Faculty of Automotive and Construction Machinery Engineering
WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control

Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

Theory of Machines and Automatic Control - project class

The Faculty of Automotive and Construction Machinery Engineering

Winter 2019/2020

2.1 EHVE – Wednesday, 8:15-10:00, room 3.14, S. Korczak, P. Wawrzyniak
2.2 EHVE – Wednesday, 10:15-12:00, room 3.11, S. Korczak, P. Wawrzyniak
2.1 MTR – Friday, ~~8:15-10:00~~, room 0.3, M. Parafiniak

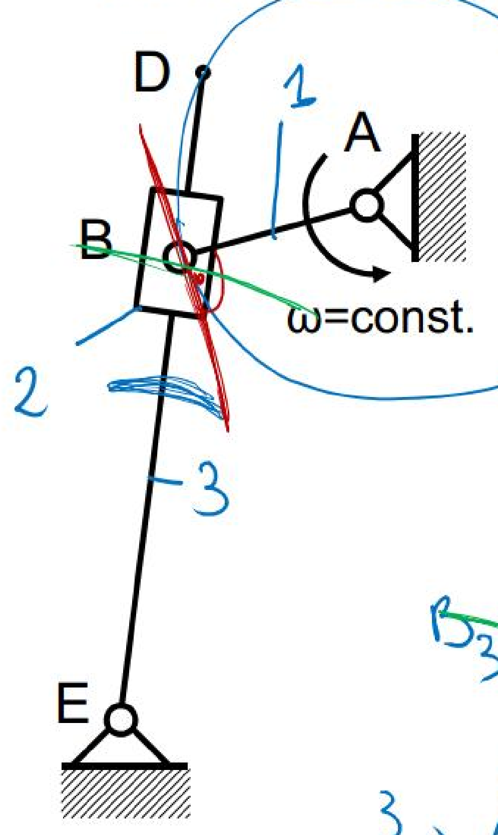
! 10-12

Group / date		topics	assessment
2.1 EHVE 2.2 EHVE	**2.1 MTR**		
23.10.2019	~~25.10.2019~~ 8.11.2019	Introduction. 1st project topics distribution. Graphical methods.	---
30.10.2019	---	---	---
6.11.2019	~~8.11.2019~~ 15.11.2019	1st project consultations. Analytical method.	---
13.11.2019	~~15.11.2019~~	---	---
20.11.2019	22.11.2019	---	---
27.11.2019	29.11.2019	1st project commitment. 2nd project topics distribution.	1st project evaluation.
4.12.2019	6.12.2019		
11.12.2019	13.12.2019	2nd project consultations.	---
18.12.2019	20.12.2019	2nd project commitment. 3rd project topics distribution. Characteristics of basic automatic control elements. Block diagram algebra.	2nd project evaluation.
Winter break (23.12.2019 – 6.02.2020)			
8.01.2020	10.01.2020	---	---
15.01.2020	17.01.2020	3rd project consultations. PID control.	---
22.01.2020	24.01.2020	3rd project consultations & commitment.	3rd project evaluation.
23.01.2020	31.01.2020	Final class evaluation.	
1.01.2020 – 14.02.2020: exam session			

Lecture 3 cont.

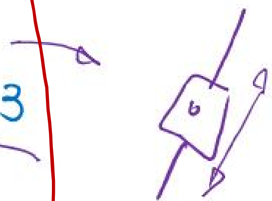
Relative motion – example.

Velocities and accelerations in relative motion – example

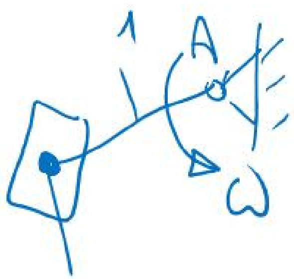


$$\overline{V}_{B1} = \overline{V}_{B3} + \overline{V}_{B1B3}$$

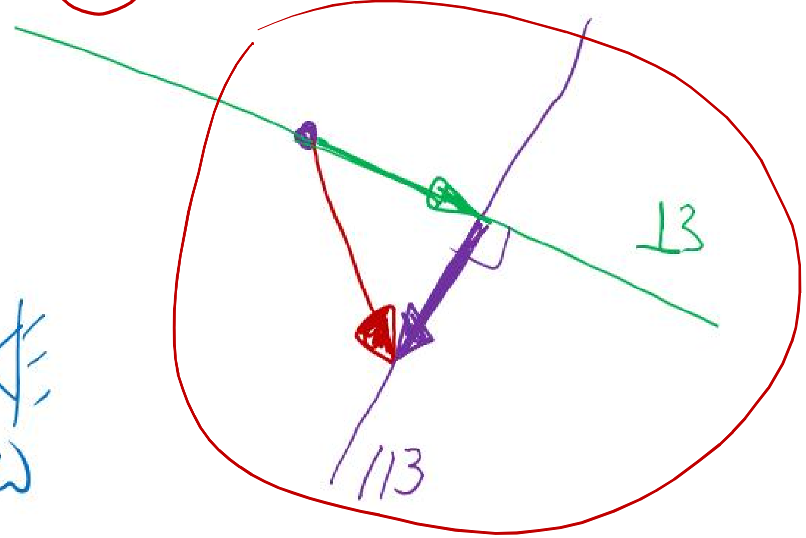
$\perp 1 \qquad \perp 3 \qquad \parallel 13$



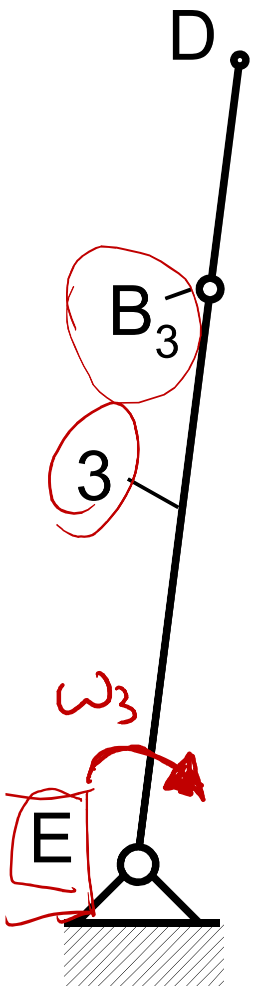
$$|\overline{V}_{B1}| = \omega |AB|$$

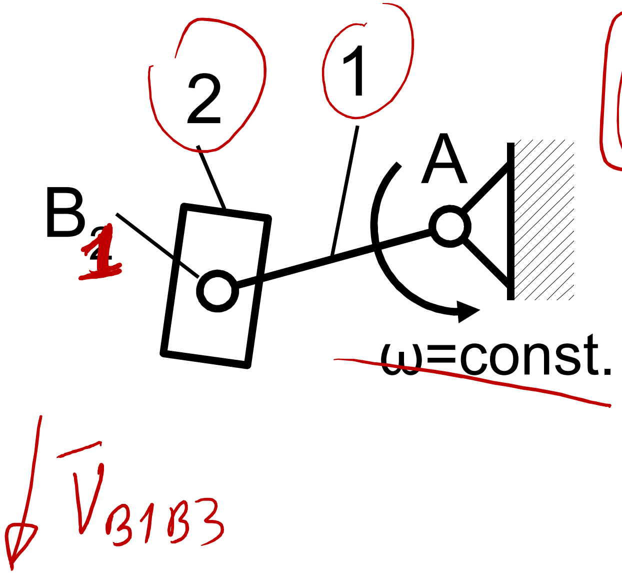


$B_1 = B_2$



Velocities and accelerations in relative motion – example





$$\bar{a}_{B1} = \bar{a}_{B3} + \bar{a}^{rel.}_{B1B3} + \bar{a}^{cor.}$$

$\bar{a}^{rel.}_{B1B3} \parallel 3$, $\bar{a}^{cor.} \perp 3$

$$\bar{a}_{B1} = \bar{a}^{n}_{B1A} + \bar{a}^{t}_{B1A}$$

$\bar{a}^{n}_{B1A} \parallel 1 \rightarrow \omega^2|BA|$

$\bar{a}^{t}_{B1A} \perp 1 \rightarrow \varepsilon|BA| = 0$

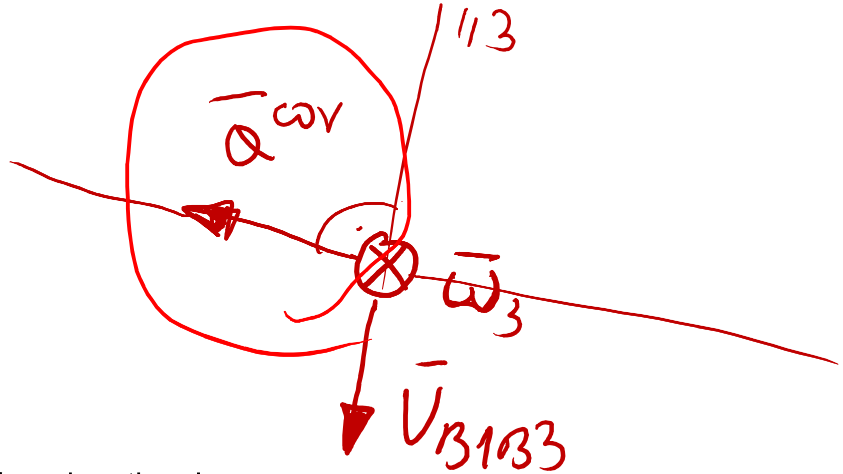
$$\varepsilon = \frac{d\omega}{dt} = 0$$

$$\bar{a}_{B3} = \bar{a}^{n}_{B3E} + \bar{a}^{t}_{B3E}$$

$\bar{a}^{n}_{B3E} \parallel 3$, $\bar{a}^{t}_{B3E} \perp 3 \rightarrow \varepsilon_3 = ?$

