



Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control Winter 2019/2020

Lecturer: Sebastian Korczak, PhD, Eng.

Lecture 1 cont.

kinematic pairs, mechanisms, mobility

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Kinematic chain mobility

kinematic chain mobility – structural formula

(the Chebychev–Grübler–Kutzbach criterion)

$$(3D \text{ chain}) \quad F = 6N - p_1 - 2p_2 - 3p_3 - 4p_4 - 5p_5$$

$$(2D \text{ chain}) \quad F = 3N - p_4 - 2p_5$$

N – number of moving bodies

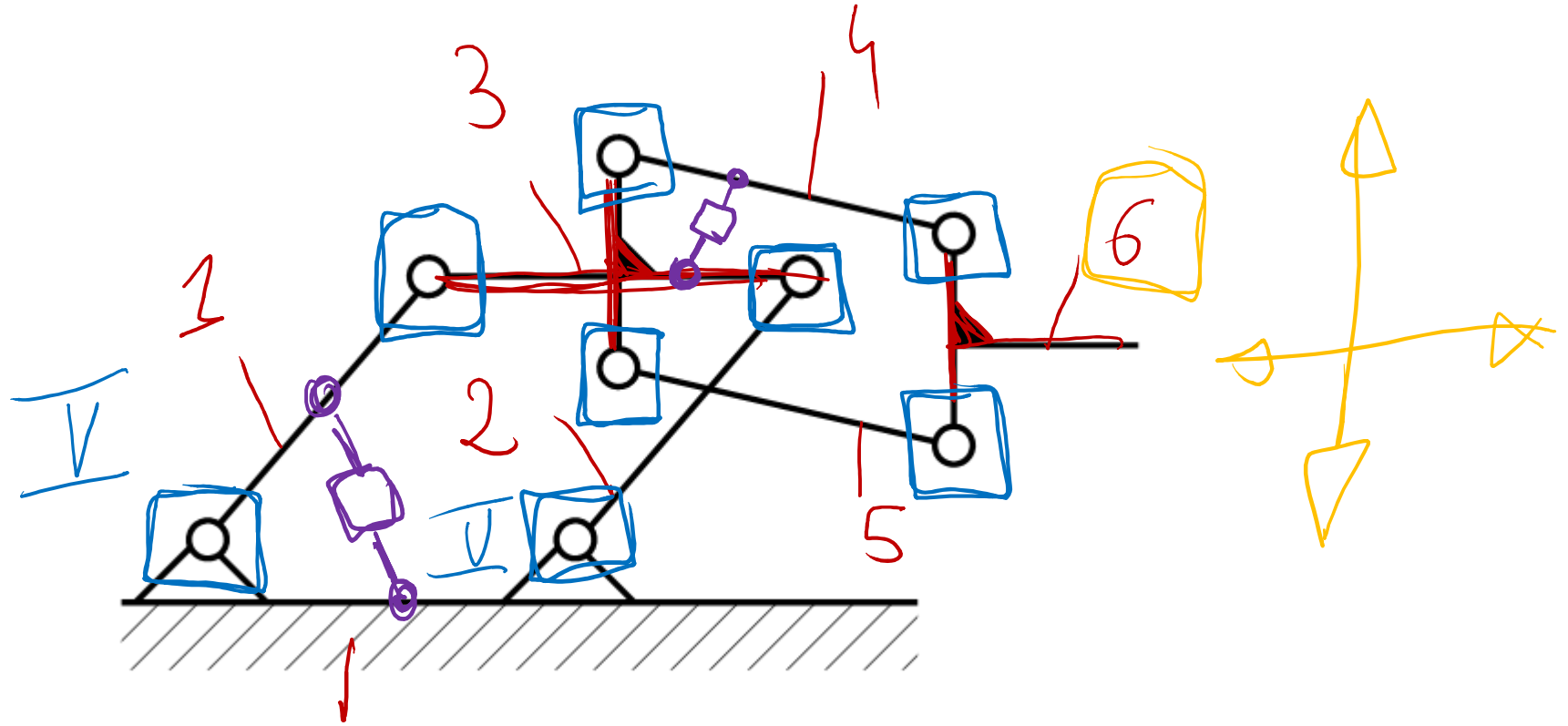
p_i – number of i -type classes

$F \geq 1$ – movable

$F < 1$ – locked or overconstrained

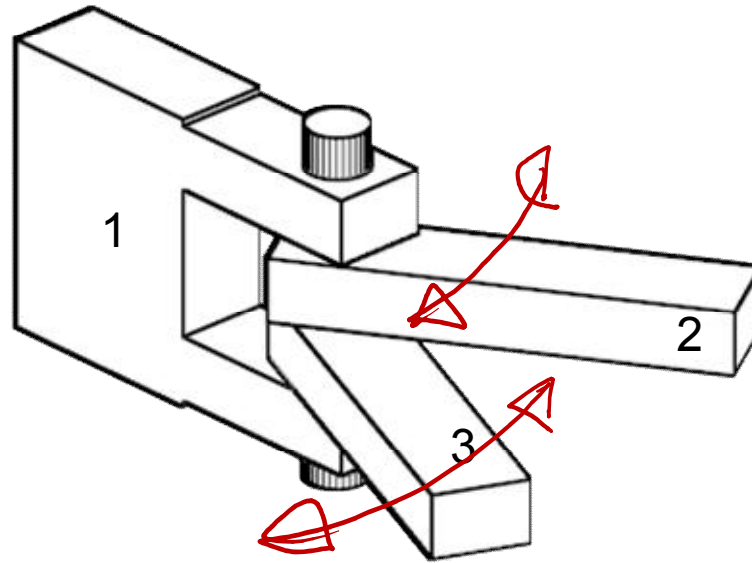
Kinematic chain - examples

$$N=6$$
$$p_5=8$$



$$F = 3 \cdot N - 2p_5 = 3 \cdot 6 - 2 \cdot 8 = 2$$

Multiple kinematic pairs



IV 1-2

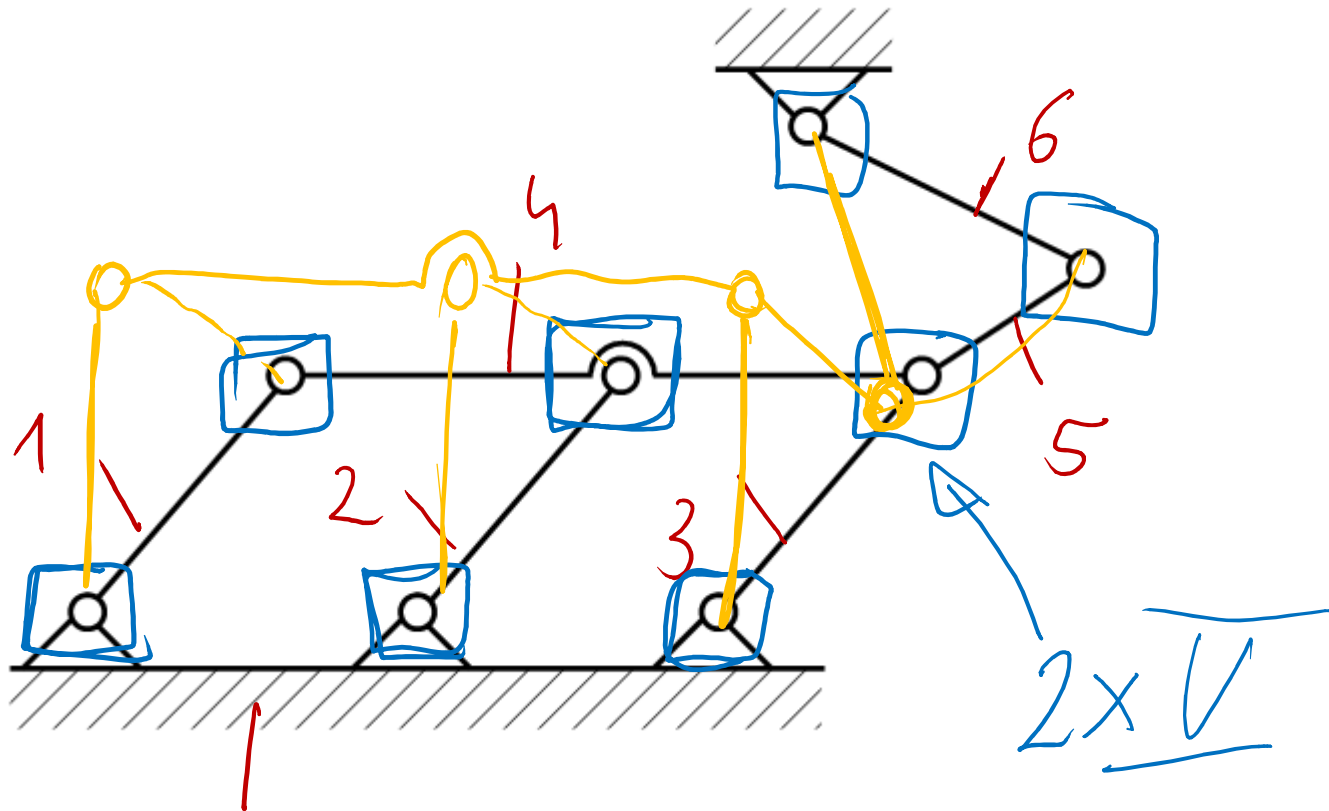
V 1-3

~~IV 2-3~~

Kinematic chain - examples

$$N=6$$

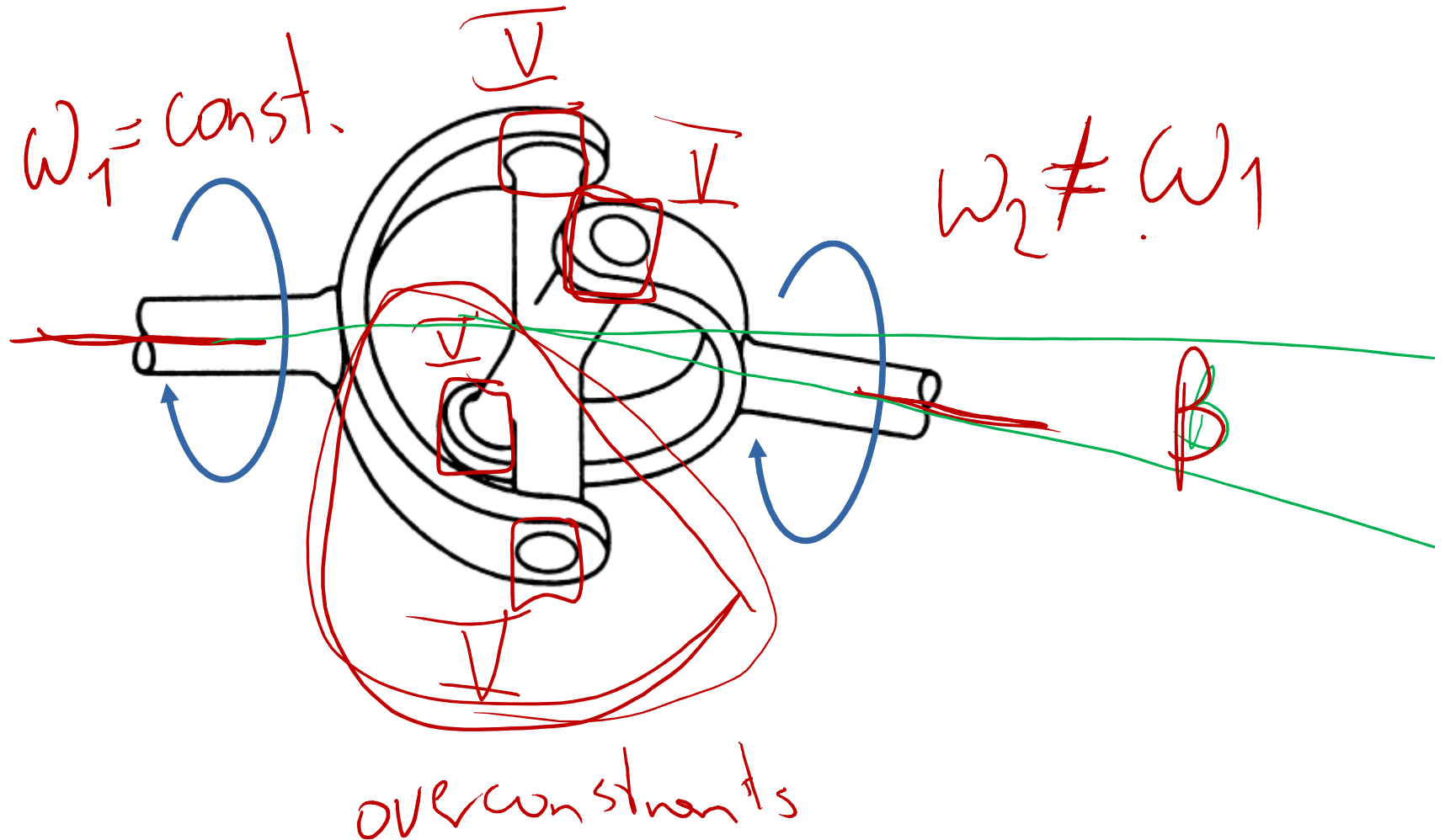
$$P_5=9$$



$$F = 3 \cdot 6 - 2 \cdot 9 = 0$$

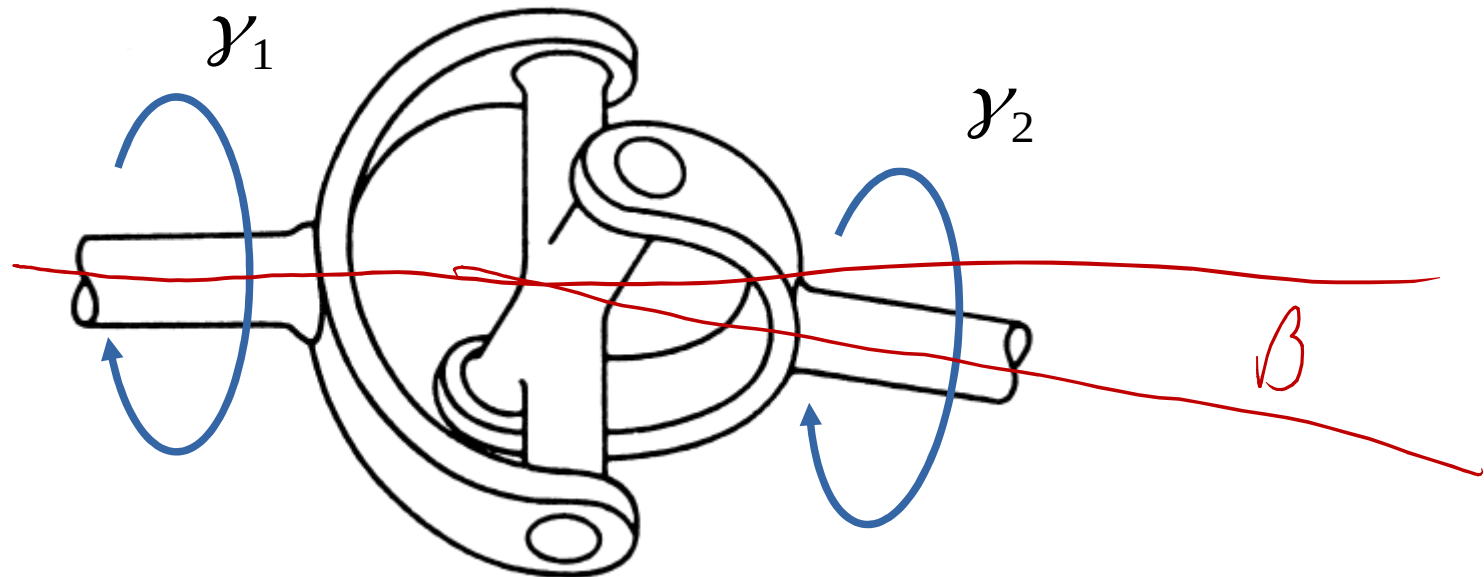
Kinematic chain - examples

Universal joint (Cardan, Hooke's, Hardy Spicer)



Kinematic chain - examples

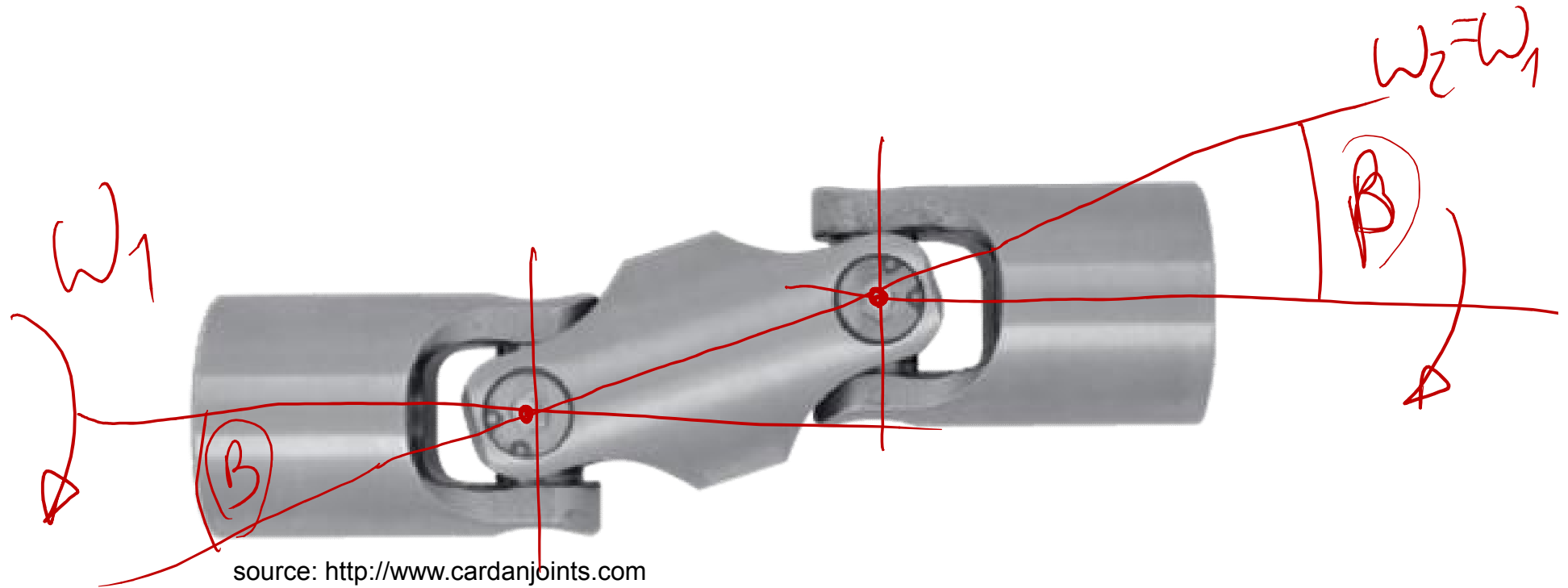
Universal joint (Cardan, Hooke's, Hardy Spicer)



$$\omega_2 = \frac{\omega_1 \cos \beta}{1 - \sin^2 \beta \cos^2 \gamma_1}, \quad \omega_1 = \frac{d \gamma_1}{dt}, \quad \omega_2 = \frac{d \gamma_2}{dt}$$

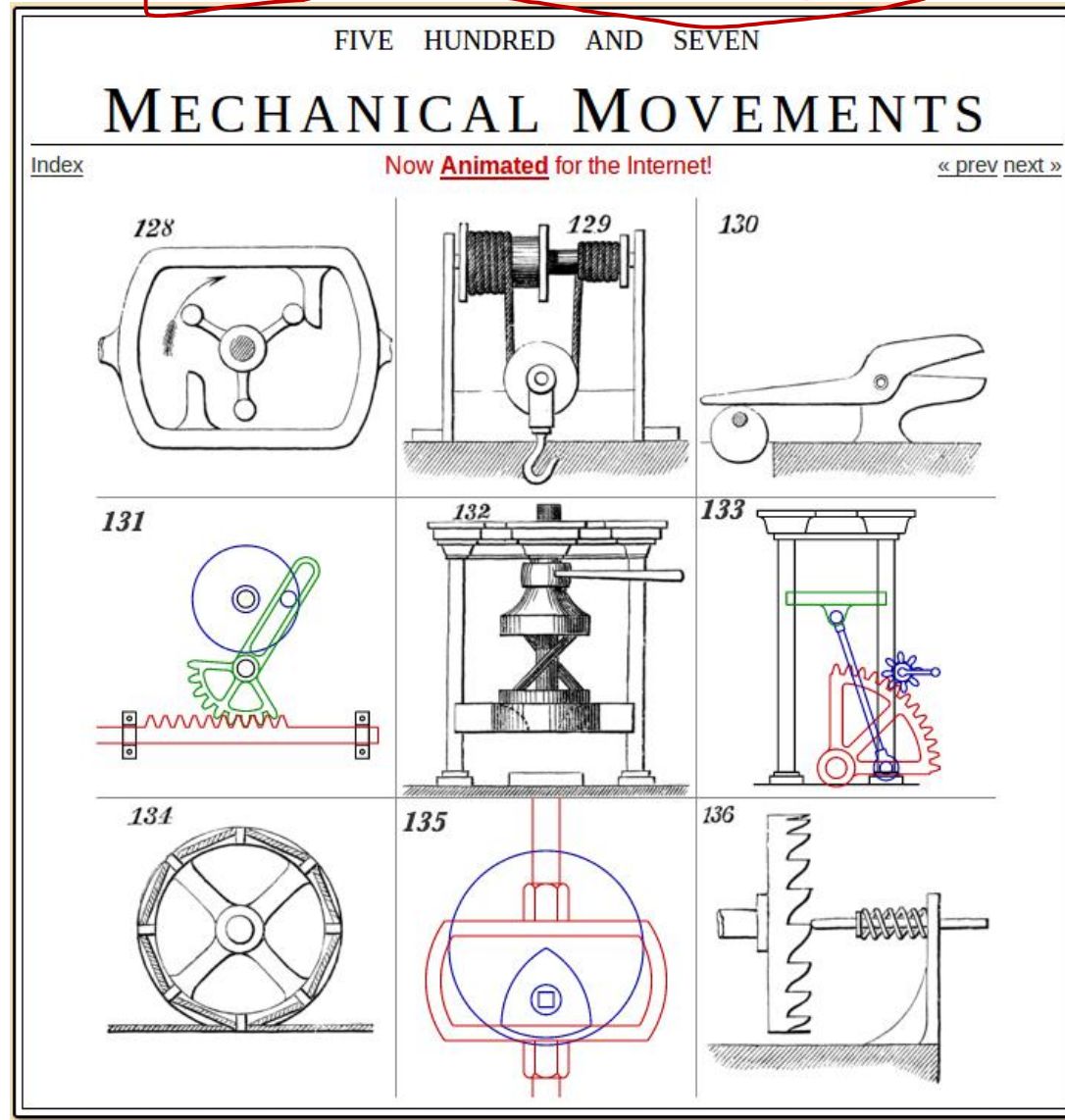
Kinematic chain - examples

Constant-velocity joint (homokinetic, Double Cardan)



