Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control

Winter 2019/2020

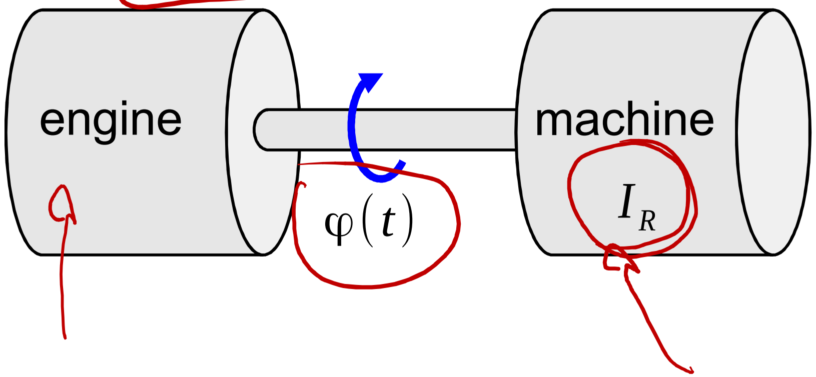
Lecturer: Sebastian Korczak, PhD Eng.

Lecture 7

Non-uniformity of machine motion.
Introduction to automatic control.

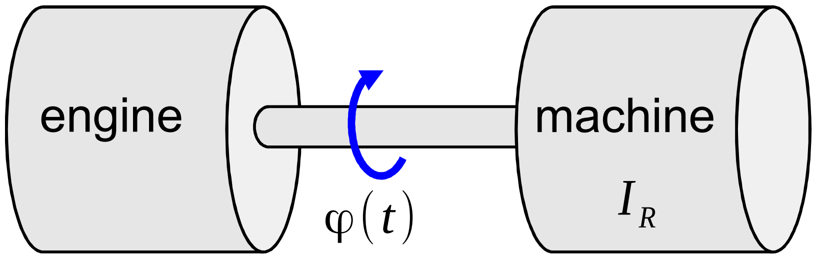
Non-uniformity of machine motion

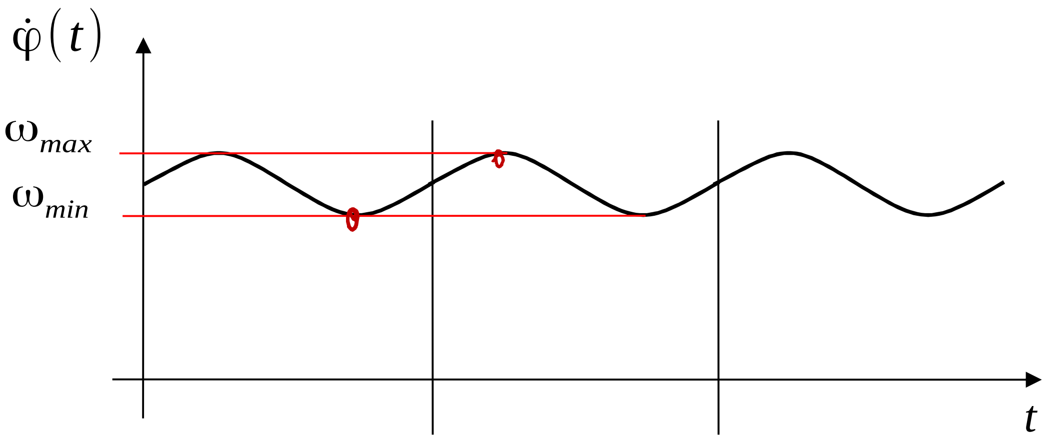
Steady-state motion



Non-uniformity of machine motion

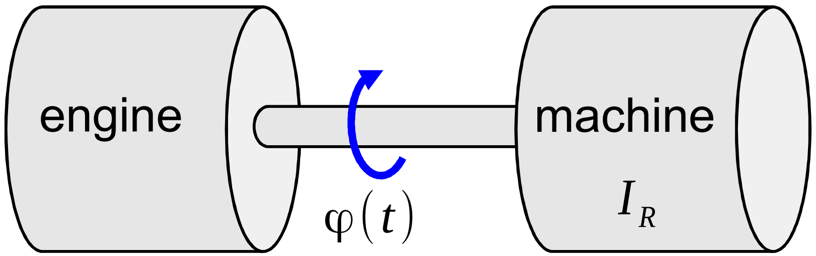
<u>Steady-state motion</u>

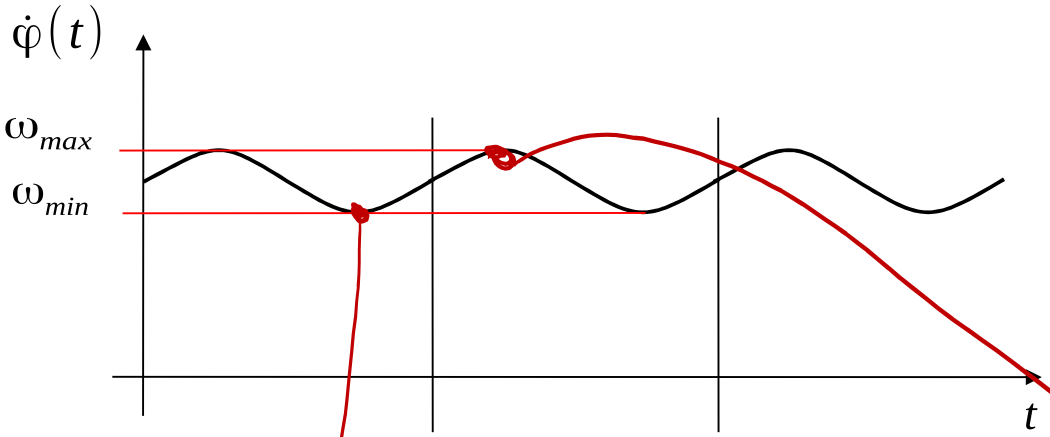




Non-uniformity of machine motion

Steady-state motion





Non-uniformity of machine motion

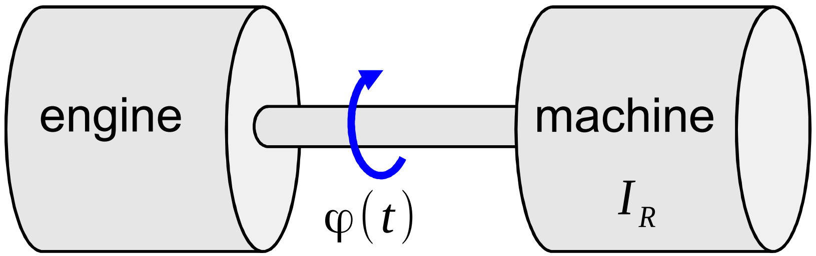
$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \qquad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

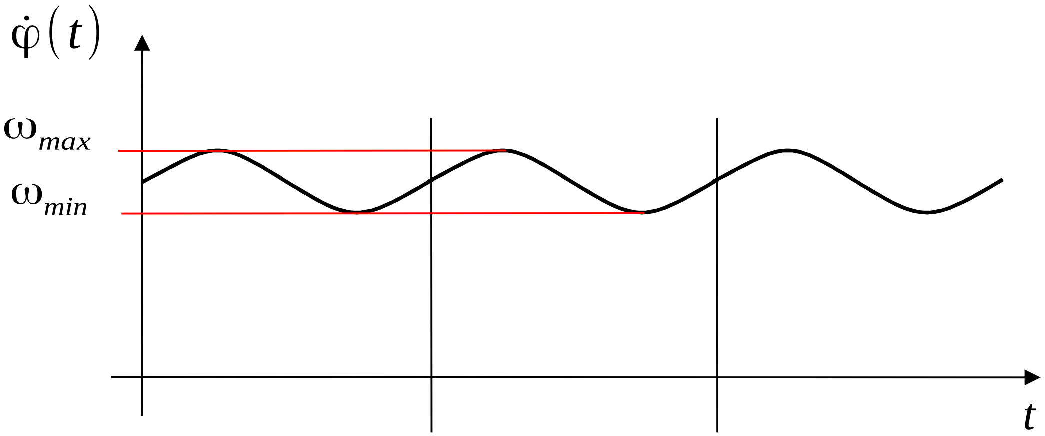
$$T_{min} = \frac{1}{2} I_R \omega_{min}^2 \qquad T_{max} = \frac{1}{2} I_R \omega_{max}^2$$

$$W = T_{max} - T_{min} = \frac{1}{2} I_R (\omega_{max}^2 - \omega_{min}^2) = \delta I_R \omega_{mean}^2$$

Non-uniformity of machine motion

Steady-state motion





Non-uniformity of machine motion

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \qquad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

pumps

$\delta = 1/5 \div 1/30$

combustion engines

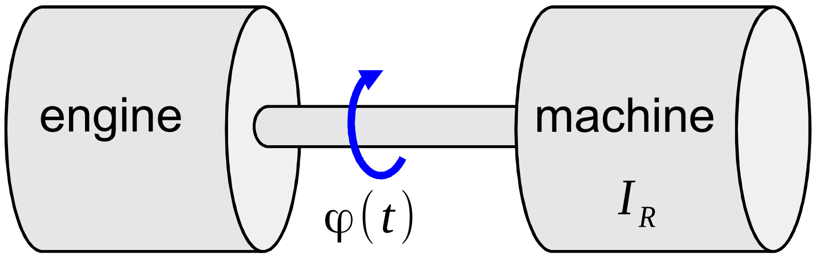
$\delta = 1/50 \div 1/150$

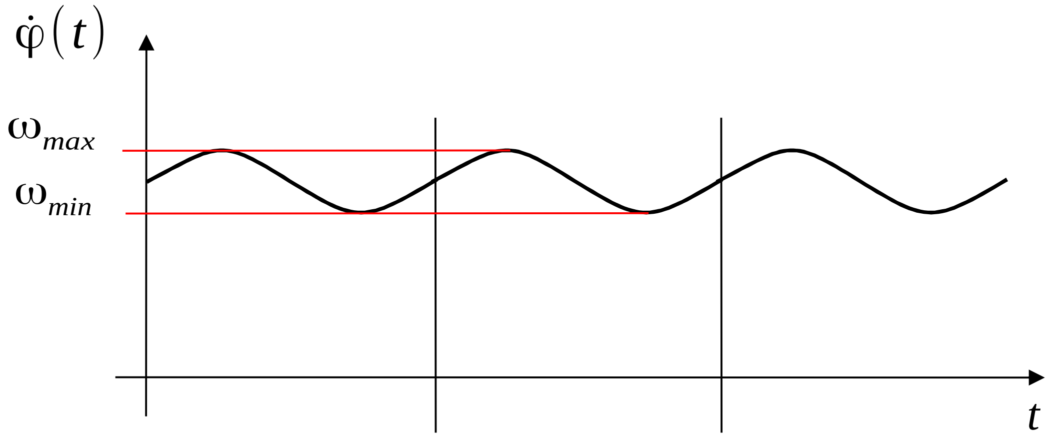
generators

$\delta = 1/200 \div 1/300$

Non-uniformity of machine motion

Steady-state motion





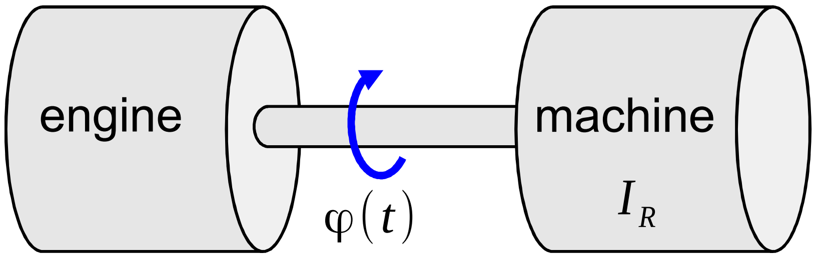
Non-uniformity of machine motion

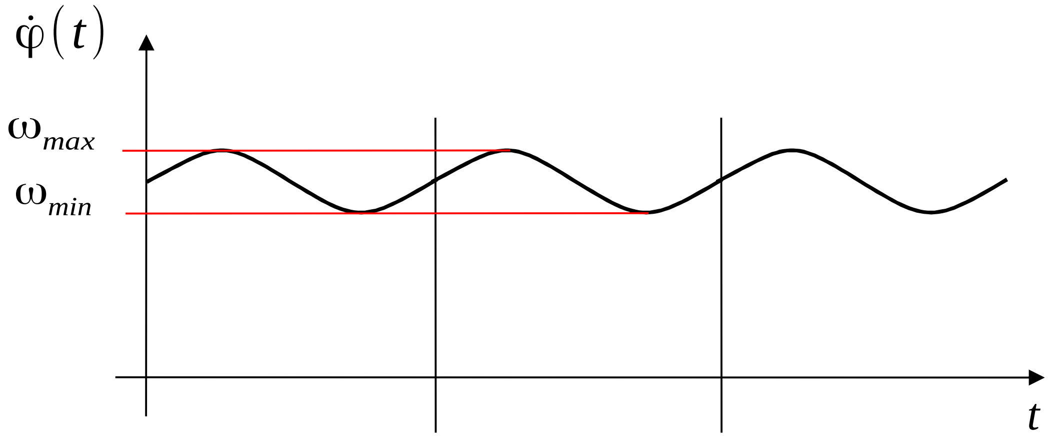
$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \qquad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

$$T_{max} = \frac{1}{2} I_R \omega_{max}^2 \qquad T_{min} = \frac{1}{2} I_R \omega_{min}^2$$

Non-uniformity of machine motion

Steady-state motion





Non-uniformity of machine motion

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}}$$

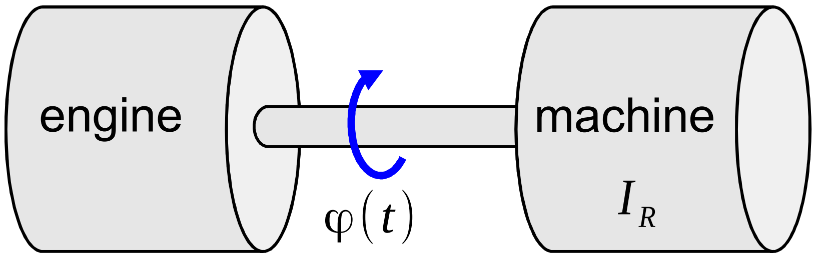
$$\omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

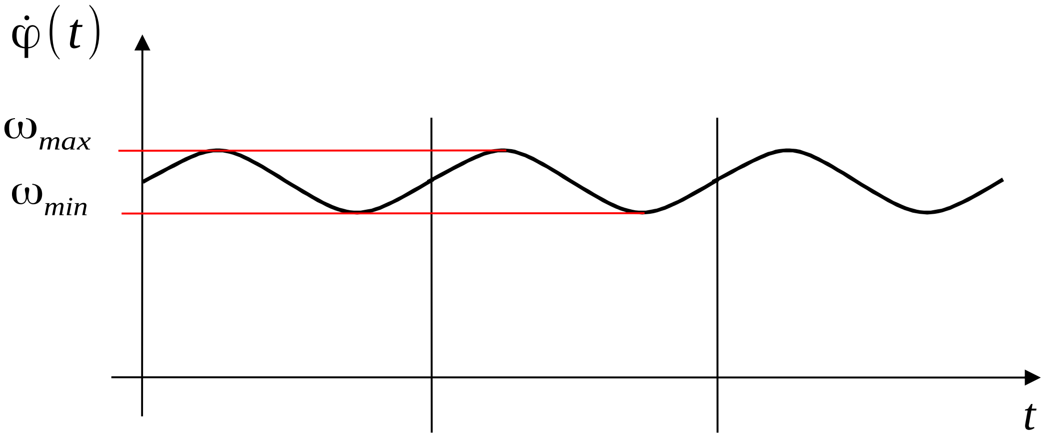
$$T_{max} = \frac{1}{2} I_R \omega_{max}^2 \qquad T_{min} = \frac{1}{2} I_R \omega_{min}^2$$

$$W = T_{max} - T_{min}$$

Non-uniformity of machine motion

Steady-state motion





Non-uniformity of machine motion

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \qquad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

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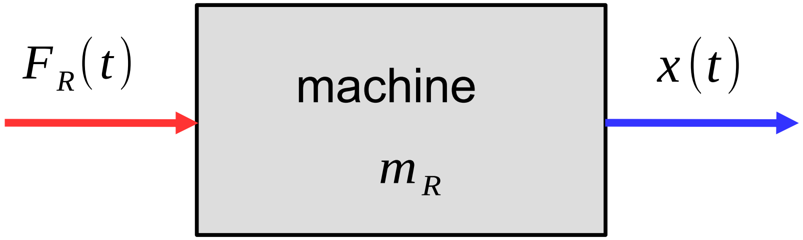
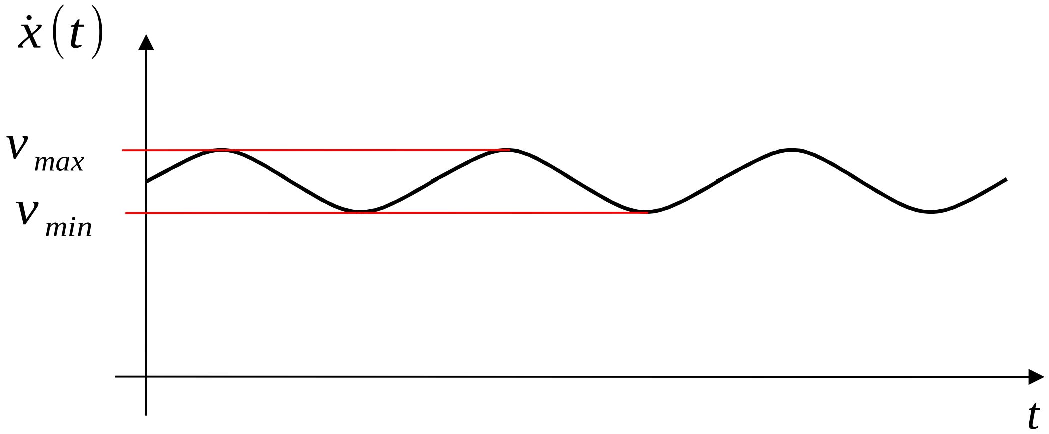
$$W = T_{max} - T_{min} = \delta I_R \omega_{mean}^2$$