$$|\bar{a}^{n}_{B3E}| = \omega_3^2|BE|$$

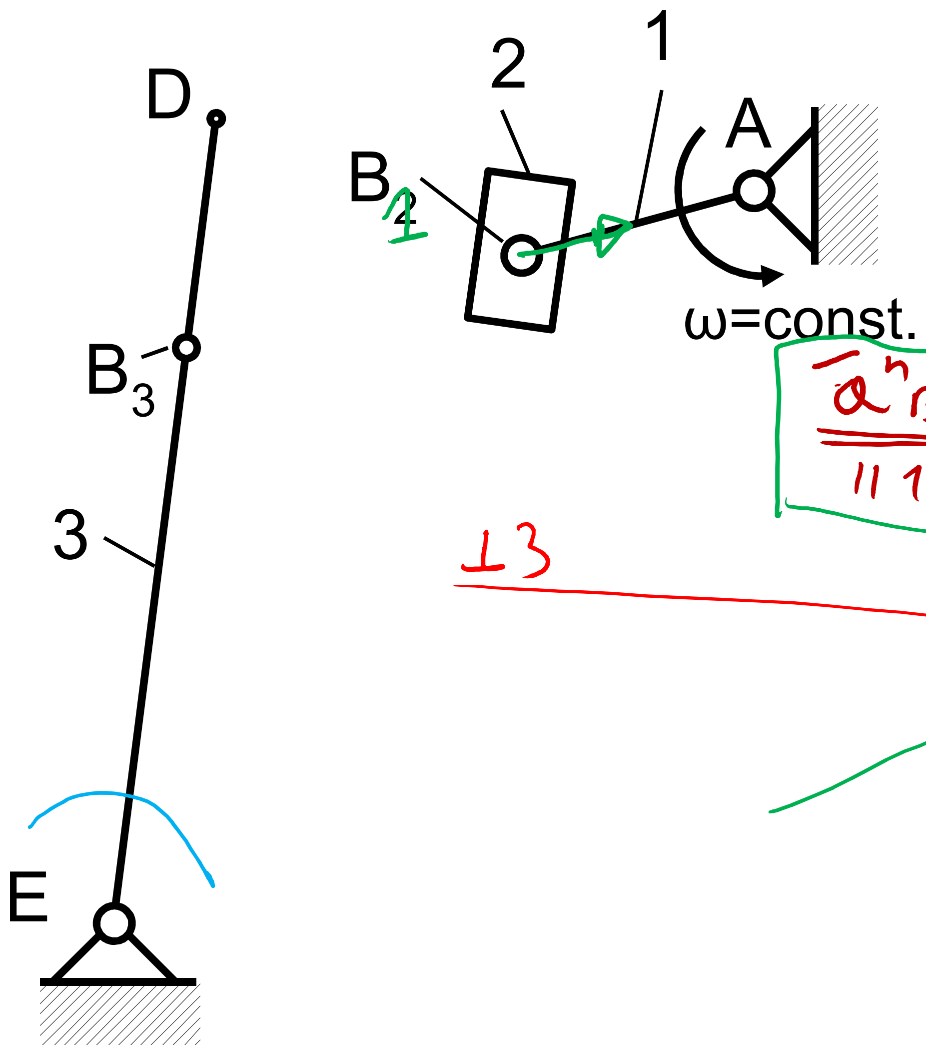
$$\bar{a}^{cor.} = 2\,\vec{\omega}_3 \times \bar{V}_{B1B3}$$



Velocities and accelerations in relative motion – example

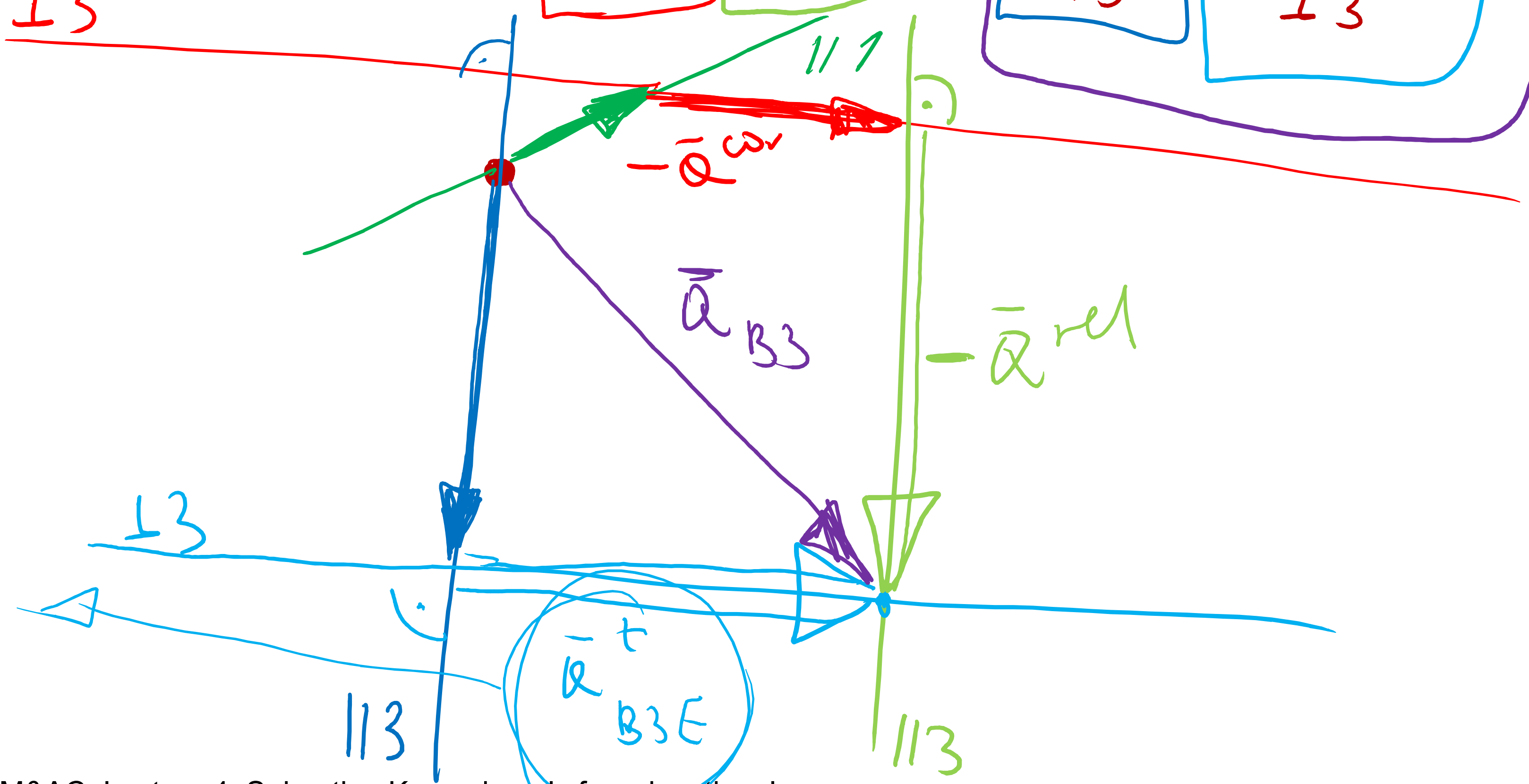
$$\bar a^{n}_{B1A} = \bar a^{n}_{B3E} + \bar a^{t}_{B3E} + \bar a^{rel}_{B1B3} + \bar a^{cor.}$$

$\parallel 1 \qquad \parallel 3 \qquad \perp 3 \qquad \parallel 3 \qquad \perp 3$



$$\bar a^{n}_{B1A} - \bar a^{cor} - \bar a^{rel} = \bar a^{n}_{B3E} + \bar a^{t}_{B3E}$$

$\parallel 1 \qquad \perp 3 \qquad \parallel 3 \qquad \parallel 3 \qquad \perp 3$



$$\varepsilon_3 = \frac{|\bar a^{t}_{B3E}|}{|B_3E|}$$

Lecture 4

Analytical method. Cam mechanisms.

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

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1. Set up Cartesian coordinate system O_{XY}.

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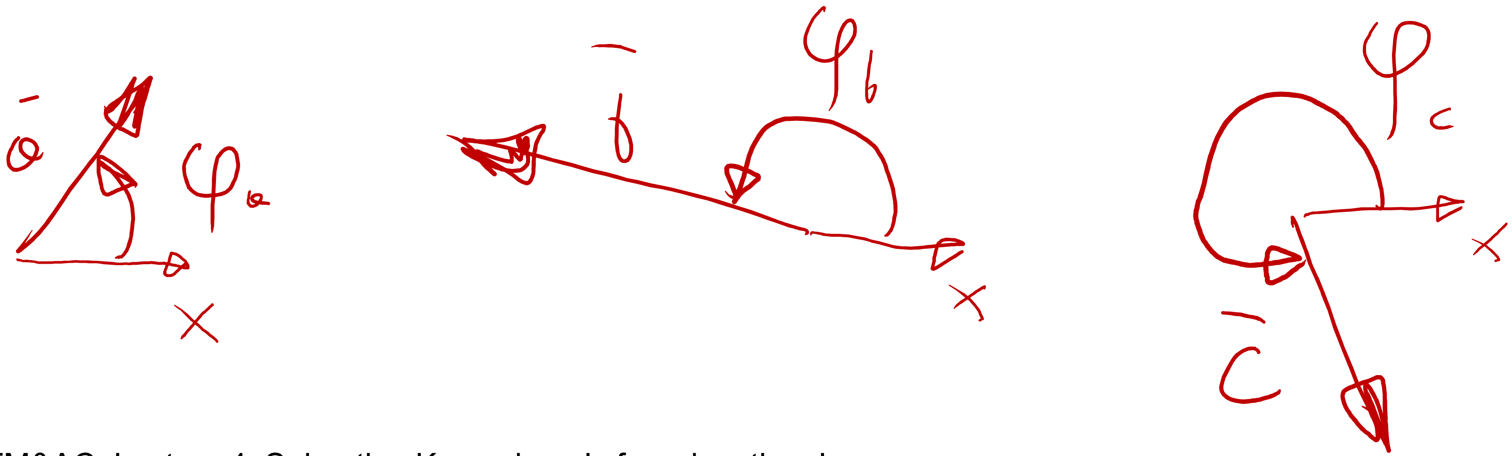
2. Substitute the mechanism's members with set of vectors. All vectors can move with mechanism's elements, change their size, location and orientation.