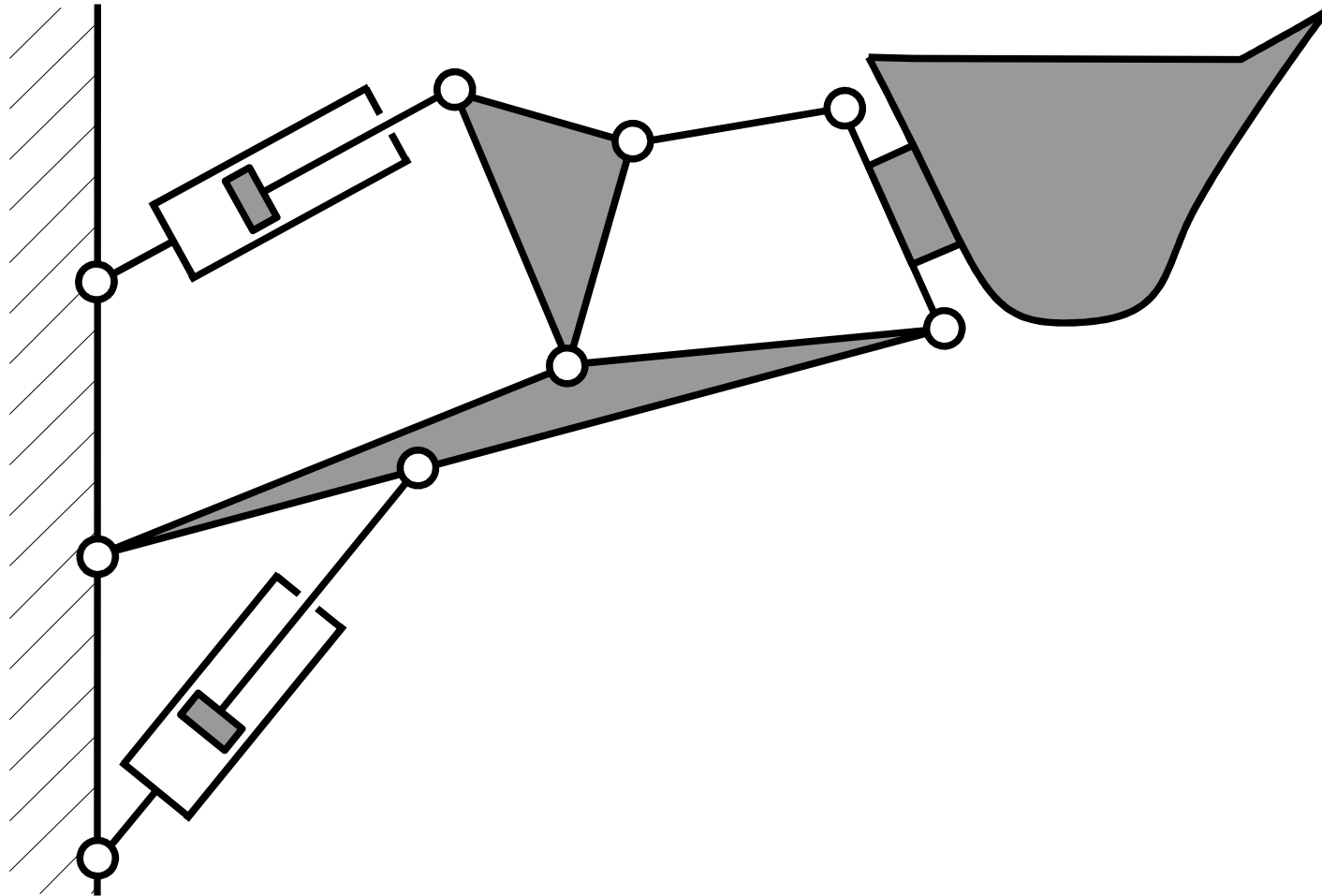
Additional materials

<http://507movements.com/>

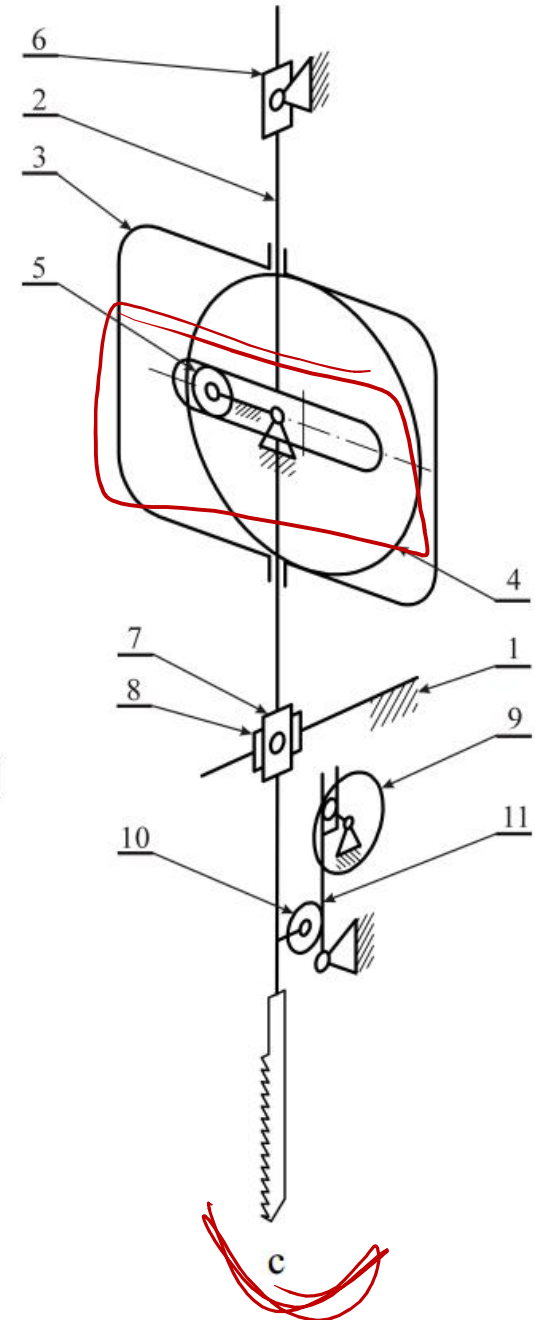
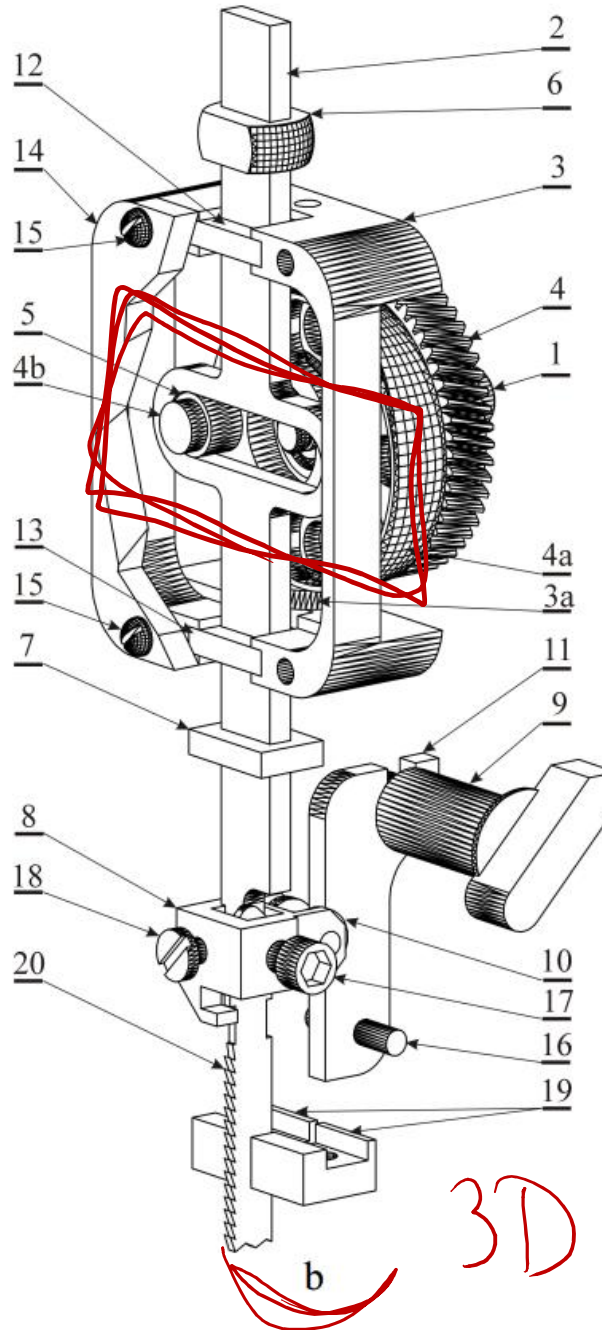


Examples – digger arm

howe mobility = ?



Examples – oscillatory fretsaw

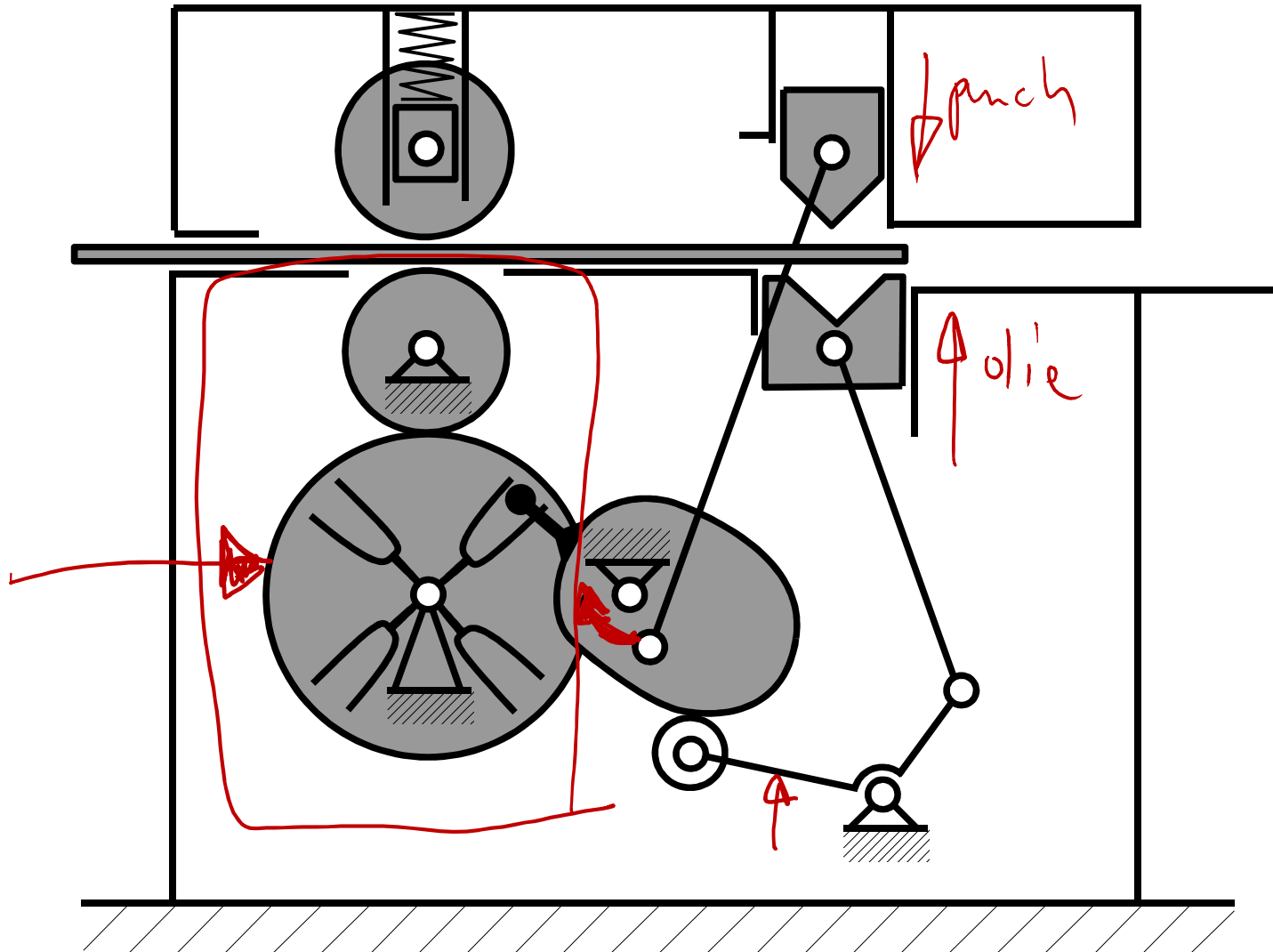


a

b

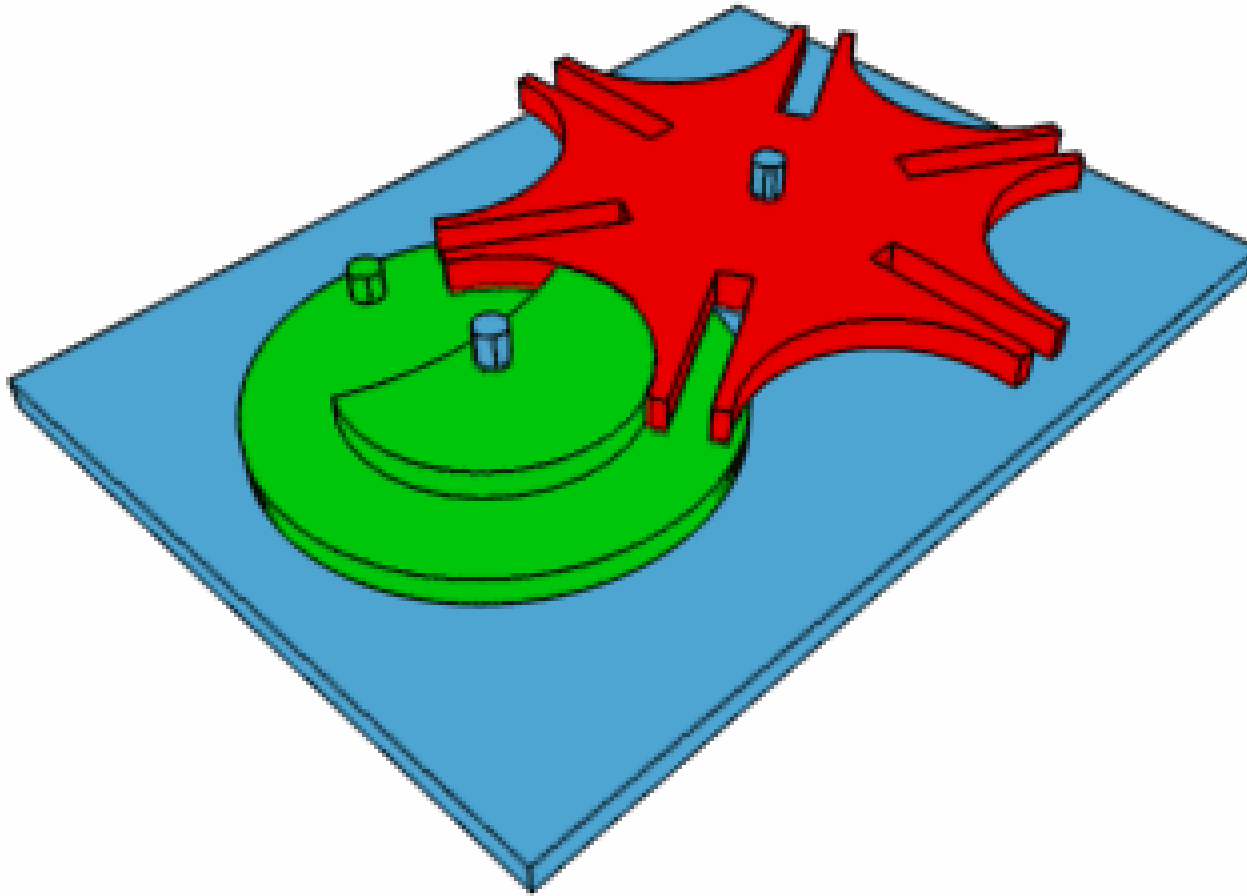
c

Examples - corrugated sheet metal forming machine



Examples

Geneva drive (Maltese cross)



Lecture 2

Structural classification, velocities in planar mechanisms.

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Classification of kinematic chains



Simple kinematic chain – every member has maximum two kinematic pairs.

Complex kinematic chain – at least one member has three kinematic pairs.

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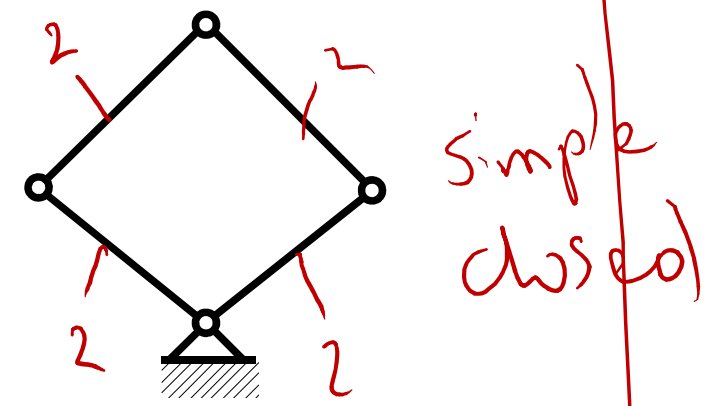
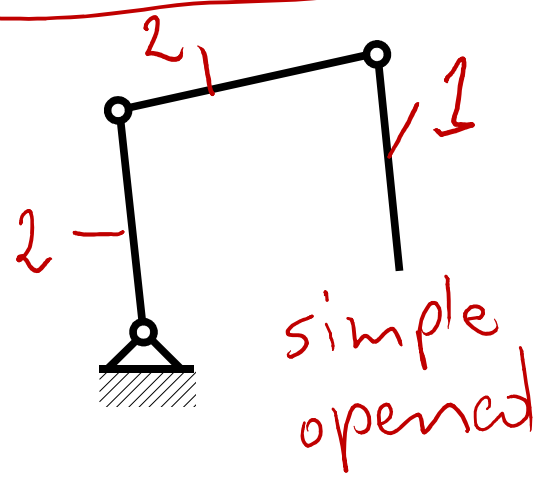
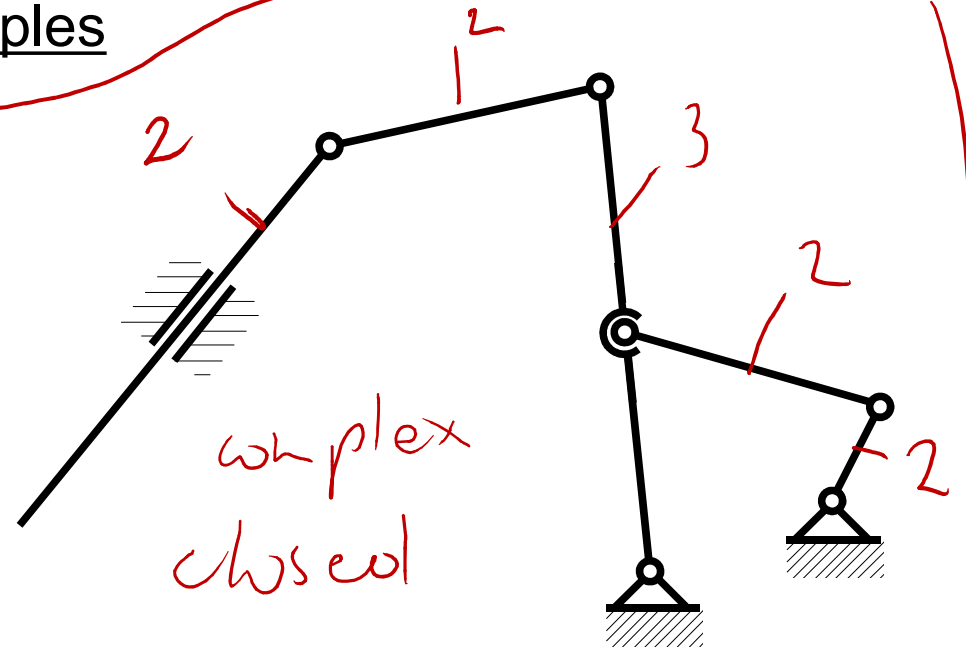
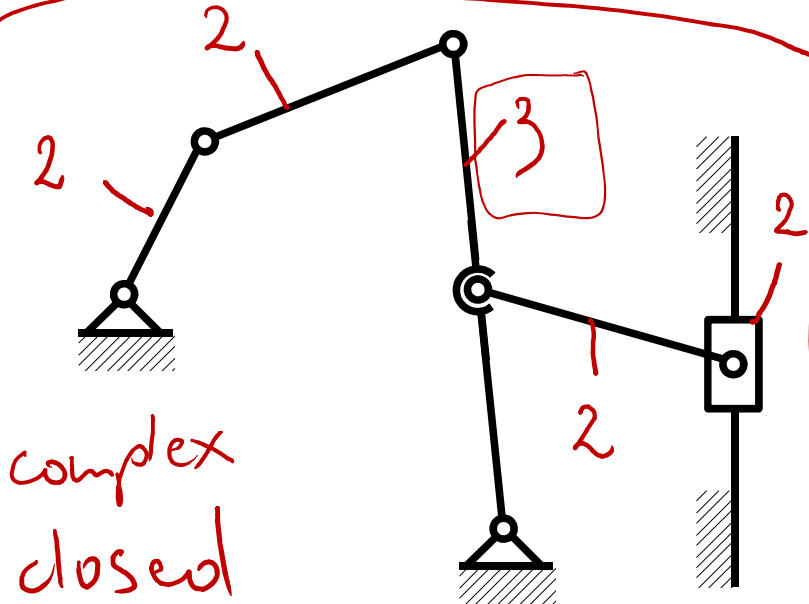
Complex kinematic chain – at least one member has three kinematic pairs.

Open kinematic chain – at least one member has only one kinematic pair.

Closed kinematic chain – every member has minimum two kinematic pairs.

Classification of kinematic chains

Examples



Structural classification of mechanisms



Structural group – the simplest part of mechanism that has zero mobility.

