Faculty of Automotive
and Construction
Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control

Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

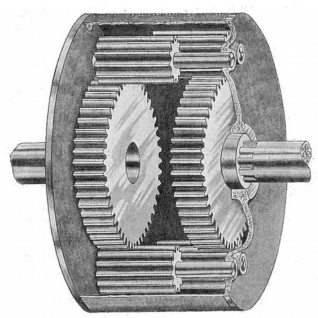
Lecture 14

Repeat of material.
Information about the exam.
~~WUT questionnaires~~.

Lecture 1

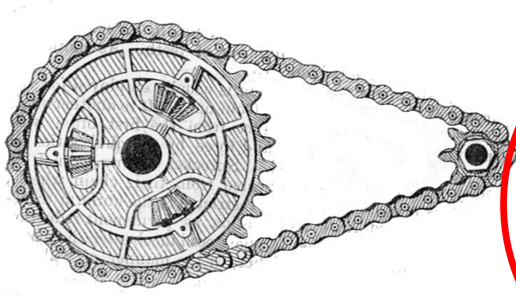
kinematic pairs, mechanisms, mobility

Components of machines



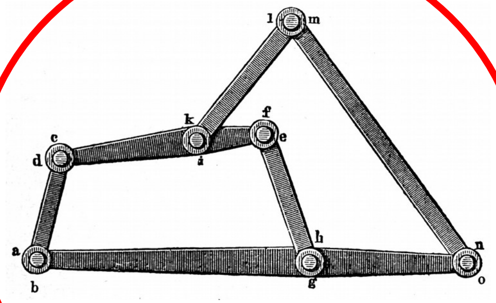
gear train

belt drive

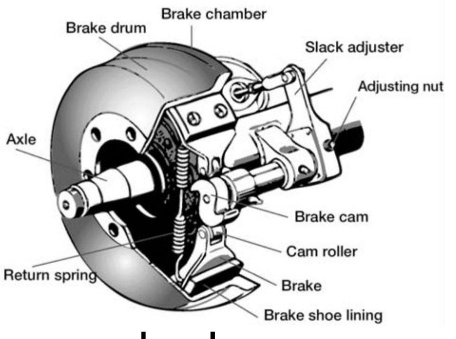


chain drive

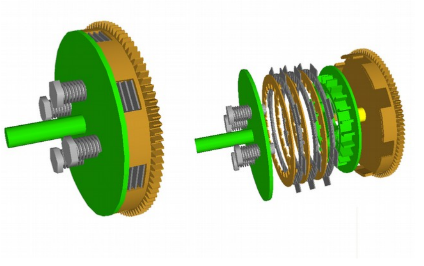
cam



linkage



brake



clutch

fastener

graphics source: https://en.wikipedia.org

Members of mechanisms

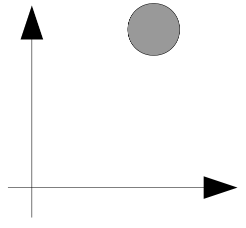
member = part = element = segment = link

Rigid members – described by material points (*Theoretical Mechanics I, 2nd semester lecture*) or rigid bodies (*Theoretical Mechanics II, 3rd semester lecture).*

Deformable members – springs, ropes, belts, air etc.

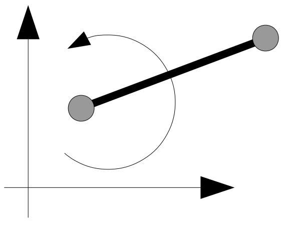
Degrees of freedom

material point (2D)



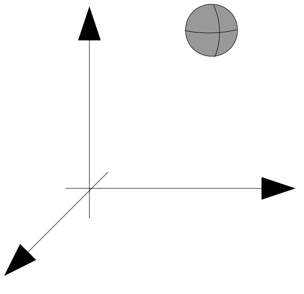
2 DoF

rigid body (2D)



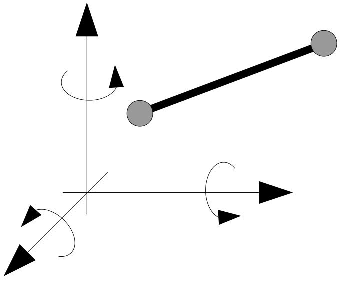
3 DoF

material point (3D)



3 DoF

rigid body (3D)



6 DoF

Kinematic pairs & chains

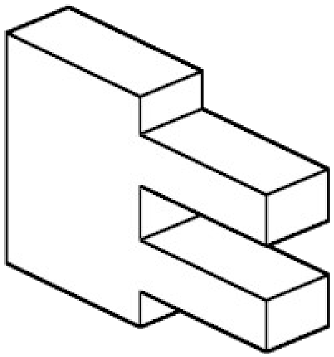
A kinematic pair is a movable coupling of two rigid members that imposes restraints on the relative motion of the members by the conditions of linkage.

A kinematic chain is an assembly of kinematic pairs.

A base is a fixed (motionless) member of mechanism.

Kinematic pairs (3D)

not connected



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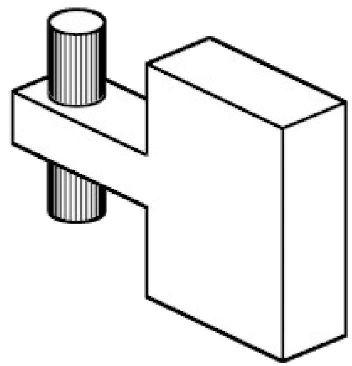

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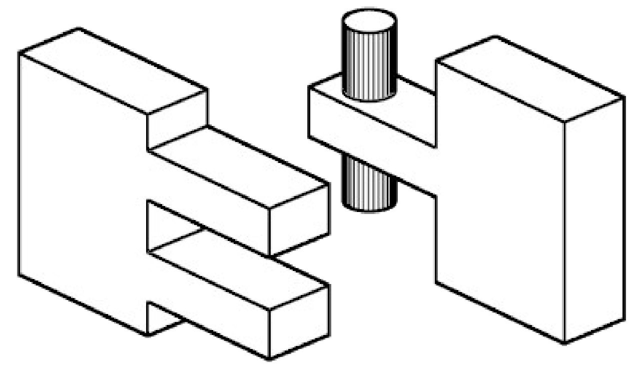


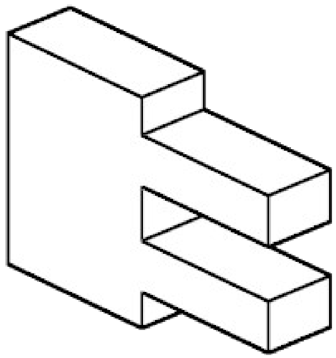
6 DoF

6 DoF

total: 12 DoF

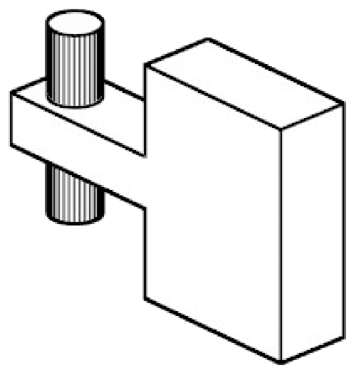
relative motion: 6DoF

Kinematic pairs (3D)



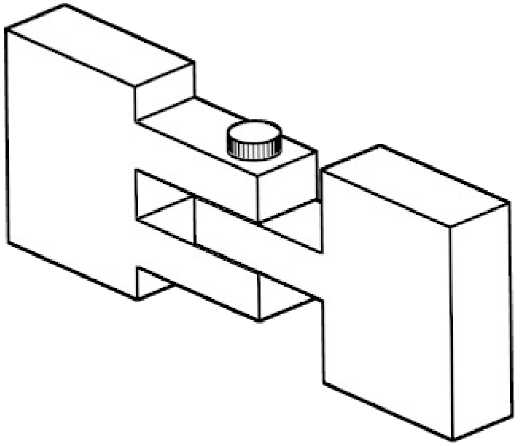
6 DoF

+



6 DoF

=



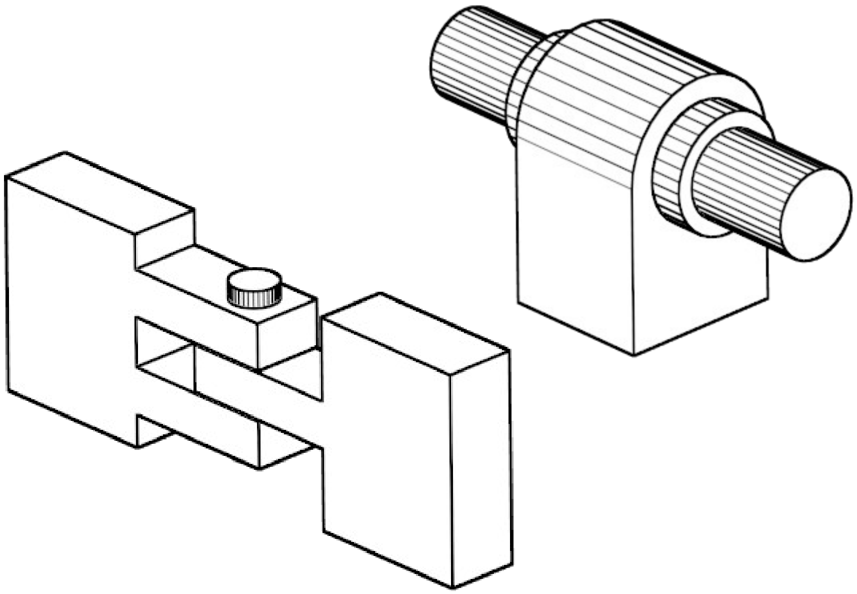
<u>relative motion: 1DoF</u>

total: 7DoF

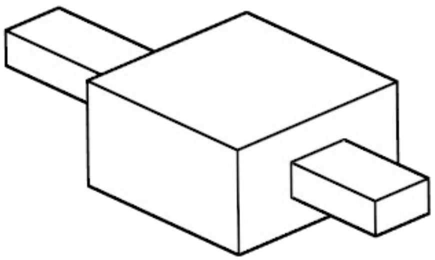
Kinematic pairs (3D)

Class V **= 6 - 1**

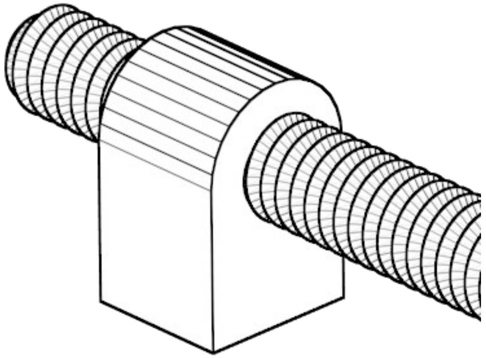
rotary



translatory



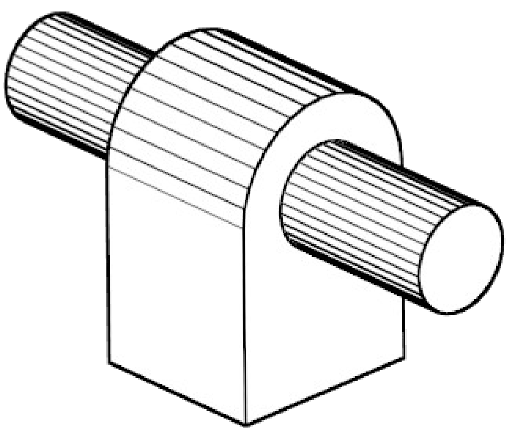
screw-type

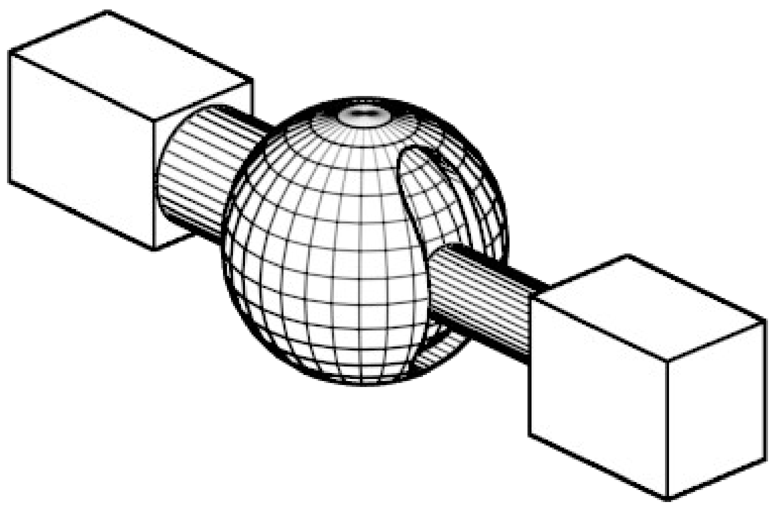


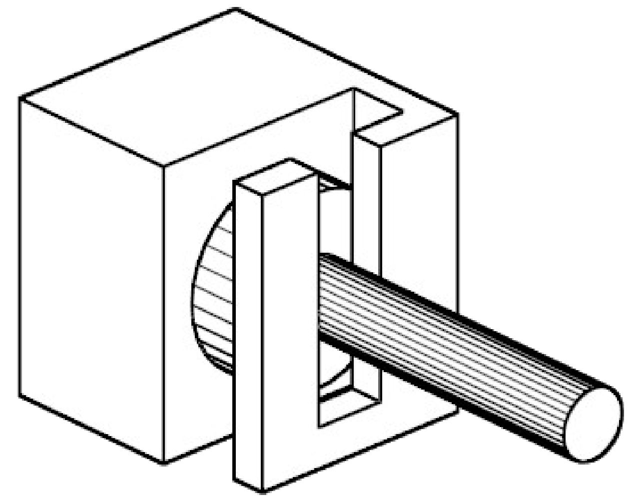
Kinematic pairs (3D)

Class IV **= 6 - 2**

cylindrical



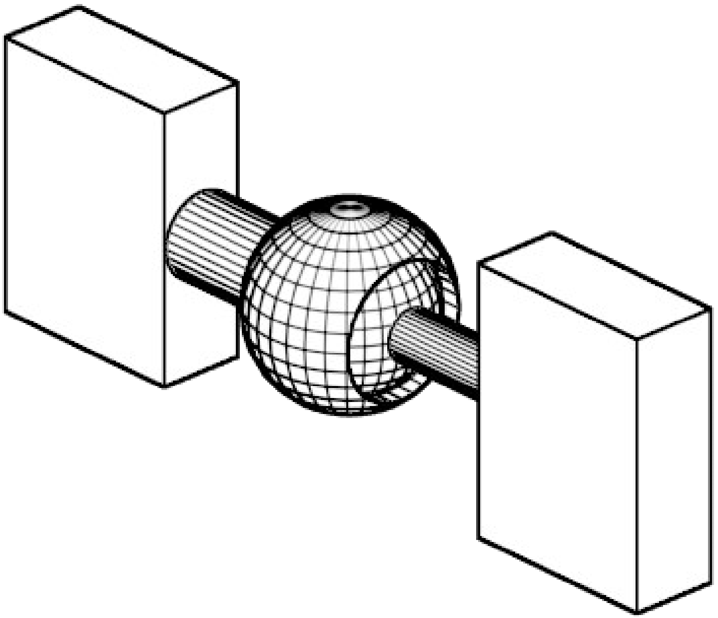


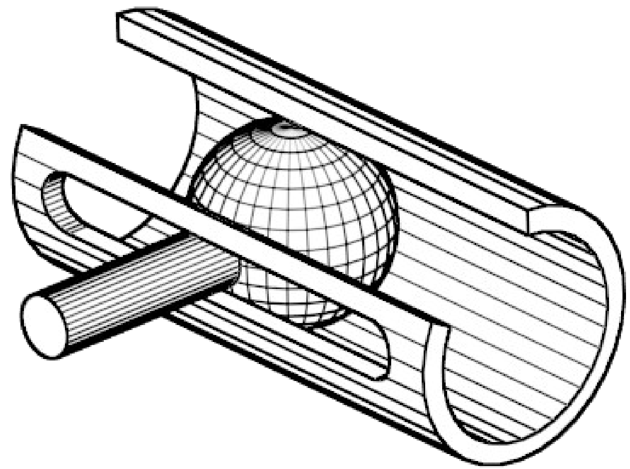


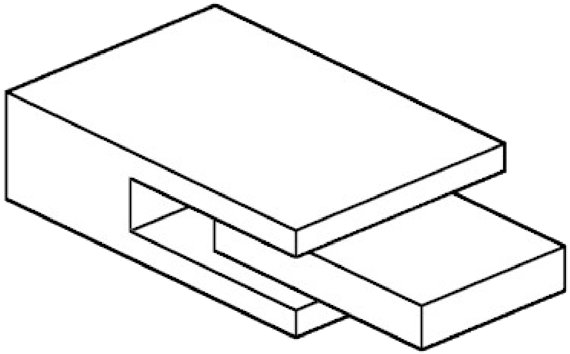
Kinematic pairs (3D)

Class III **= 6 - 3**

spherical

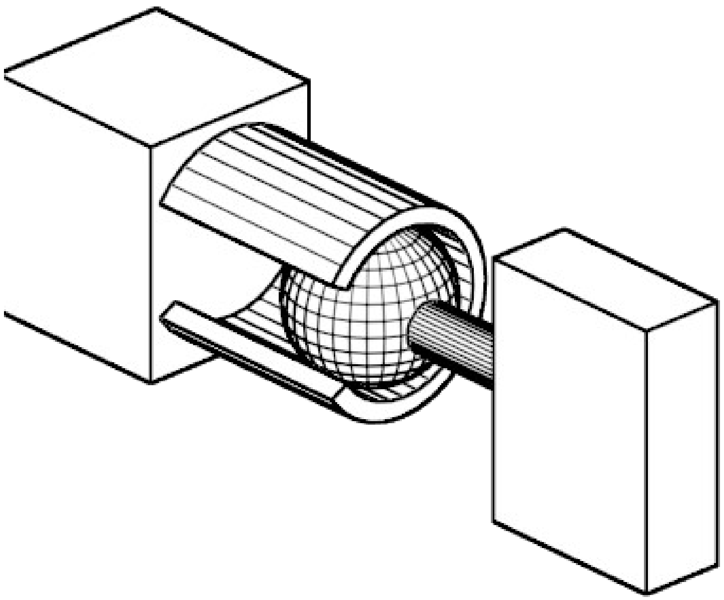


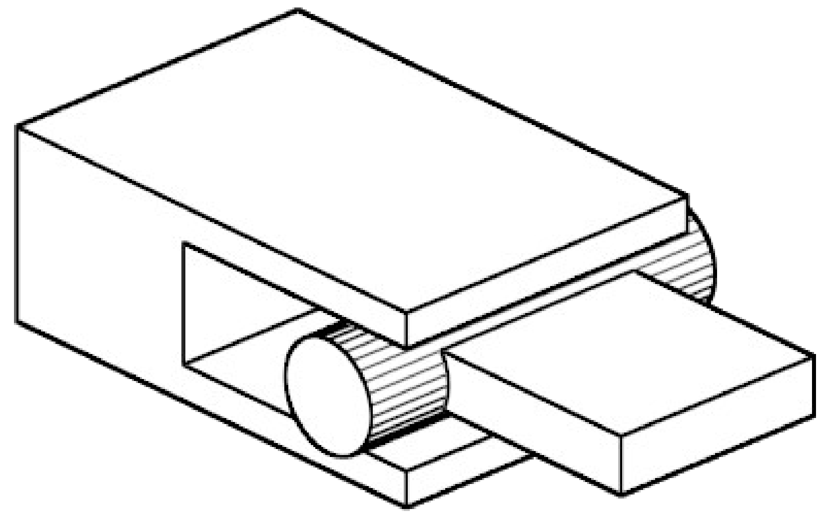




Kinematic pairs (3D)

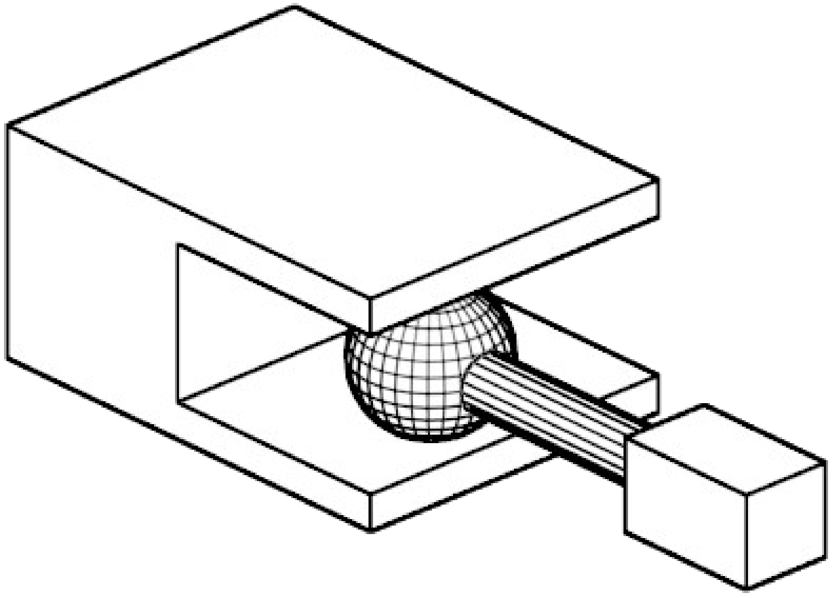
Class II **= 6 - 4**





Kinematic pairs (3D)

Class I **= 6 – 5**



Kinematic pairs (2D)

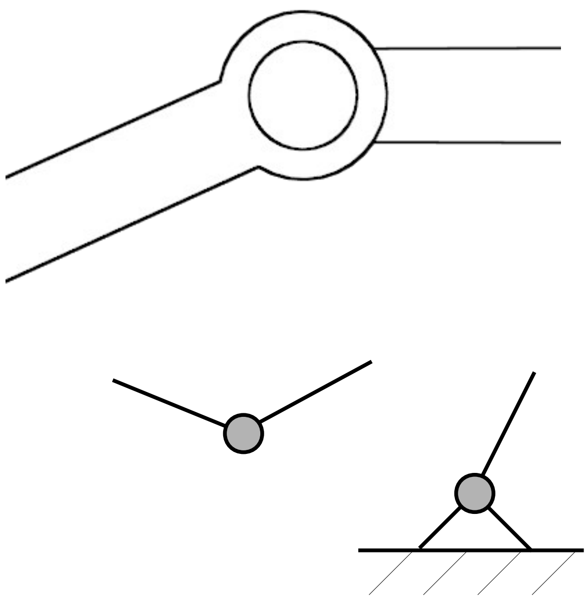
Class I, class II → not possible in 2D

Class III → free body in 2D

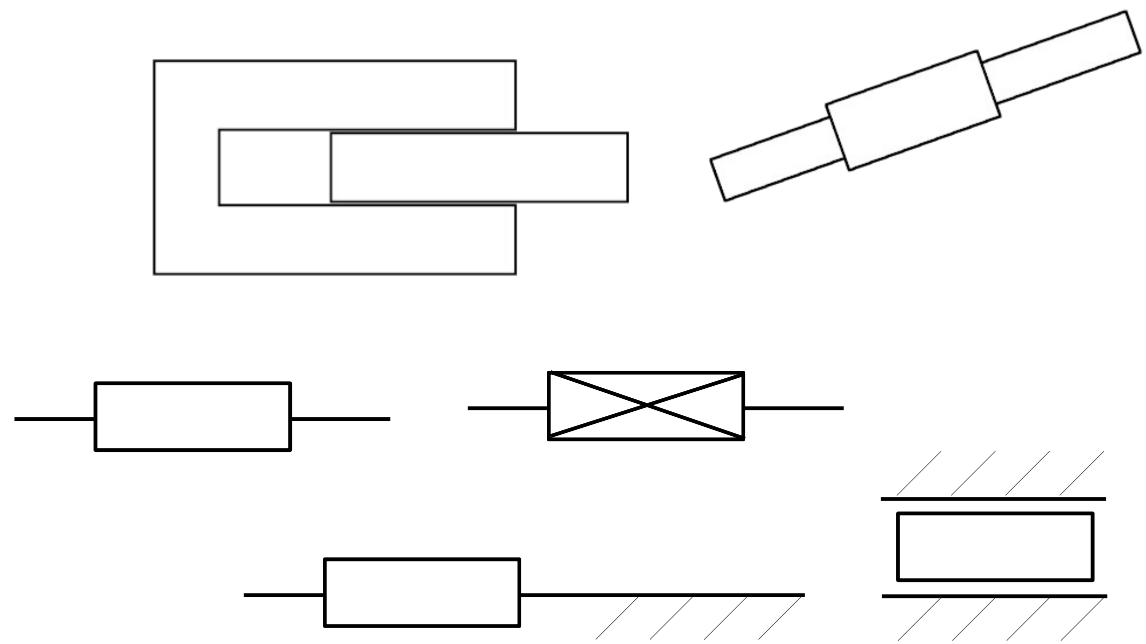
Kinematic pairs (2D)

Class V **= 6 - 1**

rotary

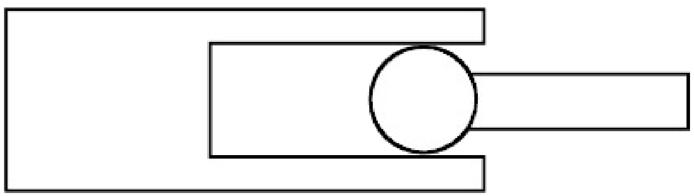


translatory



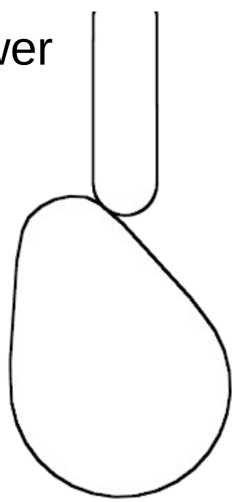
Kinematic pairs (2D)

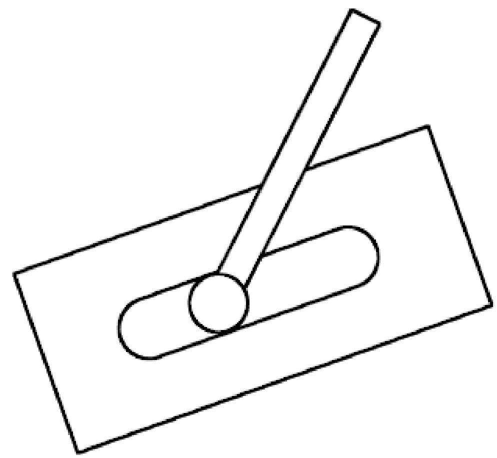
Class IV **= 6 - 2**

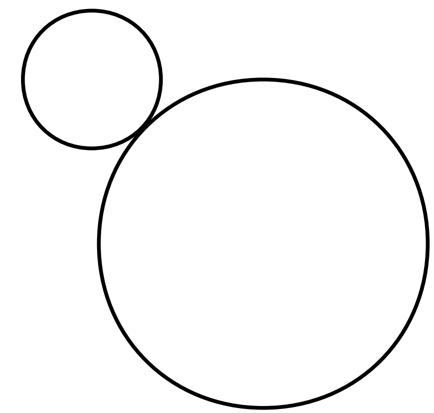


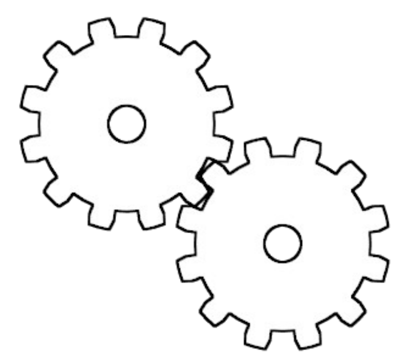
cam follower
(tapper)

cam joint









Kinematic pairs

lower kinematic pair – surface contact

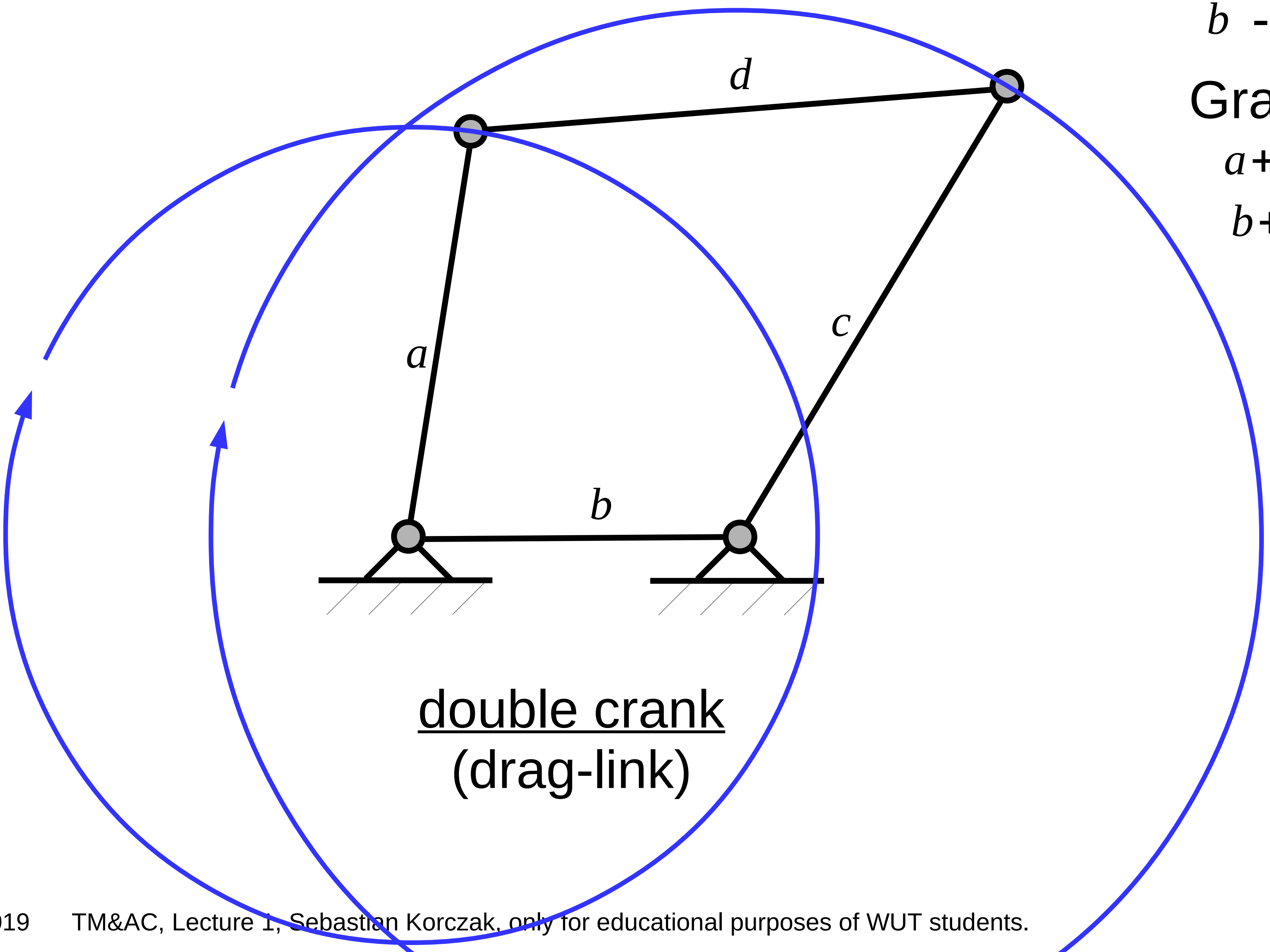
higher kinematic pair – line or point contact

closed pair (self-closed pair) – contact because of shape

open pair (force-closed pair) – force required for constant contact

Kinematic chain - examples

Four-bar chain



double crank
(drag-link)

b - the shortes

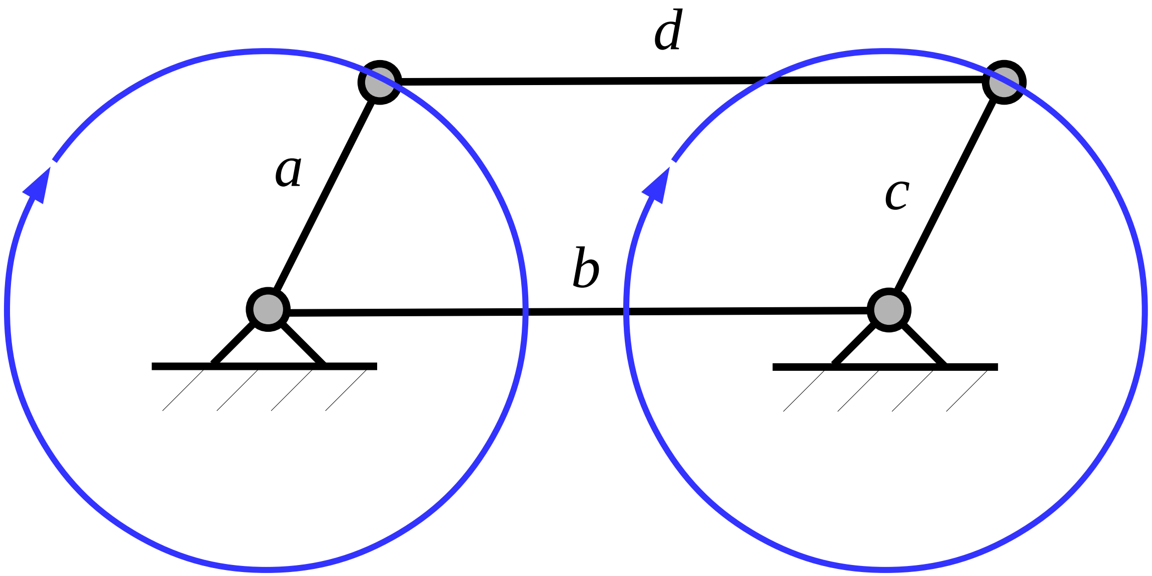
Grashof conditi

$a+b \leqslant c+d$

$b+c \leqslant a+d$

Kinematic chain - examples

Four-bar chain

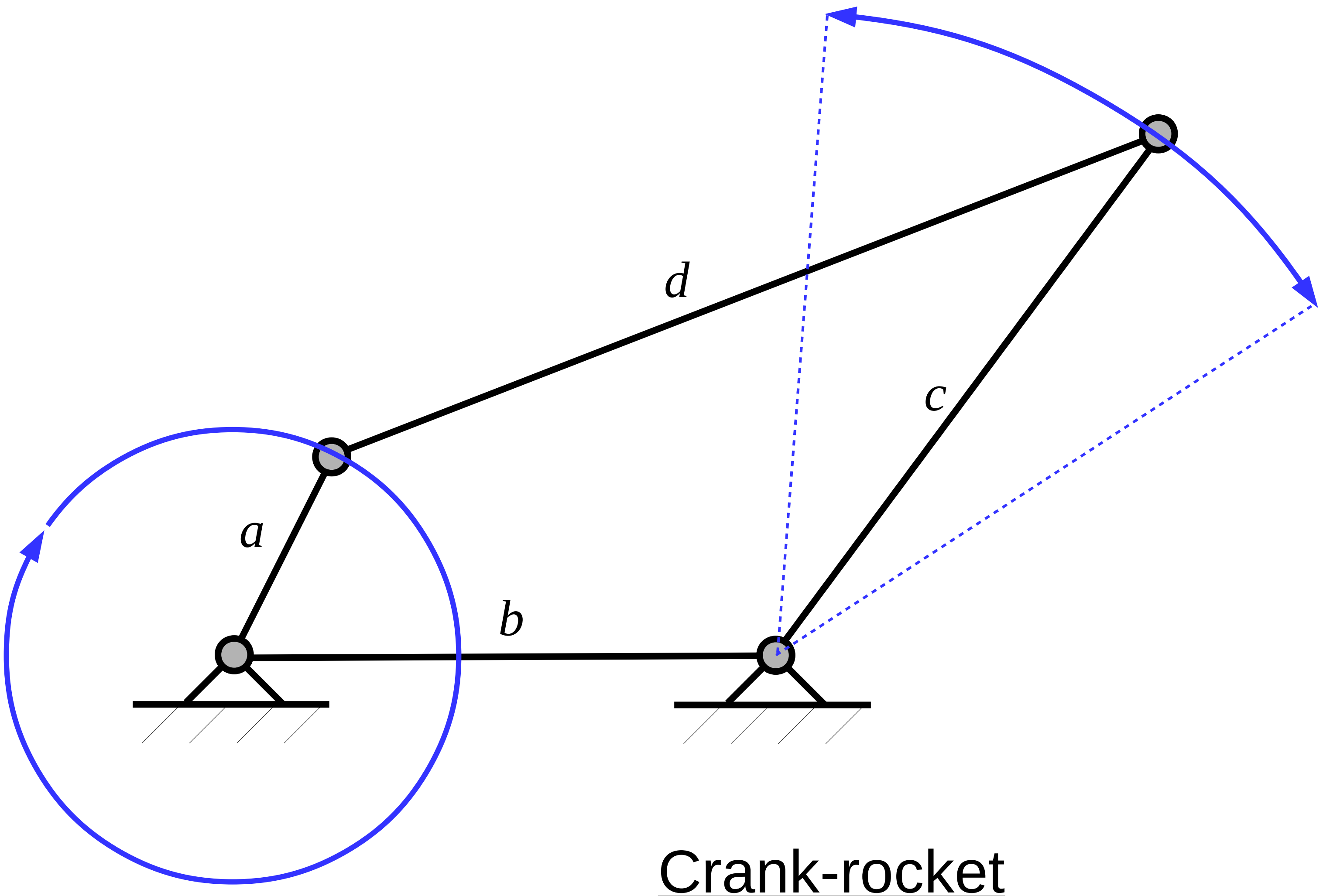


$$a+b=c+d$$
$$a=c$$

Parallelogram linkage
(double crank mechanism)