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

1. Set up Cartesian coordinate system O_{XY}.
2. Substitute the mechanism's members with set of vectors. All vectors can move with mechanism's elements, change their size, location and orientation.
3. Vectors must to create closed polygons.

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

1. Set up Cartesian coordinate system O_{XY}.
2. Substitute the mechanism's members with set of vectors. All vectors can move with mechanism's elements, change their size, location and orientation.
3. Vectors must to create closed polygons.
4. Define “directed angles” for all vectors defined in the same manner. Assume that this angles are created by the positive x axis counter-clockwise rotation.



Procedure of analytical determination of velocities and accelerations in planar mechanisms.

1. Set up Cartesian coordinate system O_{XY}.
2. Substitute the mechanism's members with set of vectors. All vectors can move with mechanism's elements, change their size, location and orientation.
3. Vectors must to create closed polygons.
4. Define “directed angles” for all vectors defined in the same manner. Assume that this angles are created by the positive x axis counter-clockwise rotation.
5. Fore each of polygon write down sum of vectors, e.g.:

$$\bar{a} + \bar{b} + \bar{c} = 0$$

$$\bar{e} + \bar{f} = \bar{g} - h$$

$$\sum_{i=1}^{i=n} \vec{l}_i = 0$$

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

6a. Write down projections of each polygon onto coordinate system's axes:

$$x:\ \sum_{i=1}^{i=n} l_i \cos\varphi_i = 0 \qquad y:\ \sum_{i=1}^{i=n} l_i \sin\varphi_i = 0$$

(we do not need to analyze signs because of „directed angles" setup procedure)

$$\begin{cases} \bar{x}:\ |\bar{a}|\cos\varphi_a + |b|\cos\varphi_b + |\bar{c}|\cos\varphi_c + \ldots = 0 \\ \bar{y}:\ |\bar{a}|\sin\varphi_a + |b|\sin\varphi_b \ldots \end{cases}$$

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

6a. Write down projections of each polygon onto coordinate system's axes:

$$\text{x:}\quad \sum_{i=1}^{i=n} l_i \cos\varphi_i = 0 \qquad\qquad \text{y:}\quad \sum_{i=1}^{i=n} l_i \sin\varphi_i = 0$$

(we do not need to analyze signs because of „directed angles" setup procedure)

6b. Define which vectors' lengths and angles are given and/or constant (related to geometry), and which are variable in time and unknown.

(for a proper defined system number of unknown variables is equal to the number of equations)

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

6a. Write down projections of each polygon onto coordinate system's axes:

$$\text{x:}\ \sum_{i=1}^{i=n} l_i \cos\varphi_i = 0 \qquad \text{y:}\ \sum_{i=1}^{i=n} l_i \sin\varphi_i = 0$$

(we do not need to analyze signs because of „directed angles” setup procedure)

6b. Define which vectors' lengths and angles are given and/or constant (related to geometry), and which are variable in time and unknown.

(for a proper defined system number of unknown variables is equal to the number of equations)

7. Solve the equations. The resulting functions describe motion of the mechanism.

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

8. Differentiate functions achieved in p.7 to obtain velocities. Differentiate once again to obtain accelerations.

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

8. Differentiate functions achieved in p.7 to obtain velocities. Differentiate once again to obtain accelerations.

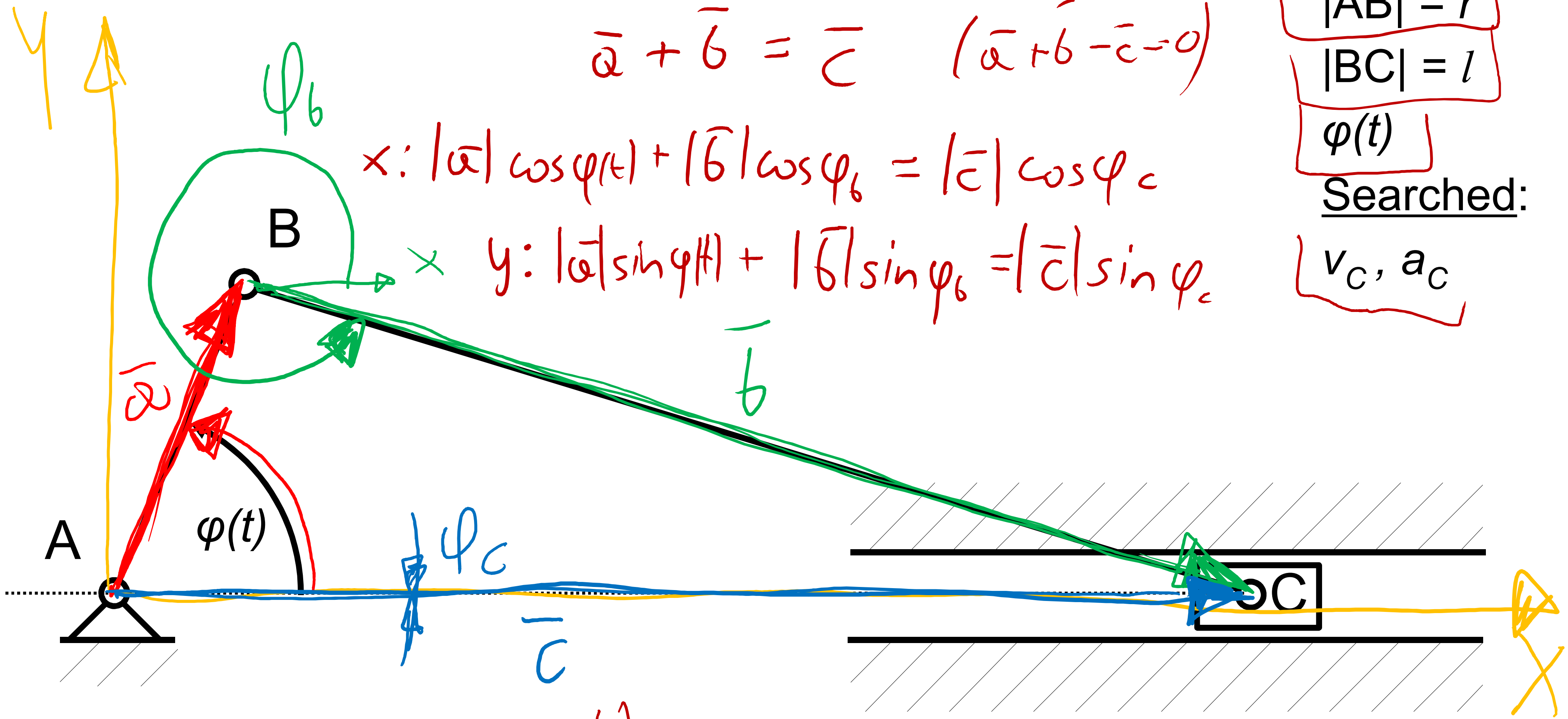
9. If desired information was not obtained in p.8, differentiate equations from p.6. Sometimes rotation of the coordinate system is useful here.

Analytical method – example:
crank-slider mechanism

$$\bar{a} + \bar{b} = \bar{c} \quad (\bar{a} + \bar{b} - \bar{c} = 0)$$

$$x:\ |\bar{a}|\cos\varphi(t) + |\bar{b}|\cos\varphi_b = |\bar{c}|\cos\varphi_c$$

$$y:\ |\bar{a}|\sin\varphi(t) + |\bar{b}|\sin\varphi_b = |\bar{c}|\sin\varphi_c$$



Given:

$|AB| = r$

$|BC| = l$

$\varphi(t)$

Searched:

v_C, a_C

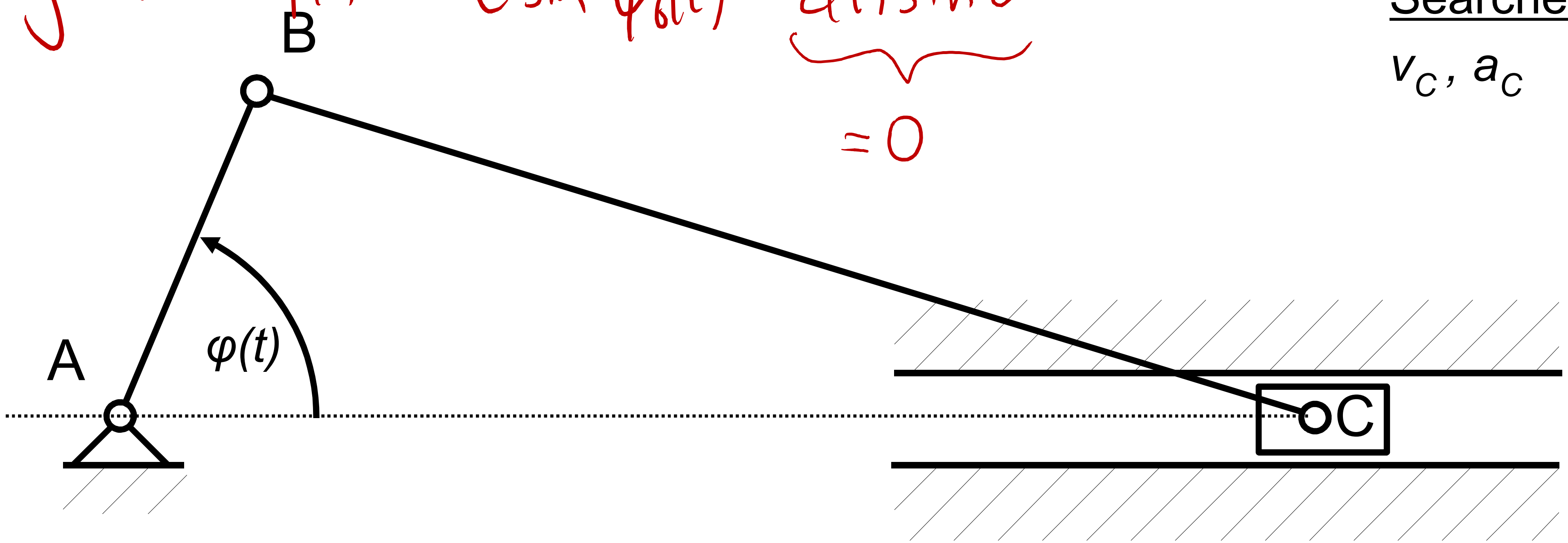
$|\bar{a}| = r$ - given, const. $\quad \varphi(t)$ - given

$|\bar{b}| = l$ - given, const. $\quad \varphi_b(t)$ - unknown

$|\bar{c}| = c(t)$ - unknown $\quad \varphi_c = 0$ - const.

Analytical method – example:
crank-slider mechanism

$$\begin{cases} x: \; r\cos\varphi(t) + l\cos\varphi_b(t) = d(t)\overbrace{\cos 0^\circ}^{1} \\ y: \; r\sin\varphi(t) + l\sin\varphi_b(t) = \underbrace{d(t)\sin 0^\circ}_{=0} \end{cases}$$



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Searched:

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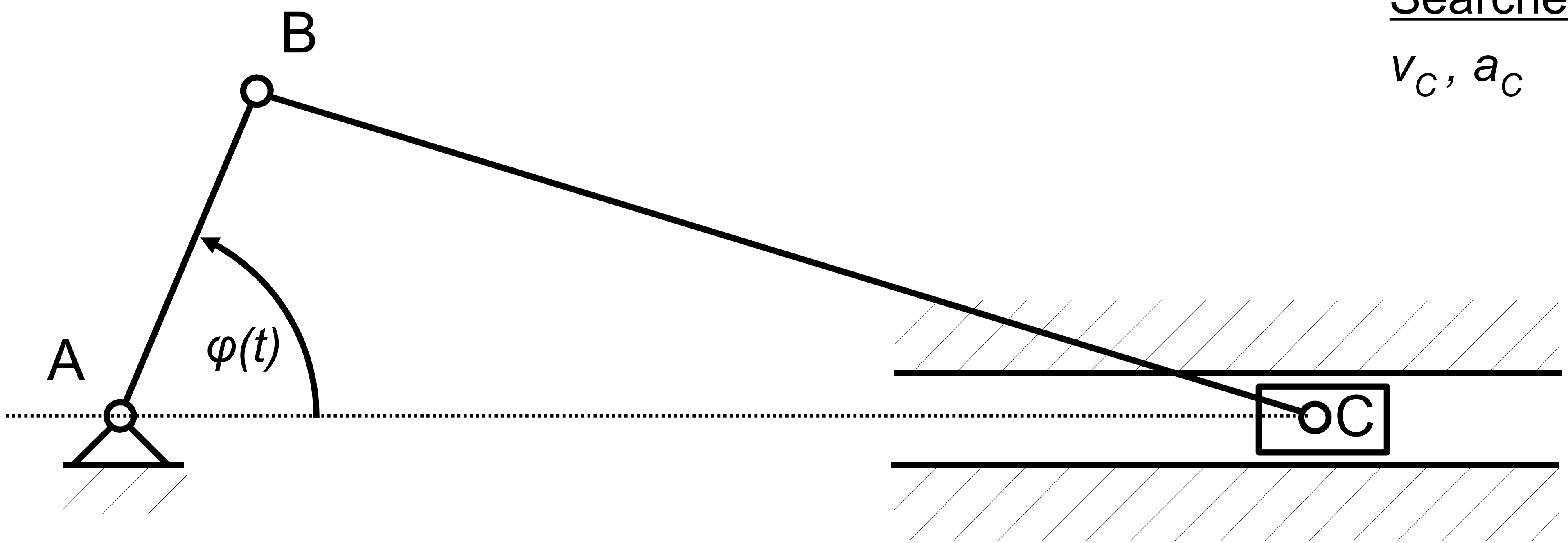
Analytical method – example:
crank-slider mechanism



Given:

|AB| = r

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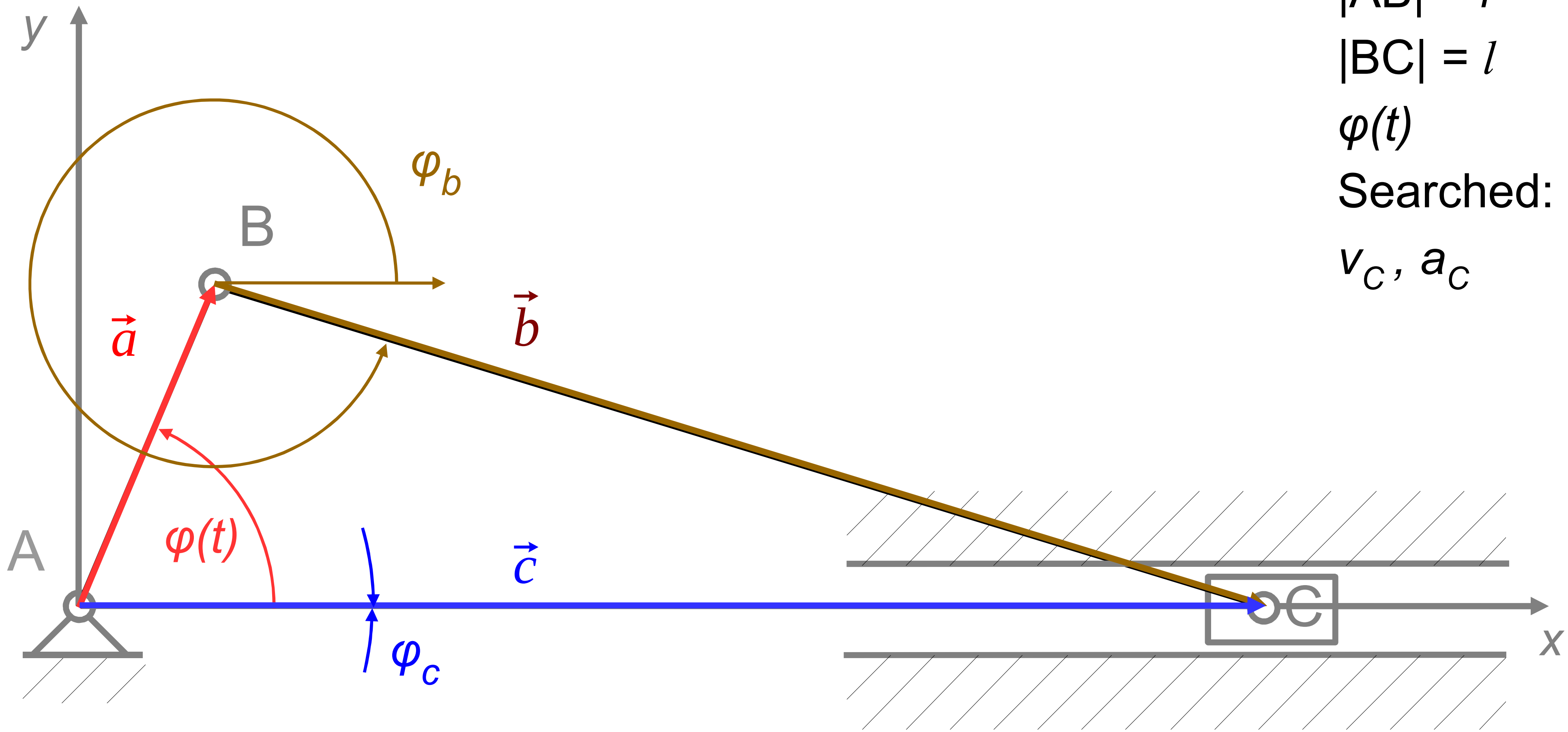
$\varphi(t)$