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Planar mechanism with only 5th class pairs: $F = 3n - 2p_5 \neq 0$

Structural classification of mechanisms

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$$\boxed{\frac{p_5}{n}} = \frac{3}{2} = \frac{6}{4} = \frac{9}{6} = \dots$$

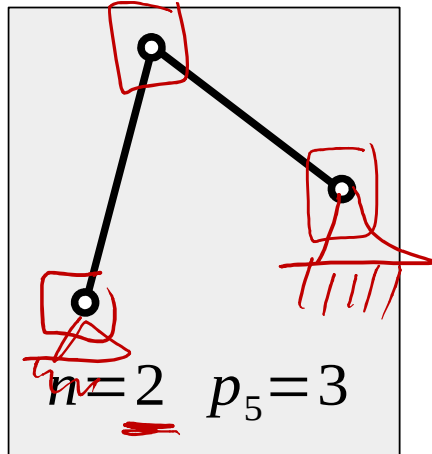
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IInd structural
group



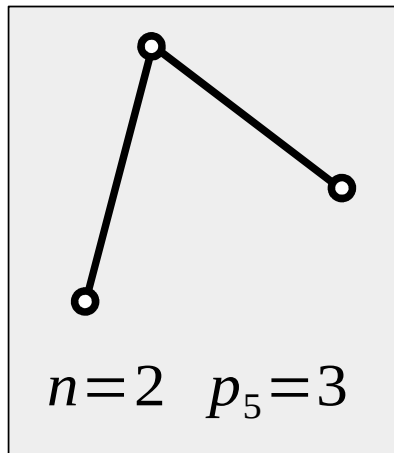
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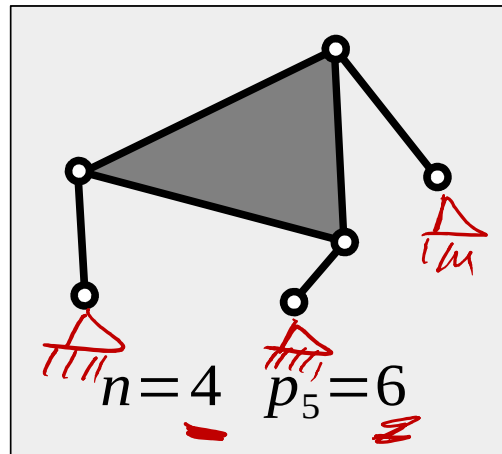
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IIIrd structural group



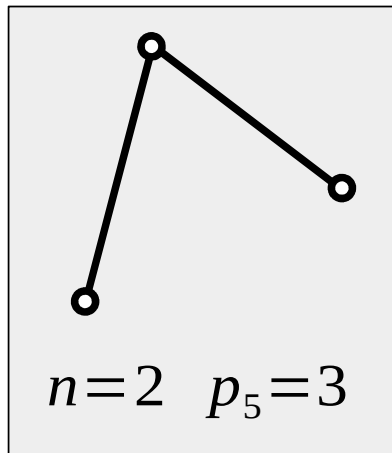
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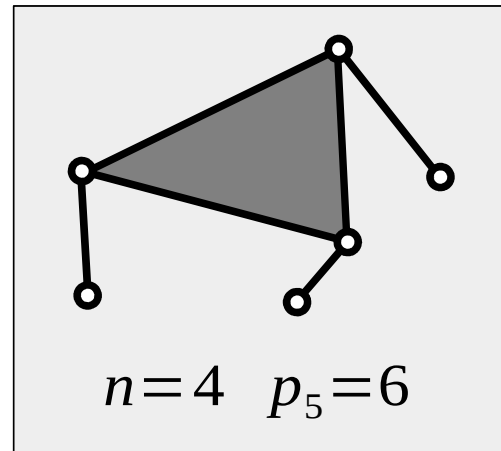
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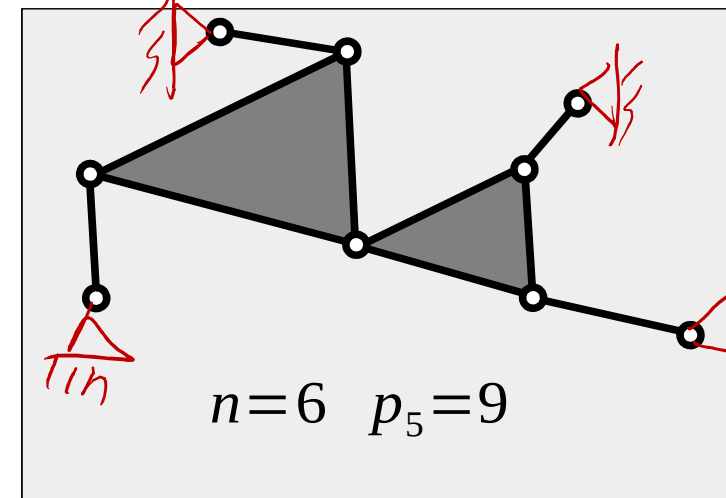
IInd structural group



IIIrd structural group



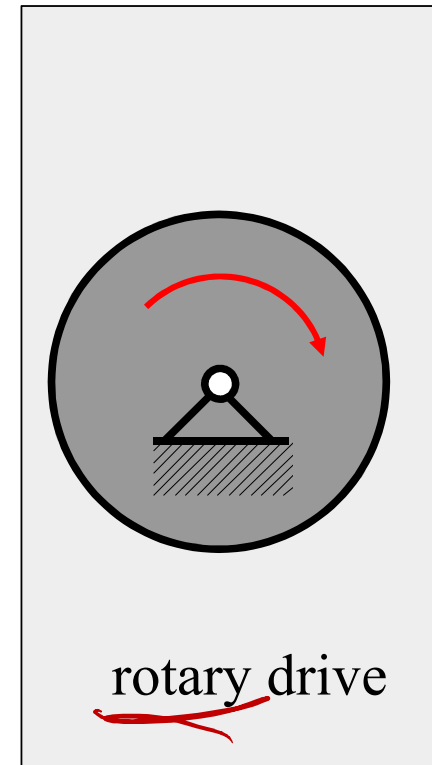
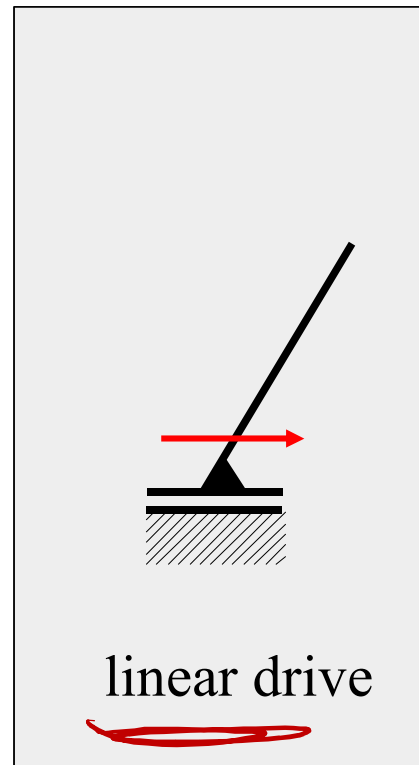
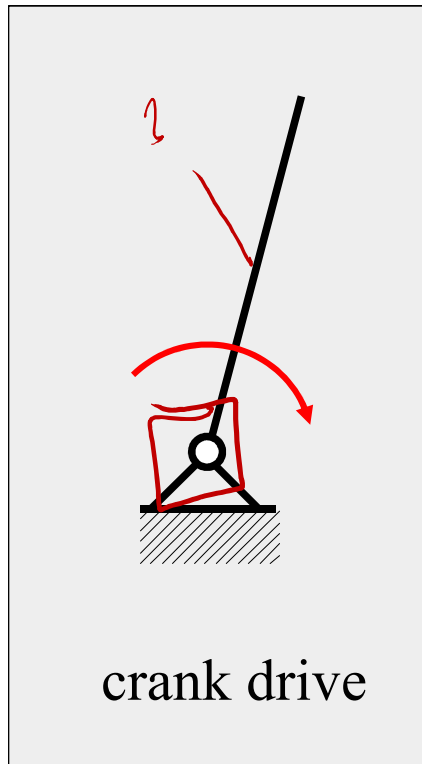
IVth structural group



Structural classification of mechanisms

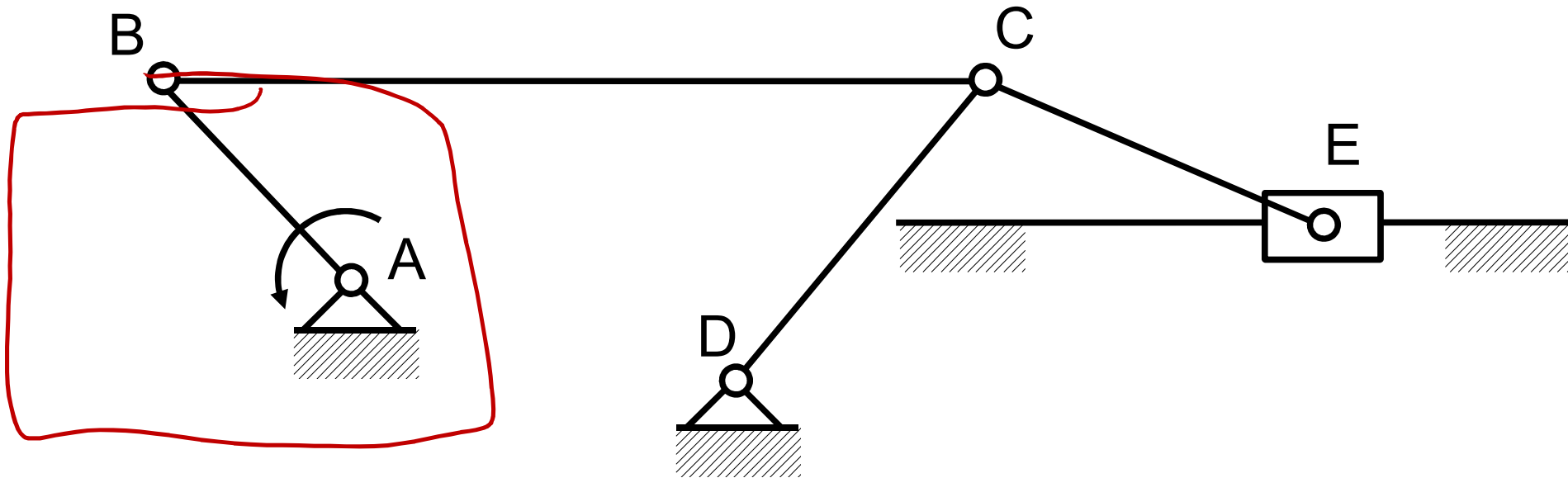
Ist structural group – drive

$$n=1 \quad p_5=1 \quad + \text{ drive}$$



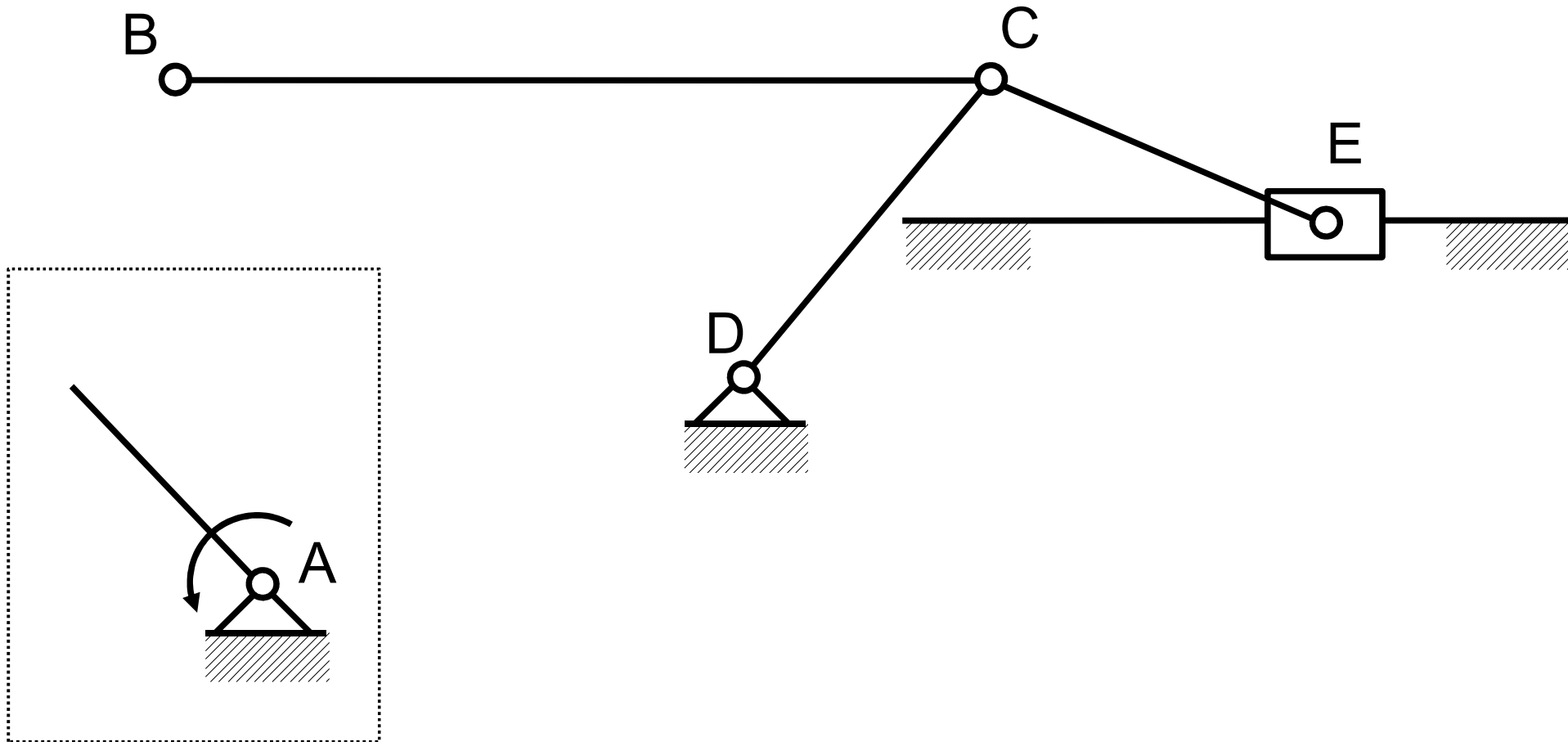
Structural classification of mechanisms

Example



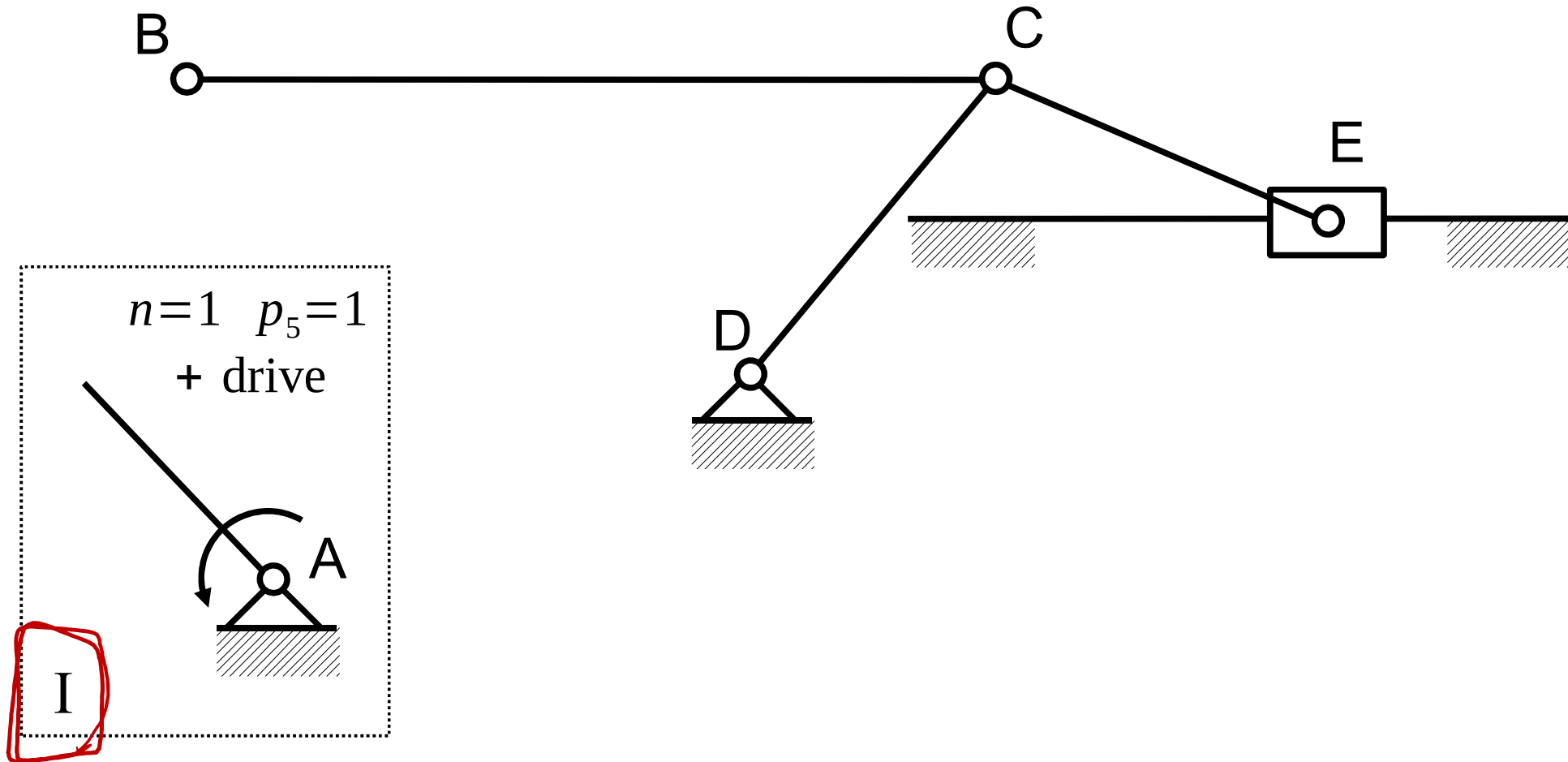
Structural classification of mechanisms

Example



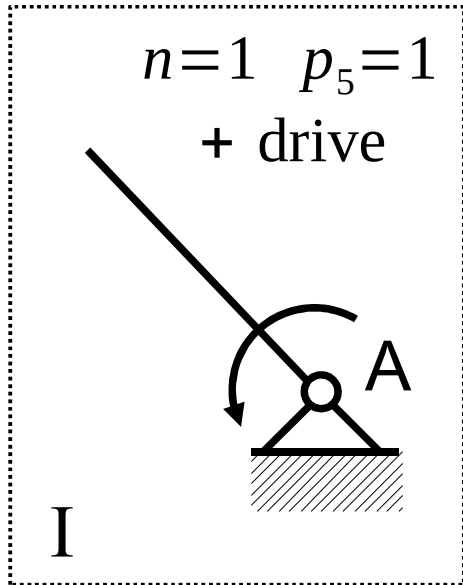
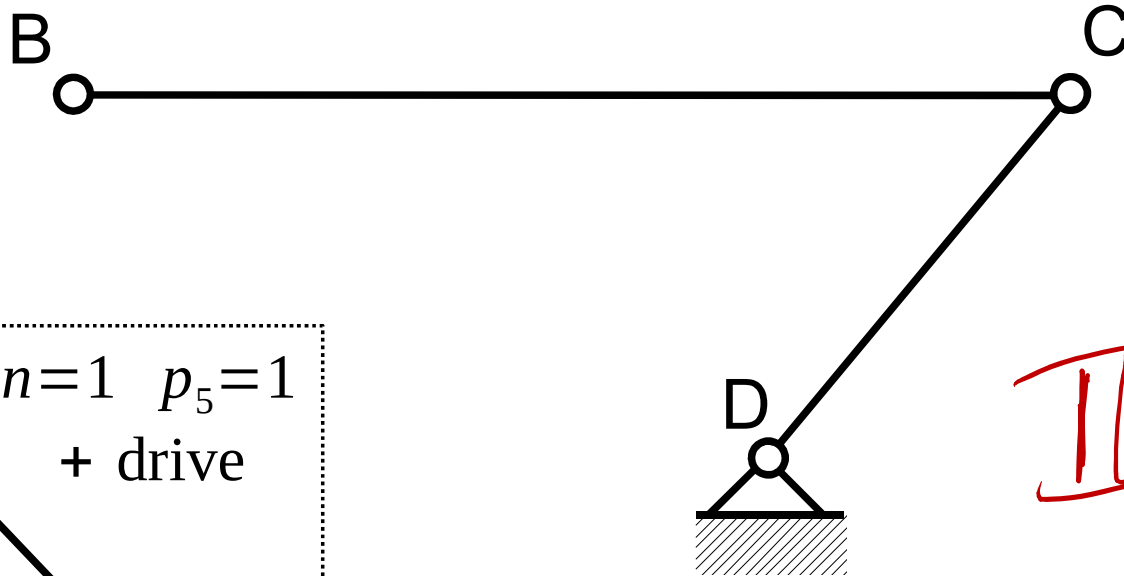
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Example

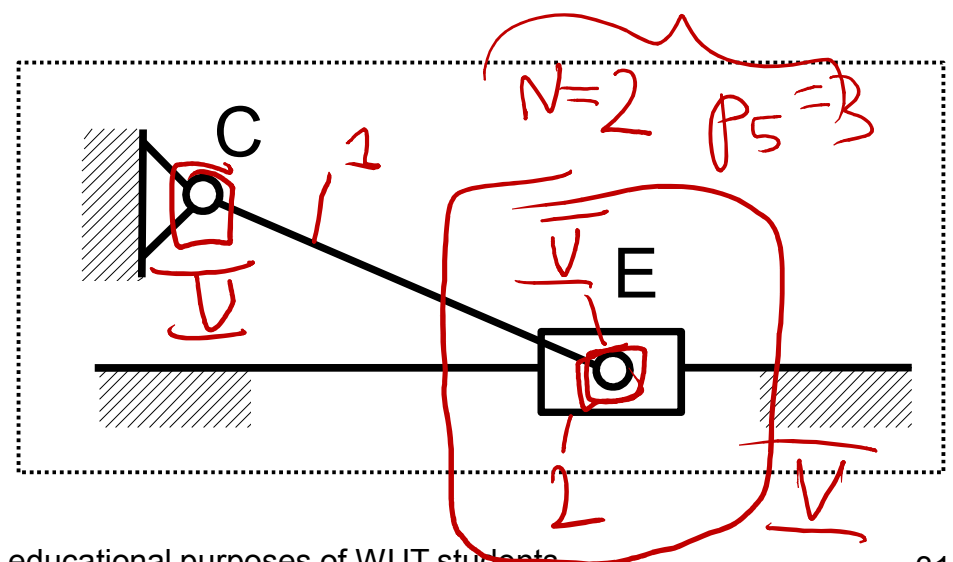


Structural classification of mechanisms

Example

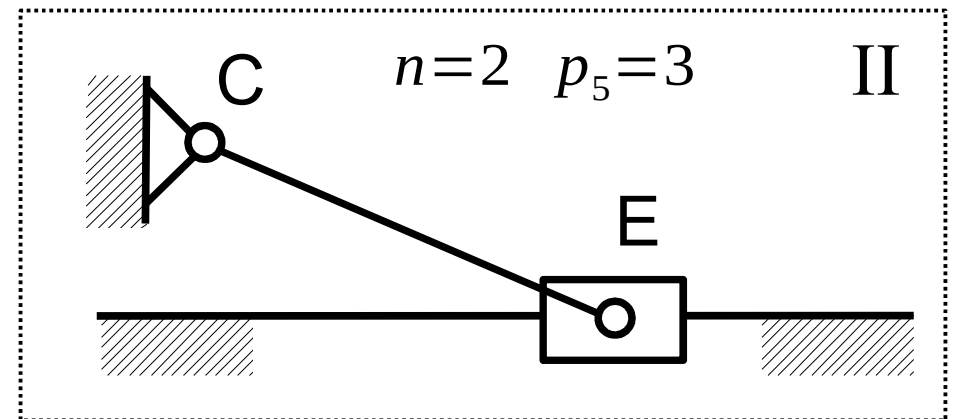
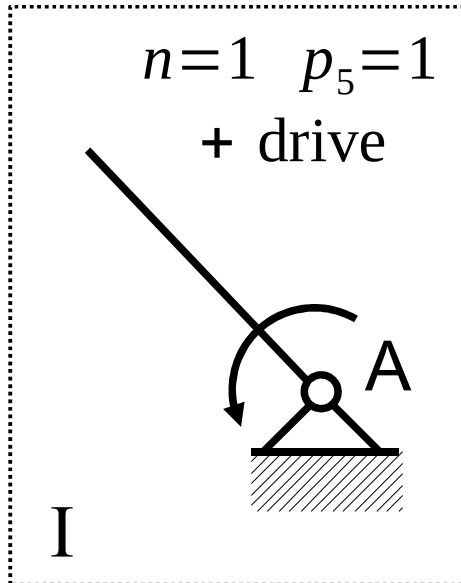
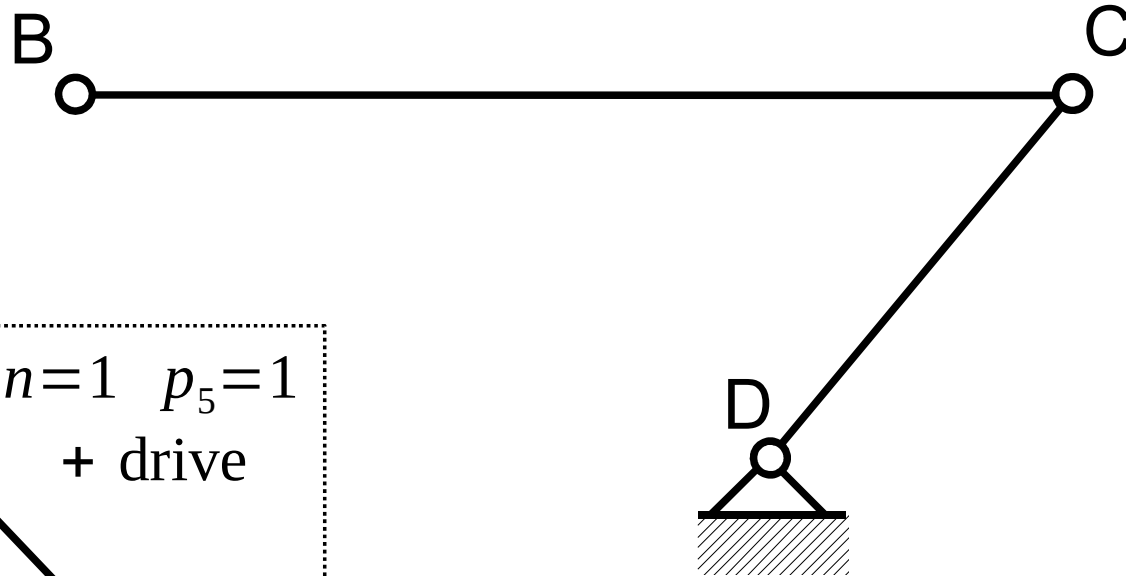


