Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control

Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

Lecture 3

Accelerations in planar mechanisms.

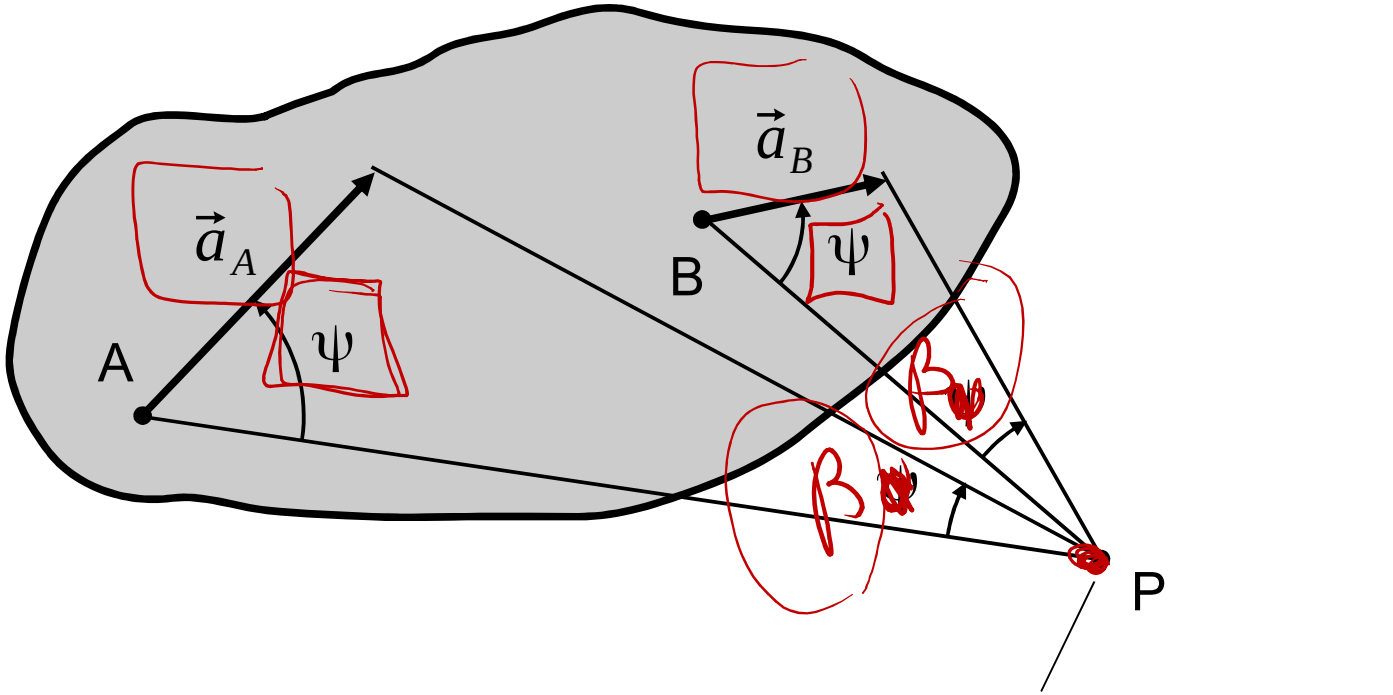
Methods of velocities and accelerations determination

Graphical methods

- velocity projection method,
- instantaneous center of rotation method,
- instantaneous center of acceleration method,
- method of rotated velocities,
- velocity decomposition method,
- acceleration decomposition method,
- velocity scheme (diagram) method,
- accelerations scheme (diagram) method.

Analytical method

Instantaneous center of acceleration



$$\psi = \operatorname{atan} \frac{\varepsilon}{\omega^2}$$

ε - angular acceleration

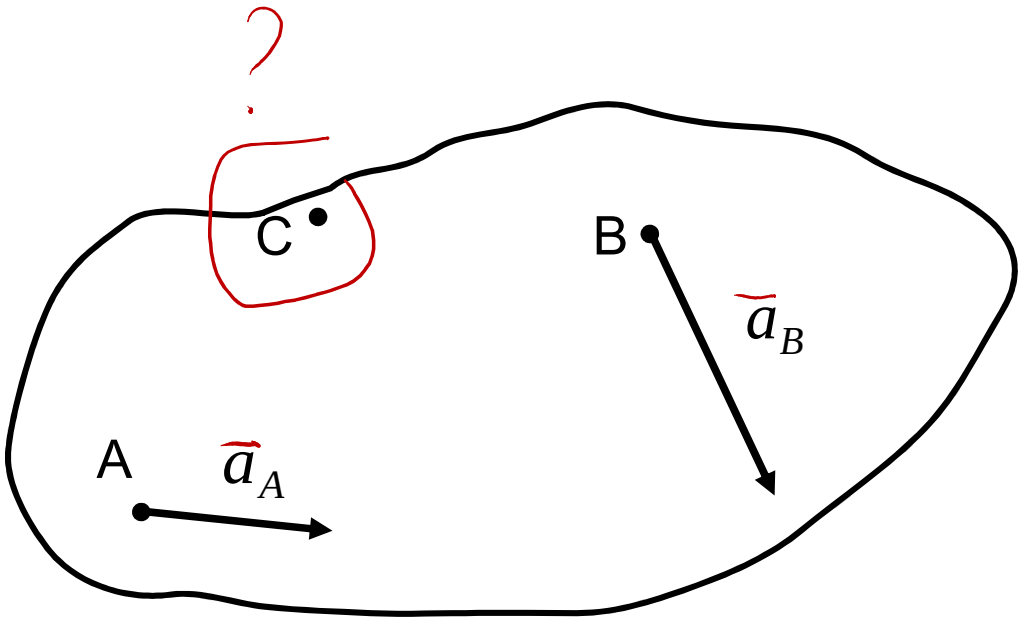
ω - angular velocity

Instantaneous center of acceleration method

Example

Given: a_A and a_B

Searched: a_C



Instantaneous center of acceleration method

Example

Given: a_A and a_B

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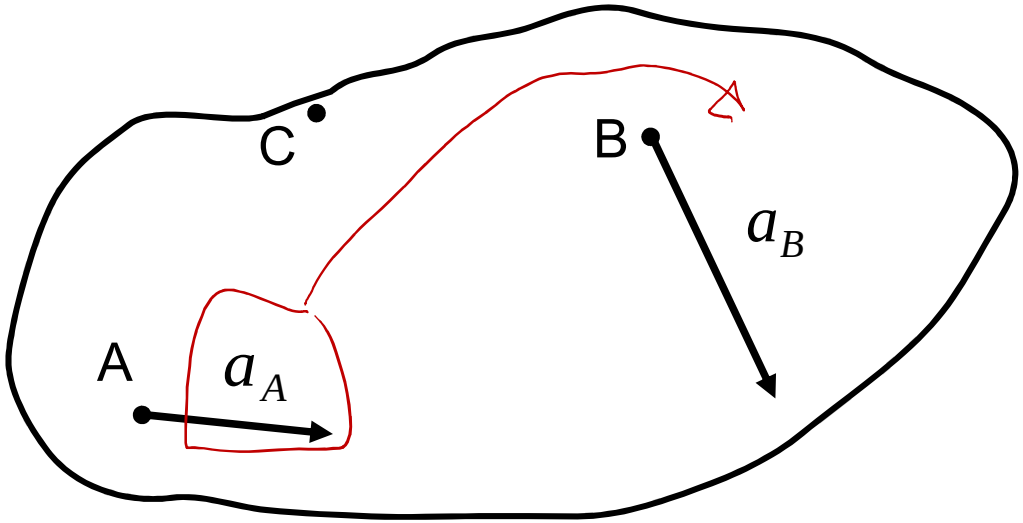
1. STEP:
construction of ψ



Instantaneous center of acceleration method

Example

Given: a_A and a_B

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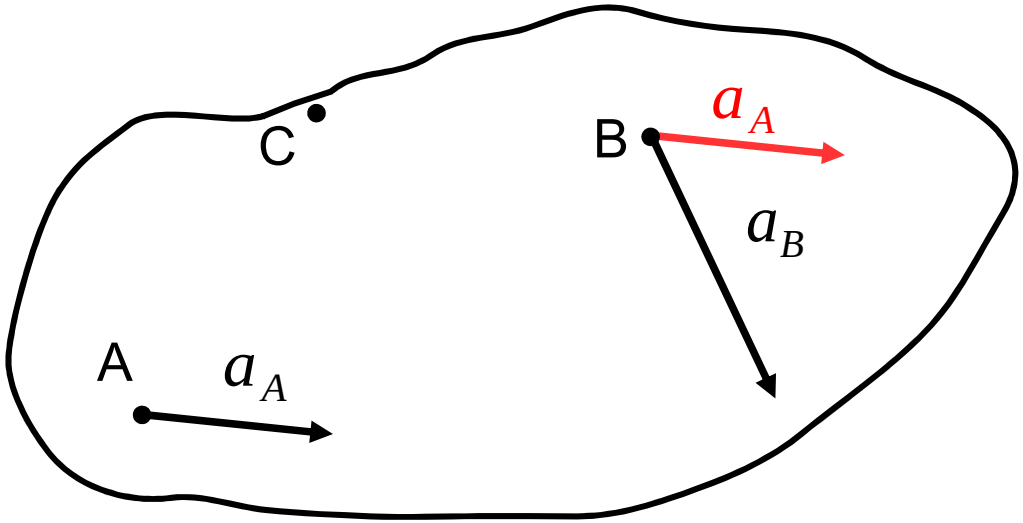


Instantaneous center of acceleration method

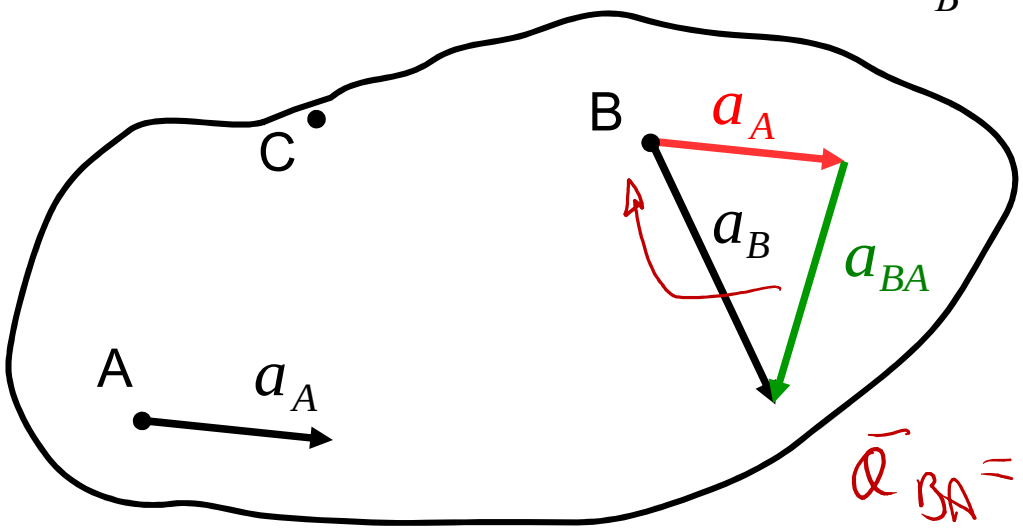
Example

$$a_B = a_A + a_{BA}$$

Given: a_A and a_B

Searched: a_C

1. STEP:
construction of ψ



$\tilde{a}_{BA} = \tilde{a}_B - \tilde{a}_A$

Instantaneous center of acceleration method

Example

$$a_B = a_A + a_{BA}$$

Given: a_A and a_B

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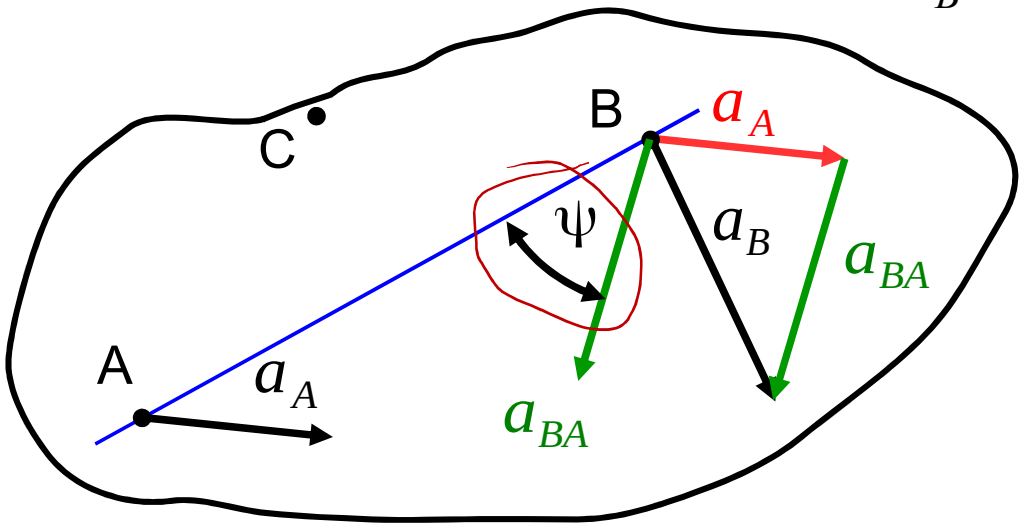

Instantaneous center of acceleration method

Example

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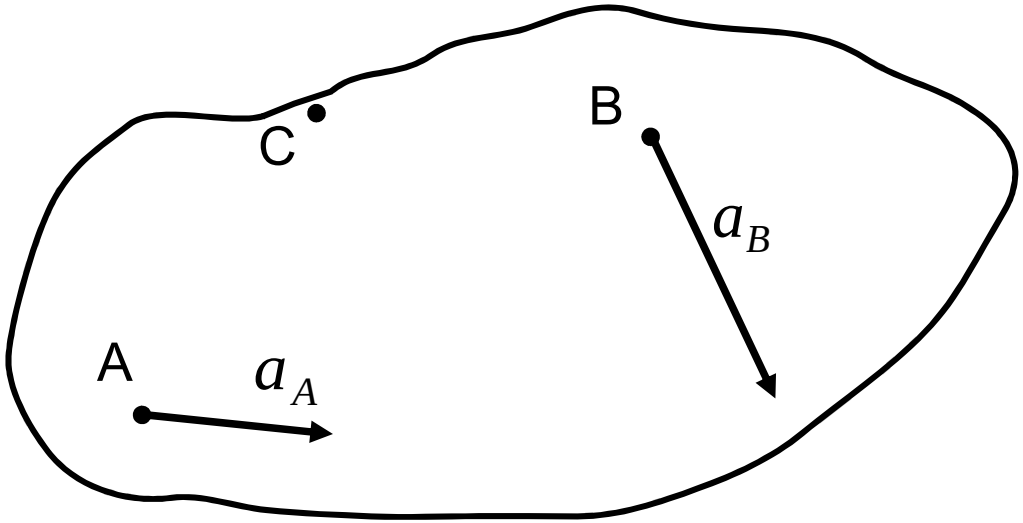
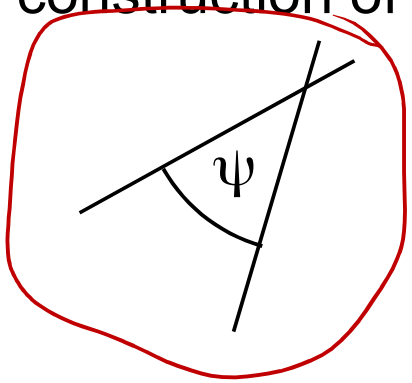
Searched: a_C