Searched:

v_C , a_C

Analytical method – example:
crank-slider mechanism



Given:

|AB| = r

|BC| = l

$\varphi(t)$

Searched:

v_C , a_C

$$|\vec{a}| = r \qquad \varphi(t)$$

$$|\vec{b}| = l \qquad \varphi_b(t)$$

$$|\vec{c}| = c(t) \qquad \varphi_c = 0$$

$$\vec{a} + \vec{b} = \vec{c}$$

x: $r\cos\varphi(t) + l\cos\varphi_b(t) = c(t)\cos 0$

y: $r\sin\varphi(t) + l\sin\varphi_b(t) = c(t)\sin 0$

Analytical method – example:

crank-slider mechanism

$$\lambda = \frac{r}{l}$$

$$r\cos\varphi(t) + l\cos\varphi_b(t) = c(t)\cos 0 \quad (\cos 0 = 1)$$

$$r\sin\varphi(t) + l\sin\varphi_b(t) = c(t)\sin 0 \quad (\sin 0 = 0)$$

2 unknowns

$$\sin\varphi_b(t) = -r\sin\varphi(t)/l = -\lambda\sin\varphi(t)$$

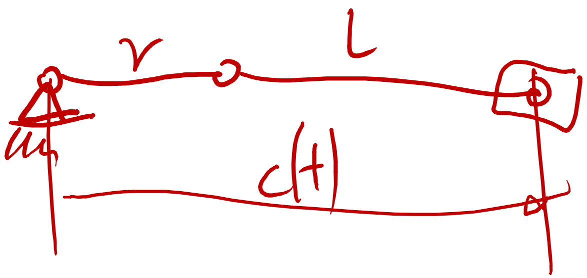
$$\varphi_b(t) = \arcsin(-\lambda\sin\varphi(t)) = -\arcsin(\lambda\sin\varphi(t))$$

$$\sin^2\varphi_b(t) + \cos^2\varphi_b(t) = 1 \rightarrow \cos\varphi_b(t) = \pm\sqrt{1-\sin^2\varphi_b(t)}$$

$$\cos\varphi_b(t) = \pm\sqrt{1-\lambda^2\sin^2\varphi(t)}$$

$$c(t) = r\cos\varphi(t) \pm l\sqrt{1-\lambda^2\sin^2\varphi(t)}$$

if $\varphi(t) = 0$



Analytical method – example:
crank-slider mechanism

$$r\cos\varphi(t)+l\cos\varphi_b(t)=c(t)\cos 0$$
$$r\sin\varphi(t)+l\sin\varphi_b(t)=c(t)\sin 0$$

2 unknowns

Analytical method – example:
crank-slider mechanism

$$r\cos\varphi(t) + l\cos\varphi_b(t) = c(t)\cos 0$$
$$r\sin\varphi(t) + l\sin\varphi_b(t) = c(t)\sin 0$$

2 unknowns

Analytical method – example:
crank-slider mechanism

$$r\cos\varphi(t)+l\cos\varphi_b(t)=c(t)\cos 0$$
$$r\sin\varphi(t)+l\sin\varphi_b(t)=c(t)\sin 0$$

2 unknowns

$$r\cos\varphi(t)+l\cos\varphi_b(t)=c(t)$$
$$r\sin\varphi(t)+l\sin\varphi_b(t)=0$$

$$\sin\varphi_b(t)=-\frac{r}{l}\sin\varphi(t)=-\lambda\sin\varphi(t)$$

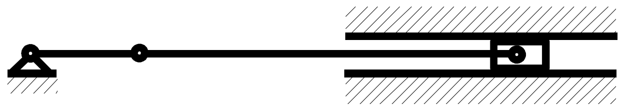
$$\varphi_b(t)=-\arcsin(\lambda\sin\varphi(t))$$

$$\sin^2\varphi_b(t)+\cos^2\varphi_b(t)=1$$

$$\cos\varphi_b(t)=\pm\sqrt{1-\sin^2\varphi_b(t)}$$

$$\cos\varphi_b(t)=\pm\sqrt{1-\lambda^2\sin^2\varphi(t)}$$

$$c(t)=r\cos\varphi(t)\pm l\sqrt{1-\lambda^2\sin^2\varphi(t)}$$



for $\varphi(t)=0$

$c(t)=r+l$

$$\boxed{c(t)=r\cos\varphi(t)+l\sqrt{1-\lambda^2\sin^2\varphi(t)}}$$

Analytical method – example:
crank-slider mechanism

slider movement

$$c(t) = r\cos\varphi(t) + l\sqrt{1-\lambda^2\sin^2\varphi(t)}$$

$$v_C(t) = \frac{dc(t)}{dt} = -r\frac{d\varphi(t)}{dt}\sin\varphi(t) - \frac{-2l\lambda^2\frac{d\varphi(t)}{dt}\sin\varphi(t)\cos\varphi(t)}{2\sqrt{1-\lambda^2\sin^2\varphi(t)}}$$

$$a_C(t) = \frac{dv_C(t)}{dt} = \ldots$$

?

Analytical method – example:
crank-slider mechanism

calculations with wxmaxima

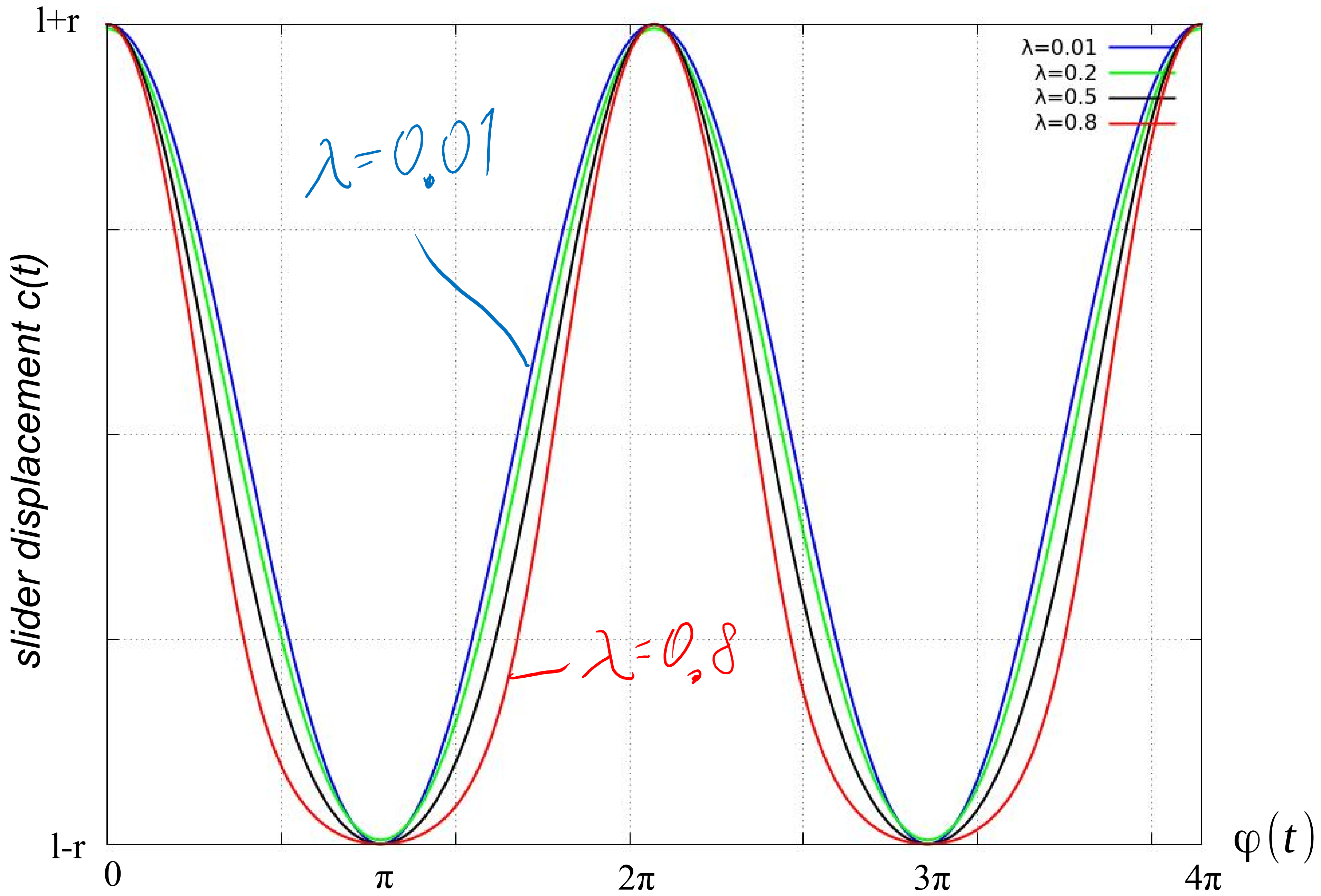
```
(%i14) c: r*cos(%phi(t))+l*sqrt(1-%lambda^2*(sin(%phi(t)))^2);
       v: diff(c,t,1);
       a: diff(v,t,1);
```

(c) $$l\sqrt{1-\lambda^2\sin(\varphi(t))^2}+r\cos(\varphi(t))$$

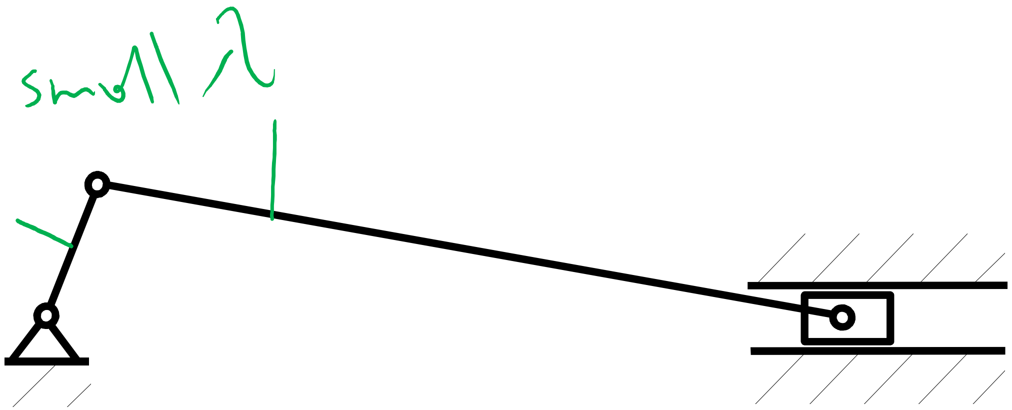
(v) $$-\frac{\lambda^2 l\cos(\varphi(t))\sin(\varphi(t))\left(\frac{d}{d\,t}\varphi(t)\right)}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}}-r\sin(\varphi(t))\left(\frac{d}{d\,t}\varphi(t)\right)$$