II s.a. $F=0$



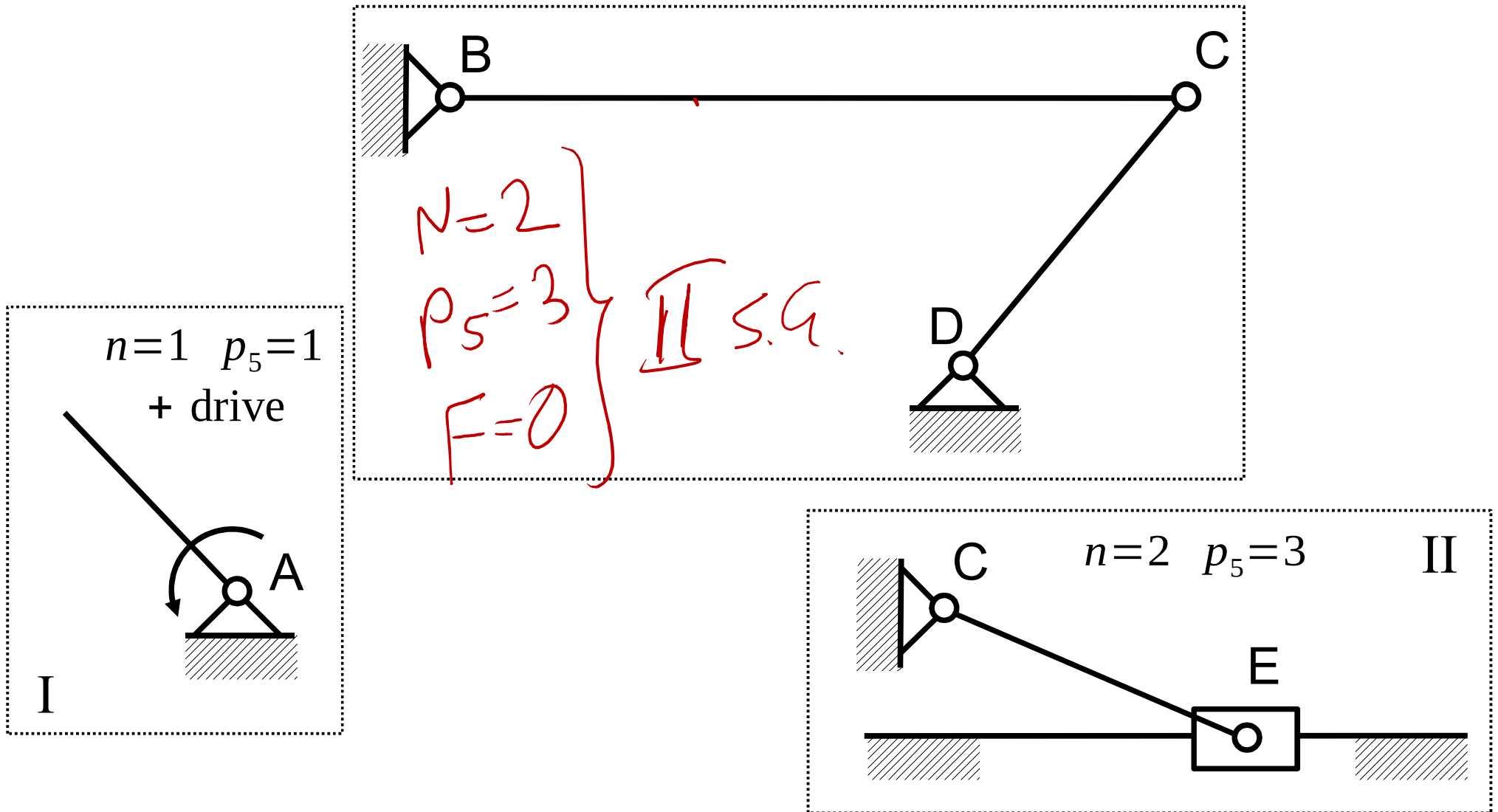
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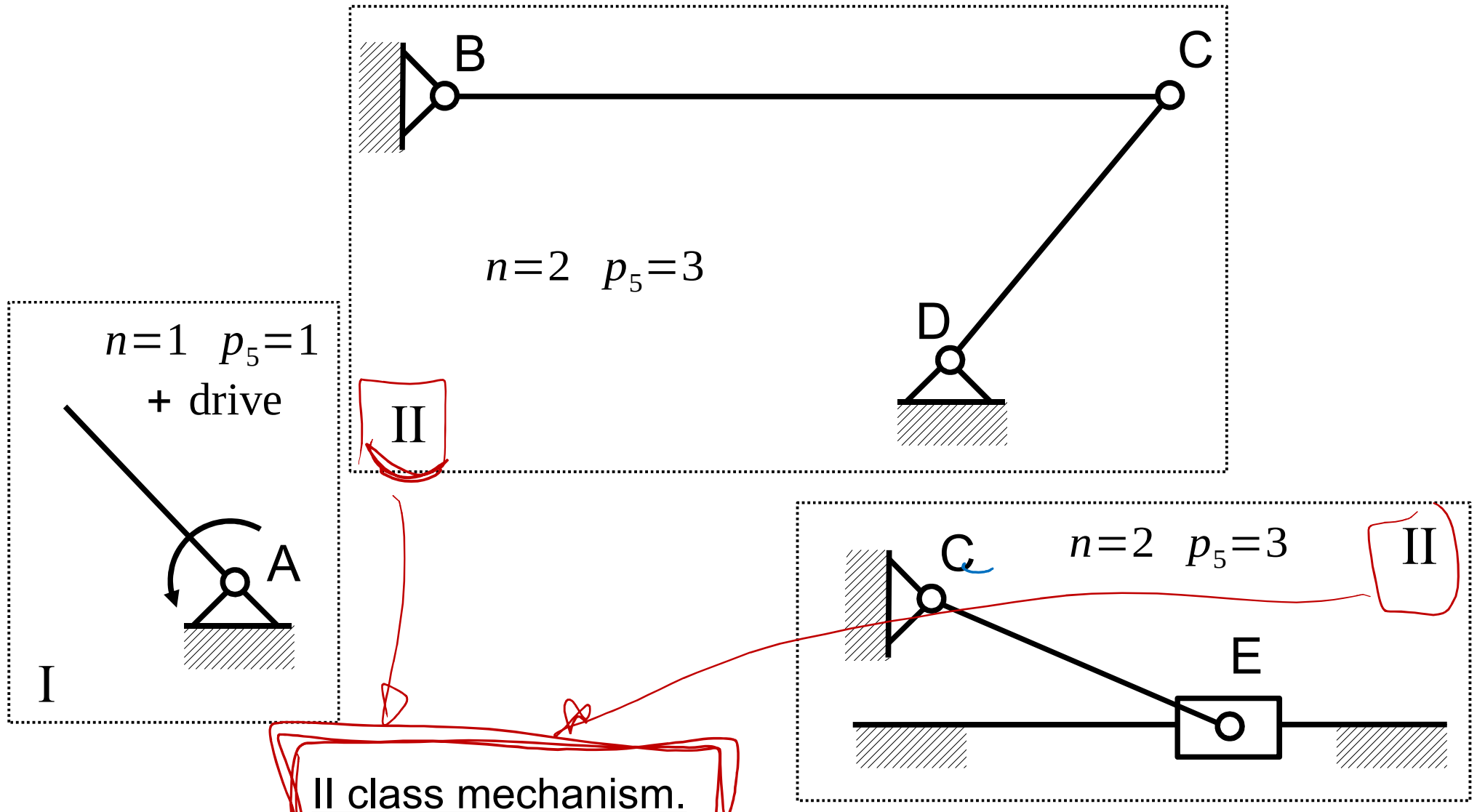
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Example



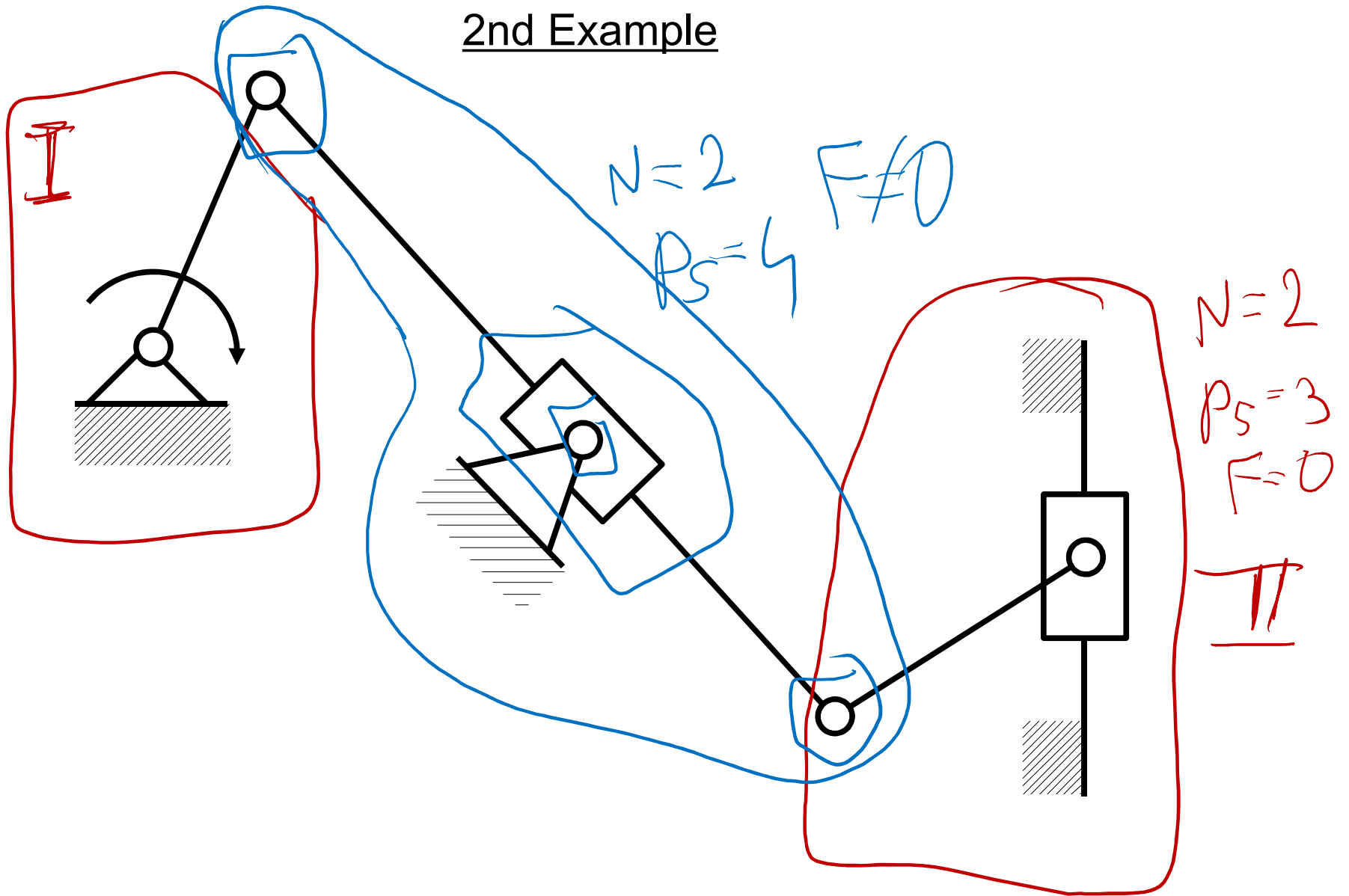
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Example



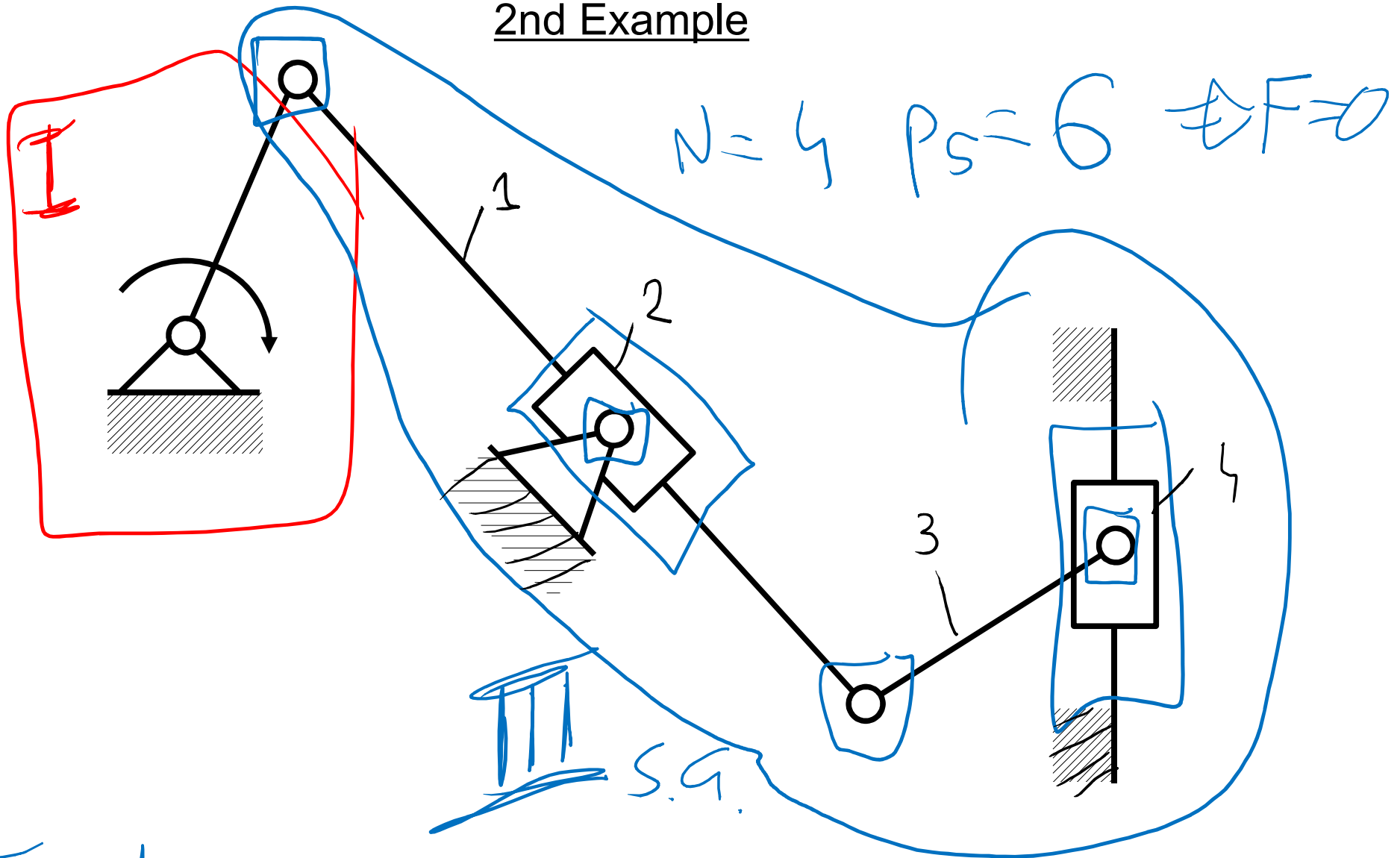
Structural classification of mechanisms

2nd Example



Structural classification of mechanisms

2nd Example



III class mech.

Kinematics of mechanisms

Kinematic analysis of a mechanism – determination of velocities and accelerations of selected mechanism members' points at considered configuration. Mechanism structure must be given (geometry of members, kinematic pairs) and drive method must be known.

Methods of velocities and acceleration determination

Graphical methods

- velocity projection method,
- instantaneous center of rotation method,
- instantaneous center of acceleration method,
- method of rotated velocities,
- velocity decomposition method,
- acceleration decomposition method,
- velocity scheme method,
- accelerations scheme method.

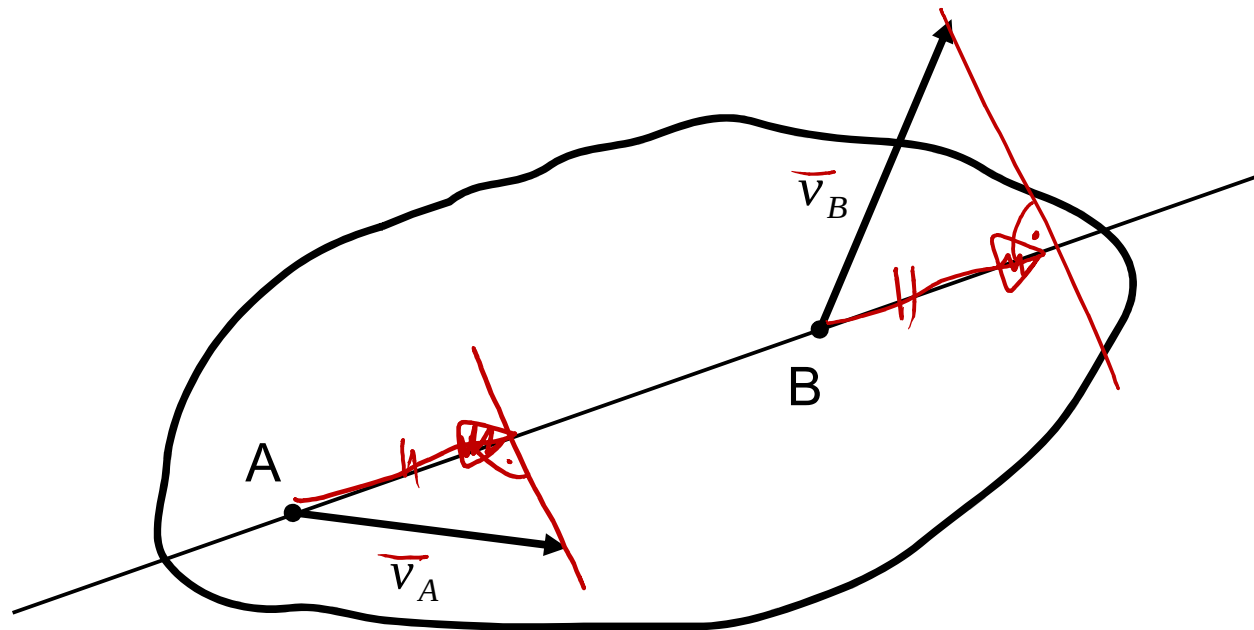
Analytical method

Methods of velocities and acceleration determination

	Graphical methods	Analytical method
advantage	<ul style="list-style-type: none">• better understanding of mechanism motion,• analysis of very complicated mechanisms,• computers not needed,	<ul style="list-style-type: none">• functions of configuration as a solution,• analysis of very complicated mechanisms,
disadvantage	<ul style="list-style-type: none">• great workload,• needs to repeat graphs for every configuration,• graphical errors.	<ul style="list-style-type: none">• computer needed for complicated mechanisms,• complicated systems of equations to solve,• solution interpretation may be complicated.

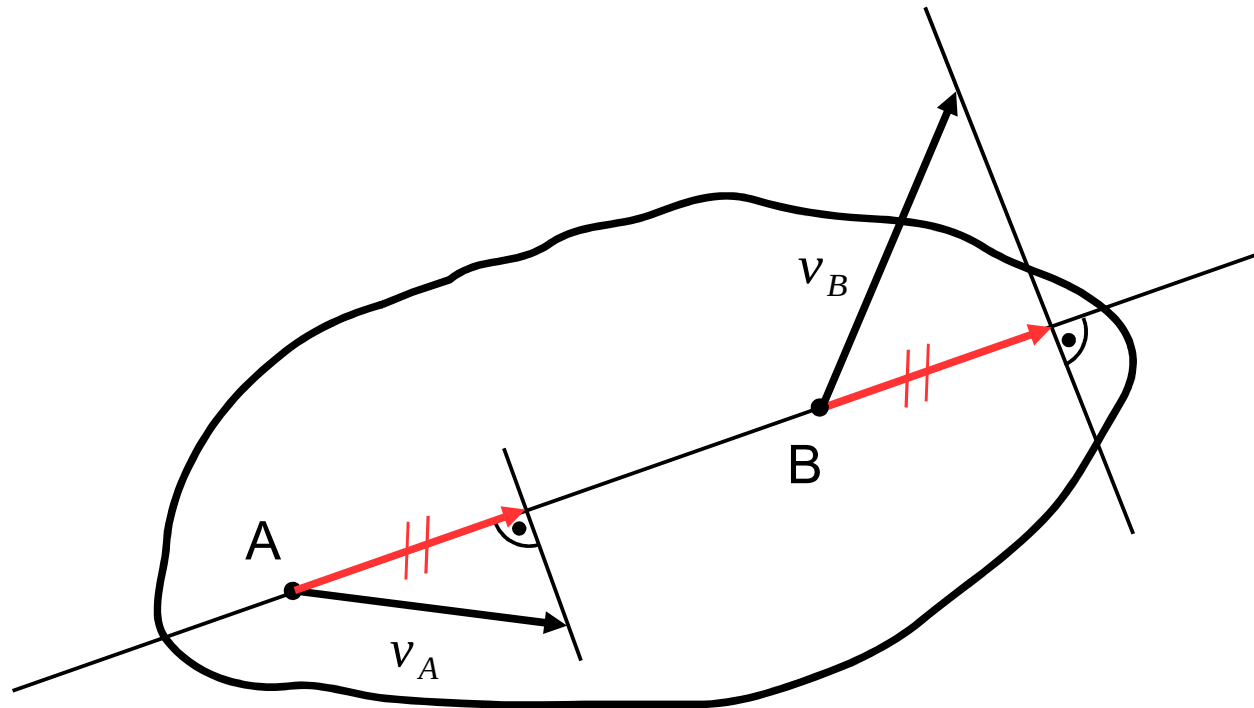
Velocity projection method

Projections of velocities of two rigid body's points onto common line are equal.