Non-uniformity of machine motion

Steady-state motion





Non-uniformity of machine motion

$$\delta = \frac{v_{max} - v_{min}}{v_{mean}}$$

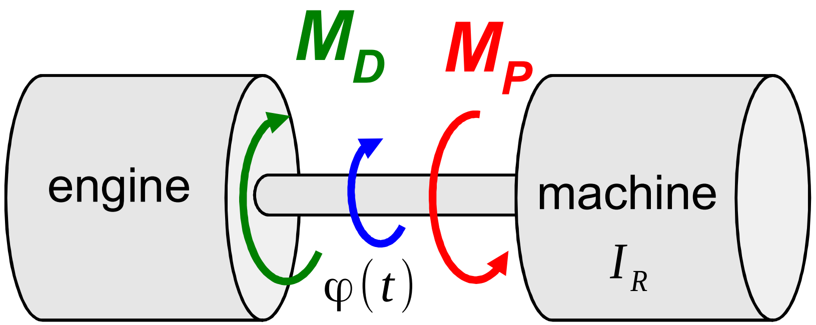
$$v_{mean} = \frac{v_{max} + v_{min}}{2}$$

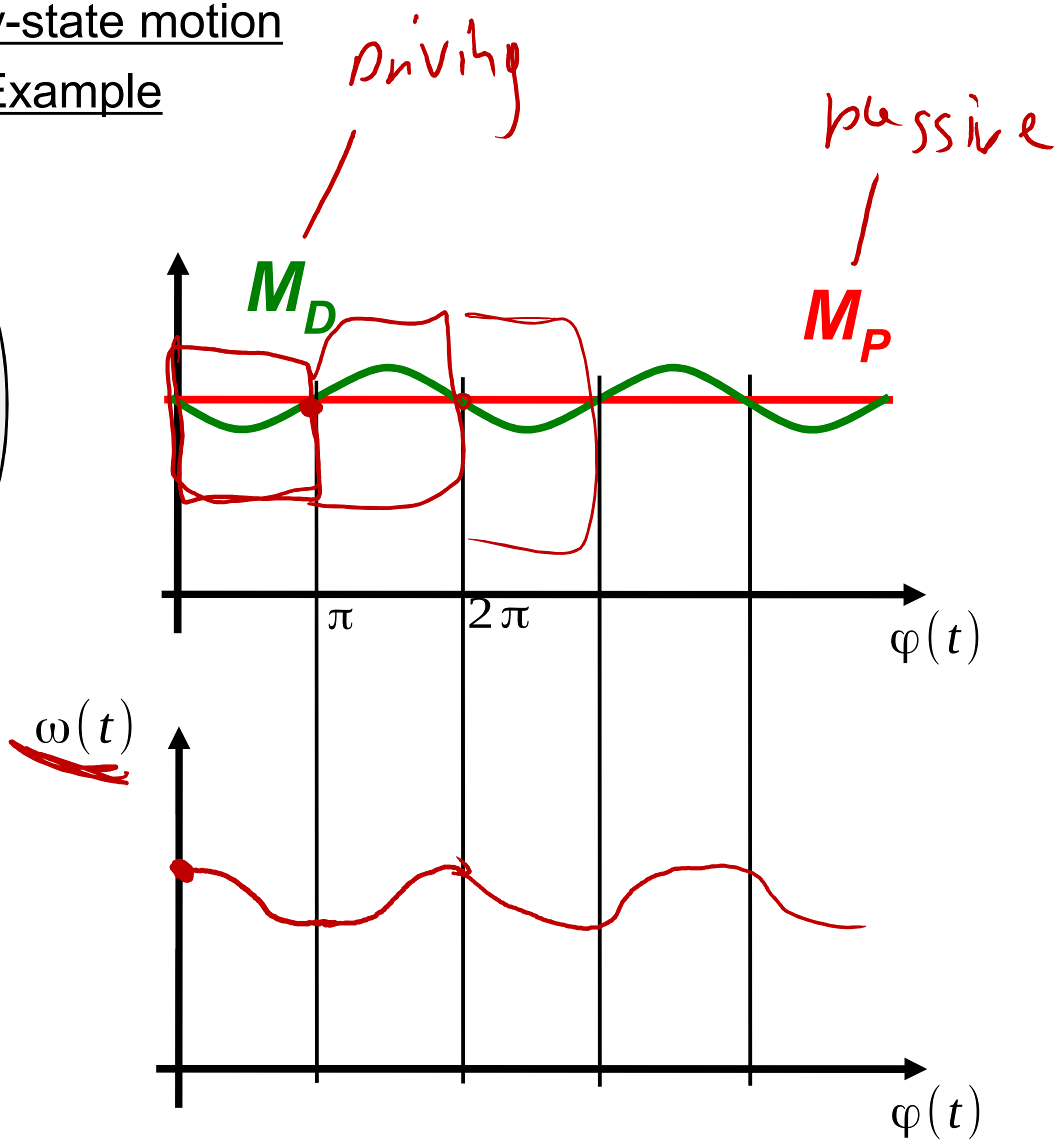
$$W = \delta \, m_R \, v_{mean}^2$$

Non-uniformity of machine motion

Steady-state motion

Example

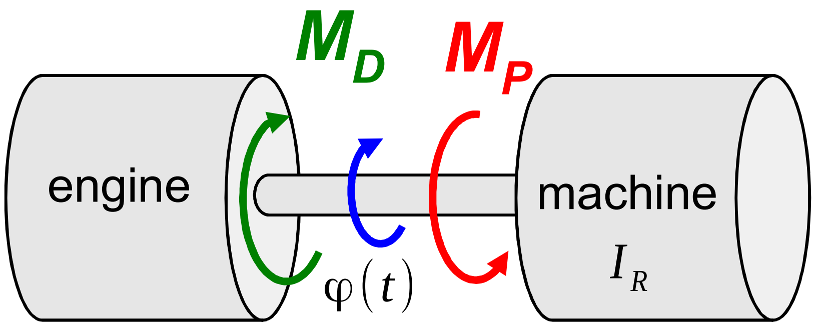




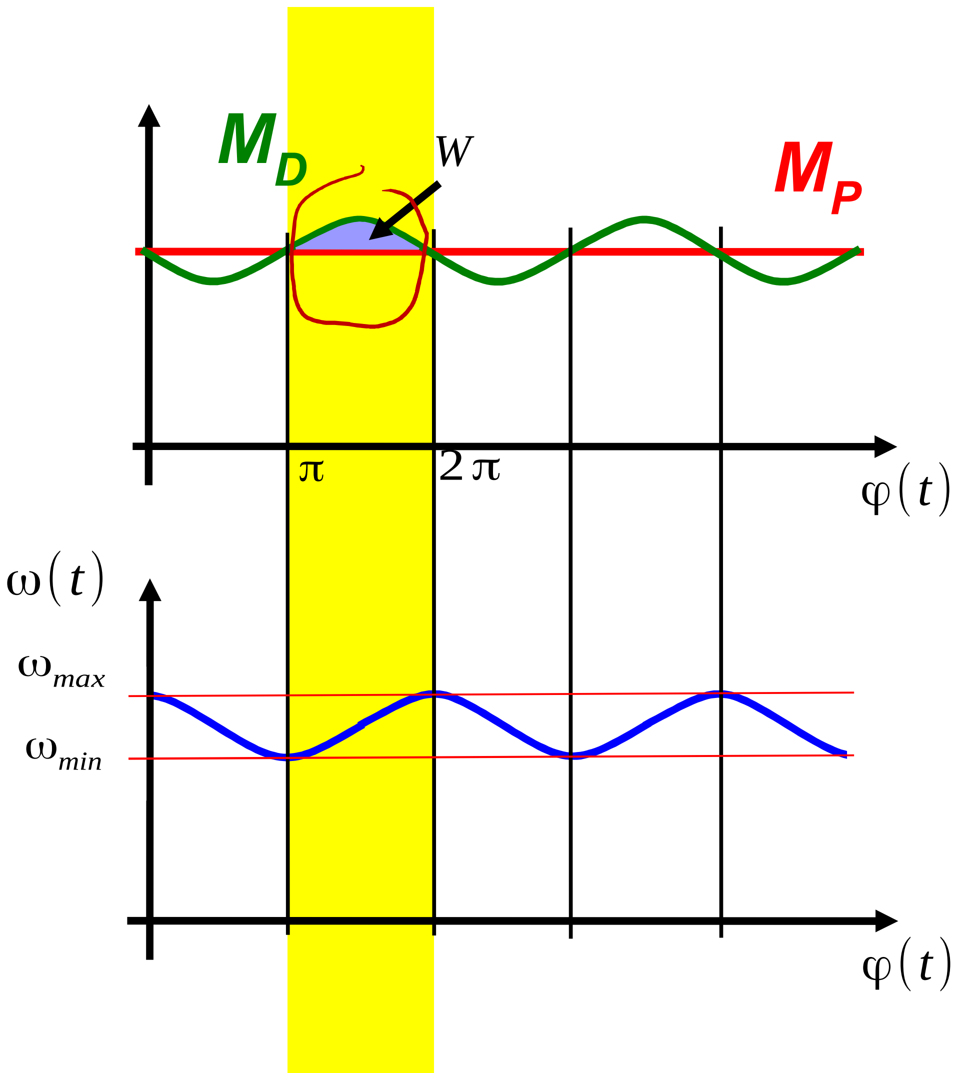
Non-uniformity of machine motion

Steady-state motion

Example



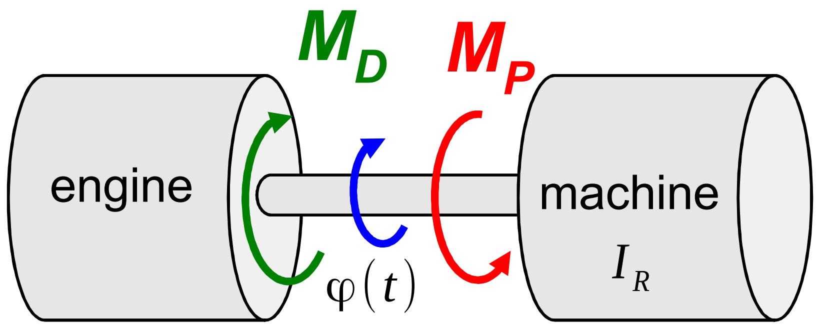
$$W = \int_{\varphi_{min}}^{\varphi_{max}} (M_D - M_P)\, d\varphi$$

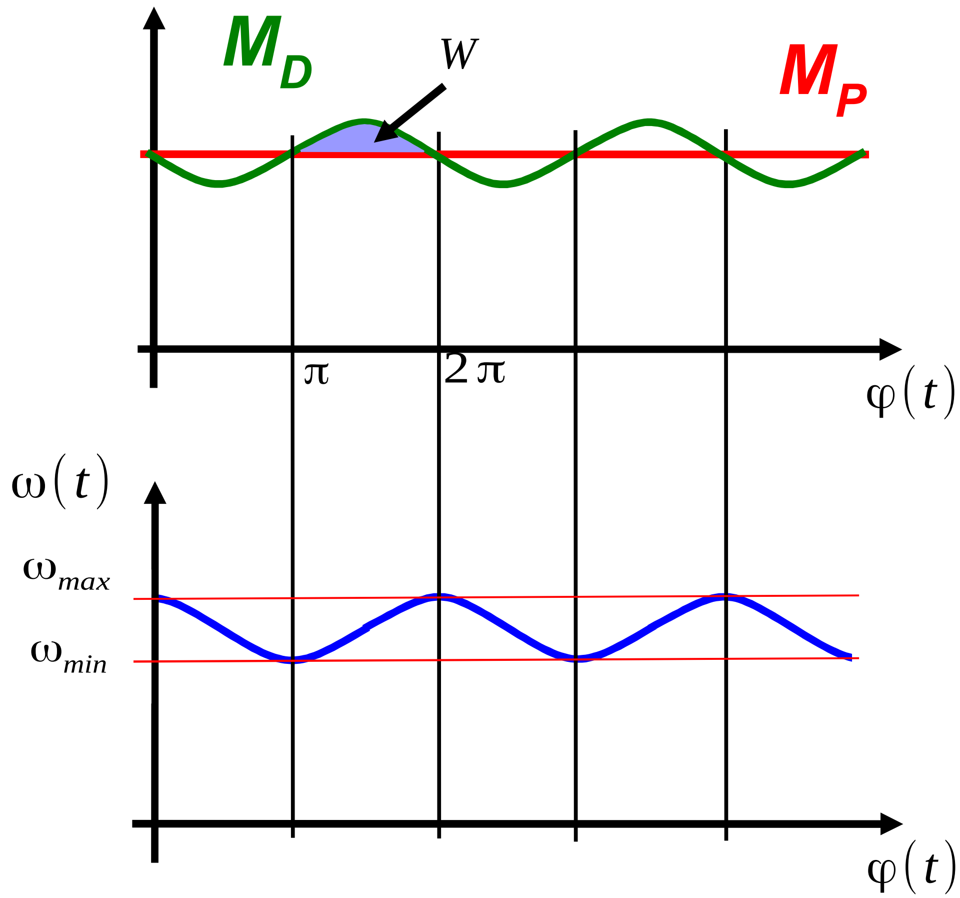


Non-uniformity of machine motion

Steady-state motion

Example





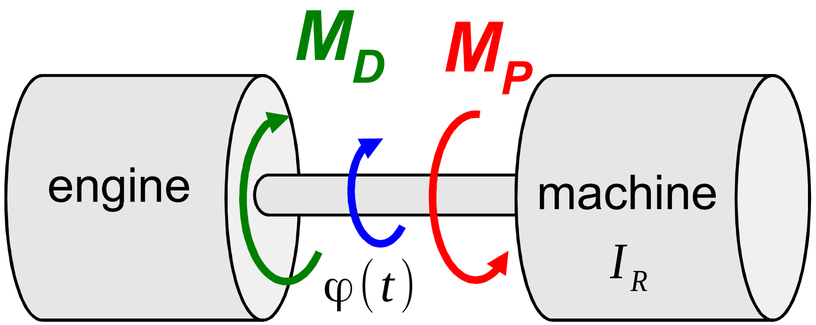
$$W = \int_{\varphi_{min}}^{\varphi_{max}} (M_D - M_P)\, d\varphi$$

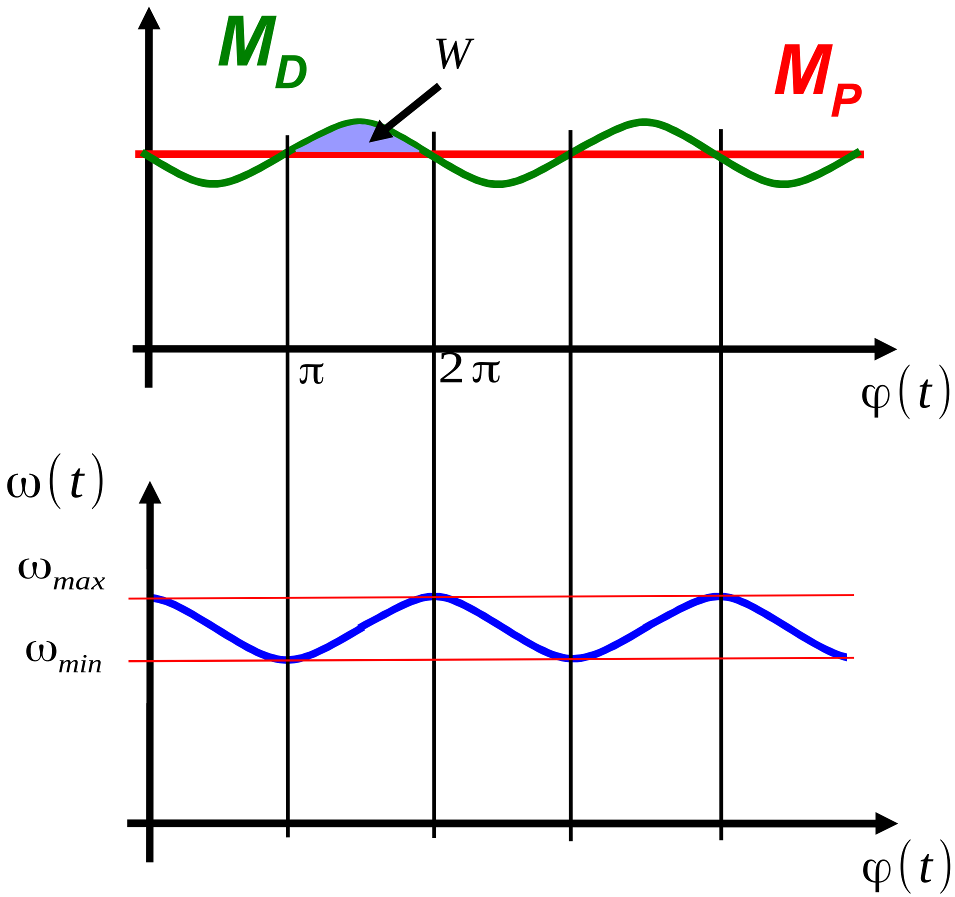
$$W = T_{max} - T_{min} = \delta I_R \omega_{mean}^2$$