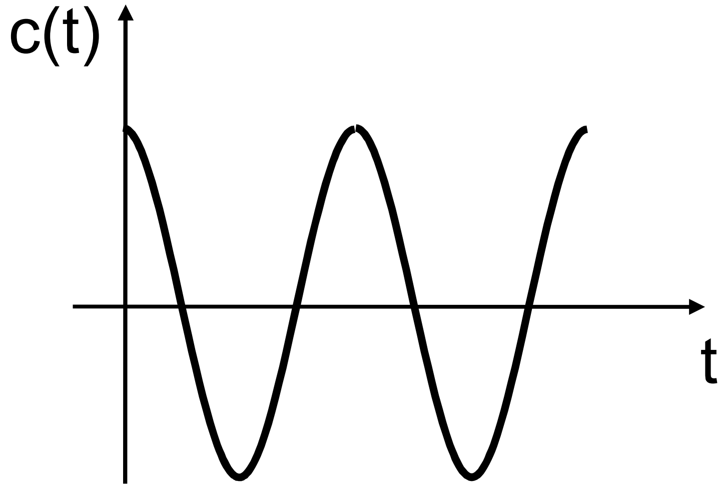
(a) $$-\frac{\lambda^2 l\cos(\varphi(t))\sin(\varphi(t))\left(\frac{d^2}{d\,t^2}\varphi(t)\right)}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}}-r\sin(\varphi(t))\left(\frac{d^2}{d\,t^2}\varphi(t)\right)+\frac{\lambda^2 l\sin(\varphi(t))^2\left(\frac{d}{d\,t}\varphi(t)\right)^2}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}}-\frac{\lambda^2 l\cos(\varphi(t))^2\left(\frac{d}{d\,t}\varphi(t)\right)^2}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}}-\frac{\lambda^4 l\cos(\varphi(t))^2\sin(\varphi(t))^2\left(\frac{d}{d\,t}\varphi(t)\right)^2}{\left(1-\lambda^2\sin(\varphi(t))^2\right)^{3/2}}-r\cos(\varphi(t))\left(\frac{d}{d\,t}\varphi(t)\right)^2$$

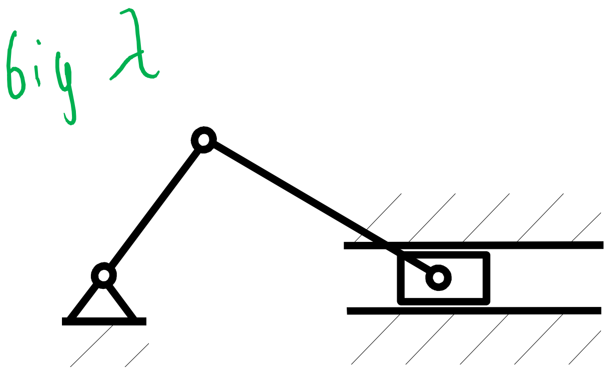
Analytical method – example:
crank-slider mechanism

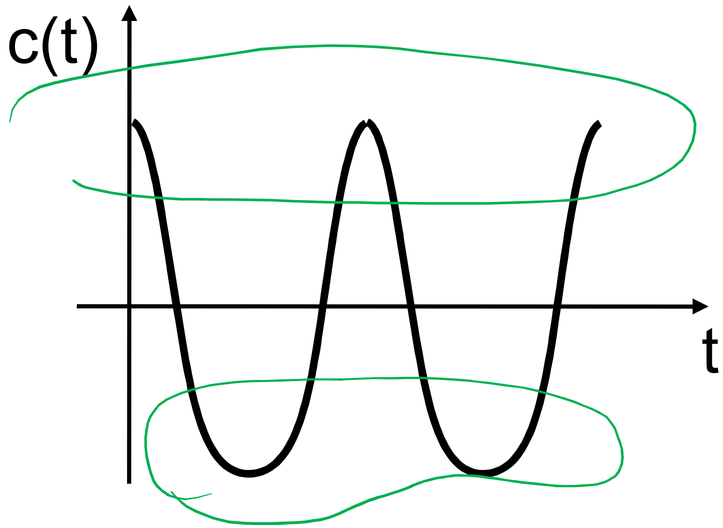


Analytical method – example:
crank-slider mechanism









Interesting reeding: http://www.enginebuildermag.com/2016/08/understanding-rod-ratios/

Analytical method – example:
crank-slider mechanism

connecting rod motion

$$\varphi_b(t) = -\arcsin(\lambda \sin\varphi(t))$$

$$\omega_b(t) = \frac{d\varphi_b(t)}{dt} = \frac{-\lambda\dot{\varphi}(t)\cos\varphi(t)}{\sqrt{1-\lambda^2\sin^2\varphi(t)}}$$

$$\varepsilon_b(t) = \frac{d\omega_b(t)}{dt} = -\frac{\lambda\cos(\varphi(t))\left(\frac{d^2}{dt^2}\varphi(t)\right)}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}} + \frac{\lambda\sin(\varphi(t))\left(\frac{d}{dt}\varphi(t)\right)^2}{\sqrt{1-\lambda^2\sin(\varphi(t))^2}} - \frac{\lambda^3\cos(\varphi(t))^2\sin(\varphi(t))\left(\frac{d}{dt}\varphi(t)\right)^2}{\left(1-\lambda^2\sin(\varphi(t))^2\right)^{3/2}}$$

Analytical method – example:
slider-yoke

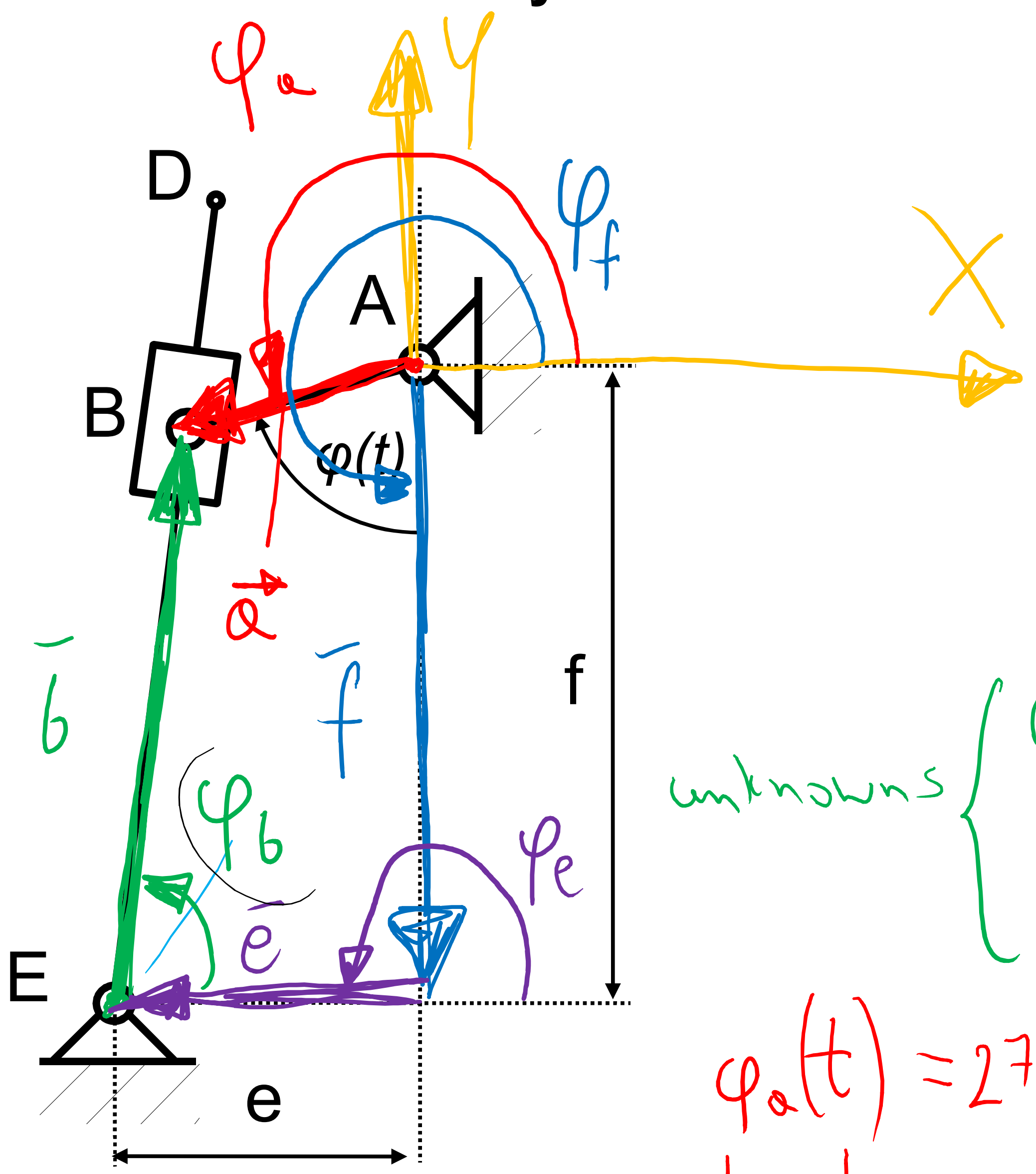
<u>Given</u>:

|AB| = *r*

e, f, *φ(t)*

<u>Searched</u>:

angular velocity ω_2 and acceleration ε_2 of ED element



$\varphi_f = 270^\circ$

$\varphi_e = 180^\circ$

$|\bar{e}| = e$

$|\bar{f}| = f$

unknowns $\begin{cases} \varphi_b(t) \\ |\bar{b}| = b(t) \end{cases}$

$\varphi_a(t) = 270^\circ - \varphi(t)$ – given

$|\bar{a}| = r$

$\bar{a} = \bar{f} + \bar{e} + \bar{b}$

Analytical method – example:
slider-yoke

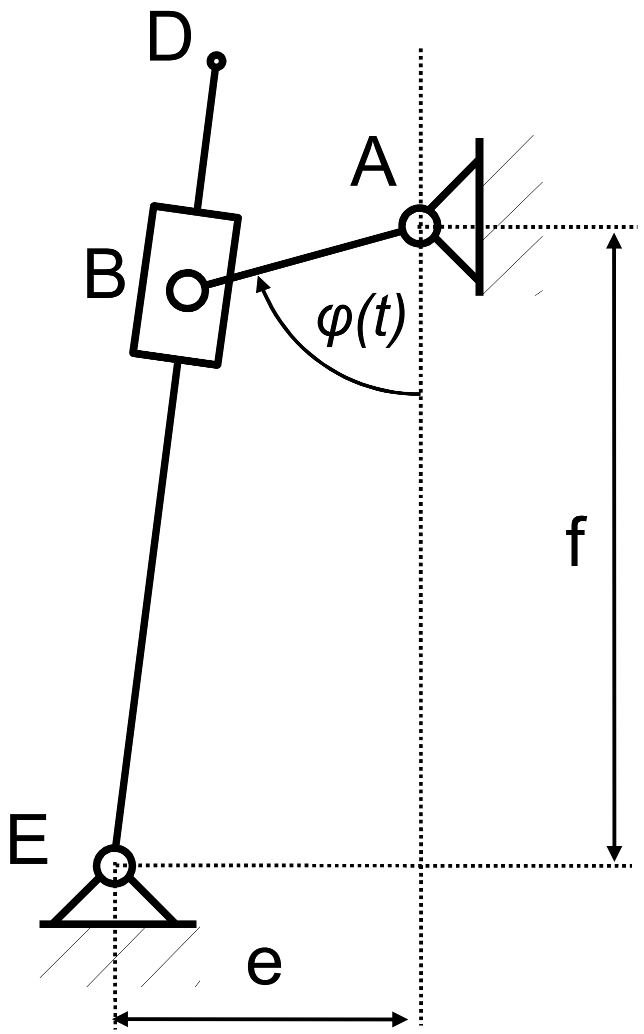
Given:

|AB| = *r*

e, f, *φ(t)*

Searched:

angular velocity ω_2 and acceleration ε_2 of ED element



Analytical method – example:
slider-yoke

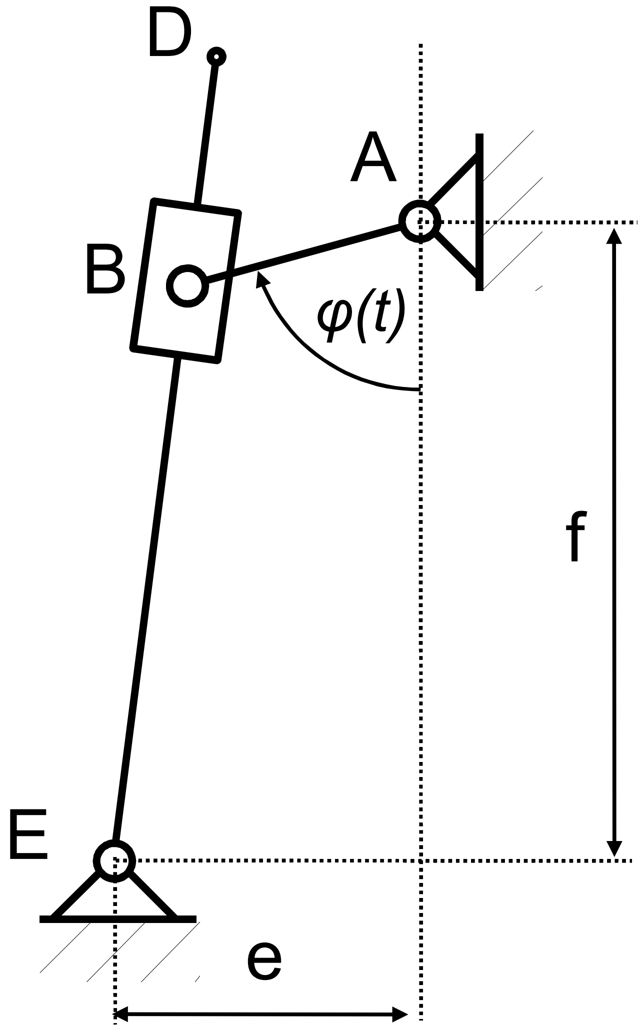
Given:

|AB| = *r*

e, f, $\varphi(t)$

Searched:

angular velocity ω_2 and acceleration ε_2 of ED element



Analytical method – example:
slider-yoke

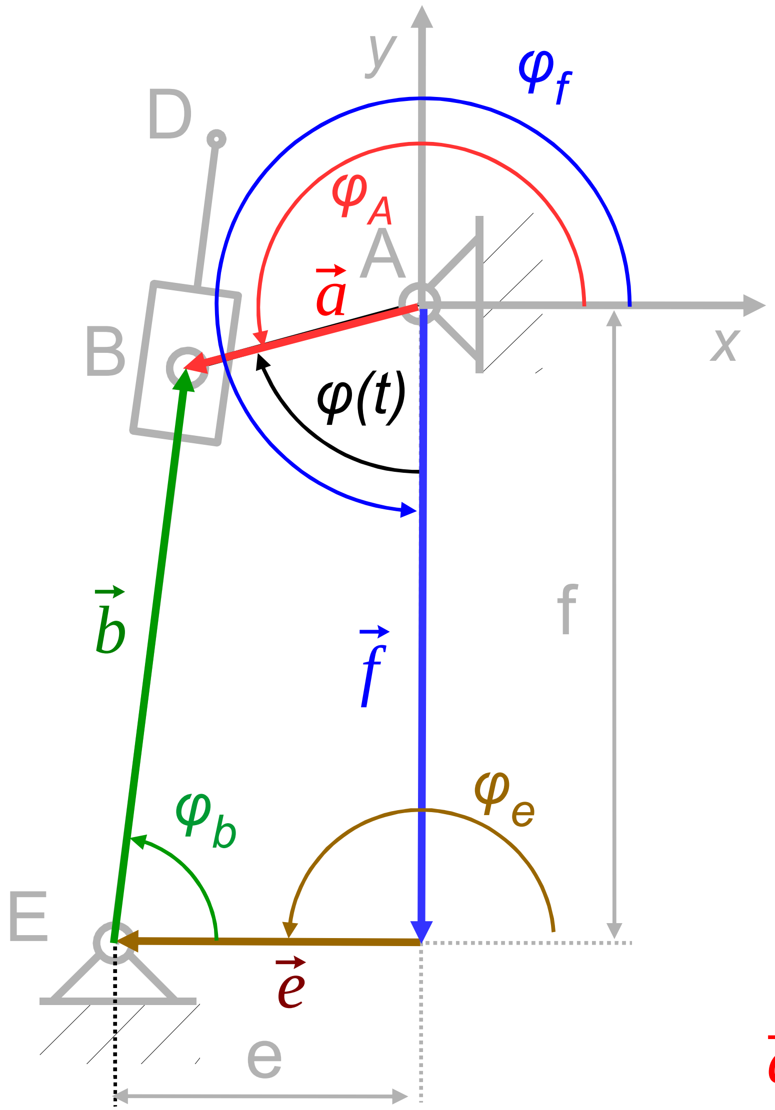
<u>Given</u>:

|AB| = *r*

e, f, $\varphi(t)$

<u>Searched</u>:

angular velocity ω_2 and acceleration ε_2 of ED element



$|\vec{a}| = r$

$\varphi_a(t) = 270^o - \varphi(t)$

$|\vec{b}| = b(t)$

$\varphi_b(t)$

$|\vec{e}| = e$

$\varphi_e = 180^o$

$|\vec{f}| = f$

$\varphi_f = 270^o$

$$\vec{a} = \vec{b} + \vec{e} + \vec{f}$$

Analytical method – example:
slider-yoke

sin(α+β)=
cos(α+β)=

Given:

|AB| = r

e, f, $\varphi(t)$

Searched:

angular velocity ω_2 and acceleration ε_2 of ED element

$$|\vec{a}|=r$$
$$\varphi_a(t)=270^o-\varphi(t)$$
$$|\vec{b}|=b(t)$$
$$\varphi_b(t)$$
$$|\vec{e}|=e$$
$$\varphi_e=180^o$$
$$|\vec{f}|=f$$
$$\varphi_f=270^o$$

$$\vec{a}=\vec{b}+\vec{e}+\vec{f}$$

$$x:\ r\cos(270^o-\varphi(t))=b(t)\cos\varphi_b(t)+e\cos 180^o+f\cos 270^o$$
$$y:\ r\sin(270^o-\varphi(t))=b(t)\sin\varphi_b(t)+e\sin 180^o+f\sin 270^o$$

$$x:\ -r\sin\varphi(t)=b(t)\cos\varphi_b(t)-e$$
$$y:\ -r\cos\varphi(t)=b(t)\sin\varphi_b(t)-f$$

