

Laplace transform of x(t): $$X(s)=L\{x(t)\}=\int_0^{\infty} x(t)e^{-st}dt$$

where: $s \in \mathbb{C}, \quad s=\sigma+j\omega, \quad j=\sqrt{-1}$

Inverse Laplace transform of x(t): $$x(t)=L^{-1}\{X(s)\}=\frac{1}{2\pi j}\lim_{\omega\to\infty}\int_{\gamma-j\omega}^{\gamma+j\omega} X(s)e^{st}ds$$

A necessary condition for existence of the integral is that x(t) must be locally integrable on t in <0, ∞).

Laplace transform

$$\int e^{at}dt = \frac{e^{at}}{a}$$

Example 1

Calculate Laplace transform of x(t) function from definition.

$$\boxed{x(t)=e^{-2t}}$$

$$X(s) = \mathcal{L}\{x(t)\} = \mathcal{L}\{e^{-2t}\} =$$

$$= \int_0^\infty e^{-2t} e^{-st} dt = \int_0^\infty e^{-(2+s)t} dt = \left[\frac{e^{-(2+s)t}}{-(2+s)}\right]_0^\infty$$

$$= \lim_{t\to\infty}\left(\frac{e^{-(2+s)t}}{-(2+s)}\right) - \frac{e^{-(2+s)\cdot 0}}{-(2+s)} =$$

$$= 0 + \frac{1}{2+s}$$

$$\mathcal{L}\{e^{-2t}\} = \frac{1}{s+2}$$

Laplace transform

Example 1

Calculate Laplace transform of x(t) function from definition.

$$x(t)=e^{-2t}$$

$$X(s)=L\{e^{-2t}\}=\int_0^{\infty} e^{-2t}e^{-st}dt=\int_0^{\infty} e^{-(2+s)t}dt=\left[\frac{e^{-(2+s)t}}{-(2+s)}\right]_0^{\infty}=\frac{1}{s+2}$$

Laplace transform

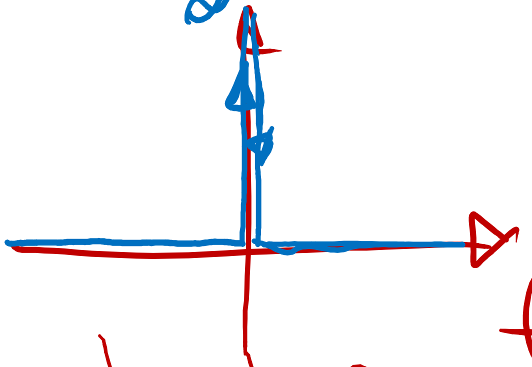
$f(t),\ t\geqslant 0$	$F(s)$
$\delta(t)$ unit impulse	1
$1(t)$ unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-bt}	$\frac{1}{s+b}$
$1-e^{-bt}$	$\frac{b}{s(s+b)}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega}$
$\cosh(\omega t)$	$\frac{s}{s^2-\omega}$
$a\cdot f(t)$	$a\cdot F(s)$
$x(t)+y(t)$	$X(s)+Y(s)$
$x(t)*y(t)$ convolution	$X(s)\cdot Y(s)$
$\frac{dy(t)}{dt}$	$sY(s)-y(0)$
$\frac{d^2y(t)}{dt^2}$	$s^2Y(s)-s\,y(0)-\frac{dy(0)}{dt}$
$\frac{d^ny(t)}{dt^n}$	$s^nY(s)-\frac{d^{n-1}y(0)}{dt^{n-1}}-s\frac{d^{n-2}y(0)}{dt^{n-2}}-\ldots-s^{n-1}y(0)$

table on the website

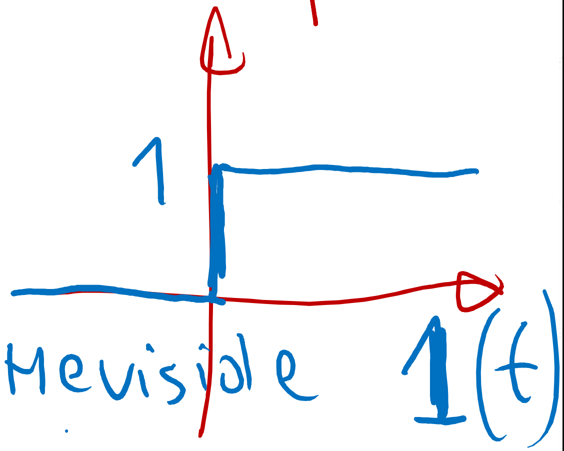
Laplace transform pairs

$f(t),\ t \geqslant 0$	$F(s)$
$\delta(t)$ unit impulse	1
$1(t)$ unit step	$\frac{1}{s}$
t ramp	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-bt}	$\frac{1}{s+b}$
$1-e^{-bt}$	$\frac{b}{s(s+b)}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega}$
$\cosh(\omega t)$	$\frac{s}{s^2-\omega}$

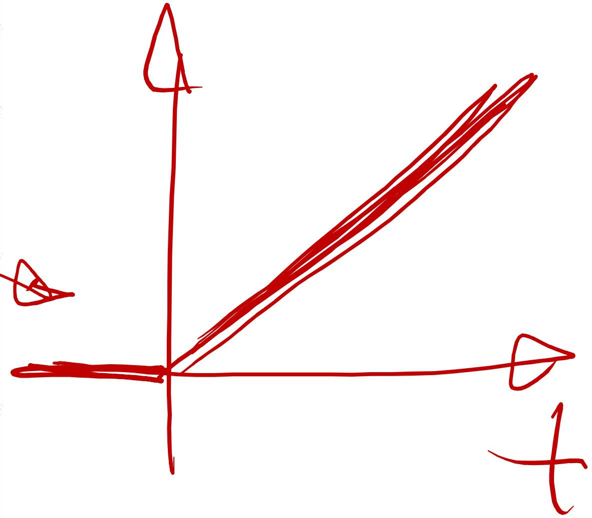
delta dirac



unit step



Heviside $1(t)$



Properties of Laplace transform

$f(t),\quad t \geqslant 0$	$F(s)$
$a \cdot f(t)$	$a \cdot F(s)$
$x(t)+y(t)$	$X(s)+Y(s)$
$x(t)*y(t)$ convolution	$X(s) \cdot Y(s)$

$$\mathcal{L}\{x(t)+y(t)\} = \mathcal{L}\{x(t)\} + \mathcal{L}\{y(t)\}$$

$$\mathcal{L}\{x(t)*y(t)\} = \mathcal{L}\{x(t)\} \cdot \mathcal{L}\{y(t)\}$$

Properties of Laplace transform cont.

$$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = s\cdot\mathcal{L}\{y(t)\} - y(0)$$

$$\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\} = s\cdot\left(\quad\right) - \frac{dy(0)}{dt}$$

$\dfrac{dy(t)}{dt}$	$sY(s)-y(0)$
$\dfrac{d^2y(t)}{dt^2}$	$s^2Y(s)-sy(0)-\dfrac{dy(0)}{dt}$
$\dfrac{d^ny(t)}{dt^n}$	$s^nY(s)-\dfrac{d^{n-1}y(0)}{dt^{n-1}}-s\dfrac{d^{n-2}y(0)}{dt^{n-2}}-\ldots-s^{n-1}y(0)$

Properties of Laplace transform cont.

$\int\limits_{t=0}^{\infty} f(t)\,dt$	$\dfrac{F(s)}{s}$
$\underbrace{\int\int\ldots\int}_{n} f(t)\,dt$	$\dfrac{F(s)}{s^{n}}$
$f(t-\tau)$	$e^{-\tau s}F(s)$

time shift

Properties of Laplace transform

$f(t),\ t \geqslant 0$	$F(s)$
$a \cdot f(t)$	$a \cdot F(s)$
$x(t) + y(t)$	$X(s) + Y(s)$
$x(t) * y(t)$ convolution	$X(s) \cdot Y(s)$
$\dfrac{dy(t)}{dt}$	$s\,Y(s) - y(0)$
$\dfrac{d^2 y(t)}{dt^2}$	$s^2 Y(s) - s\,y(0) - \dfrac{dy(0)}{dt}$
$\dfrac{d^n y(t)}{dt^n}$	$s^n Y(s) - \dfrac{d^{n-1} y(0)}{dt^{n-1}} - s\dfrac{d^{n-2} y(0)}{dt^{n-2}} - \ldots - s^{n-1} y(0)$
$\int\limits_{t=0}^{\infty} f(t)\,dt$	$\dfrac{F(s)}{s}$
$\underbrace{\int \int \ldots \int}_{n} f(t)\,dt$	$\dfrac{F(s)}{s^n}$
$f(t-\tau)$	$e^{-\tau s} F(s)$

Laplace transform

Example 2

Solve equation for a given initial conditions using Laplace transform.

$$\frac{d^2 y(t)}{dt^2} - 3\frac{dy(t)}{dt} + 2y(t) = 1(t), \quad \frac{dy(0)}{dt} = 2, \quad y(0) = 3, \quad t \geqslant 0$$

$$\mathcal{L}\{1(t)\} = \frac{1}{s}; \quad \mathcal{L}\{2y(t)\} = 2\cdot\mathcal{L}\{y(t)\} = 2\cdot Y(s)$$

$$\mathcal{L}\left\{-3\frac{dy(t)}{dt}\right\} = -3\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = -3\left(s\cdot Y(s) - y(0)\right) =$$

$$= -3sY(s) + 9$$

$$\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\} = s^2Y(s) - sy(0) - \frac{dy(0)}{dt} = s^2Y(s) - 3s - 2$$

Laplace transform

Example 2

Solve equation for a given initial conditions using Laplace transform.

$$\frac{d^2 y(t)}{dt^2} - 3\frac{dy(t)}{dt} + 2y(t) = 1(t)\,,\quad \frac{dy(0)}{dt} = 2,\ y(0) = 3,\ t \geqslant 0$$

$$s^2 Y(s) - 3s - 2 - 3sY(s) + 9 + 2Y(s) = \frac{1}{s} \qquad =$$

$$Y(s)\left(s^2 - 3s + 2\right) = \frac{1}{s} + 3s - 7 \quad /\cdot s$$

$$Y(s)\left(s^3 - 3s^2 + 2s\right) = 1 + 3s^2 - 7s$$

$$\boxed{Y(s) = \frac{3s^2 - 7s + 1}{s^3 - 3s^2 + 2s}} \rightarrow y(t) = \mathcal{L}^{-1}\left\{Y(s)\right\}$$

$$A+B+C=3$$
$$-3A-2B-C=-7$$
$$2A=1$$

Laplace transform

$$A=\frac{1}{2}$$
$$B=3$$
$$C=-\frac{1}{2}$$

Example 2

Solve equation for a given initial conditions using Laplace transform.

$$\frac{d^2y(t)}{dt^2}-3\frac{dy(t)}{dt}+2y(t)=1(t),\ \frac{dy(0)}{dt}=2,\ y(0)=3,\ t\geqslant 0$$

$$Y(s)=\frac{\boxed{3}s^2\boxed{-7}s+1}{s(s-1)(s-2)}=\frac{A}{s}+\frac{B}{s-1}+\frac{C}{s-2}=$$

$$=\frac{A(s-1)(s-2)+Bs(s-2)+Cs(s-1)}{s(s-1)(s-2)}=$$

$$=\frac{s^2\boxed{(A+B+C)}+s\boxed{(-3A-2B-C)}+2A}{s(s-1)(s-2)}$$

Laplace transform

<u>Example 2</u>

Solve equation for a given initial conditions using Laplace transform.