Kinematic chain - examples

Four-bar chain



Crank-rocket

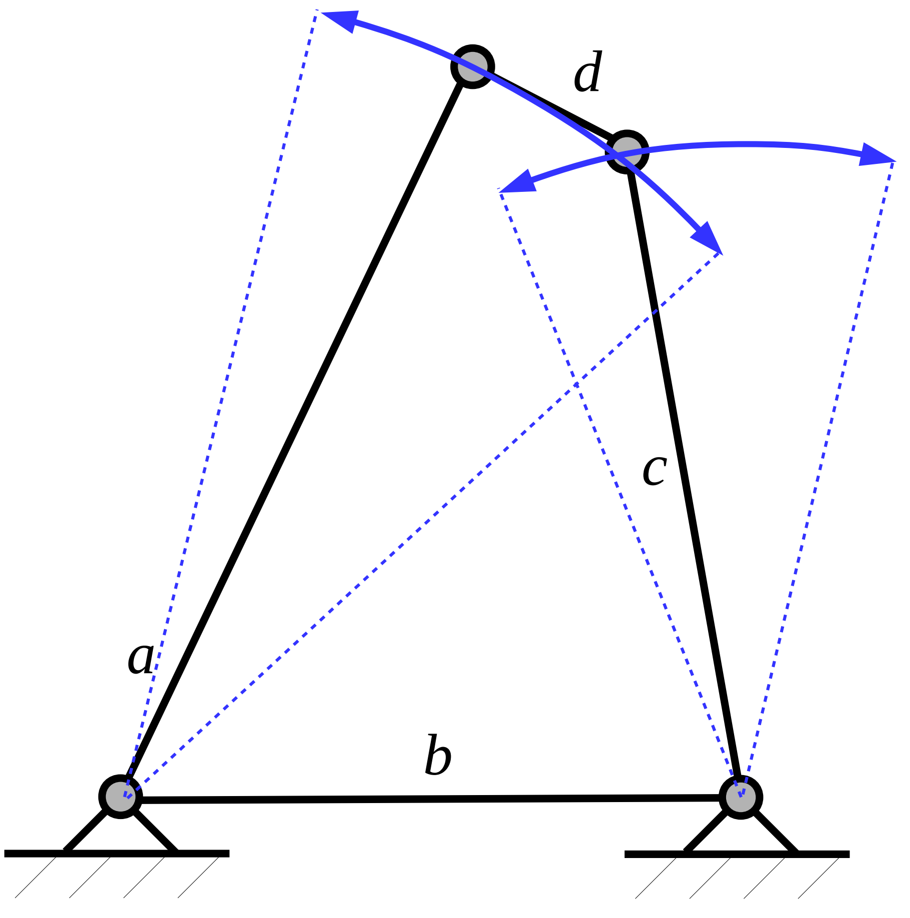
Grashof conditio

$a + d < b + c$

a - the shortest

Kinematic chain - examples

Four-bar chain



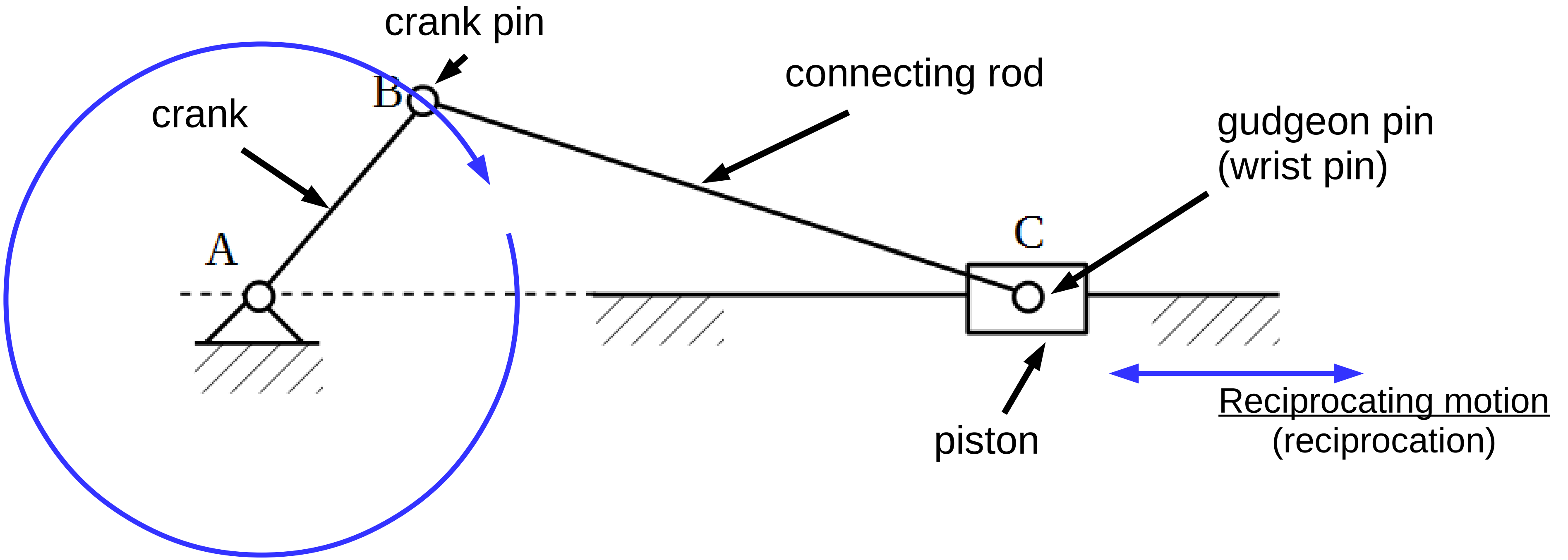
$a+d>b+c$

d - the shortest

<u>Double-rocket</u>

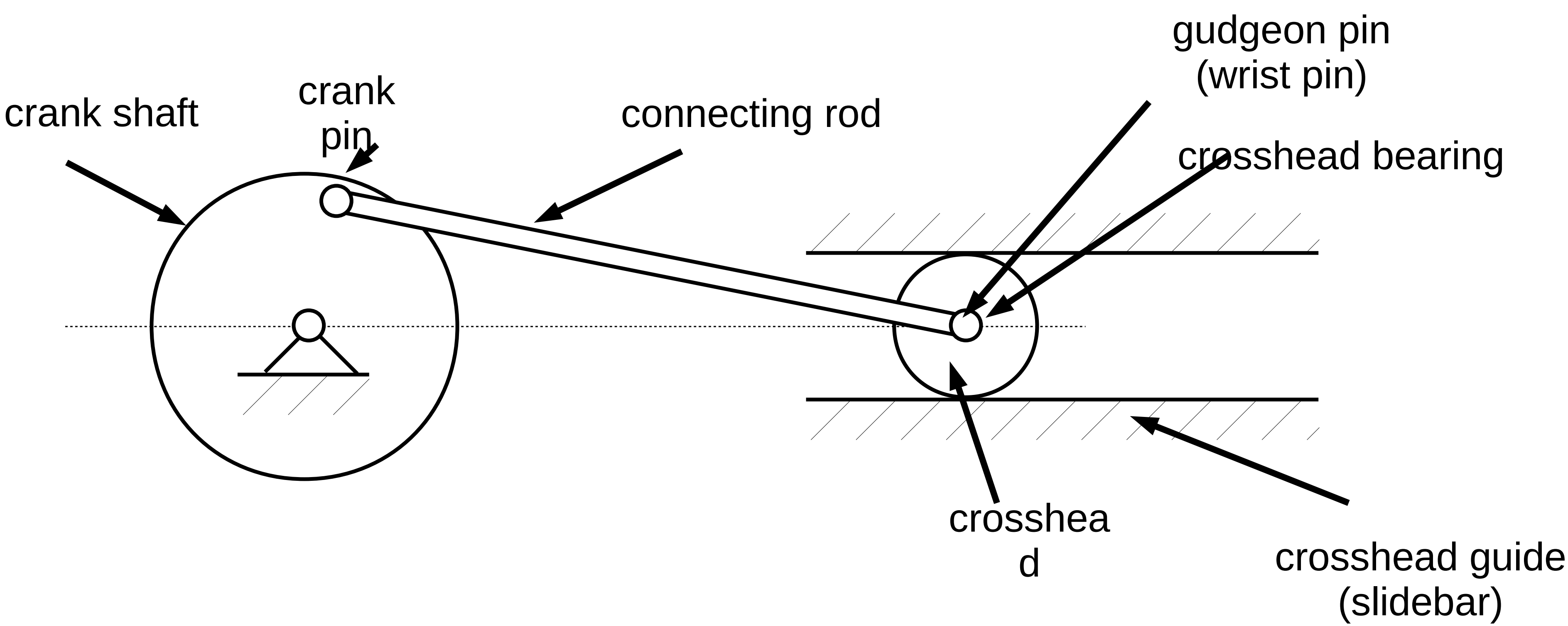
Kinematic chain - examples

Crank-slider mechanism



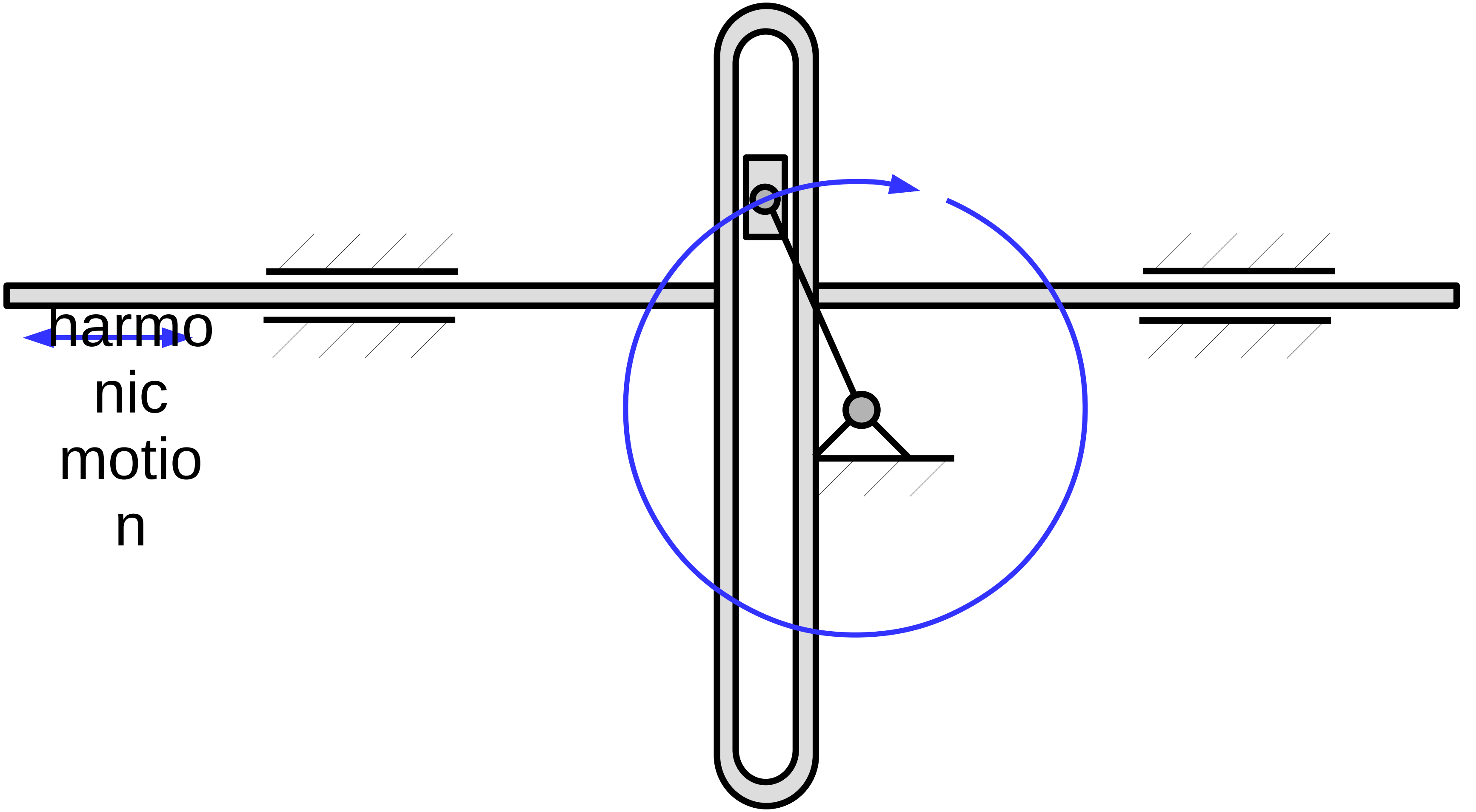
Kinematic chain - examples

Crank-slider mechanism

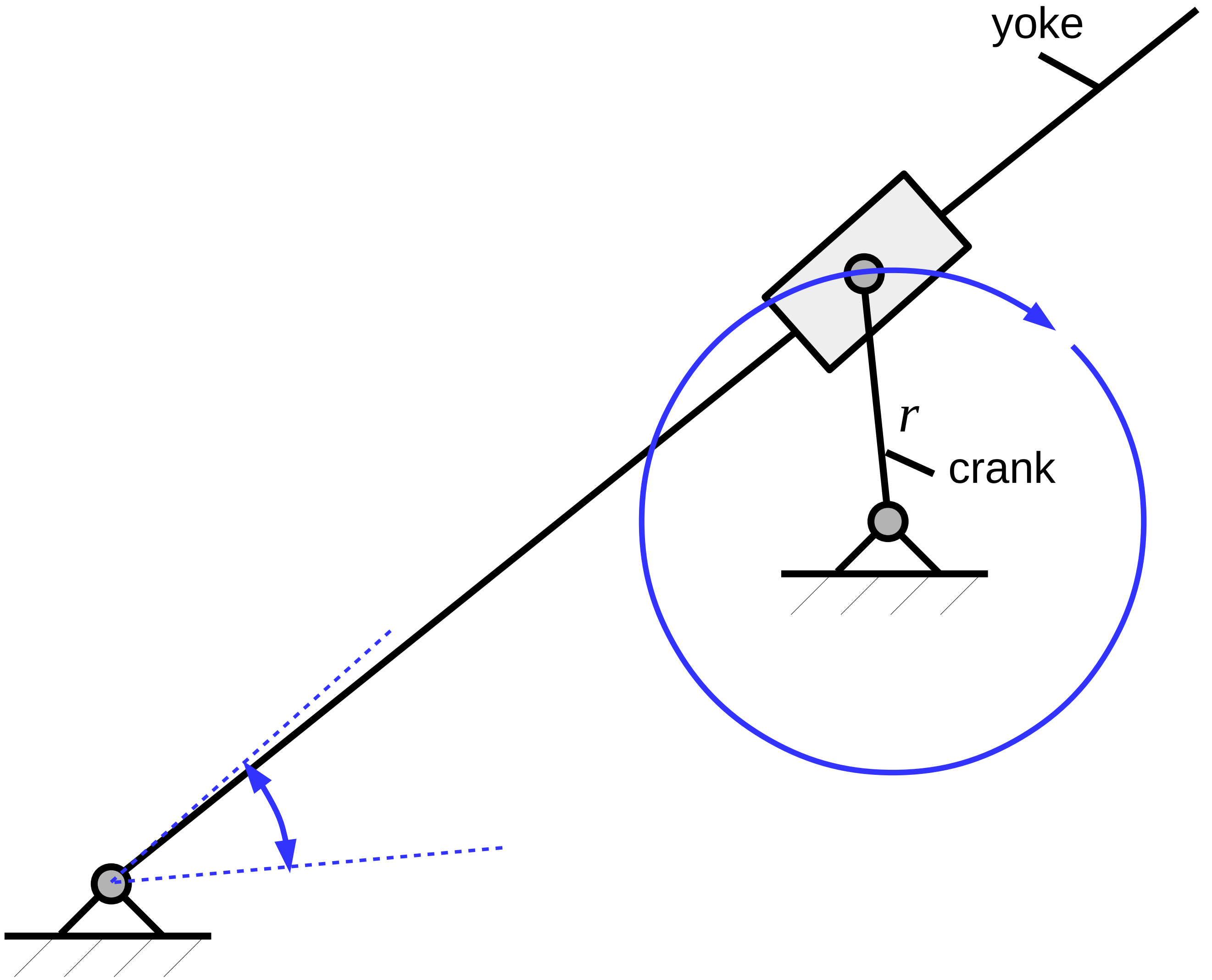


Kinematic chain - examples

Scotch yoke mechanism

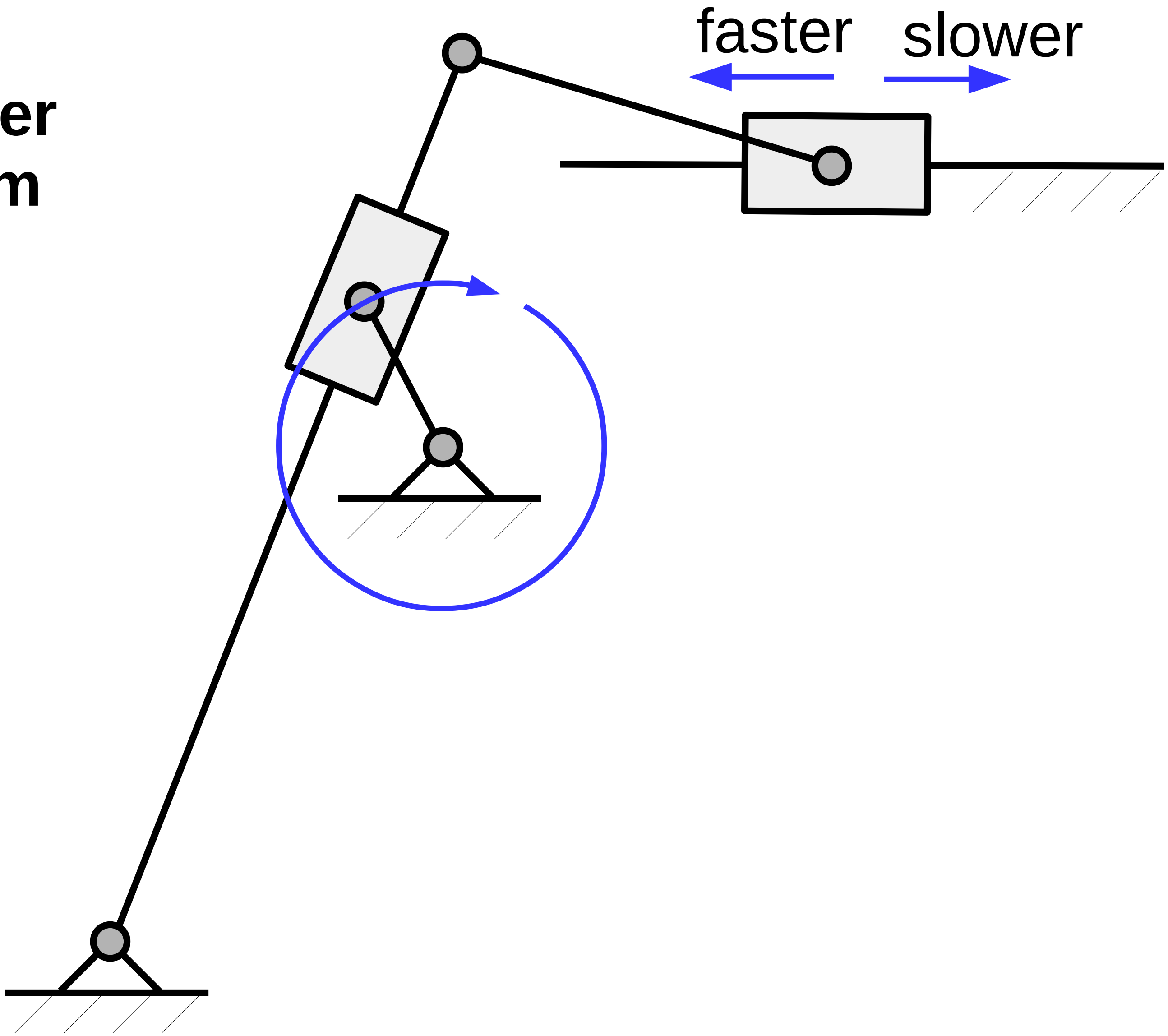


Kinematic chain - examples



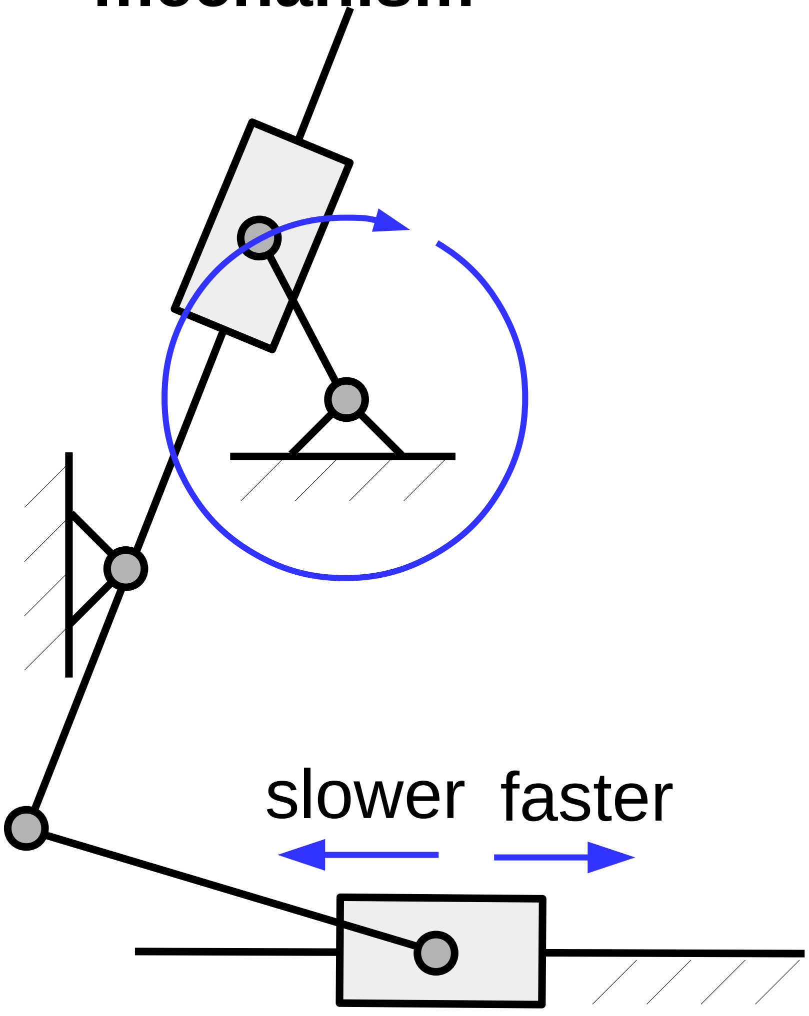
Kinematic chain - examples

Slotted lever mechanism



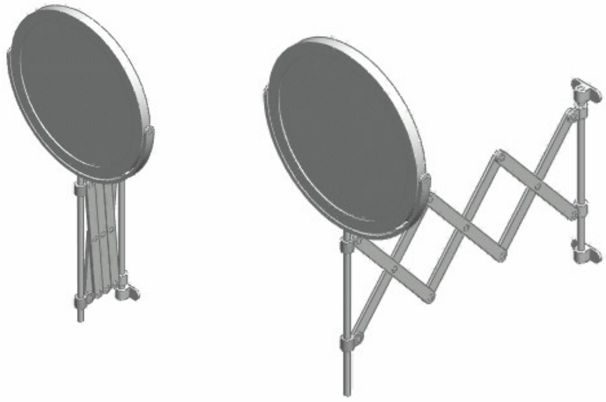
Kinematic chain - examples

Whitworth Quick Return mechanism

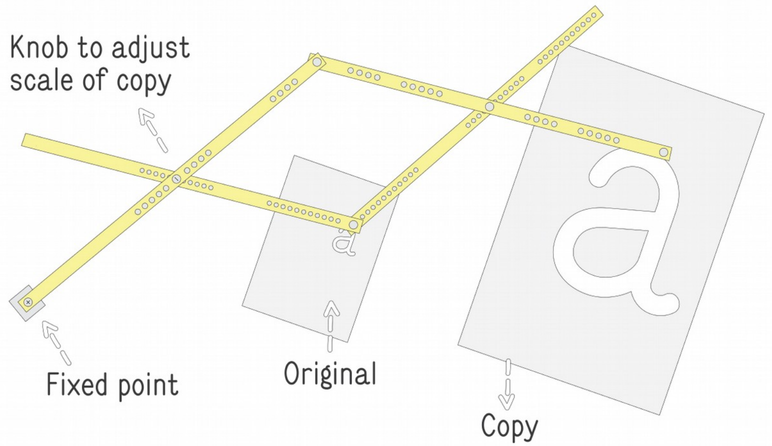


Kinematic chain - examples

Four-bar chain examples



Pantograph

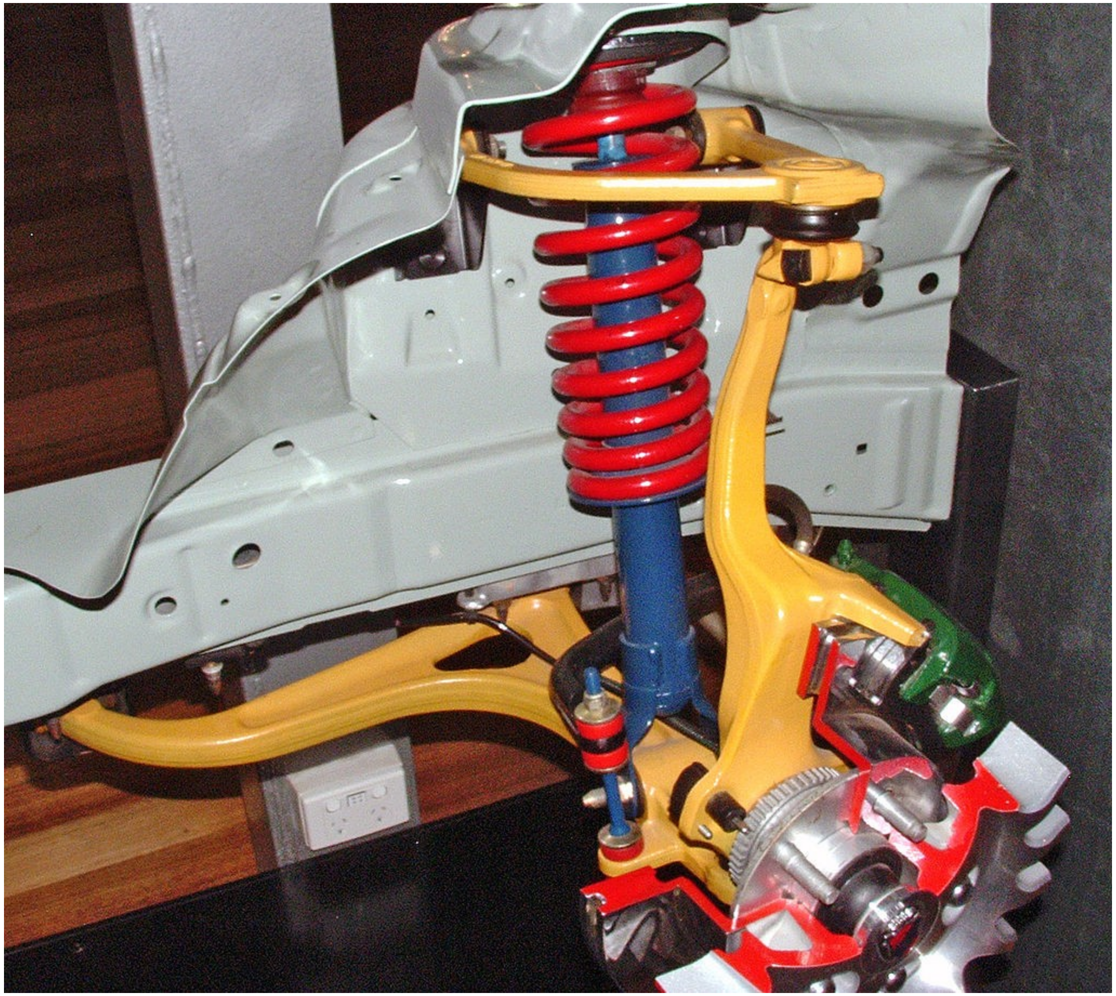


source: http://en.wikipedia.org/wiki/Pantograph

Kinematic chain - examples

Four-bar chain examples

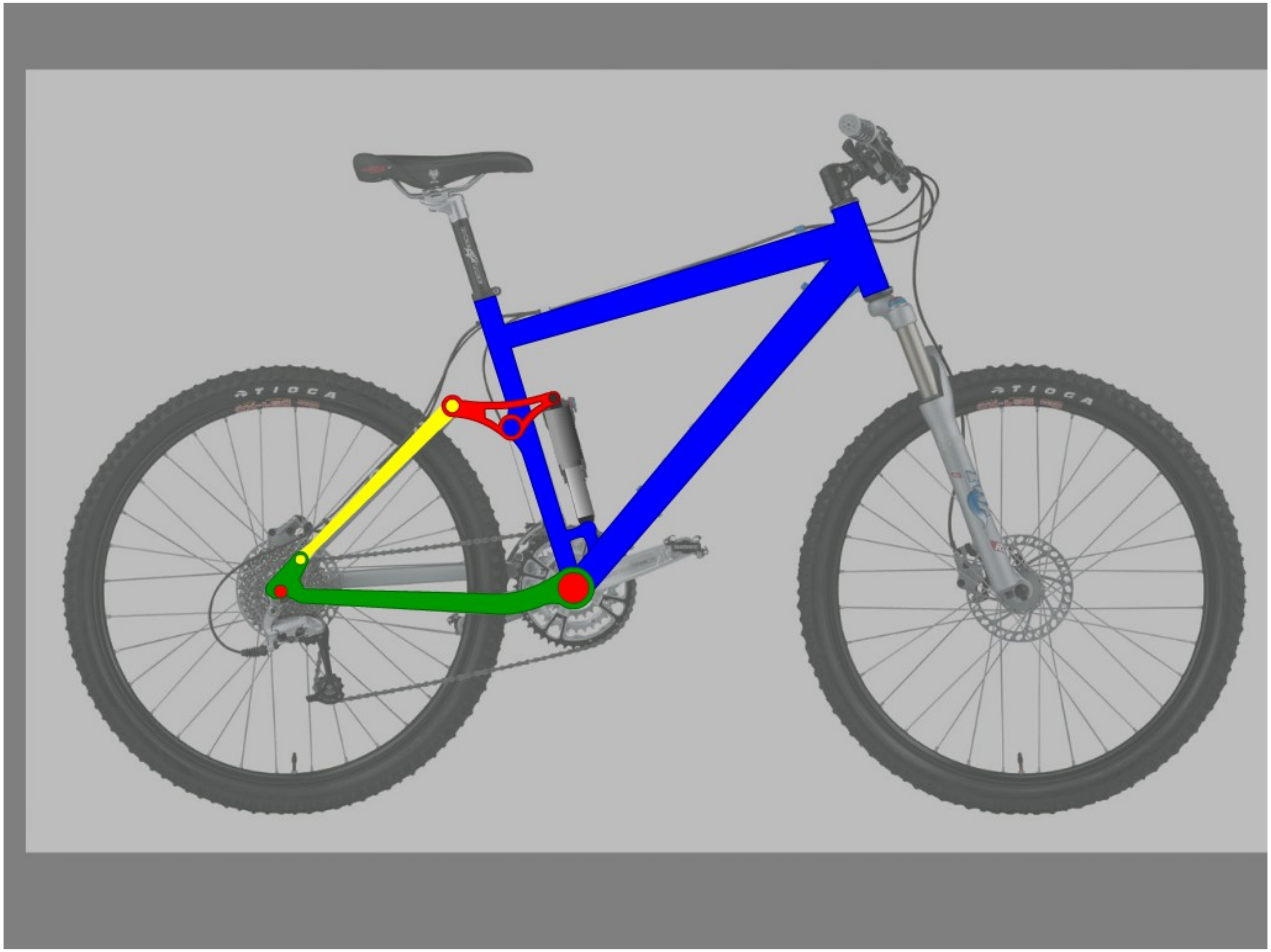
Double wishbone suspension



source: http://en.wikipedia.org/wiki/Double_wishbone_suspension

Kinematic chain - examples

Four-bar chain examples

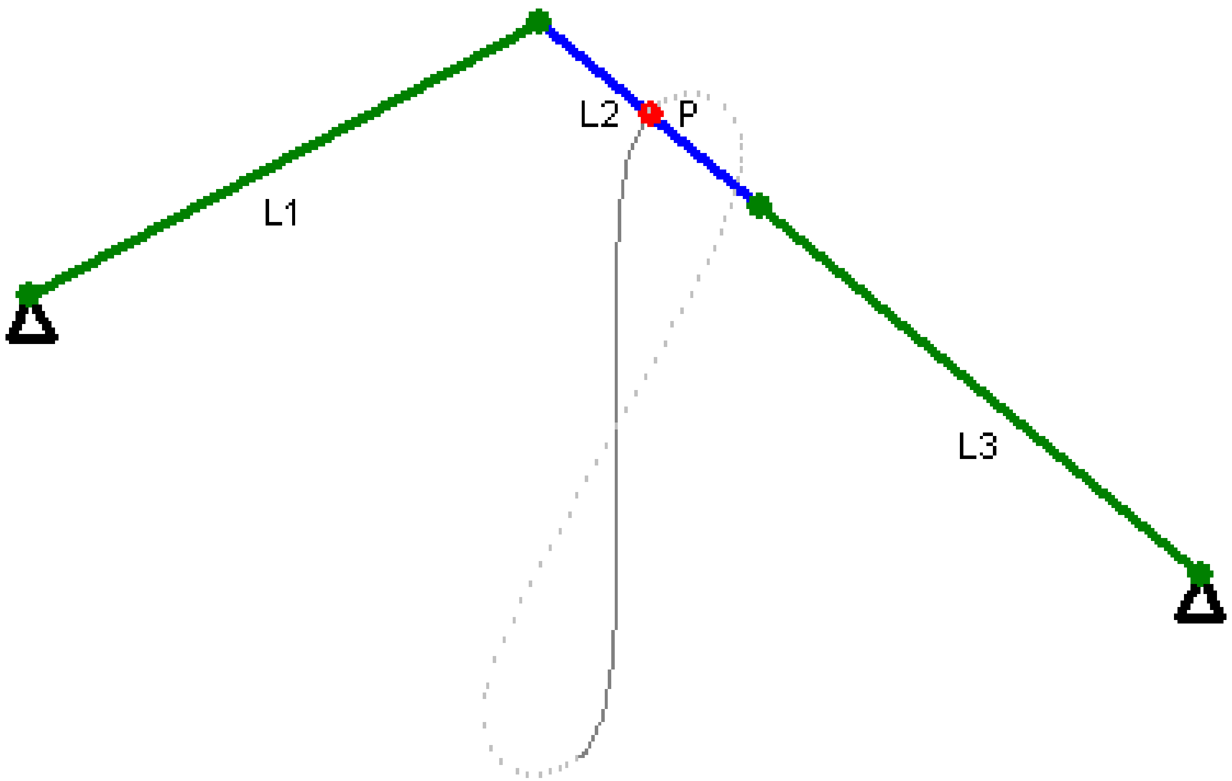


http://en.wikipedia.org/wiki/Bicycle_susper

Kinematic chain - examples

Four-bar chain examples

Watt's linkage (parallel linkage)

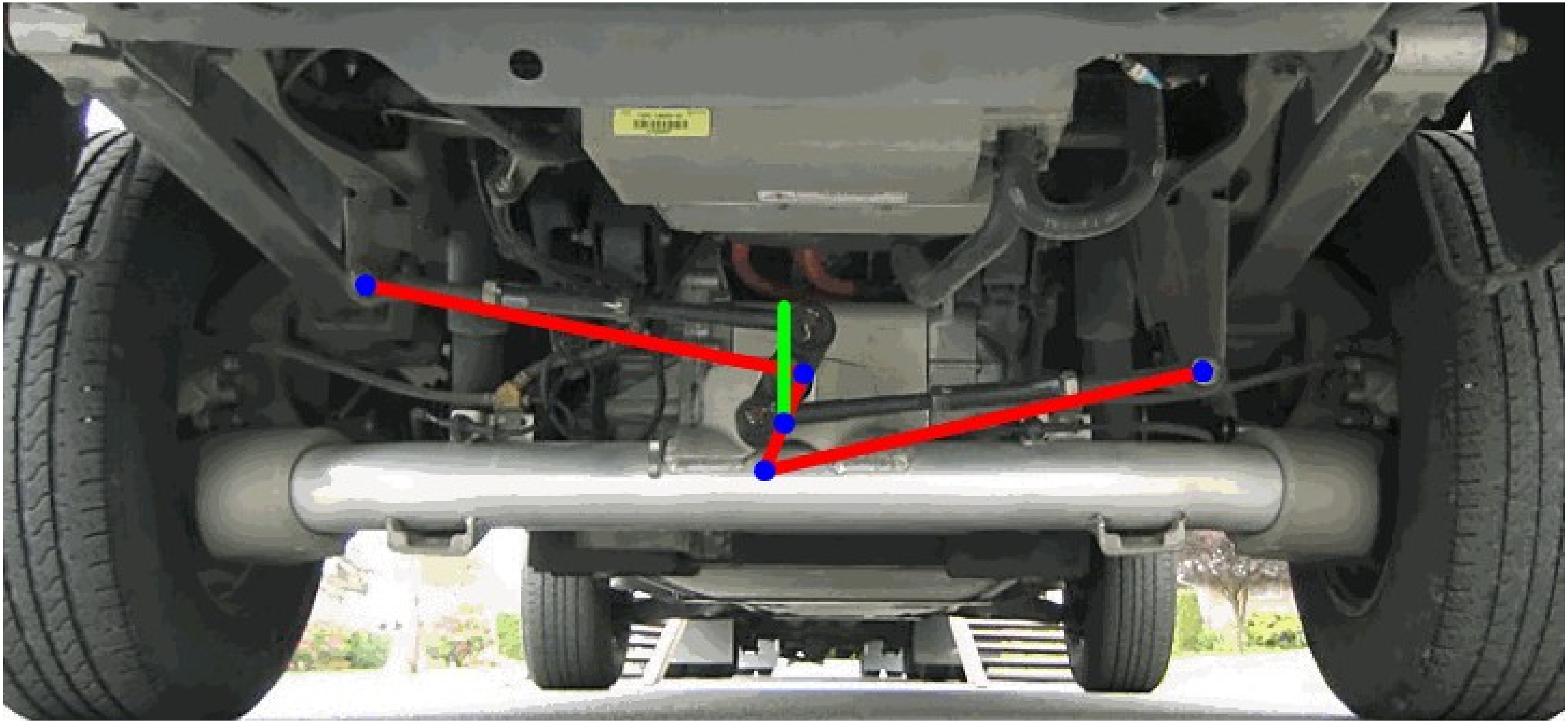


http://en.wikipedia.org/wiki/Watt%27s_linkage

Kinematic chain - examples

Four-bar chain examples

Watt's linkage (parallel linkage)