Analytical method – example:
slider-yoke

$$e - r\sin\varphi(t) = b(t)\cos\varphi_b(t) \quad (1)$$
$$f - r\cos\varphi(t) = b(t)\sin\varphi_b(t) \quad (2)$$

$$(1)^2 + (2)^2 \quad (e - r\sin\varphi)^2 + (f - r\cos\varphi)^2 = \underbrace{b^2\cos^2\varphi_b + b^2\sin^2\varphi_b}_{b^2}$$

$$b(t) = (+)\sqrt{(e - r\sin\varphi(t))^2 + (f - r\cos\varphi(t))^2}$$

$$\frac{(2)}{(1)} \quad \frac{b(t)\sin\varphi_b(t)}{b(t)\cos\varphi_b(t)} = \frac{f - r\cos\varphi(t)}{e - r\sin\varphi(t)} = \tan\varphi_b(t)$$

$$\varphi_b(t) = \operatorname{atan}\left(\frac{f - r\cos\varphi(t)}{e - r\sin\varphi(t)}\right) \rightarrow \left\langle -\frac{\pi}{2}; \frac{\pi}{2}\right\rangle$$

$$\varphi_b(t) = \operatorname{atan2}(f - r\cos\varphi;\ e - r\sin\varphi) \rightarrow \langle 0; 2\pi\rangle$$

Analytical method – example:
slider-yoke

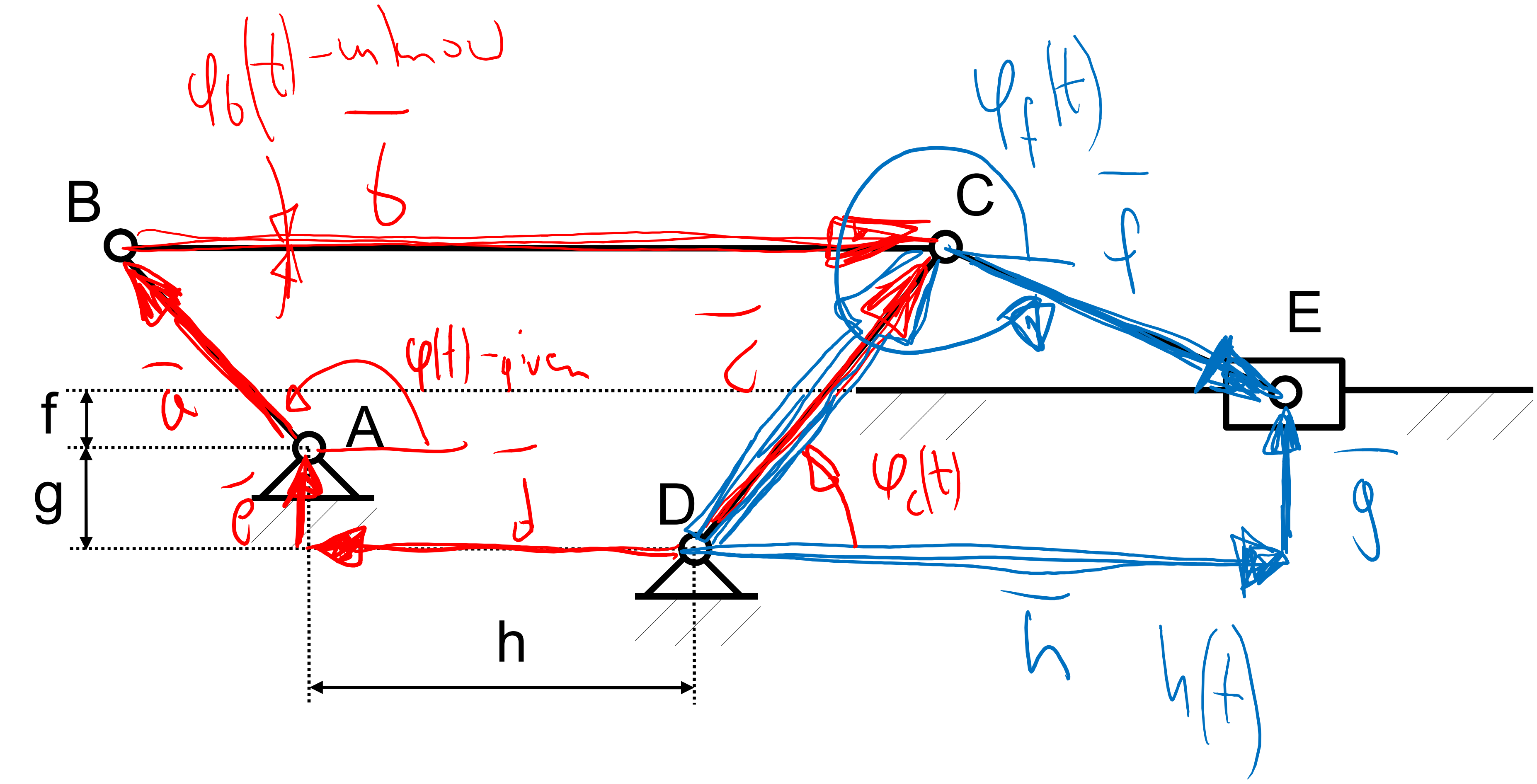
$$e - r\sin\varphi(t) = b(t)\cos\varphi_b(t)$$
$$f - r\cos\varphi(t) = b(t)\sin\varphi_b(t)$$

$$\omega_2(t) = \frac{d}{dt}\varphi_b(t)$$

$$\varepsilon_2(t) = \frac{d}{dt}\omega_2(t)$$

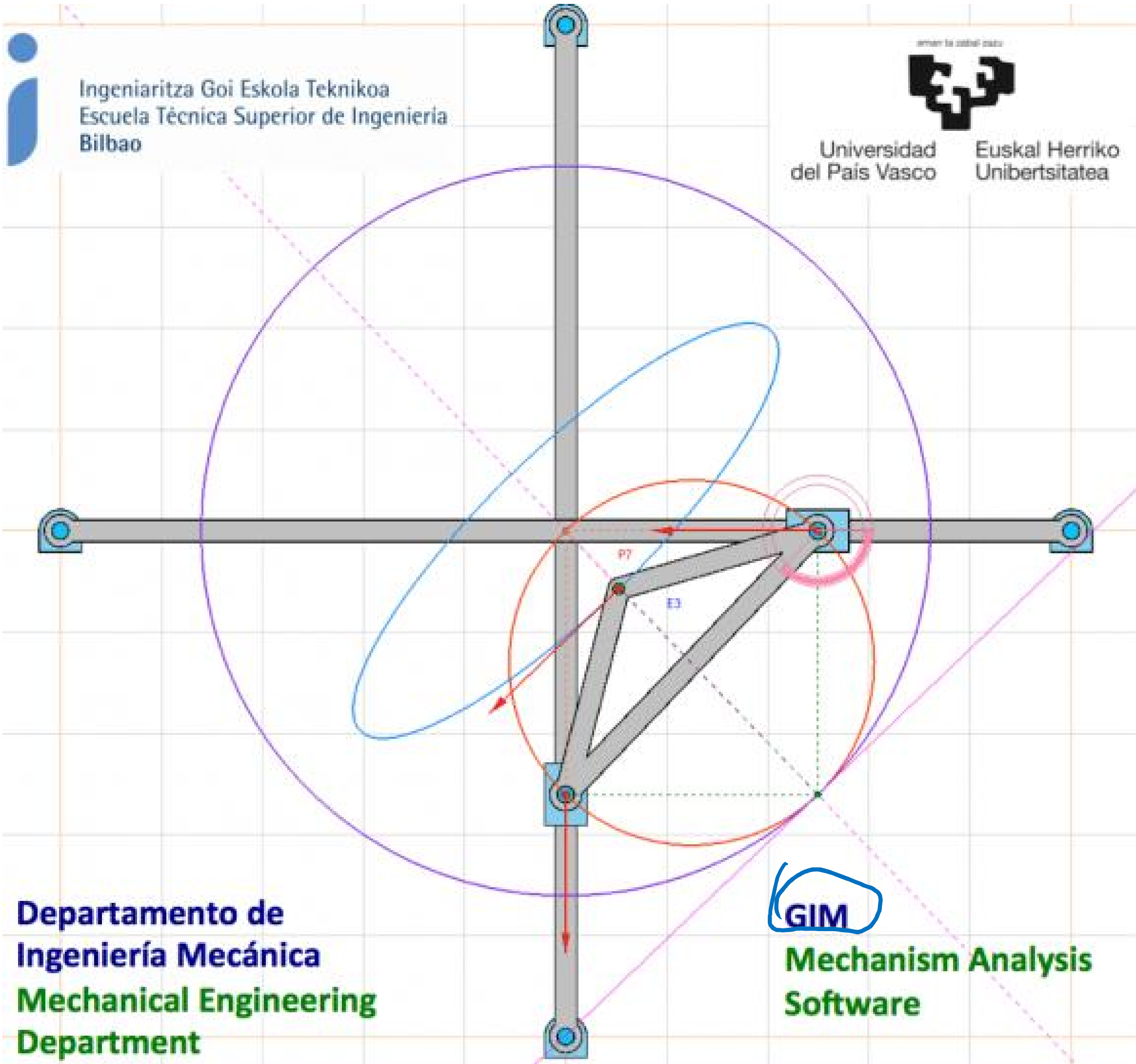
t	$\varphi(t)$	$\varphi_b(t)$	ω_2	ε_2
0	0°	φ_{b0}	?	?
Δt	1°	φ_{b1}	$=\frac{\varphi_{b1}-\varphi_{b0}}{\Delta t}$	?
$2\Delta t$	2°	φ_{b2}	=	$=\frac{\omega_2-\omega_1}{\Delta t}$
$3\Delta t$	3°	⋮	⋮	⋮
⋮	4°	⋮	⋮	⋮

Analytical method – example



$a + b - c = d + e \qquad \bar{c} + \bar{f} = \bar{h} + \bar{g}$

Software



http://www.ehu.eus/compmech/software/