$$\frac{d^2 y(t)}{dt^2} - 3\frac{dy(t)}{dt} + 2y(t) = 1(t), \quad \frac{dy(0)}{dt} = 2, \; y(0) = 3, \; t \geqslant 0$$

after Laplace transformation

$$Y(s) = \frac{1 - 7s + 3s^2}{s(s-1)(s-2)}$$

after partial fraction decomposition/expansion

$$Y(s) = \frac{1}{2}\frac{1}{s} + 3\frac{1}{s-1} - \frac{1}{2}\frac{1}{s-2}$$

after inverse Laplace transformation

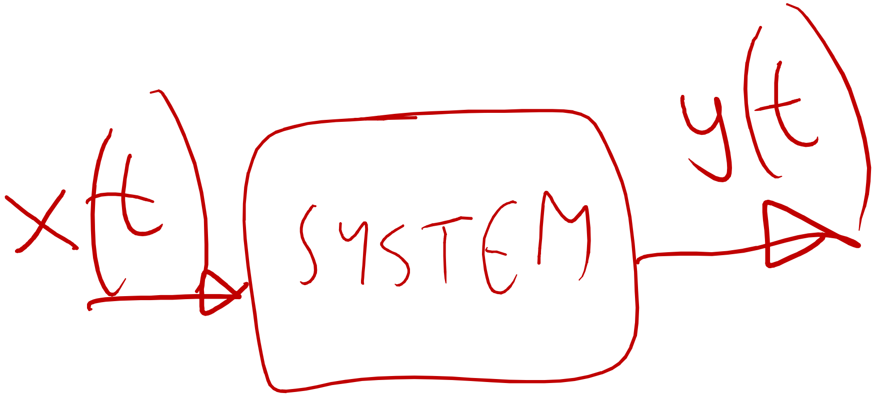
$$y(t) = \frac{1}{2}1(t) + 3e^t - \frac{1}{2}e^{2t}$$

Transfer function – definition

For LTI SISO system with continuous input x(t) and output y(t) for zero initial conditions, transfer function is a ratio of the output of a system to the input of a system described in complex domain by the Laplace transformation.

$$\boxed{H(s)=\frac{L\{y(t)\}}{L\{x(t)\}}=\frac{Y(s)}{X(s)}}$$

$$\frac{\text{OUTPUT}}{\text{INPUT}}$$



Transfer function form

Standard form: $$H(s)=\frac{b^m s^m+b_{m-1}s^{m-1}+\ldots+b_1 s+b_0}{a_n s^n+a_{n-1}s^{n-1}+\ldots+a_1 s+a_0}$$

Factored form: $$H(s)=\frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

$H(s=z_i) \to 0$

$z_1, z_2, \ldots, z_m$ - zeroes

$p_1, p_2, \ldots, p_n$ - poles ← STABILITY

$H(s=p_i) \to \infty$

Drawing of a transfer function

$$\boxed{H(s)}$$

$$\boxed{s = \sigma + j\omega}$$

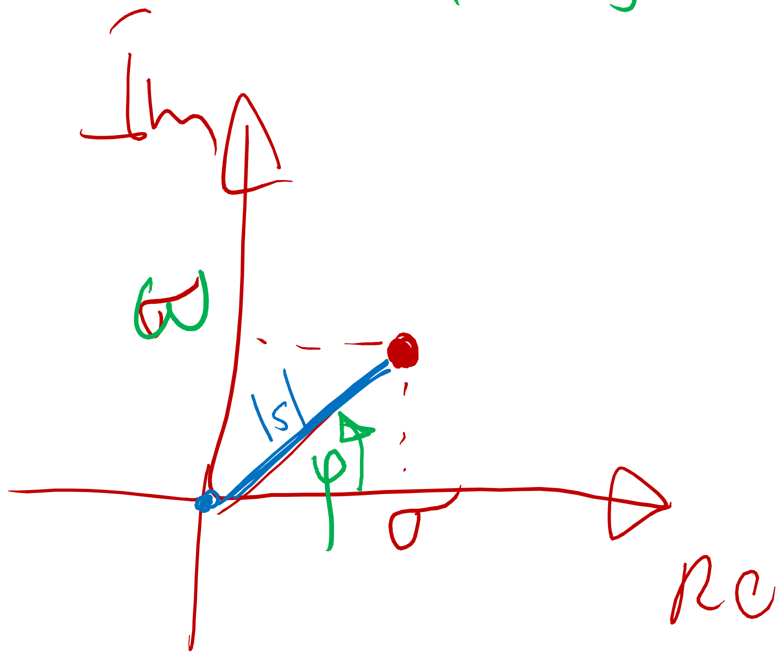
$$|s| = \sqrt{\sigma^2 + \omega^2}$$

$$\varphi = \mathrm{Ang}\, s = \mathrm{atan}\frac{\omega}{\sigma}$$

for every $\underline{s \in \mathbb{C}}$

there is $\underline{H(s) \in \mathbb{C}}$

4)



Drawing of a transfer function

$$H(s) = |H(s)|\, e^{j \arg H(s)}$$

$$s = \sigma + j\omega$$

for every $s \in \mathbb{C}$
there is $H(s) \in \mathbb{C}$

4D

for every $s \in \mathbb{C}$
there is $|H(s)| \in \mathbb{R}$

3D

for every $s \in \mathbb{C}$
there is $\operatorname{Arg} H(s) \in \mathbb{R}$

3D

Drawing of a transfer function

Example

$$\boxed{H(s)=\frac{2-s}{s^3+s^2-2}}=\boxed{\frac{s-2}{(s-1)(s+j+1)(s-j+1)}}$$

$z_1 = 2$

poles: $p_1 = 1$; $p_2 = -j-1$; $p_3 = j-1$

Drawing of a transfer function

Example

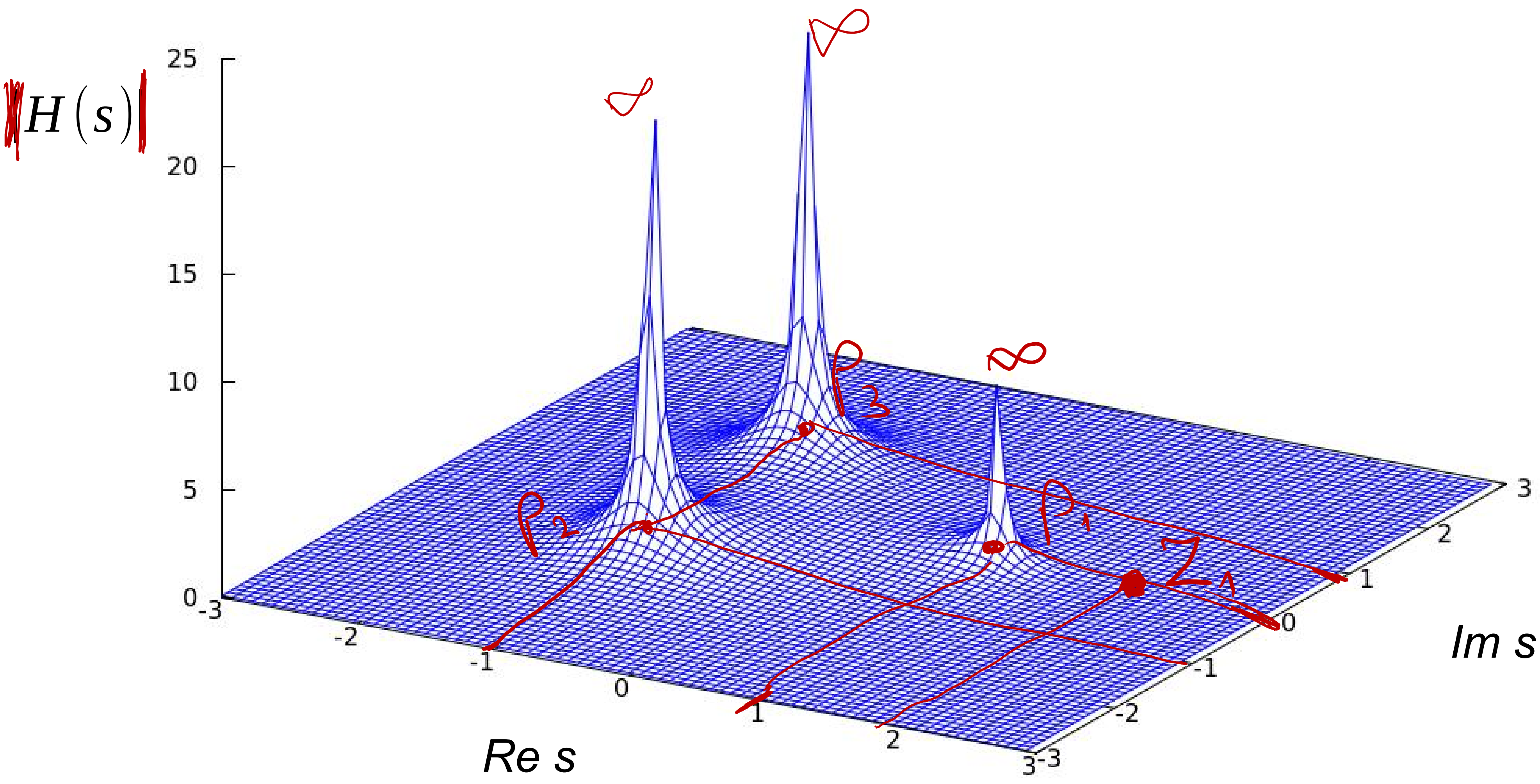
$$H(s) = \frac{2-s}{s^3+s^2-2} = \frac{s-2}{(s-1)(s+j+1)(s-j+1)}$$

Poles: $p_1 = 1,\ p_2 = -1-j,\ p_3 = -1+j$ Zeroes: $z_1 = 2$

Drawing of a transfer function

Example

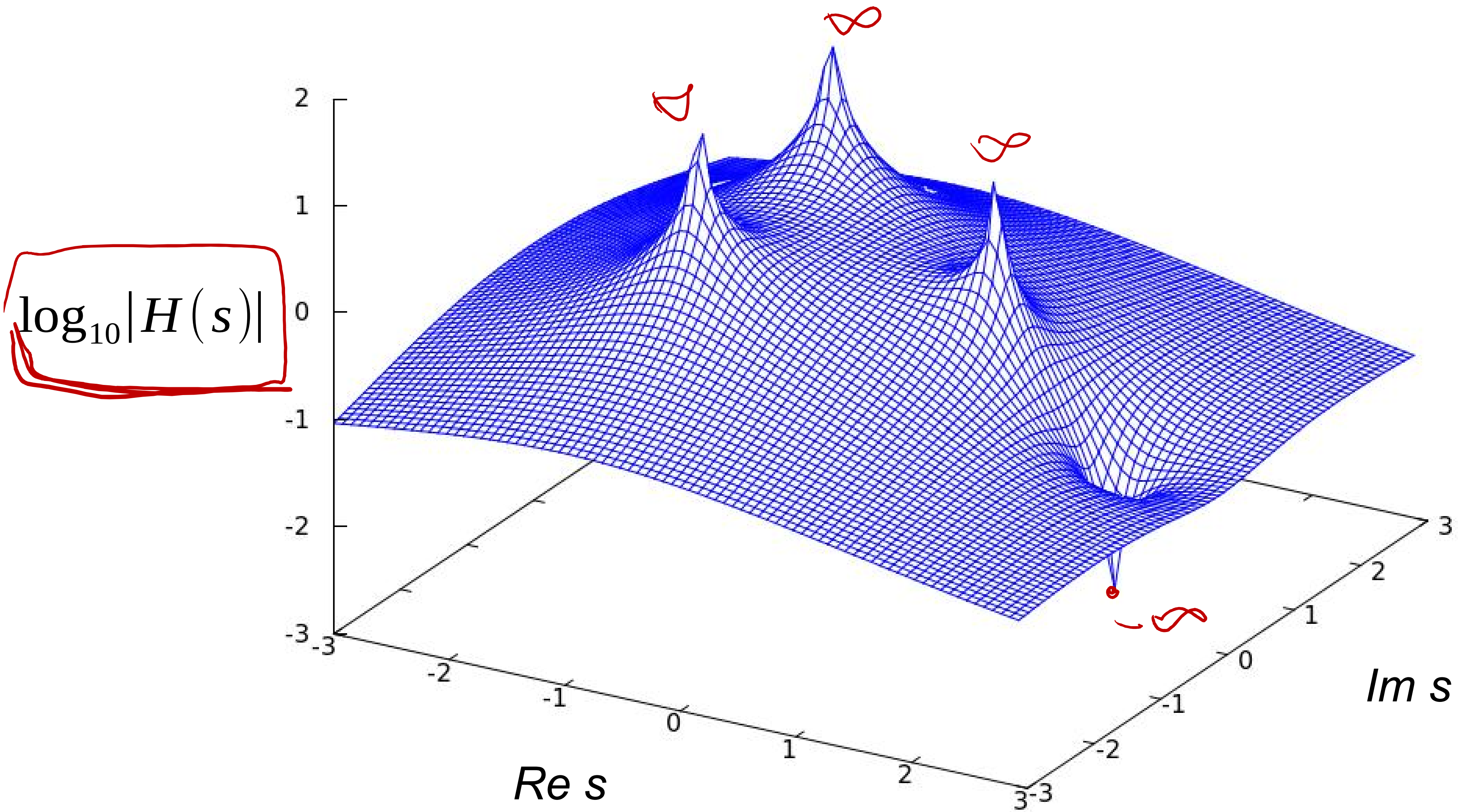
Poles: $p_1 = 1, \; p_2 = -1 - j, \; p_3 = -1 + j$ Zeroes: $z_1 = 2$