1. STEP:
construction of ψ



Instantaneous center of acceleration method

Example

Given: a_A and a_B

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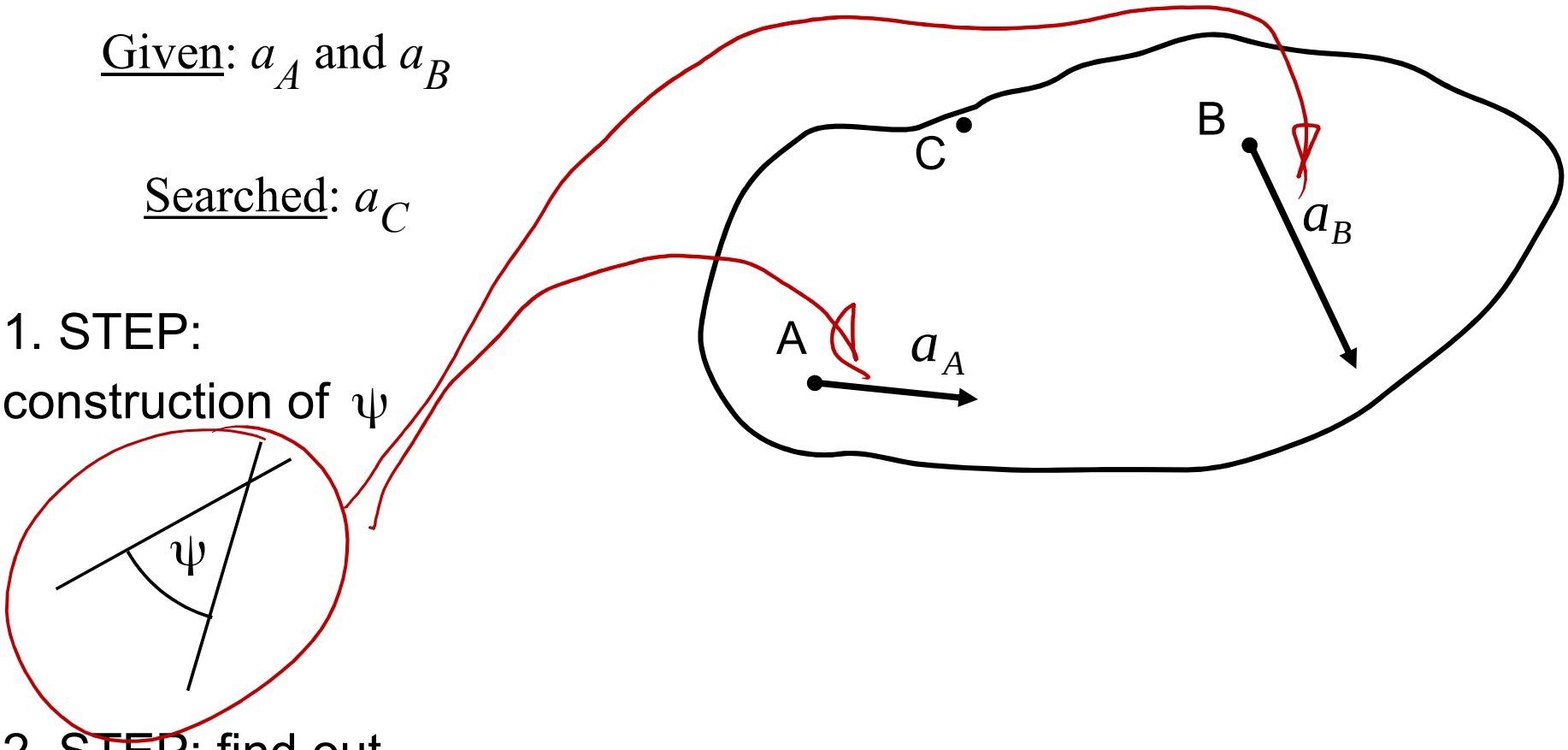
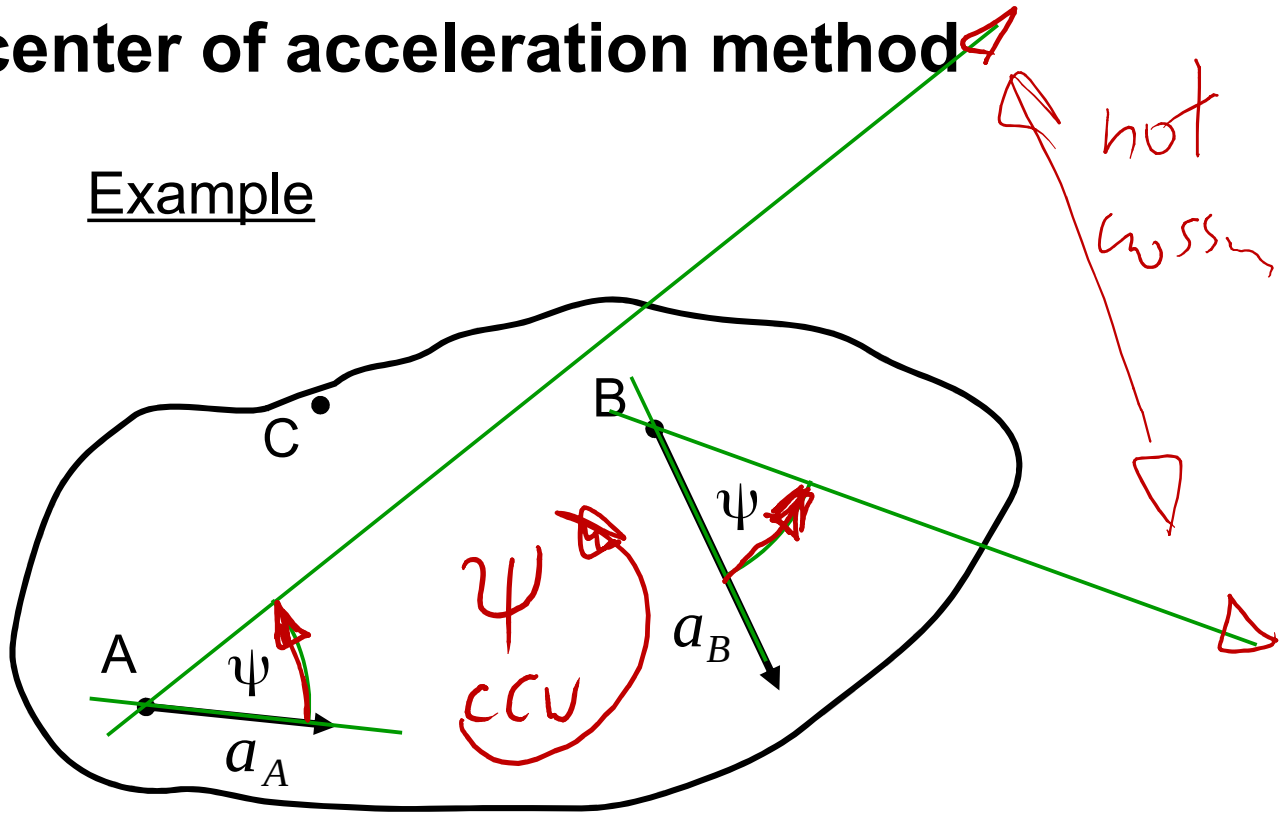


1. STEP: construction of ψ

2. STEP: find out the center of acceleration

Instantaneous center of acceleration method

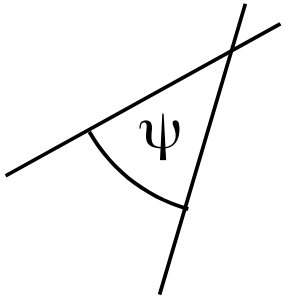
Example



Given: a_A and a_B

Searched: a_C

1. STEP:
construction of ψ



2. STEP: find out the center of acceleration

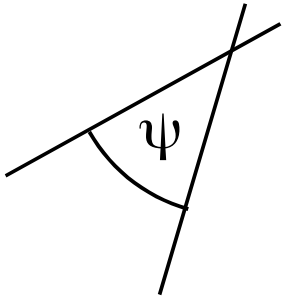
Instantaneous center of acceleration method

Example

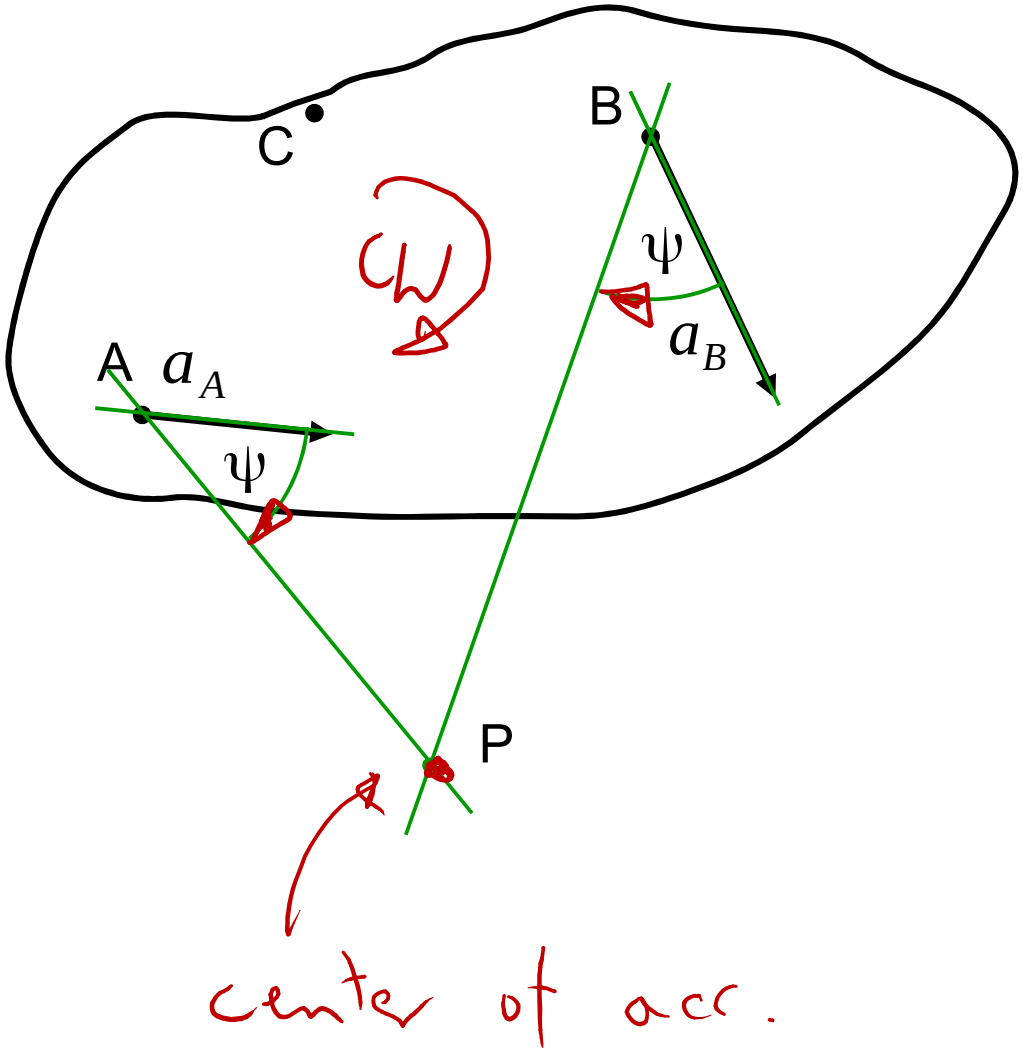
Given: a_A and a_B

Searched: a_C

1. STEP:
construction of ψ



2. STEP: find out the center of acceleration



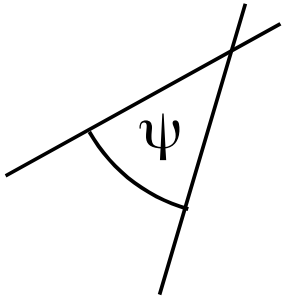
Instantaneous center of acceleration method

Example

Given: a_A and a_B

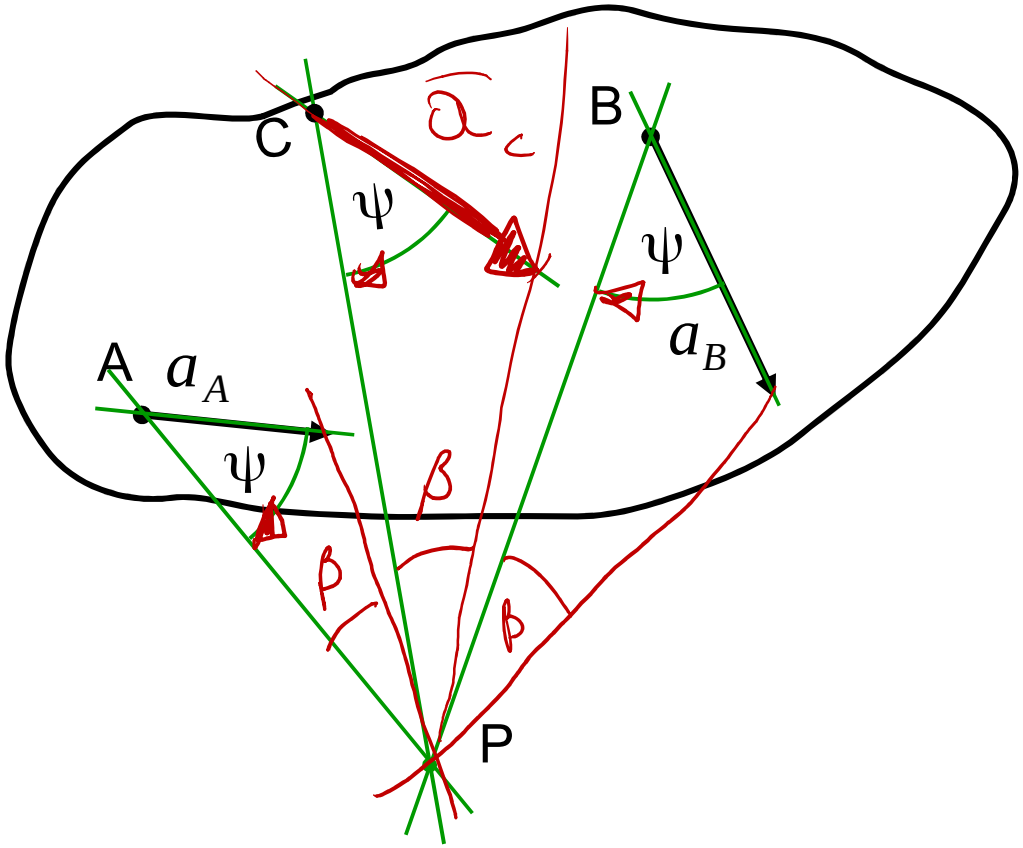
Searched: a_C

1. STEP:
construction of ψ



2. STEP: find out the center of acceleration

3. STEP: find out a_C



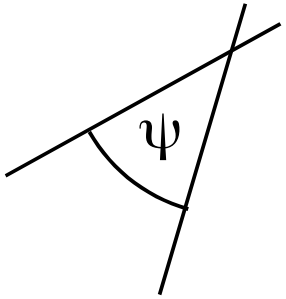
Instantaneous center of acceleration method