Non-uniformity of machine motion

Steady-state motion

Example





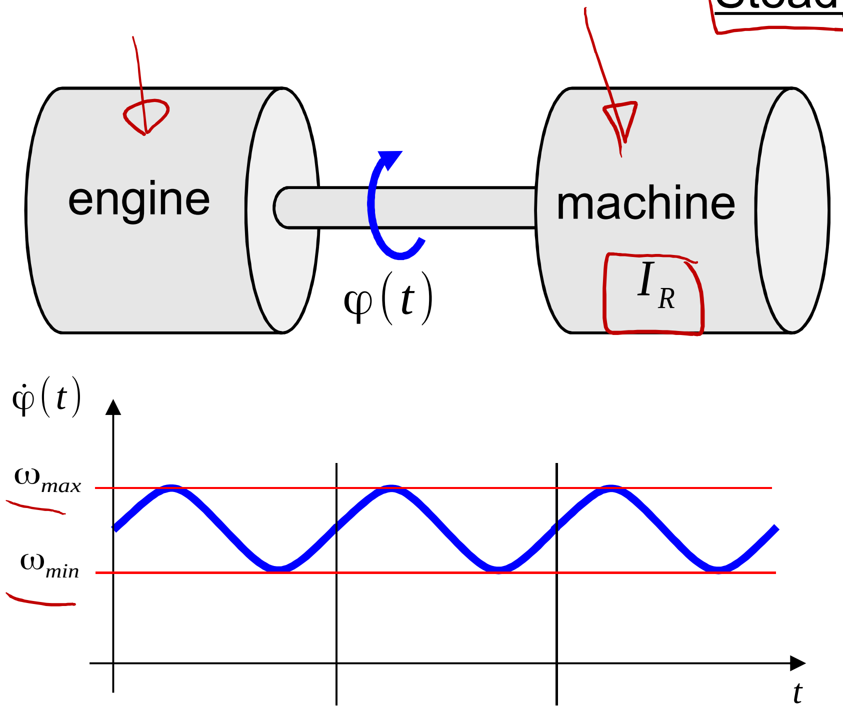
$$W=\int_{\varphi_{min}}^{\varphi_{max}} (M_D - M_P)\, d\varphi$$

$$W = T_{max} - T_{min} = \delta I_R \omega_{mean}^2$$

$$\delta = \frac{W}{I_R \omega_{mean}^2}$$

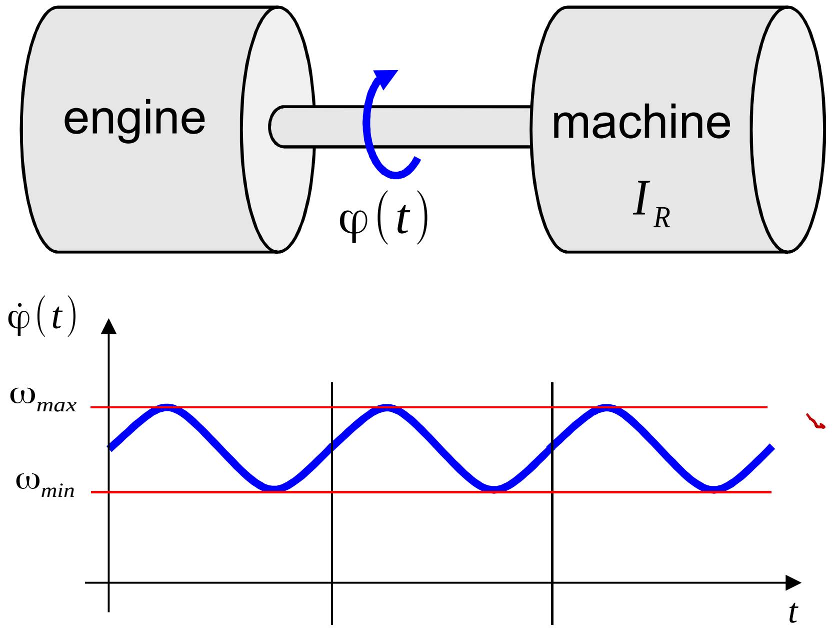
Flywheel

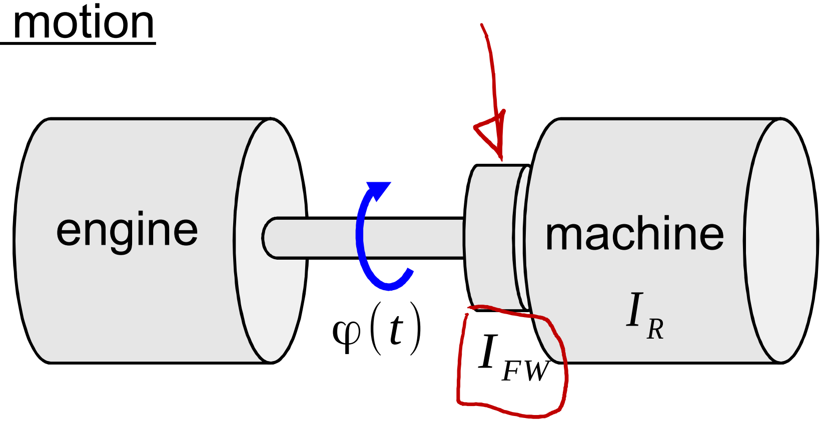
Steady-state motion



Flywheel

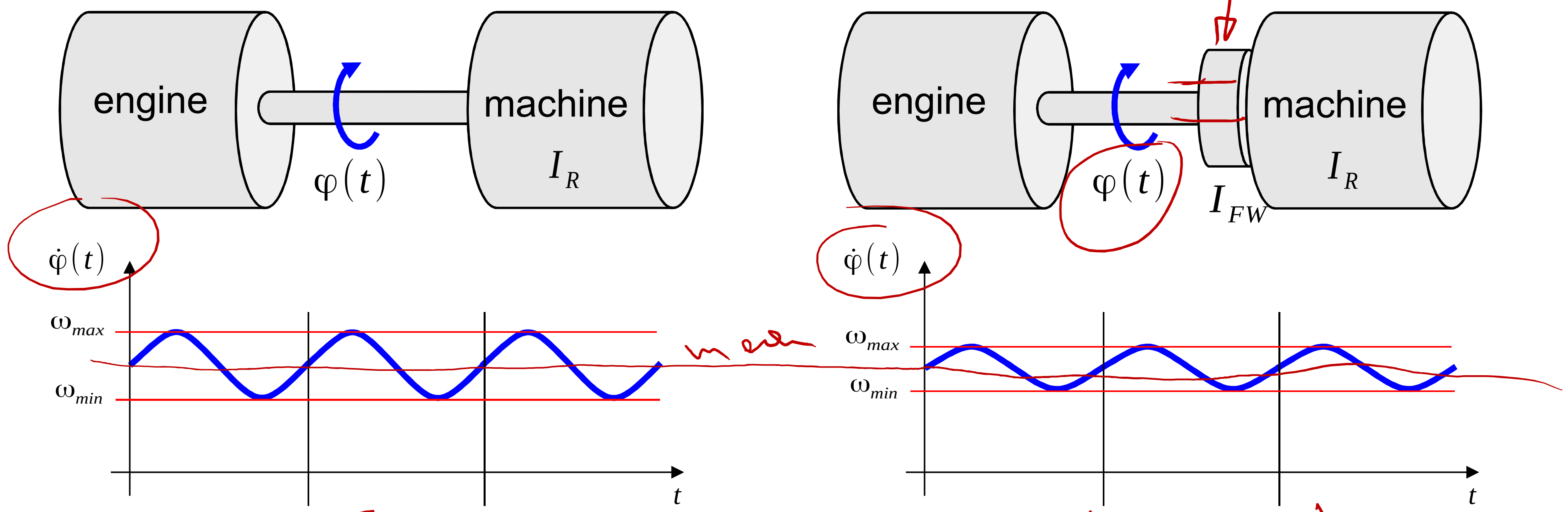
Steady-state motion





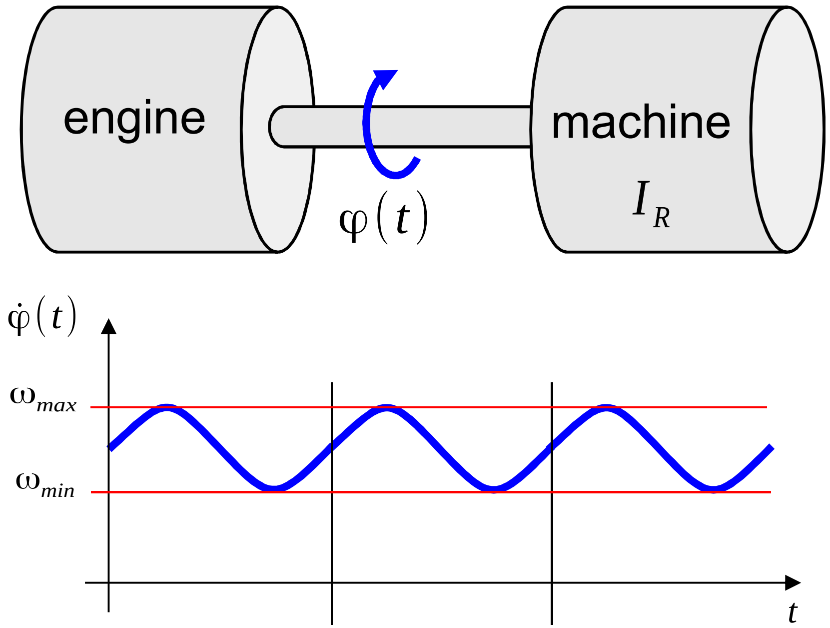
Flywheel

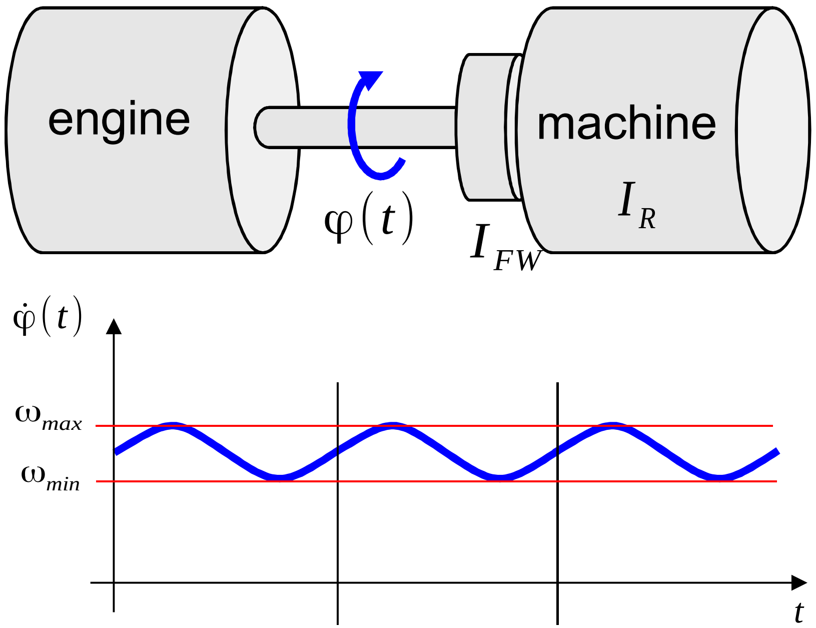
Steady-state motion



Flywheel

Steady-state motion





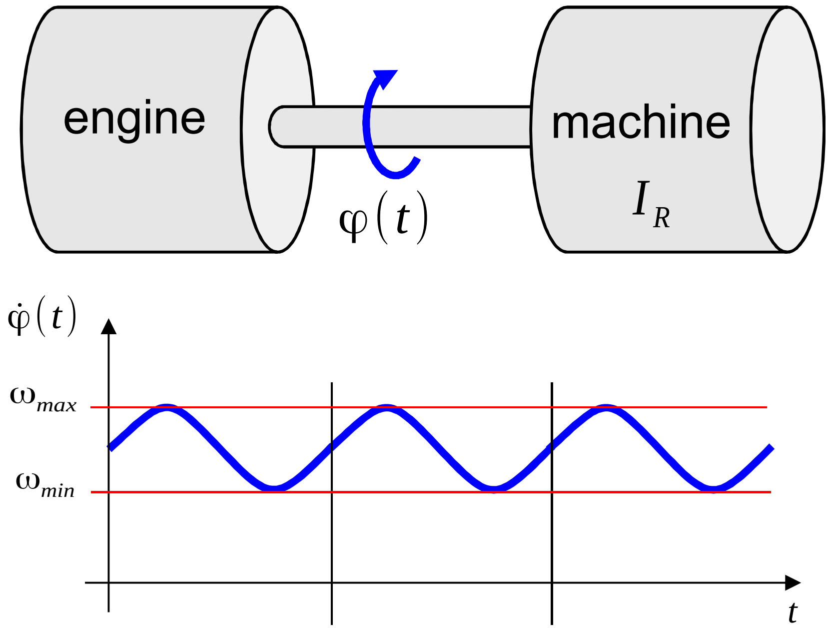
$$W = \delta_1 I_R \omega_{mean}^2$$

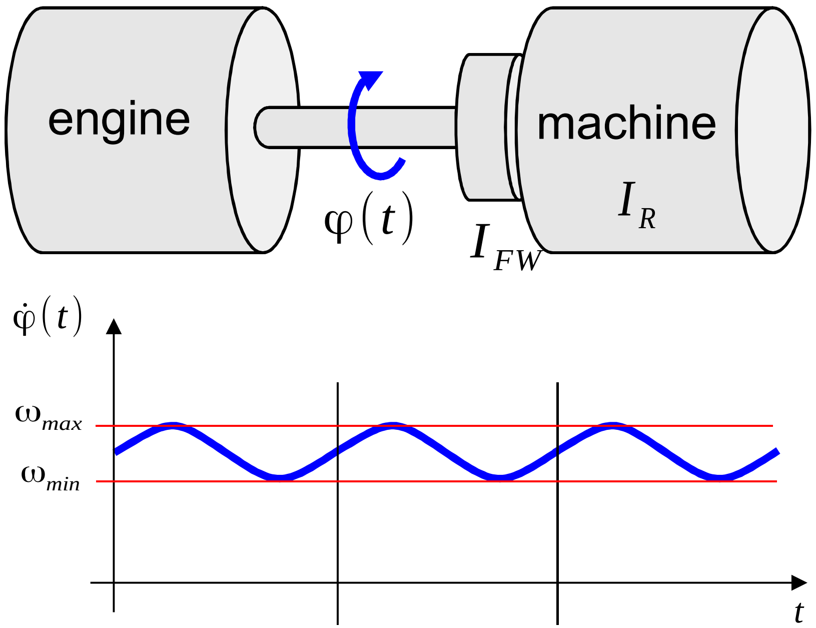
assume

$$I_R \approx const.$$

Flywheel

Steady-state motion





$$W = \delta_1 I_R \omega_{mean}^2$$

assume

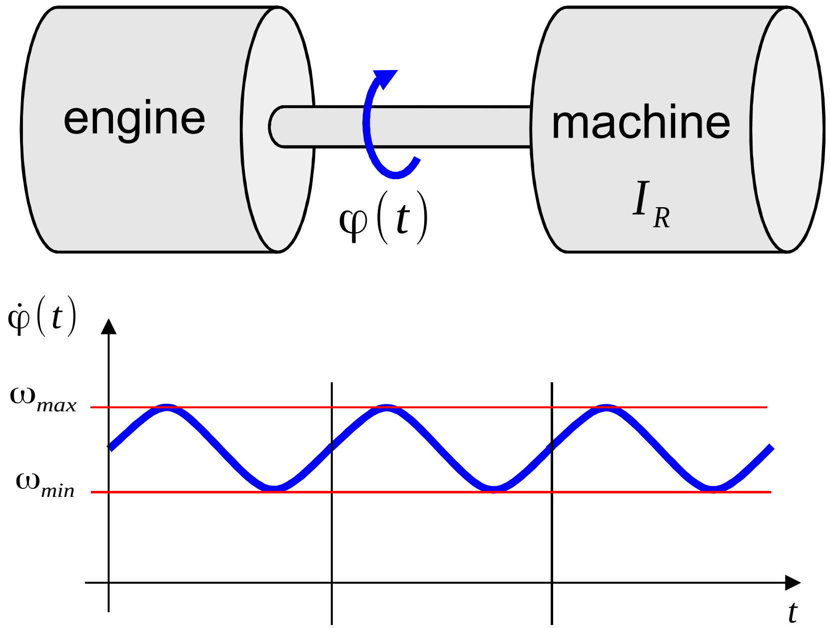
$I_R \approx const.$

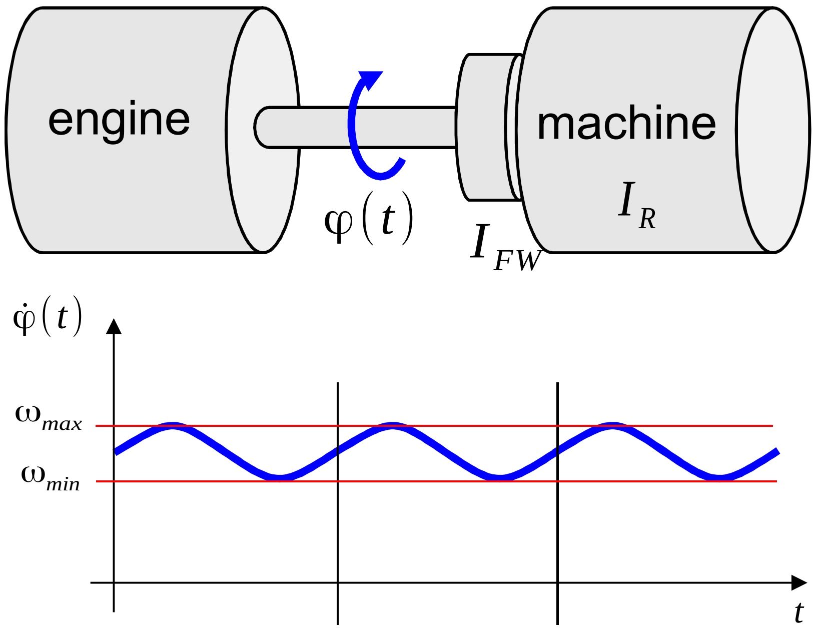
$$W = \delta_2 (I_R + I_{FW}) \omega_{mean}^2$$