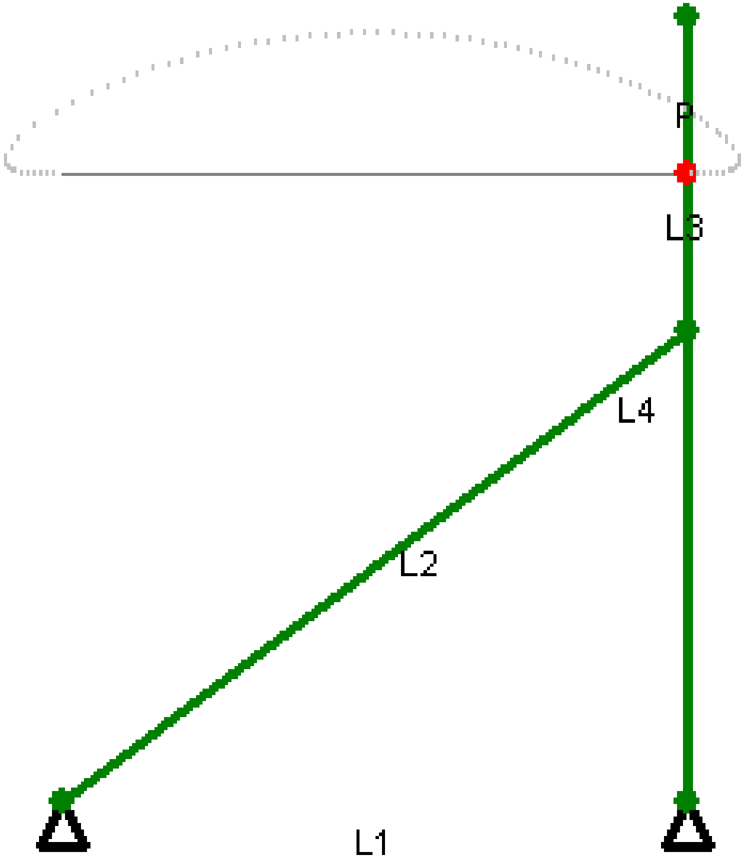


http://en.wikipedia.org/wiki/Watt%27s_linkage

Kinematic chain - examples

Four-bar chain examples

Chebyshev

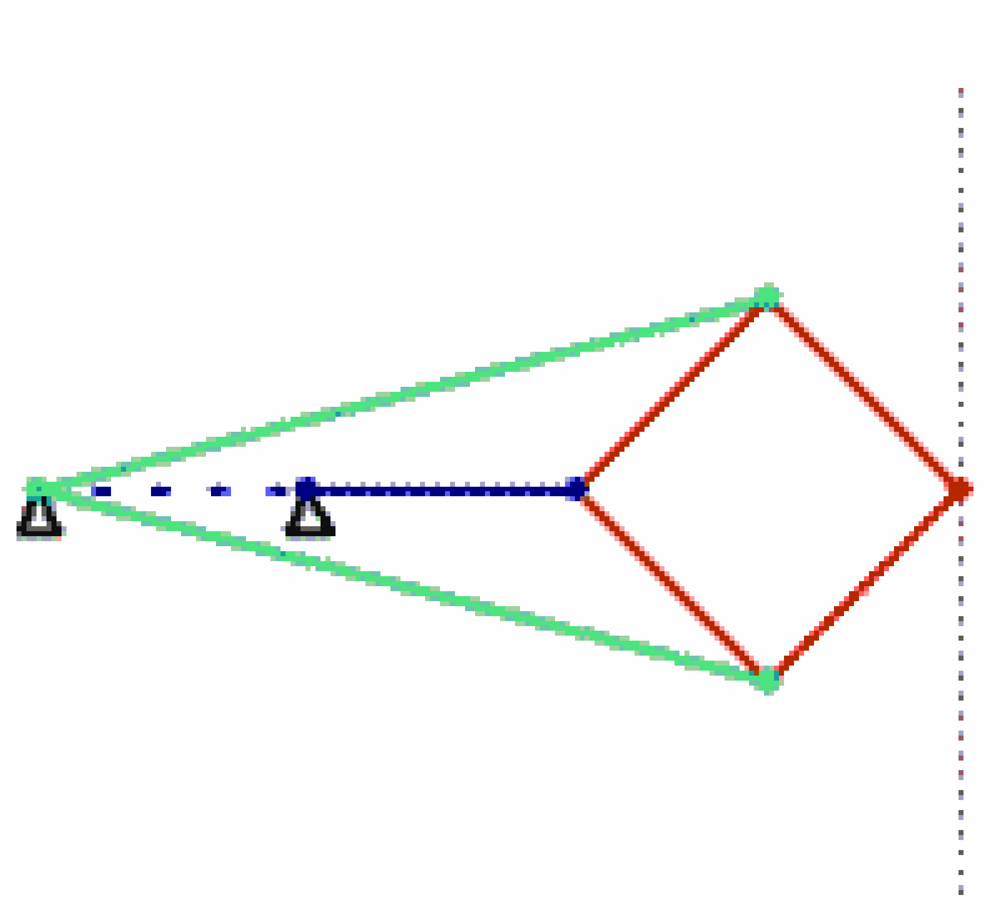


http://en.wikipedia.org/wiki/Chebyshev_linkage

Kinematic chain - examples

Four-bar chain examples

Peaucellier–Lipkin linkage

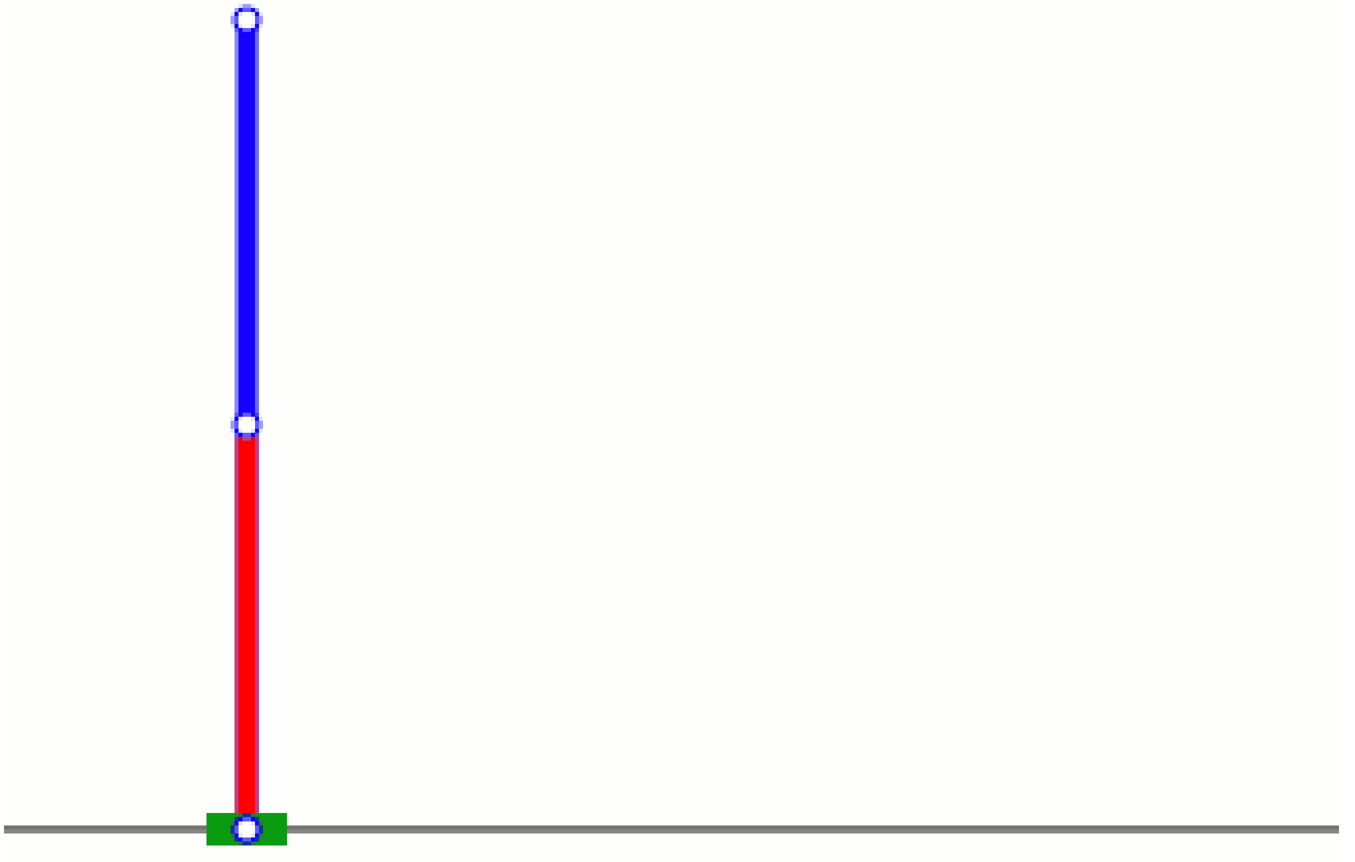


http://en.wikipedia.org/wiki/Peaucellier%E2%80%93Lipkin_linkage

Kinematic chain - examples

Four-bar chain examples

Scott-Russell linkage

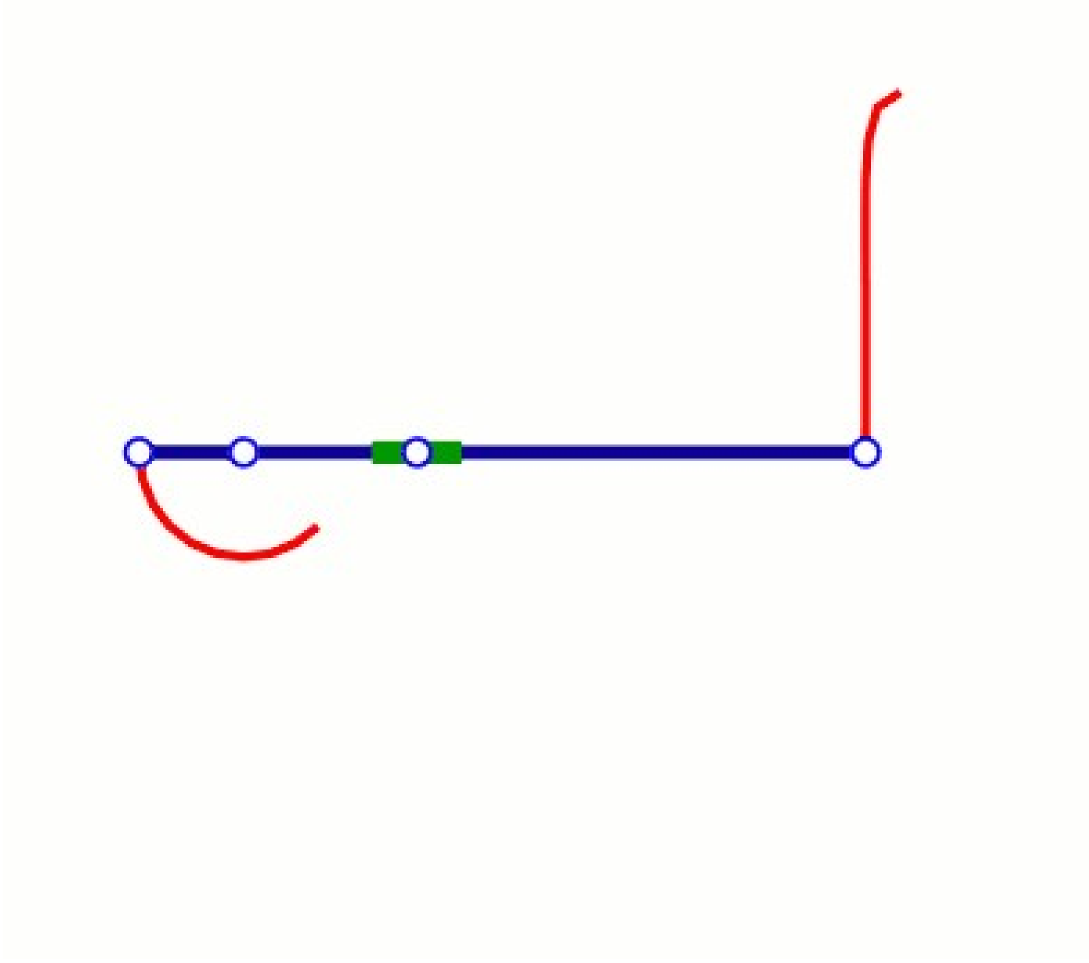


http://en.wikipedia.org/wiki/Scott_Russell_linkage

Kinematic chain - examples

Four-bar chain examples

Hoeckens linkage



http://en.wikipedia.org/wiki/Hoeckens_linkage

Kinematic chain - examples

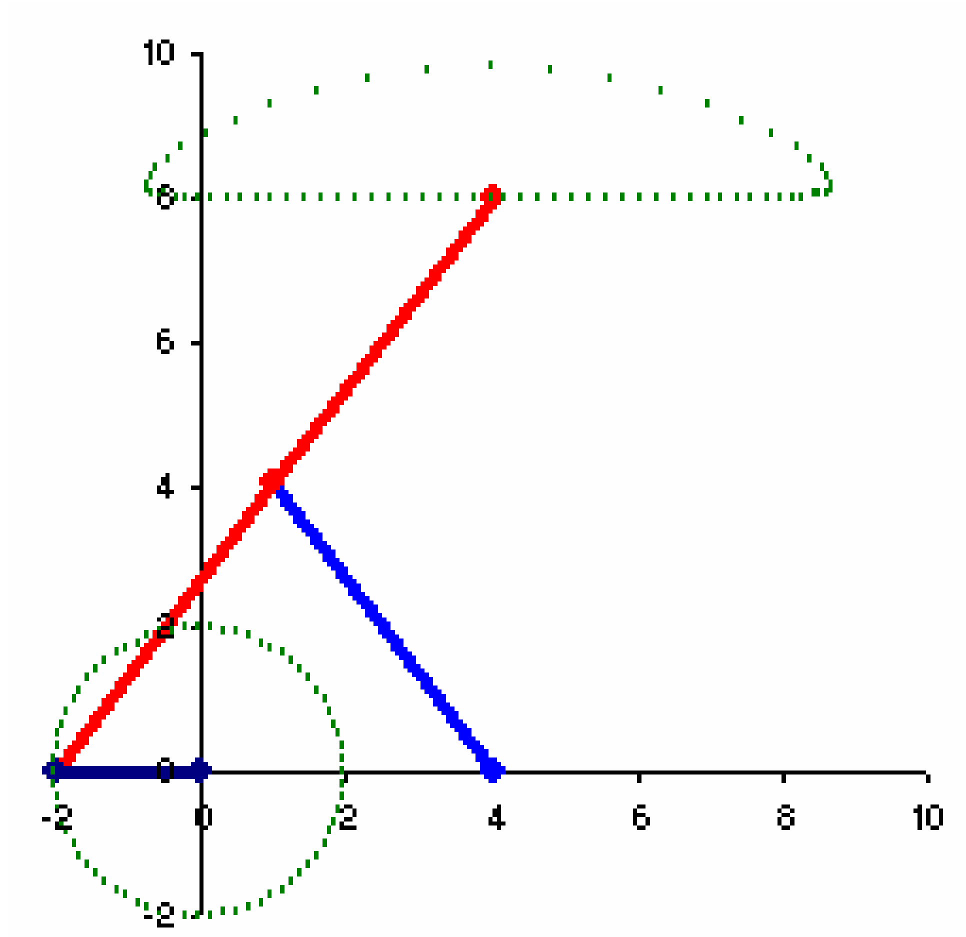
Four-bar chain examples

Sarrus linkage linkage

http://en.wikipedia.org/wiki/Sarrus_linkage

Kinematic chain - examples

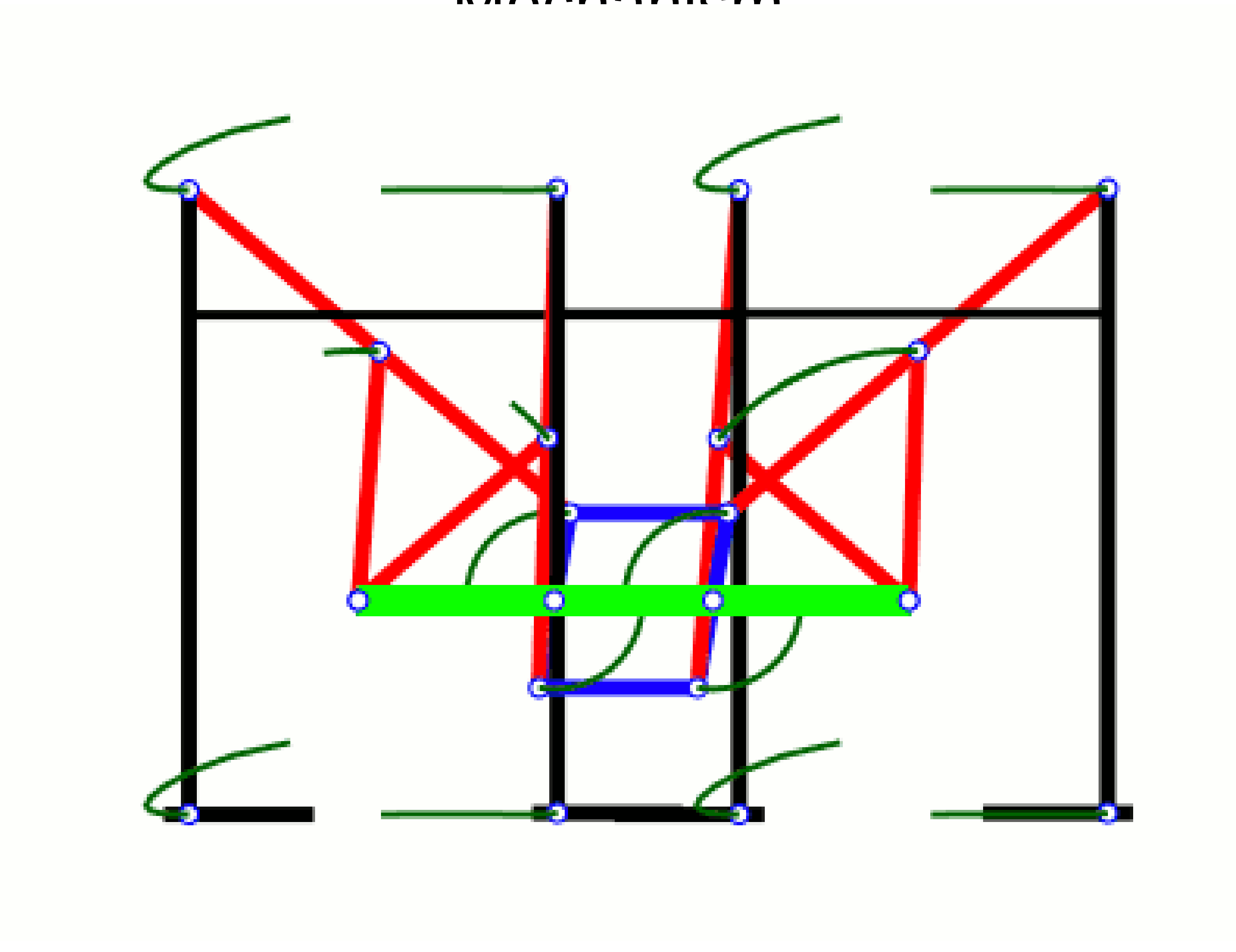
Chebyshev's Lambda Mechanism



https://en.wikipedia.org/wiki/Chebyshev%27s_Lambda_Mechanism

Kinematic chain - examples

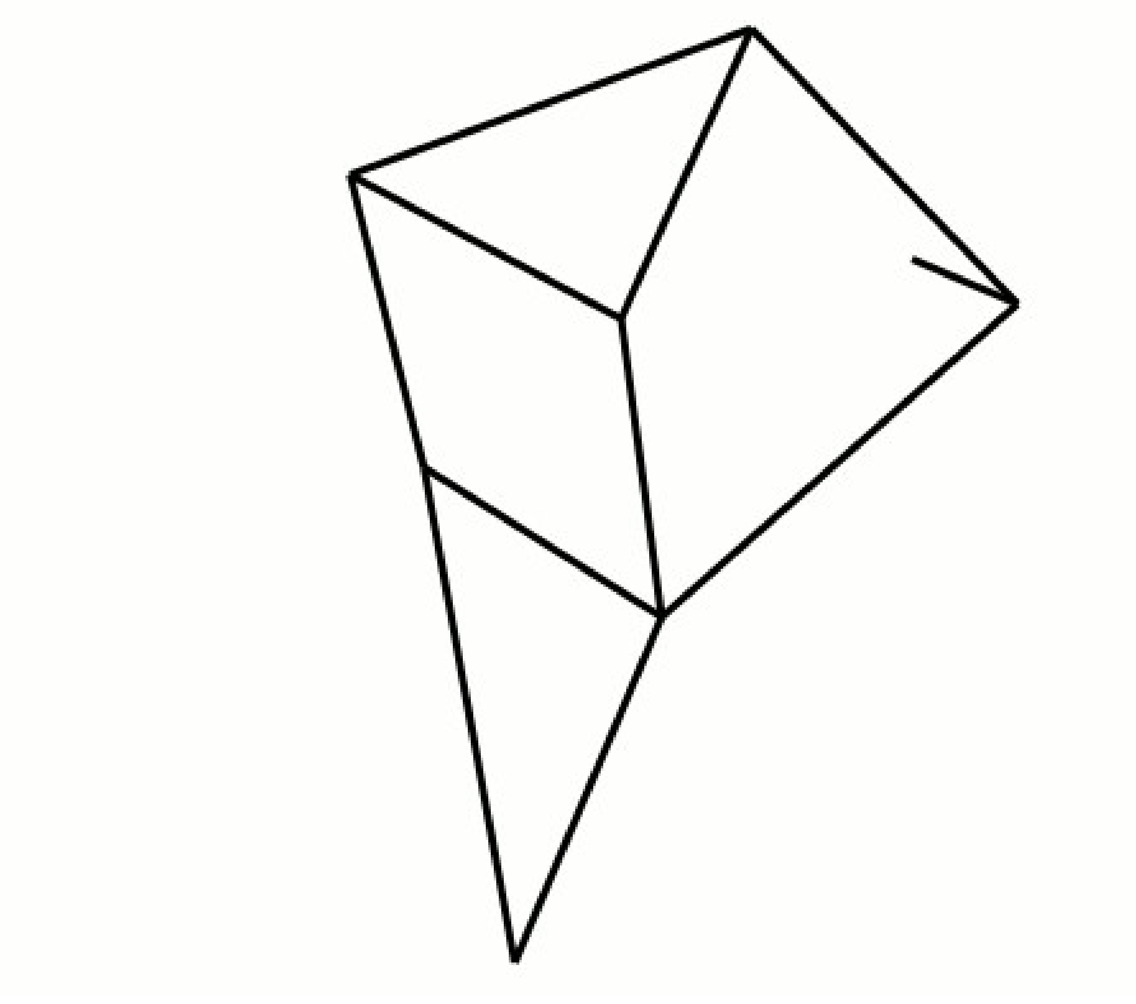
Chebyshev's Lambda Mechanism



https://en.wikipedia.org/wiki/Chebyshev%27s_Lambda_Mechanism

Kinematic chain - examples

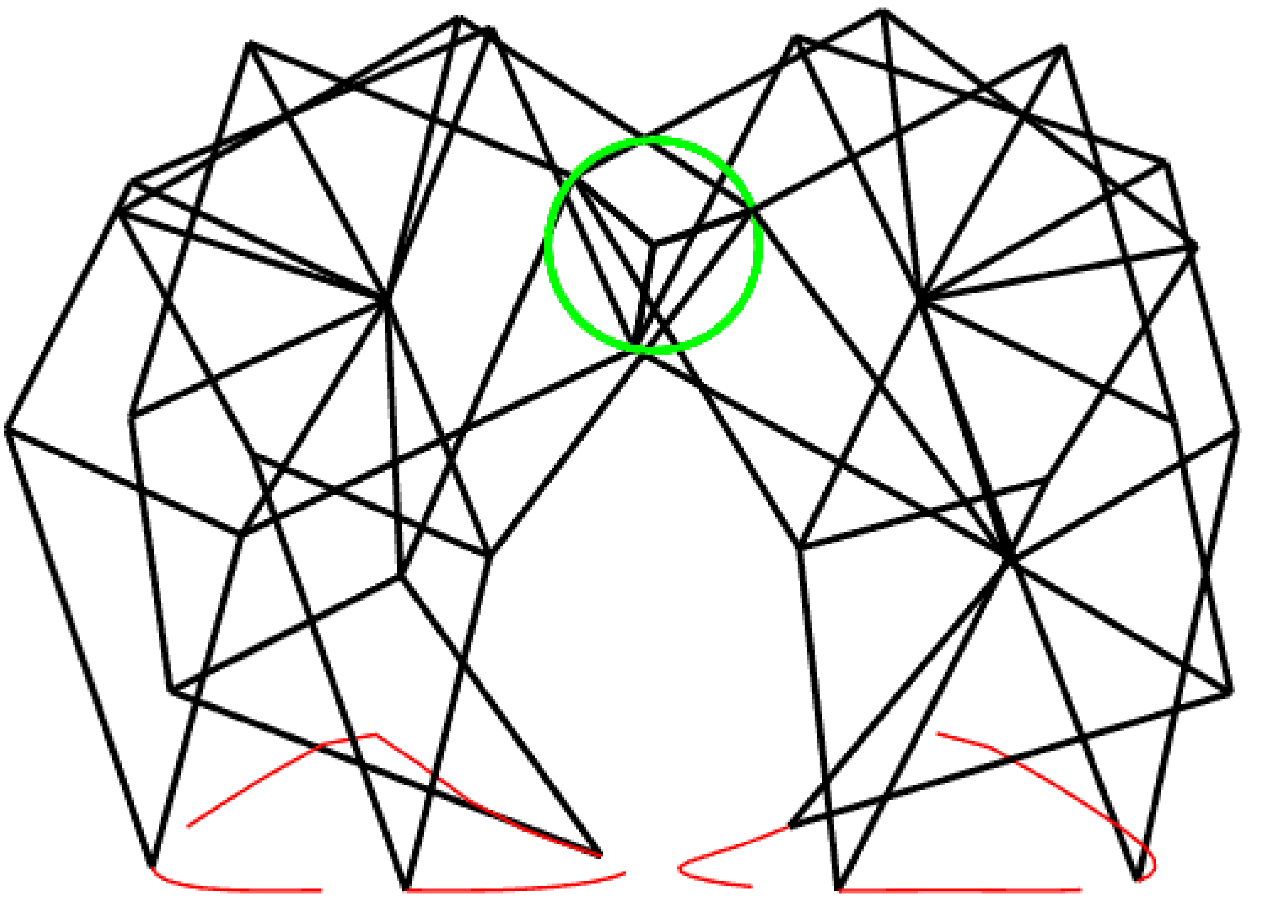
Jansen's linkage



http://en.wikipedia.org/wiki/Jansen%27s_linkage

Kinematic chain - examples

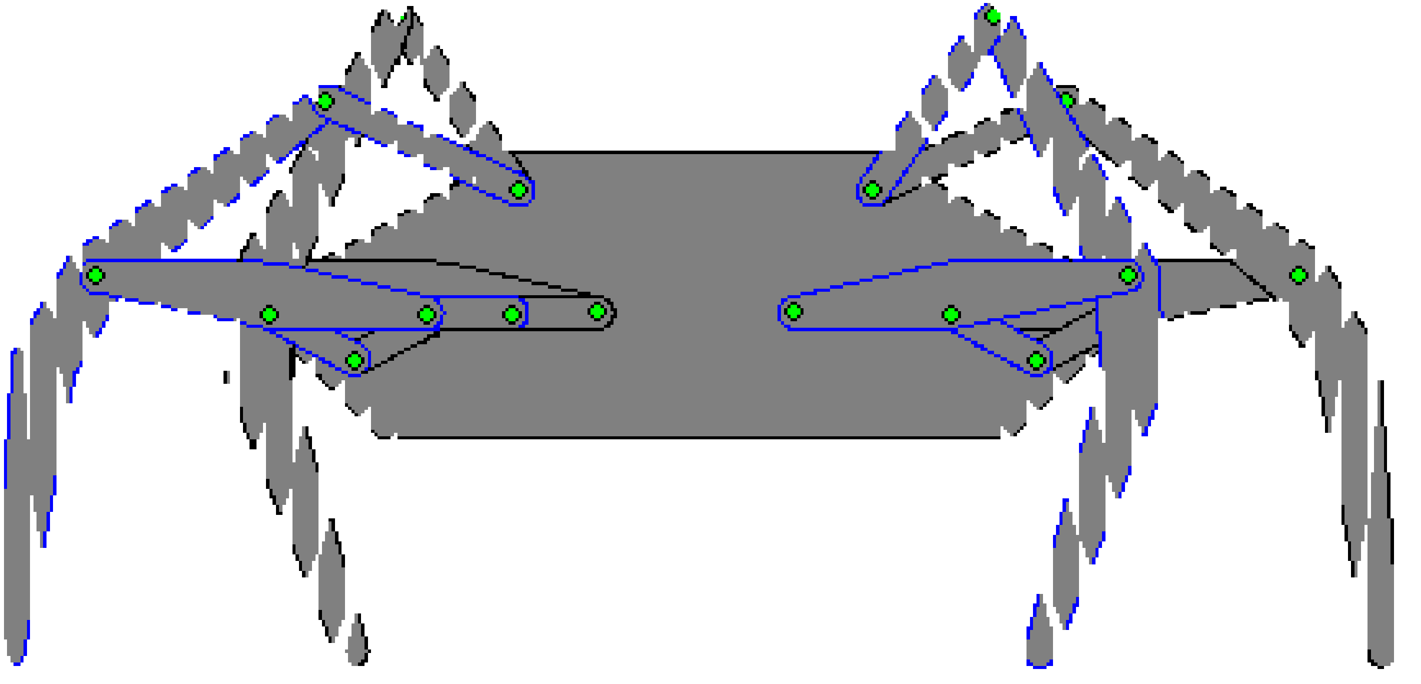
Jansen's linkage



http://en.wikipedia.org/wiki/Jansen%27s_linkage

Kinematic chain - examples

Klann linkage



http://en.wikipedia.org/wiki/Klann_linkage

Kinematic chain mobility

kinematic chain mobility – structural formula

(the Chebychev–Grübler–Kutzbach criterion)

$$(3\,D\ chain)\quad F=6\,N-p_1-2\,p_2-3\,p_3-4\,p_4-5\,p_5$$

$$(2\,D\ chain)\quad F=3\,N-p_4-2\,p_5$$

$$N-number\,of\ moving\ bodies$$

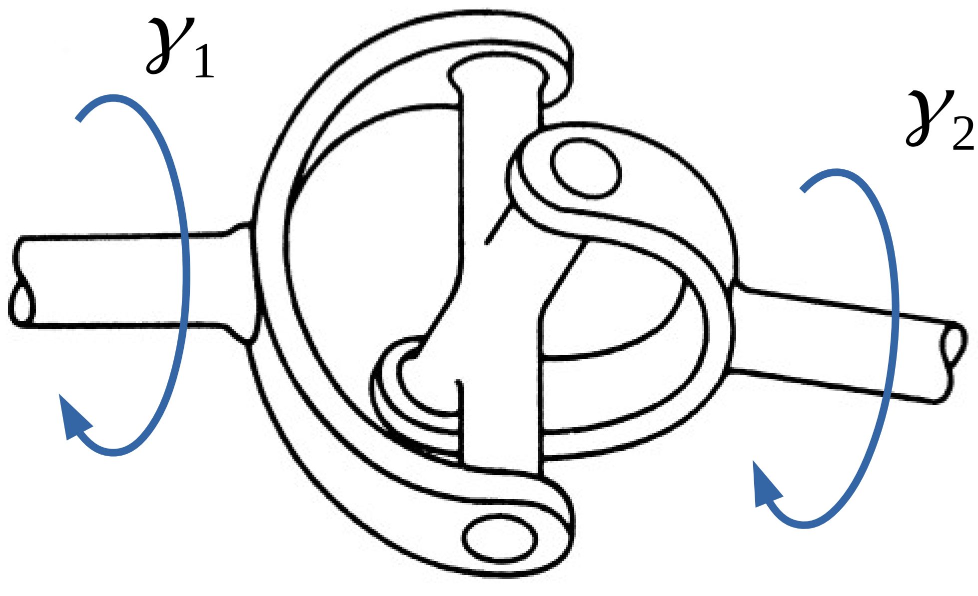
$$p_i-number\,of\,i-type\ classes$$

F >= 1 – movable

F < 1 – locked or overconstrained

Kinematic chain - examples

Universal joint (Cardan, Hooke's, Hardy Spicer)



$$\omega_2 = \frac{\omega_1 \cos\beta}{1 - \sin^2\beta \cos^2\gamma_1}, \quad \omega_1 = \frac{d\gamma_1}{dt}, \quad \omega_2 = \frac{d\gamma_2}{dt}$$

Kinematic chain - examples

Constant-velocity joint (homokinetic, Double Cardan)

source: http://www.cardanjoints.com

Lecture 2

Structural classification, velocities in planar mechanisms.

Classification of kinematic chains

Simple kinematic chain – every member has maximum two kinematic pairs.

Complex kinematic chain – at least one member has three kinematic pairs.

Open kinematic chain – at least one member has only one kinematic pair.

Closed kinematic chain – every member has minimum two kinematic pairs.

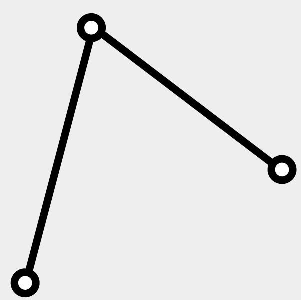
Structural classification of mechanisms

Structural group – the simplest part of mechanism that has zero mobility.

Planar mechanism with only 5th class pairs: $F=3n-2p_5=0$

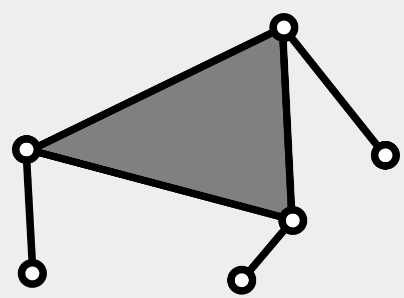
$$\frac{p_5}{n}=\frac{3}{2}=\frac{6}{4}=\frac{9}{6}=\ldots$$

IInd structural group



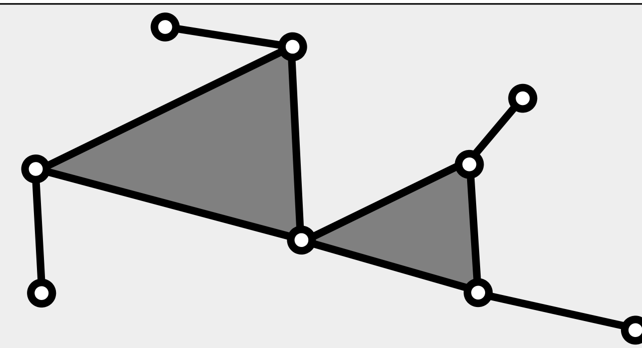
$n=2 \quad p_5=3$

IIIrd structural group



$n=4 \quad p_5=6$

IVth structural group

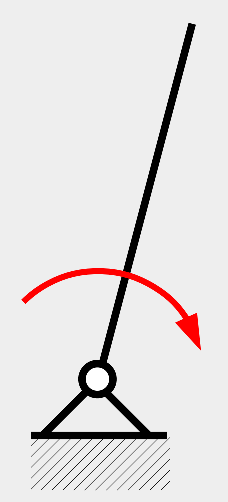


$n=6 \quad p_5=9$

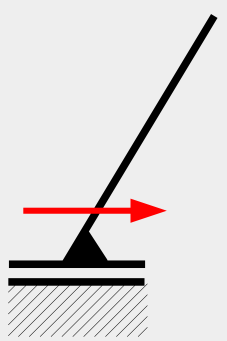
Structural classification of mechanisms

Ist structural group – drive

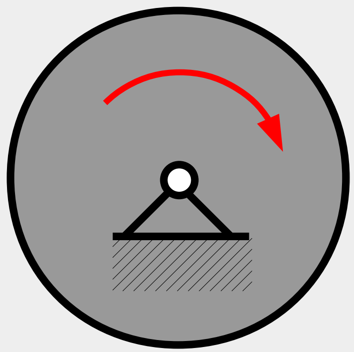
$$n=1 \quad p_5=1 \; + \text{ drive}$$



crank drive



linear drive



rotary drive

Kinematics of mechanisms

Kinematic analysis of a mechanism – determination of velocities and accelerations of selected mechanism members' points at considered configuration. Mechanism structure must be given (geometry of members, kinematic pairs) and drive method must be known.