Example

Given: a_A and a_B

Searched: a_C

1. STEP:
construction of ψ



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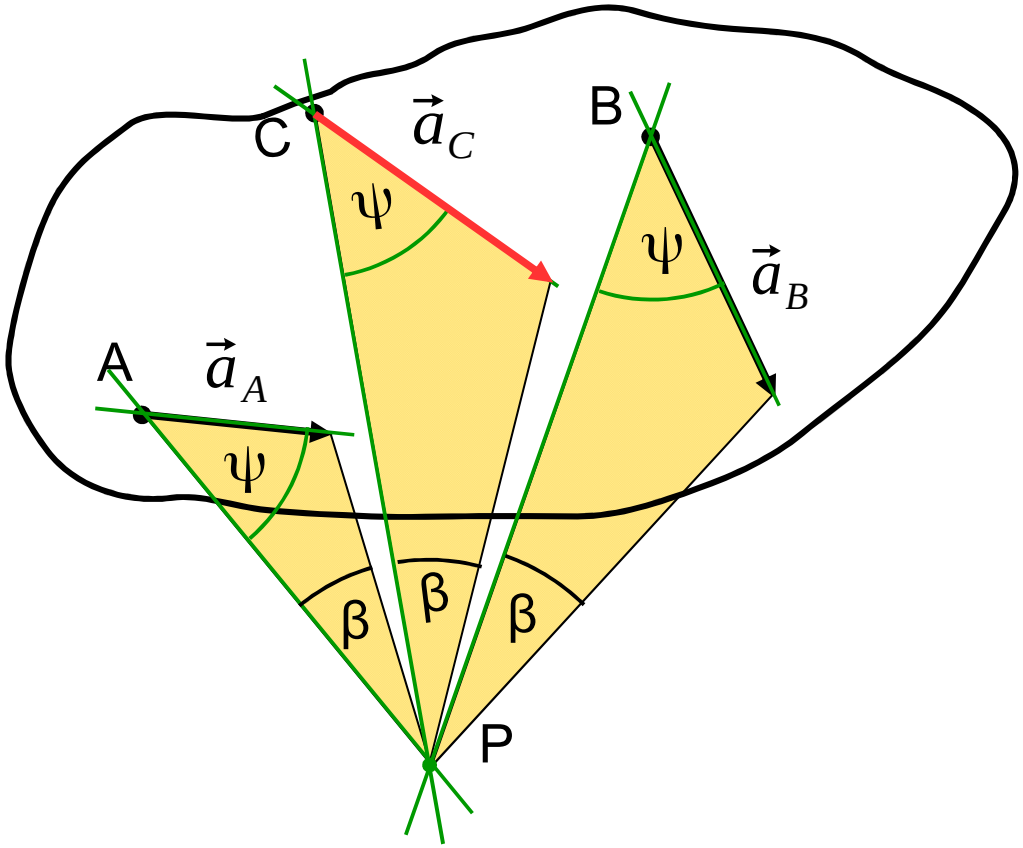
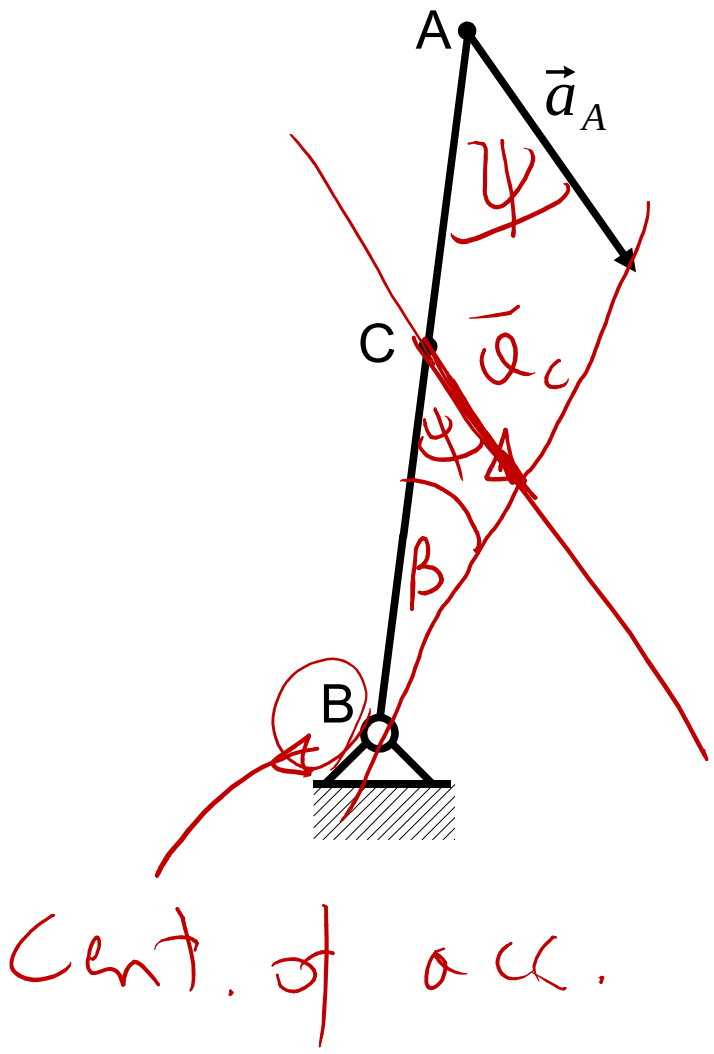


Instantaneous center of acceleration method

Example 2

Given: a_A

Searched: a_C



Instantaneous center of acceleration method

Example 2

Given: a_A

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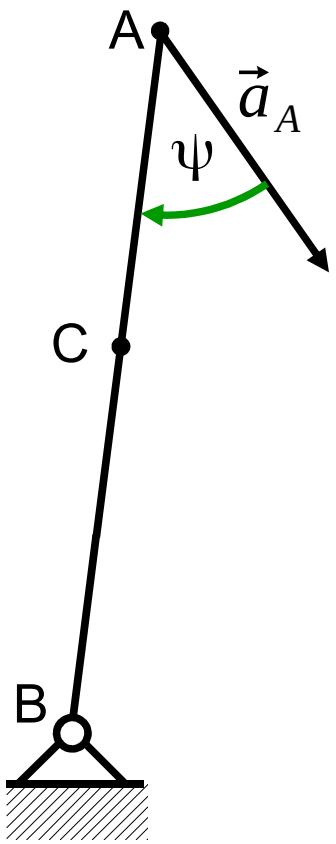

Instantaneous center of acceleration method

Example 2

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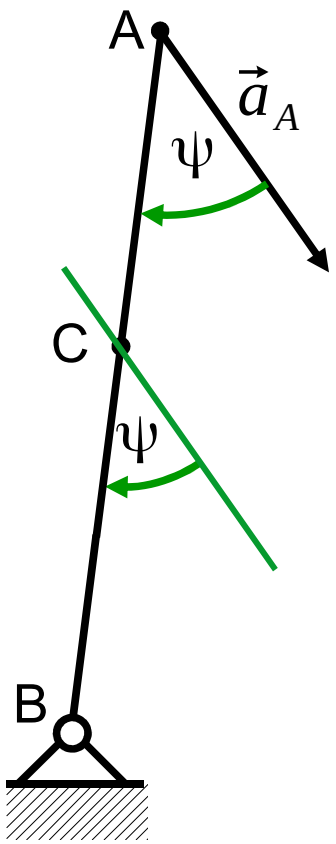

Instantaneous center of acceleration method

Example 2

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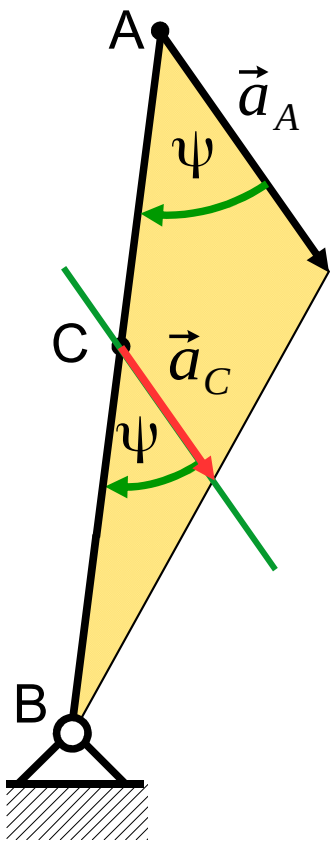
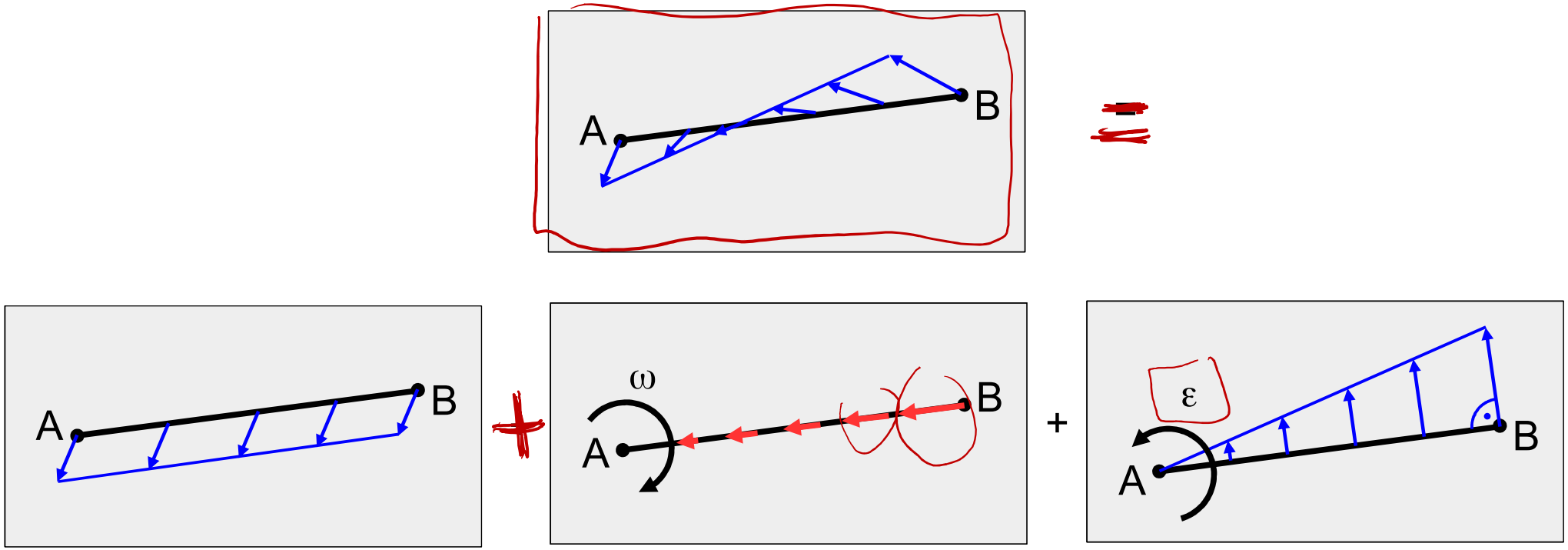


Acceleration decomposition method

Example



LINEAR

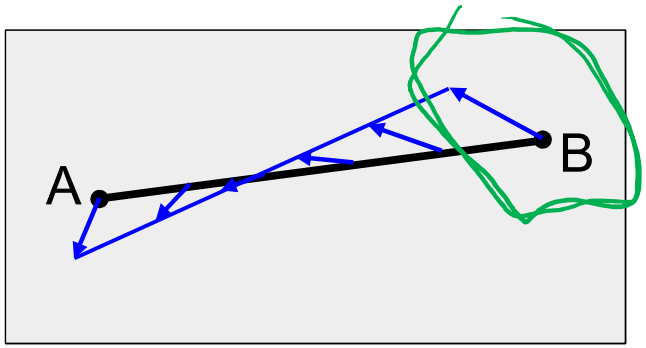
NORMAL

TANGENTIAL

ANGULAR

Acceleration decomposition method

Example



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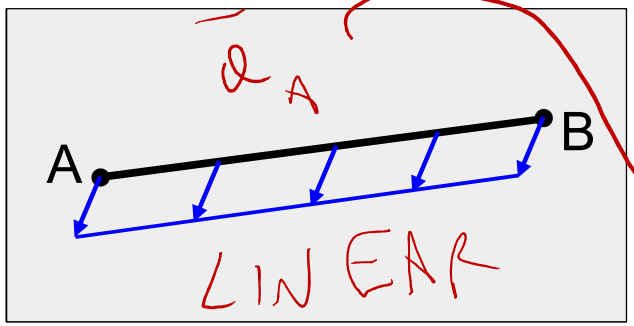

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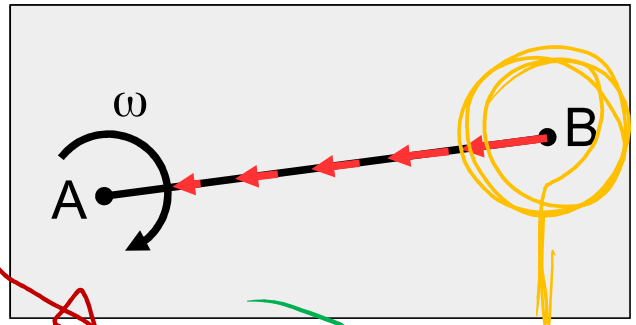

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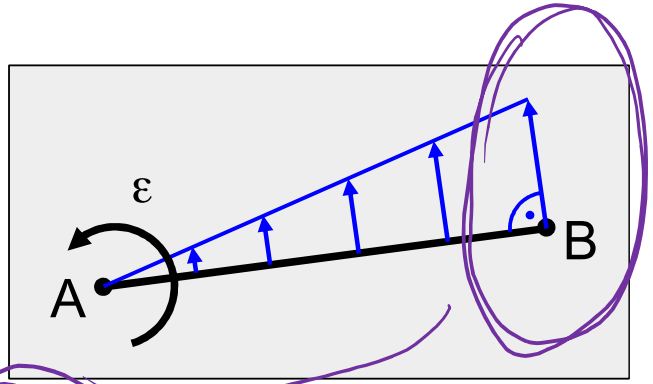


$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}^{\,n}_{BA} + \vec{a}^{\,t}_{BA}$$

absolute acceleration of point B

absolute acceleration of point A

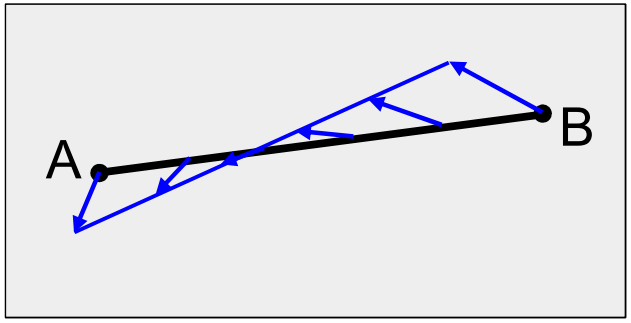
Angular acceleration of point B in rotation around point A.