(if velocity of a flywheel same as analyzed velocity)

Flywheel

Steady-state motion





$$W = \delta_1 I_R \omega_{mean}^2$$

assume

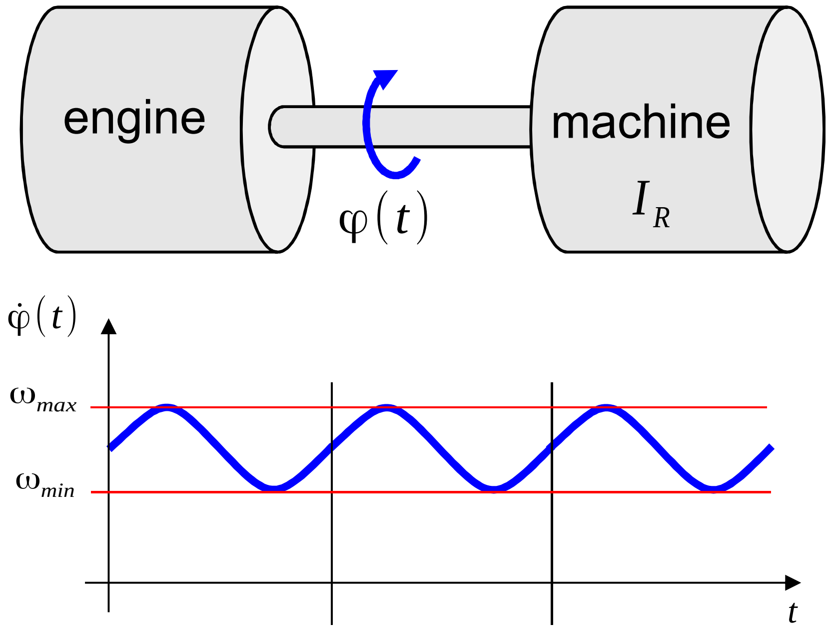
$$I_R \approx const.$$

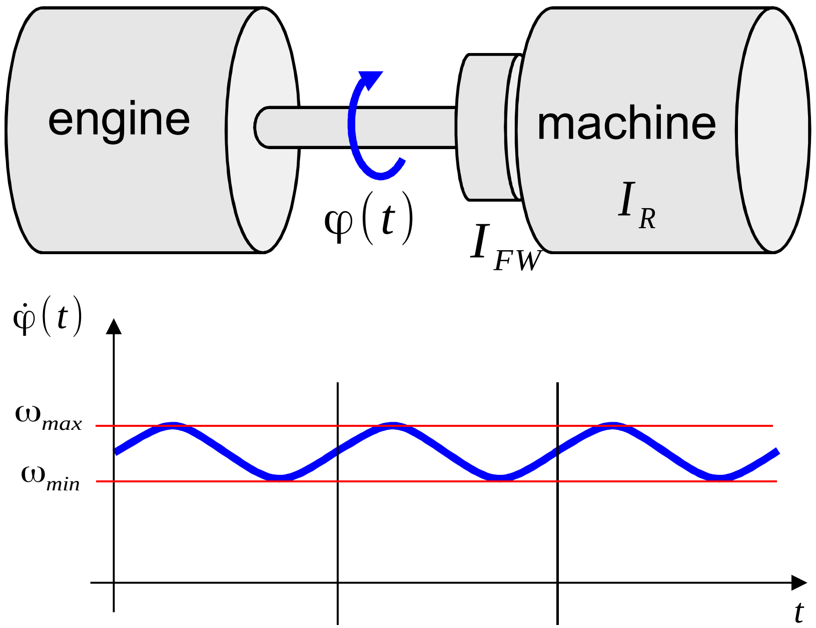
$$W = \delta_2 (I_R + I_{FW}) \omega_{mean}^2$$

$$\delta_1 I_R \omega_{mean}^2 = \delta_2 (I_R + I_{FW}) \omega_{mean}^2$$

Flywheel

Steady-state motion





$$W=\delta_1 I_R \omega_{mean}^2$$

assume $I_R \approx const.$

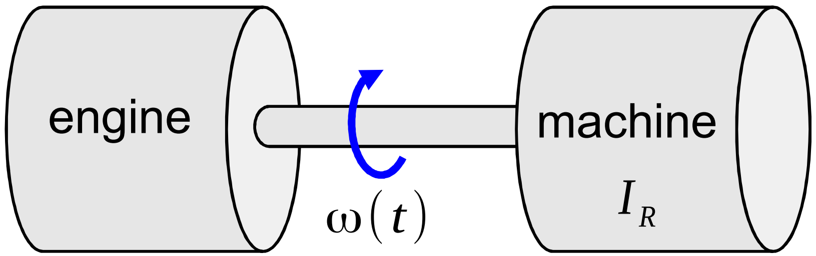
$$W=\delta_2 (I_R+I_{FW}) \omega_{mean}^2$$

$$\delta_1 I_R \omega_{mean}^2=\delta_2 (I_R+I_{FW}) \omega_{mean}^2$$

$$\boxed{I_{FW}=\left(\frac{\delta_1}{\delta_2}-1\right) I_R}$$

Non-uniformity of machine motion

Example 1



Given:

$\omega_{max}(t) = 1000\,\text{rpm}$

$\omega_{min}(t) = 950\,\text{rpm}$

$I_R = 10\,\text{kgm}^2$

Design flywheel to obtain 10rpm velocity variations.

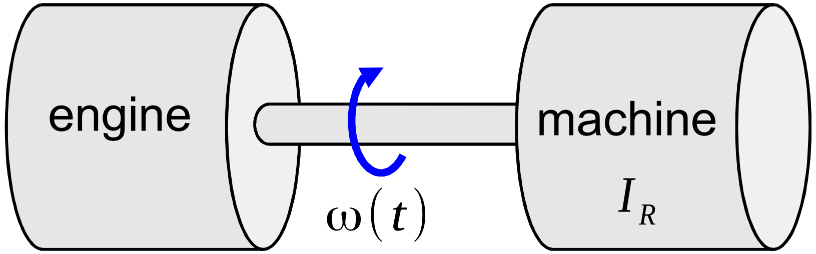
WITHOUT F.W.: $\delta = \dfrac{\omega_{max} - \omega_{min}}{\omega_{mean}} = \dfrac{50\,\text{rpm}}{975\,\text{rpm}}$

WITH FW: $\delta_{FW} = \dfrac{10\,\text{rpm}}{975\,\text{rpm}}$

$$I_{FW} = \left(\frac{\delta}{\delta_{FW}} - 1\right) I_R = 4 I_R = 40\,\text{kgm}^2$$

Non-uniformity of machine motion

Example 1



Given:

$\omega_{max}(t) = 1000\,\text{rpm}$

$\omega_{min}(t) = 950\,\text{rpm}$

$I_R = 10\,\text{kgm}^2$

Design flywheel to obtain 10rpm velocity variations.

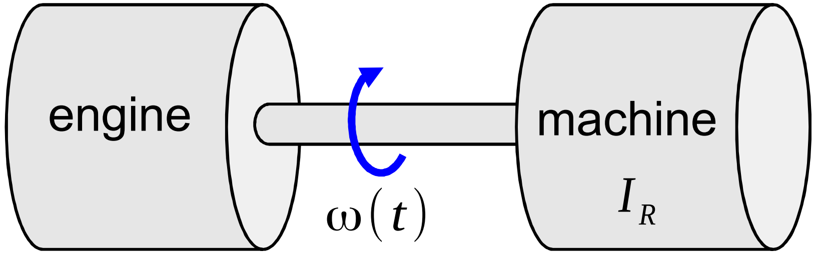
Non-uniformity without a flywheel:

Desired non-uniformity with a flywheel:

Mass moment of inertia of a flywheel:

Non-uniformity of machine motion

Example 1



Given:

$$\omega_{max}(t)=1000\,\mathrm{rpm}$$
$$\omega_{min}(t)=950\,\mathrm{rpm}$$
$$I_R=10\,\mathrm{kgm}^2$$

Design flywheel to obtain 10rpm velocity variations.

Non-uniformity without a flywheel:

$$\delta=\frac{\omega_{max}-\omega_{min}}{\omega_{mean}}=\frac{50}{975}=0{,}0513$$

Desired non-uniformity with a flywheel:

$$\delta_{FW}=\frac{10}{975}=0{,}010256$$

Mass moment of inertia of a flywheel:

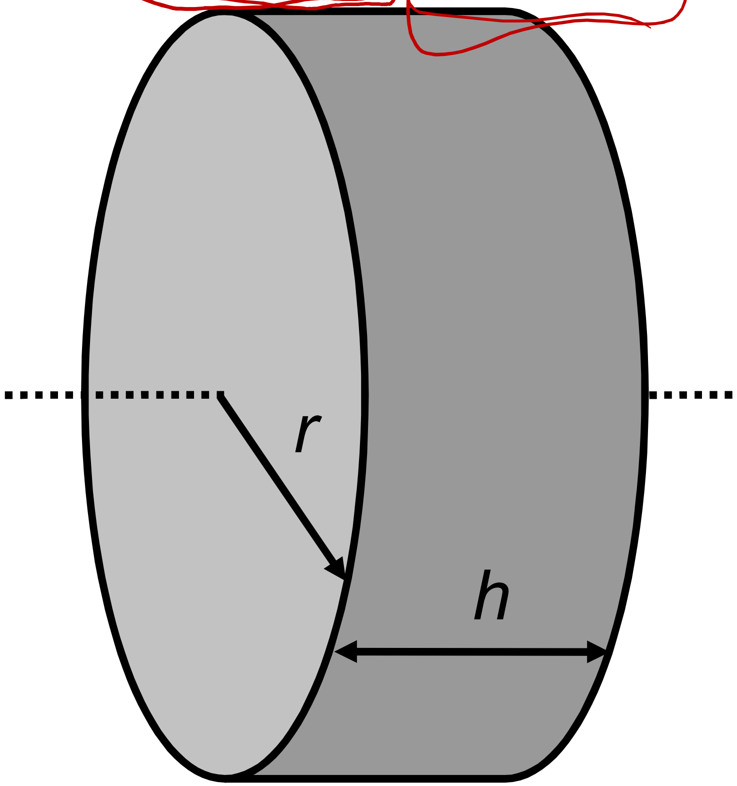
$$I_{FW}=\left(\frac{\delta}{\delta_{FW}}-1\right)I_R=40\,kg\,m^2$$