Methods of velocities and acceleration determination

Graphical methods

- velocity projection method,
- instantaneous center of rotation method,
- instantaneous center of acceleration method,
- method of rotated velocities,
- velocity decomposition method,
- acceleration decomposition method,
- velocity scheme method,
- accelerations scheme method.

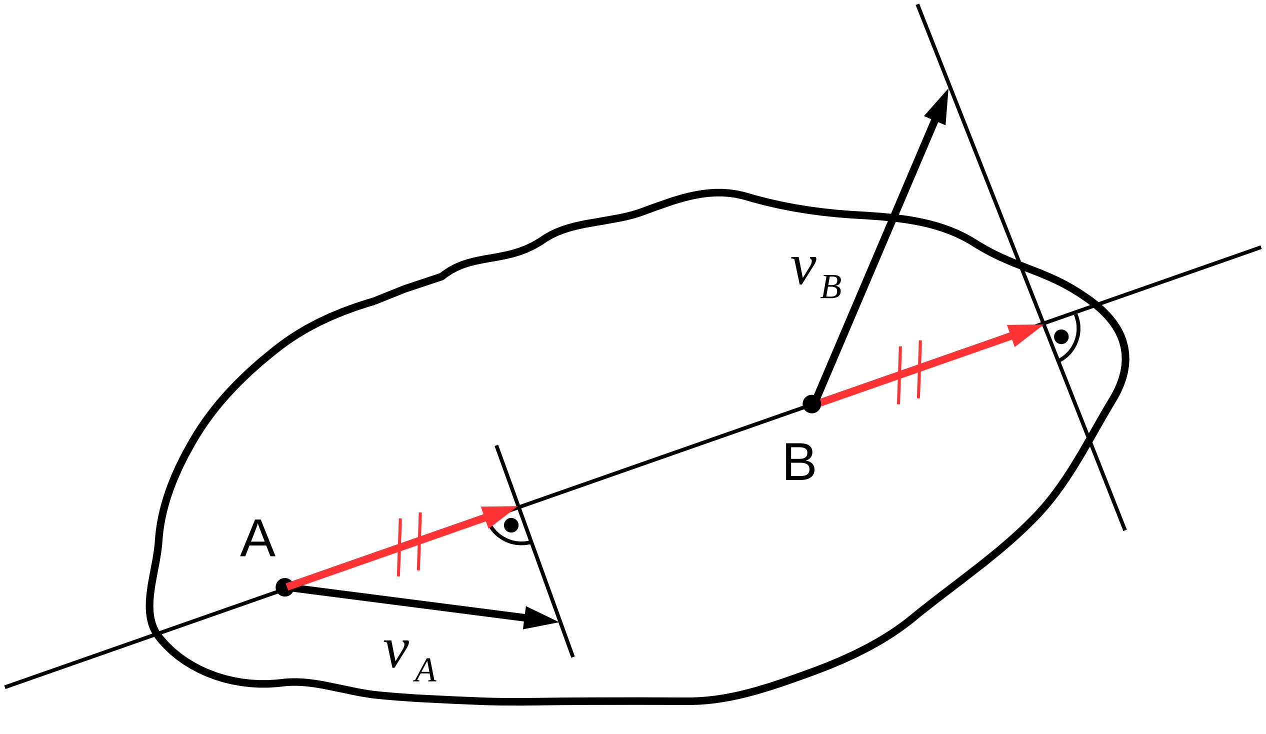
Analytical method

Methods of velocities and acceleration determination

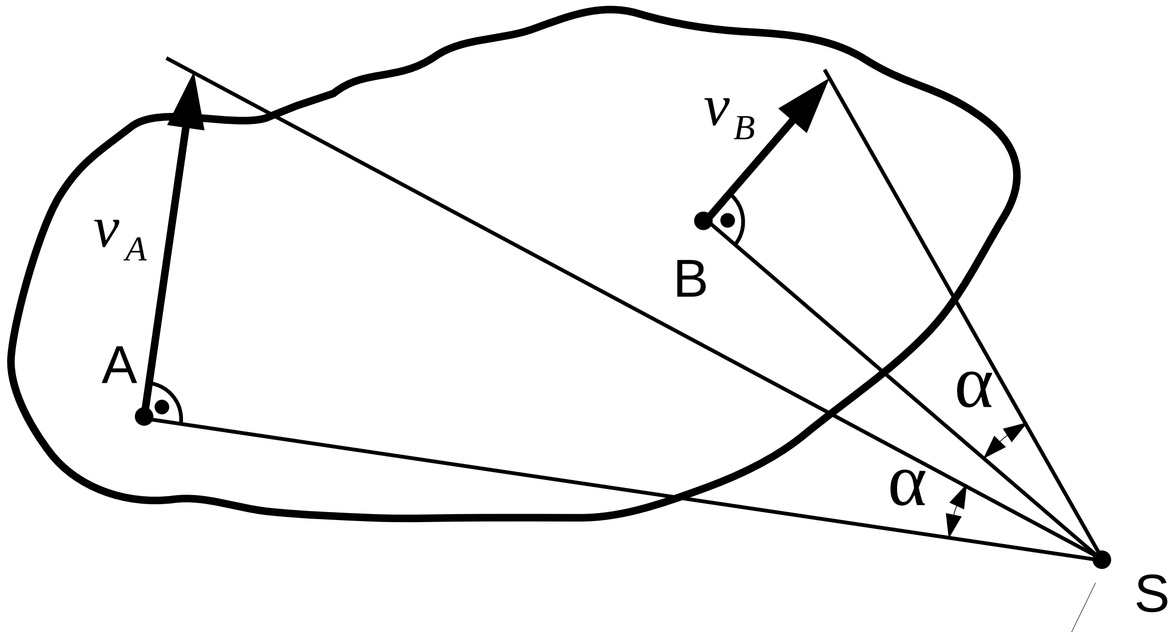
	Graphical methods	Analytical method
advantage	• better understanding of mechanism motion, • analysis of very complicated mechanisms, • computers not needed,	• functions of configuration as a solution, • analysis of very complicated mechanisms,
disadvantage	• great workload, • needs to repeat graphs for every configuration, • graphical errors.	• computer needed for complicated mechanisms, • complicated systems of equations to solve, • solution interpretation may be complicated.

Velocity projection method

Projections of velocities of two rigid body's points onto common line are equal.



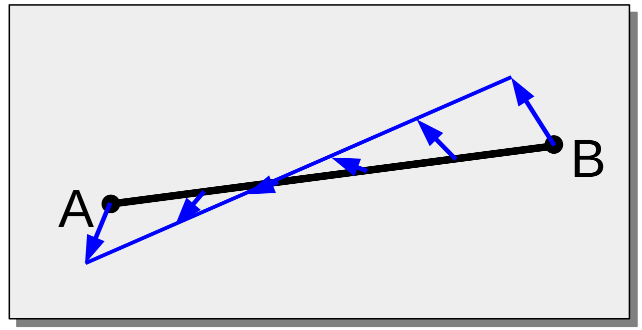
Instantaneous center of rotation method



center of instantaneous rotation
(center of velocities)

Velocity decomposition method

Every planar motion can be described by a superposition of a linear motion and an angular motion.



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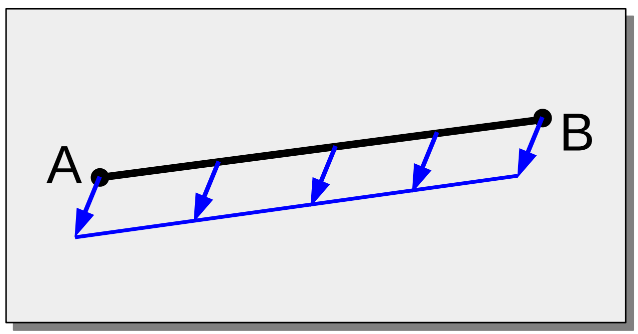

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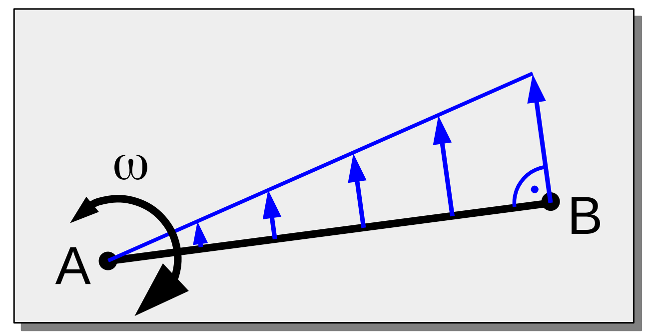


$$\vec{v}_B = \vec{v}_A + \vec{v}_{BA}$$

absolute velocity of point B

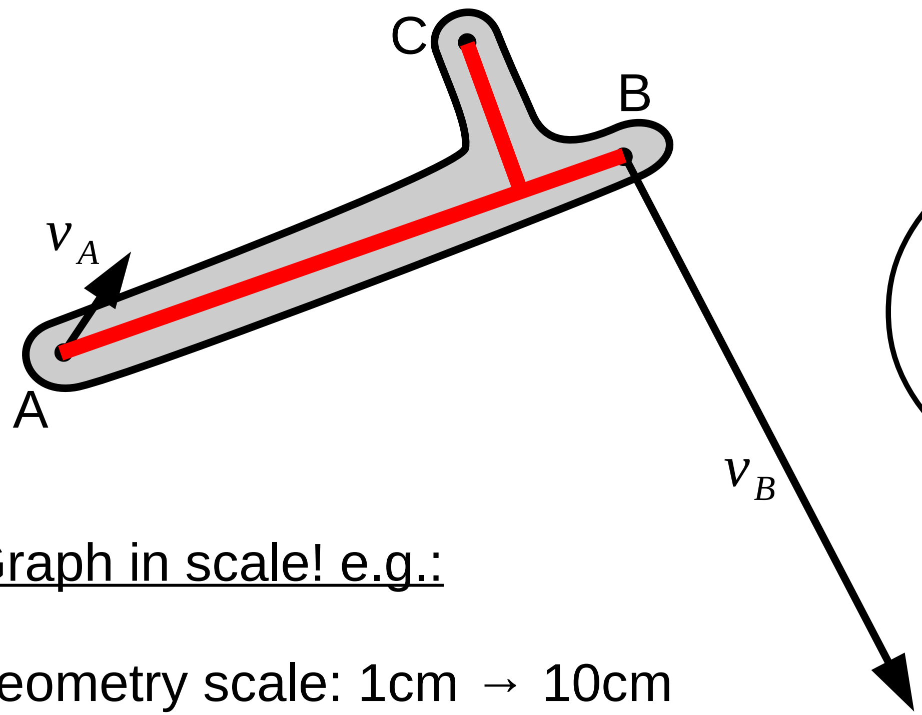
velocity of a linear motion

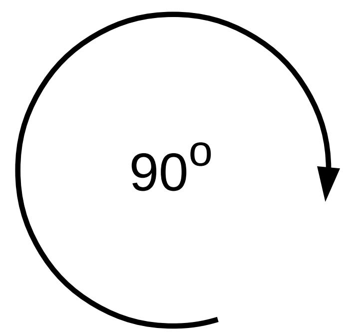
Angular velocity of point B in rotation around point A.

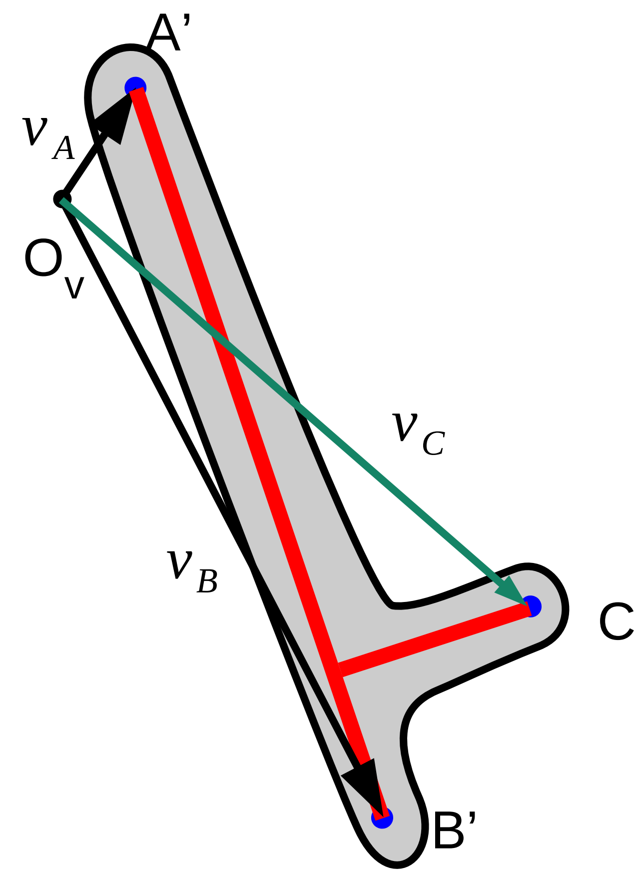
$$\vec{v}_{BA} = \vec{\omega} \times \overrightarrow{AB}$$

Velocity scheme method

<u>Velocity scheme of a rigid body</u> – geometry created by the ends of it's velocity vectors moved to the common starting point (pole). Velocity scheme is similar to the corresponding rigid body: it is scaled and rotated by an 90^o angle in the direction of body's angular velocity.





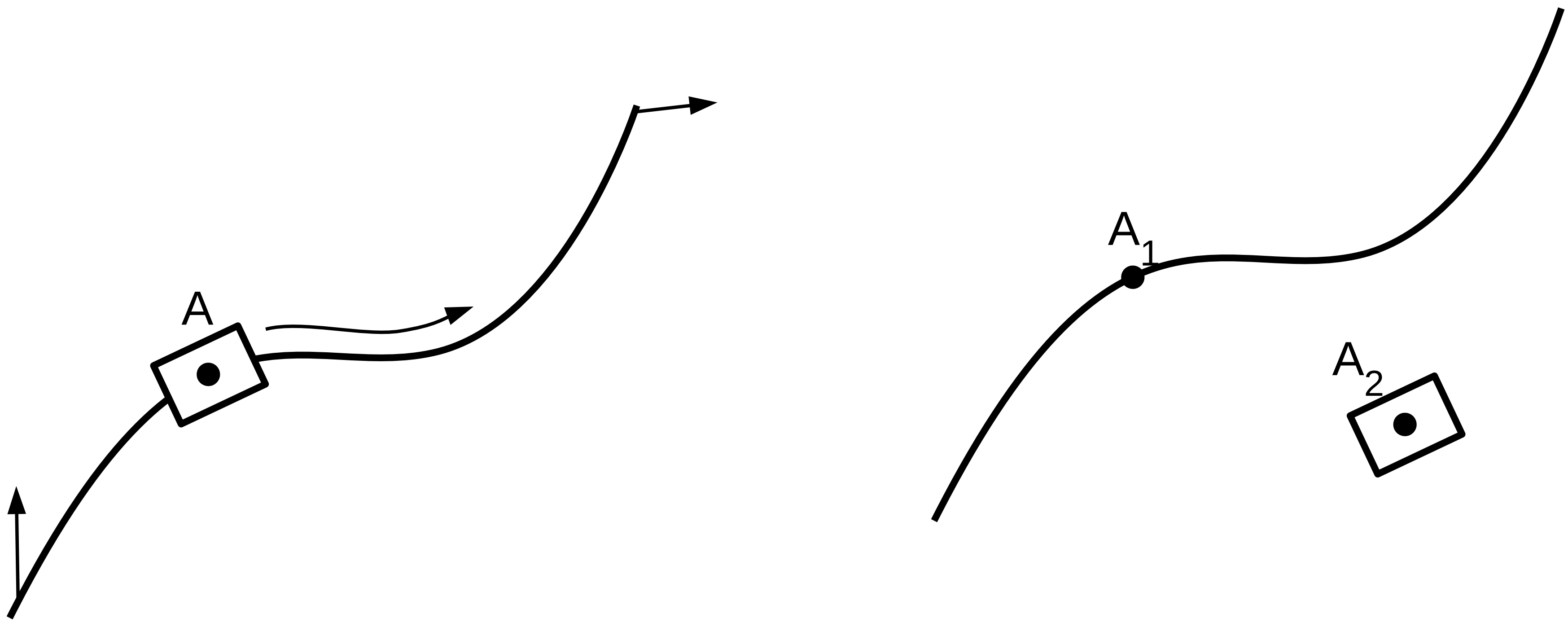


<u>Graph in scale! e.g.:</u>

geometry scale: 1cm → 10cm
velocity scale: 1cm → 1m/s

velocity scale: the same
geometry scale: new!

Velocities in relative motion



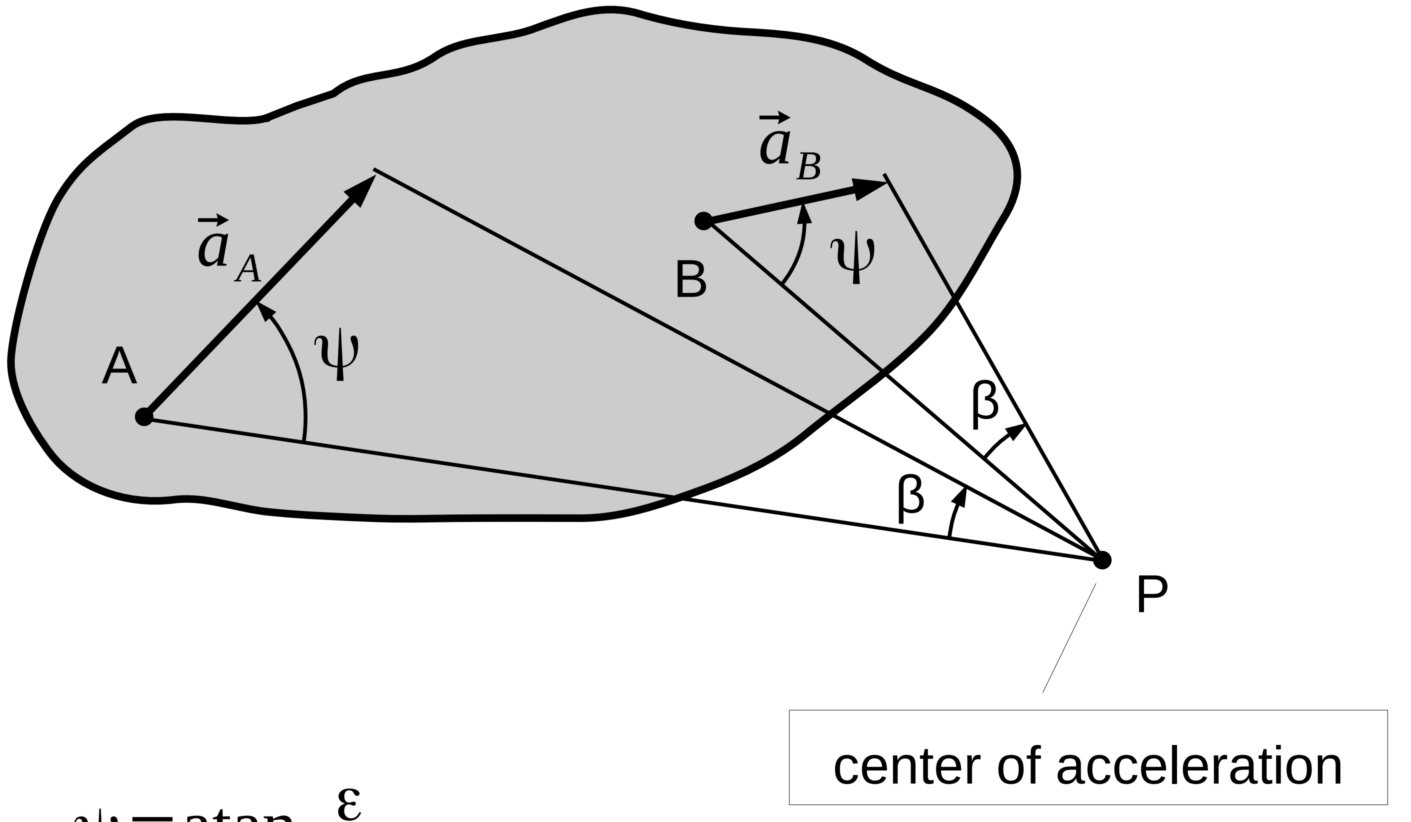
$$\vec{v}_{A2} = \vec{v}_{A1} + \vec{v}_{A2A1}$$

- absolute velocity of point A_2 ($\vec{v}_{A2}$)
- transportation velocity ($\vec{v}_{A1}$)
- relative velocity ($\vec{v}_{A2A1}$)

Lecture 3

Accelerations in planar mechanisms.

Instantaneous center of acceleration



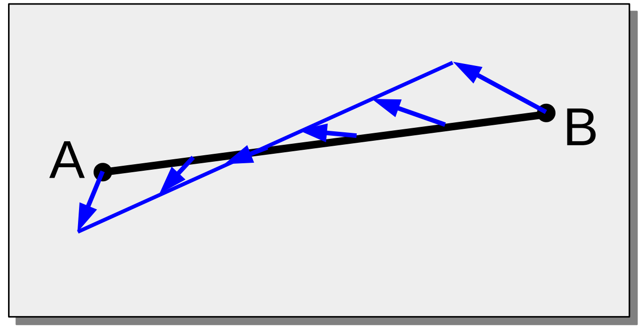
$$\psi = \operatorname{atan}\frac{\varepsilon}{\omega^2}$$

ε - angular acceleration

ω - angular velocity

Acceleration decomposition method

Example



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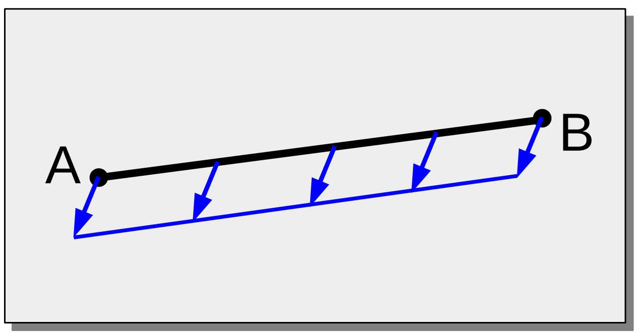

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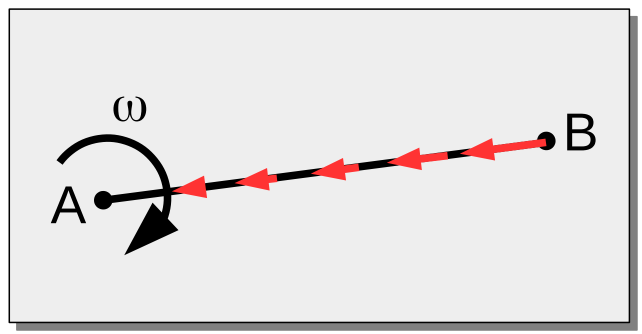

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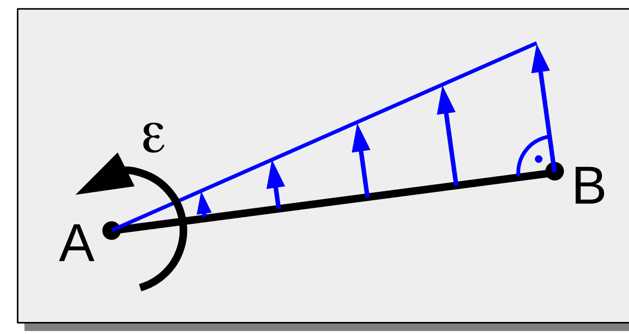


$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}^n_{BA} + \vec{a}^t_{BA}$$

$$\vec{a}^t_{BA} = \vec{\varepsilon} \times \overrightarrow{AB}$$

- $\vec{a}_B$ – absolute acceleration of point B
- $\vec{a}_A$ – absolute acceleration of point A
- $\vec{a}_{BA}$ – Angular acceleration of point B in rotation around point A.
- $\vec{a}^n_{BA}$ – Centripetal acceleration (normal)
- $\vec{a}^t_{BA}$ – Rotary acceleration (tangential)

$$\vec{a}^n_{BA} = \vec{\omega} \times \left(\vec{\omega} \times \overrightarrow{AB} \right) = -\omega^2 \overrightarrow{AB}$$

Acceleration scheme (diagram)

Acceleration scheme of a rigid body – geometry created by the ends of it's acceleration vectors moved to the common starting point (acceleration scheme's pole).

Acceleration scheme is similar to the corresponding rigid body:

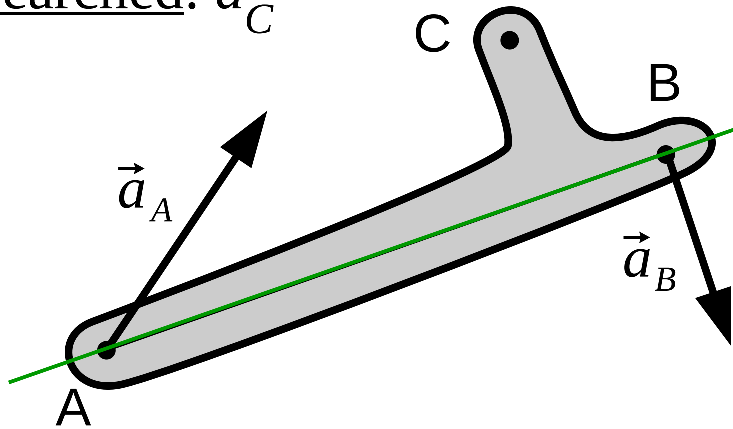
it is scaled and rotated by $(180^{o}-\psi)$ *angle in the direction of body's angular velocity if* $\mathrm{sgn}\,\omega=\mathrm{sgn}\,\varepsilon$ *(or opposite direction if* $\mathrm{sgn}\,\omega\neq\mathrm{sgn}\,\varepsilon$*).*

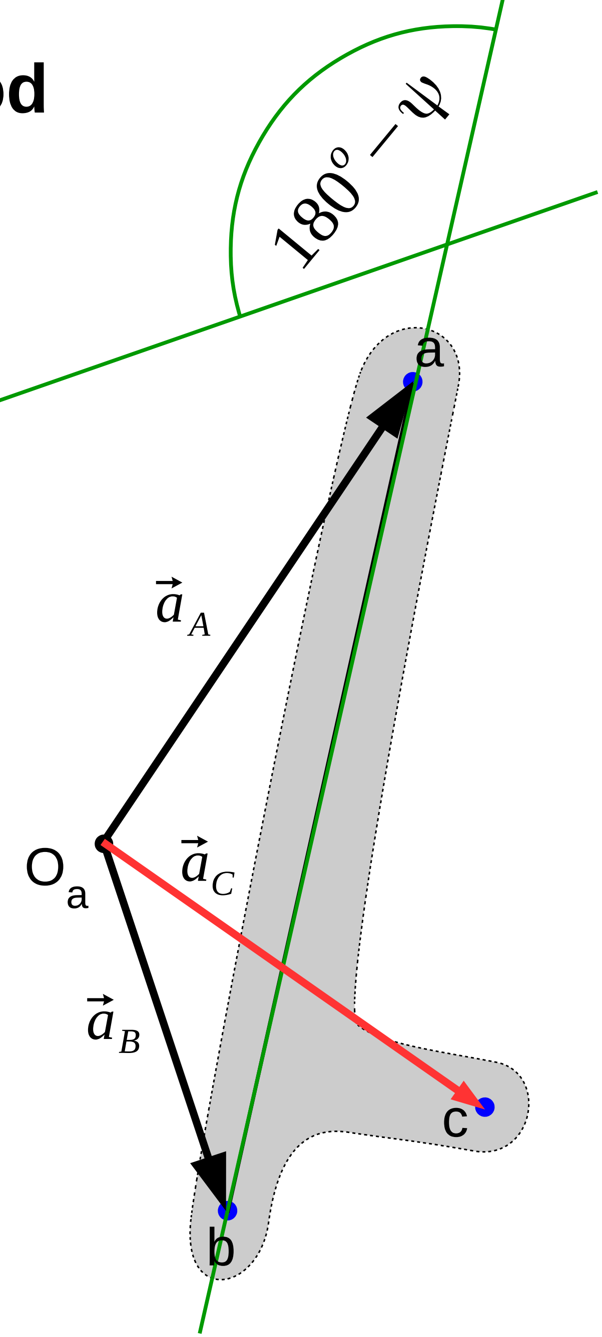
Acceleration scheme method

Example

Given: $\bar{a}_A$ and $\bar{a}_B$ + geometry

Searched: $\bar{a}_C$

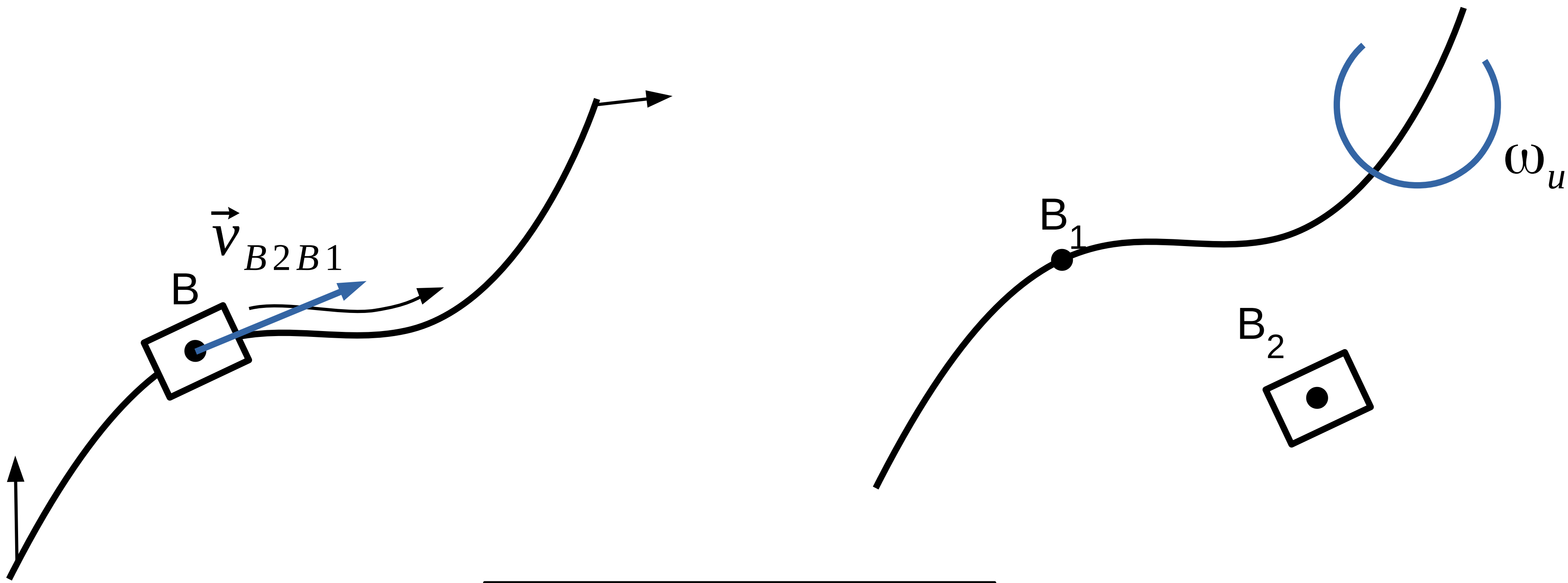




Acceleration scheme of a rigid body – geometry created by the ends of it's acceleration vectors moved to the common starting point (acceleration scheme's pole).
Acceleration scheme is similar to the corresponding rigid body: it is scaled and rotated by (180°-) angle in the direction of body's angular velocity if sgnω=sgnε *(or opposite direction if* sgn$\omega \neq$sgnε*).*

acceleration scale, e.g.: 1cm → 1m/s

Accelerations in relative motion



$$\vec{a}_{B2}=\vec{a}^{u}_{B1}+\vec{a}^{rel}_{B2B1}+\vec{a}^{c}$$

- $\vec{a}_{B2}$ — absolute acceleration of point B2
- $\vec{a}^{u}_{B1}$ — Transportation acceleration (absolute acceleration of point B1)
- $\vec{a}^{rel}_{B2B1}$ — Relative acceleration
- $\vec{a}^{c}$ — Coriolis acceleration

$$\vec{a}^{c}=2\vec{\omega}_u\times\vec{v}_{B2B1}$$

Lecture 4

Analytical method. Cam mechanisms.

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

1. Set up Cartesian coordinate system O_{XY}.
2. Substitute the mechanism's members with set of vectors. All vectors can move with mechanism's elements, change their size, location and orientation.
3. Vectors must to create closed polygons.
4. Define “directed angles” for all vectors defined in the same manner. Assume that this angles are created by the positive x axis counter-clockwise rotation.
5. Fore each of polygon write down sum of vectors, e.g.:

$$\sum_{i=1}^{i=n} \vec{l}_i = 0$$

Procedure of analytical determination of velocities and accelerations in planar mechanisms.

6a. Write down projections of each polygon onto coordinate system's axes:

$$\text{x:}\quad \sum_{i=1}^{i=n} l_i \cos\varphi_i = 0 \qquad\qquad \text{y:}\quad \sum_{i=1}^{i=n} l_i \sin\varphi_i = 0$$

(we do not need to analyze signs because of „directed angles" setup procedure)

6b. Define which vectors' lengths and angles are given and/or constant (related to geometry), and which are variable in time and unknown.

(for a proper defined system number of unknown variables is equal to the number of equations)

7. Solve the equations. The resulting functions describe motion of the mechanism.

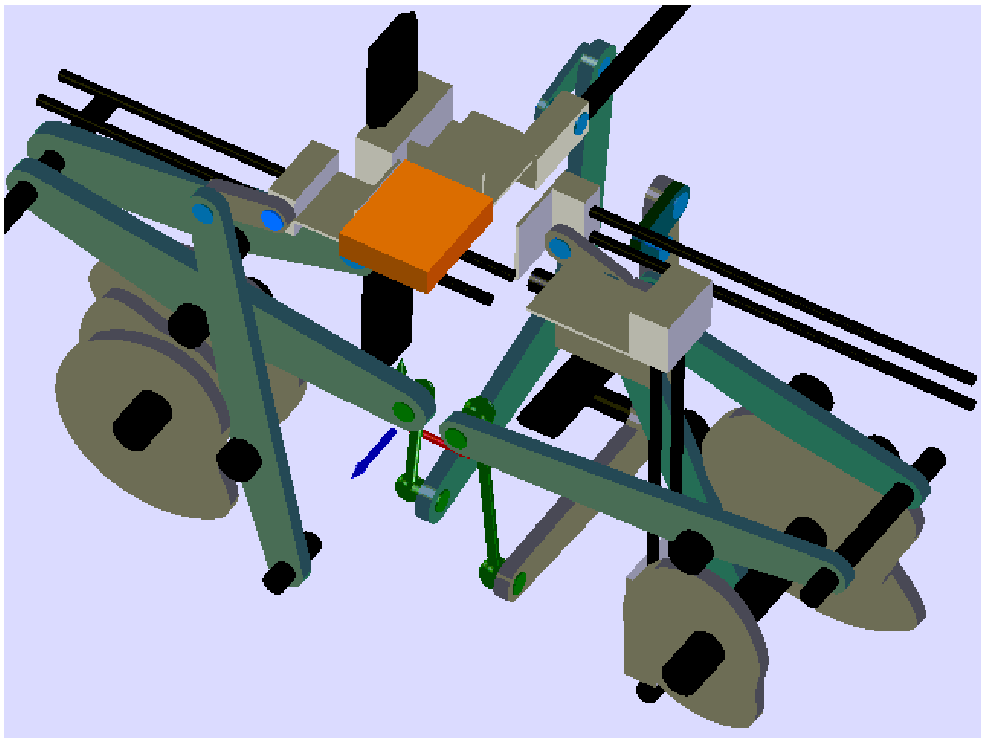
Procedure of analytical determination of velocities and accelerations in planar mechanisms.