Drawing of a transfer function

Example

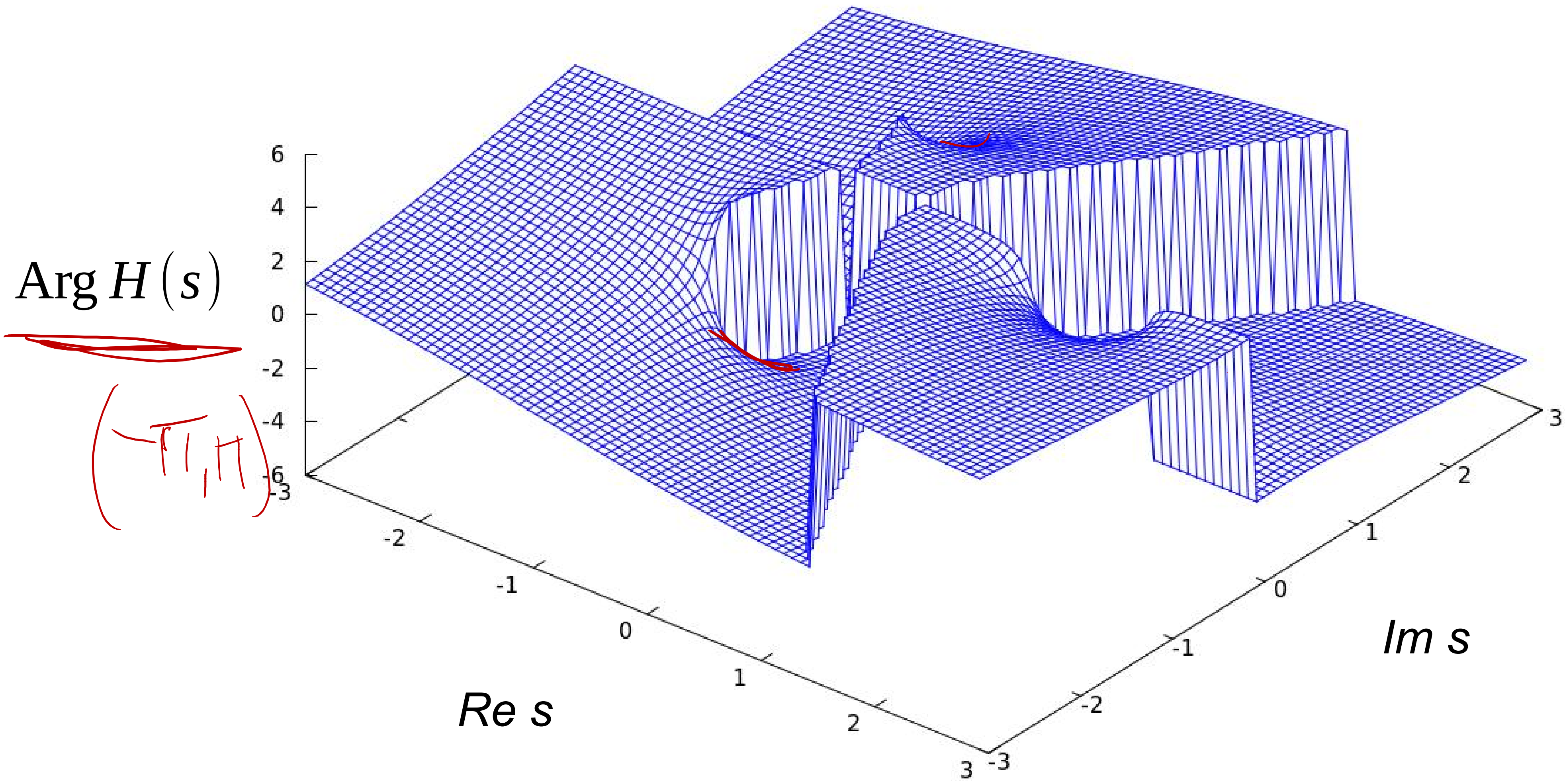
Poles: $p_1=1,\ p_2=-1-j,\ p_3=-1+j$ Zeroes: $z_1=2$



Drawing of a transfer function

Example

Poles: $p_1=1,\ \ p_2=-1-j,\ \ p_3=-1+j$ Zeroes: $z_1=2$



Input and output

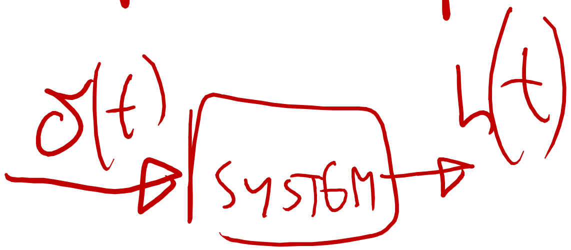
Transfer function: $$\boxed{H(s) = \frac{Y(s)}{X(s)}}$$

$$Y(s) = H(s) \cdot X(s)$$

$$y(t) = \mathcal{L}^{-1}\{H(s) \cdot X(s)\} = \underbrace{\mathcal{L}^{-1}\{H(s)\}} * \mathcal{L}^{-1}\{X(s)\} =$$

$$= h(t) * x(t)$$

impulse response



Input and output

Transfer function: $$H(s)=\frac{Y(s)}{X(s)}$$

Laplace transform of output: $$Y(s)=H(s)X(s)$$

Input and output

Transfer function: $H(s)=\frac{Y(s)}{X(s)}$

Laplace transform of output: $Y(s)=H(s)X(s)$

Output in time domain: $y(t)=L^{-1}\{Y(s)\}$

Input and output

Transfer function: $H(s)=\frac{Y(s)}{X(s)}$

Laplace transform of output: $Y(s)=H(s)X(s)$

Output in time domain: $y(t)=L^{-1}\{Y(s)\}$

$$y(t)=L^{-1}\{H(s)X(s)\}=L^{-1}\{H(s)\}*L^{-1}\{X(s)\}=h(t)*x(t)$$

Input and output

Transfer function: $H(s)=\frac{Y(s)}{X(s)}$

Laplace transform of output: $Y(s)=H(s)X(s)$

Output in time domain: $y(t)=L^{-1}\{Y(s)\}$

$$y(t)=L^{-1}\{H(s)X(s)\}=L^{-1}\{H(s)\}* L^{-1}\{X(s)\}=h(t)* x(t)$$

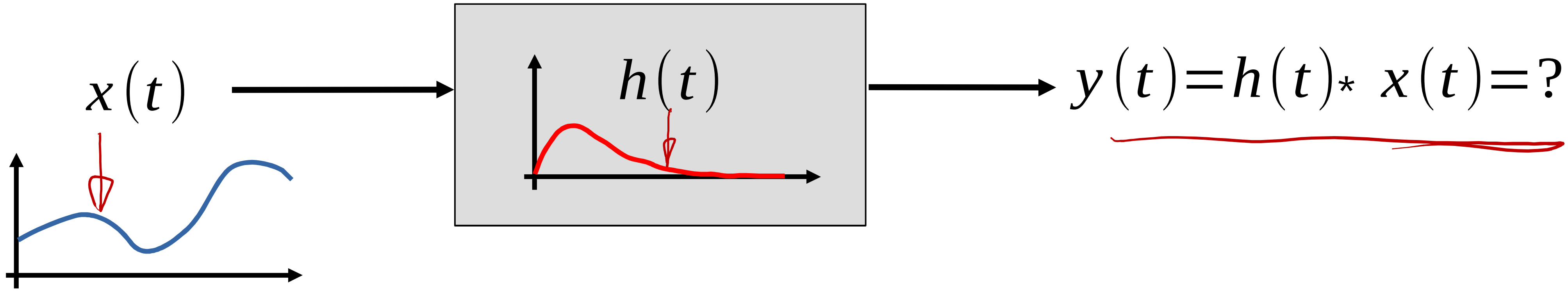
$h(t)$ - system impulse response ($y(t)$ when $x(t)=\delta(t)$)