Non-uniformity of machine motion

Example 1

Solid cylinder

$$I_{FW} = \frac{1}{2} m r^2 = \frac{1}{2} \rho \pi h r^4$$



ρ – density

Non-uniformity of machine motion

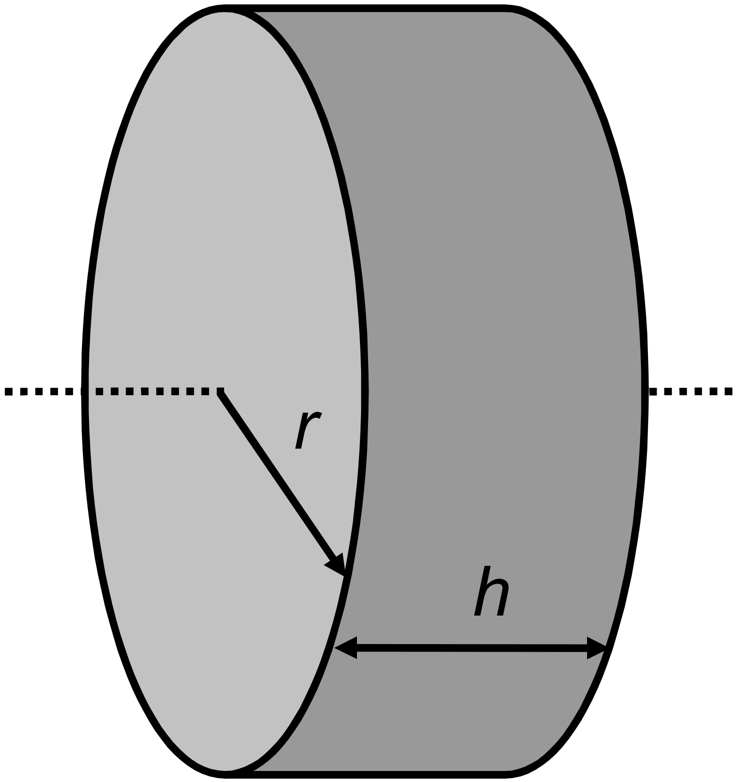
Example 1

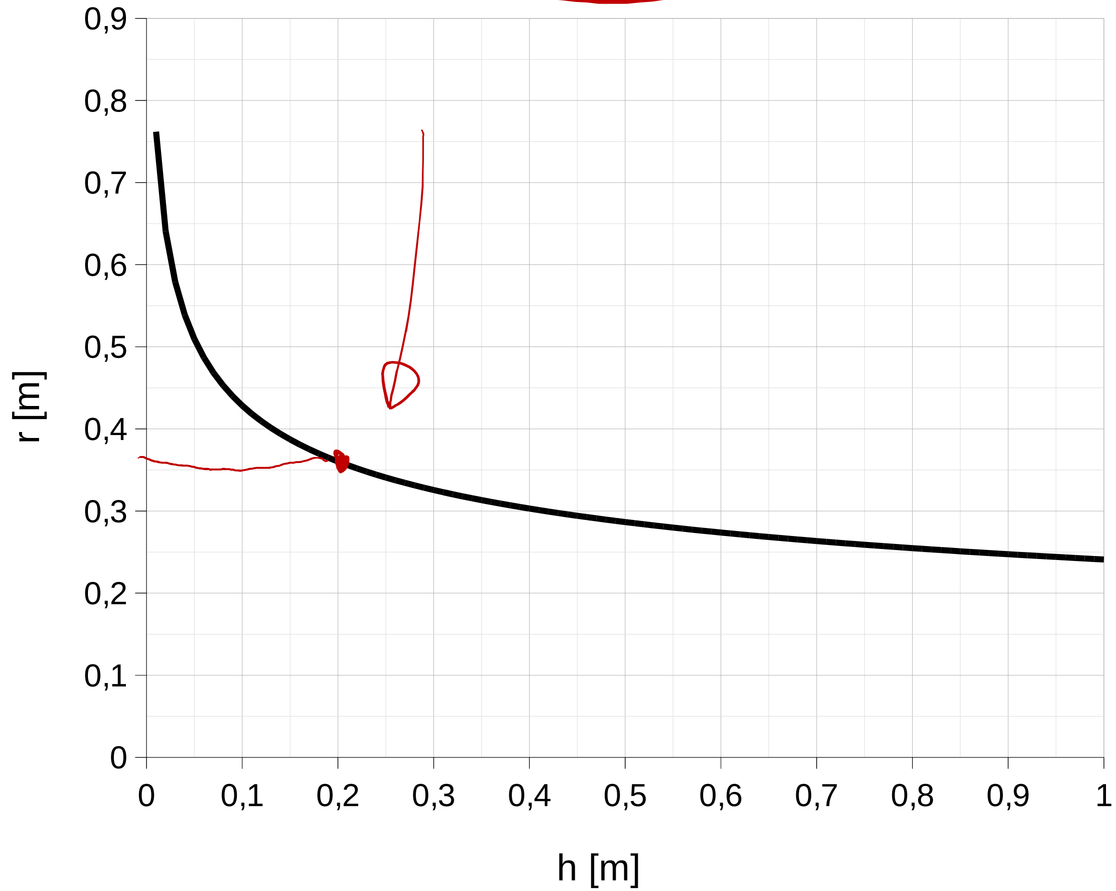
$$I_{FW} = 40\,\mathrm{kgm}^2$$

$$\rho_{steel} = 7800\ kg/m^3$$

Solid cylinder

$$I_{FW} = \frac{1}{2} m r^2 = \frac{1}{2} \rho \pi h r^4$$





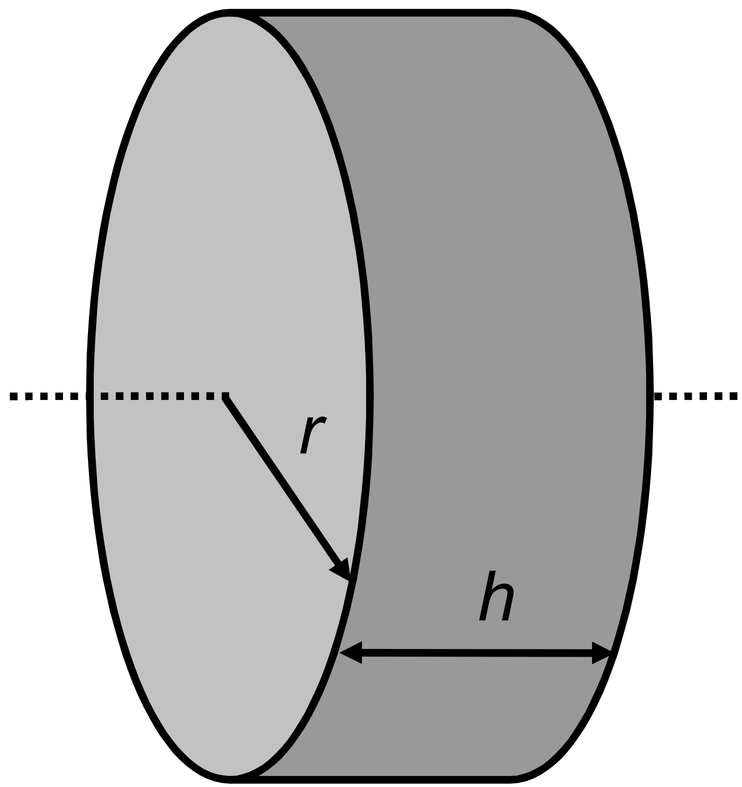
Non-uniformity of machine motion

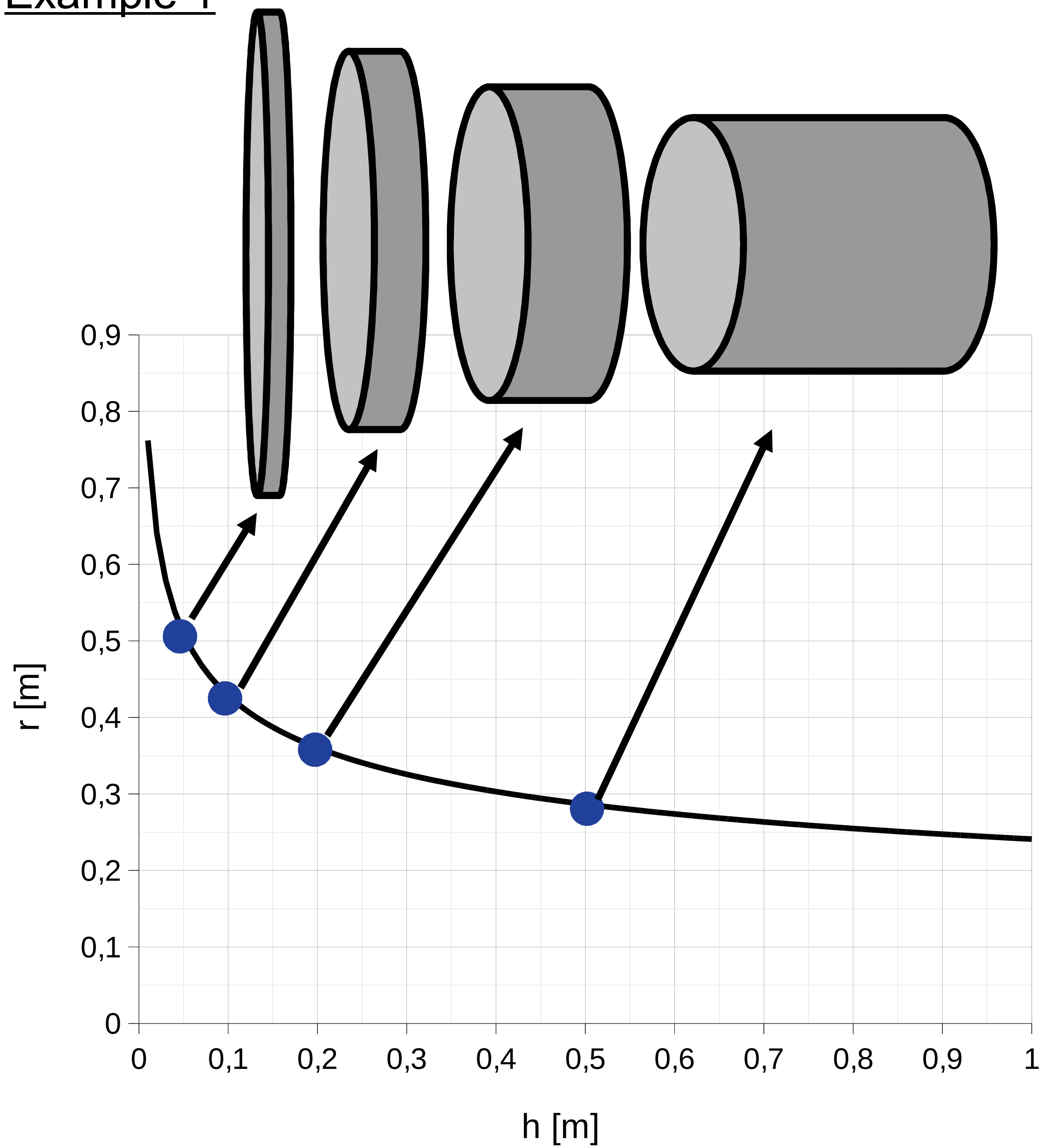
Example 1

Solid cylinder

$$I_{FW}=\frac{1}{2}m r^2=\frac{1}{2}\rho\pi h r^4=40\,\text{kgm}^2$$

$$\rho_{steel}=7800\,kg/m^3$$





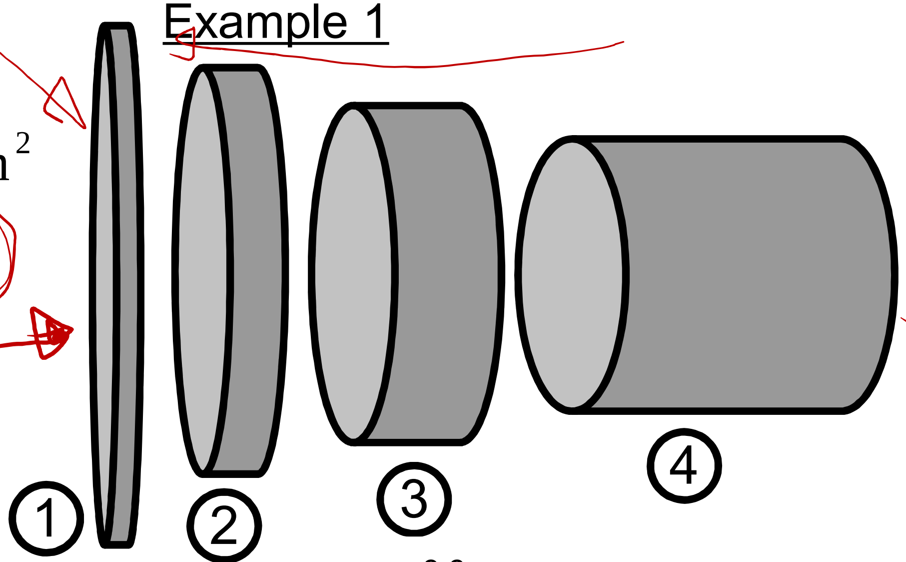
Non-uniformity of machine motion

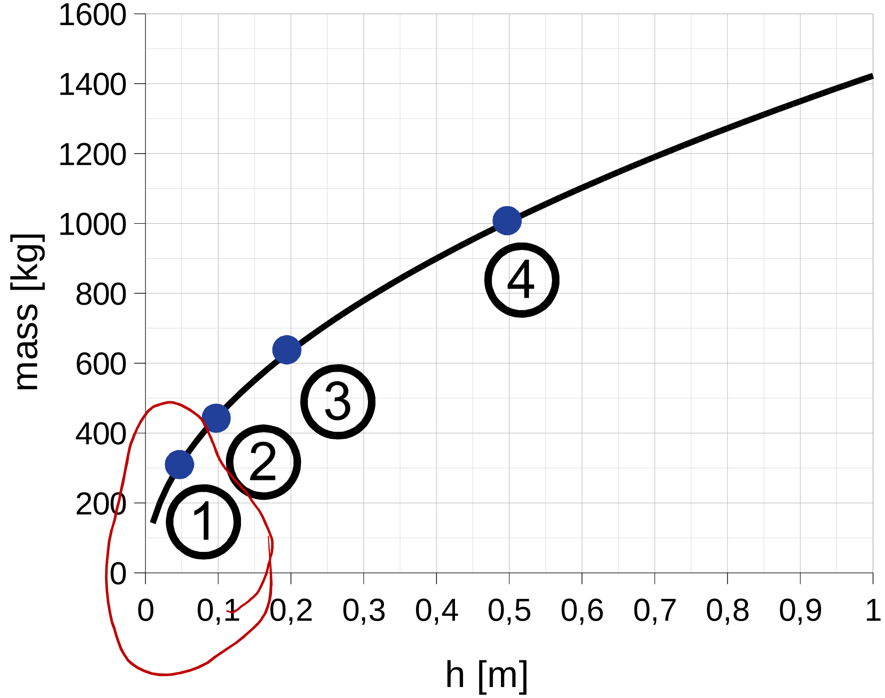
Example 1

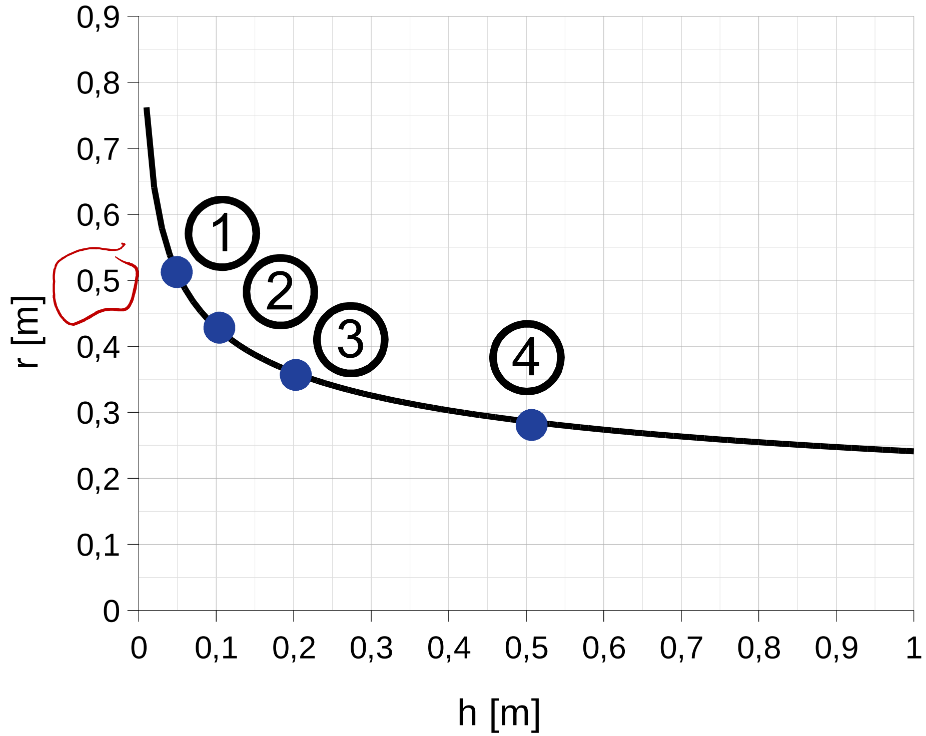
Solid cylinder

$$I_{FW} = \frac{1}{2} m r^2 = \frac{1}{2} \rho \pi h r^4 = 40\,\mathrm{kgm}^2$$

$$\rho_{steel} = 7800\, kg/m^3$$





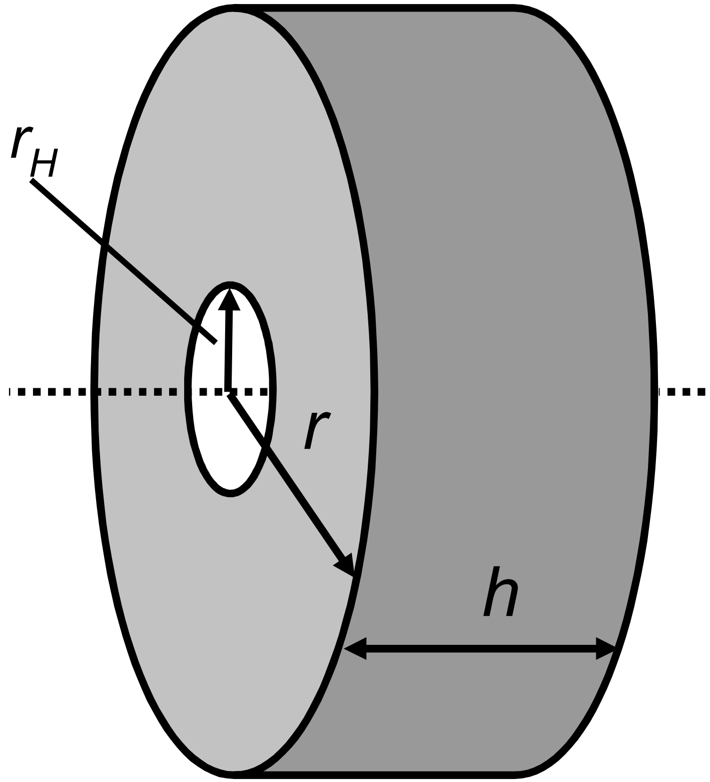


Non-uniformity of machine motion

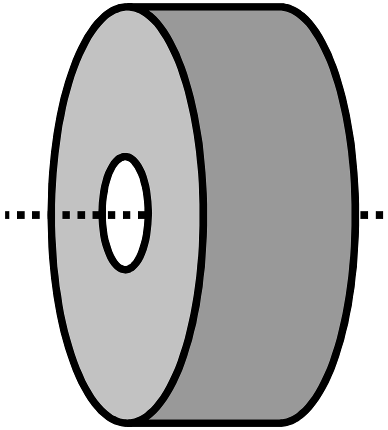
Example 2

Cylinder with a hole

$$I_{FW}=\frac{1}{2}\rho\pi h r^4-\frac{1}{2}\rho\pi h r_H^4$$

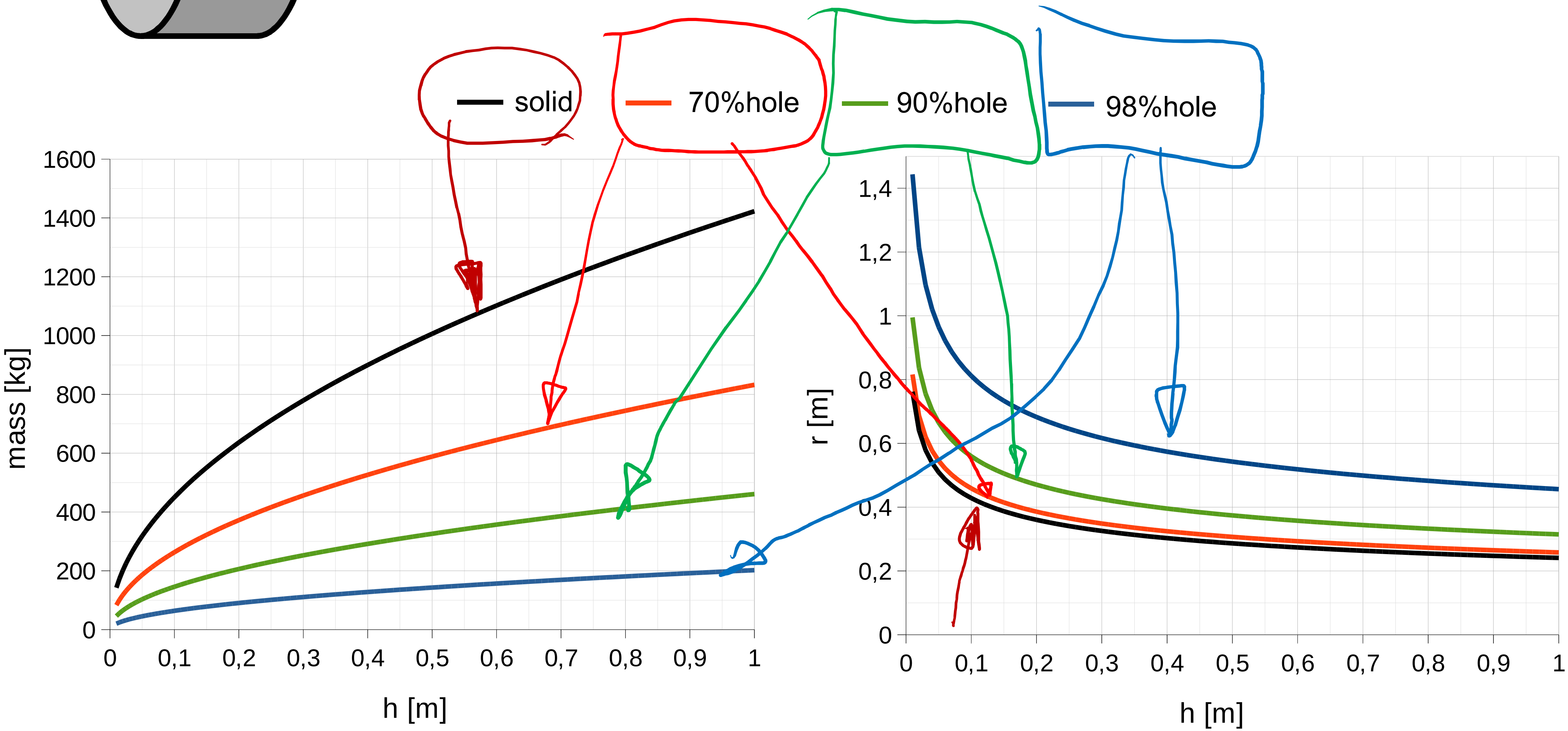


Non-uniformity of machine motion



Example 2 – cylinder with a hole

$$I_{FW}=\frac{1}{2}\rho\pi h r^4 - \frac{1}{2}\rho\pi h r_H^4 = 40\ \mathrm{kgm^2}$$