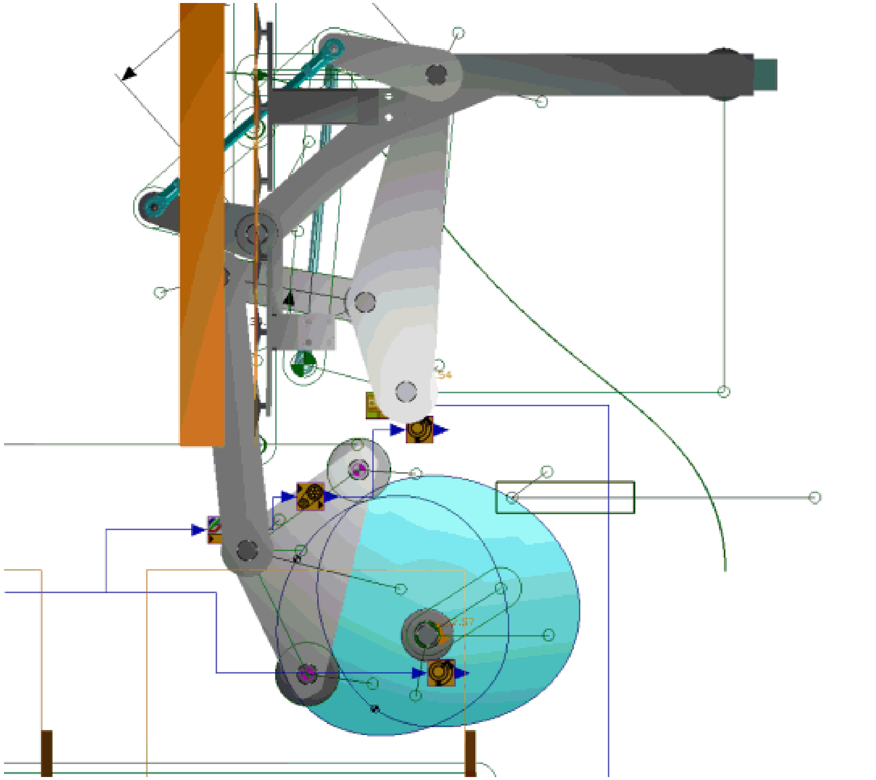
8. Differentiate functions achieved in p.7 to obtain velocities. Differentiate once again to obtain accelerations.

9. If desired information was not obtained in p.8, differentiate equations from p.6. Sometimes rotation of the coordinate system is useful here.

Cam-follower

Inspirations





source: psmotion.com

Cam-follower

Cam-follower mechanism – mechanism build of a cam and a follower (tappet) connected as a IV class kinematic pair.

Cam is rotating (sometimes is translating)
follower is reciprocating (sometimes is swinging/oscillating)

advantages

- simple to construction,
- simple to create,
- any dimensions,
- simple to create advanced motions.

disadvantages

- no adaptation possible.

Classification:

flat / spatial
with in-line (central) follower / with offset (eccentric) follower
closed with geometry / closed with force

Analysis and synthesis of cam-follower mechanism

Analysis – calculation of displacement, velocity and acceleration functions for a follower motion with respect to a cam's rotations angle for arbitrary given geometry.

Synthesis – calculation of a cam geometry needed to obtain given displacement/velocity/acceleration functions. Limitations must be included, i.e. some maximum values, geometry limitations and jerk values (third derivative).

Lecture 5

Cam-follower mechanisms.
Dynamics of planar mechanisms.

Analysis and synthesis of cam-follower mechanisms

Analysis	Syntesis
• substitution of IV. class kinematic pair with V. class kinematic pairs + graphical method (velocity and acceleration scheme) • graphical determination of a follower movement and graphical differentiation • analytical method (substitution with polygones of vectors)	• graphical determination of cam outline by a base circle rotation with follower movement • analytical designing with a function description

Synthesis of cam-follower mechanisms

Analytical method

For a given function of velocity or acceleration, function of a follower displacement can be found by integration.

Follower displacement as a function of cam angle could be used to obtain cam outline directly (or after change of coordinates).

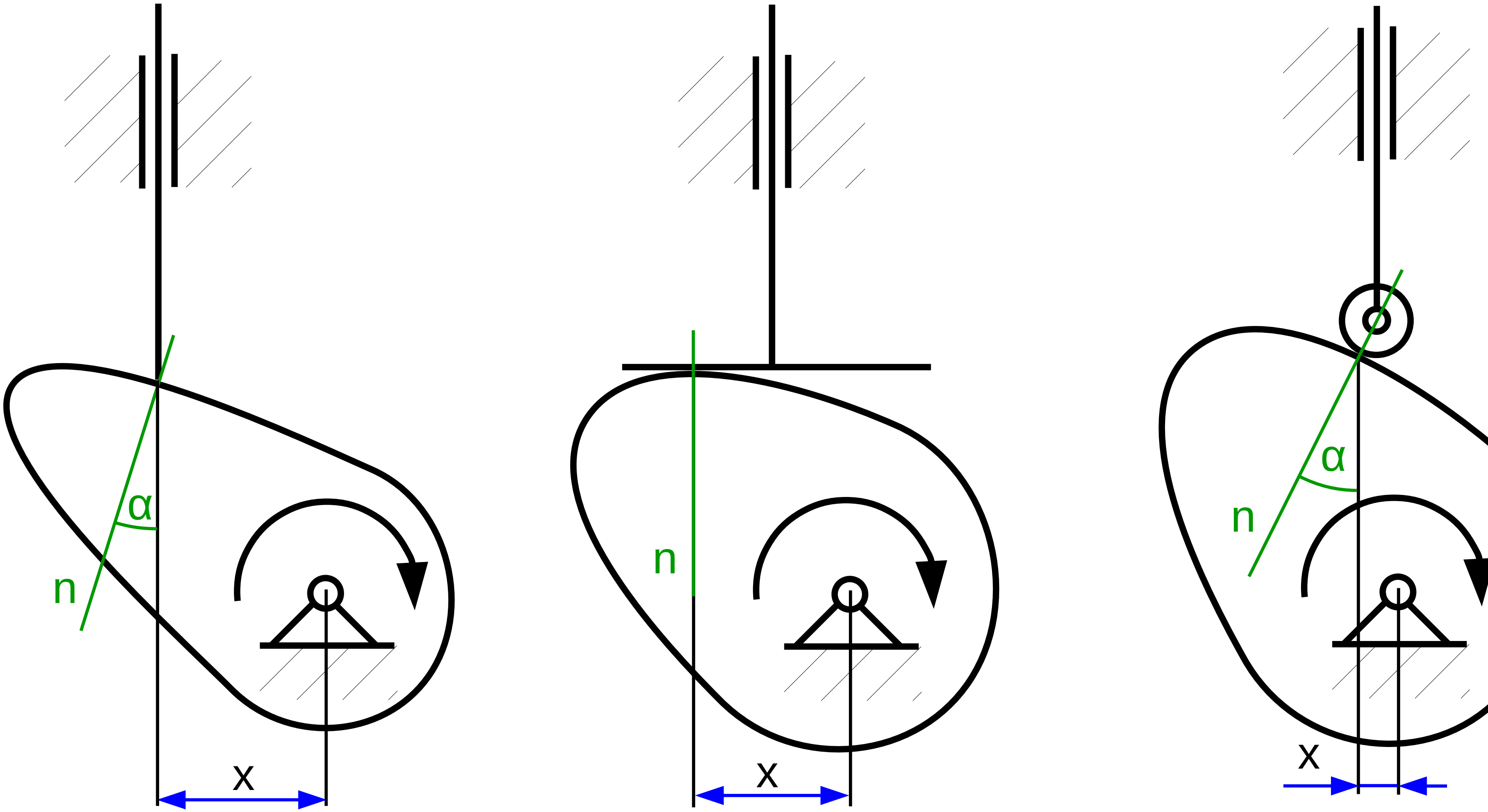
For a knife-edge follower we will obtain exact real displacement.
For a roller-ended follower some errors are possible.

Roller-ended follower give us limitation of a maximum velocity (there is a relation between roller radius and cam size).

Usually we are designing symmetric and smooth cams.

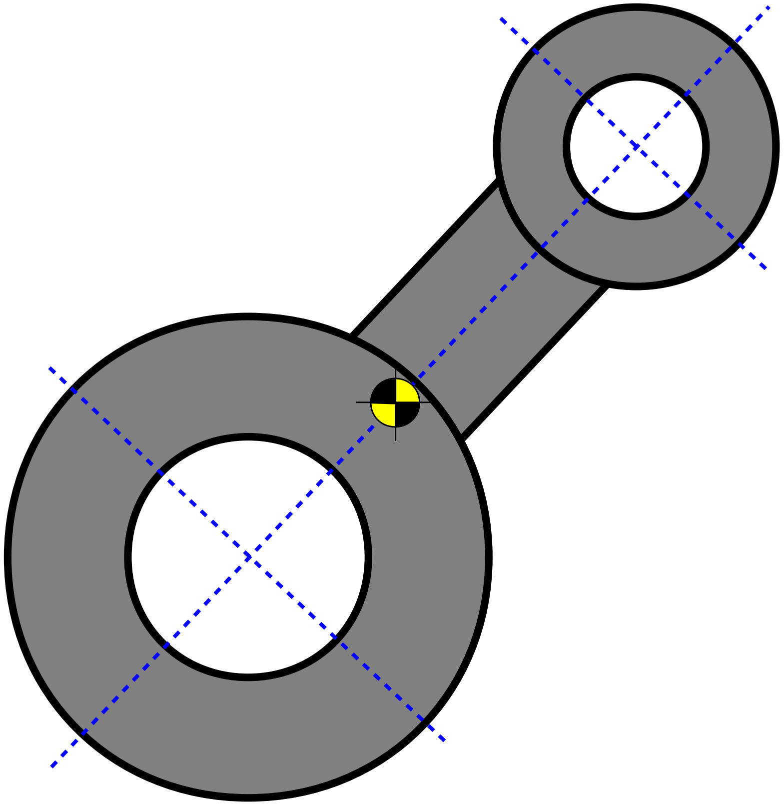
Cam-follower mechanisms

Distance of contact and angle of contact



Dynamics of planar mechanisms

Members description



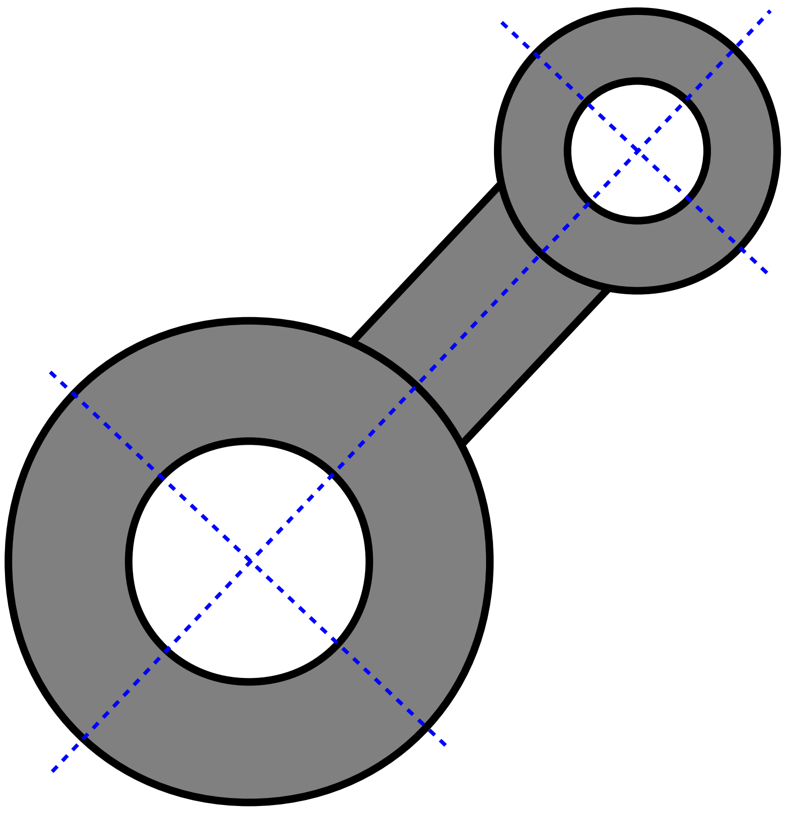
For a planar mechanism member represented by a rigid body:

- mass
- location of a center of a mass
- mass moment of inertia wrt the axis perpendicular to the motion plane in center of a mass
- location of connection points

Dynamics of planar mechanisms

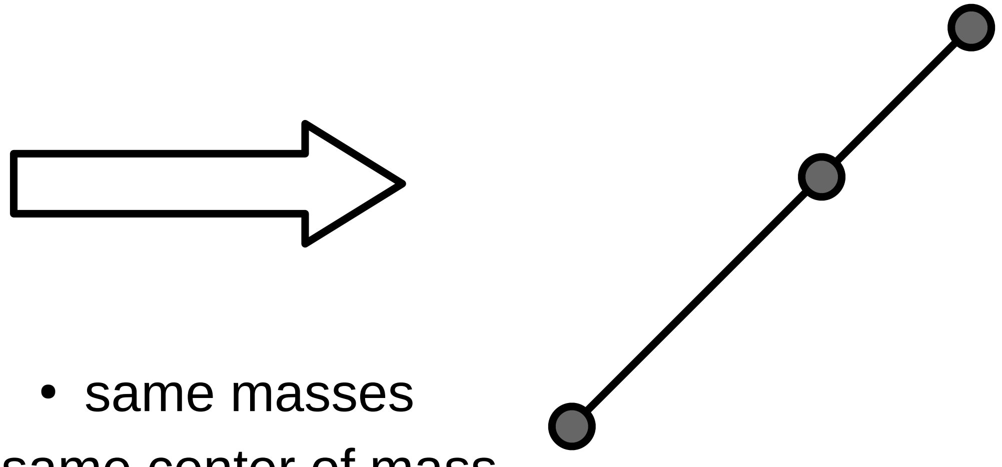
Members description

Material points method



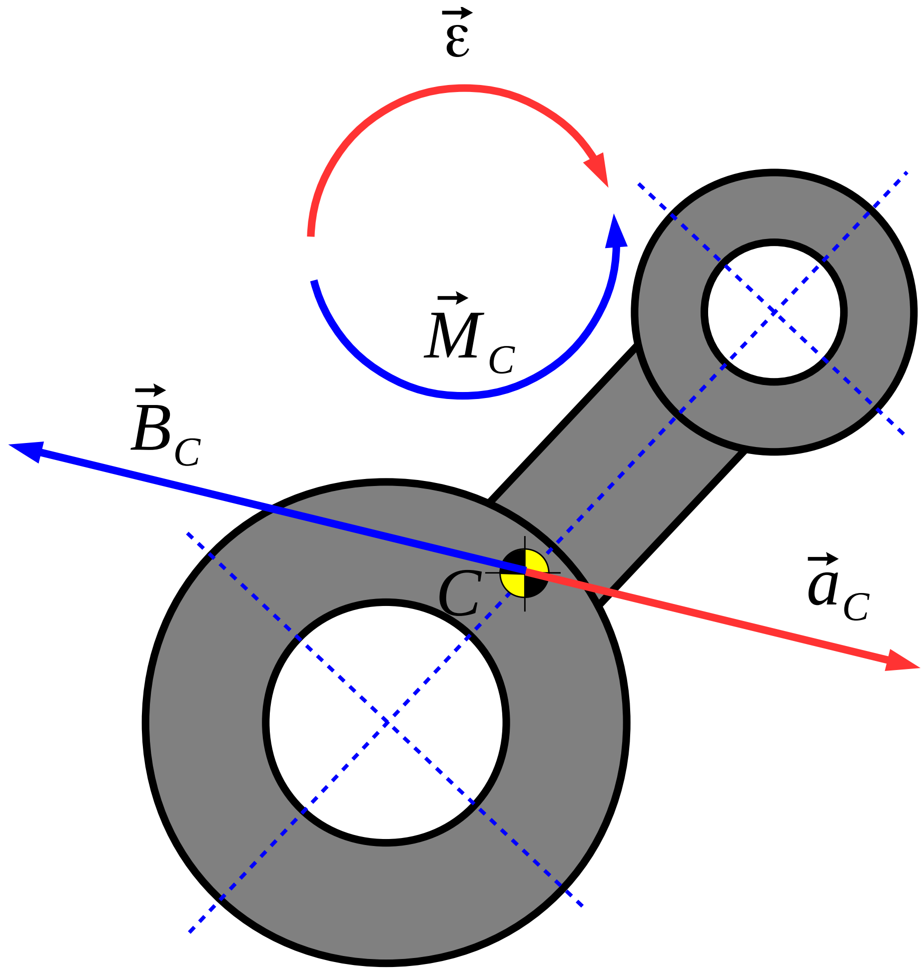
a set of material points

- same masses
- same center of mass
- same inertia



Dynamics of planar mechanisms

Inertia forces and torques



inertia force

$$\vec{B}_C = -m\vec{a}_C$$

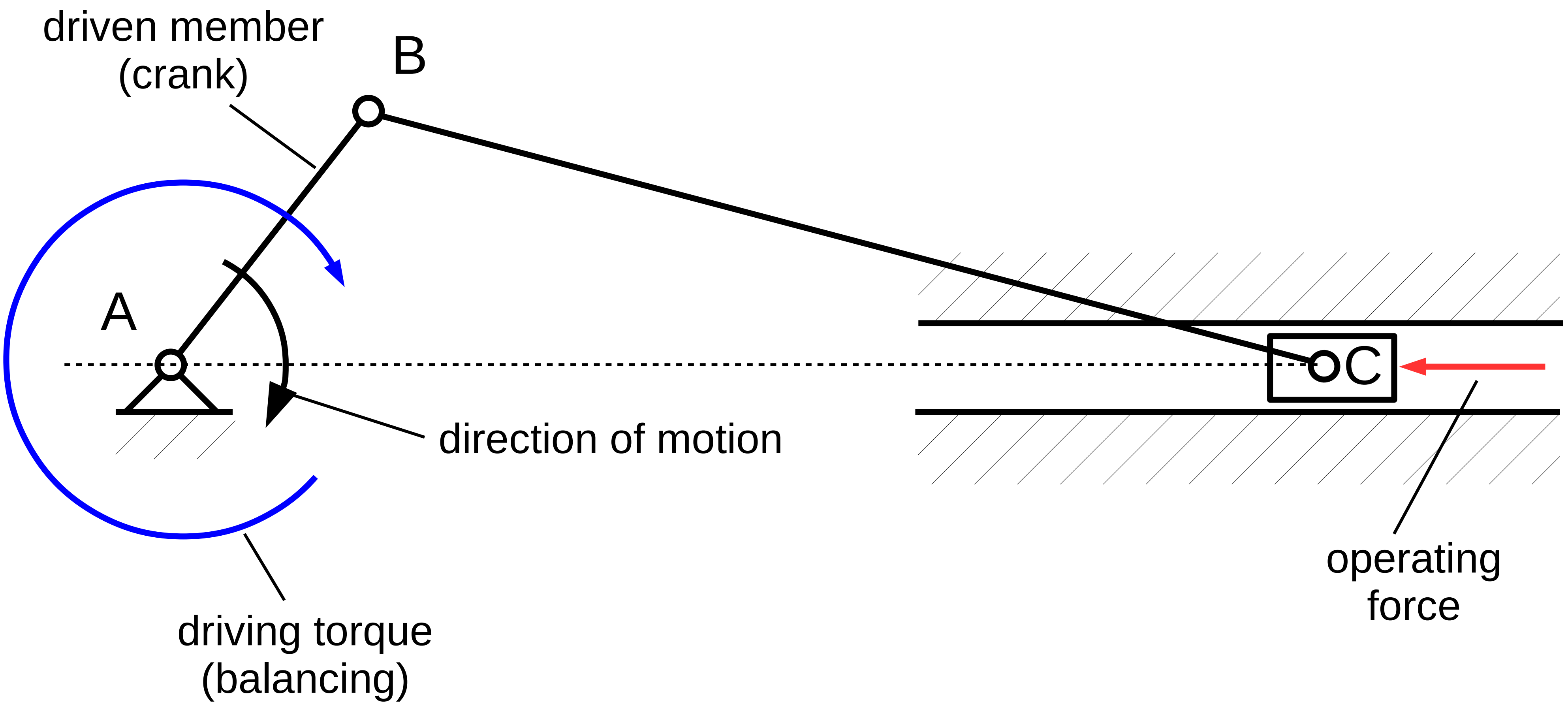
Inertia torque

$$\vec{M}_C = -I_C\vec{\varepsilon}$$

Dynamics of planar mechanisms

Driving and operating forces/torques

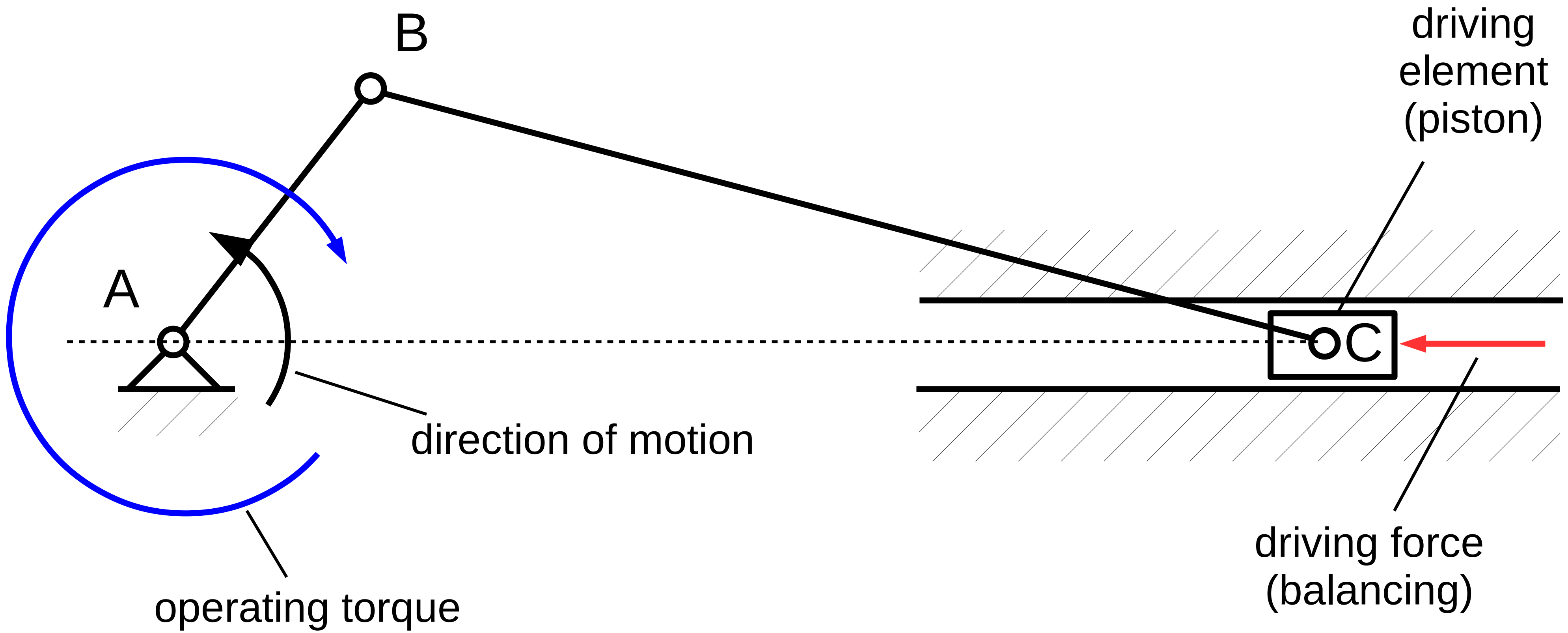
example – compressor



Dynamics of planar mechanisms

Driving and operating forces/torques

example – engine



Dynamics of planar mechanisms

Inverse dynamics problem – calculation of forces and torques that cause given motion of a mechanism.

Direct dynamics problem – calculation of mechanism's motion caused by external forces and torques.

Dynamics of planar mechanisms

Inverse dynamics problem

Calculation of forces and torques that cause given motion of a mechanism (kinetostatics)

0. Mechanism and it's geometry, driving and operating forces/torques, displacement, velocity and acceleration functions are given.

1. Calculation of inertia forces and torques acting moving members of the mechanism.

2. Decomposition of the mechanism with reaction disclosure.

3. Write down vector sums of external forces, reactions and inertia forces (d'Alembert equations).

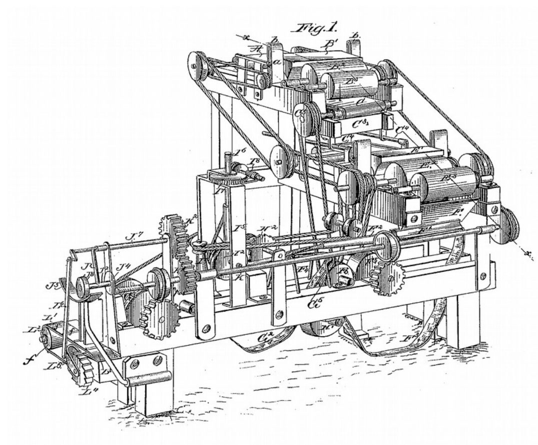
4. Solve the equations with graphical and/or analytical method.

Lecture 6

Machine dynamics.
Reduction of masses and forces.
Machine equation of motion.

Reduction of masses

Kinetic energy



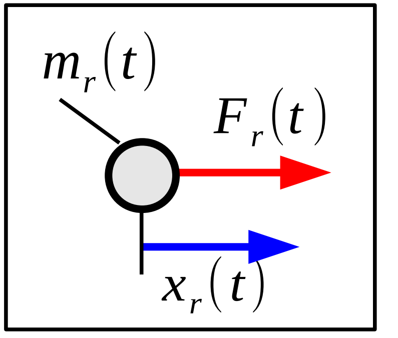
Total kinetic energy

$$T=\sum_{i=1}^{n}\left(\frac{1}{2}m_i v_i^2+\frac{1}{2}I_i\omega_i^2\right)$$

or

$$T=\frac{1}{2}\,m_r\,v_r^2$$

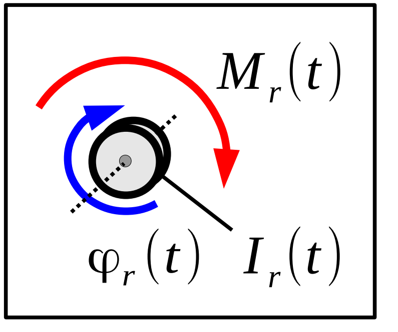
reduced mass



$$v_r=\frac{dx_r(t)}{dt}$$

$$T=\frac{1}{2}\,I_r\,\omega_r^2$$

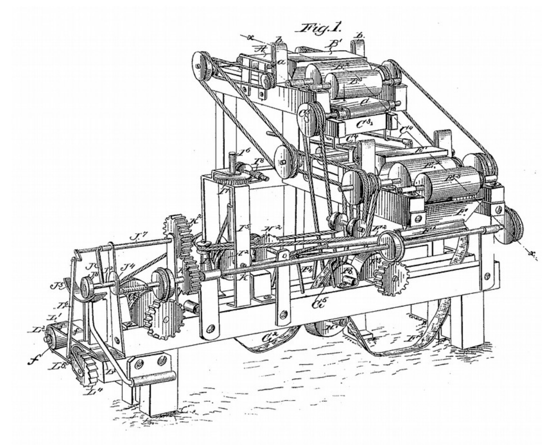
reduced moment of inertia



$$\omega_r=\frac{d\varphi_r(t)}{dt}$$

Reduction of forces

System power



Total system's power

$$P\left(F_i, M_i, \omega_i, v_i, \ldots\right)$$

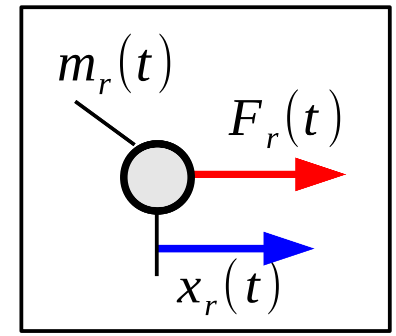
or

$$P = F_r v_r$$

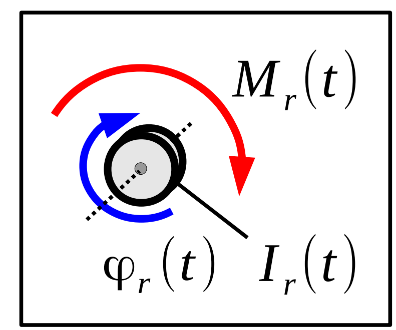
reduced force

$$P = M_r \omega_r$$

reduced torque



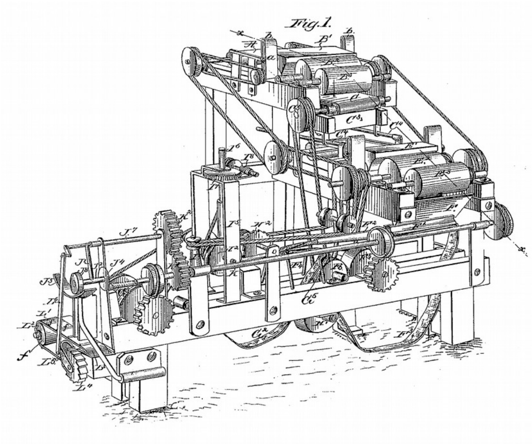
$$v_r = \frac{dx_r(t)}{dt}$$



$$\omega_r = \frac{d\varphi_r(t)}{dt}$$

Reduction of masses – details

Kinetic energy



$$T = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2 + \sum_{j=1}^{k} \frac{1}{2} I_j \omega_j^2$$

n – translating elements
k – rotating elements

$$\frac{1}{2} m_r v_r^2 = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2 + \sum_{j=1}^{k} \frac{1}{2} I_j \omega_j^2$$

$$m_r = \sum_{i=1}^{n} m_i \frac{v_i^2}{v_r^2} + \sum_{j=1}^{k} I_j \frac{\omega_j^2}{v_r^2}$$

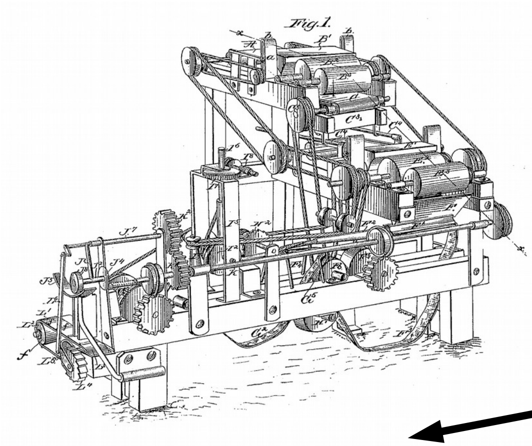
$$\frac{1}{2} I_r \omega_r^2 = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2 + \sum_{j=1}^{k} \frac{1}{2} I_j \omega_j^2$$

$$I_r = \sum_{i=1}^{n} m_i \frac{v_i^2}{\omega_r^2} + \sum_{j=1}^{k} I_j \frac{\omega_j^2}{\omega_r^2}$$

v_r, ω_r – arbitrary chosen velocities

Reduction of forces – details

Work



$$dW=\sum_{i=1}^{n} P_i\, ds_i \cos\alpha_i + \sum_{j=1}^{k} M_j\, d\varphi_j$$

n – translating elements
k – rotating elements

$$\alpha_i = \sphericalangle\left(P_i, ds_i\right)$$

$$P_r\, ds_r=\sum_{i=1}^{n} P_i\, ds_i \cos\alpha_i + \sum_{j=1}^{k} M_j\, d\varphi_j$$

$$P_r=\sum_{i=1}^{n} P_i \frac{ds_i}{ds_r} \cos\alpha_i + \sum_{j=1}^{k} M_j \frac{d\varphi_j}{ds_r}$$

$$P_r=\sum_{i=1}^{n} P_i \frac{v_i\, dt}{v_r\, dt} \cos\alpha_i + \sum_{j=1}^{k} M_j \frac{\omega_j\, dt}{v_r\, dt}$$

$$P_r=\sum_{i=1}^{n} P_i \frac{v_i}{v_r} \cos\alpha_i + \sum_{j=1}^{k} M_j \frac{\omega_j}{v_r}$$

$$M_r\, d\varphi_r=\sum_{i=1}^{n} P_i\, ds_i \cos\alpha_i + \sum_{j=1}^{k} M_j\, d\varphi_j$$

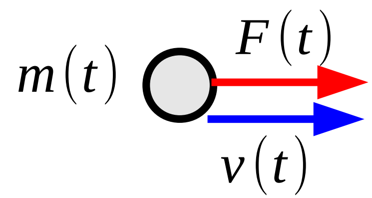
$$M_r=\sum_{i=1}^{n} P_i \frac{ds_i}{d\varphi_r} \cos\alpha_i + \sum_{j=1}^{k} M_j \frac{d\varphi_j}{d\varphi_r}$$

$$M_r=\sum_{i=1}^{n} P_i \frac{v_i\, dt}{\omega_r\, dt} \cos\alpha_i + \sum_{j=1}^{k} M_j \frac{\omega_j\, dt}{\omega_r\, dt}$$

$$M_r=\sum_{i=1}^{n} P_i \frac{v_i}{\omega_r} \cos\alpha_i + \sum_{j=1}^{k} M_j \frac{\omega_j}{\omega_r}$$

Machine equation of motion

Linear motion



elementary change of kinetic energy

elementary work

$$dT = dW$$

complete differential of kinetic energy

$$d\left(\frac{1}{2} m(t)\, v(t)^2\right) = F(t)\, dx$$

$$\frac{1}{2} dm(t)\, v(t)^2 + m(t)\, v(t)\, dv(t) = F(t)\, dx$$

$$\frac{1}{2} dm(t)\, v(t)^2 + m(t) \frac{dx(t)}{dt} dv(t) = F(t)\, dx$$

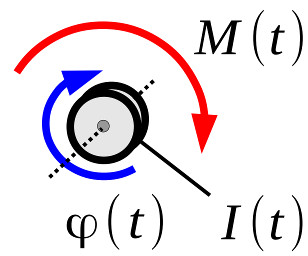
$$\frac{dm(t)}{dx} \frac{v(t)^2}{2} + m \frac{dv(t)}{dt} = F(t)$$

$$\boxed{\frac{dm(t)}{dt} \frac{v(t)}{2} + m \frac{dv(t)}{dt} = F(t)}$$

$$if \;\; m = const. \;\Rightarrow\; m \frac{dv(t)}{dt} = P(t) \quad or \quad m\ddot{x}(t) = F(t)$$

Machine equation of motion

Angular motion



$$dT = dW$$

$$d\left(I\frac{\omega(t)^2}{2}\right) = M(t)\,d\varphi$$

...

...

$$\frac{dI(t)}{d\varphi}\frac{\omega(t)^2}{2} + I(t)\frac{d\omega(t)}{dt} = M(t)$$

$$\boxed{\frac{dI(t)}{dt}\frac{\omega(t)}{2} + I(t)\frac{d\omega(t)}{dt} = M(t)}$$

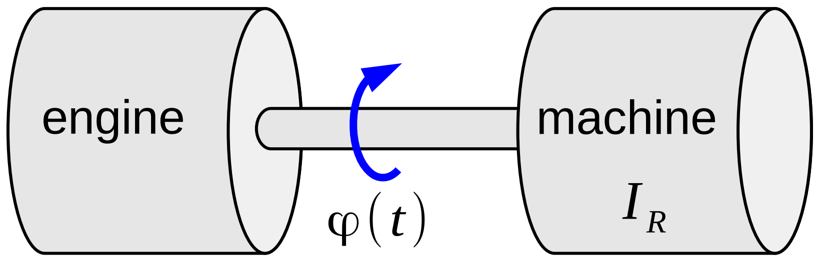
$$if \quad I = const. \Rightarrow \; I\frac{d\omega(t)}{dt} = M(t) \quad or \quad I\ddot{\varphi}(t) = M(t)$$

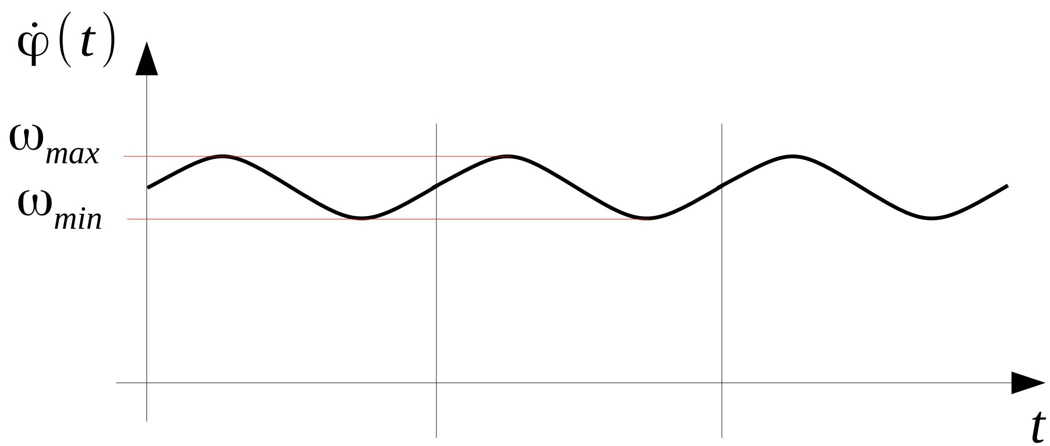
Lecture 7

Non-uniformity of machine motion.
Introduction to automatic control.