Input and output

Convolution $h(t) * x(t) = \int_{0}^{\infty} x(\tau) h(t-\tau) d\tau$

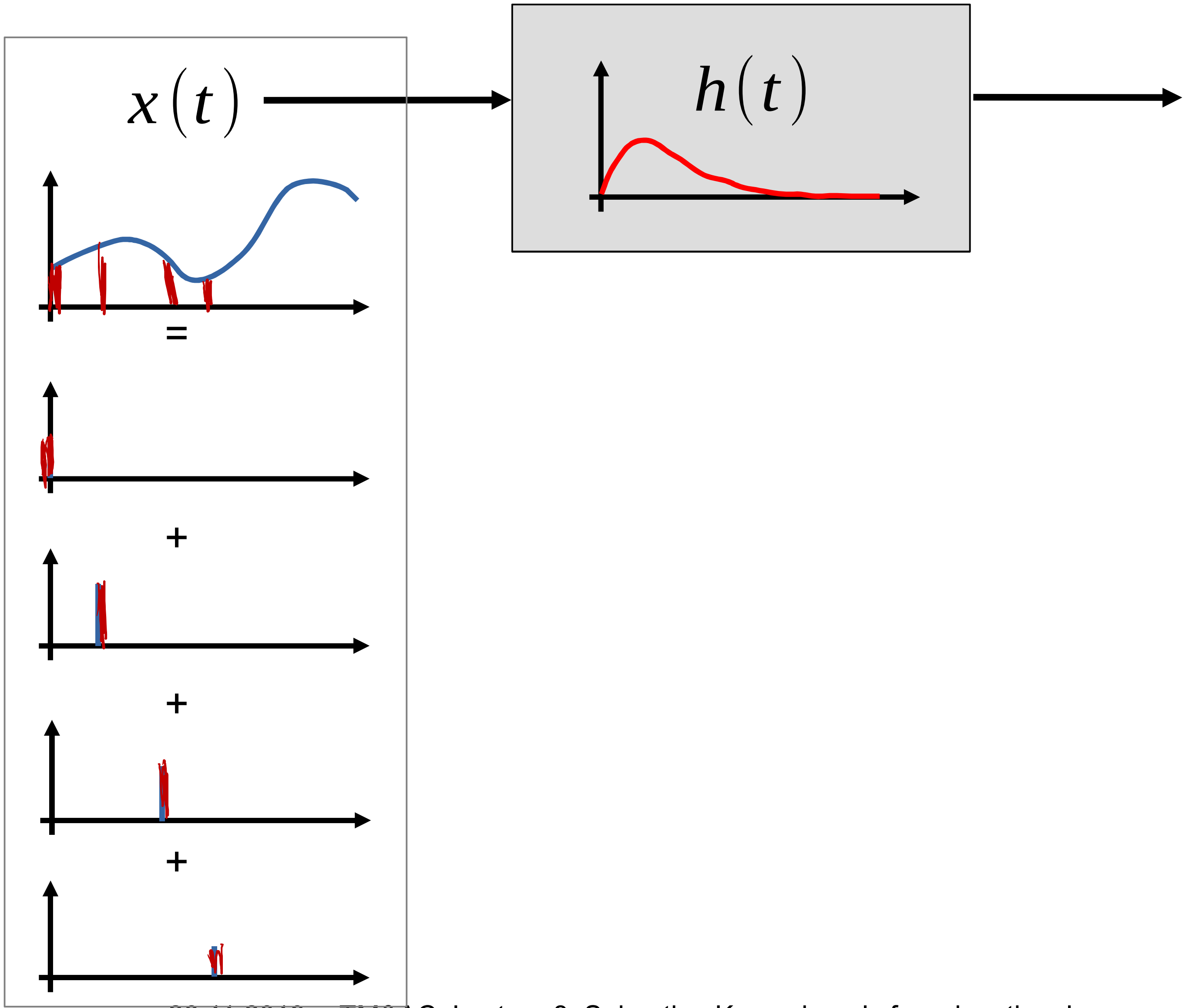
Convolution

$$h(t) * x(t) = \int_0^{\infty} x(\tau) h(t-\tau) \, d\tau$$



Convolution

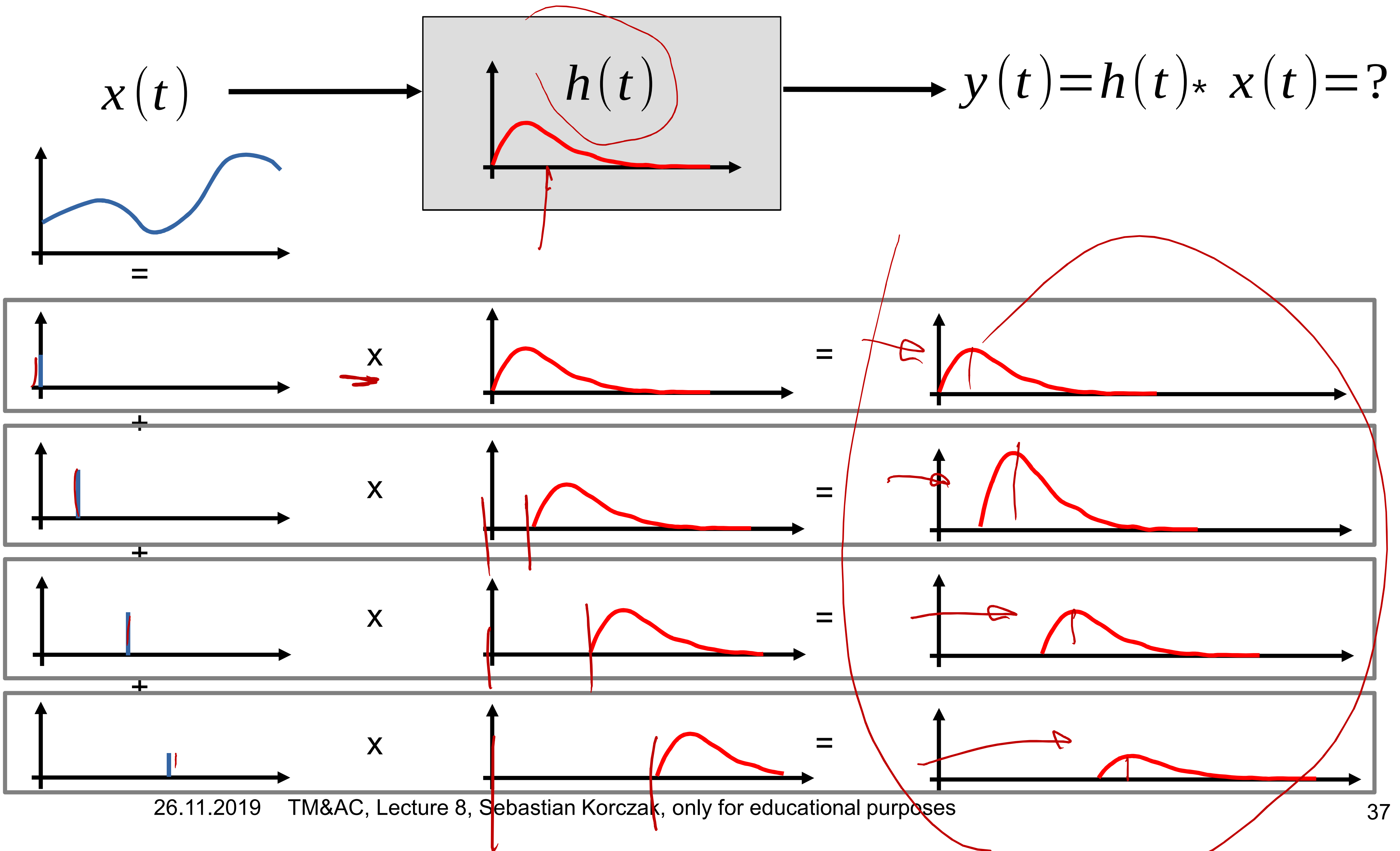
$$h(t) * x(t) = \int_0^{\infty} x(\tau) h(t-\tau) \, d\tau$$



$$y(t) = h(t) * x(t) = ?$$

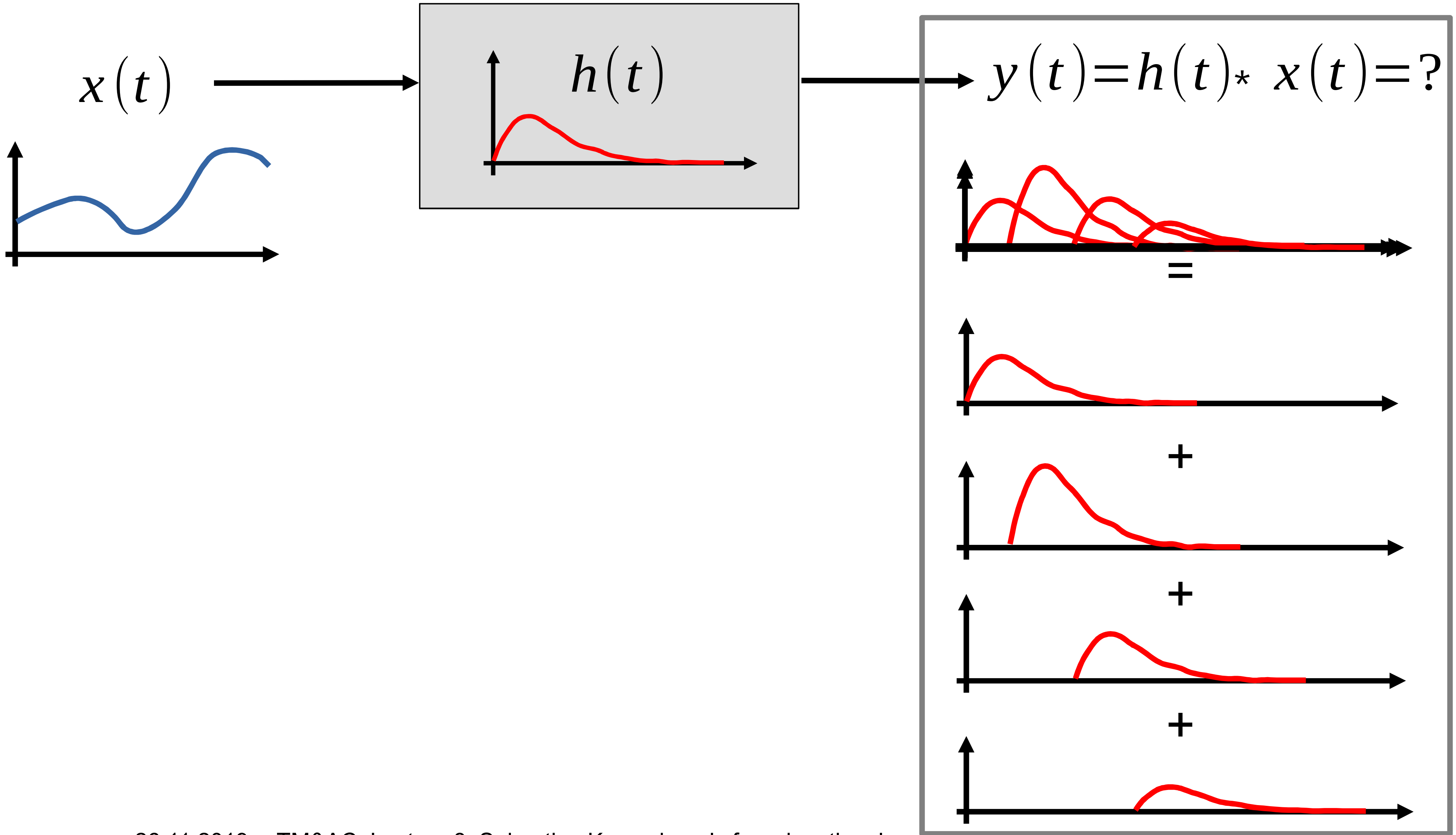
Convolution

$$h(t) * x(t) = \int_0^{\infty} x(\tau) h(t-\tau) d\tau$$



Convolution

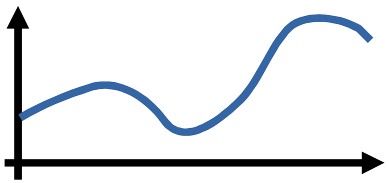
$$h(t)*x(t)=\int_{0}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

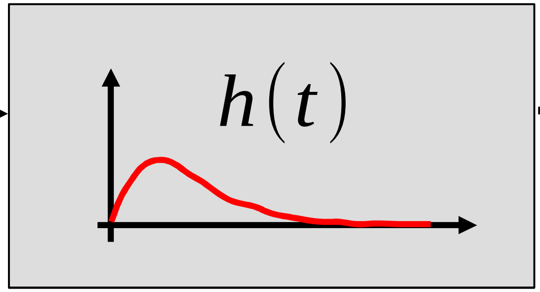


Convolution

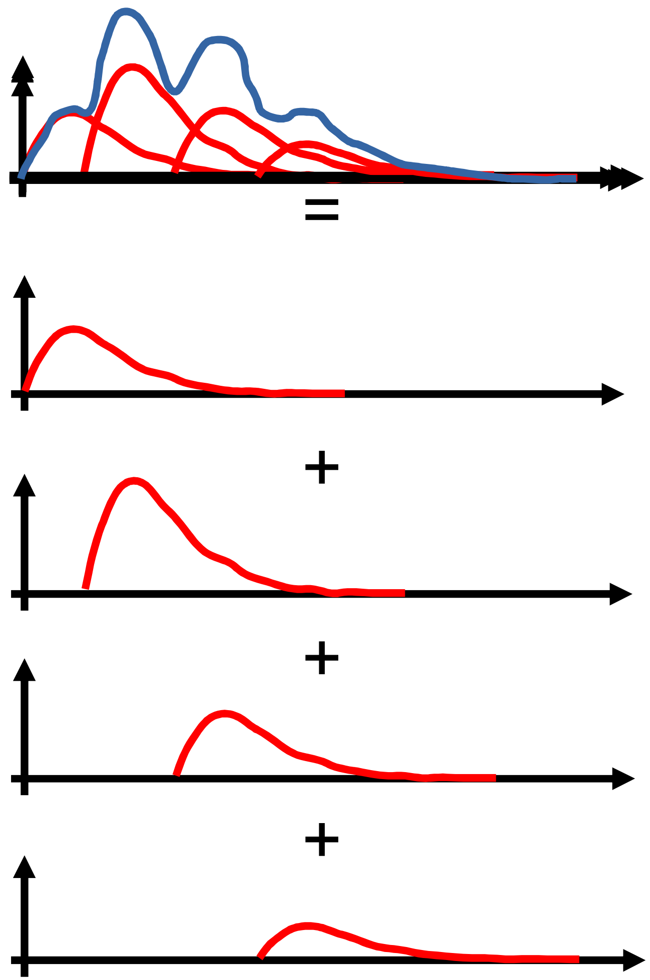
$$h(t) * x(t) = \int_0^{\infty} x(\tau) h(t - \tau) \, d\tau$$