Velocity projection method

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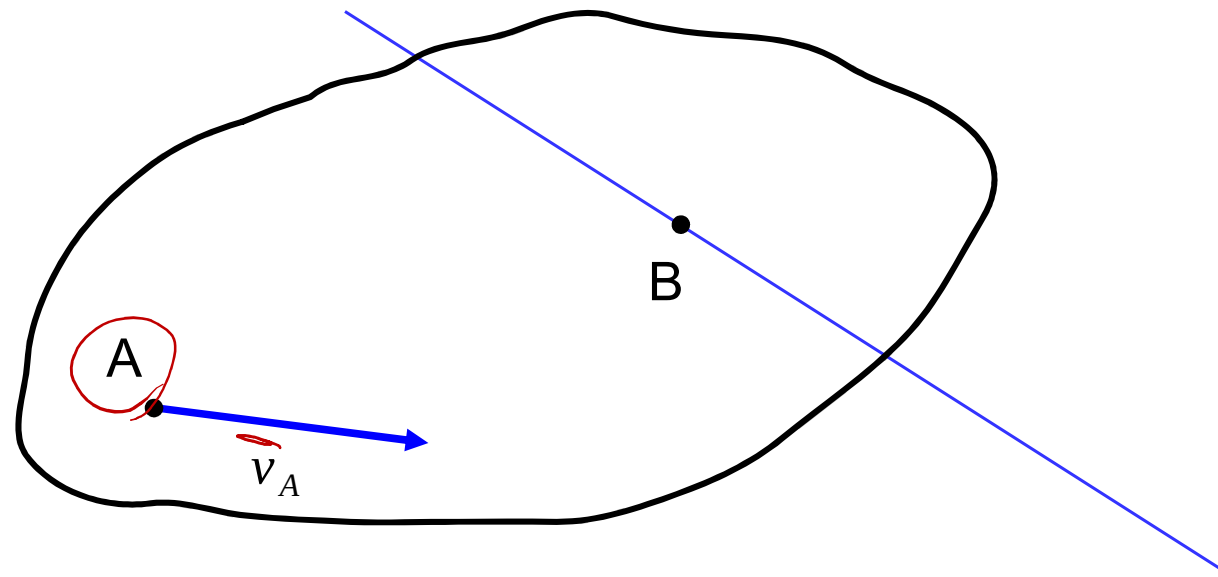


Velocity projection method

Example

Given: v_A and direction of v_B

Searched: v_B

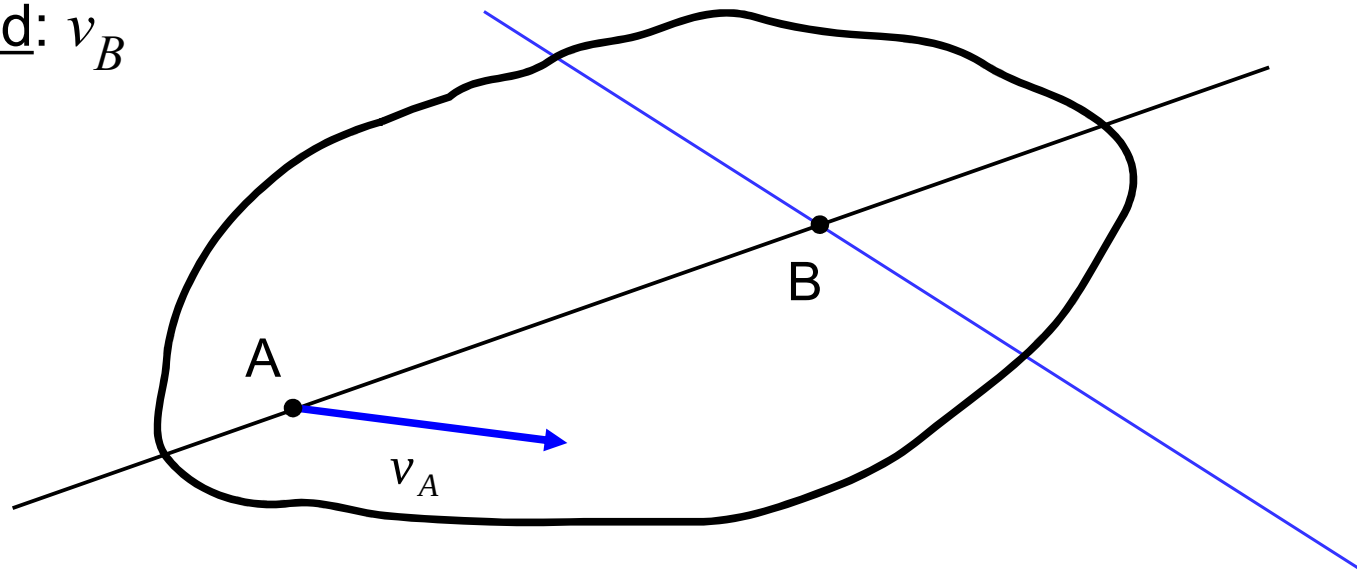


Velocity projection method

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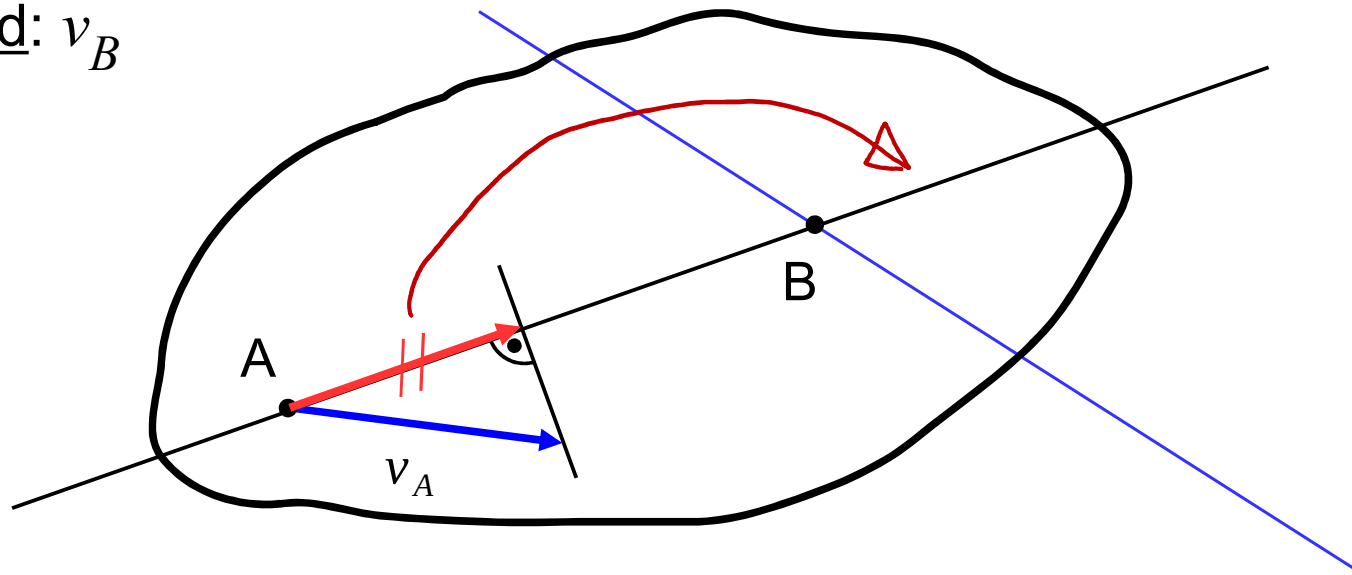


Velocity projection method

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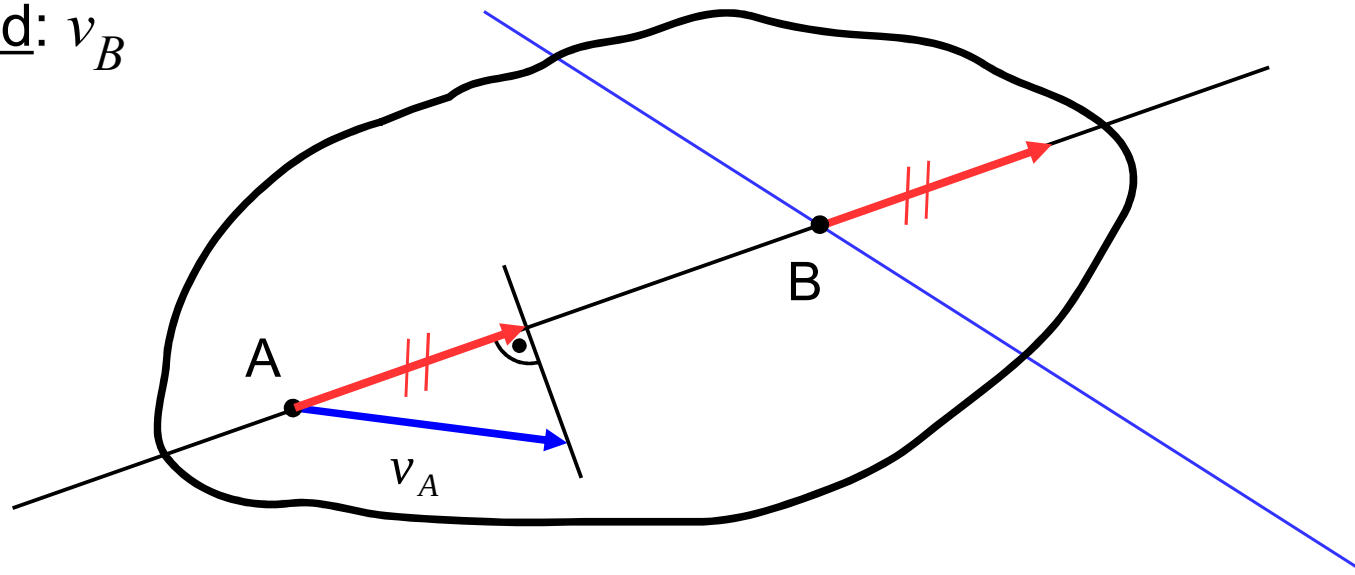


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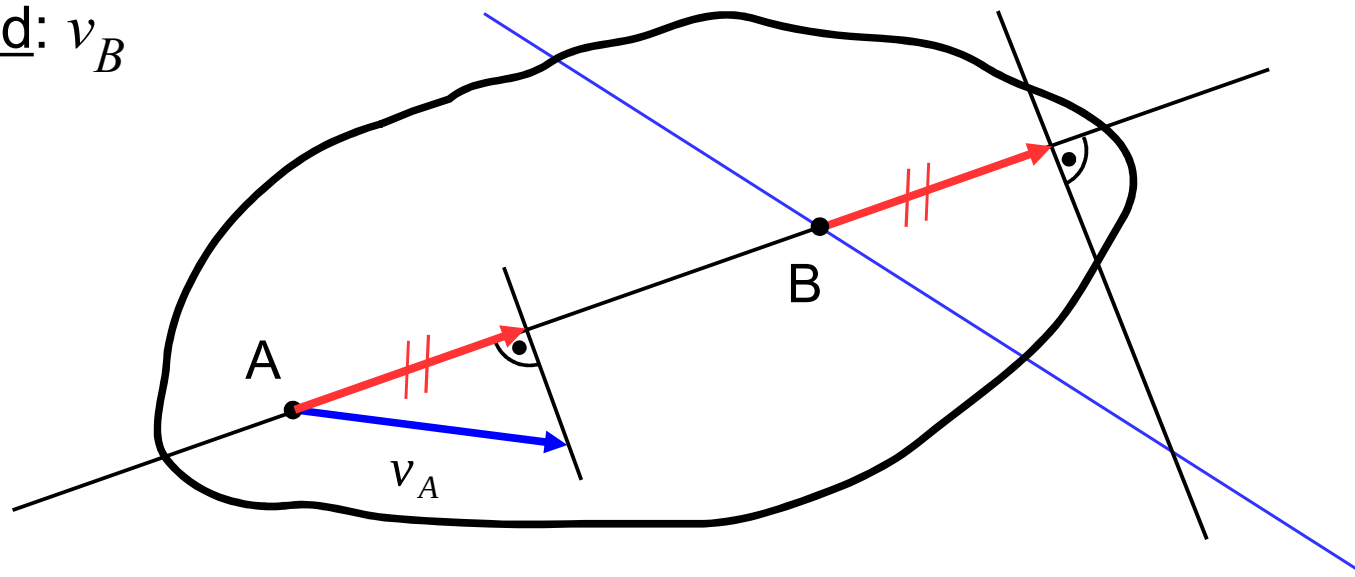


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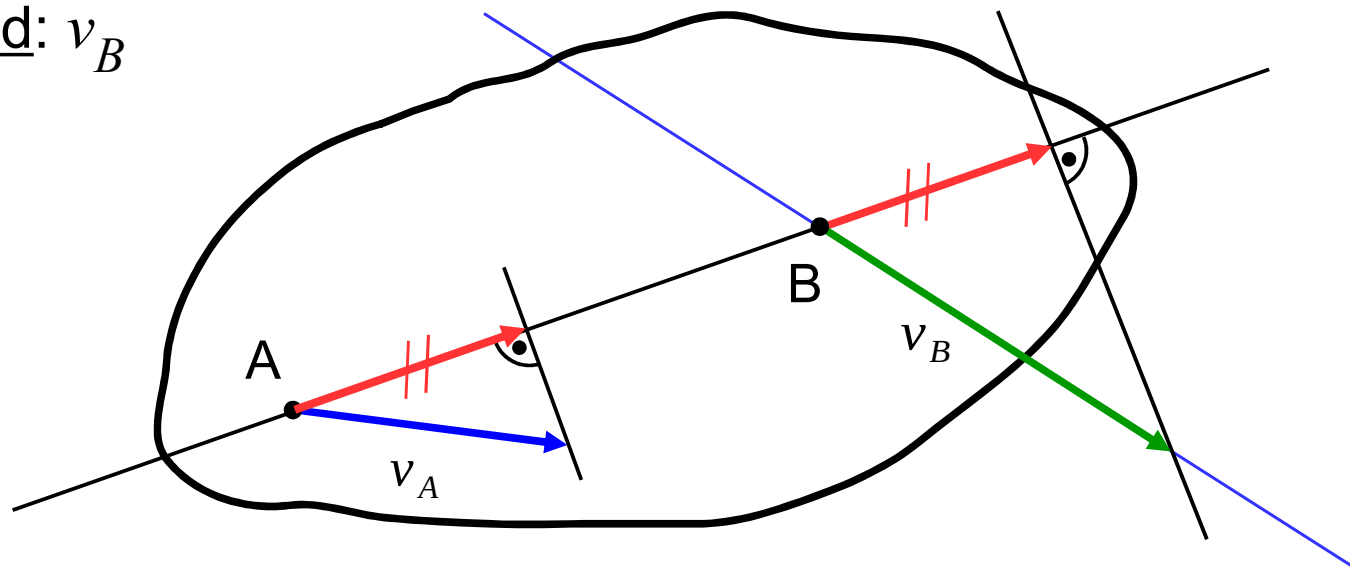


Velocity projection method

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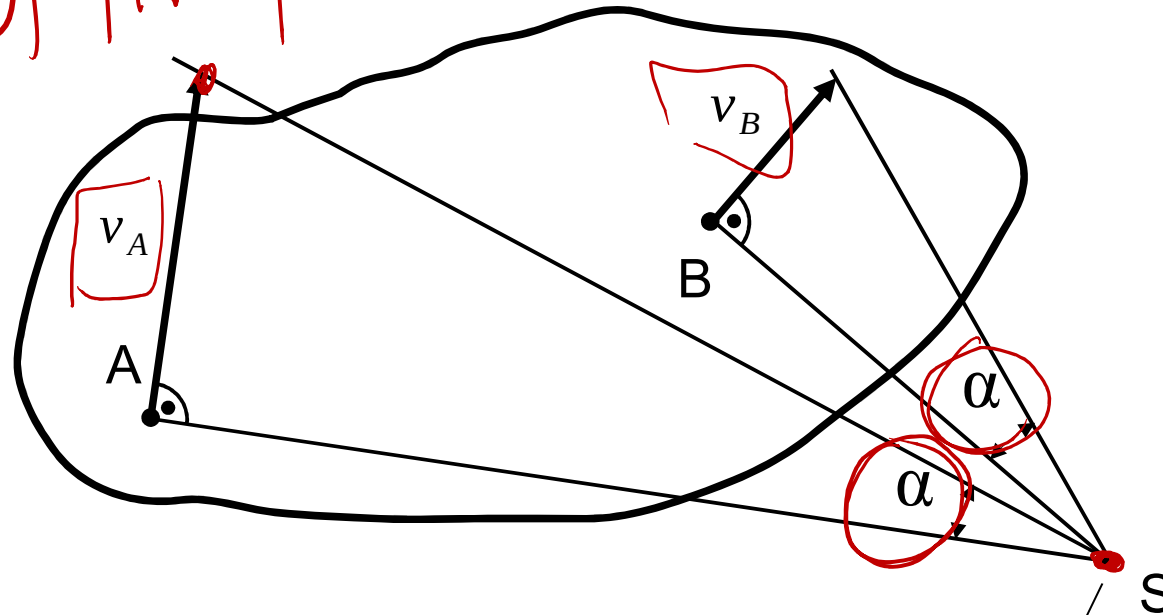
Searched: v_B



Instantaneous center of rotation method

$$|\vec{v}_A| = |\omega| \cdot |AS|$$

$$|\vec{v}_B| = |\omega| \cdot |BS|$$



center of instantaneous rotation
(center of velocities)