Non-uniformity of machine motion

Example 2 – cylinder with a hole

$$I_{FW}=\frac{1}{2}\rho\pi h r^4-\frac{1}{2}\rho\pi h r_H^4=40\,\mathrm{kgm}^2$$

	Solid cylinder	Cylinder 90% hole	Solid cylinder	Cylinder 98% hole
h=	10 cm	10 cm	5 cm	5 cm
r=	43 cm	56 cm	50 cm	96 cm
r_H=	--	50.4 cm	--	94 cm
m=	442.8 kg	143.5 kg	313 kg	44.5 kg

r: +30% (Solid cylinder → Cylinder 90% hole); +92% (Solid cylinder → Cylinder 98% hole)

m: -68% (Solid cylinder → Cylinder 90% hole); -86% (Solid cylinder → Cylinder 98% hole)

Non-uniformity of machine motion

To minimize flywheel's mass moment of inertia:

- you should mount a flywheel on a shaft that rotates with the highest angular velocity

- You can add extra transmission to increase angular velocity of a flywheel

Automatic control

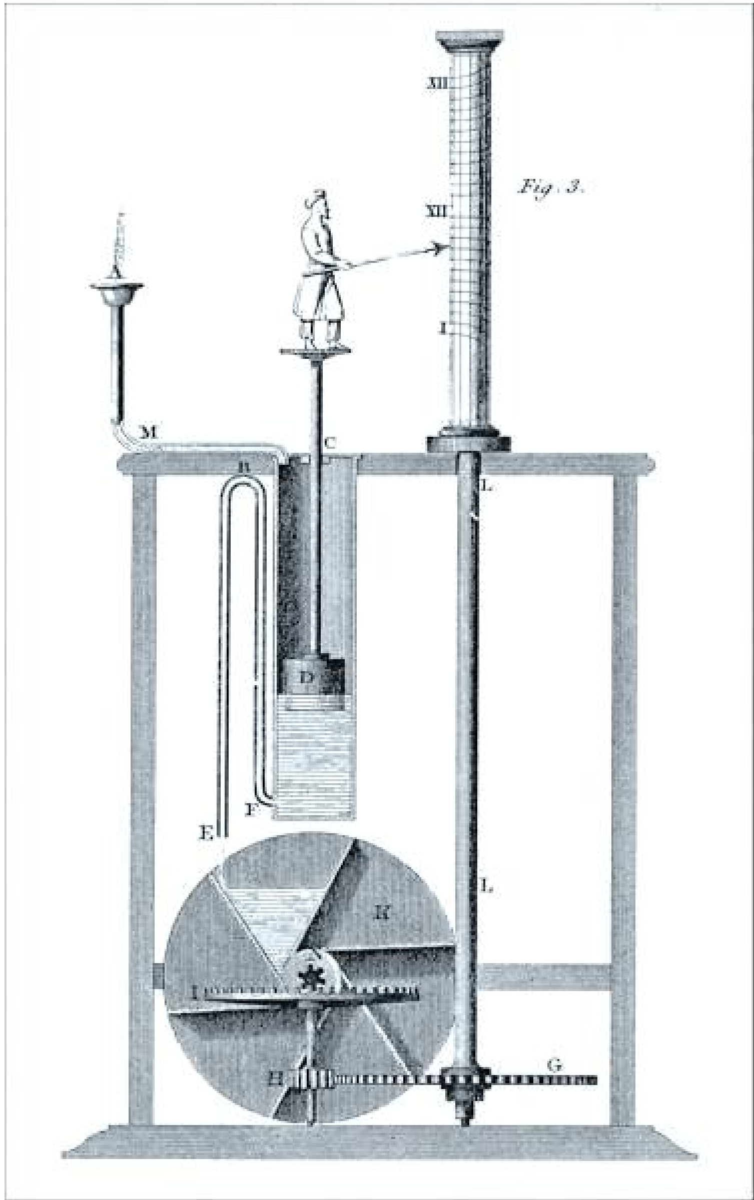
Automatic control

"Automatic control in engineering and technology is a wide generic term covering the application of mechanisms to the operation and regulation of processes without continuous direct human intervention." - *wikipedia*

Control theory – branch of mathematics and cybernetics that deals with analysis and mathematical modeling of objects and processes threated as dynamical systems with **feedback**.

Automatic control history

Ancient Greece, Arabs



water clocks,

automatic wine metering,

door opening in temples

Ctesibius's clepsydra (3rd century BC).

Source-wikipedia: Abraham Rees (1819) "Clepsydra" in Cyclopædia: or, a New Universal Dictionary of Arts and Sciences The image is the JPEG reproduction published 2007-02-01 by the Horological Foundation.

Automatic control history

XVII-XVIII

Temperature regulators for fireplaces and boilers
pressure regulators for pressure cookers

XVIII-XIX

float regulators for water distribution and steam engines
velocity and force regulators for grain mills
Watt's regulator for steam engines (centrifugal governor)

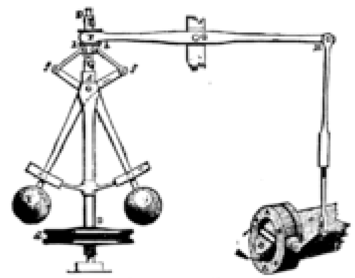


FIG. 4.—Governor and Throttle-Valve.

XIX-XX

Laplace transform and Z-transform
Lyapunov stability theory
Routh stability criterion
Hurwitz stability criterion
Nyquist stability criterion and frequency domain analysis
Bode & Nichols frequency domain analysis
Evans root locus analysis

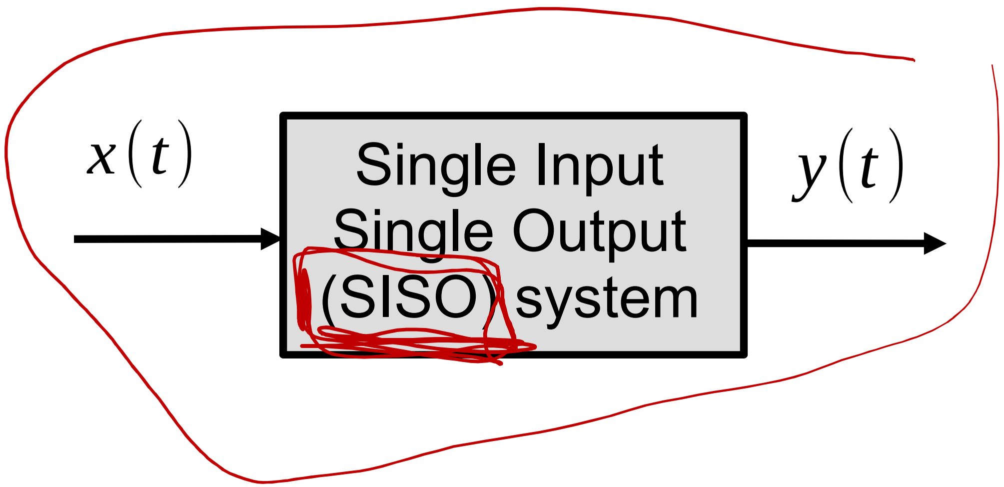
Automatic control

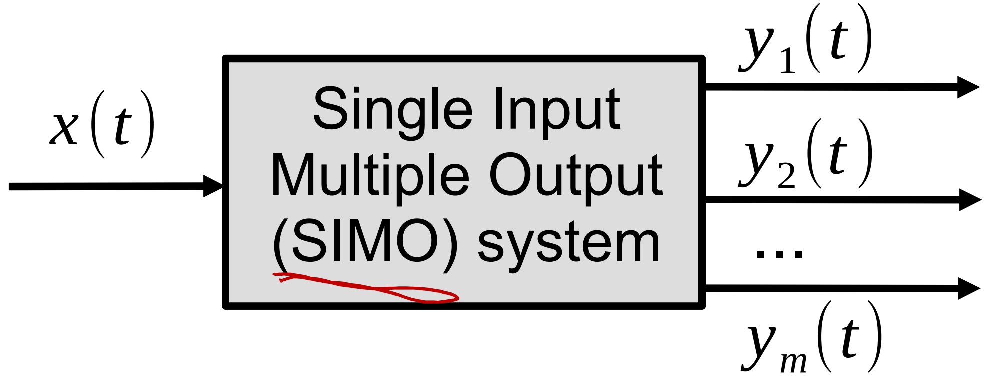
Classical control theory	
single input, single output (SISO)	
usually linear systems	
time independent systems	
description by a transfer functions	
time and frequency domain analysis	
system response is the most important	

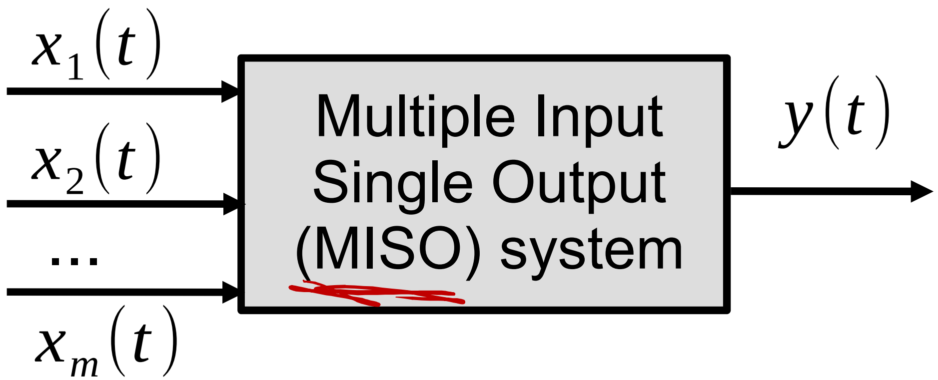
Automatic control

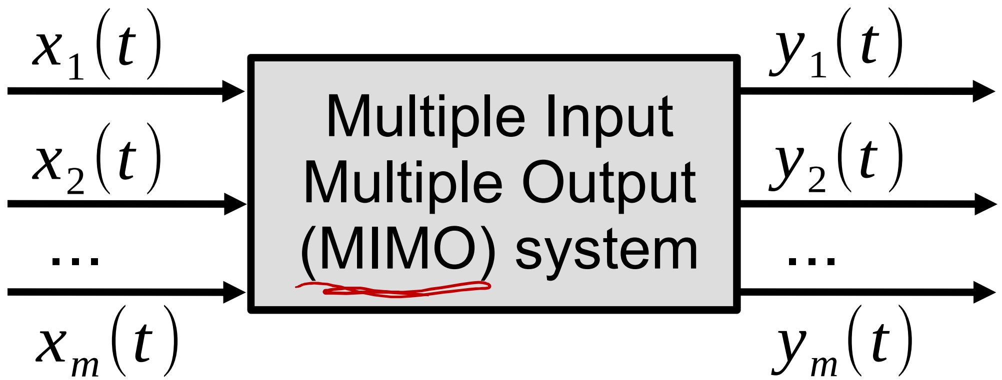
Classical control theory	modern control theory (1950-now)
single input, single output (SISO)	multiple input, multiple output (MIMO)
usually linear systems	often nonlinear systems
time independent systems	time dependent systems
description by a transfer functions	description by a state equations
time and frequency domain analysis	time domain analysis
system response is the most important	system state is the most important

Number of inputs and outputs









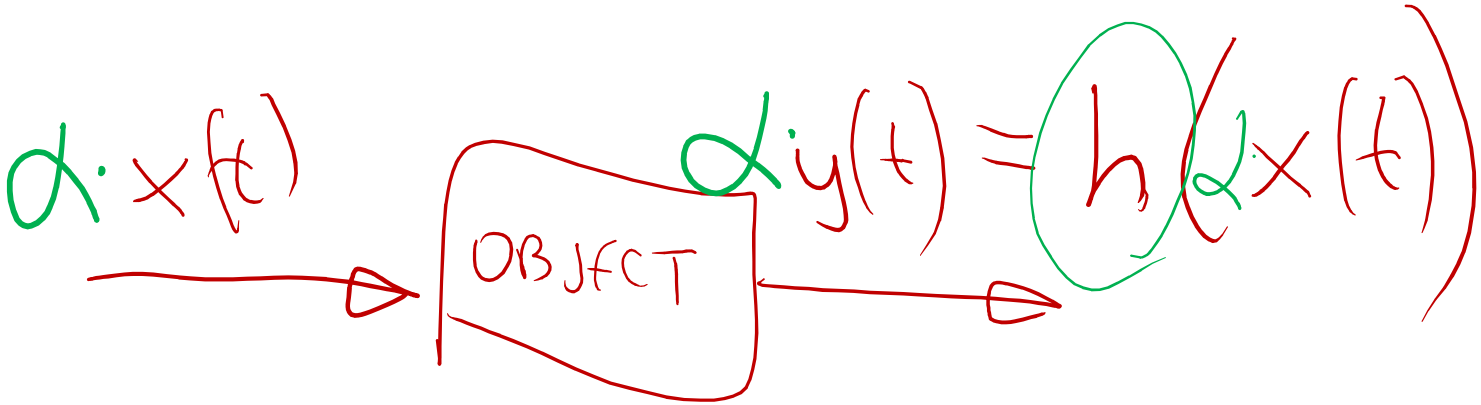
Linear time-invariant (LTI) system

Linear system

$x(t)$ - input, $y(t) = h(x(t))$ - output

$h(\alpha x(t)) = \alpha h(x(t)) = \alpha y(t)$ scaling

$h(x_1(t) + x_2(t)) = h(x_1(t)) + h(x_2(t))$ superposition



Linear time-invariant (LTI) system

Time-invariant system

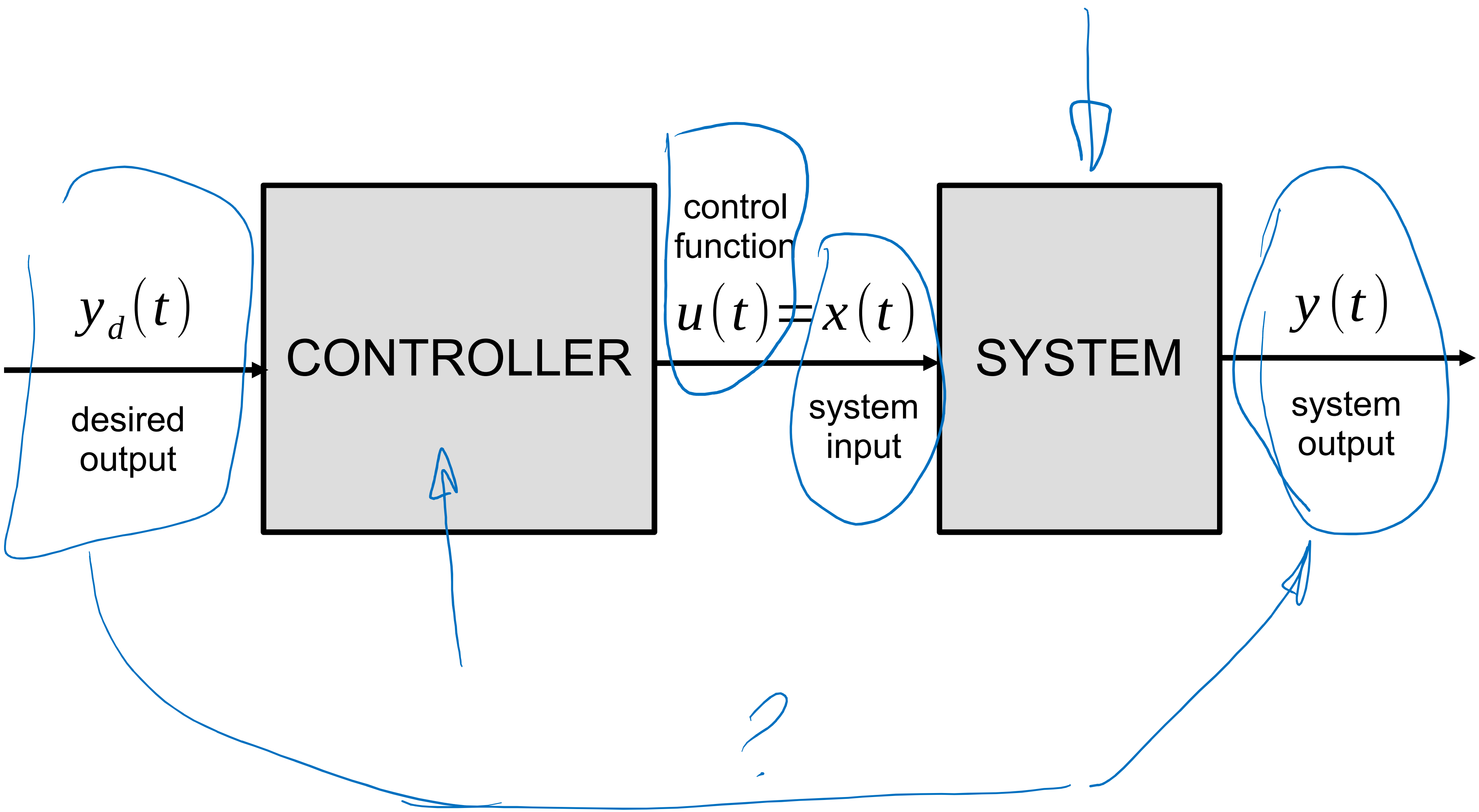
output does not depend explicitly on time