Centripetal acceleration (normal)

Rotary acceleration (tangential)

Acceleration decomposition method

Example



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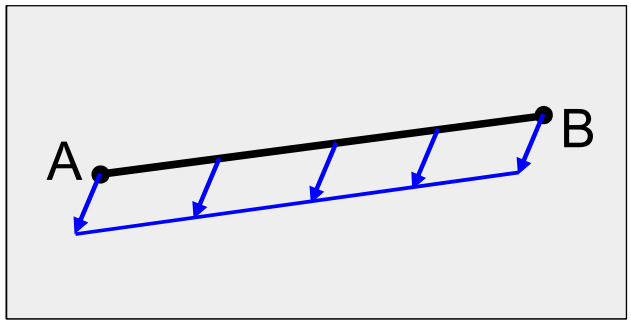

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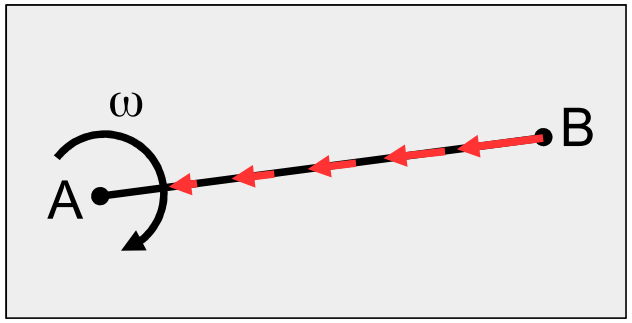

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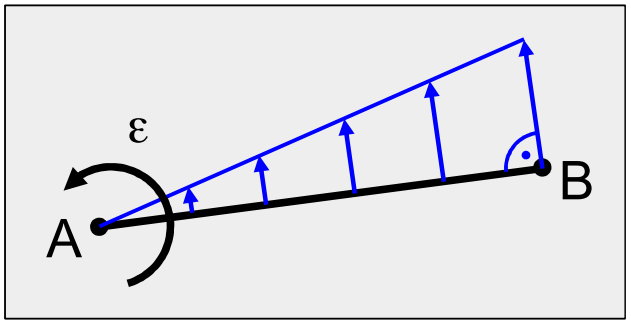


$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}^{\,n}_{BA} + \vec{a}^{\,t}_{BA}$$

Centripetal acceleration (normal)

Rotary acceleration (tangential)

$$\vec{a}^{\,n}_{BA} = \vec{\omega} \times (\vec{\omega} \times \overrightarrow{AB}) = -\,\omega^2 \overrightarrow{AB}$$

2D

$$|\vec{a}^{\,n}_{BA}| = \omega^2 |AB|$$

$$\vec{a}^{\,t}_{BA} = \vec{\varepsilon} \times \overrightarrow{AB}$$

2D $|\vec{a}^{\,t}_{BA}| = |\varepsilon| \cdot |AB|$

Acceleration scheme (diagram)

Acceleration scheme of a rigid body – geometry created by the ends of it's acceleration vectors moved to the common starting point (acceleration scheme's pole).

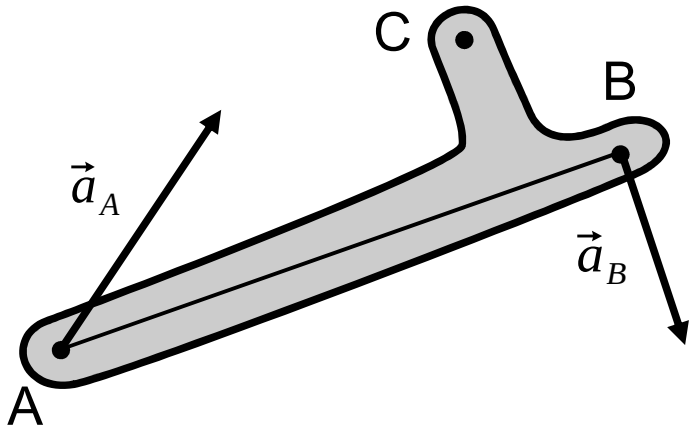
Acceleration scheme is similar to the corresponding rigid body: it is scaled and rotated by $(180^{o}-\psi)$ *angle in the direction of body's angular velocity if* $\mathrm{sgn}\omega=\mathrm{sgn}\varepsilon$ *(or opposite direction if* $\mathrm{sgn}\omega\neq\mathrm{sgn}\varepsilon$*).*

Acceleration scheme method

Example

Given: $\vec{a}_A$ and $\vec{a}_B$ + geometry

Searched: a_C

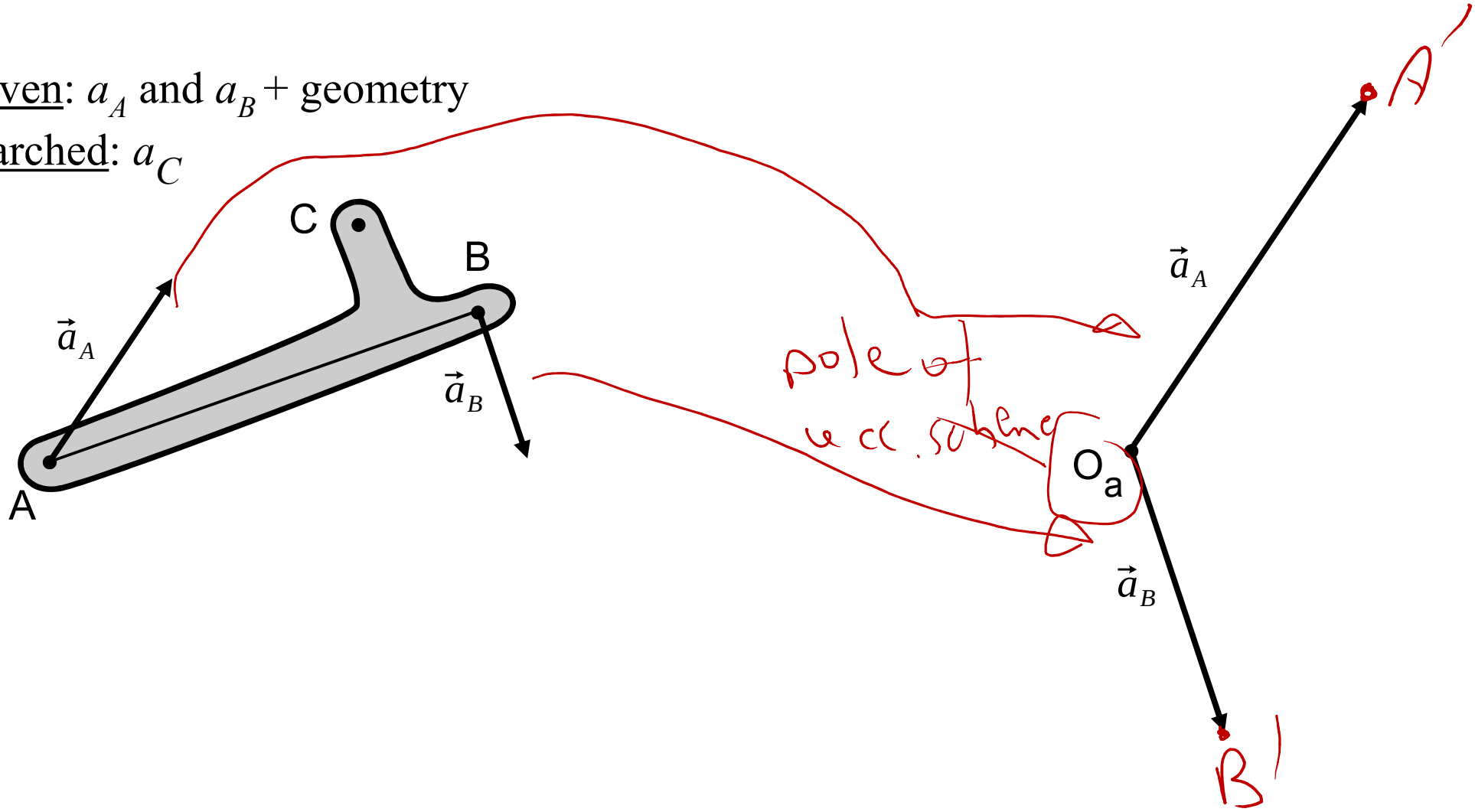


Acceleration scheme method

Example

Given: a_A and a_B + geometry

Searched: a_C



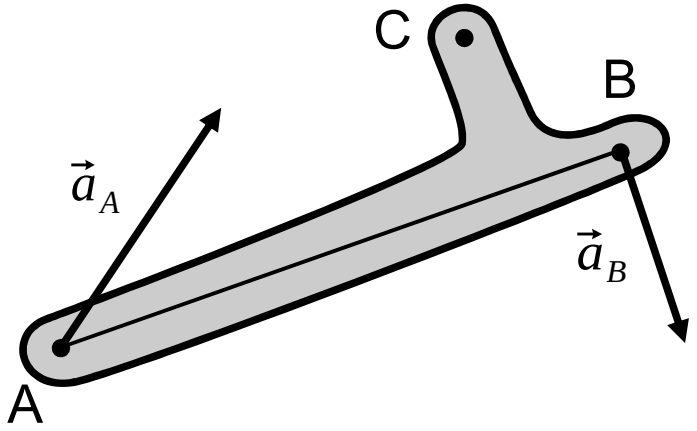
acceleration scale, e.g.: 1cm → 1m/s^2

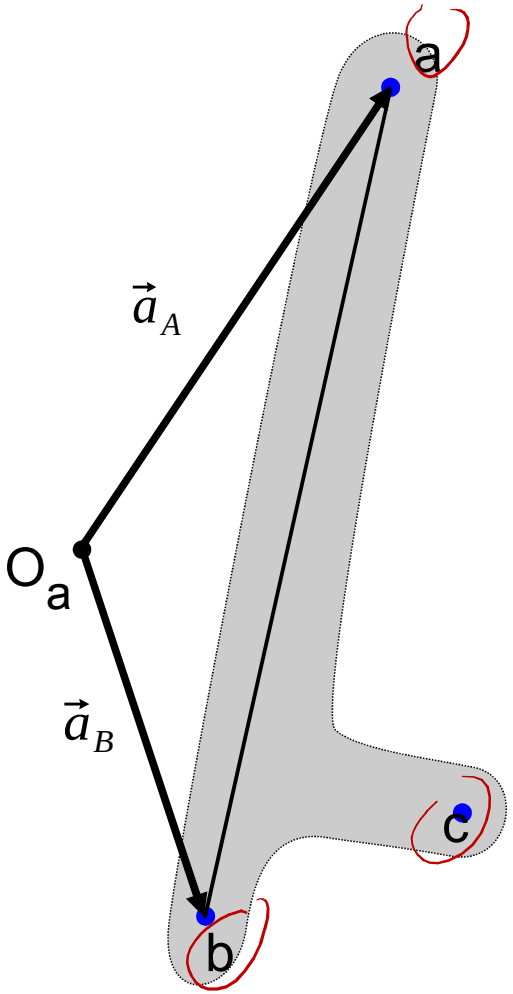
Acceleration scheme method

Example

Given: a_A and a_B + geometry

Searched: a_C





acceleration scale, e.g.: 1cm → 1m/s

Acceleration scheme method

Example

Given: a_A and a_B + geometry

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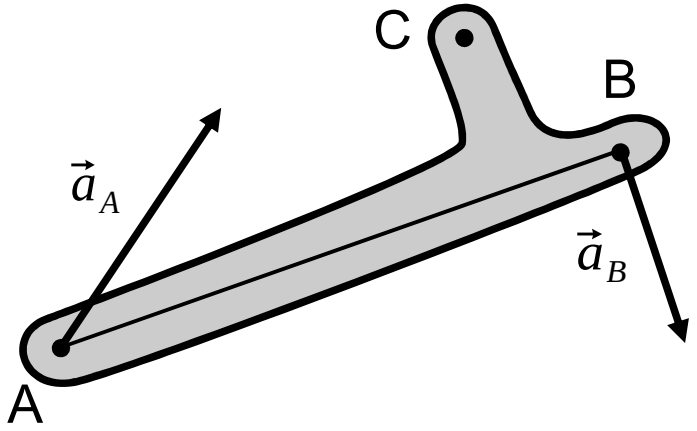