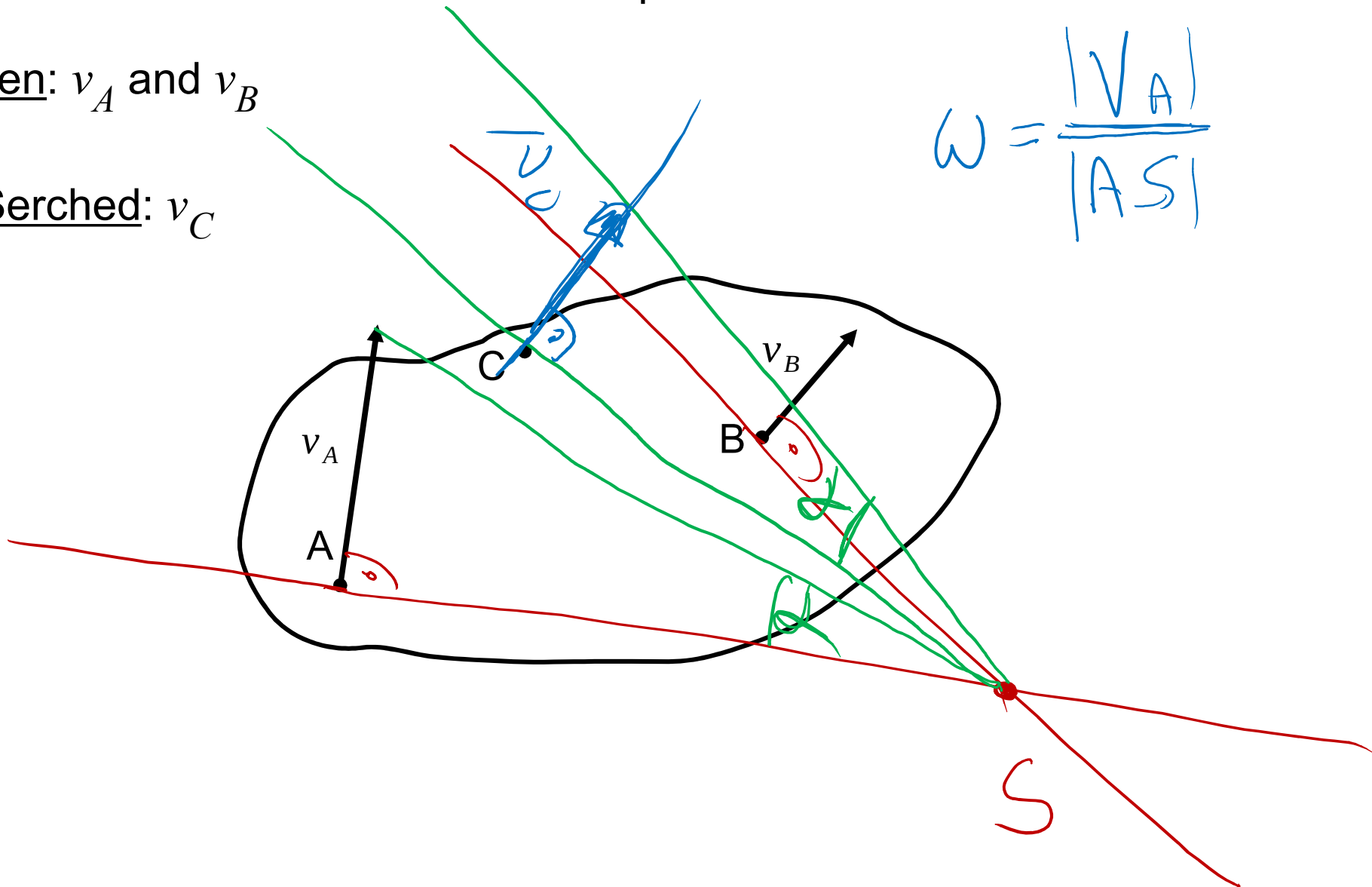
Instantaneous center of rotation method

Example

Given: v_A and v_B

Serched: v_C

$$\omega = \frac{|v_A|}{|AS|}$$

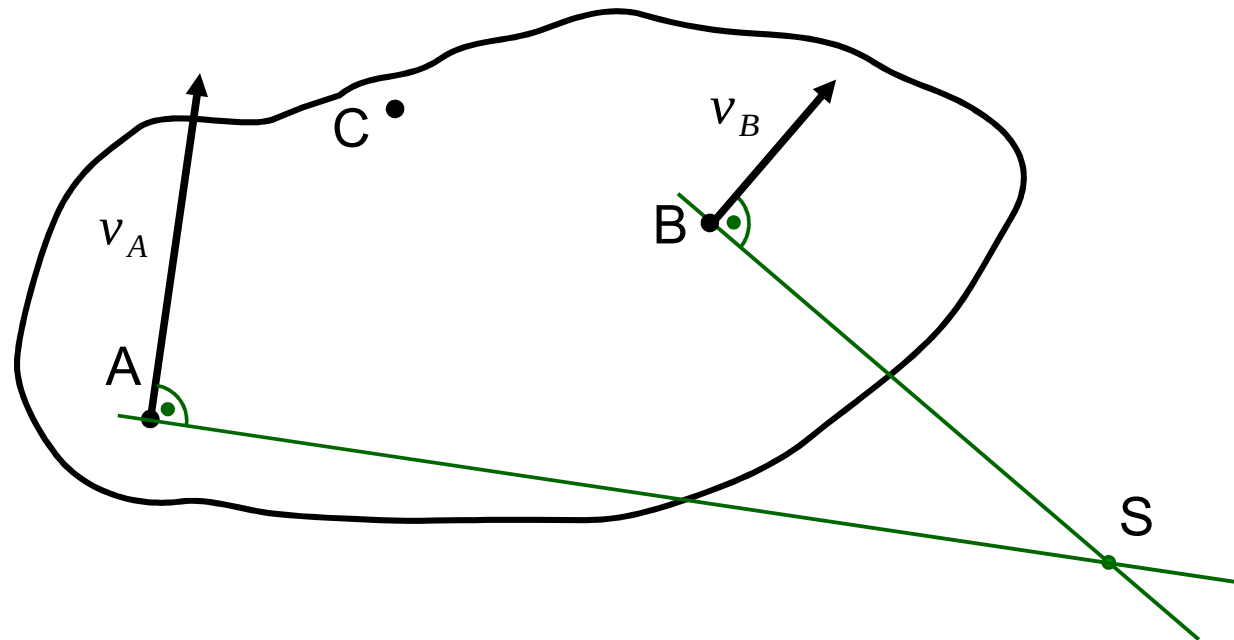


Instantaneous center of rotation method

Example

Given: v_A and v_B

Serched: v_C

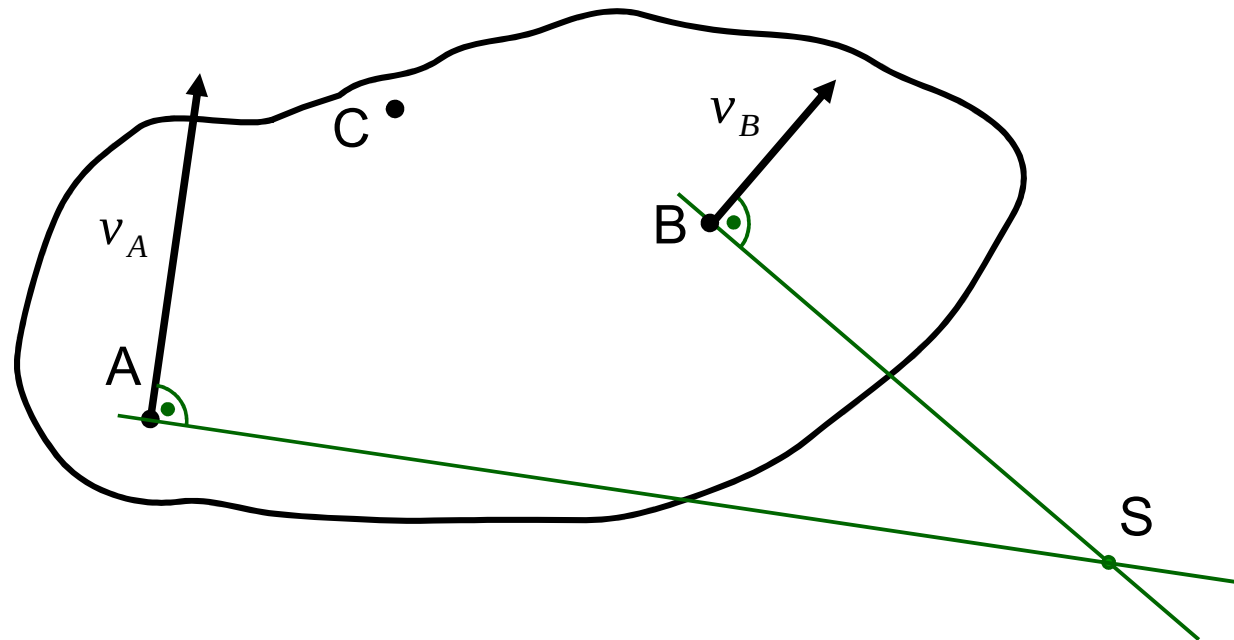


Instantaneous center of rotation method

Example

Given: v_A and v_B

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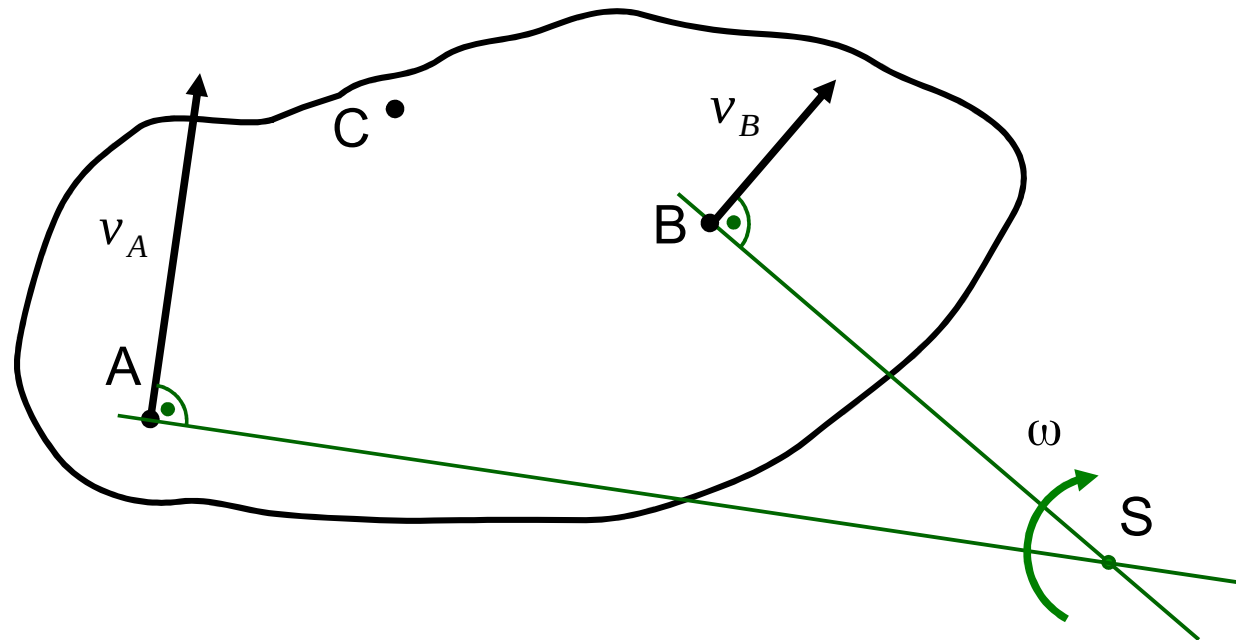


Instantaneous center of rotation method

Example

Given: v_A and v_B

Serched: v_C



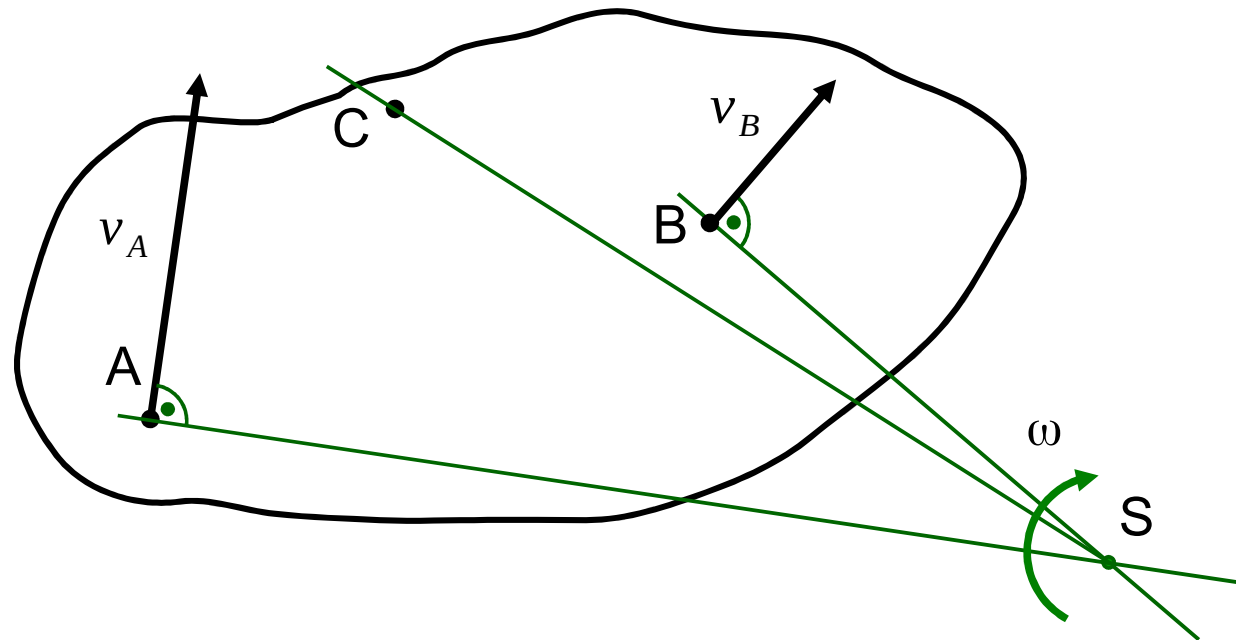
$$\omega = \frac{v_A}{|SA|} = \frac{v_B}{|SB|}$$

Instantaneous center of rotation method

Example

Given: v_A and v_B

Serched: v_C



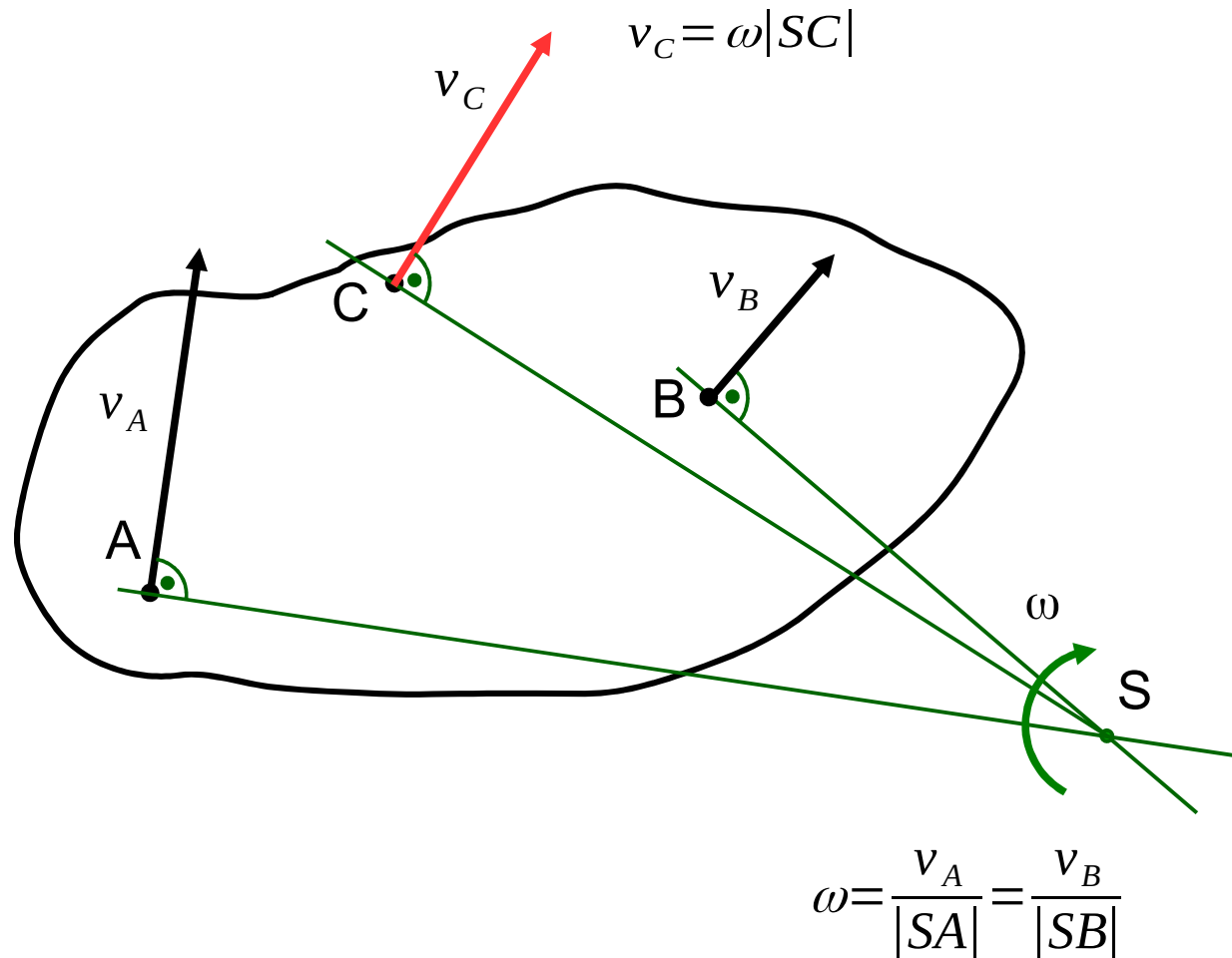
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Instantaneous center of rotation method

Example

Given: v_A and v_B

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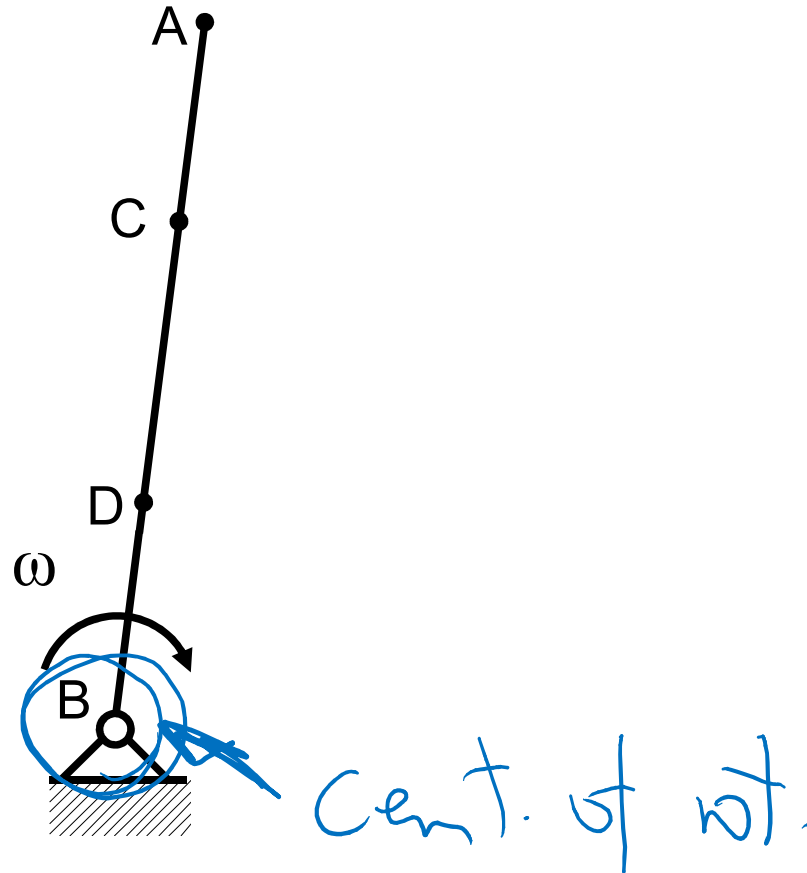


Instantaneous center of rotation method

Example 2

Given: ω

Serched: v_A, v_C, v_D

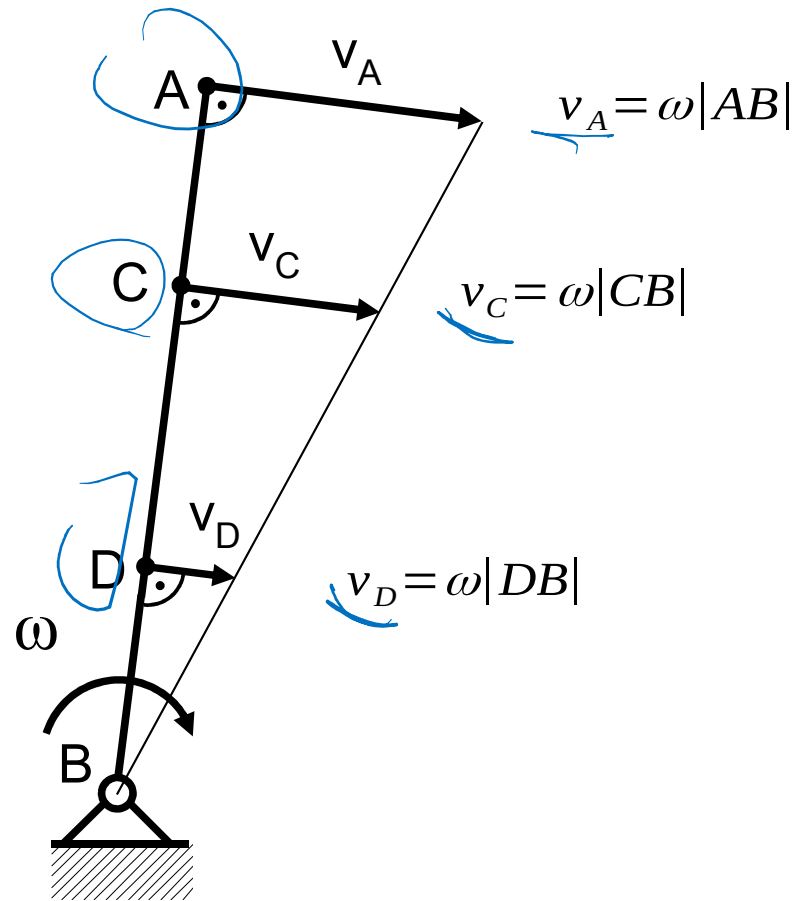


Instantaneous center of rotation method

Example 2

Given: ω

Serched: v_A, v_C, v_D

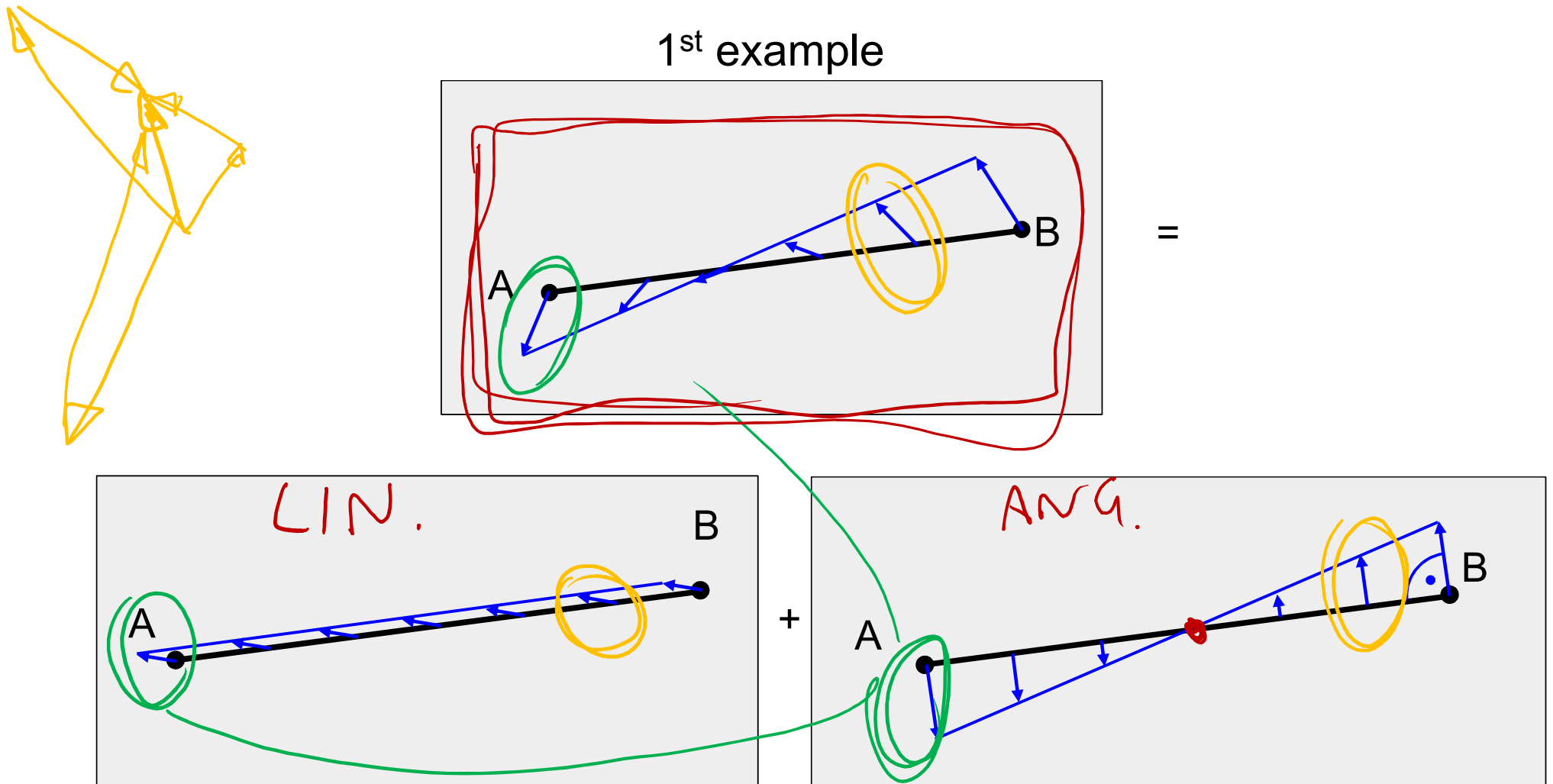


Velocity decomposition method

Every planar motion can be described by a superposition of a linear motion and an angular motion.