Non-uniformity of machine motion

Steady-state motion





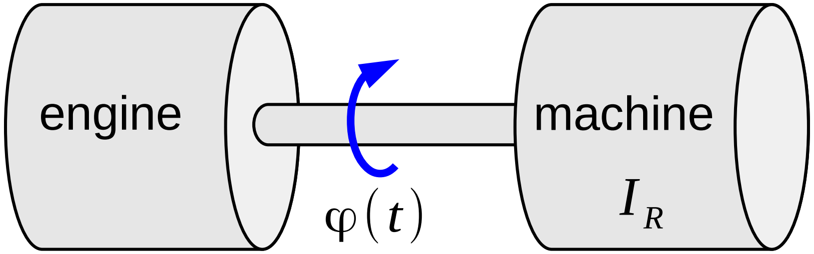
Non-uniformity of machine motion

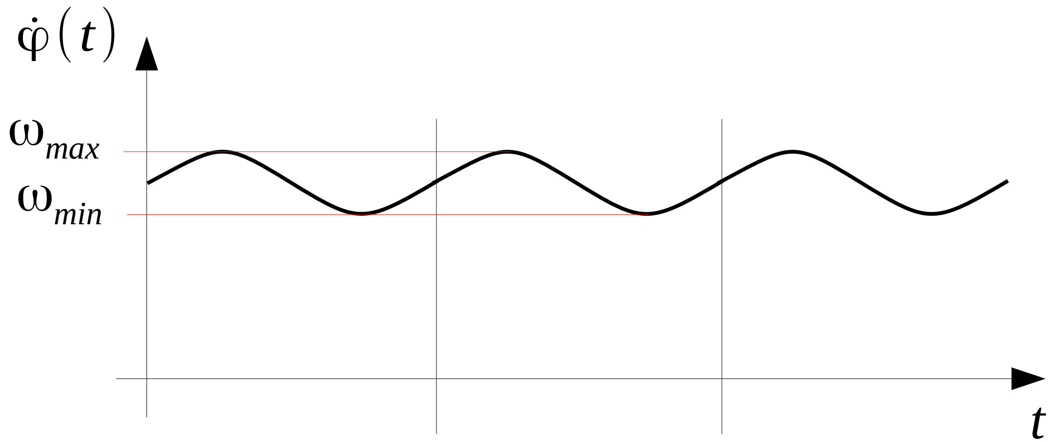
$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \qquad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

pumps	combustion engines	generators
$\delta = 1/5 \div 1/30$	$\delta = 1/50 \div 1/150$	$\delta = 1/200 \div 1/300$

Non-uniformity of machine motion

Steady-state motion





Non-uniformity of machine motion

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \qquad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

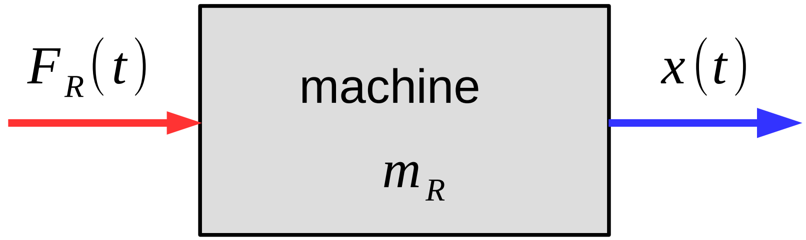
pumps	combustion engines	generators
$\delta = 1/5 \div 1/30$	$\delta = 1/50 \div 1/150$	$\delta = 1/200 \div 1/300$

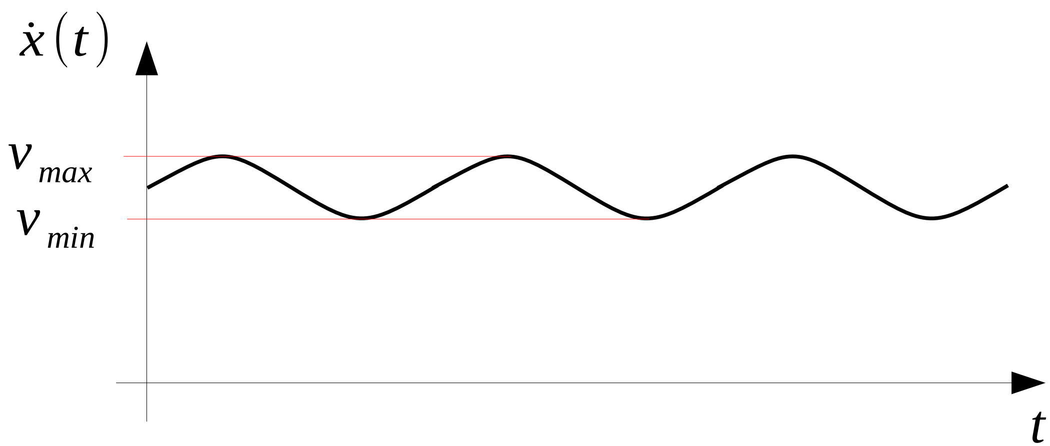
$$T_{max} = \frac{1}{2} I_R \omega_{max}^2 \qquad T_{min} = \frac{1}{2} I_R \omega_{min}^2$$

$$W = T_{max} - T_{min} = \delta I_R \omega_{mean}^2$$

Non-uniformity of machine motion

Steady-state motion





Non-uniformity of machine motion

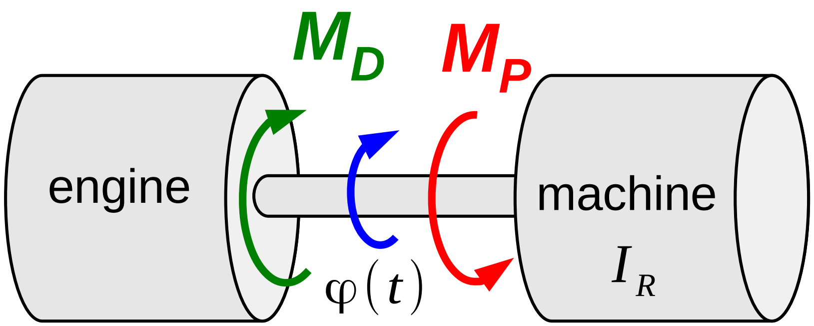
$$\delta = \frac{v_{max} - v_{min}}{v_{mean}} \qquad v_{mean} = \frac{v_{max} + v_{min}}{2}$$

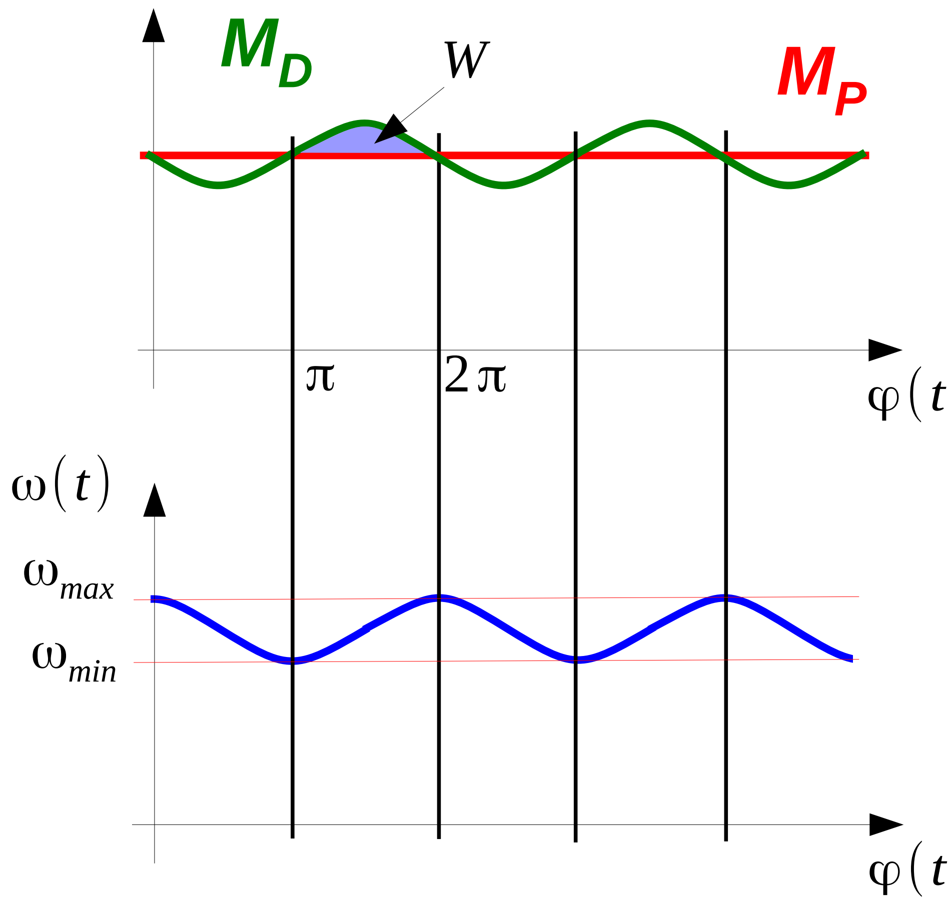
$$W = \delta \, m_R \, v_{mean}^2$$

Non-uniformity of machine motion

Steady-state motion

Example





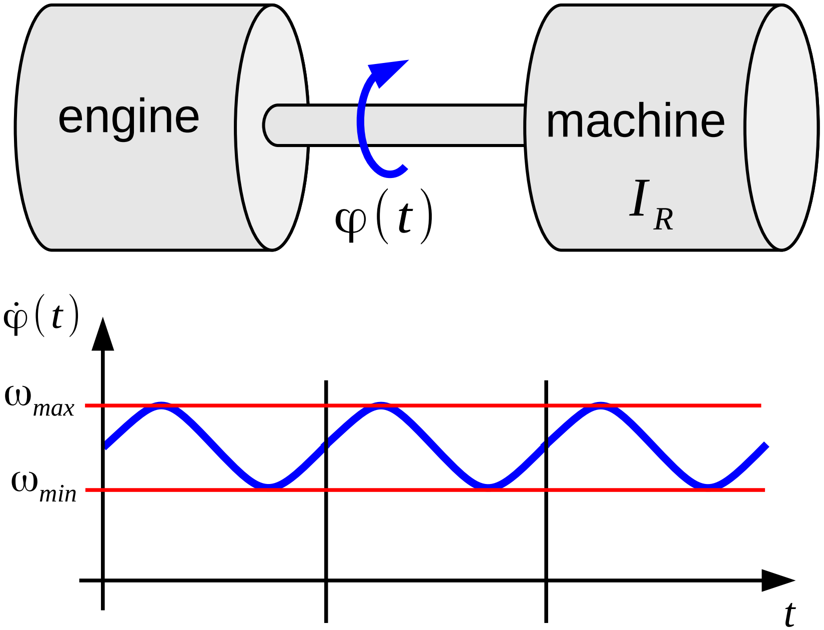
$$W = \int_{\varphi_{min}}^{\varphi_{max}} (M_D - M_P)\, d\varphi$$

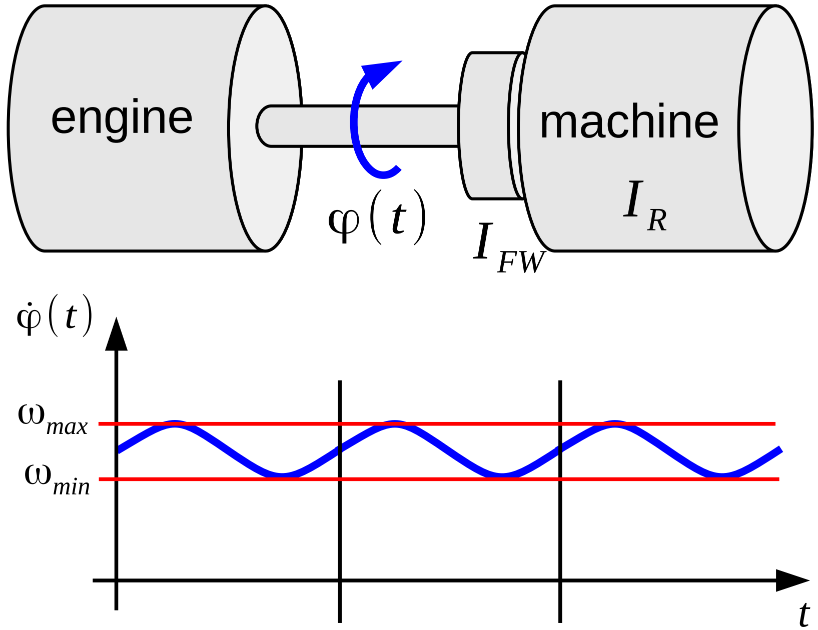
$$W = T_{max} - T_{min} = \delta I_R \omega_{mean}^2$$

$$\delta = \frac{W}{I_R \omega_{mean}^2}$$

Flywheel

Steady-state motion





$$W=\delta_1 I_R \omega_{mean}^2$$

assume $I_R \approx const.$

$$W=\delta_2 (I_R+I_{FW}) \omega_{mean}^2$$

$$\delta_1 I_R \omega_{mean}^2=\delta_2 (I_R+I_{FW}) \omega_{mean}^2$$

$$I_{FW}=\left(\frac{\delta_1}{\delta_2}-1\right) I_R$$

Non-uniformity of machine motion

To minimize flywheel’s mass moment of inertia:

- you should mount a flywheel on a shaft that rotates with the highest angular velocity

- You can add extra transmission to increase angular velocity of a flywheel

Automatic control

Automatic control

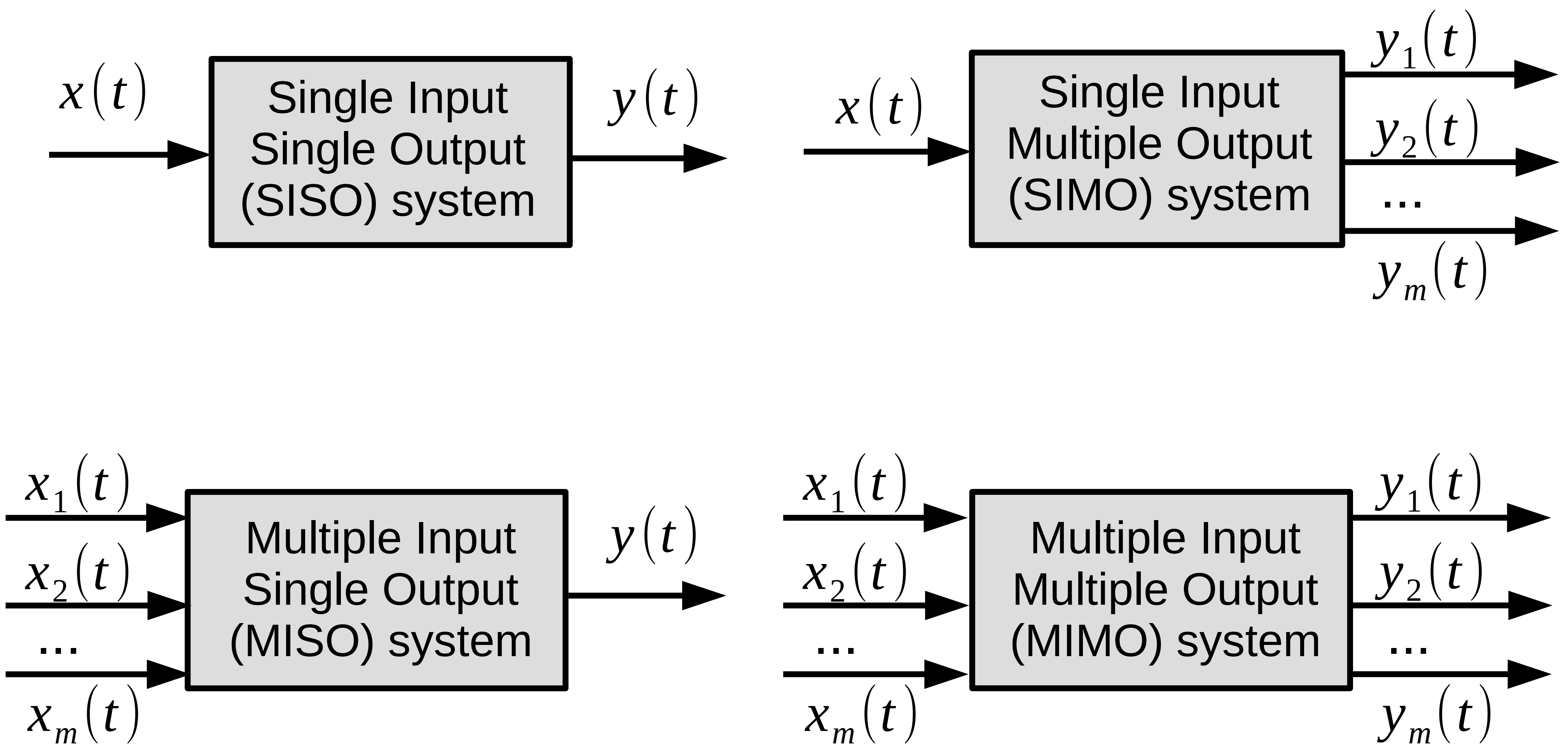
"Automatic control in engineering and technology is a wide generic term covering the application of mechanisms to the operation and regulation of processes without continuous direct human intervention." - *wikipedia*

Control theory – branch of mathematics and cybernetics that deals with analysis and mathematical modeling of objects and processes threated as dynamical systems with **feedback**.

Automatic control

Classical control theory	modern control theory (1950-now)
single input, single output (SISO)	multiple input, multiple output (MIMO)
usually linear systems	often nonlinear systems
time independent systems	time dependent systems
description by a transfer functions	description by a state equations
time and frequency domain analysis	time domain analysis
system response is the most important	system state is the most important

Number of inputs and outputs



Linear time-invariant (LTI) system

Linear system

$x(t)$ - input, $y(t) = h(x(t))$ - output

$h(\alpha x(t)) = \alpha h(x(t)) = \alpha y(t)$ scaling

$h(x_1(t) + x_2(t)) = h(x_1(t)) + h(x_2(t))$ superposition

Linear time-invariant (LTI) system

Time-invariant system

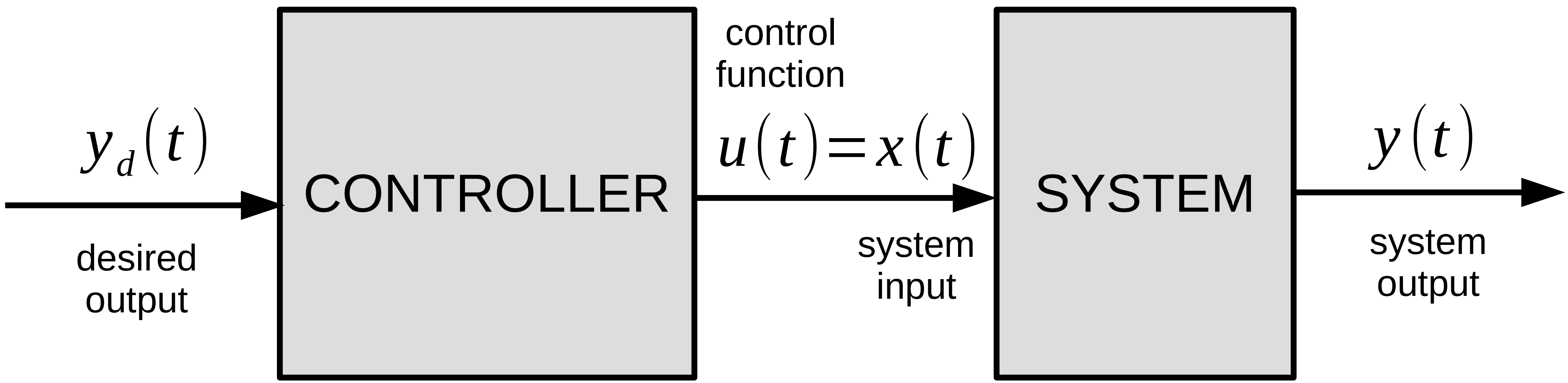
output does not depend explicitly on time

$$\text{if } y(t)=h(x(t)) \text{ then } y(t-\tau)=h(x(t-\tau))$$

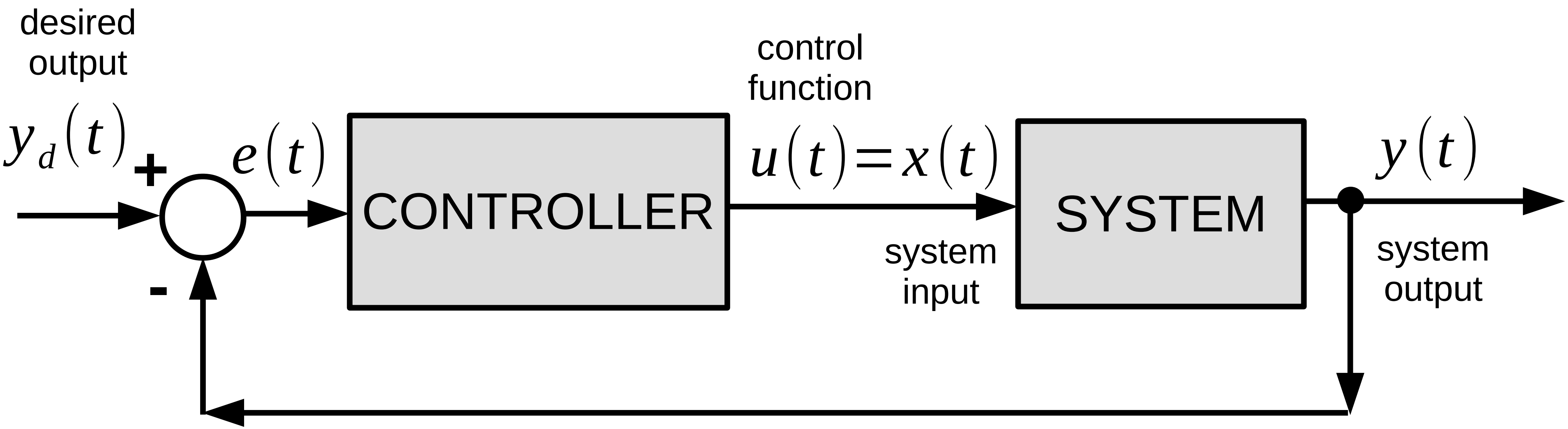
Time-varying system

$$\text{if } y(t)=h(x(t)) \text{ then } y(t-\tau)\neq h(x(t-\tau))$$

Open loop control



Closed loop control



Lecture 8

Laplace transform.
Transfer function.
Inputs and outputs in time domain.

Laplace transform

Assumption: $x(t)$ - signal such that for $t<0 \quad x(t)=0$

Laplace transform of x(t): $$X(s)=L\{x(t)\}=\int_0^\infty x(t)e^{-st}dt$$

where: $s\in\mathbb{C}, \quad s=\sigma+j\omega, \quad j=\sqrt{-1}$

Inverse Laplace transform of x(t): $$x(t)=L^{-1}\{X(s)\}=\frac{1}{2\pi j}\lim_{\omega\to\infty}\int_{\gamma-j\omega}^{\gamma+j\omega} X(s)e^{st}ds$$

A necessary condition for existence of the integral is that x(t) must be locally integrable on t in <0, ∞).

Laplace transform pairs

$f(t),\ t \geqslant 0$	$F(s)$
$\delta(t)$ unit impulse	1
$1(t)$ unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-bt}	$\frac{1}{s+b}$
$1-e^{-bt}$	$\frac{b}{s(s+b)}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega}$
$\cosh(\omega t)$	$\frac{s}{s^2-\omega}$

Properties of Laplace transform

$f(t),\ \ t\geqslant 0$	$F(s)$
$a\cdot f(t)$	$a\cdot F(s)$
$x(t)+y(t)$	$X(s)+Y(s)$
$x(t)*y(t)$ convolution	$X(s)\cdot Y(s)$
$\frac{dy(t)}{dt}$	$s\,Y(s)-y(0)$
$\frac{d^2\,y(t)}{dt^2}$	$s^2\,Y(s)-s\,y(0)-\frac{dy(0)}{dt}$
$\frac{d^n\,y(t)}{dt^n}$	$s^n\,Y(s)-\frac{d^{n-1}\,y(0)}{dt^{n-1}}-s\frac{d^{n-2}\,y(0)}{dt^{n-2}}-\ldots-s^{n-1}\,y(0)$
$\int\limits_{t=0}^{\infty} f(t)\,dt$	$\frac{F(s)}{s}$
$\underbrace{\int\int\ldots\int}_{n} f(t)\,dt$	$\frac{F(s)}{s^n}$
$f(t-\tau)$	$e^{-\tau s}\,F(s)$

Transfer function – definition

For LTI SISO system with continuous input x(t) and output y(t) for zero initial conditions, transfer function is a ratio of the output of a system to the input of a system described in complex domain by the Laplace transformation.

$$H(s)=\frac{L\{y(t)\}}{L\{x(t)\}}=\frac{Y(s)}{X(s)}$$

Transfer function form

Standard form: $$H(s)=\frac{b^m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

Factored form: $$H(s)=\frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

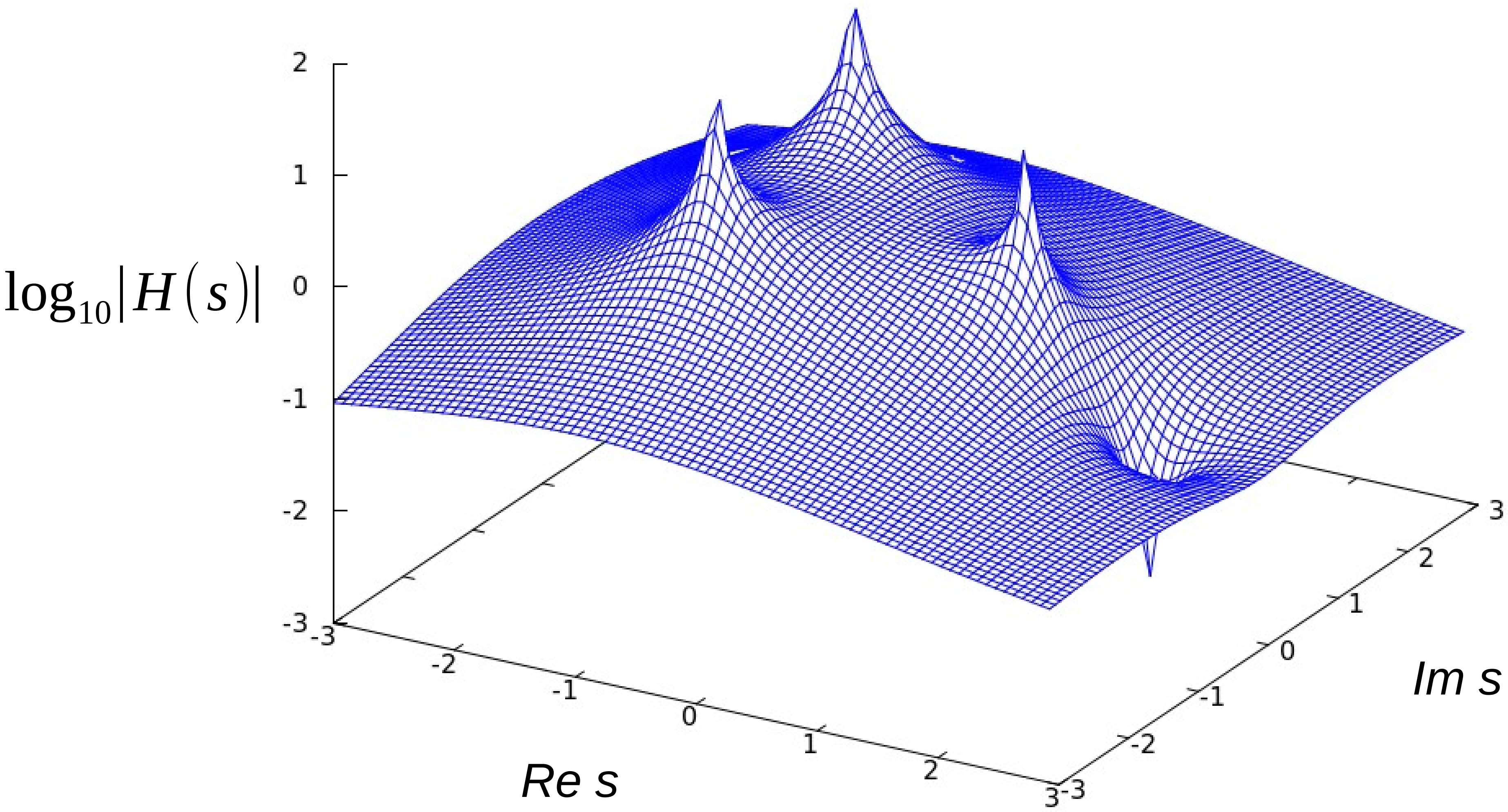
$z_{1,}\ z_2,\ \ldots,\ z_m$ - zeroes

$p_{1,}\ p_2,\ \ldots,\ p_n$ - poles

Drawing of a transfer function

<u>Example</u>

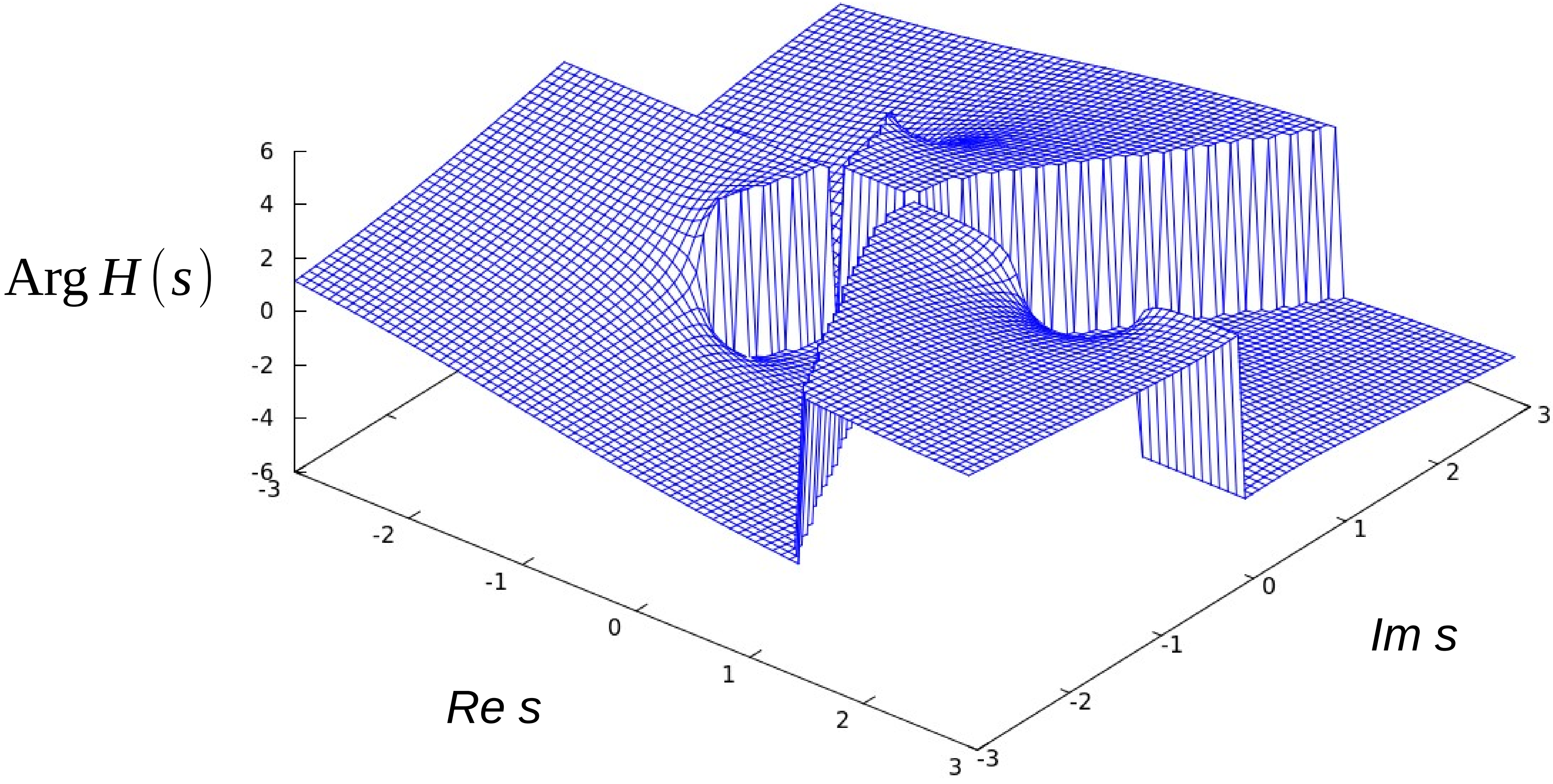
Poles: $p_1 = 1$, $p_2 = -1 - j$, $p_3 = -1 + j$ Zeroes: $z_1 = 2$



Drawing of a transfer function

Example

Poles: $p_1 = 1, \; p_2 = -1 - j, \; p_3 = -1 + j$ Zeroes: $z_1 = 2$



Input and output

Transfer function: $$H(s)=\frac{Y(s)}{X(s)}$$

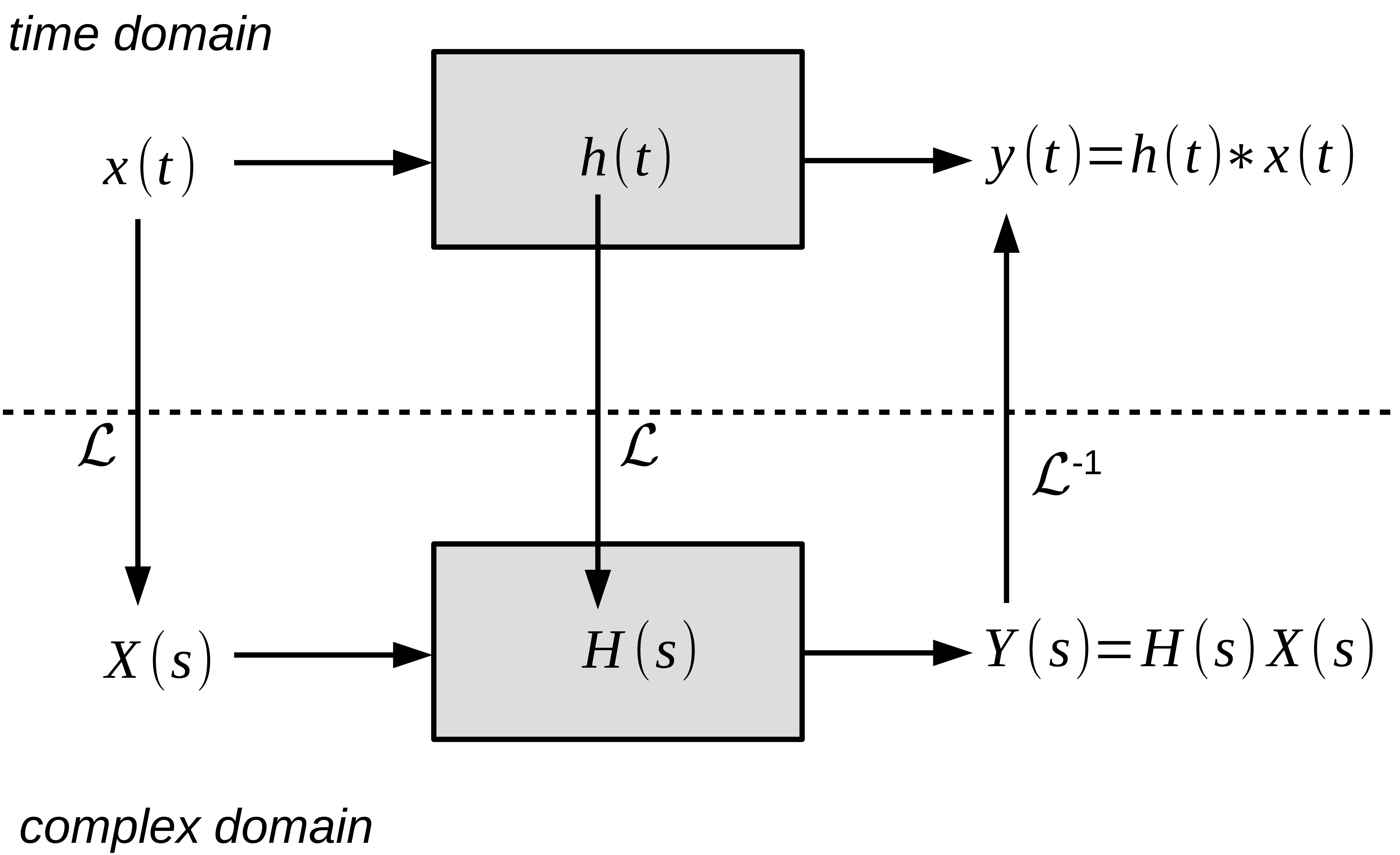
Laplace transform of output: $$Y(s)=H(s)X(s)$$

Output in time domain: $$y(t)=L^{-1}\{Y(s)\}$$

$$y(t)=L^{-1}\{H(s)X(s)\}=L^{-1}\{H(s)\}*L^{-1}\{X(s)\}=h(t)*x(t)$$

$h(t)$ - system impulse response ($y(t)$ when $x(t)=\delta(t)$)

Input and output



Exemplary input signals

No input: $x(t)=0$

Unit impulse (Dirac delta pseudofunction): $\delta(t)=\begin{cases} 0, & t<0 \\ \infty, & t=0 \\ 0, & t>0 \end{cases}$

Unit step function (Heviside step function): $1(t)=\begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}$
$H(t)$ or $1_{+}(t)$

Ramp function: $x(t)=\begin{cases} 0, & t<0 \\ t, & t>0 \end{cases}$

Harmonic function: $x(t)=a\sin(\omega t)$

Lecture 9

Frequency response.
Classification of basic automatic systems.