$$\text{if } y(t)=h(x(t)) \text{ then } y(t-\tau)=h(x(t-\tau))$$

Time-varying system

$$\text{if } y(t)=h(x(t)) \text{ then } y(t-\tau)\neq h(x(t-\tau))$$

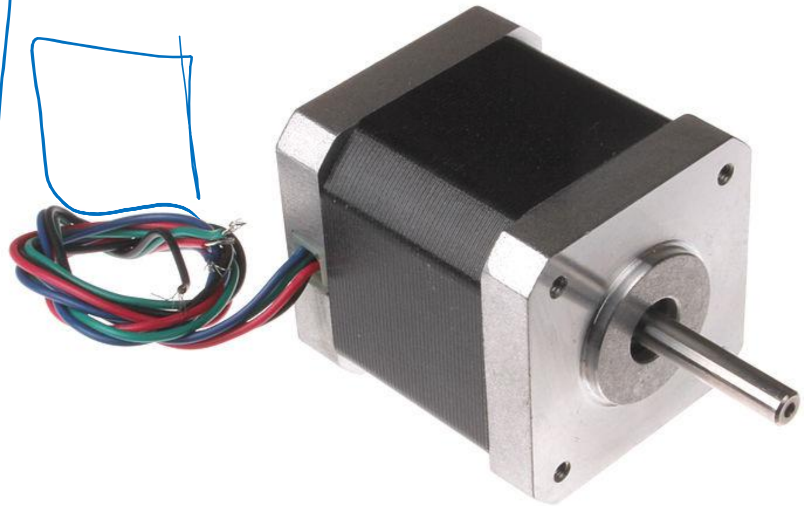
Open loop control



Open loop control

Example usages

stepper motor

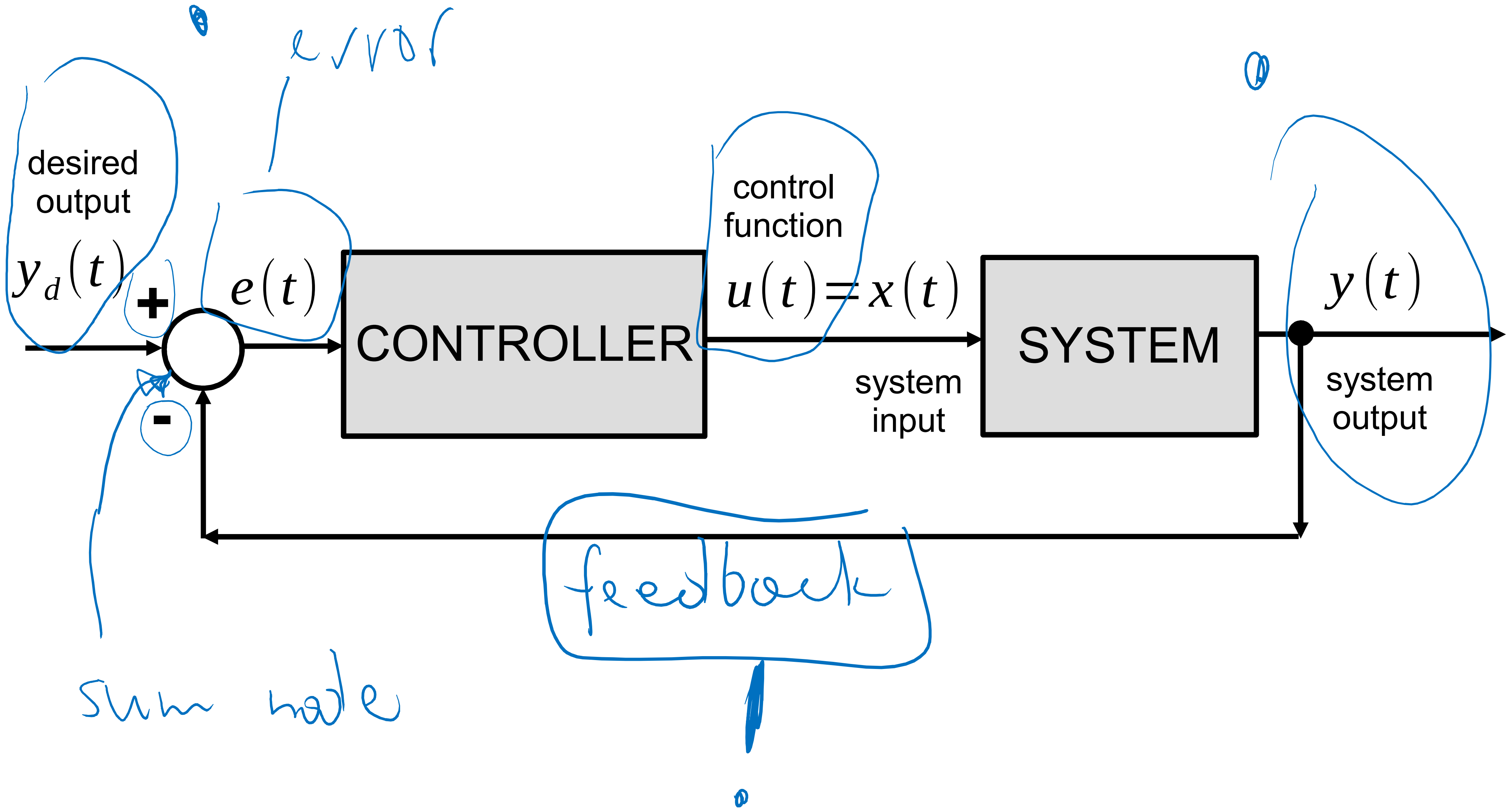


source: wikimedia.org; author: oomlout

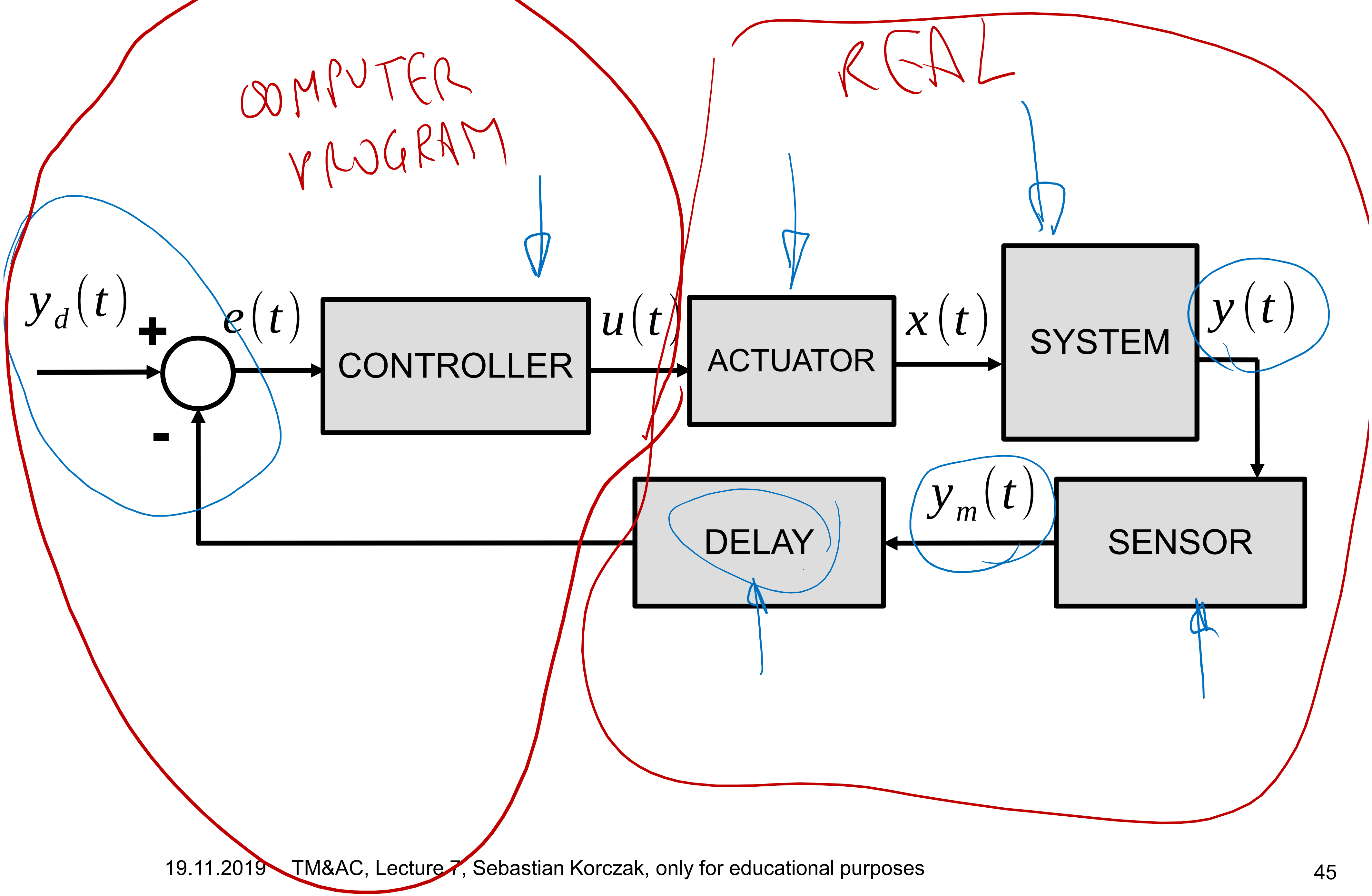
two wheeled platform
(flat surface, no slip)

source: http://www.robotliving.com

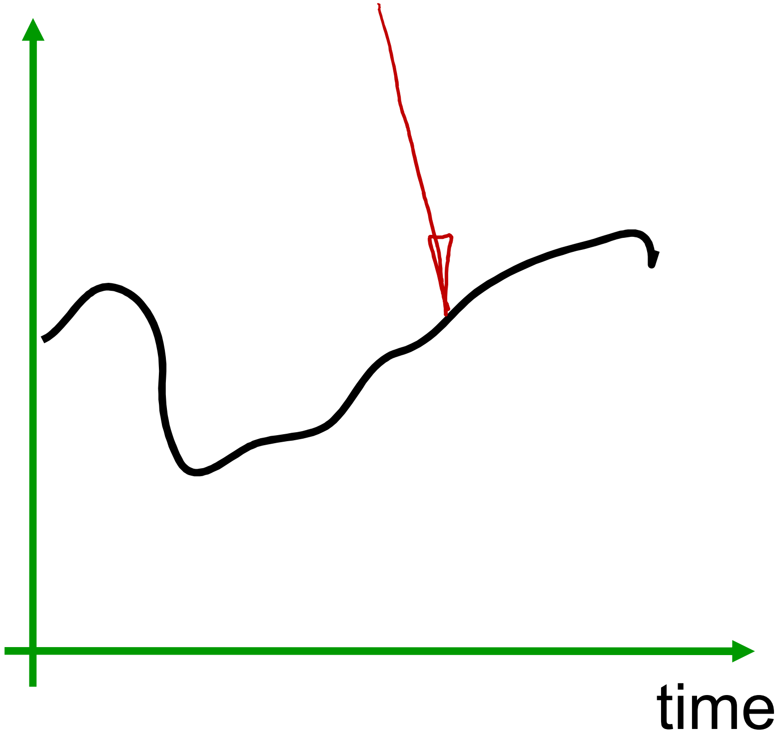
Closed loop control

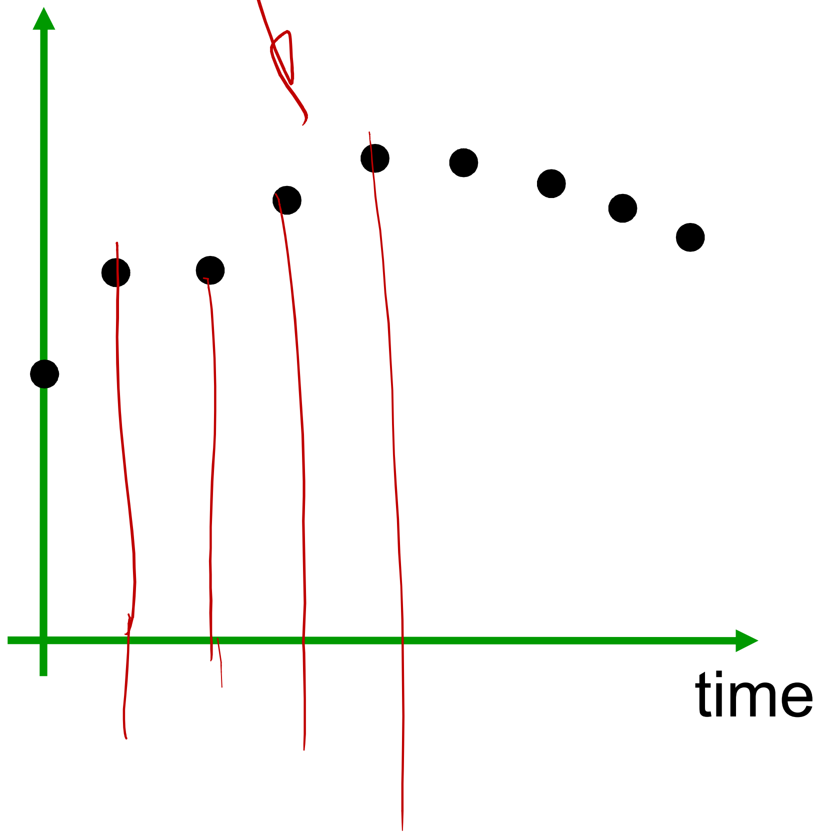


Closed loop control



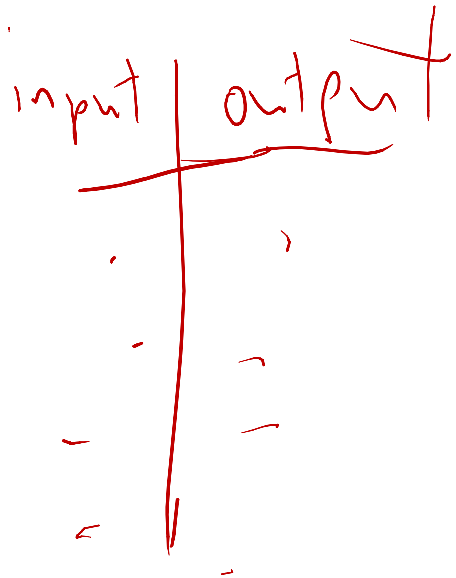
Continuous/discrete signals





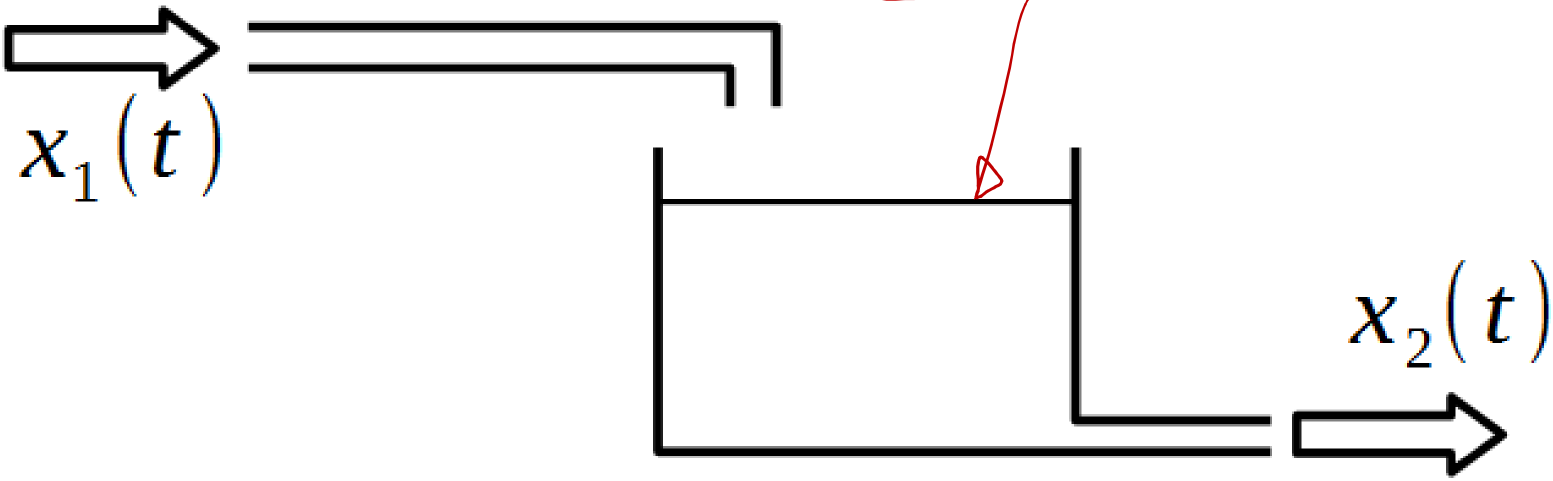
Mathematical modeling of systems

- ordinary differential equations (ODEs)
- partial differential equations (PDEs)
- table representation
- recurrence equations
- stochastic representation
- neural networks
- logical
- arithmetic
- integral equations
- combination of above
- …



Mathematical modeling of systems

Example 1 – cylindrical tank



$x_1(t)[m^3/s]$ - inflow of a liquid

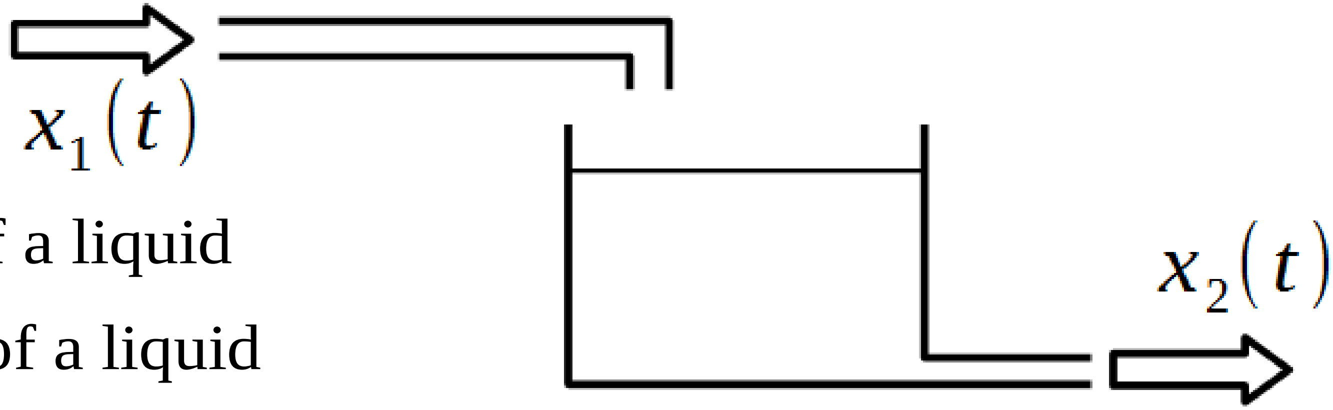
$x_2(t)[m^3/s]$ - outflow of a liquid

$v(t)[m^3]$ - volume of a liquid in a tank

Question: find out a relation between inflow, outflow and volume.