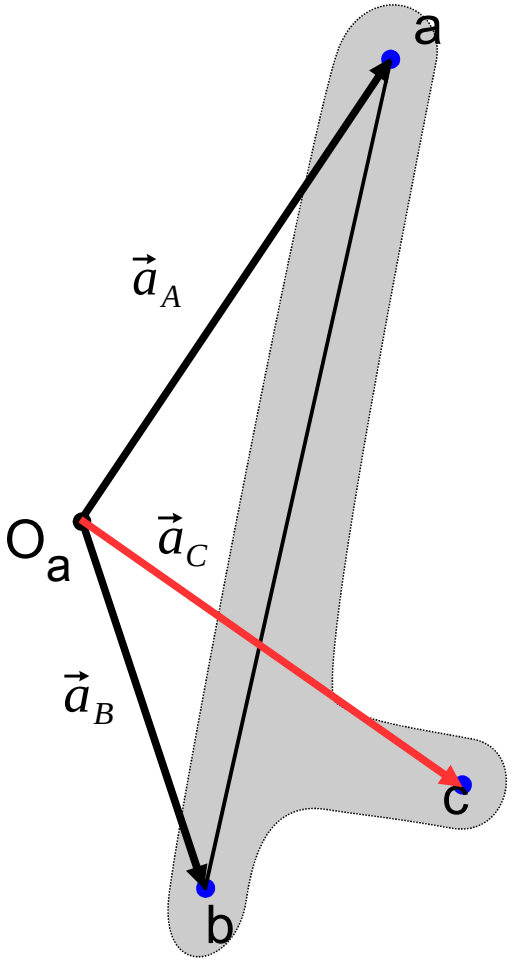

acceleration scale, e.g.: 1cm → 1m/s

Acceleration scheme method

Example

Given: a_A and a_B + geometry

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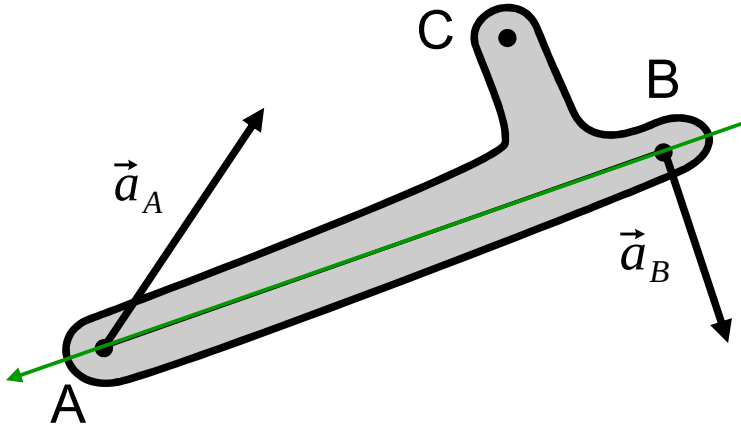
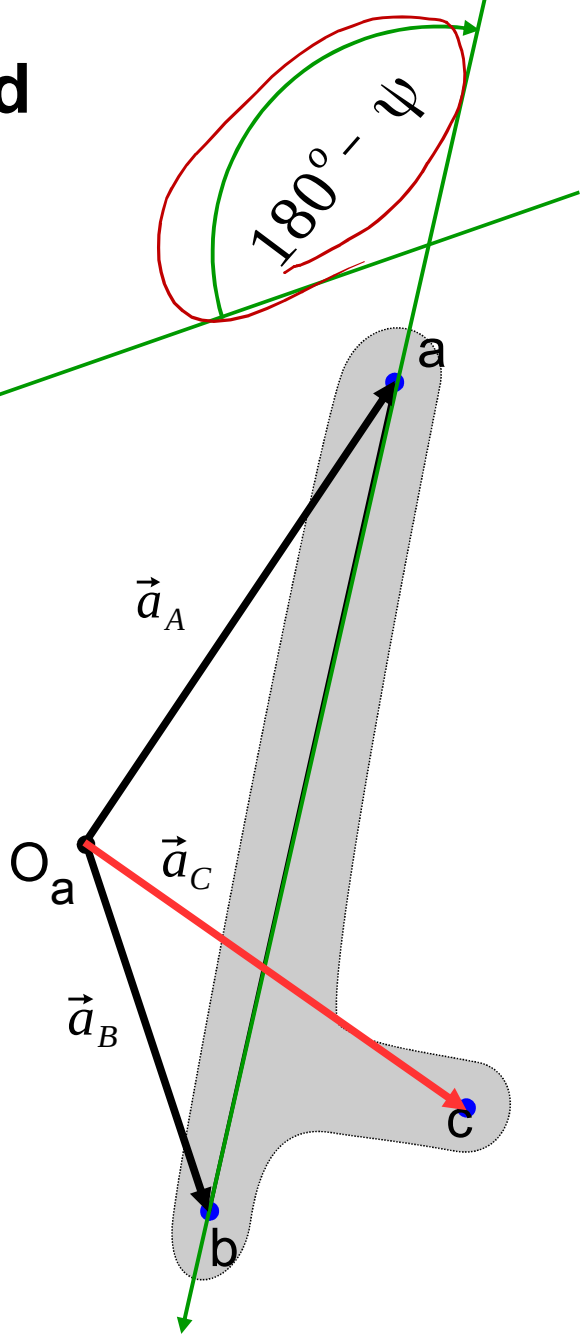
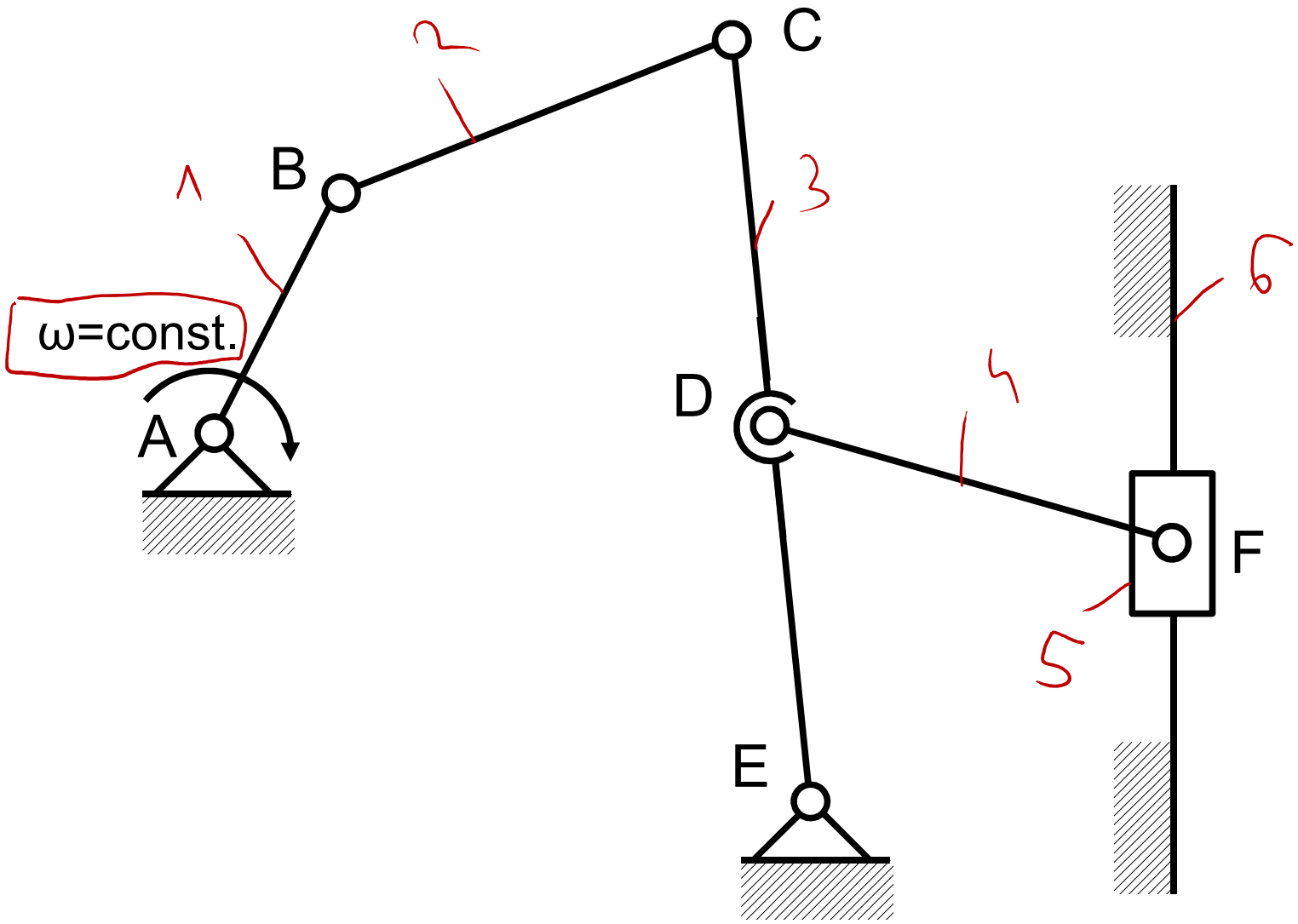




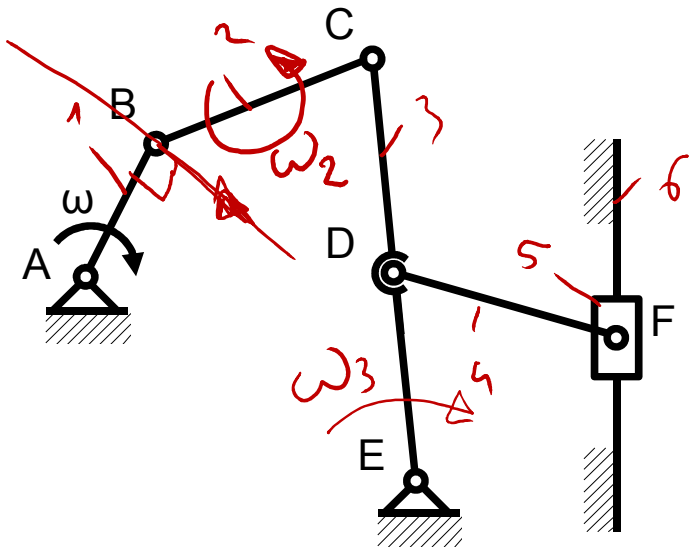
acceleration scale, e.g.: 1cm → 1m/s

Velocities and accelerations – example

Find velocity and acceleration of the point F using decompositiom methods



Velocities and accelerations – example



$$\bar{V}_C = \bar{V}_B + \bar{V}_{CB}$$

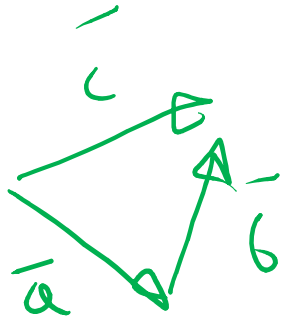
$\perp 3 \quad \perp 1 \quad \perp 2$ (decomposition for element 2)

$$|\bar{V}_B| = \omega \cdot |AB|$$

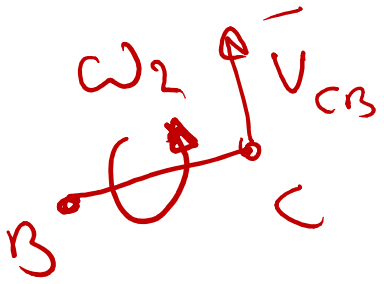
$\perp 1$

$$|\omega_3| = \frac{|\bar{V}_C|}{|CE|}$$

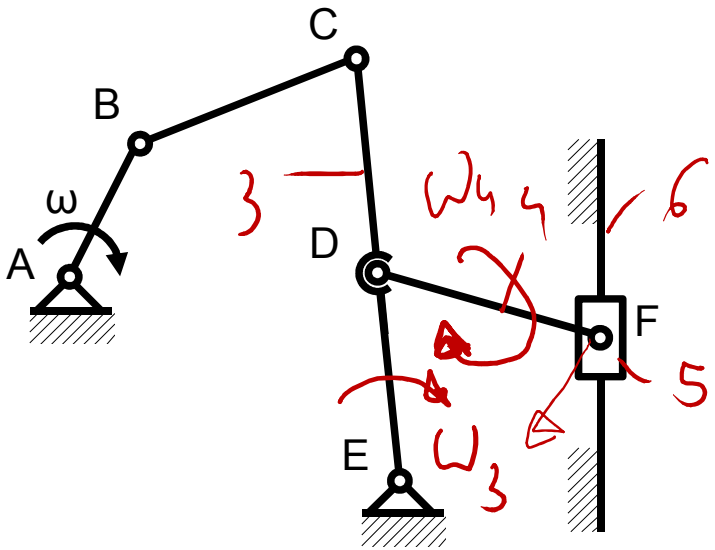
$$|\omega_2| = \frac{|\bar{V}_{CB}|}{|CB|}$$



$\bar{a} + \bar{b} = \bar{c}$



Velocities and accelerations – example



Decomposition method for mechanism 4

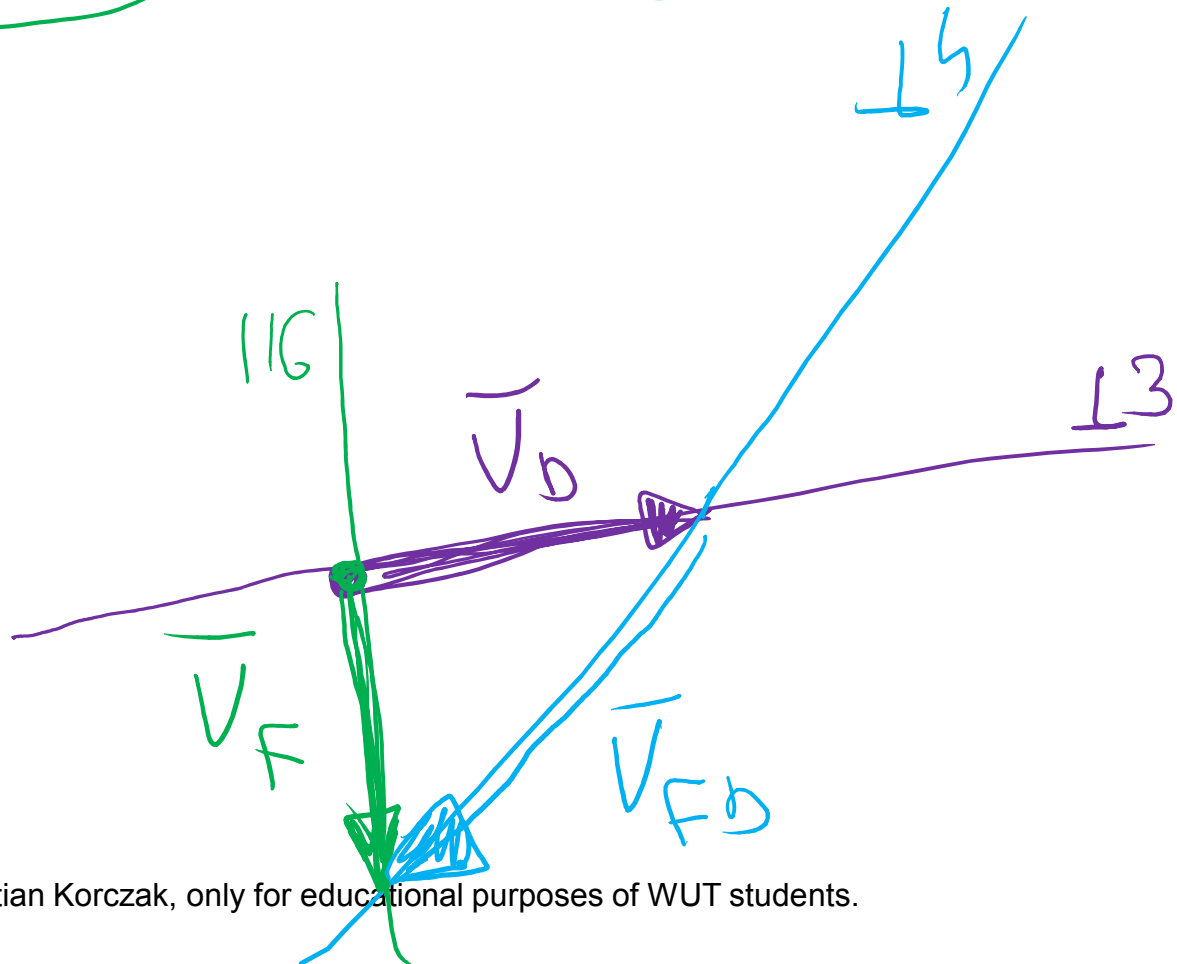
$$\bar{V}_F = \bar{V}_D + \bar{V}_{FD}$$