Transfer function & frequency response

Transfer function
(Laplace domain)

$$H(s)$$

Full system description
(for every possible input)

↓ $s = j\omega$

Frequency response
(Fourier domain)

$$H(j\omega)$$

Description of a system in <u>steady state with harmonic input</u>

Transfer function – frequency response

input: $x(t)=\sin(\omega t)$ transfer function: $H(s)$ output: $y(t)=A\sin(\omega t+\varphi)$

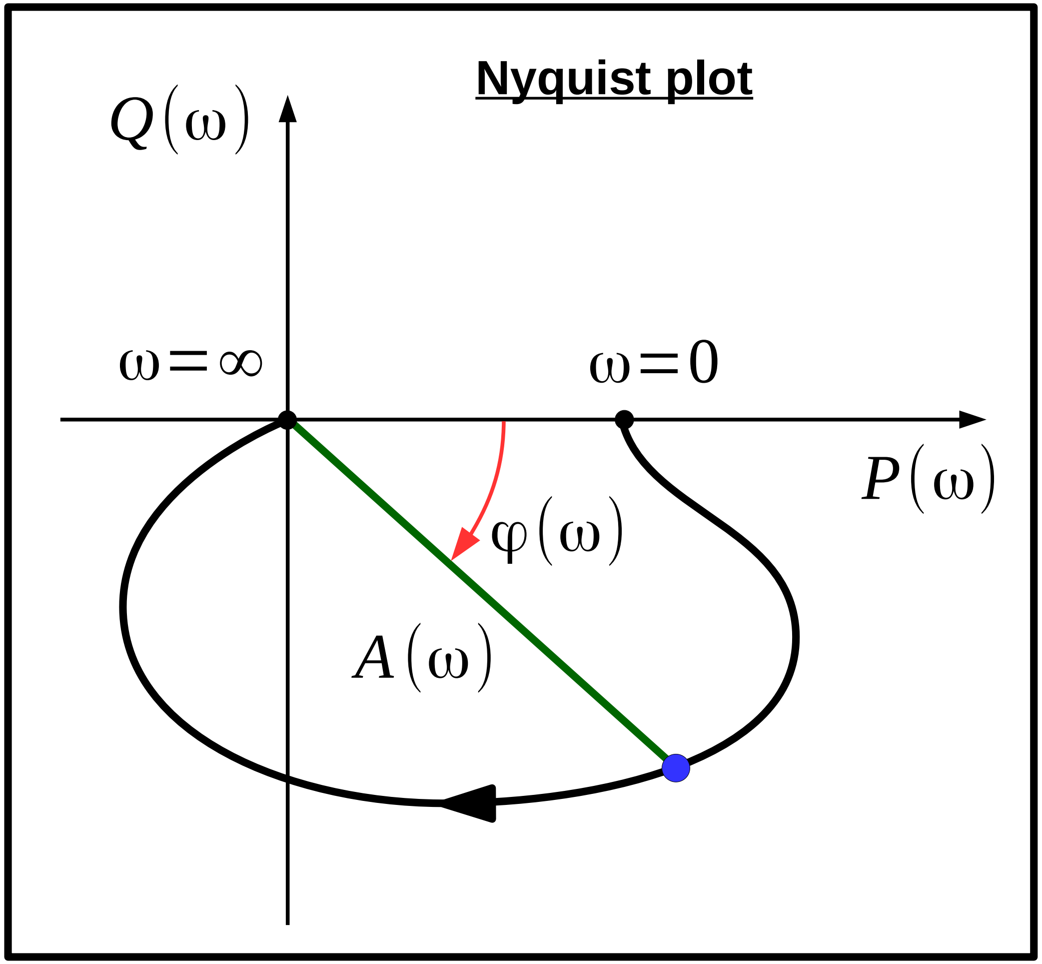
$$H(s) \xrightarrow{s=j\omega} H(j\omega)=P(\omega)+jQ(\omega)$$

$$A(\omega)=|H(j\omega)|=\sqrt{P^2(\omega)+Q^2(\omega)}$$

GAIN

$$\varphi(\omega)=\operatorname{Arg} H(j\omega)=\arctan\frac{Q}{P}$$

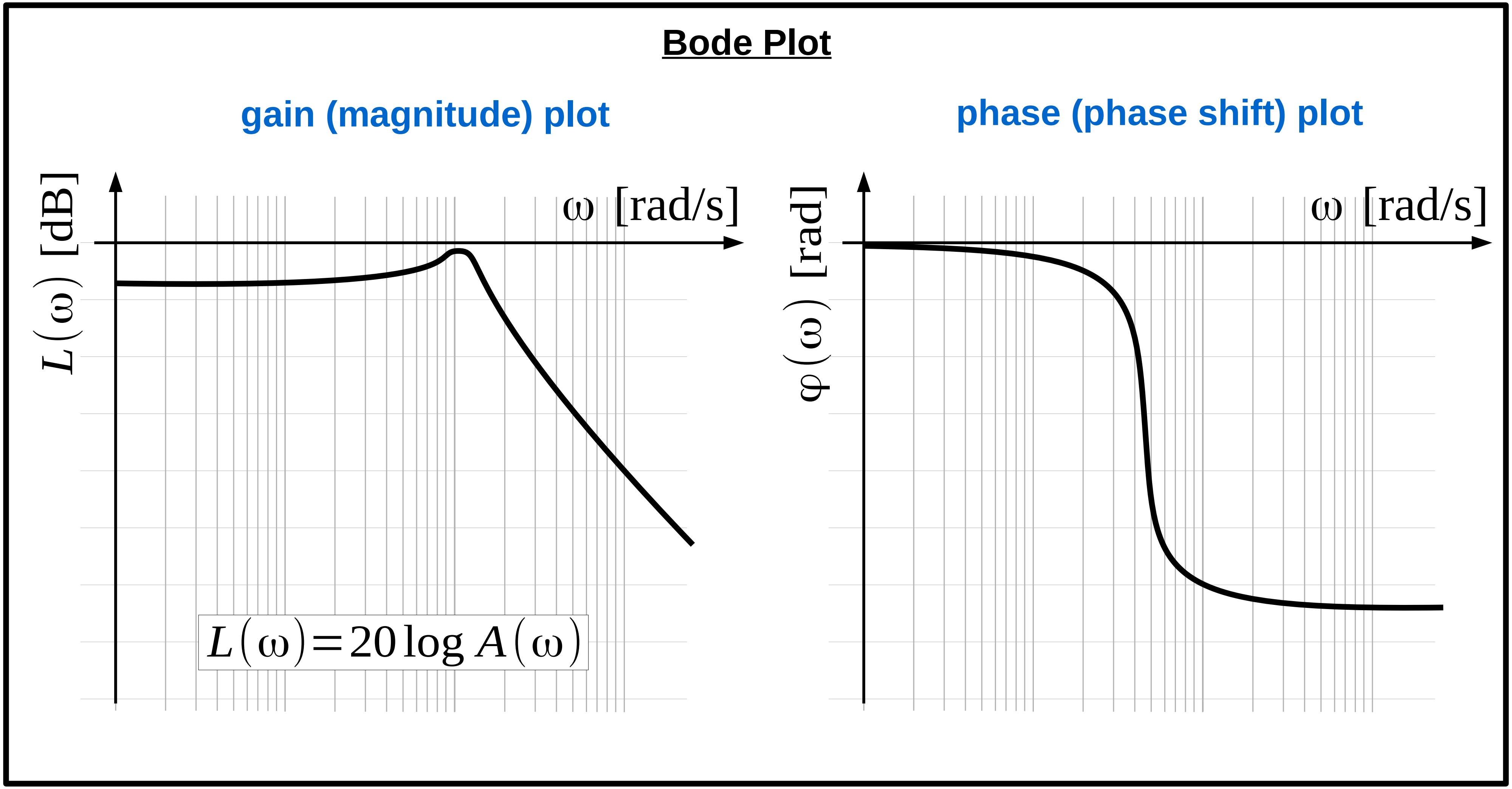
DELAY



Transfer function – frequency response

input: $x(t)=\sin(\omega t)$ transfer function: $H(s)$ output: $y(t)=A\sin(\omega t+\varphi)$

Bode Plot



Transfer function – frequency response

A (gain)	20logA [dB]
1000	60
100	40
10	20
1	0
0.1	-20
0.01	-40
0.001	-60

Classification of basic automatic systems

Element name	Equation	Transfer function
proportional	$y(t)=ku(t)$	k
first order (inertial)	$T\frac{dy(t)}{dt}+y(t)=ku(t)$	$\frac{k}{Ts+1}$
integrator	$y(t)=k\int_0^t u(t)dt$ or $\frac{dy(t)}{dt}=ku(t)$	$\frac{k}{s}$

Classification of basic automatic systems

Element name	Equation	Transfer function
derivative	$y(t) = k\dfrac{du(t)}{dt}$	ks
derivative with inertia	$T\dfrac{dy(t)}{dt} + y(t) = k\dfrac{du(t)}{dt}$	$\dfrac{ks}{Ts+1}$

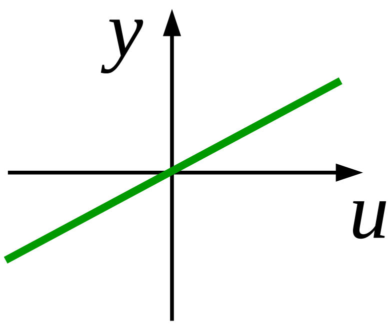
Classification of basic automatic systems

Element name	Equation	Transfer function
delay	$y(t)=u(t-\tau)$	$e^{-\tau s}$
second order (oscillator)	$T_1^2\frac{d^2y(t)}{dt^2}+T_2\frac{dy(t)}{dt}+$ $+y(t)=ku(t)$	$\frac{k}{T_1^2s^2+T_2s+1}$

Proportional element

1. Element equation: $y(t)=ku(t)$ $u(t)$ - input, $y(t)$ - output

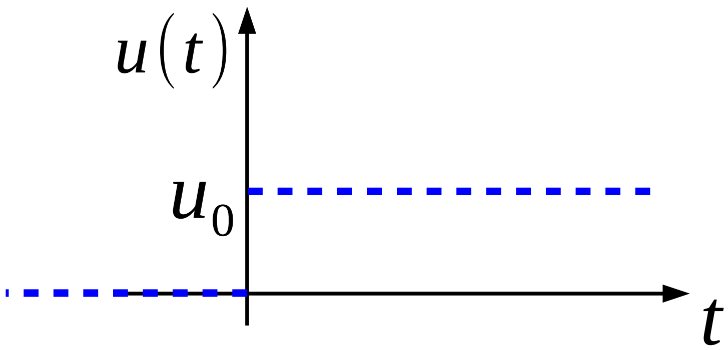
2. Static characteristic (steady state): $y=ku$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$

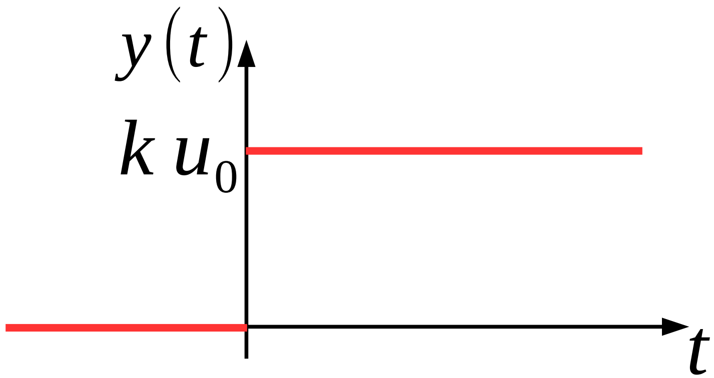


for $k>0$

3. Transfer function: $H(s)=k$

4. Step response: $y(t)=k\,u_0 1(t)$ for $u(t)=u_0 1(t)$

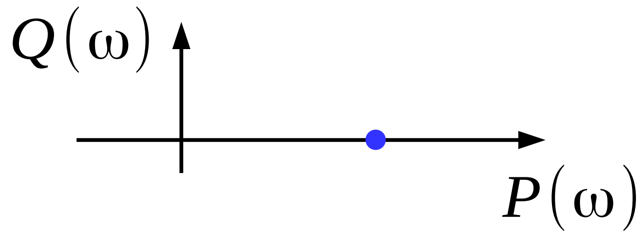




Proportional element

5. Frequency response: $H(j\omega)=k$ $P(\omega)=k,\ Q(\omega)=0$

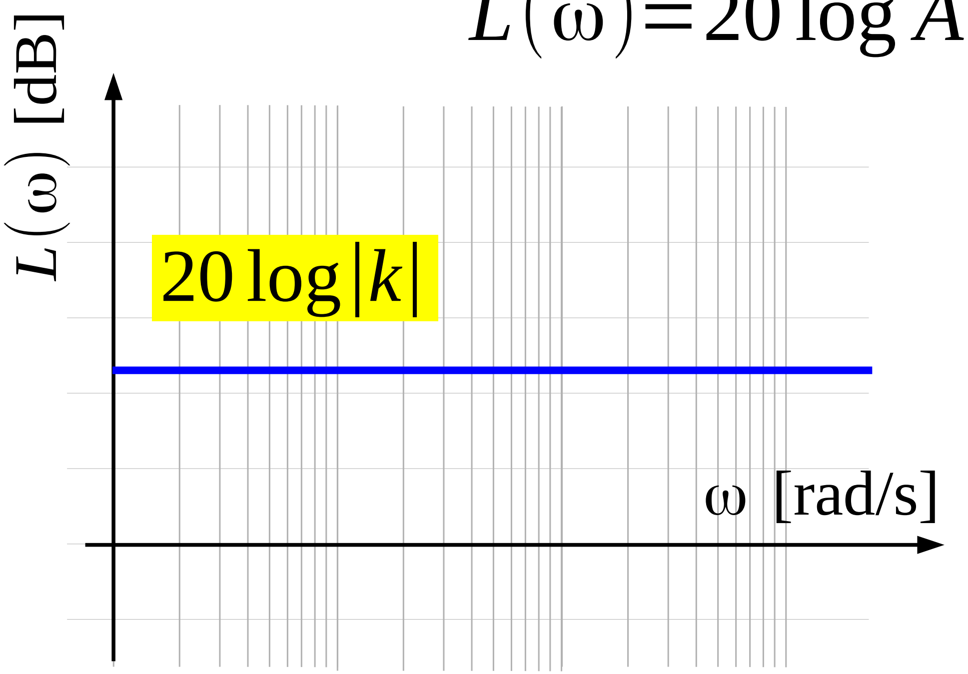
6. Nyquist plot:

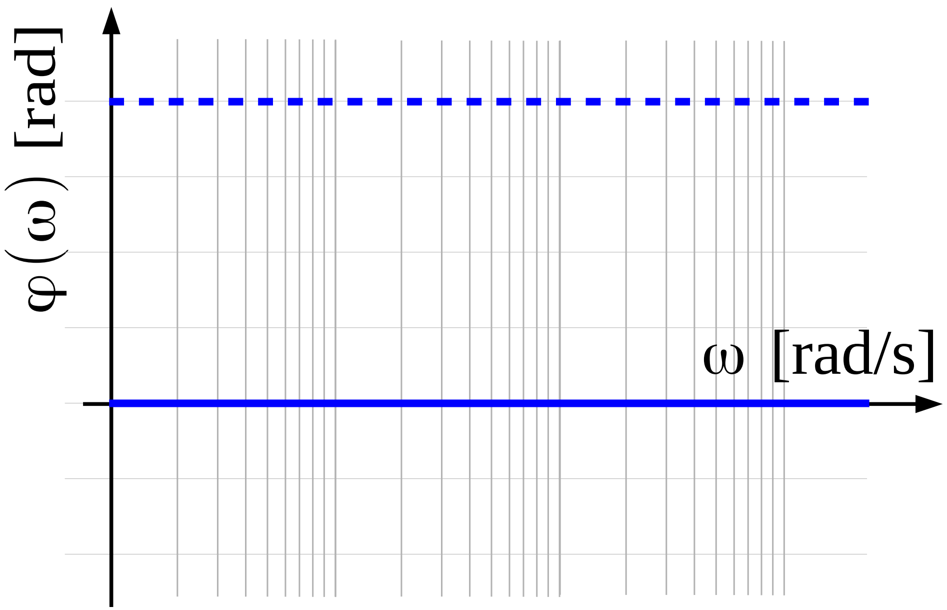


for $k>0$

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k|$ $\varphi(\omega)=\arctan\frac{Q}{P}=\begin{cases}0, \text{ dla } k\geq 0\\ \pi, \text{ dla } k<0\end{cases}$

$L(\omega)=20\log A(\omega)$

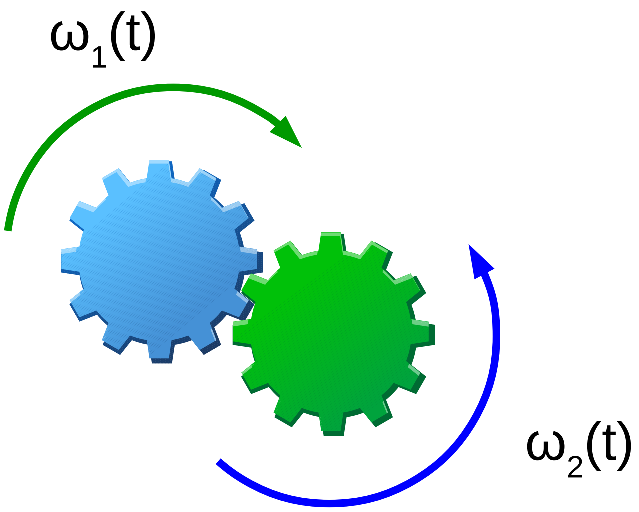




Proportional element

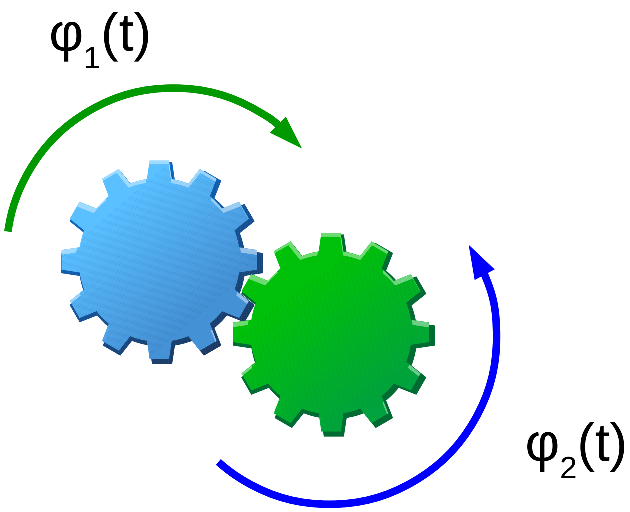
Examples

1



GEARBOX:
input – angular velocity $\omega_1(t)$
output – angular velocity $\omega_2(t)$

2

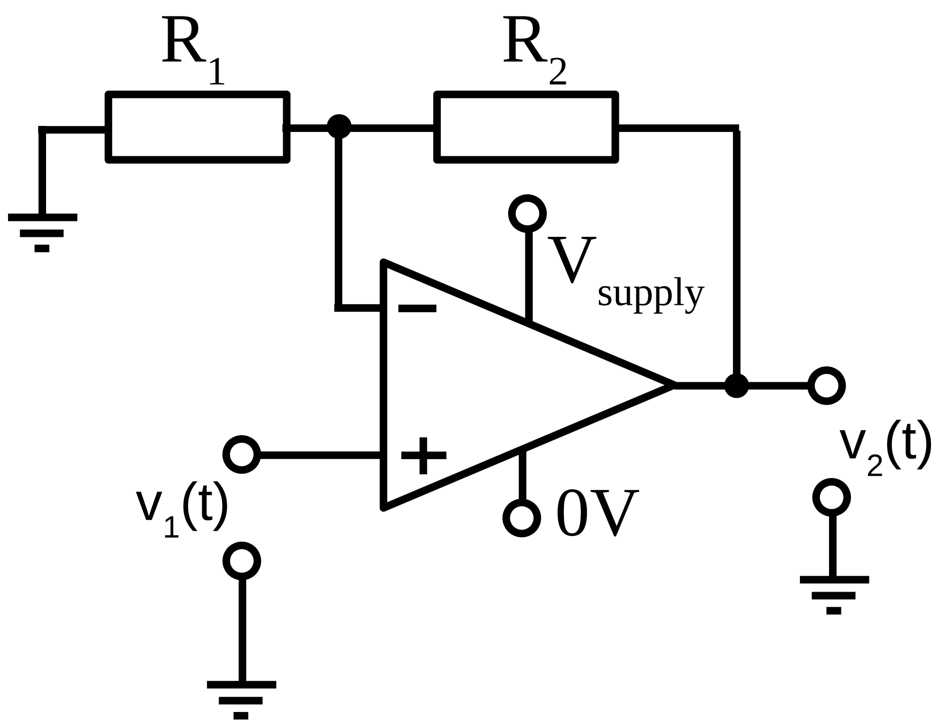


GEARBOX:
input – rotation angle $\varphi_1(t)$
output – rotation angle $\varphi_2(t)$

Proportional element

Examples

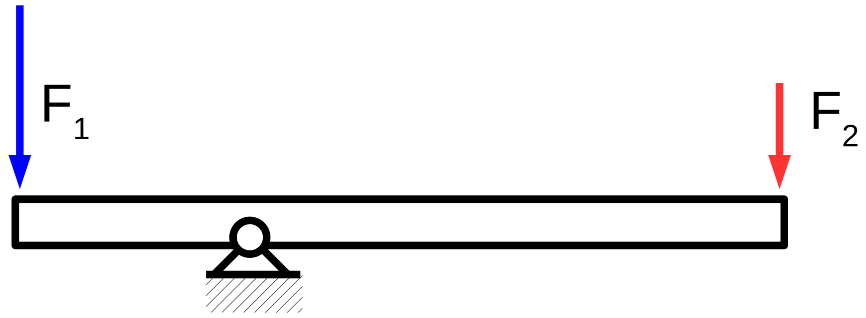
3



OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = v_1(t)\left(1 + \frac{R_2}{R_1}\right)$$

4

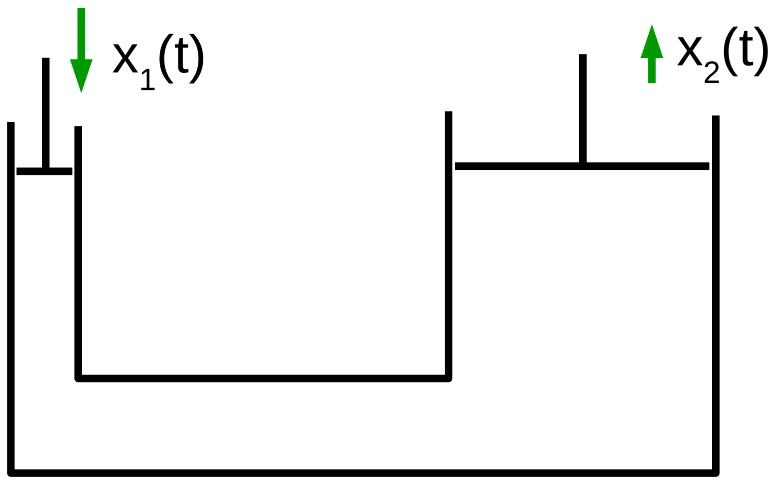


BEAM in steady state:
input – force F_1
output – force F_2

Proportional element

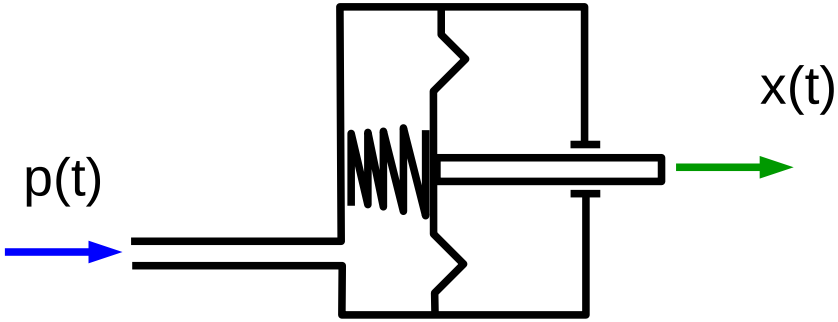
Examples

5



HYDRAULIC LEVER:
input – displacement $x_1(t)$
output – displacement $x_2(t)$

6



PRESSURE ACTUATOR:
input – pressure $p_1(t)$
output – displacement x(t)

Lecture 10

Classification of basic automatic systems with examples.

Classification of basic automatic systems

Element name	Transfer function
proportional	k
first order (inertial)	$\frac{k}{Ts+1}$
integrator	$\frac{k}{s}$
differentiator	ks
differentiator with inertia	$\frac{ks}{Ts+1}$
delay	$e^{-\tau s}$
second order (oscillator)	$\frac{k}{T_1^2 s^2 + T_2 s + 1}$

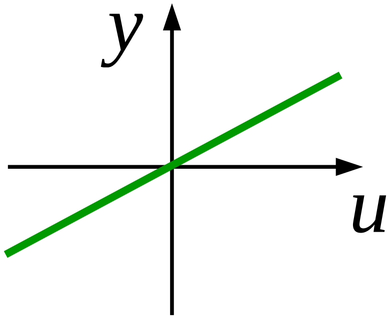
First-order inertial element

1. Element equation: $T\frac{dy(t)}{dt}+y(t)=ku(t)$

$u(t)$ - input

$y(t)$ - output

2. Static characteristic (steady state): $y=ku$ for $\frac{dy}{dt}=0\wedge\frac{du}{dt}=0$



for $k>0$

3. Transfer function: $H(s)=\frac{k}{Ts+1}$

First-order inertial element

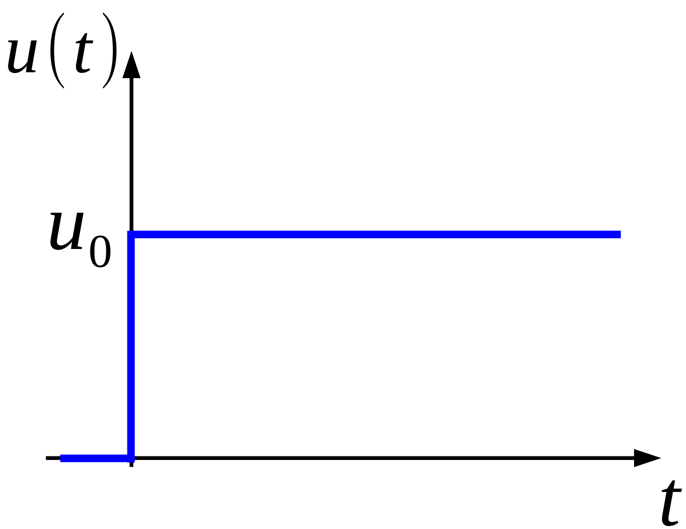
4. Step response:

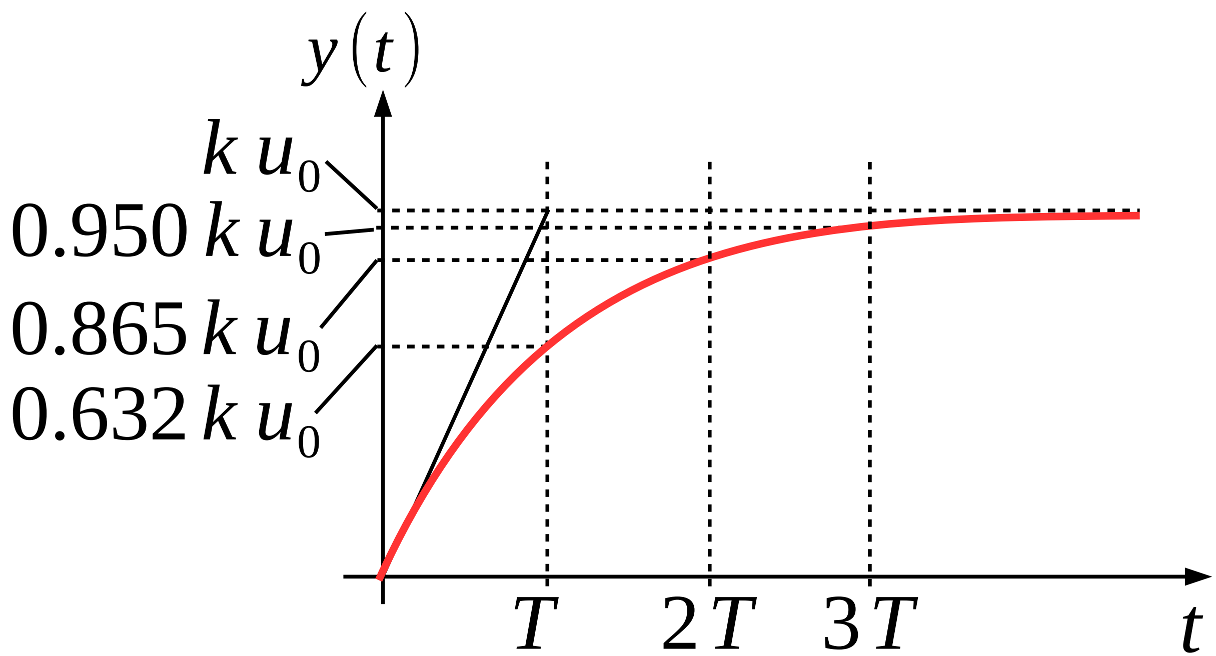
input: $u(t) = u_0 1(t)$

Laplace of input: $U(s) = u_0 \frac{1}{s}$

Laplace of output: $Y(s) = H(s)U(s) = \frac{k u_0}{s(Ts+1)}$

output: $y(t) = L^{-1}\{Y(s)\} = k u_0 (1 - e^{-t/T})$





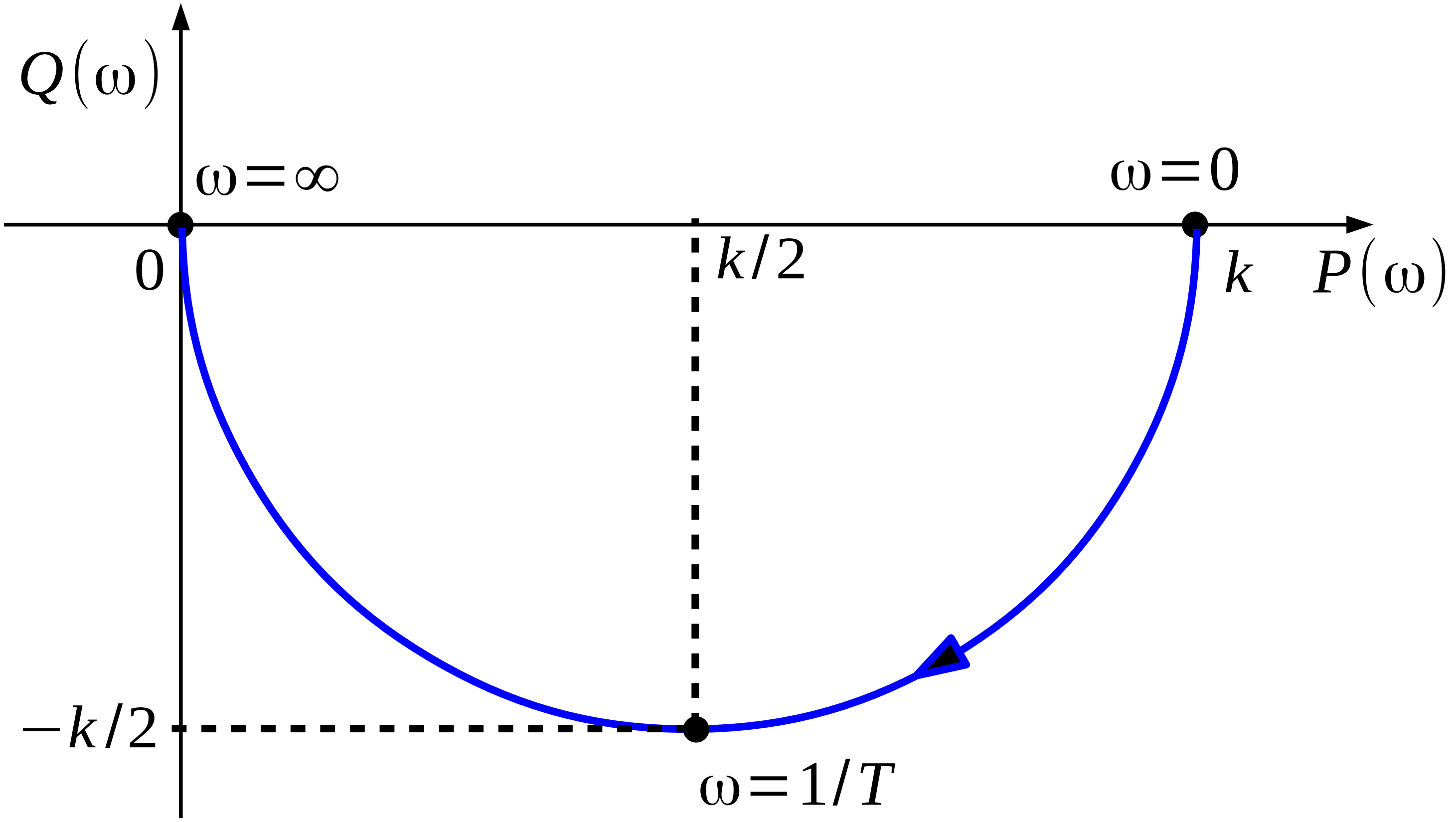
First-order inertial element

5. Frequency response: $H(j\omega)=\dfrac{k}{Tj\omega+1}$

$$P(\omega)=\frac{k}{T^2\omega^2+1},\quad Q(\omega)=\frac{-kT\omega}{T^2\omega^2+1}$$

6. Nyquist plot:

for $k>0$



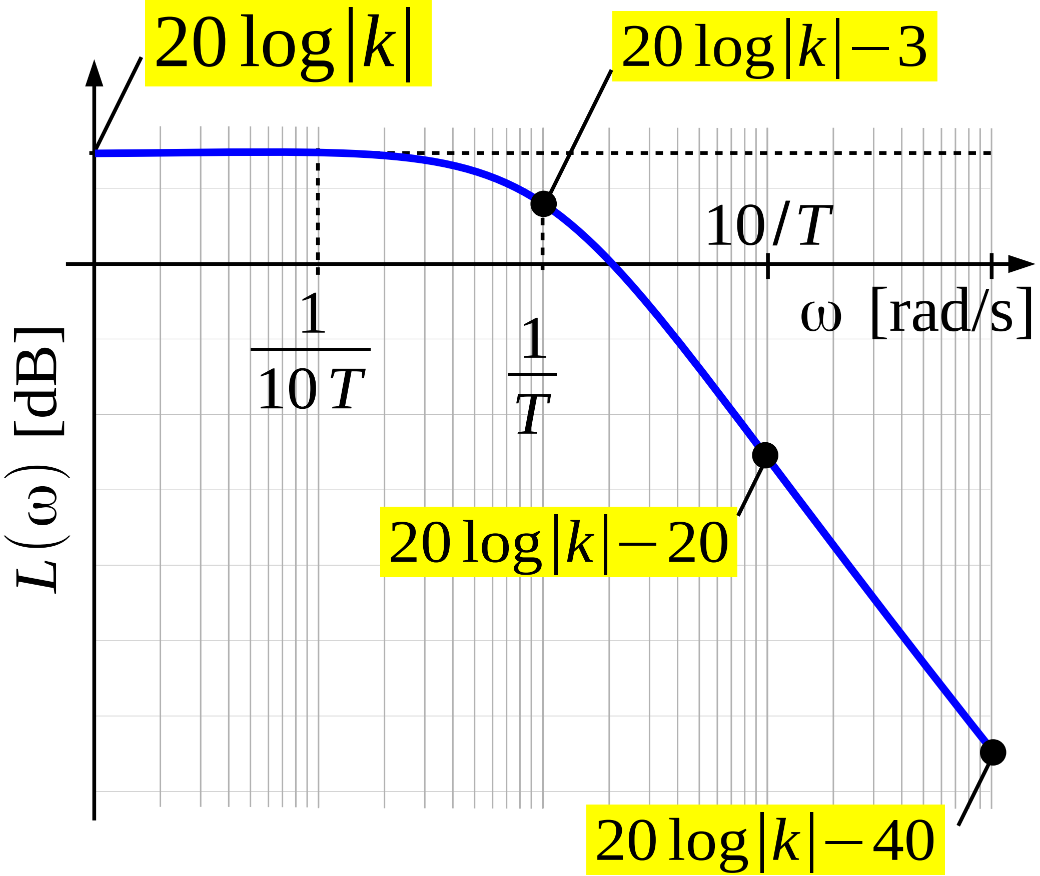
First-order inertial element

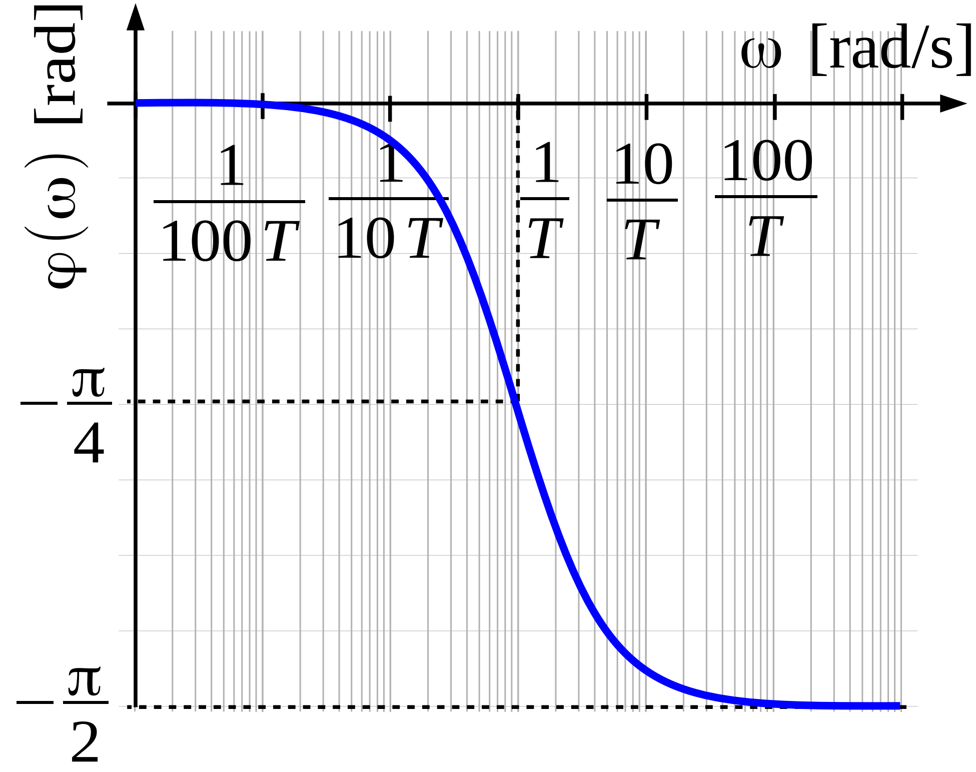
7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k|/\sqrt{T^2\omega^2+1}$

$$L(\omega)=20\log A(\omega)=20\log|k|-20\log\sqrt{T^2\omega^2+1}$$

$$\varphi(\omega)=\arctan\frac{Q}{P}=\arctan(-T\omega)$$

for $k>0$

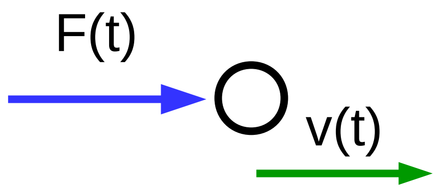




First-order inertial element

Examples

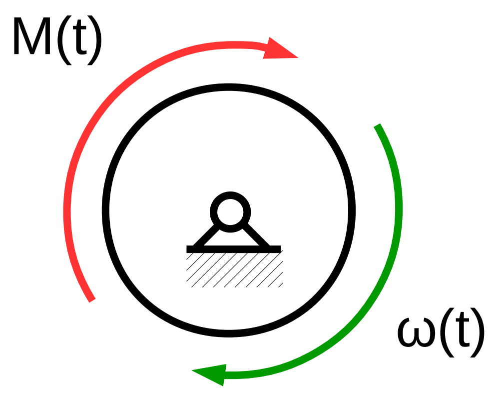
1



LINEAR MOTION OF A MATERIAL POINT WITH LINEAR DAMPING:
input – force F(t)
output – velocity v(t)

example: car is driving on a flat surface with air resistance proportional To its velocity, described using machine equation of motion, with assumption of constant reduced mass.