Mathematical modeling of systems

Example 1 – cylindrical tank



$x_1(t)[m^3/s]$ - inflow of a liquid

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$v(t)[m^3]$ - volume of a liquid in a tank

Question: find out a relation between inflow, outflow and volume.

$$v(t_1) \rightarrow v(t_1+\Delta t) = v(t_1) + (x_1(t_1) - x_2(t_1))\Delta t$$

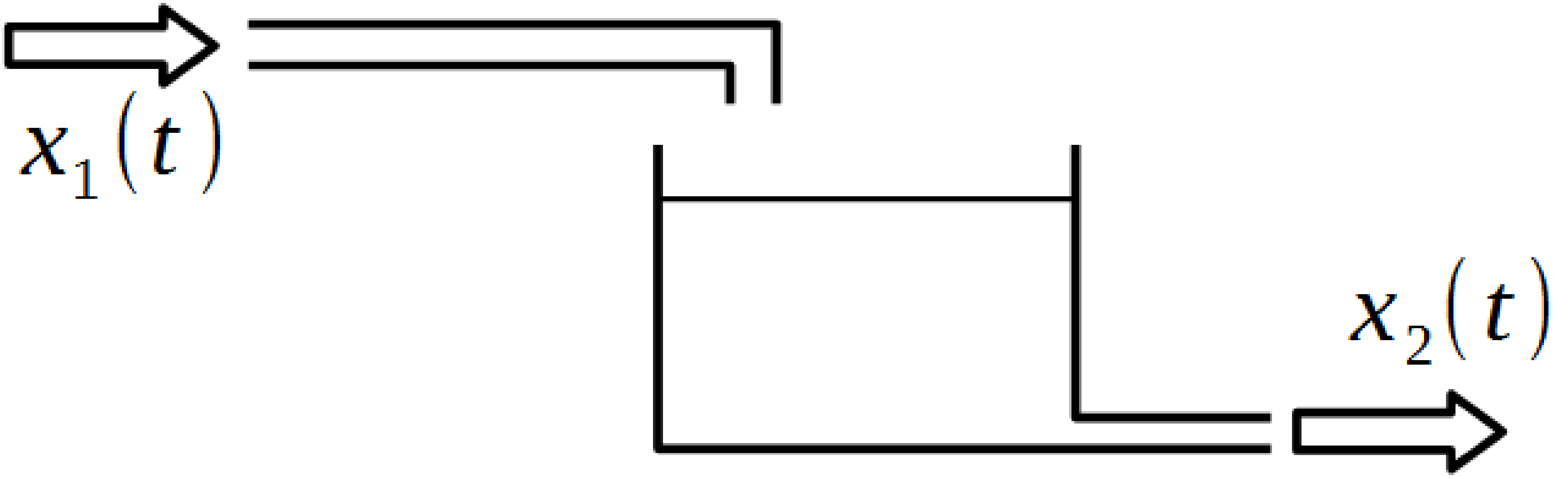
$$\frac{v(t_1+\Delta t) - v(t_1)}{\Delta t} = x_1(t_1) - x_2(t_1)$$

$\Delta t \rightarrow 0$

$$\frac{dv(t)}{dt} = x_1(t) - x_2(t)$$

Mathematical modeling of systems

Example 1



$x_1(t)[m^3/s]$ - inflow of a liquid

$x_2(t)[m^3/s]$ - outflow of a liquid

$v(t)[m^3]$ - volume of a liquid in a tank

Question: find out a relation between inflow, outflow and volume.

Answer:

$$t_2 = t_1 + \Delta$$

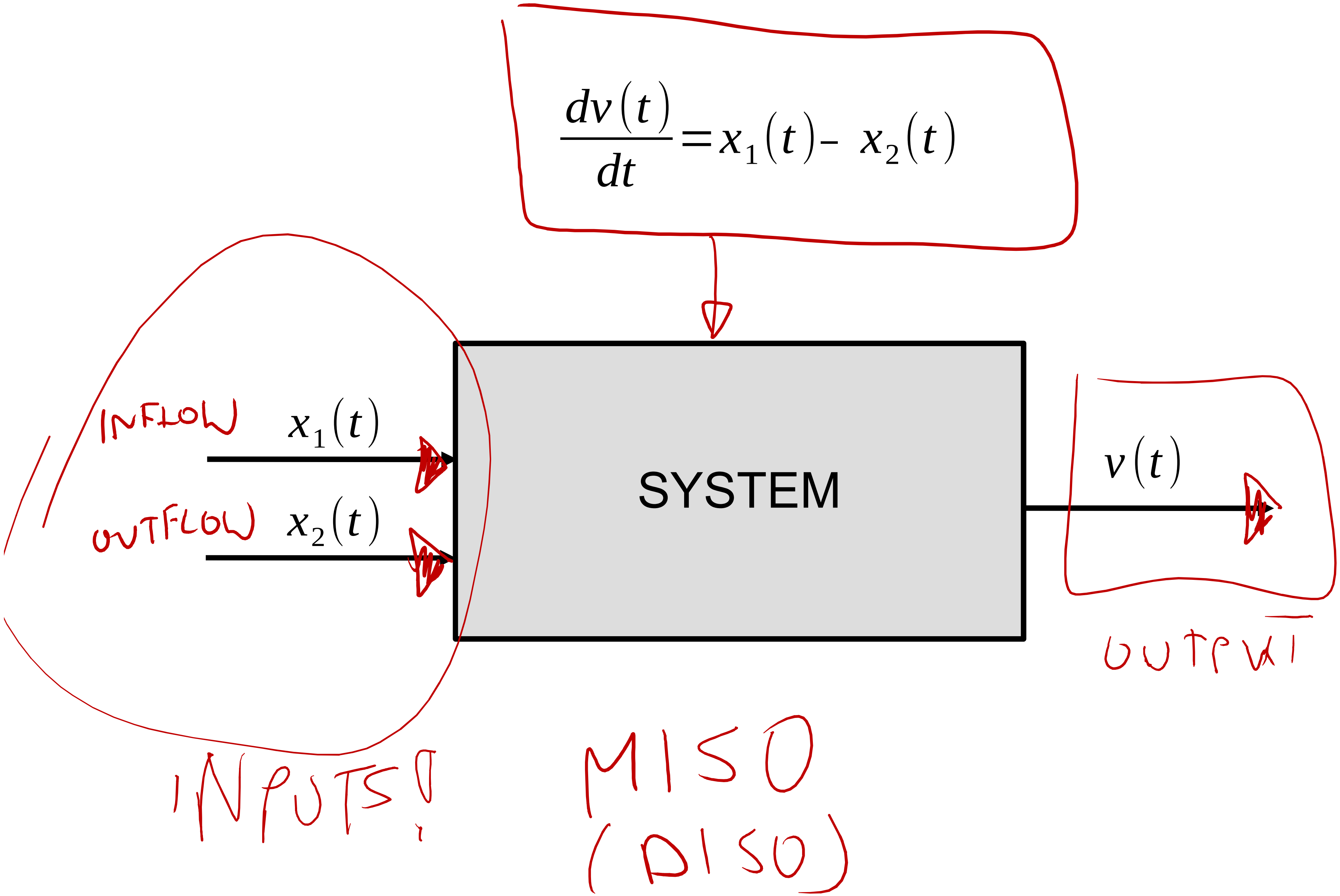
$$v(t_2) \approx v(t_1) + \Delta(x_1(t_2) - x_2(t_2))$$

$$\frac{v(t_2) - v(t_1)}{\Delta} \approx x_1(t_2) - x_2(t_2)$$

$$\frac{dv(t)}{dt} = x_1(t) - x_2(t)$$

Mathematical modeling of systems

Example 1



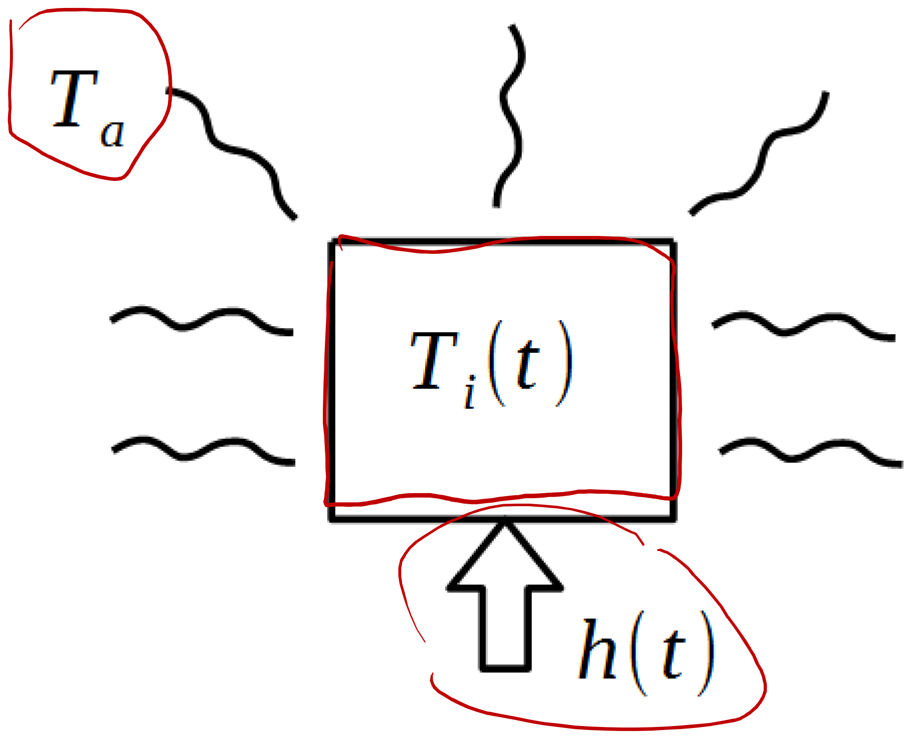
Mathematical modeling of systems

Example 2

$h(t)[W]$ - heater power

$T_a(t)[K]$ - ambient temperature

$T_i(t)[K]$ - object temperature



Question: how to obtain a relation between heater power (input) and object temperature (output)? Assume energy loss only by convection.

Mathematical modeling of systems

Example 2

Answer:

rate of change of heat = heat gain – heat loss

Mathematical modeling of systems

Example 2

Answer:

rate of change of heat = heat gain – heat loss

$$\frac{dQ(t)}{dt} = Q_H - Q_L$$

Mathematical modeling of systems

Example 2

Answer:

rate of change of heat = heat gain – heat loss

$$\frac{dQ(t)}{dt} = Q_H - Q_L$$

$Q[J] = c_p m T_i$ - heat energy inside the object

$c_p[J/kg\,K]$ - specific heat coefficient, $m[kg]$ - mass of the object

Mathematical modeling of systems

Example 2

Answer:

rate of change of heat = heat gain – heat loss

$$\frac{dQ(t)}{dt} = Q_H - Q_L$$

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$Q_H[W] = h(t)$ - increase of heat energy caused by the heater

Mathematical modeling of systems

Example 2

Answer:

rate of change of heat = heat gain – heat loss

$$\frac{dQ(t)}{dt} = Q_H - Q_L$$

$$c_p m \frac{dT_i(t)}{dt} + \alpha T_i(t) = h(t) + \alpha T_a(t)$$

$Q[J] = c_p m T_i$ - heat energy inside the object

$c_p[J/kg\,K]$ - specific heat coefficient, $m[kg]$ - mass of the object

$Q_H[W] = h(t)$ - increase of heat energy caused by the heater

$Q_L[W] = \alpha(T_i - T_a)$ - heat energy loss caused by convection

$\alpha[W/K]$ - convection coefficient (assume constant)

$$c_p m \frac{dT_i(t)}{dt} = h(t) - \alpha(T_i(t) - T_a(t))$$

Mathematical modeling of systems

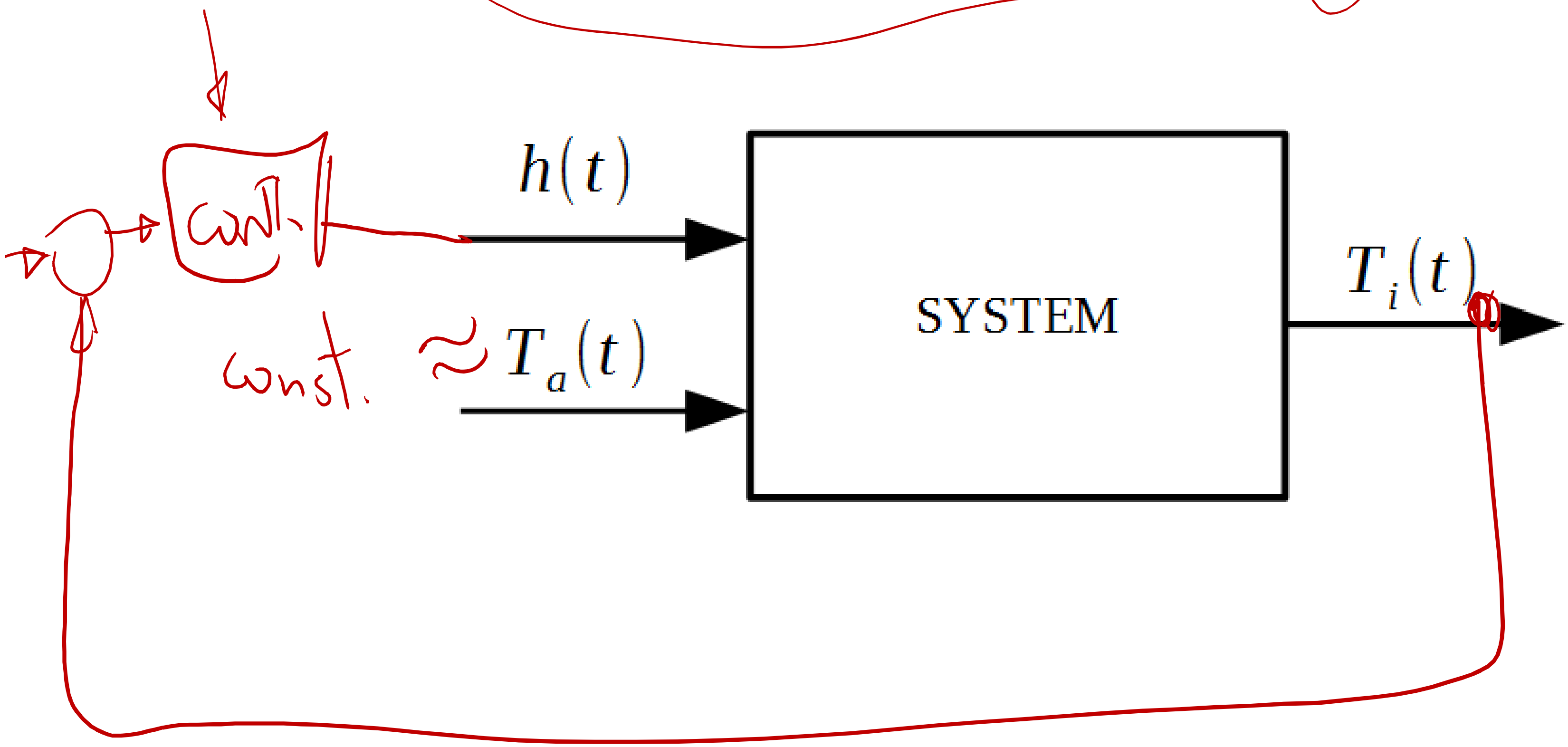
Example 2

$$c_p m \frac{dT_i(t)}{dt} = h(t) - \alpha\left(T_i(t) - T_a(t)\right)$$

Mathematical modeling of systems

Example 2

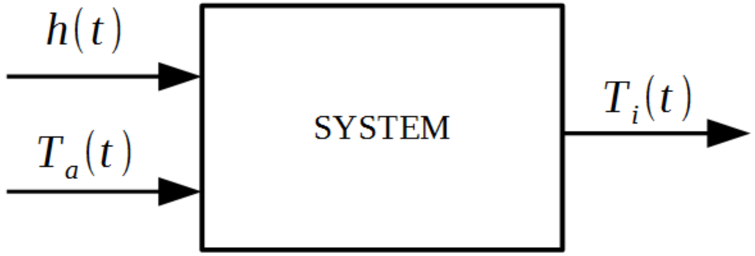
$$c_p m \frac{dT_i(t)}{dt} = h(t) - \alpha\left(T_i(t) - T_a(t)\right)$$



Mathematical modeling of systems

Example 2

$$c_p m \frac{dT_i(t)}{dt} = h(t) - \alpha\left(T_i(t) - T_a(t)\right)$$



Question: Can we convert this MISO model into SISO model?