$x(t)$



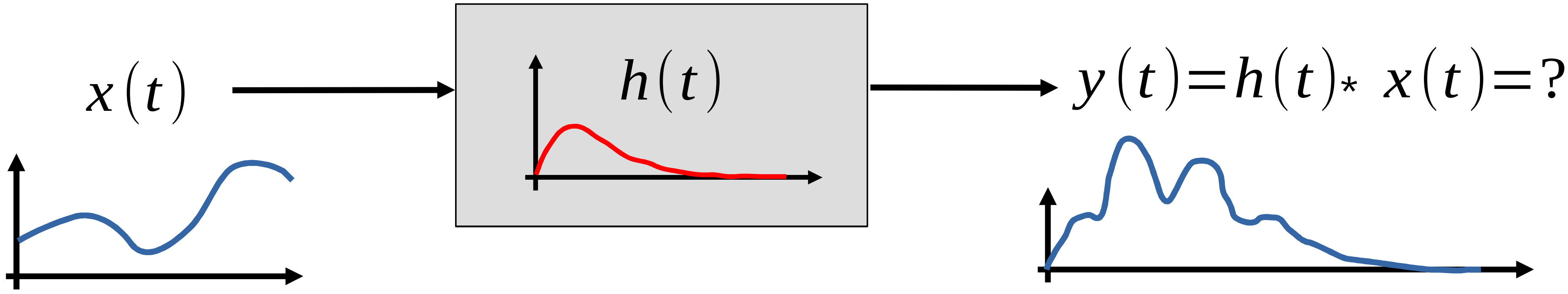


$y(t) = h(t) * x(t) = ?$



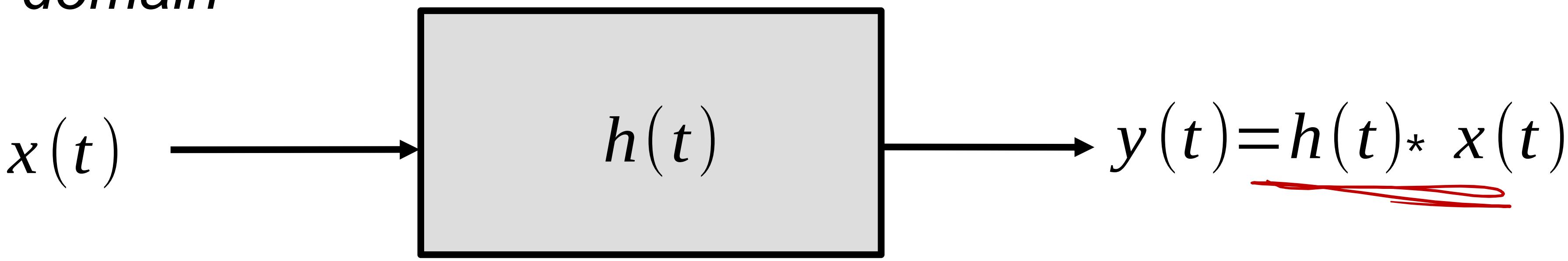
Convolution

$$h(t) * x(t) = \int_0^\infty x(\tau) h(t-\tau) d\tau$$

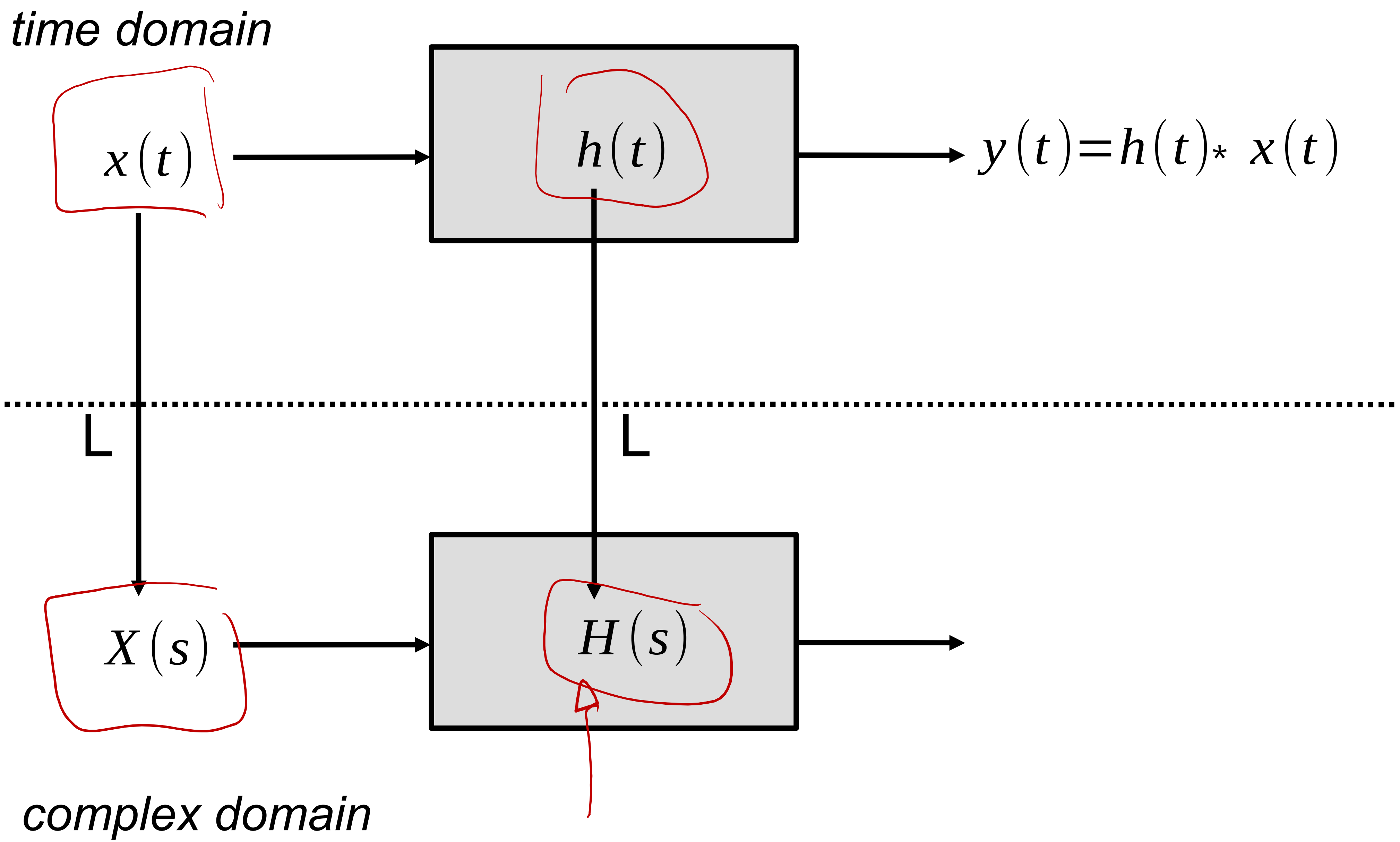


Input and output

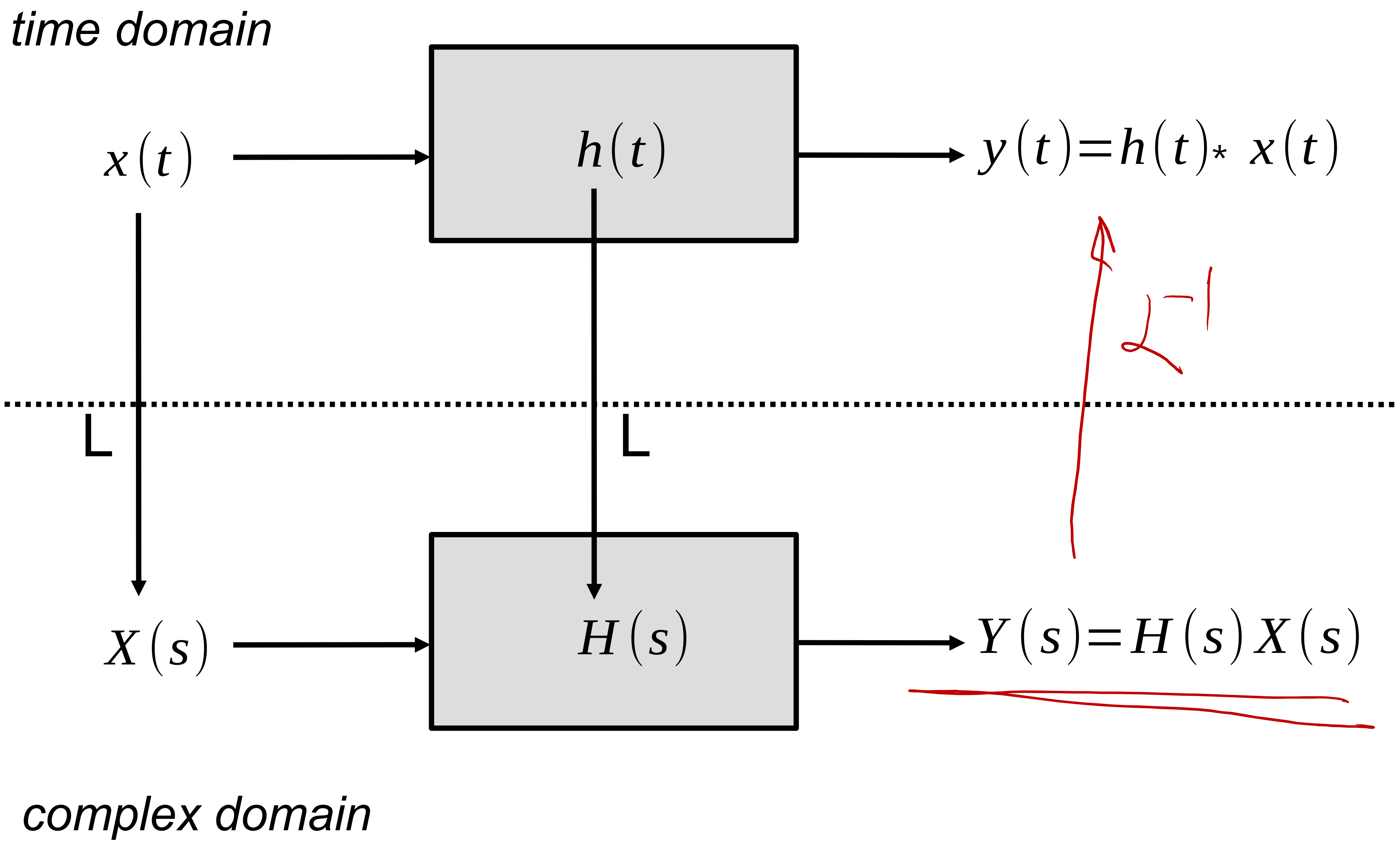
time domain



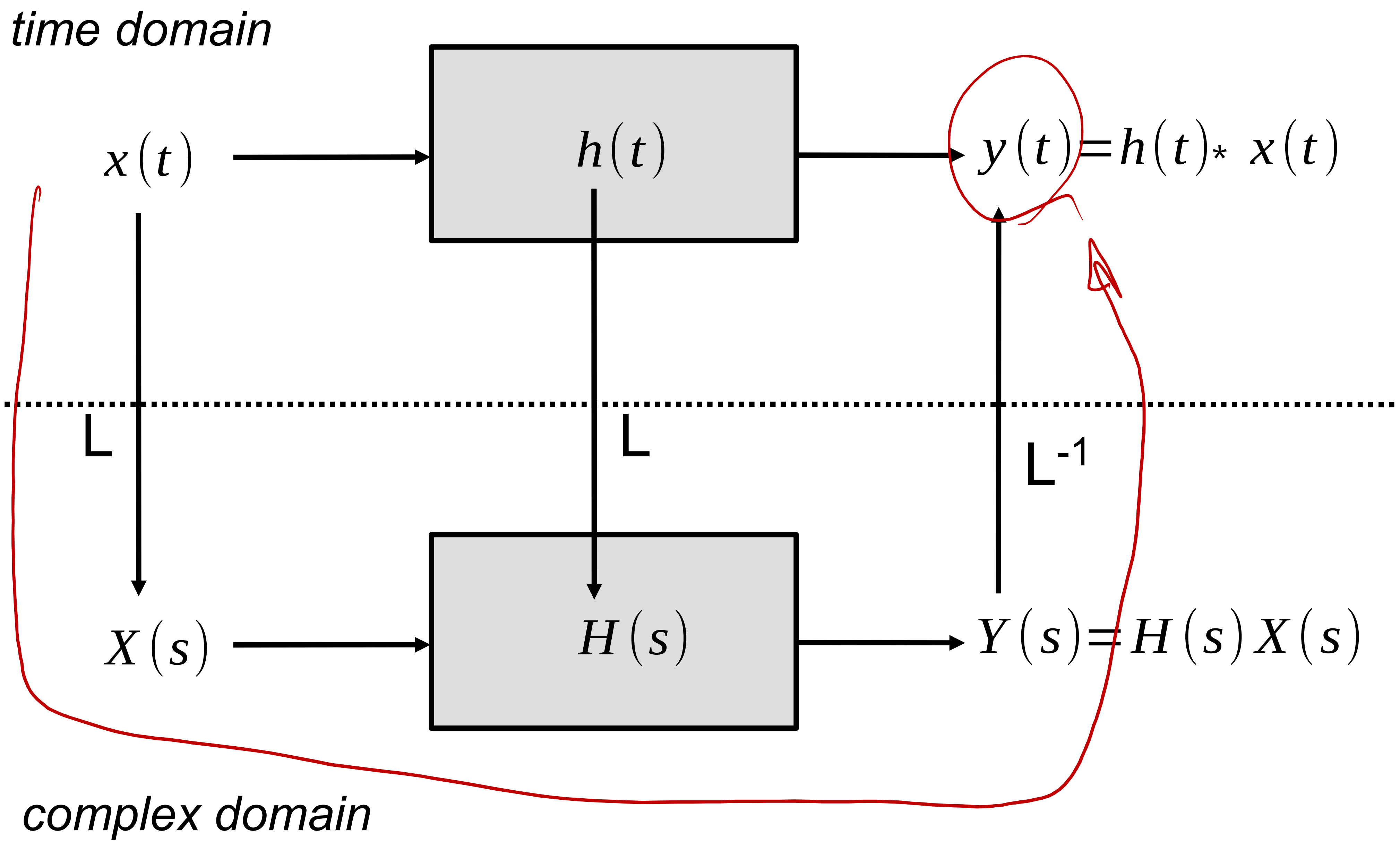
Input and output



Input and output



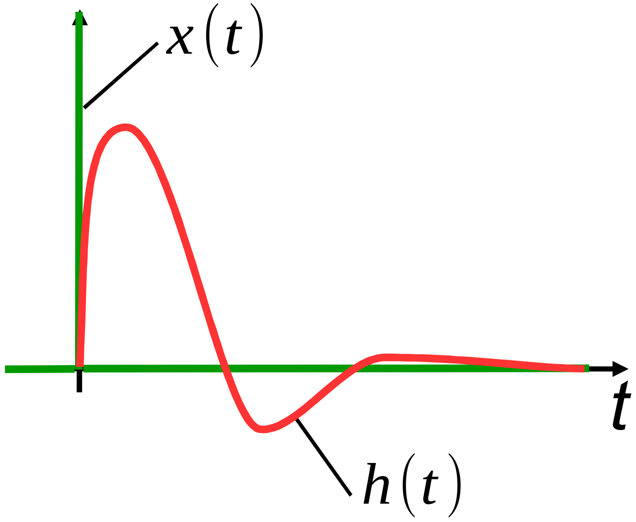
Input and output

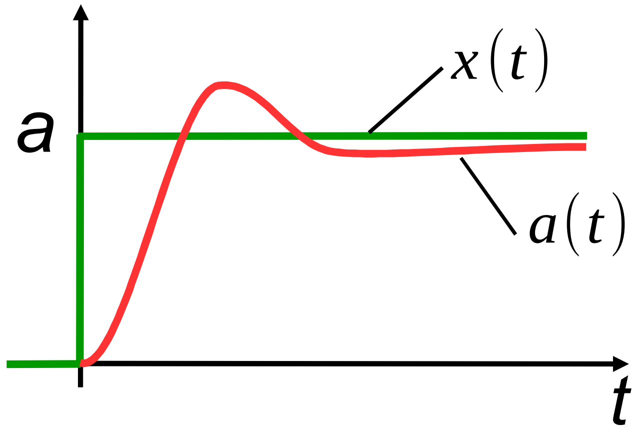


Input and output

$h(t)$
impulse response
$y(t)$ for $x(t)=\delta(t)$

$a(t)$
step response
$y(t)$ for $x(t)=1(t)$





$$\frac{d\,a(t)}{dt}=h(t)$$

Exemplary input signals

No input: $x(t)=0$

Unit impulse (Dirac delta pseudofunction): $\delta(t)=\begin{cases}0, & t<0\\ \infty, & t=0\\ 0, & t>0\end{cases}$

Unit step function (Heviside step function): $1(t)=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$

$H(t)$ or $1_+(t)$

Ramp function: $x(t)=\begin{cases}0, & t<0\\ t, & t>0\end{cases}$

Harmonic function: $x(t)=a\sin(\omega t)$

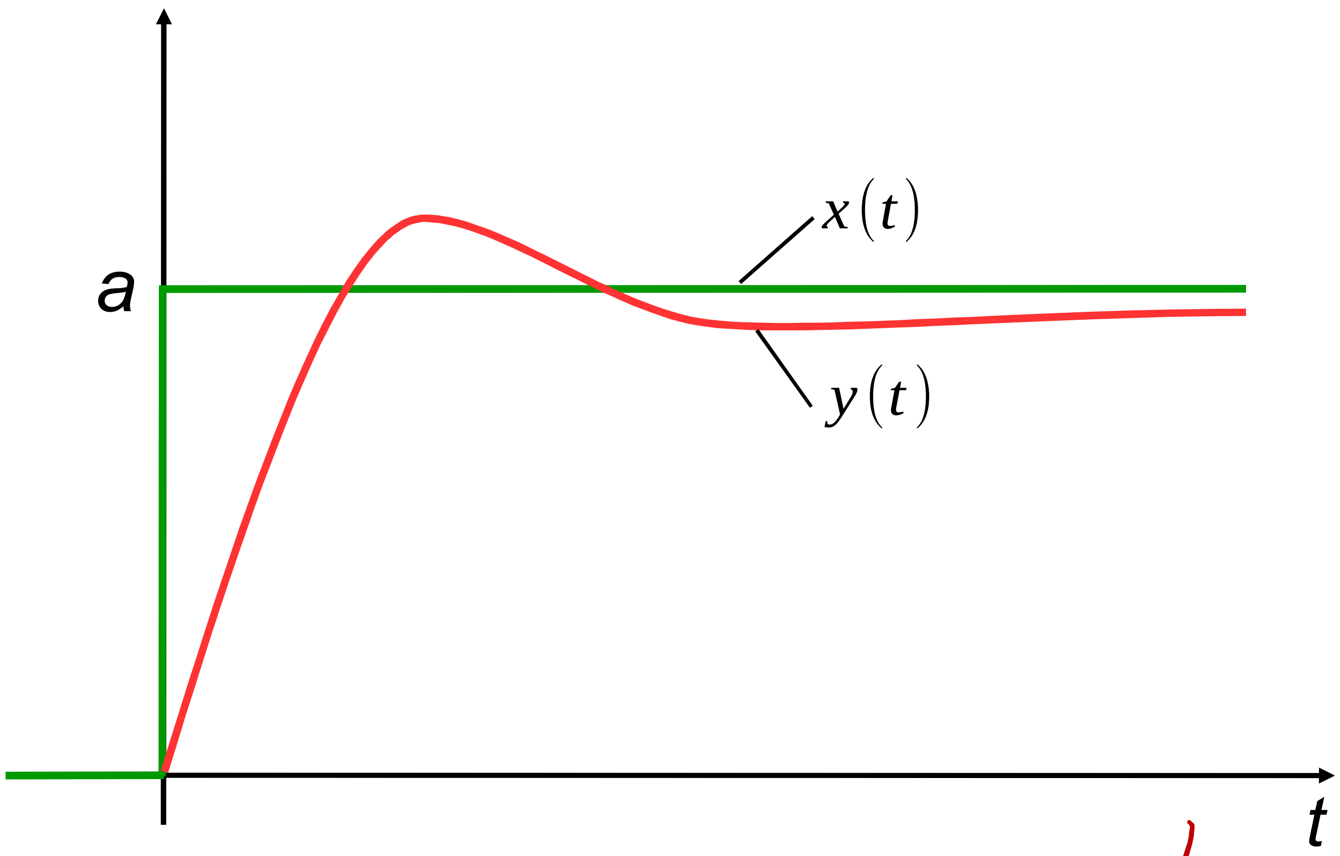
System step response

input: $x(t) = a \cdot 1(t)$ transfer function: $H(s)$ output: $y(t) = ?$

$$X(s) = L\{x(t)\} = a \cdot \frac{1}{s}$$