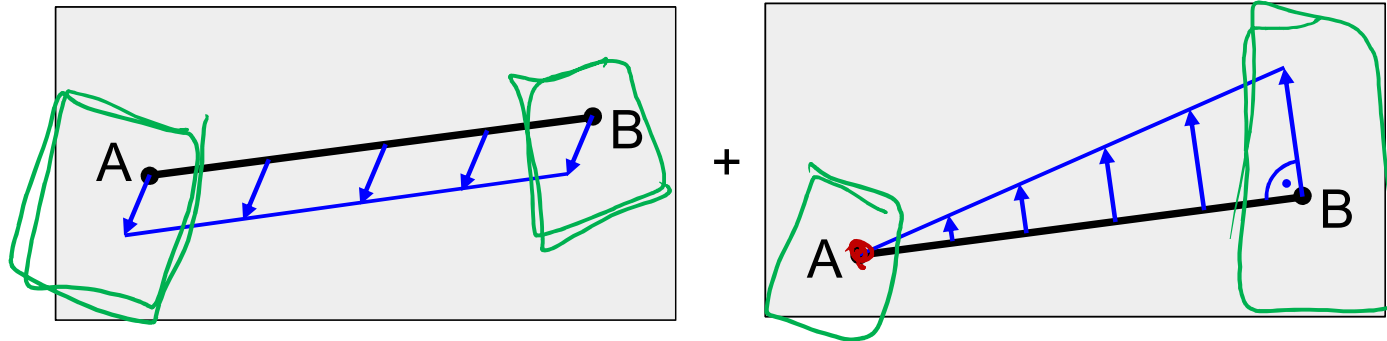
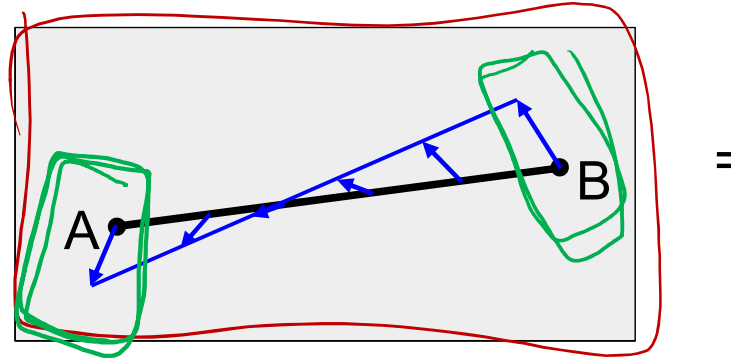
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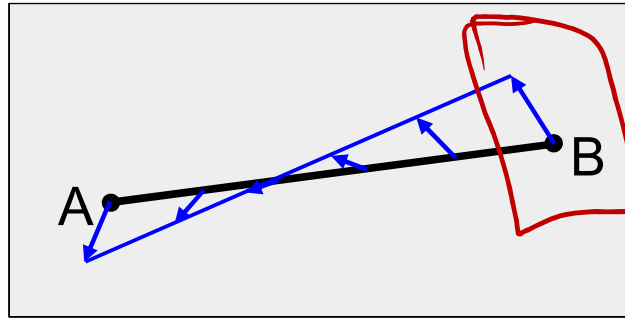
Velocity decomposition method

2nd example

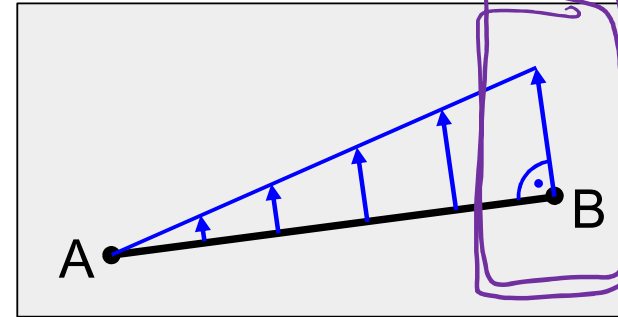
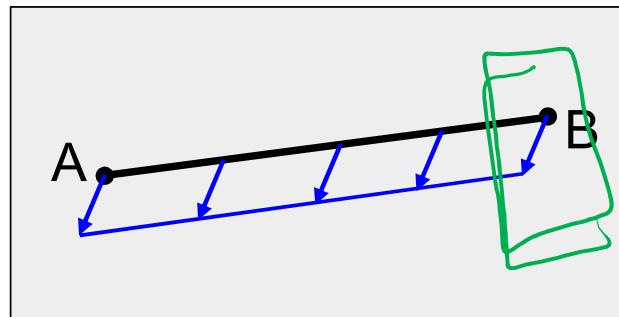


Velocity decomposition method

2nd example



$$|v_{BA}| = \omega |AB|$$



+

$$\vec{v}_B = \vec{v}_A + \vec{v}_{BA}$$

absolute velocity of point B

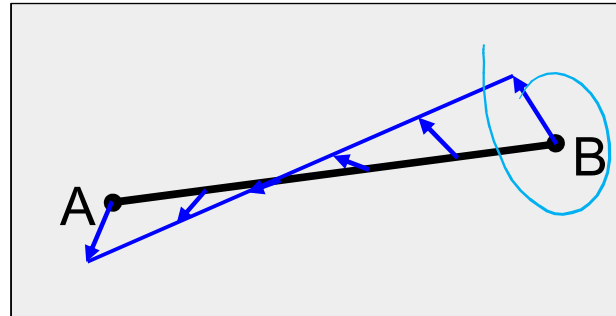
velocity of a linear motion

Angular velocity of point B in rotation around point A.

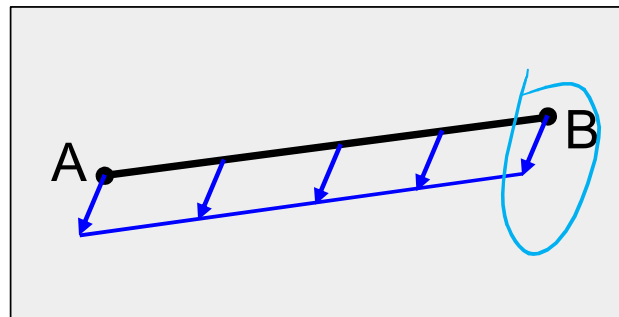
$$\vec{v}_{BA} = \vec{\omega} \times \vec{AB}$$

Velocity decomposition method

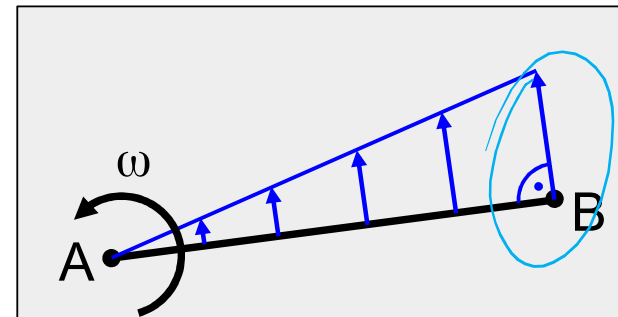
2nd example



=



+



$$\vec{v}_B = \vec{v}_A + \vec{v}_{BA}$$

absolute velocity
of point B

velocity of a linear motion

Angular velocity of point B
in rotation around point A.

$$\vec{v}_{BA} = \vec{\omega} \times \vec{AB}$$

Velocity scheme method

Velocity scheme of a rigid body – geometry created by the ends of it's velocity vectors moved to the common starting point (pole).

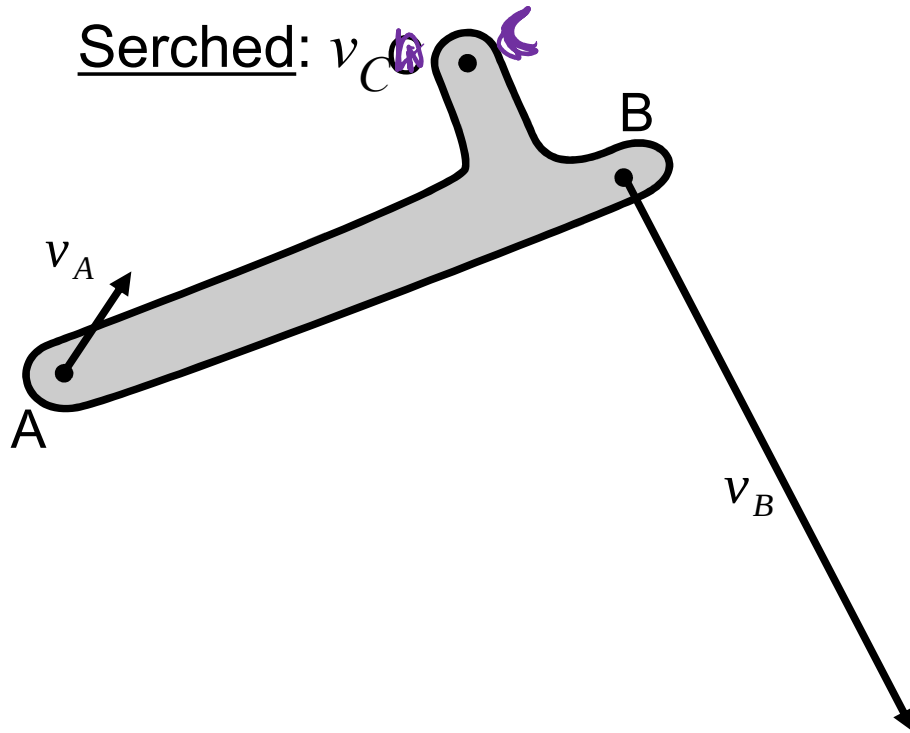
Velocity scheme is similar to the corresponding rigid body: it is scaled and rotated by an 90° angle in the direction of body's angular velocity.

Velocity scheme method

Example

Given: v_A and v_B

Serched: v_C

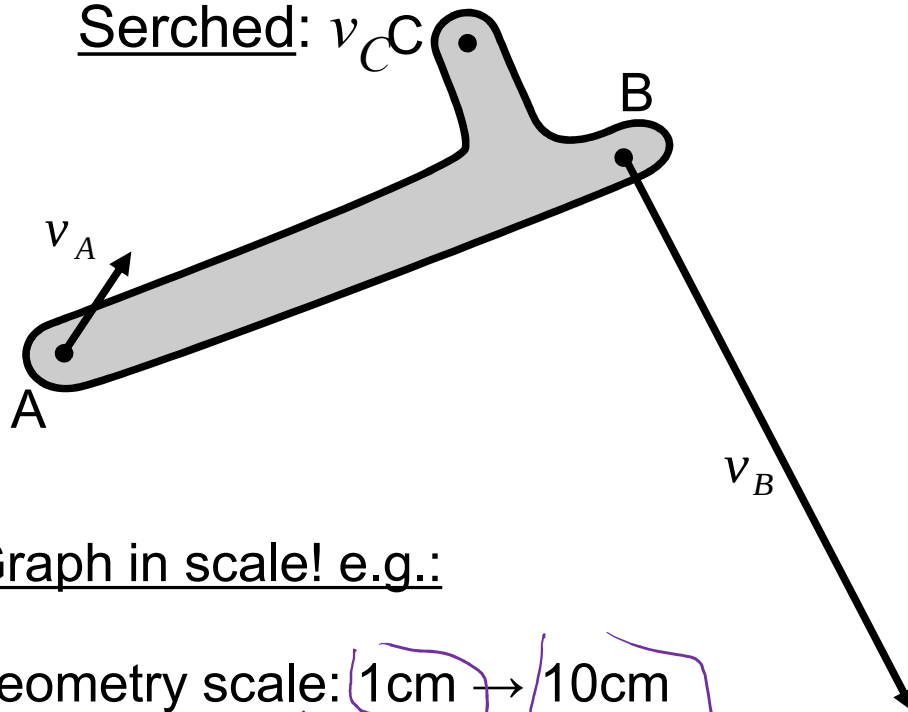


Velocity scheme method

Example

Given: v_A and v_B

Serched: v_C

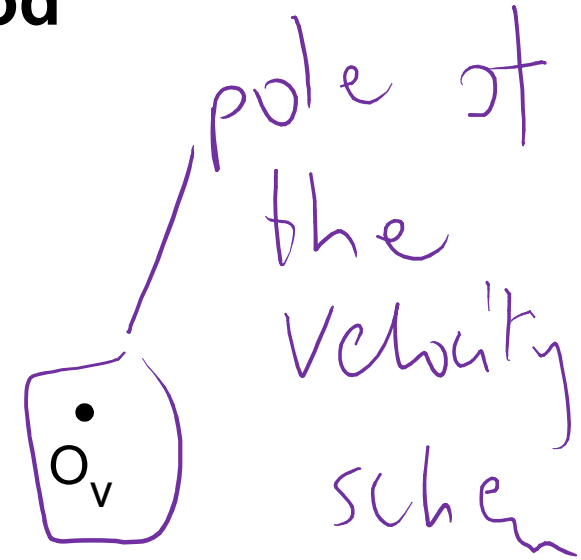
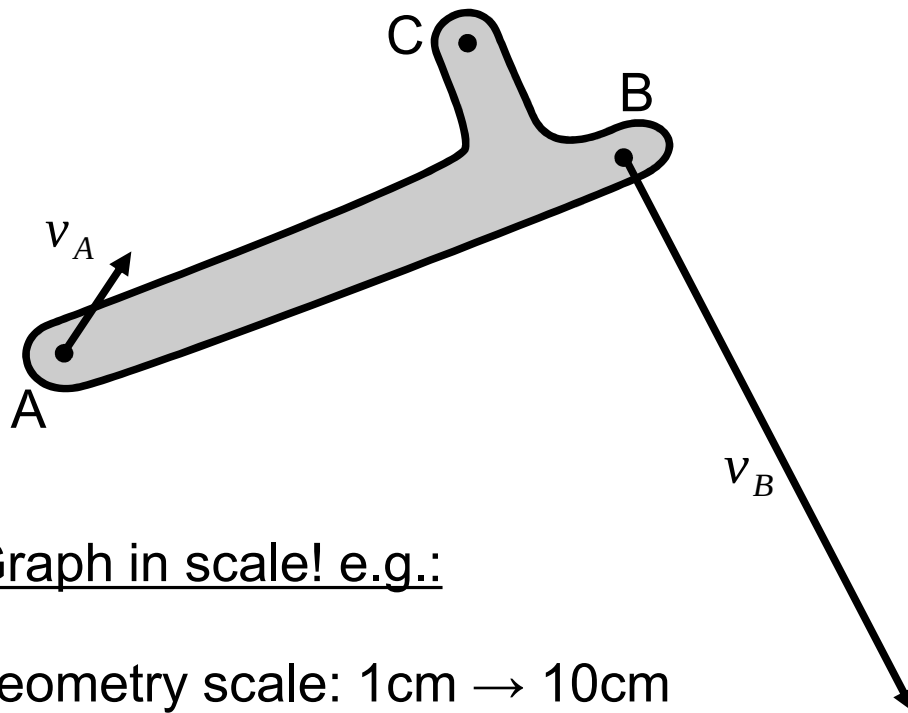


Graph in scale! e.g.:

geometry scale: 1cm → 10cm
velocity scale: 1cm → 1m/s

Velocity scheme method

Example



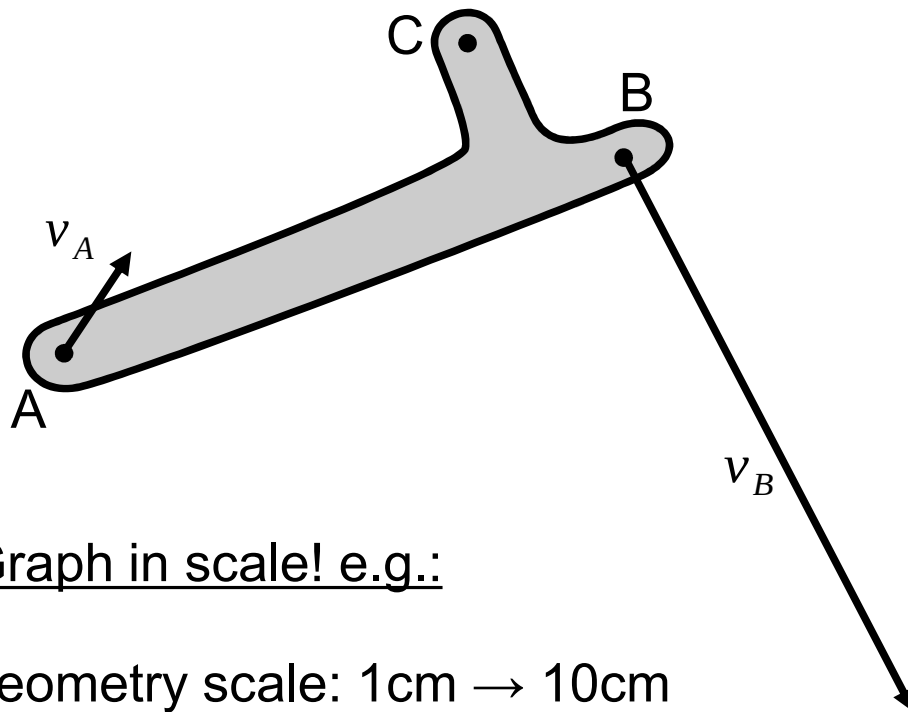
Graph in scale! e.g.:

geometry scale: 1cm \rightarrow 10cm

velocity scale: 1cm \rightarrow 1m/s

Velocity scheme method

Example



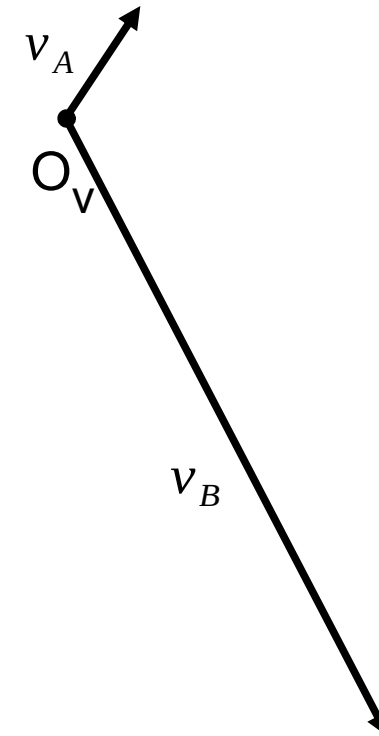
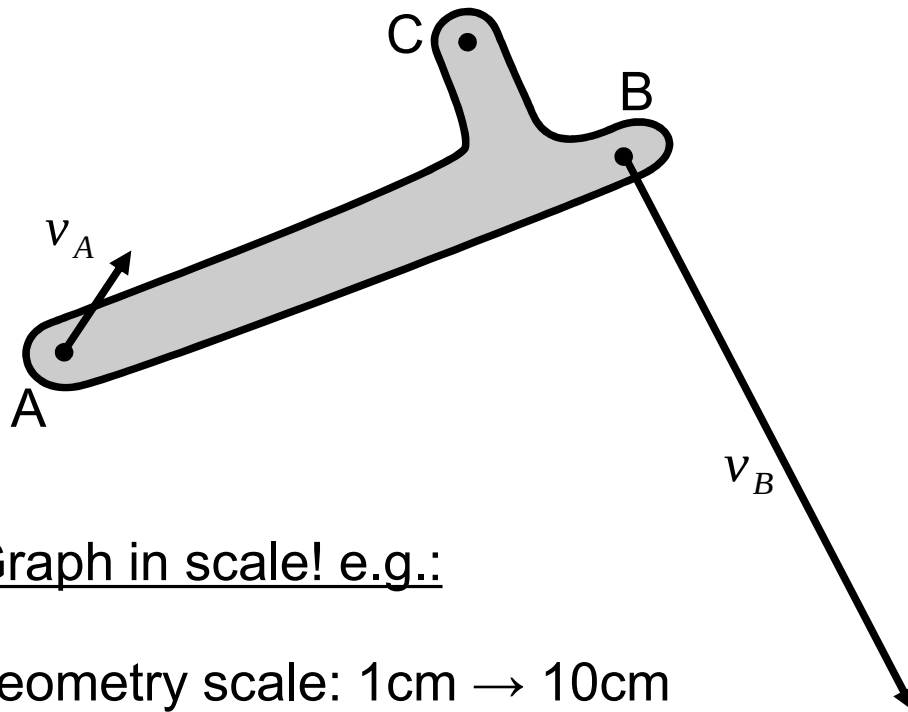
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Velocity scheme method

Example



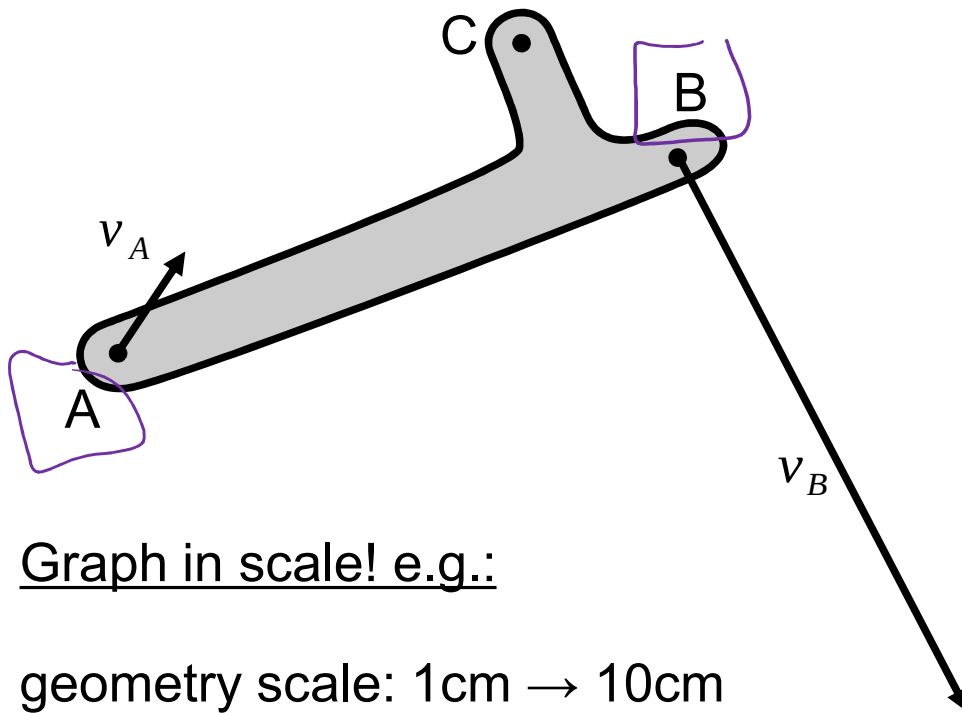
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Velocity scheme method

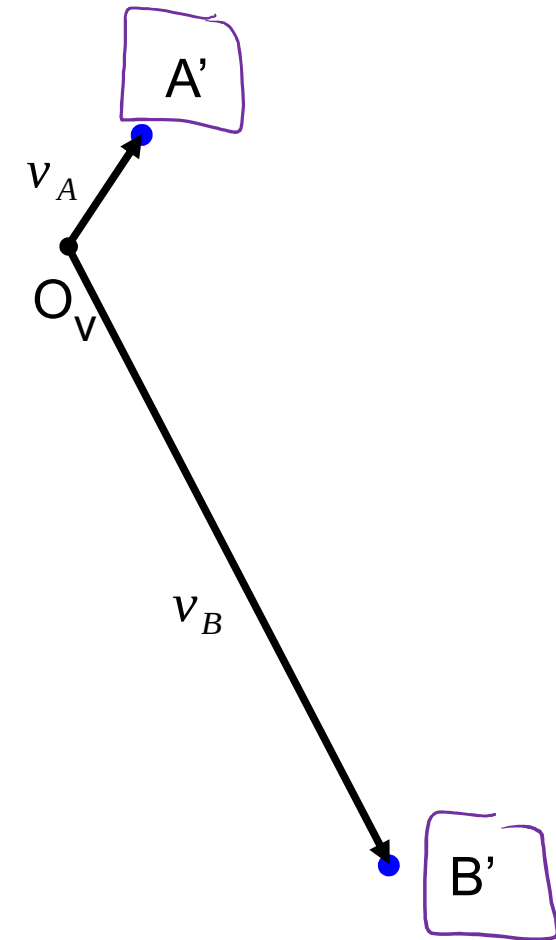
Example



Graph in scale! e.g.:

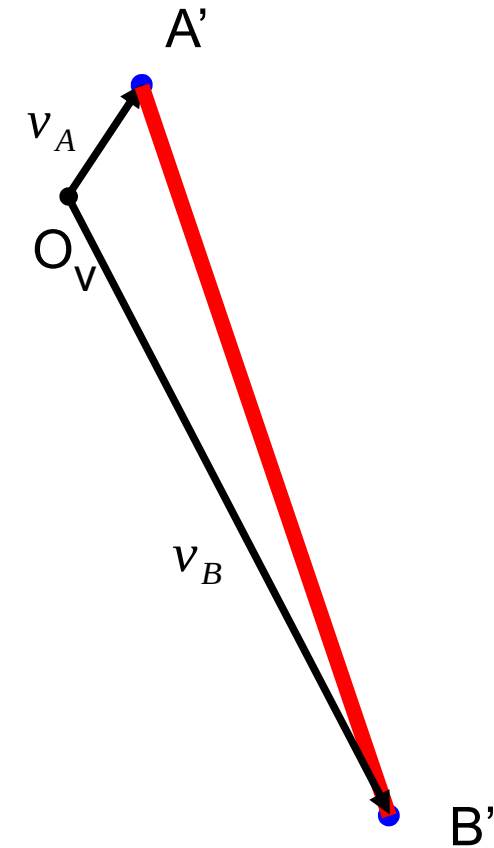
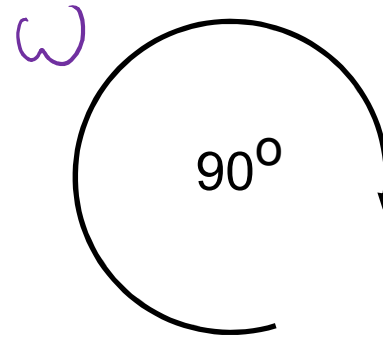
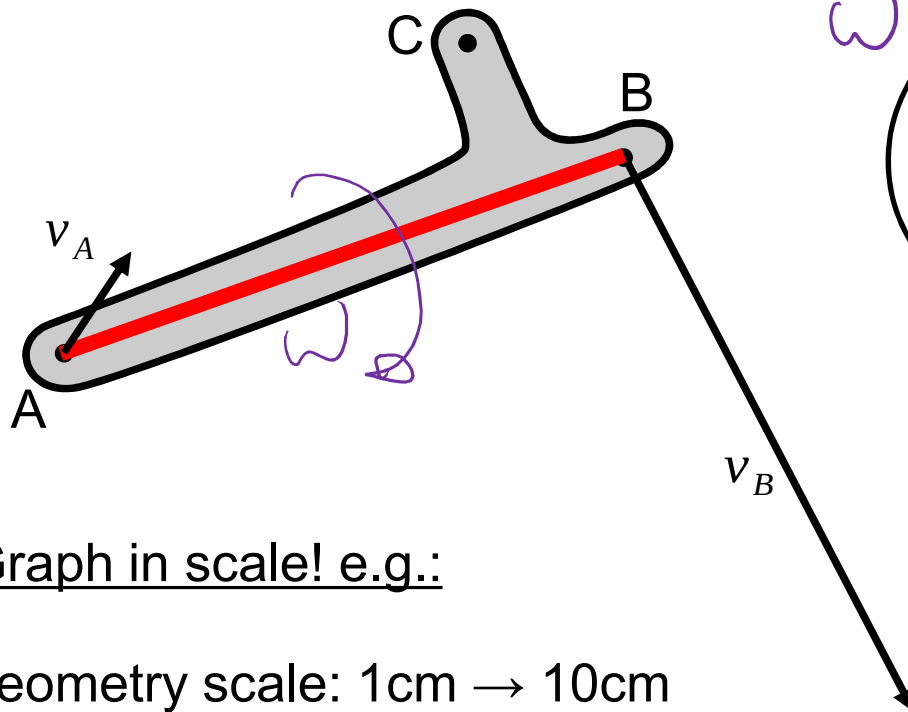
geometry scale: 1cm \rightarrow 10cm

velocity scale: 1cm \rightarrow 1m/s



Velocity scheme method

Example



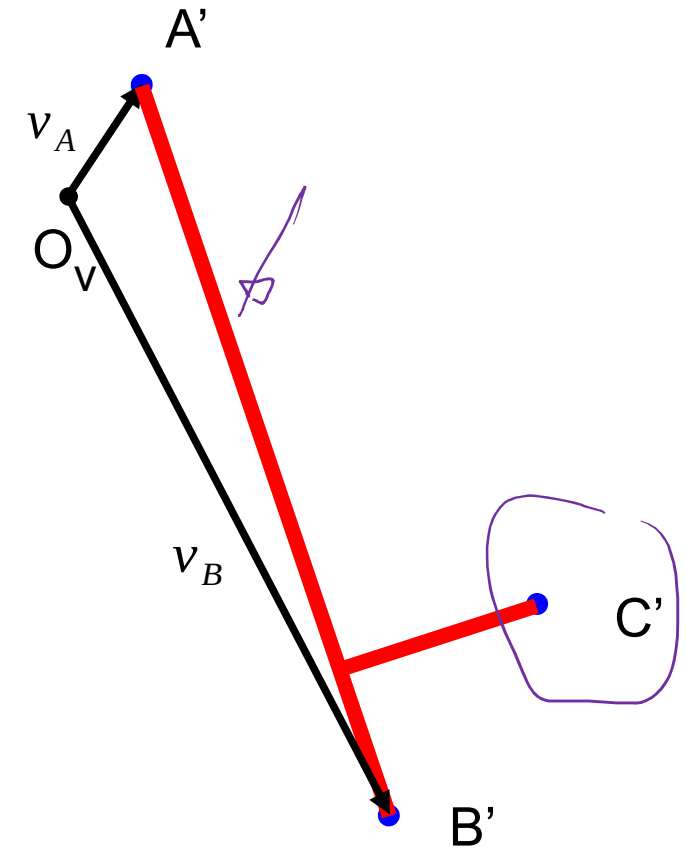
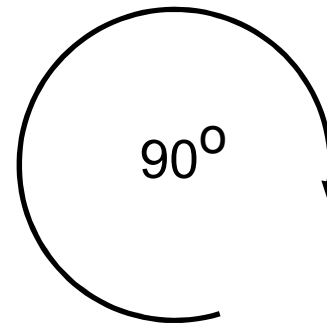
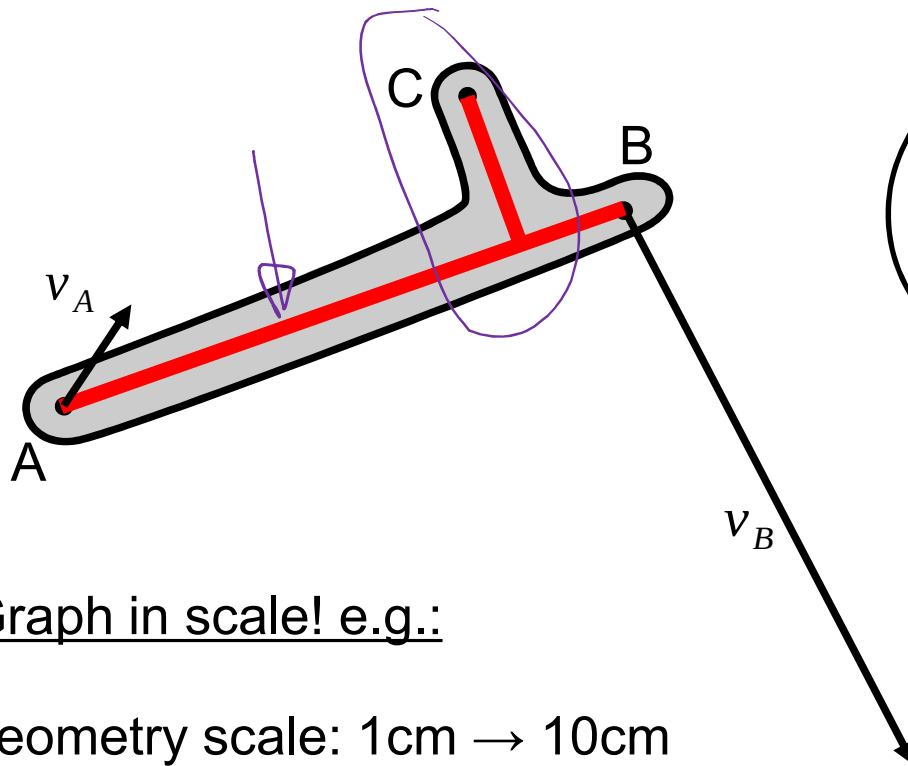
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Velocity scheme method

Example



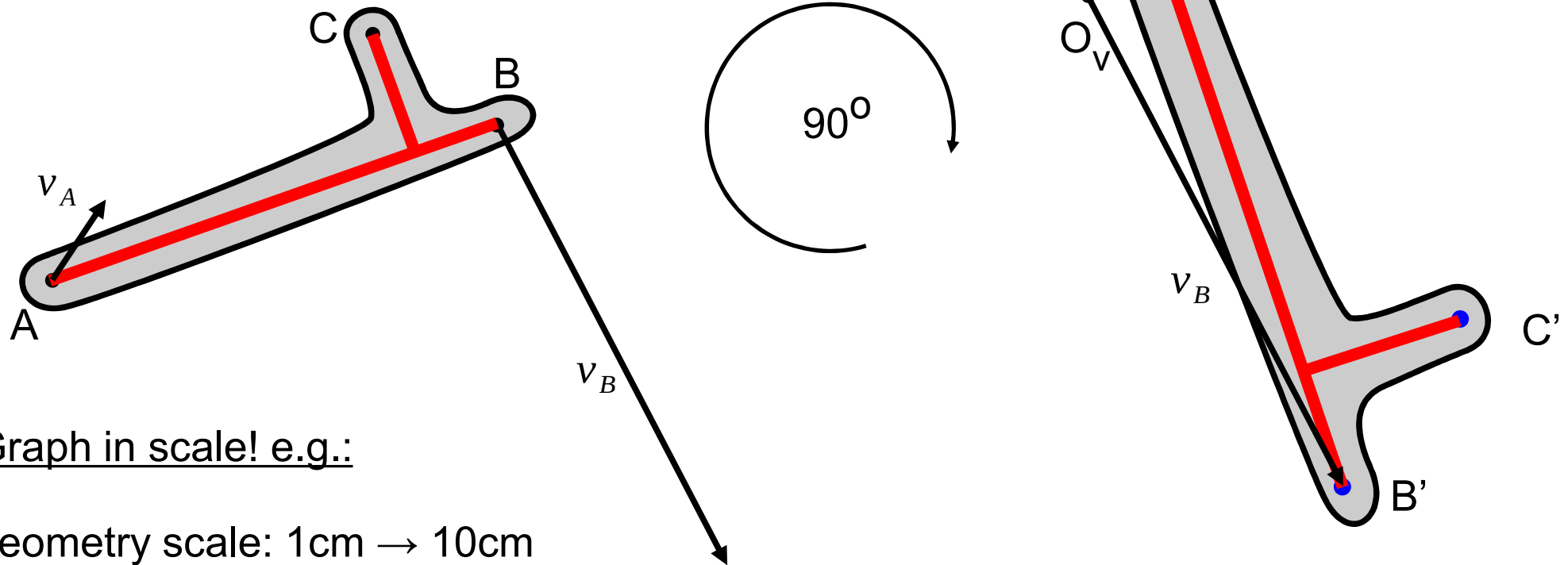
Graph in scale! e.g.:

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velocity scale: 1cm \rightarrow 1m/s

Velocity scheme method

Example



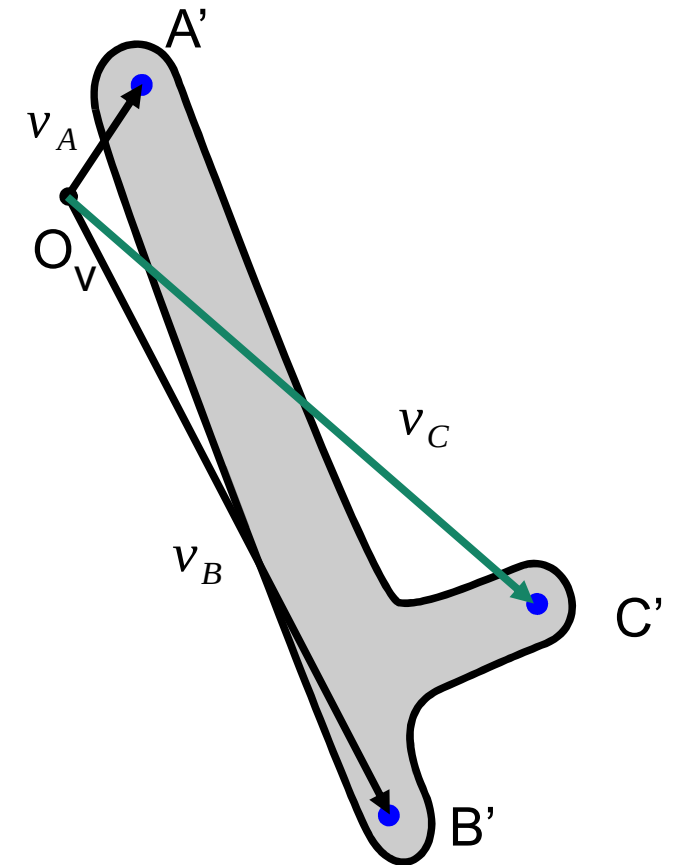
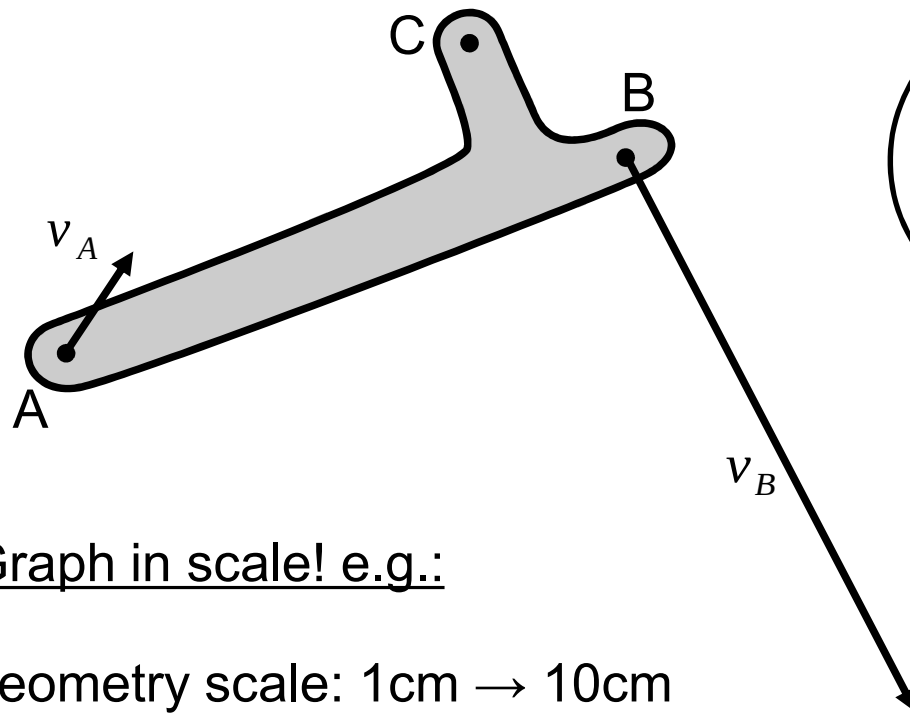
Graph in scale! e.g.:

geometry scale: 1cm \rightarrow 10cm

velocity scale: 1cm \rightarrow 1m/s

Velocity scheme method

Example



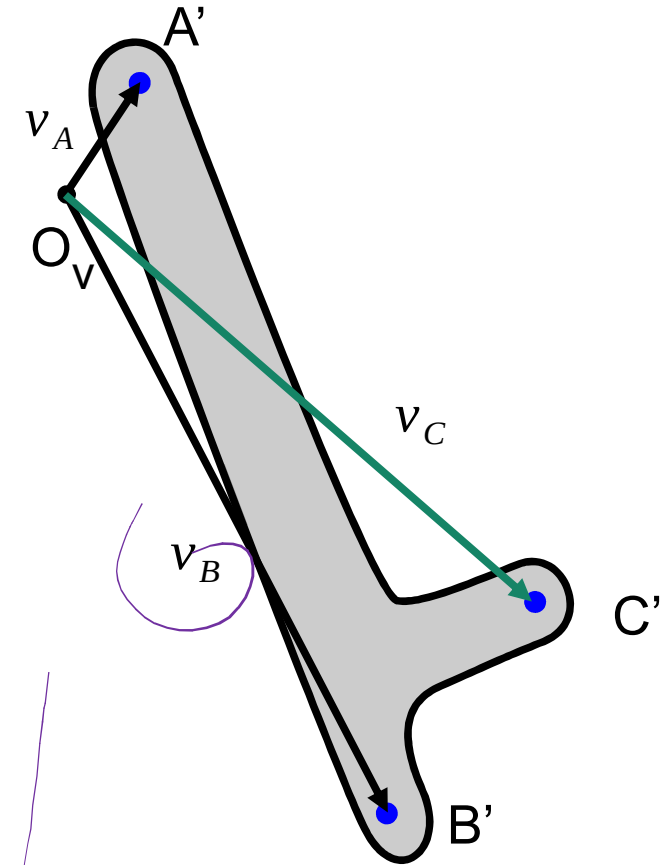
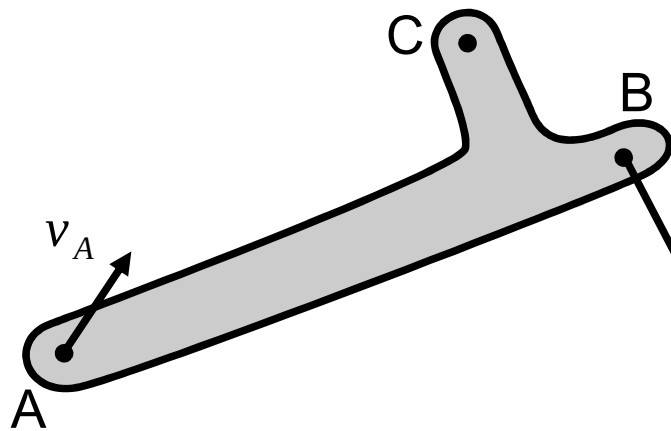
Graph in scale! e.g.:

geometry scale: 1cm \rightarrow 10cm

velocity scale: 1cm \rightarrow 1m/s

Velocity scheme method

Example



Graph in scale! e.g.:

geometry scale: 1cm \rightarrow 10cm

velocity scale: 1cm \rightarrow 1m/s

velocity scale: the same
geometry scale: new!

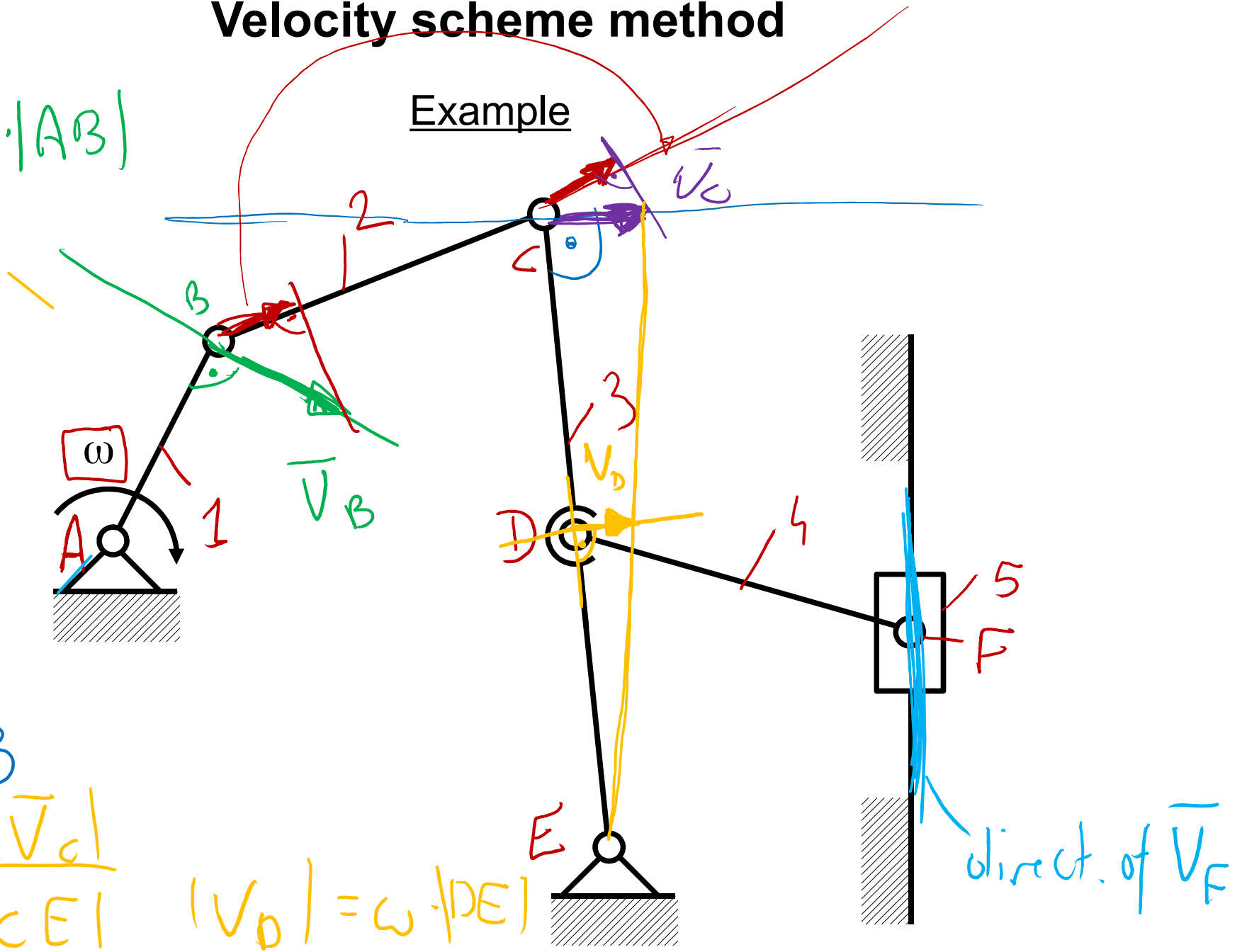
Velocity scheme method

Example

$$\vec{V}_A = 0$$

$$|\vec{V}_B| = |\omega| \cdot |AB|$$

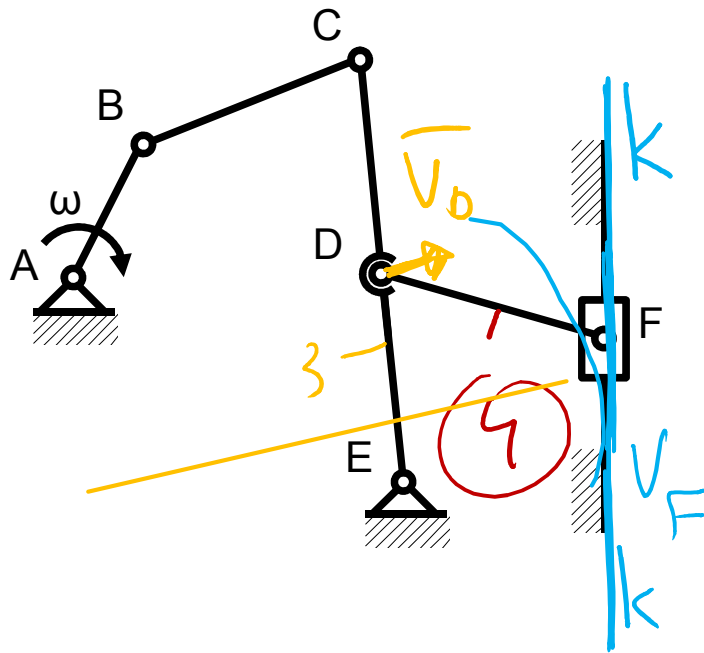
$$\vec{V}_E = 0$$



Velocity scheme method

Example

veloc. decomp. method of (4)



$$\vec{v}_F = \vec{v}_D + \vec{v}_{FD}$$

\vec{v}_F is vertical (labeled $\perp k$)
 \vec{v}_D is horizontal (labeled $\perp 3$)
 \vec{v}_{FD} is perpendicular to DF (labeled $\perp 4$)