$\bar{V}_F \perp 16$, $\bar{V}_D \perp 13$, $\bar{V}_{FD} \perp 14$

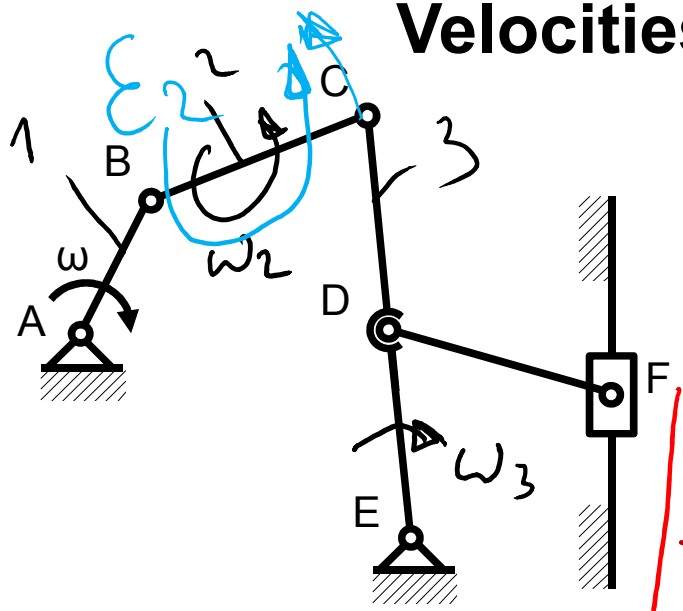
$$|V_D| = \omega_3 \cdot |DE|$$

$\bar{V}_D \perp 3$

$$\omega_4 = \frac{|\bar{V}_{FD}|}{|FD|}$$



Velocities and accelerations – example



decomposition for element 2 $\varepsilon_2 = \frac{|a^t_{CB}|}{|CB|}$

$$\bar{a}_C = \bar{a}_B + \bar{a}^n_{CB} + \bar{a}^t_{CB}$$

$$\underbrace{\bar{a}^n_{CE}}_{113} + \underbrace{\bar{a}^t_{CE}}_{13} = \underbrace{\bar{a}^n_{BA}}_{111} + \underbrace{\bar{a}^n_{CB}}_{112} + \underbrace{\bar{a}^t_{CB}}_{12}$$

$$\bar{a}_B = \underbrace{\bar{a}^n_{BA}}_{111} + \underbrace{\bar{a}^t_{BA}}_{11}$$

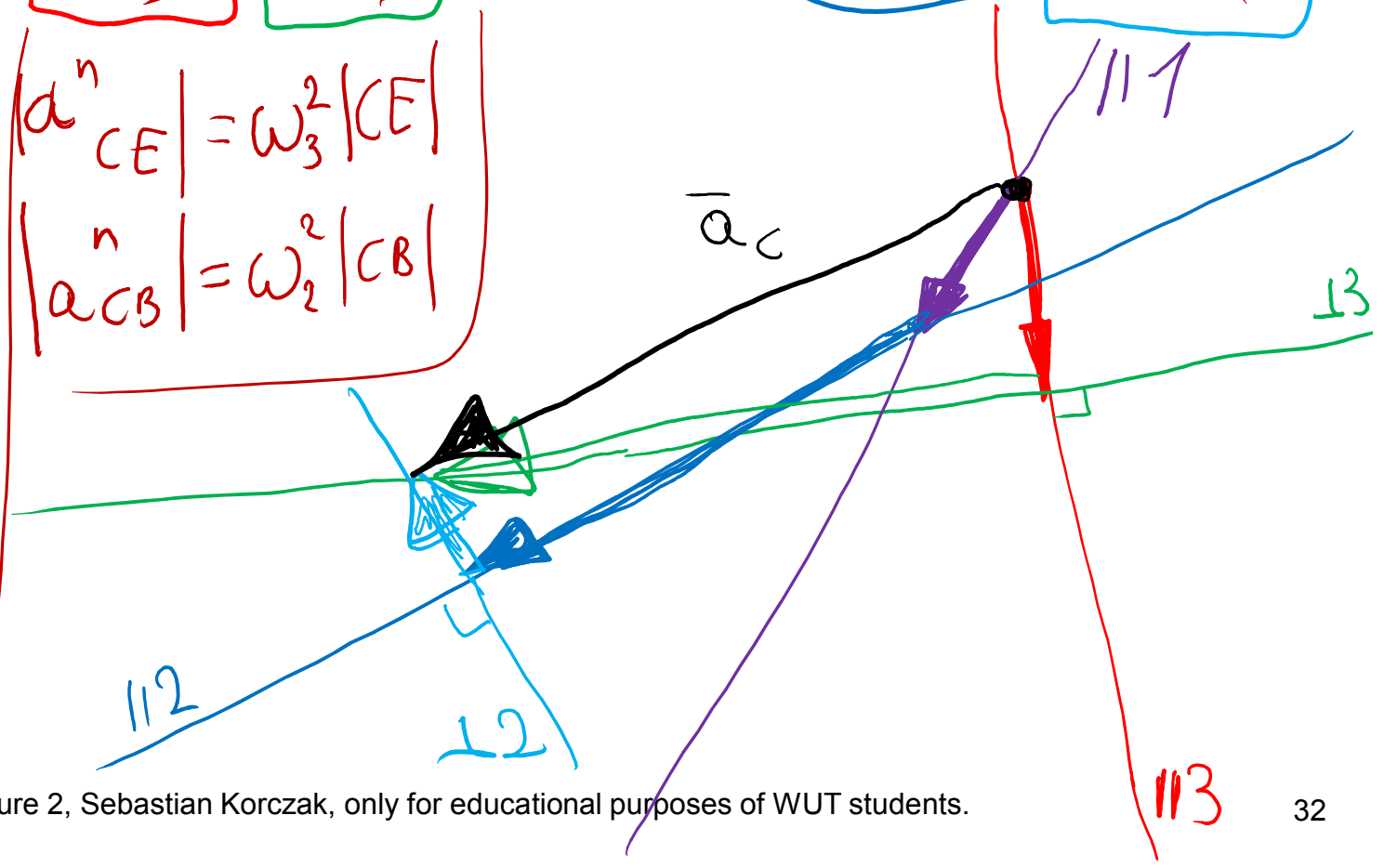
$$|a^n_{CE}| = \omega_3^2 |CE|$$

$$|a^n_{CB}| = \omega_2^2 |CB|$$

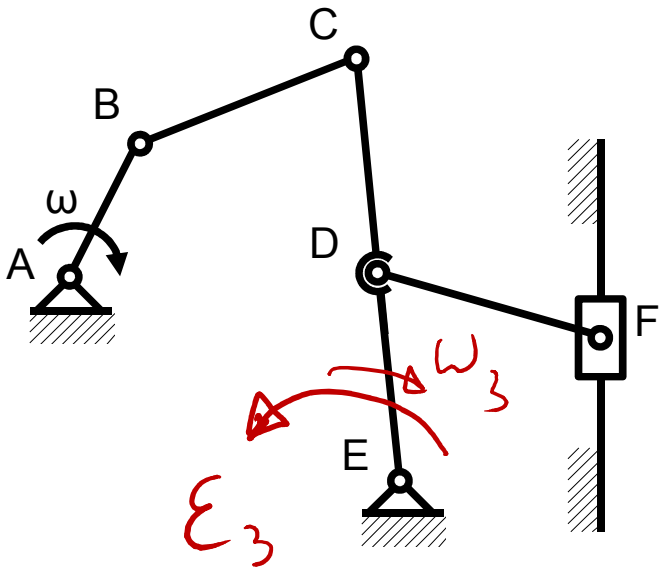
$$|a^n_{BA}| = \omega^2 |AB|$$

$$|a^t_{BA}| = \varepsilon |AB| = 0$$

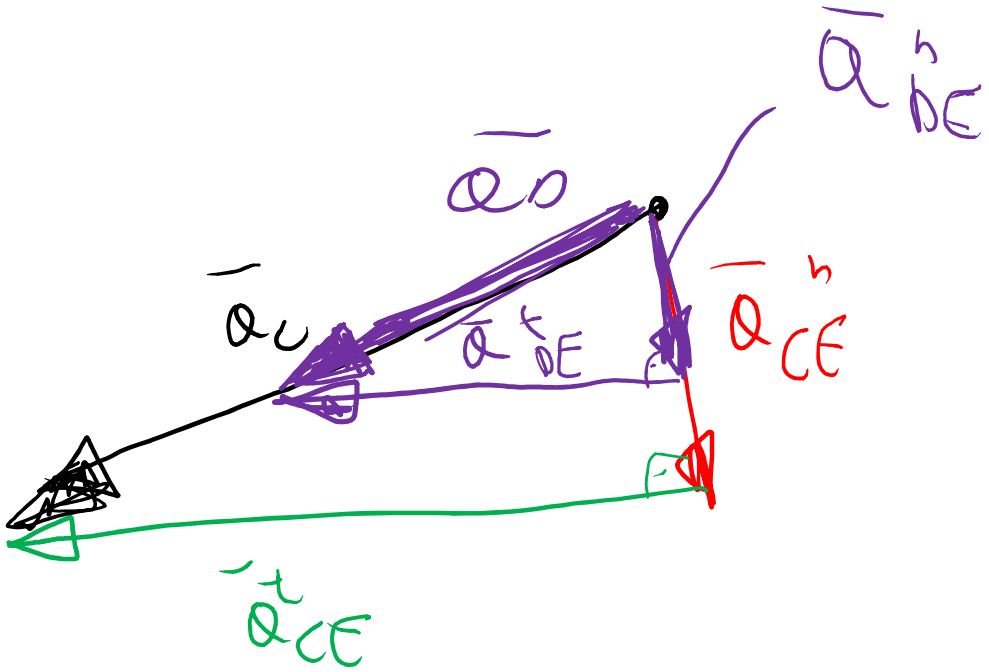
$$\varepsilon = \frac{d\omega(t)}{dt} = 0$$



Velocities and accelerations – example



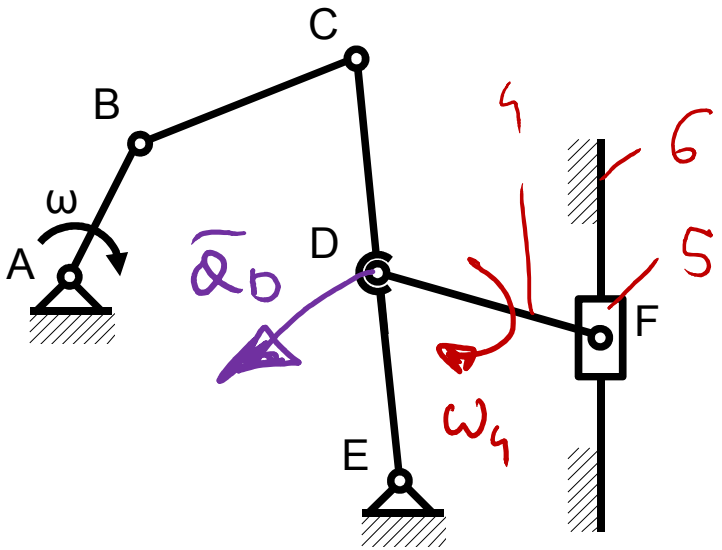
$$\frac{|\bar{a}^t_{CE}|}{|CE|} = \varepsilon_3$$



$$\bar{a}_D = \bar{a}^n_{DE} + \bar{a}^t_{DE}$$

$$\| \bar{a}_C \quad \omega_3^2 |DE| \quad \varepsilon_3 |DE|$$

Velocities and accelerations – example

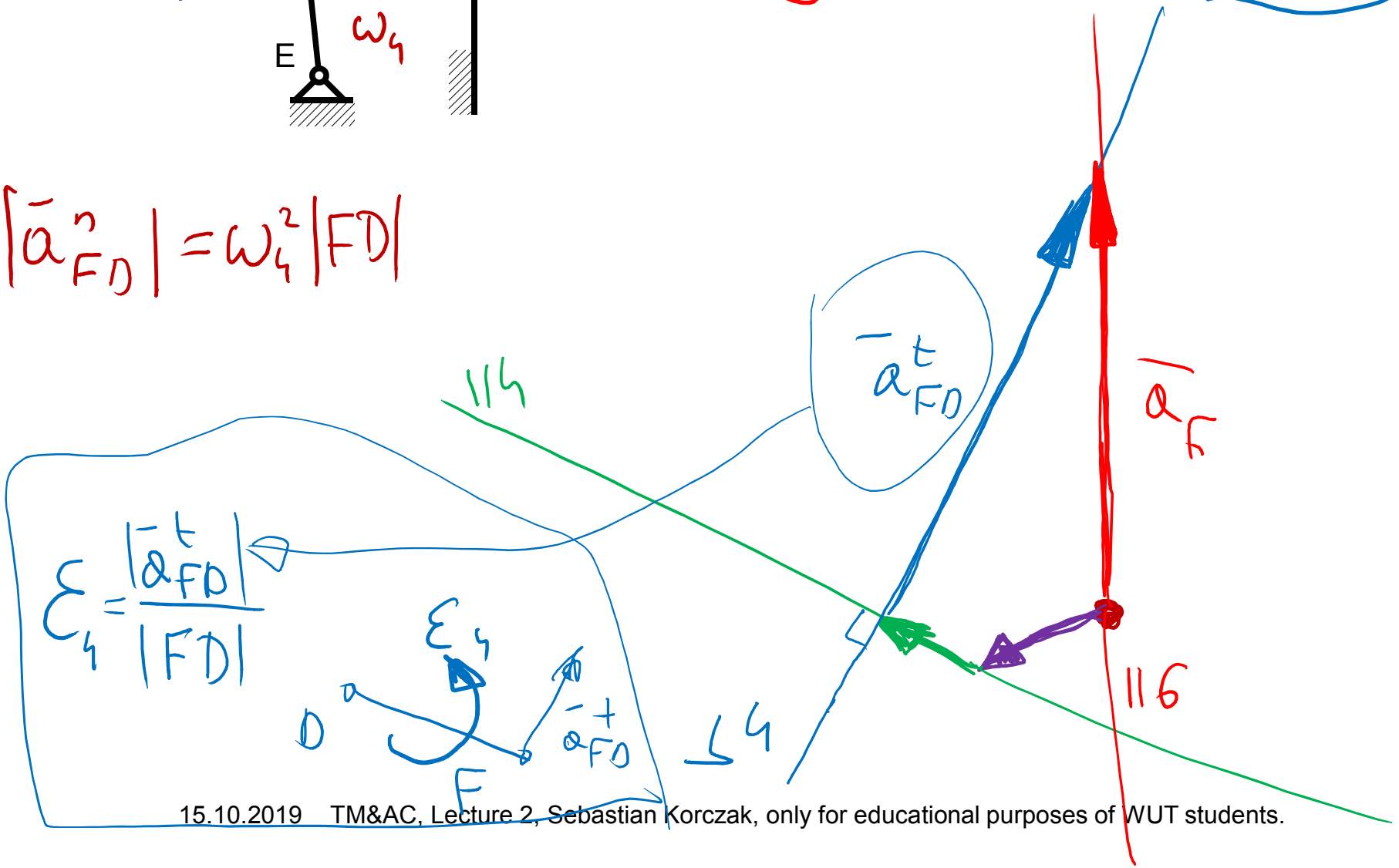


Decomposition for element nr 4

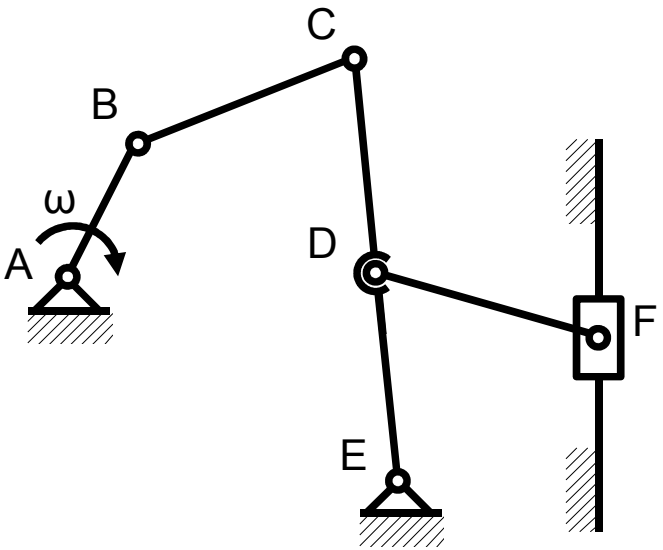
$$\bar{a}_F = \bar{a}_D + \bar{a}^n_{FD} + \bar{a}^t_{FD}$$