$$Y(s) = X(s) \cdot H(s)$$

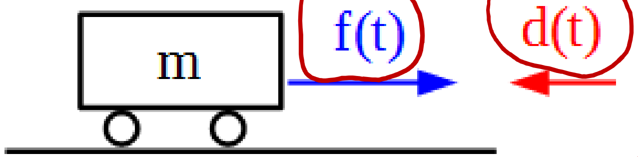
$$y(t) = L^{-1}\{Y(s)\}$$



Step response – example 1

$d(t) = c \cdot v(t)$

Car on a flat surface
m – mass,
f(t) – driving force,
d(t)=c*v(t) – air ressistance,
v(t) – velocity



$$m\frac{dv(t)}{dt} = f(t) - d(t)$$

Laplace with zero I.C.

$$m \cdot s \cdot V(s) = F(s) - c \cdot V(s)$$

$$\boxed{H(s) = \frac{V(s)}{F(s)} = \frac{1}{ms+c}}$$

$$f(t) = f_0 \cdot 1(t)$$

$$F(s) = \mathcal{L}\{f_0 \cdot 1(t)\} = f_0 \frac{1}{s}$$

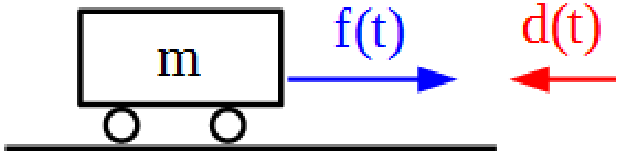
$$V(s) = H(s) \cdot F(s) = \frac{1}{ms+c} \cdot f_0 \frac{1}{s} =$$

$$= f_0 \frac{1}{s(ms+c)}$$

$$v(t) = \mathcal{L}^{-1}\{V(s)\} = \mathcal{L}^{-1}\left\{f_0 \frac{1}{s(ms+c)}\right\}$$

Step response – example 1

Car on a flat surface
m – mass,
f(t) – driving force,
d(t)=c*v(t) – air ressistance,
v(t) – velocity



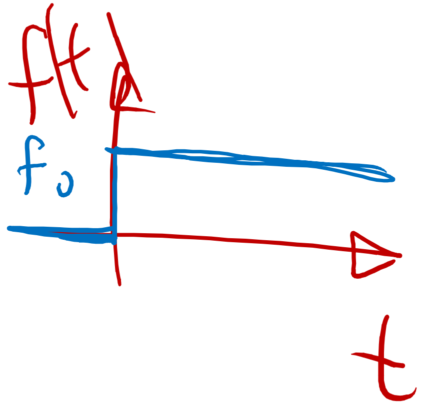
$$m\frac{dv(t)}{dt}=f(t)-d(t)$$

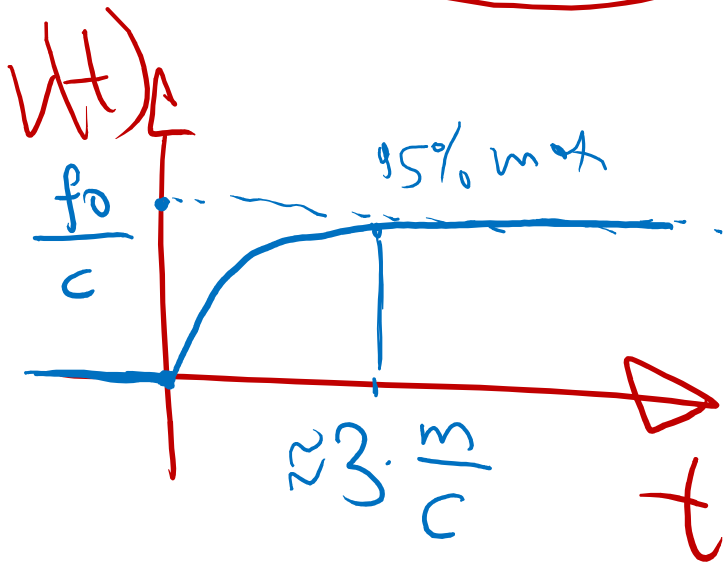
$$v(t)=\mathcal{L}^{-1}\left\{f_0\frac{1}{s(ms+c)}\right\}=$$

$$\boxed{1-e^{-bt} \;\Big|\; \frac{b}{s(s+b)}}$$

$$=\mathcal{L}^{-1}\left\{\frac{f_0}{c}\,\frac{\frac{c}{m}}{s\left(s+\frac{c}{m}\right)}\right\}=$$