$\bar{a}_F$ ⊥ 16 ; $\bar{a}_D$; $\bar{a}^n_{FD}$ ∥ 14 ; $\bar{a}^t_{FD}$ ⊥ 14

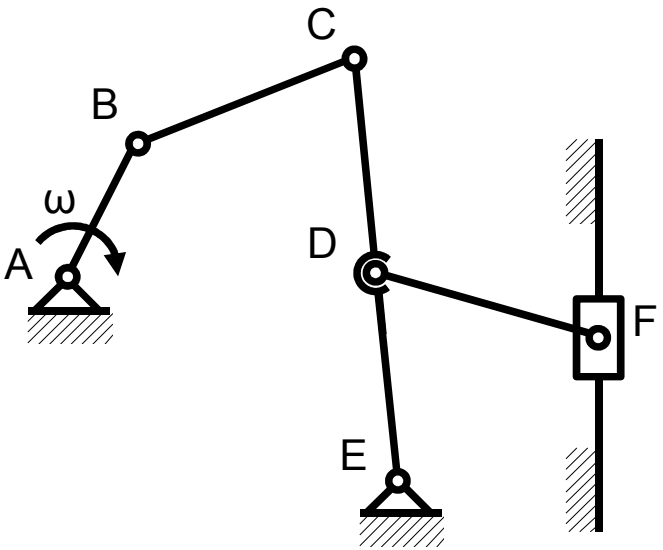
$$|\bar{a}^n_{FD}| = \omega_4^2 |FD|$$



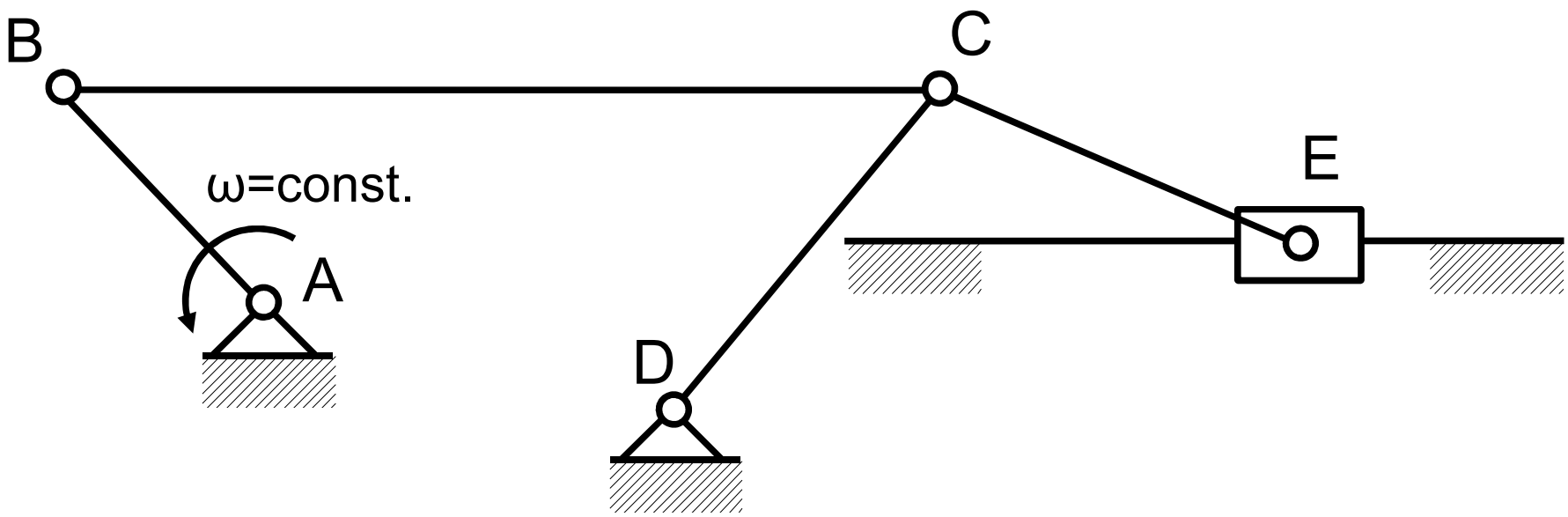
Velocities and accelerations – example



Velocities and accelerations – example



Velocities and accelerations – example 2



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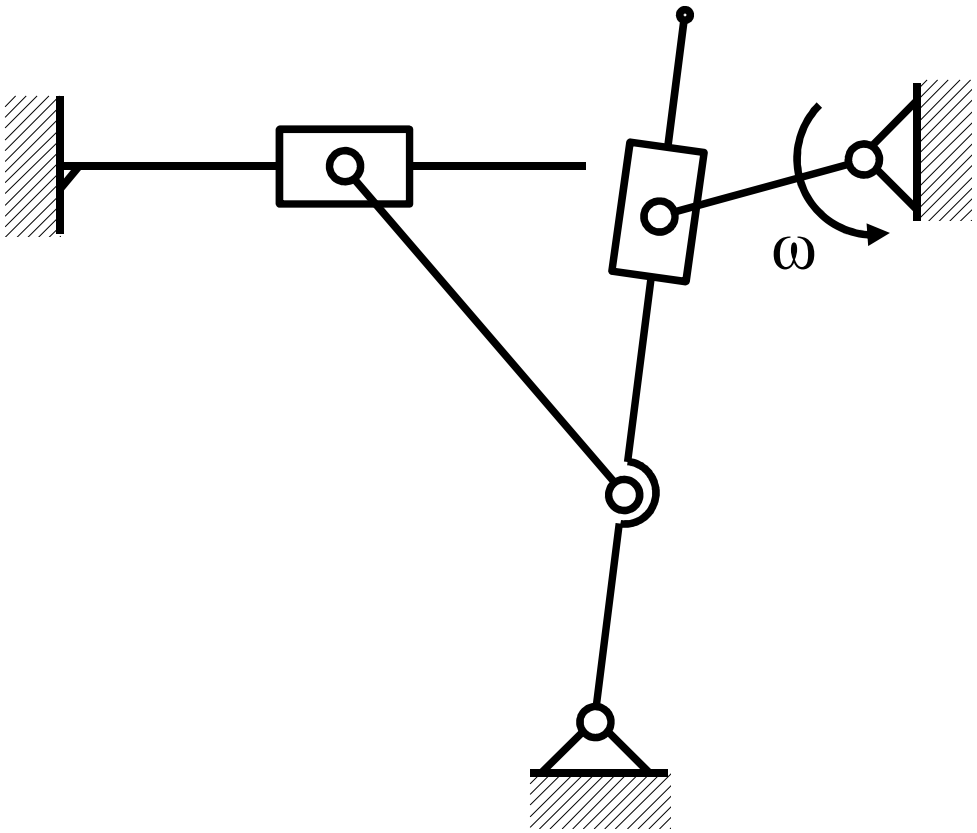
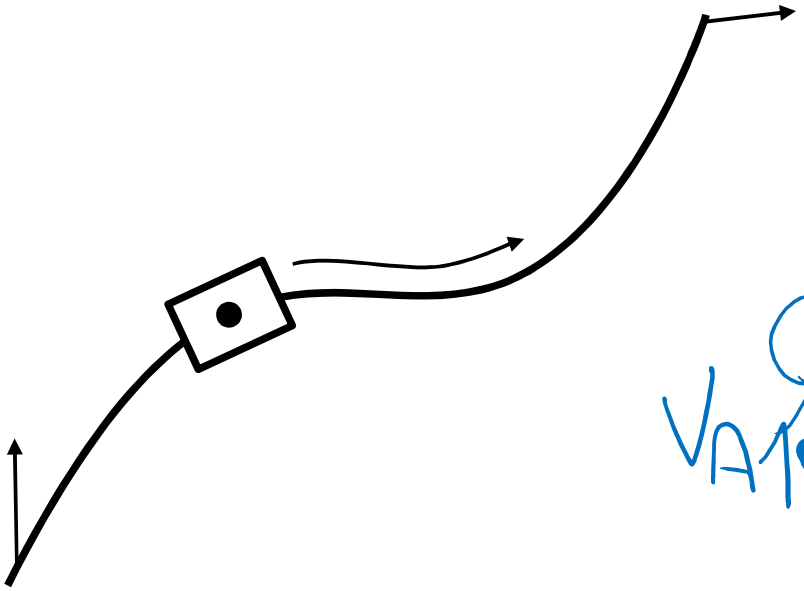
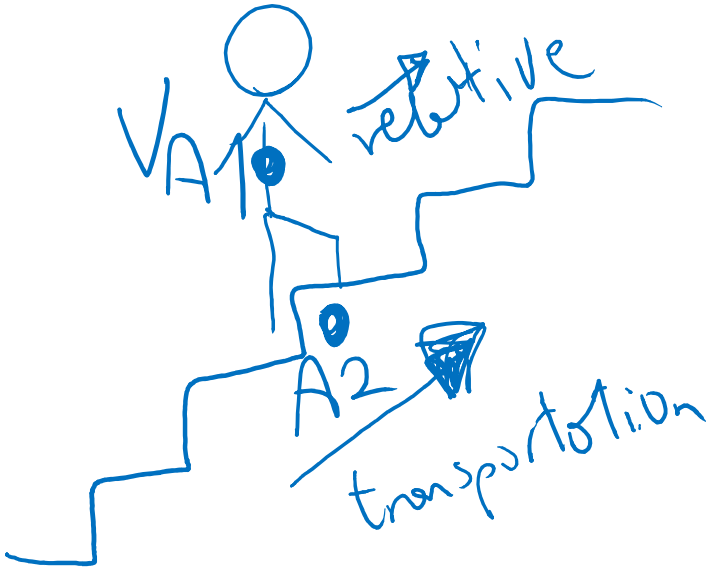
Velocities and accelerations – example 3 (appendix)