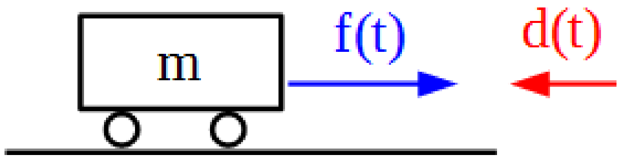
$$=\frac{f_0}{c}\left(1-e^{-\frac{c}{m}t}\right)$$





Step response – example 1

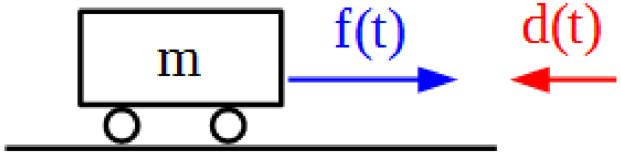
Car on a flat surface
m – mass,
f(t) – driving force,
d(t)=c*v(t) – air ressistance,
v(t) – velocity



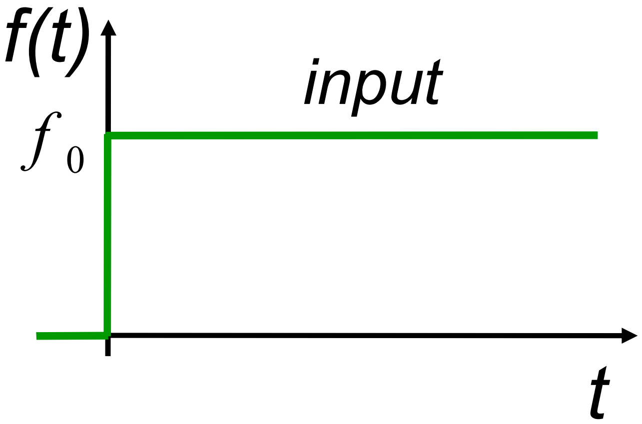
$$m\frac{dv(t)}{dt}=f(t)-d(t)$$

Step response – example 1

Car on a flat surface
m – mass,
f(t) – driving force,
d(t)=c*v(t) – air ressistance,
v(t) – velocity



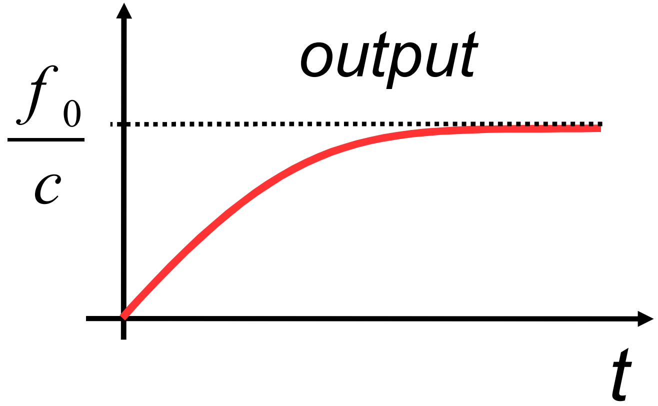
$$m\frac{dv(t)}{dt}=f(t)-d(t)$$



$$m\frac{dv(t)}{dt}=f(t)-cv(t)$$

$$msV(s)=F(s)-cV(s)$$

$$H(s)=\frac{V(s)}{F(s)}=\frac{1}{ms+c}$$



$$f(t)=f_0 1(t)$$

$$F(s)=f_0\frac{1}{s}$$

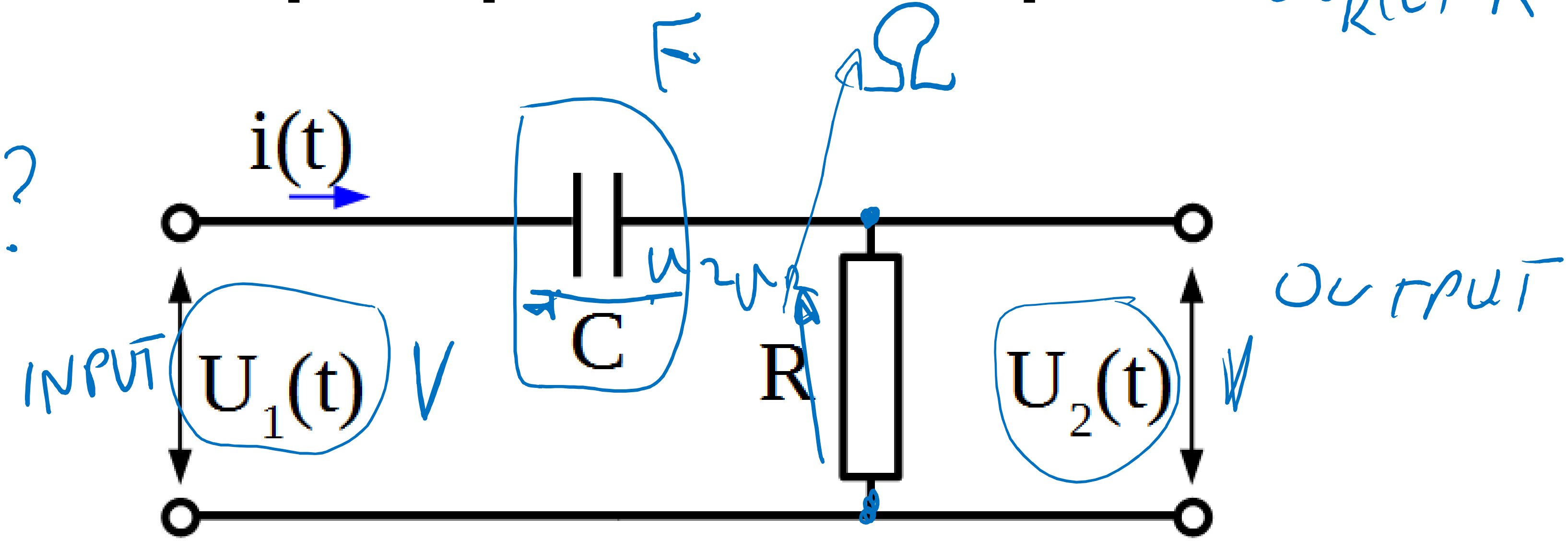
$$V(s)=H(s)F(s)=\frac{1}{ms+c}f_0\frac{1}{s}=\frac{f_0}{s(ms+c)}$$

$$v(t)=L^{-1}\left\{\frac{f_0}{s(ms+c)}\right\}=L^{-1}\left\{\frac{f_0}{c}\frac{c/m}{s(s+c/m)}\right\}=\frac{f_0}{c}\left(1-e^{-\frac{c}{m}t}\right)$$

Step response – example 2

$u_R(t) = R \cdot i(t)$

Transfer function?

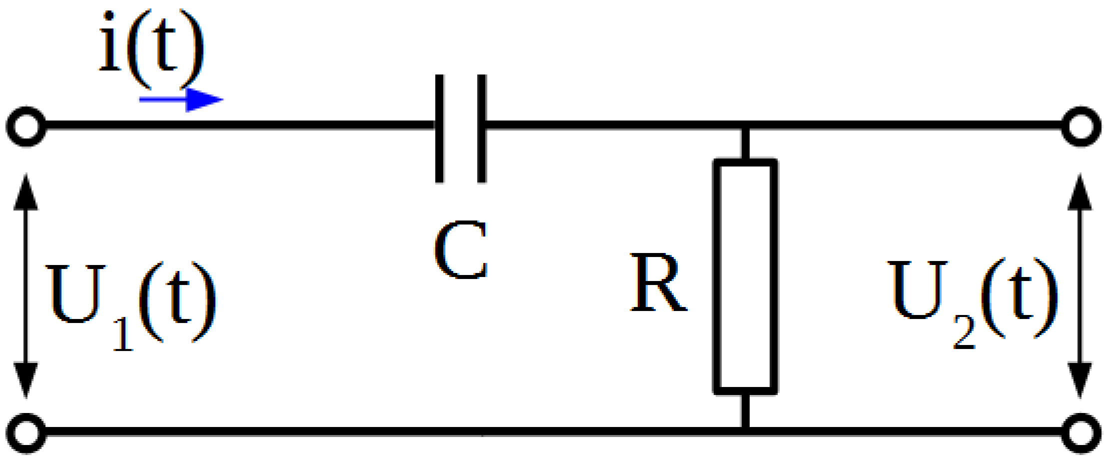


$$u_1(t) = u_C(t) + u_R(t) \qquad u_R = u_2(t)$$

$$u_C(t) = \frac{q(t)}{C} = \frac{\int i(t)\,dt}{C} = \frac{\int \frac{u_R}{R}\,dt}{C} = \frac{\int u_2\,dt}{CR} \qquad i(t) = \frac{dq(t)}{dt}$$

$$u_1(t) = \frac{1}{CR}\int u_2\,dt + u_2(t)$$

Step response – example 2

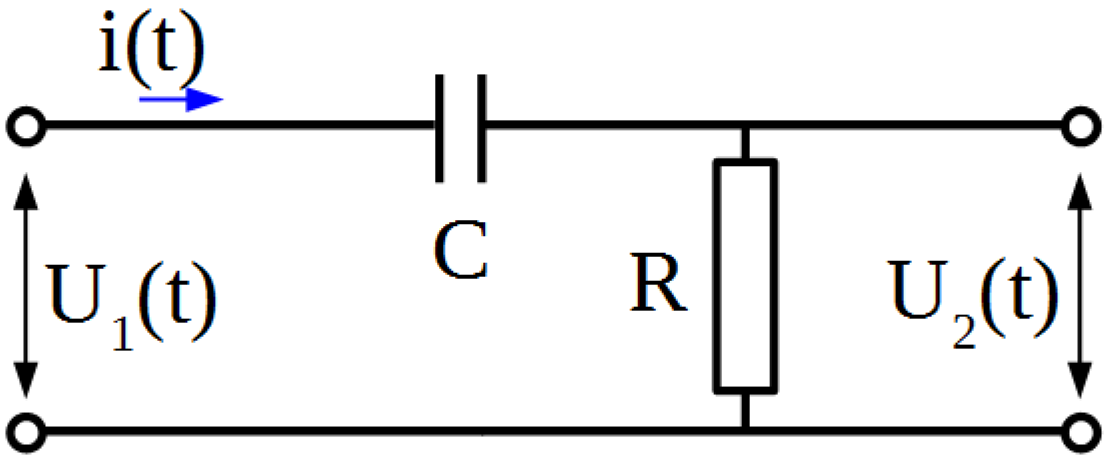


$$u_1(t) = u_C(t) + u_R(t)$$

$$u_C(t) = \frac{q(t)}{C}, \quad u_R(t) = i(t)R, \quad i = \frac{dq}{dt} \qquad u_2(t) = u_R(t)$$

$$u_C(t) = \frac{q(t)}{C} = \frac{1}{C}\int i\,dt = \frac{1}{C}\int \frac{u_R}{R}dt = \frac{1}{CR}\int u_2\,dt$$

Step response – example 2

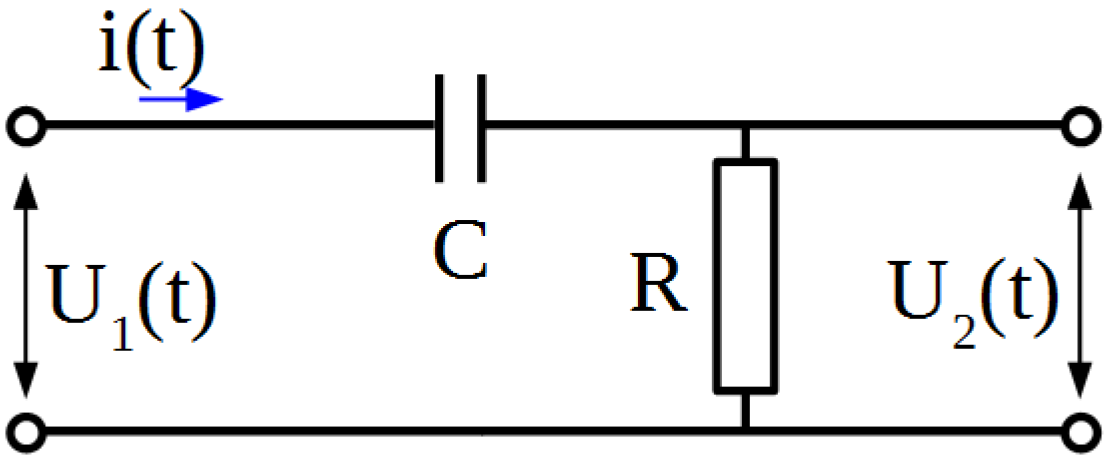


$$\frac{1}{CR}\int u_2(t)\,dt + u_2(t) = u_1(t) \qquad \mathcal{L}$$

$$\frac{1}{CR}\frac{1}{s}U_2(s) + U_2(s) = U_1(s)$$

$$H(s) = \frac{U_2(s)}{U_1(s)} = \frac{1}{\frac{1}{CRs}+1} = \boxed{\frac{s}{\frac{1}{CR}+s}}$$

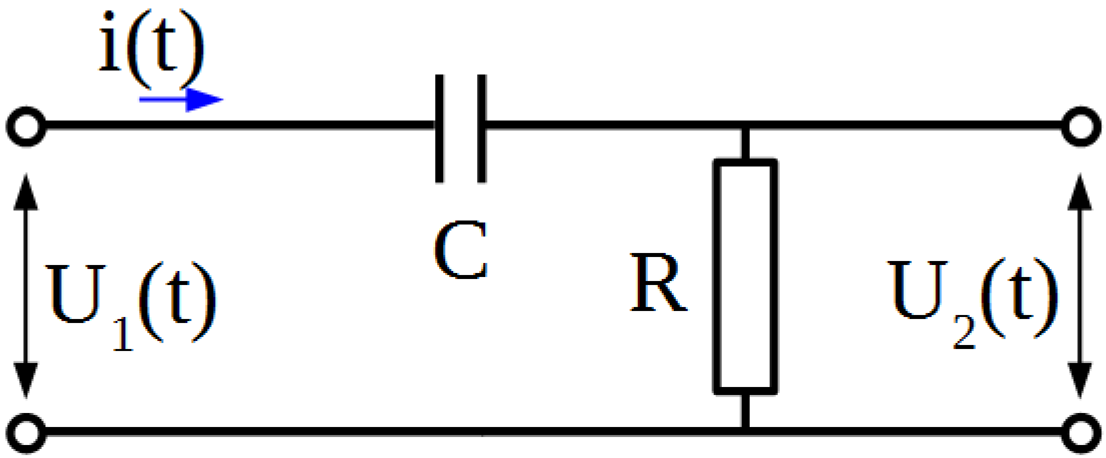
Step response – example 2



$$\frac{1}{CR}\int u_2(t)\,dt + u_2(t) = u_1(t)$$

$$\frac{1}{CR}u_2(t) + \frac{du_2(t)}{dt} = \frac{du_1(t)}{dt}$$

Step response – example 2



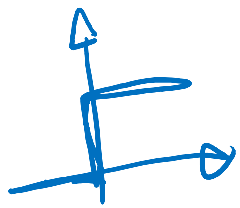
$$\frac{1}{T}U_2(s)+sU_2(s)=sU_1(s) \qquad T=CR$$

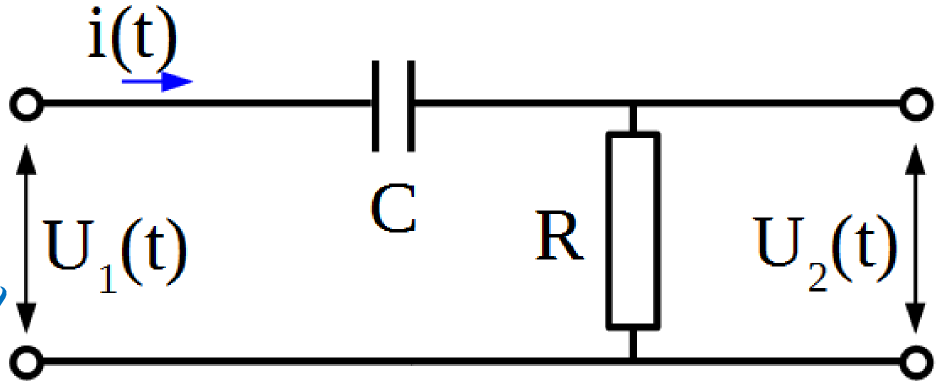
$$G(s)=\frac{U_2(s)}{U_1(s)}=\boxed{\frac{Ts}{1+Ts}}$$

Step response – example 2

$T = CR$

$$\boxed{H(s) = \frac{Ts}{1+Ts}}$$





$u_1(t) = U_0 \mathbb{1}(t)$

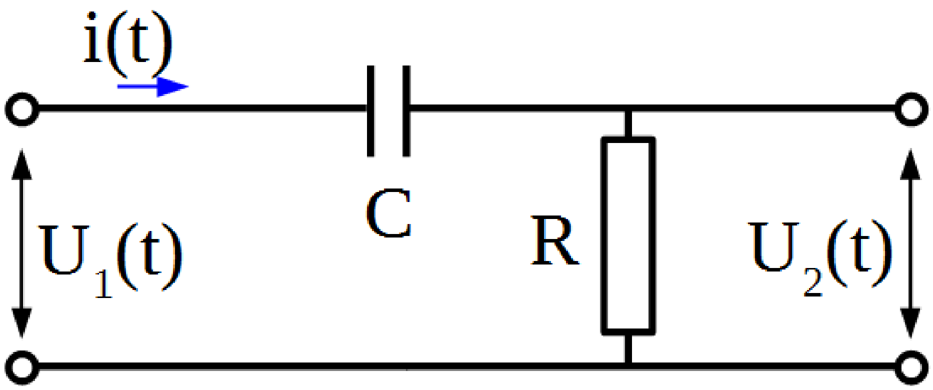
$U_1(s) = U_0 \frac{1}{s}$

$$U_2(s) = H(s) \cdot U_1(s) = \frac{T\cancel{s}}{1+Ts} \cdot U_0 \frac{1}{\cancel{s}} = \frac{T U_0}{1+Ts}$$

$$u_2(t) = \mathcal{L}^{-1}\left\{\frac{T u_0}{Ts+1}\right\} = \mathcal{L}^{-1}\left\{\frac{u_0}{s+\frac{1}{T}}\right\} = u_0 e^{-\frac{t}{T}}$$

Step response – example 2

$$G(s)=\frac{Ts}{1+Ts}$$



$$u_1(t)=a\cdot 1(t),$$

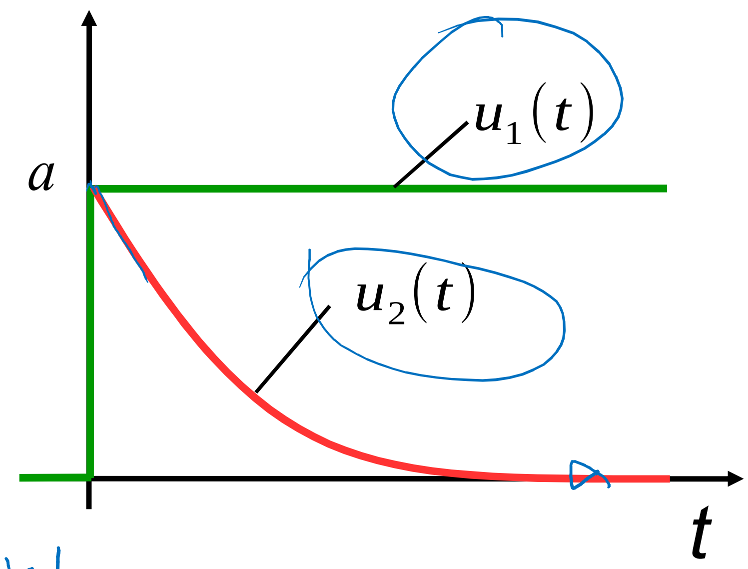
$$U_2(s)=U_1(s)\cdot G(s)=a\frac{1}{s+\frac{1}{T}}$$

Table inertial

$$u_2(t)=L^{-1}\left[U_s(s)\right]=ae^{\frac{-t}{T}}$$

$\frac{1}{s+a}$	$\frac{s}{s+a}$

diff.



Computer methods for transfer function analysis

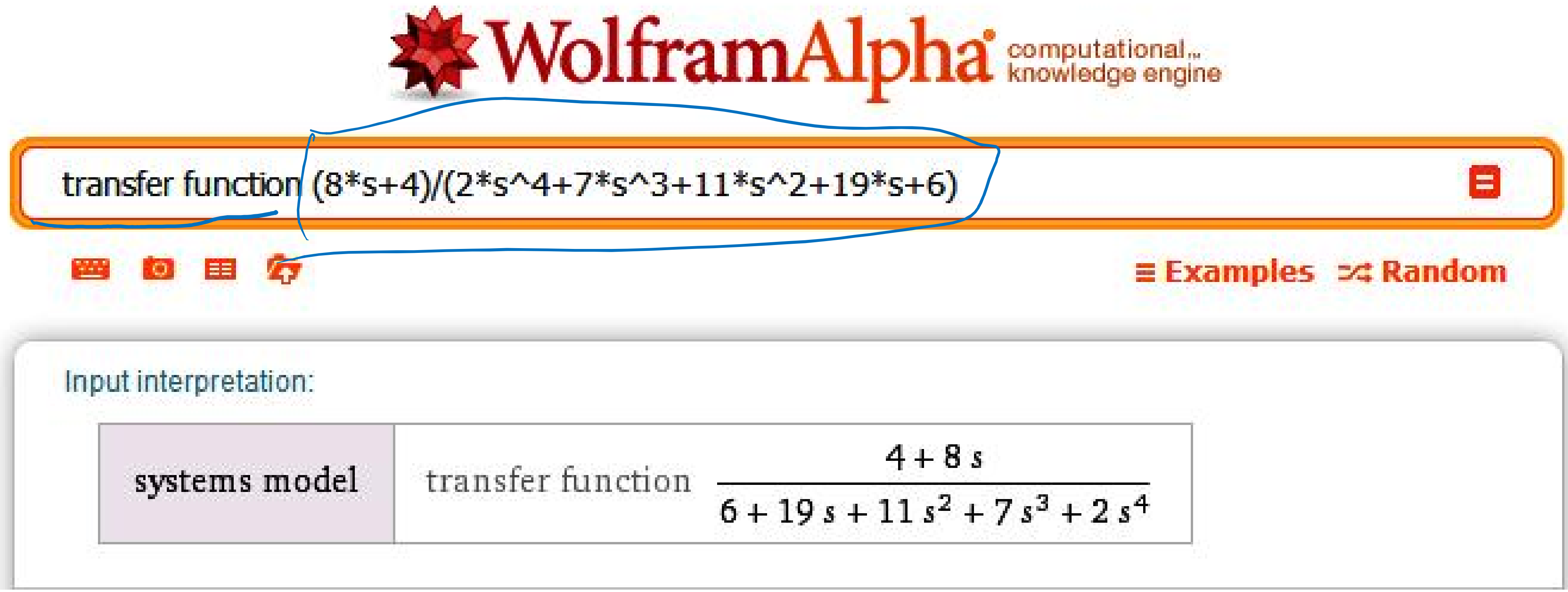
Exemplary computer algebra systems:

- Maxima/wxMaxima (free and open source)
- Wolfram Mathematica (http://www.wolfram.com/mathematica/)
- Mathcad
- Website: www.wolframalpha.com
- MatLab

(en.wikipedia.org/wiki/List_of_computer_algebra_systems)

Spreadsheet for graphs (Excel, LibreOffice Calc)

WolframAlpha



WolframAlpha