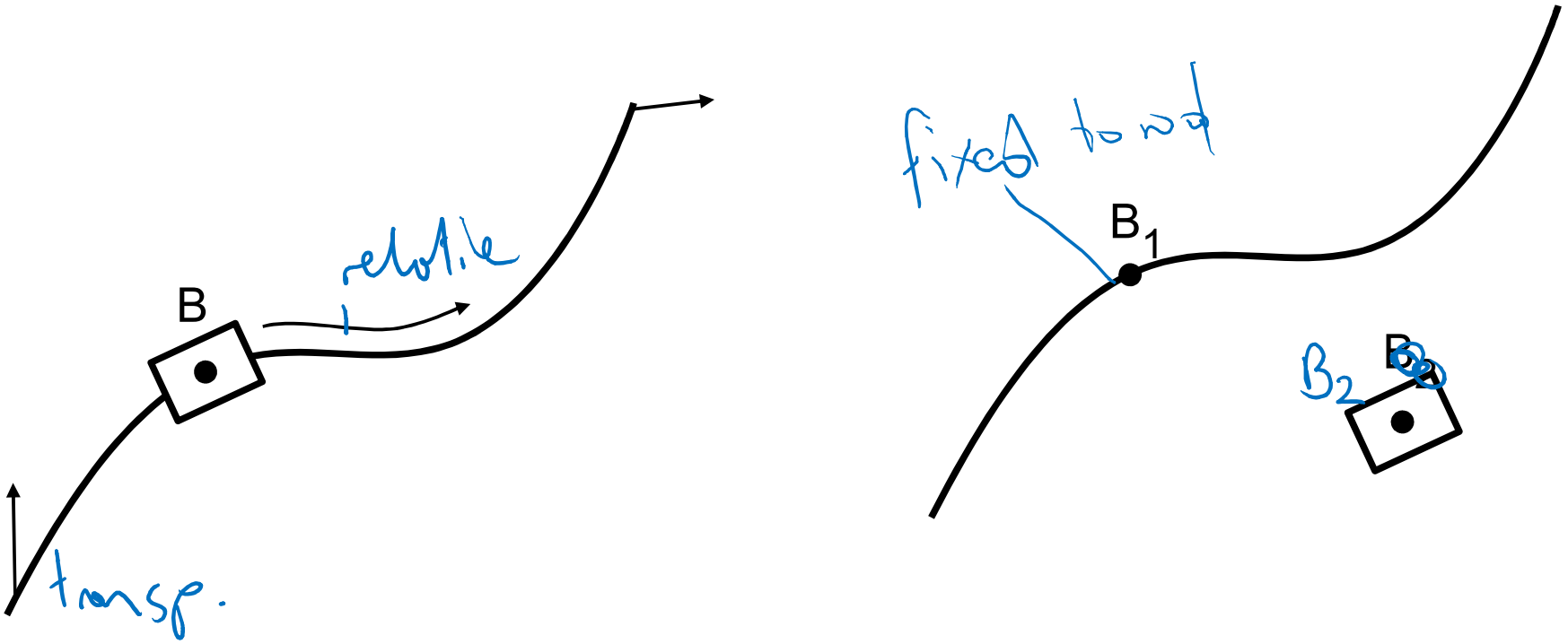
Velocities in relative motion



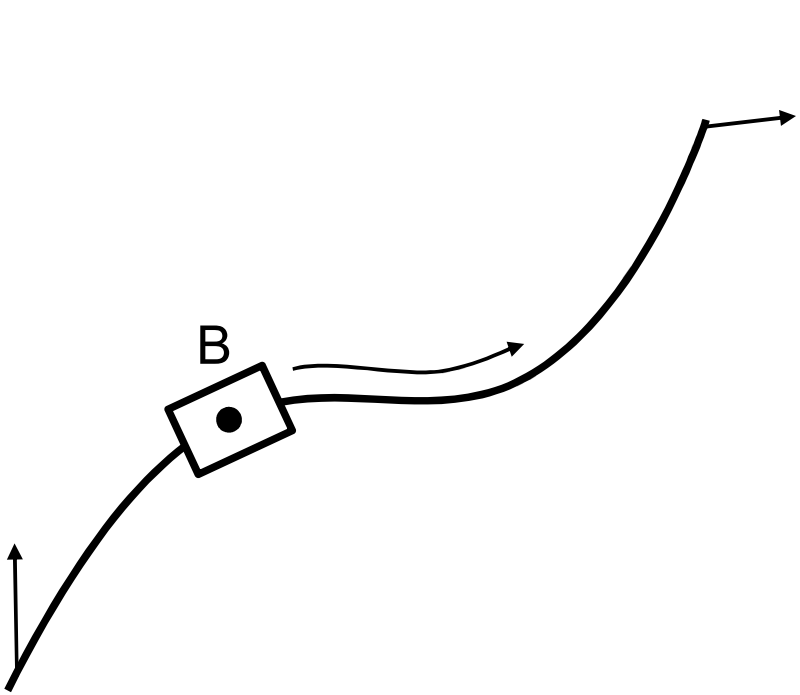
$$V_{A1} = V_{A2} + V_{A1A2}$$

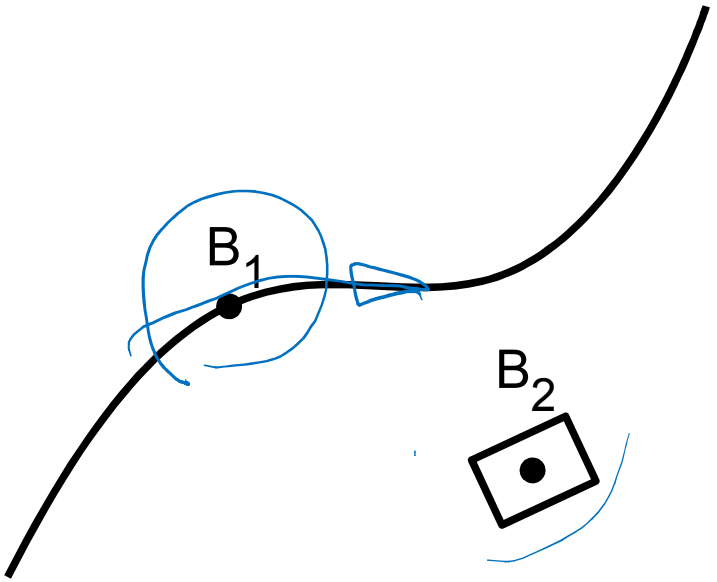


Velocities in relative motion



Velocities in relative motion





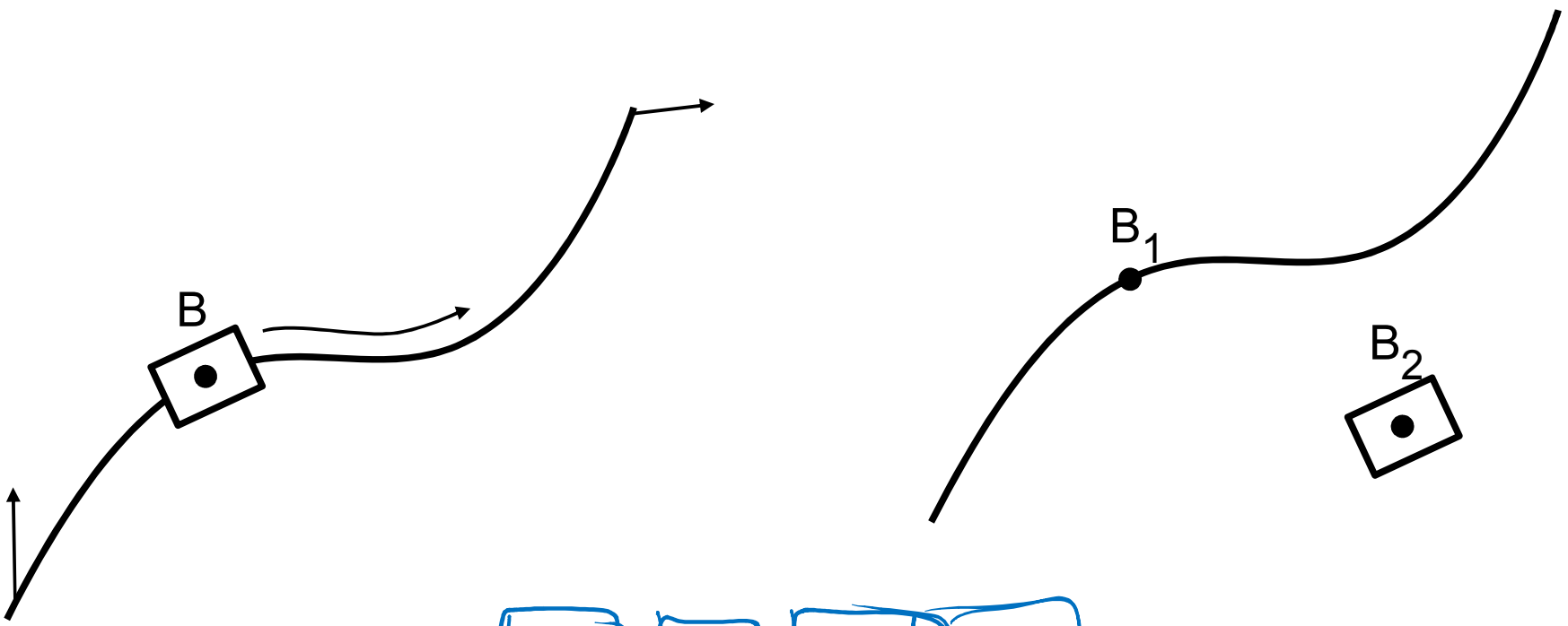
$$\vec{v}_{B2} = \vec{v}_{B1} + \vec{v}_{B2B1}$$

absolute velocity of point B_2

transportation velocity

relative velocity

Accelerations in relative motion



$$\vec{a}_{B2} = \vec{a}^{\,u}_{B1} + \vec{a}^{\,rel.}_{B2B1} + \vec{a}^{\,c}$$

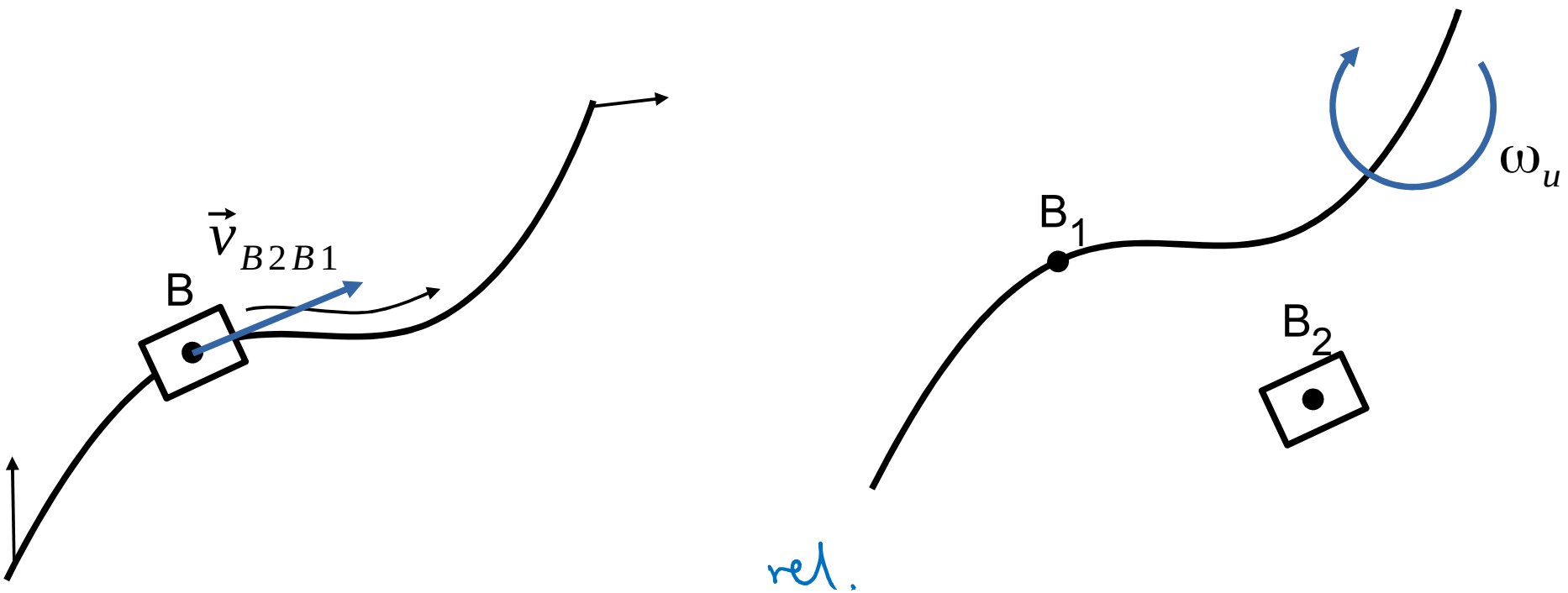
absolute acceleration of point B2

Transportation acceleration (absolute acceleration of point B1)

Relative acceleration

Coriolis acceleration

Accelerations in relative motion



$$\vec{a}_{B2} = \vec{a}^{u}_{B1} + \vec{a}^{rel}_{B2B1} + \vec{a}^{c}$$

absolute acceleration of point B2

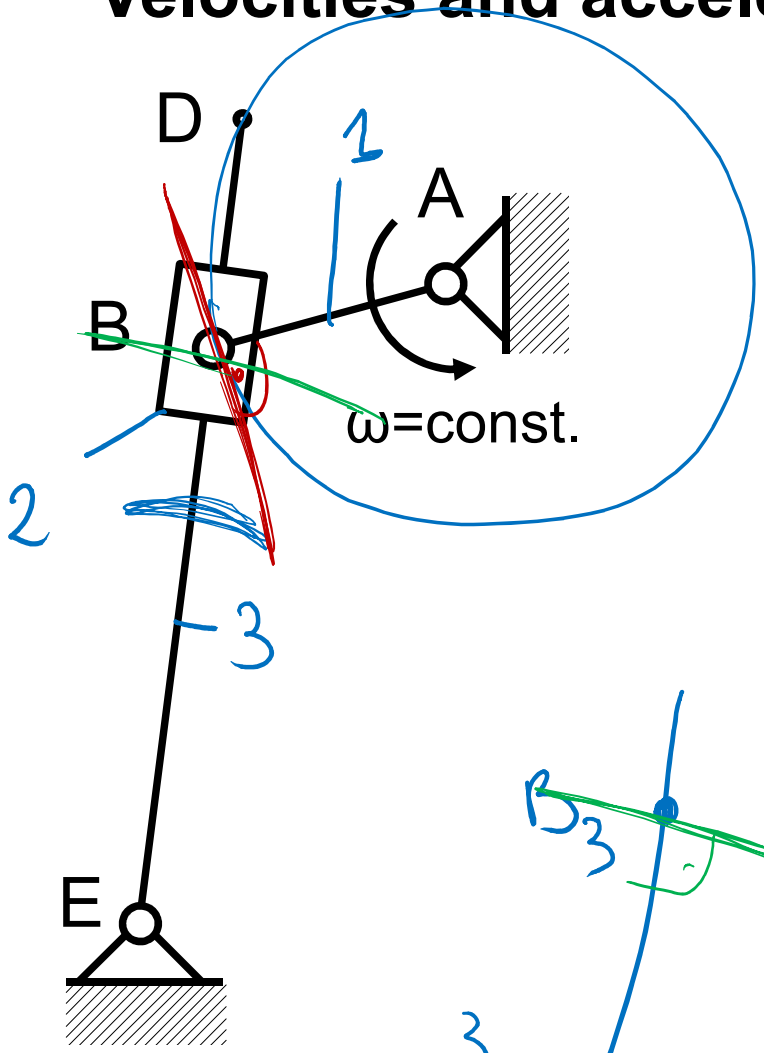
Transportation acceleration (absolute acceleration of point B1)

Relative acceleration

Coriolis acceleration

$$\vec{a}^{c} = 2\vec{\omega}_u \times \vec{v}_{B2B1}$$

Velocities and accelerations in relative motion – example

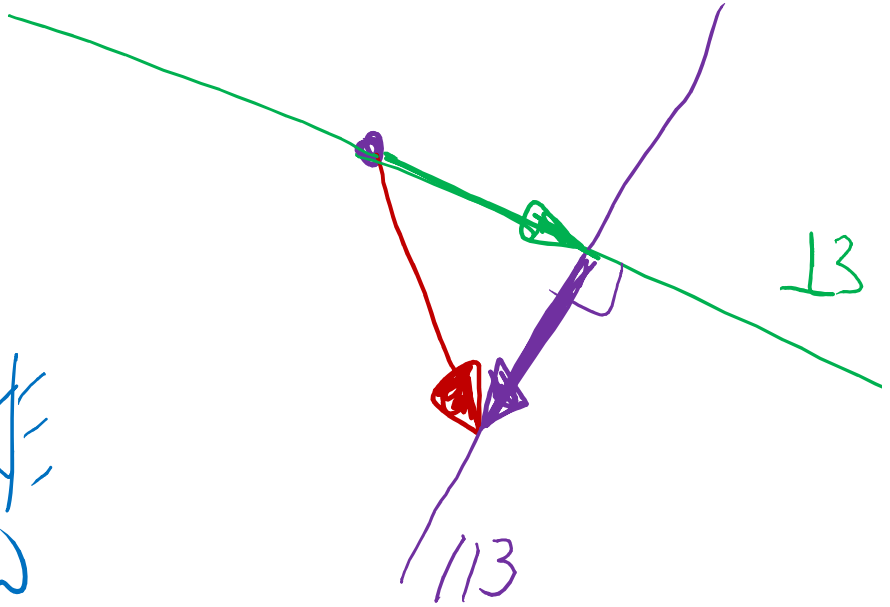


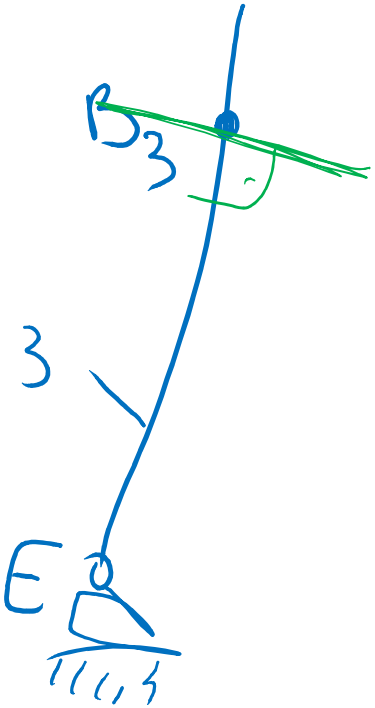
$\bar{v}_{B1} = \bar{v}_{B3} + \bar{v}_{B1B3}$

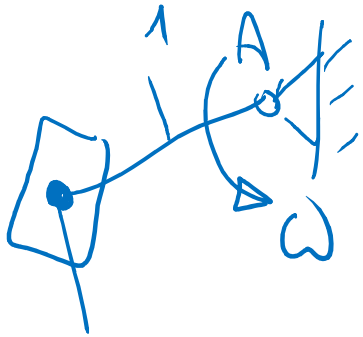
$\bar{v}_{B1} \perp l1$

$\bar{v}_{B3} \perp l3$

$\bar{v}_{B1B3} \parallel 113$







$|\bar{v}_{B1}| = \omega |AB|$

$B_1 = B_2$