2

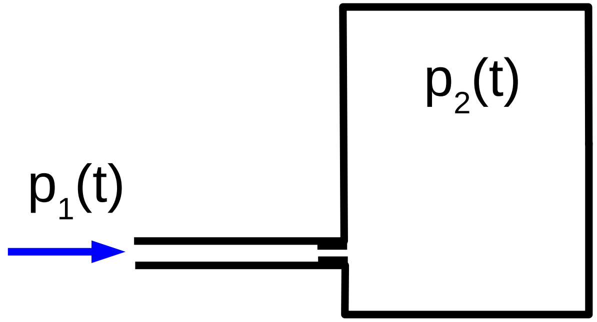


ANGULAR MOTION OF A RIGID BODY WITH LINEAR DAMPING:
input – torque M(t)
output – angular velocity ω(t)

First-order inertial element

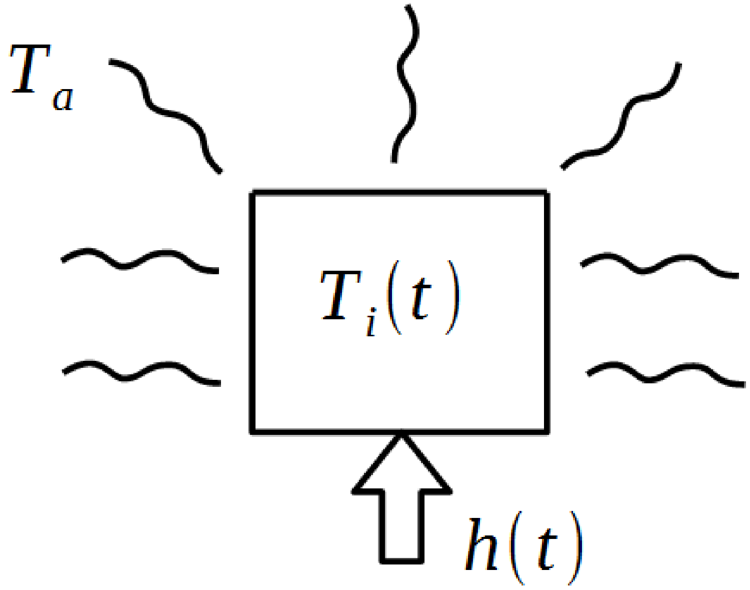
Examples

3



AIR CONTAINER:
input – pressure $p_1(t)$
output – pressure $p_2(t)$

4



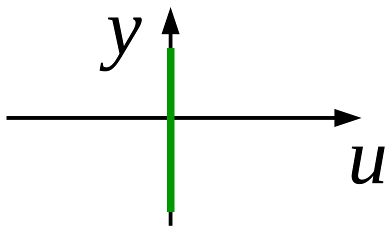
HEATED OBJECT WITH SMALL INERTIA:
input – heater power h(t)
output – object temperature $T_i(t)$

Integrator

1. Element equation: $$\frac{dy(t)}{dt} = k\,u(t)$$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $u = 0$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



3. Transfer function: $$H(s) = \frac{k}{s}$$

Integrator

4. Step response:

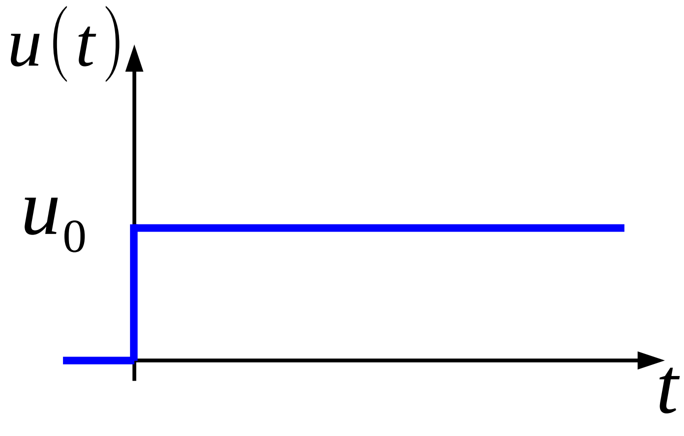
input: $u(t) = u_0 1(t)$

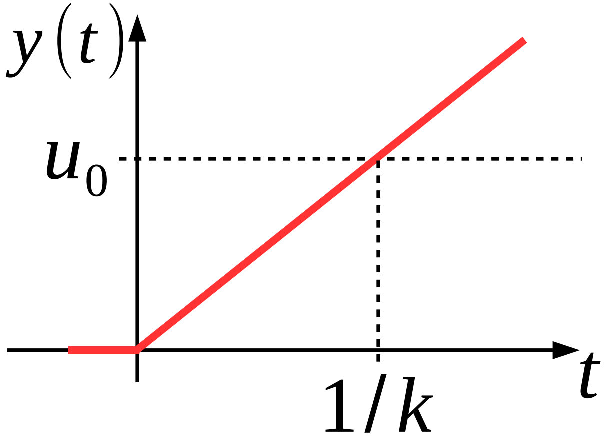
Laplace of input: $U(s) = u_0 \frac{1}{s}$

Laplace of output: $Y(s) = H(s)U(s) = \frac{k u_0}{s^2}$

output: $y(t) = L^{-1}\{Y(s)\} = k u_0 t$

for $k > 0$





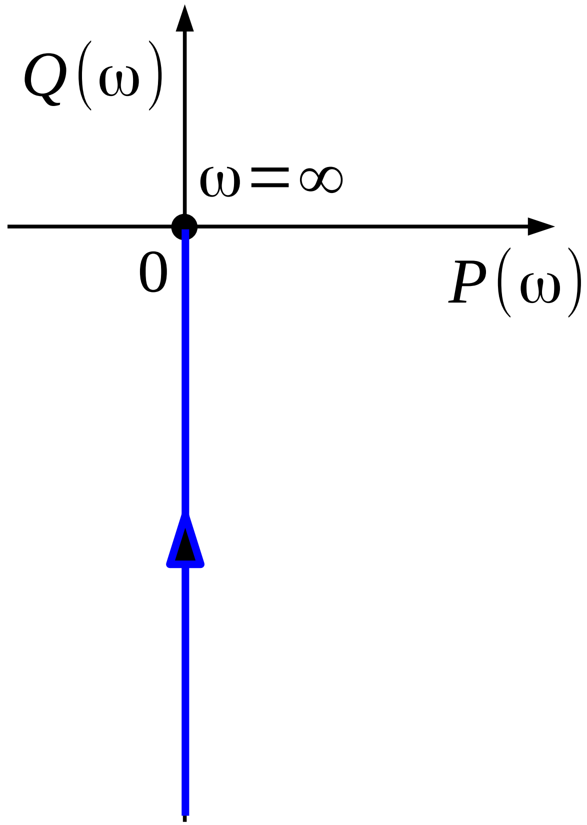
Integrator

5. Frequency response: $H(j\omega)=\dfrac{k}{j\omega}$

$$P(\omega)=0,\quad Q(\omega)=-\frac{k}{\omega}$$

6. Nyquist plot:

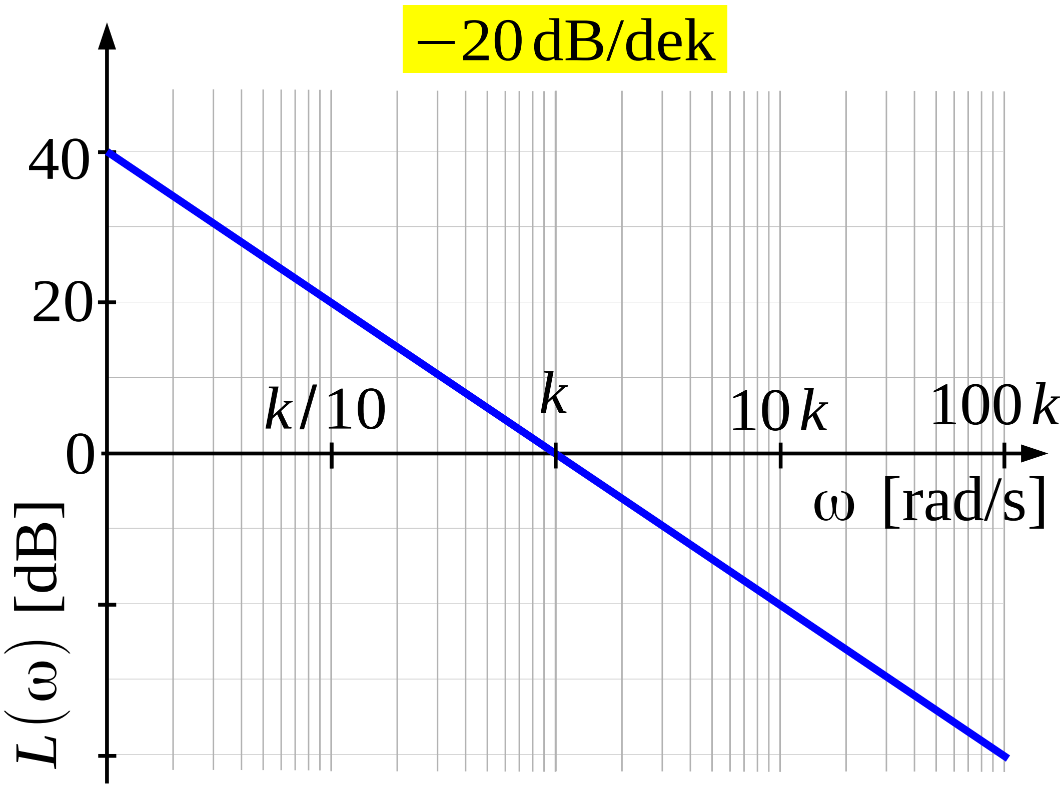
for $k>0$

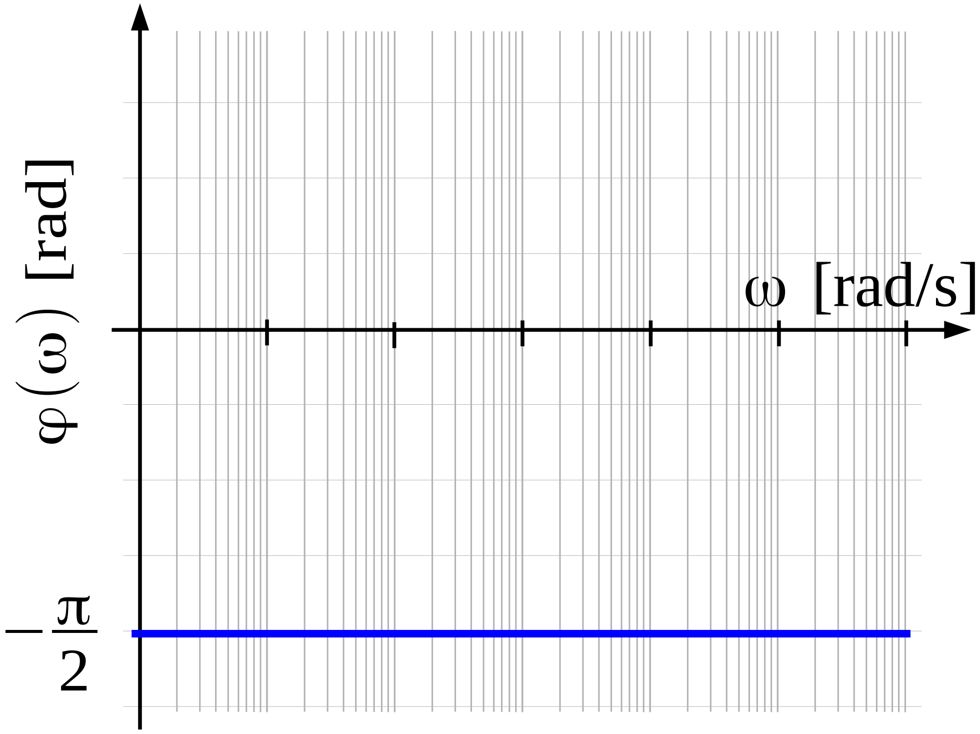


Integrator

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|\frac{k}{\omega}|$ for $k>0$

$$L(\omega)=20\log A(\omega)=20\log|\frac{k}{\omega}| \qquad \varphi(\omega)=\arctan\frac{Q}{P}=\arctan(-\infty)$$

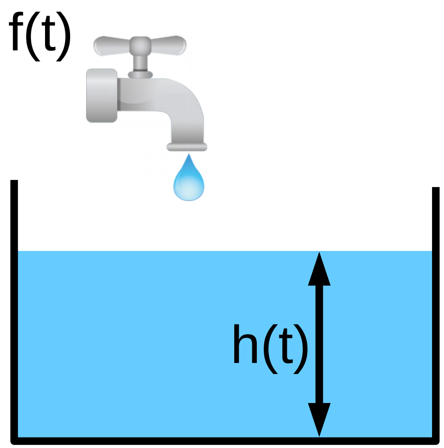




Integrator

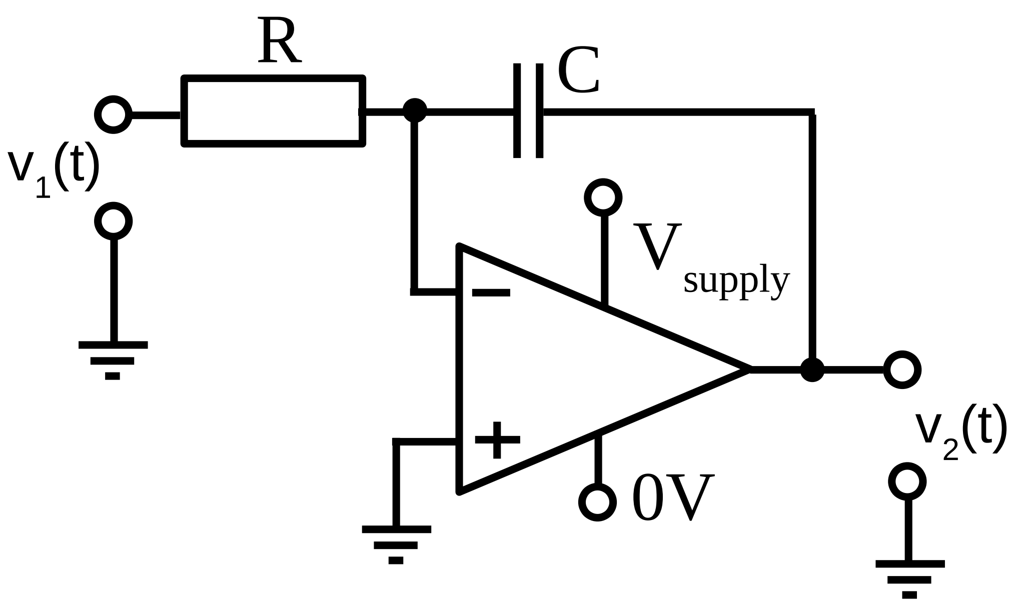
Examples

1



PRISM LIQUID TANK:
input – liquid inflow f(t)
output – liquid level h(t)

2



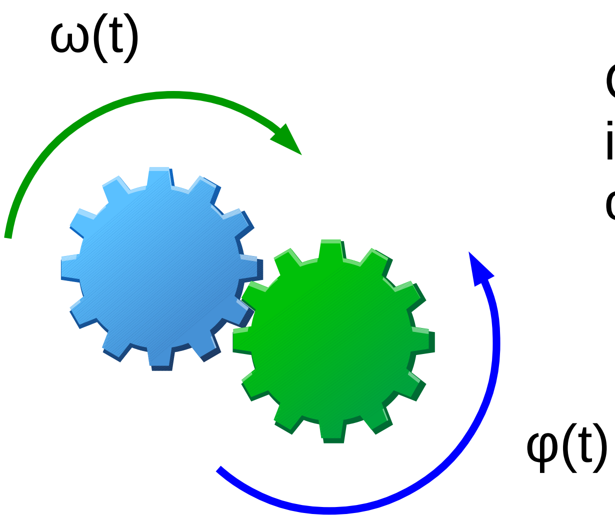
OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

$$v_2(t) = \frac{1}{RC} \int_0^t v_1(t)\, dt$$

Integrator

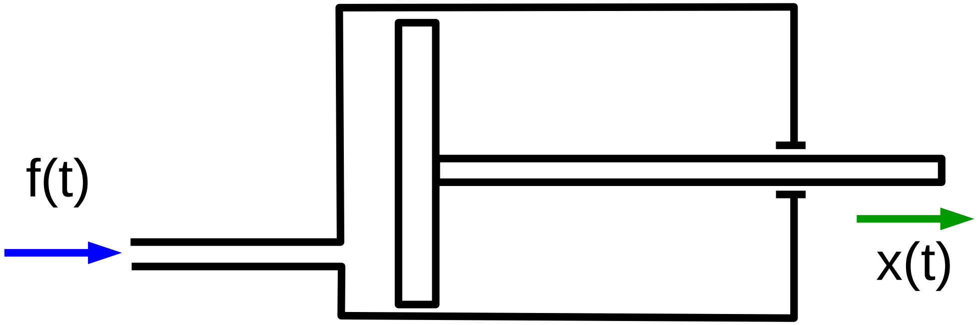
Examples

3



GEARBOX:
input – angular velocity ω(t)
output – rotation angle φ(t)

4



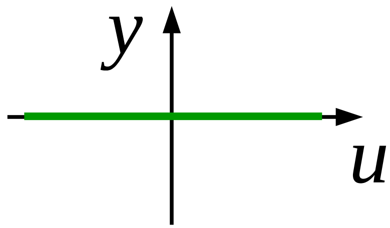
HYDRAULIC CYLINDER:
input – volume inflow f(t)
output – displacement x(t)

Differentiator

1. Element equation: $y(t)=k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $y=0$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$



3. Transfer function: $H(s)=k\,s$

Differentiator

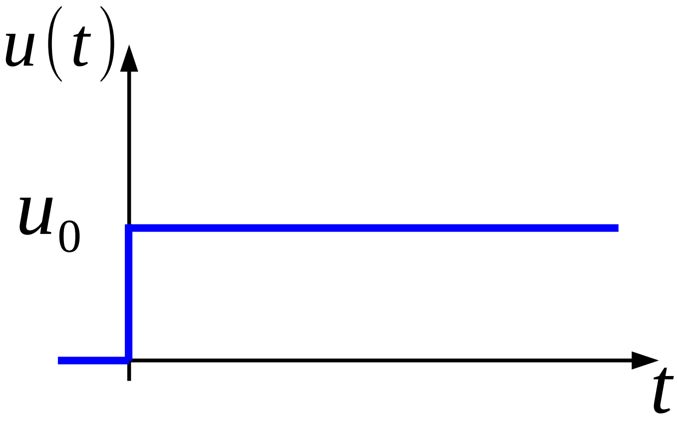
4. Step response:

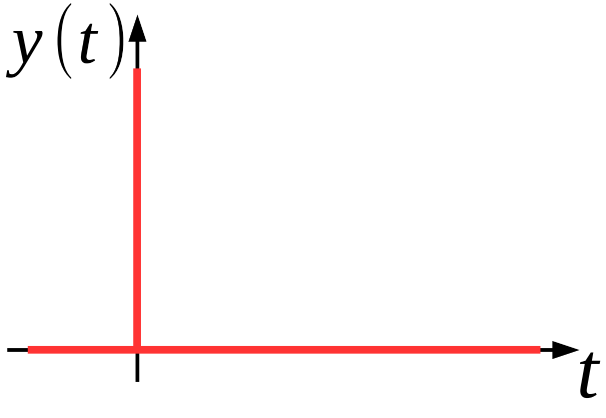
input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0 \frac{1}{s}$

Laplace of output: $Y(s)=H(s)U(s)=k u_0$

output: $y(t)=L^{-1}\{Y(s)\}=k u_0 \delta(t)$





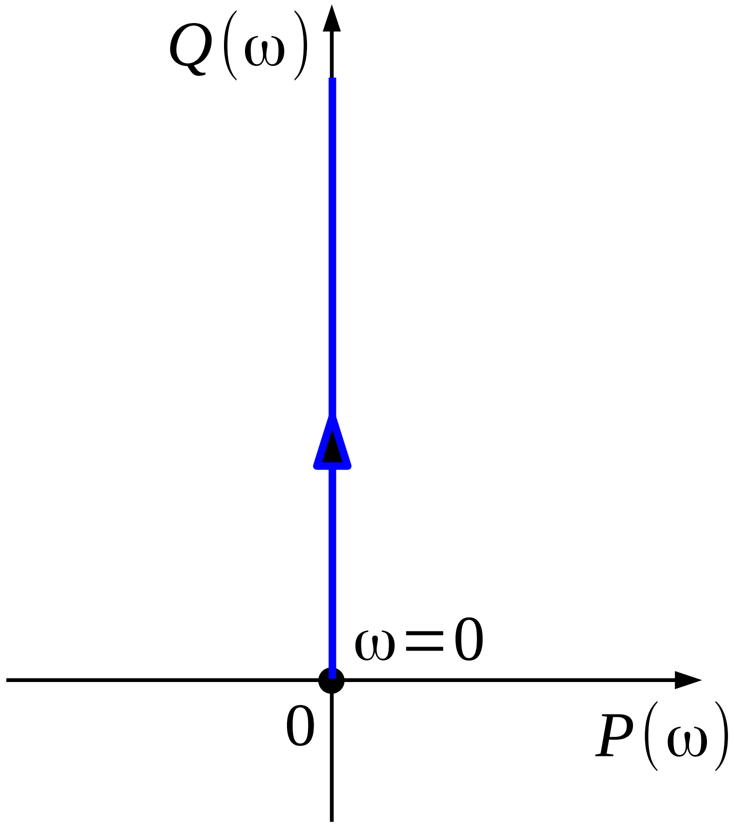
Differentiator

5. Frequency response: $H(j\omega) = jk\omega$

$$P(\omega) = 0, \quad Q(\omega) = k\omega$$

6. Nyquist plot:

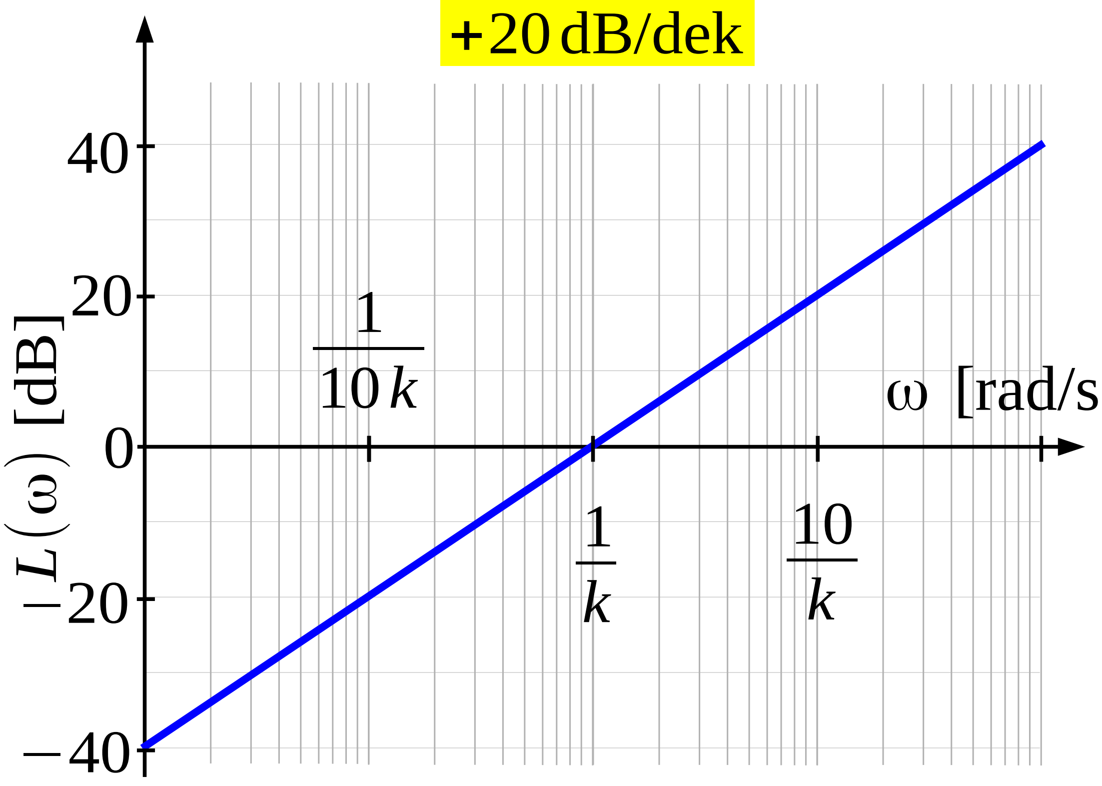
for $k > 0$



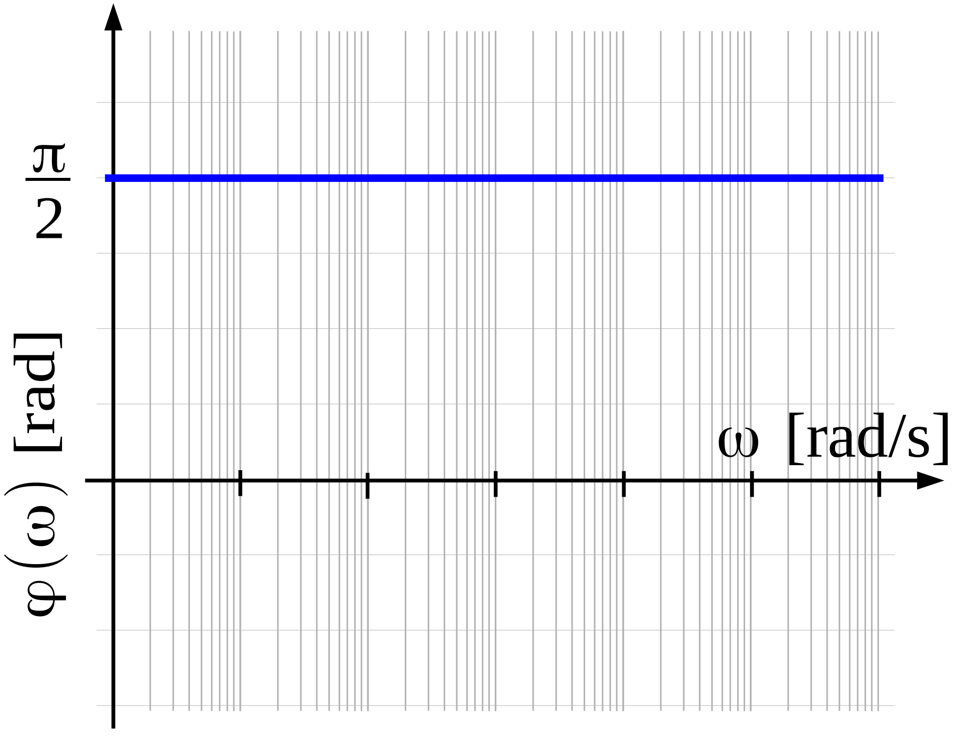
Differentiator

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k\omega|$

$$L(\omega)=20\log A(\omega)=20\log|k\omega| \qquad \varphi(\omega)=\arctan\frac{Q}{P}=\arctan(\infty)$$



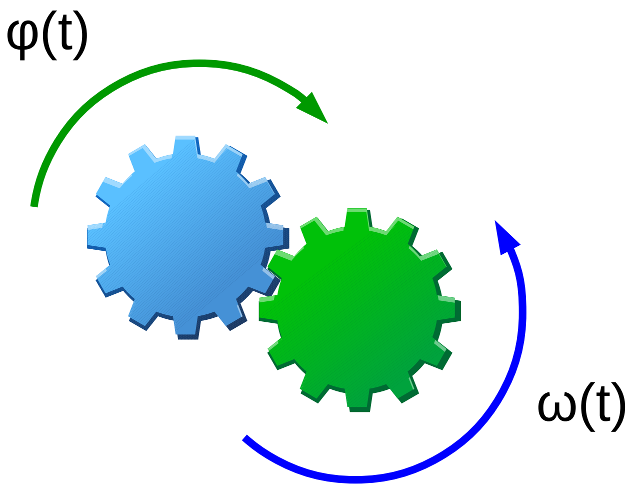
for $k>0$



Differentiator

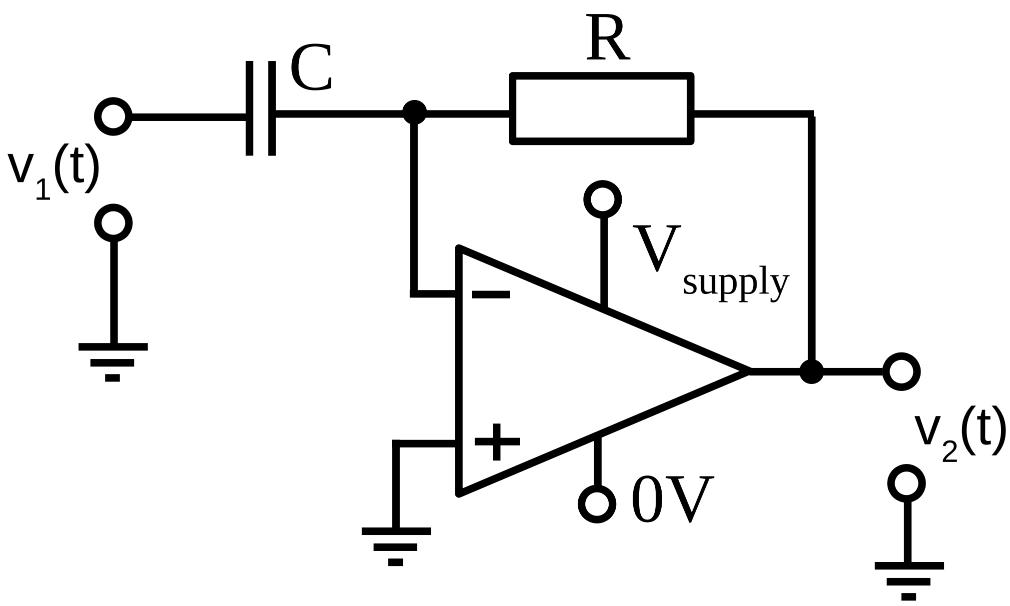
Examples

1



GEARBOX:
input – rotation angle φ(t)
output – angular velocity ω(t)

2



OPERATIONAL AMPLIFIER:
input – voltage $v_1(t)$
output – voltage $v_2(t)$

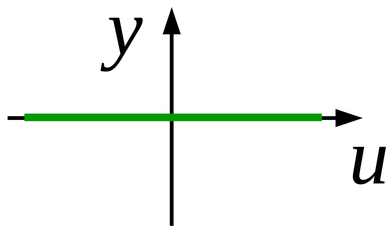
$$v_2(t) = -RC\frac{dv_1(t)}{dt}$$

Real differentiator (derivative+1st order)

1. Element equation: $T\frac{dy(t)}{dt}+y(t)=k\frac{du(t)}{dt}$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $y=0$ for $\frac{dy}{dt}=0\wedge\frac{du}{dt}=0$



3. Transfer function: $H(s)=\frac{k\,s}{Ts+1}$

Real differentiator (derivative+1st order)

4. Step response:

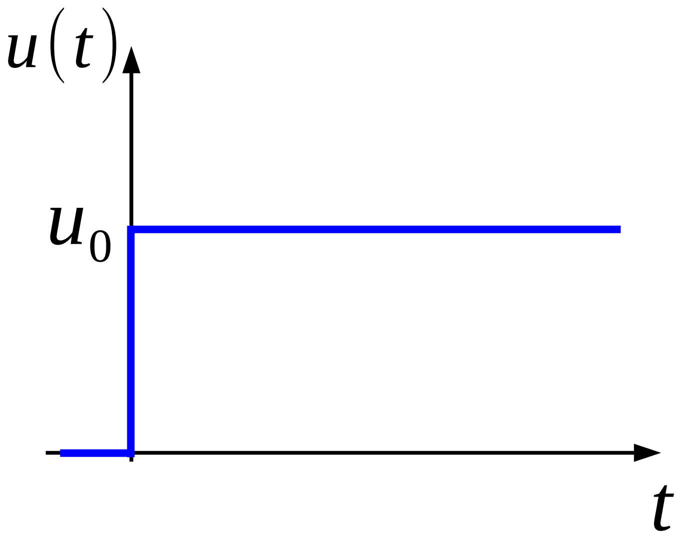
input: $u(t)=u_0 1(t)$

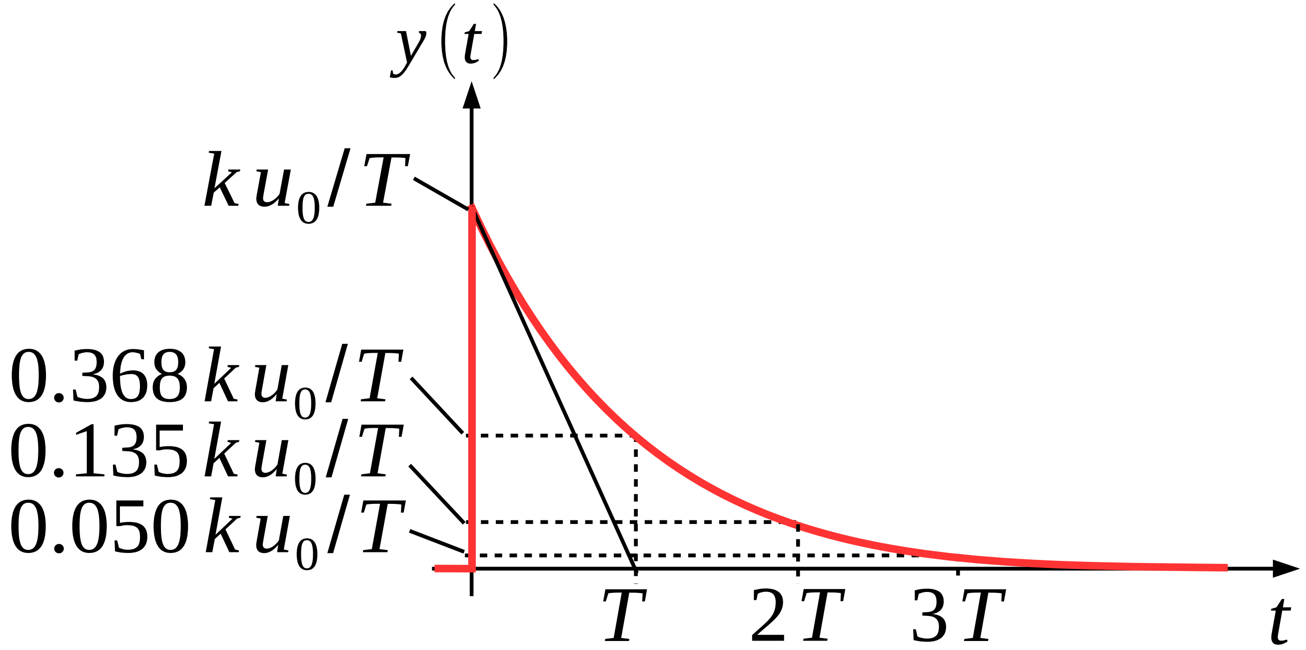
Laplace of input: $U(s)=u_0 \frac{1}{s}$

Laplace of output: $Y(s)=H(s)U(s)=\frac{k u_0}{Ts+1}$

output: $y(t)=L^{-1}\{Y(s)\}=\frac{k u_0}{T} e^{-t/T}$

for $k>0$





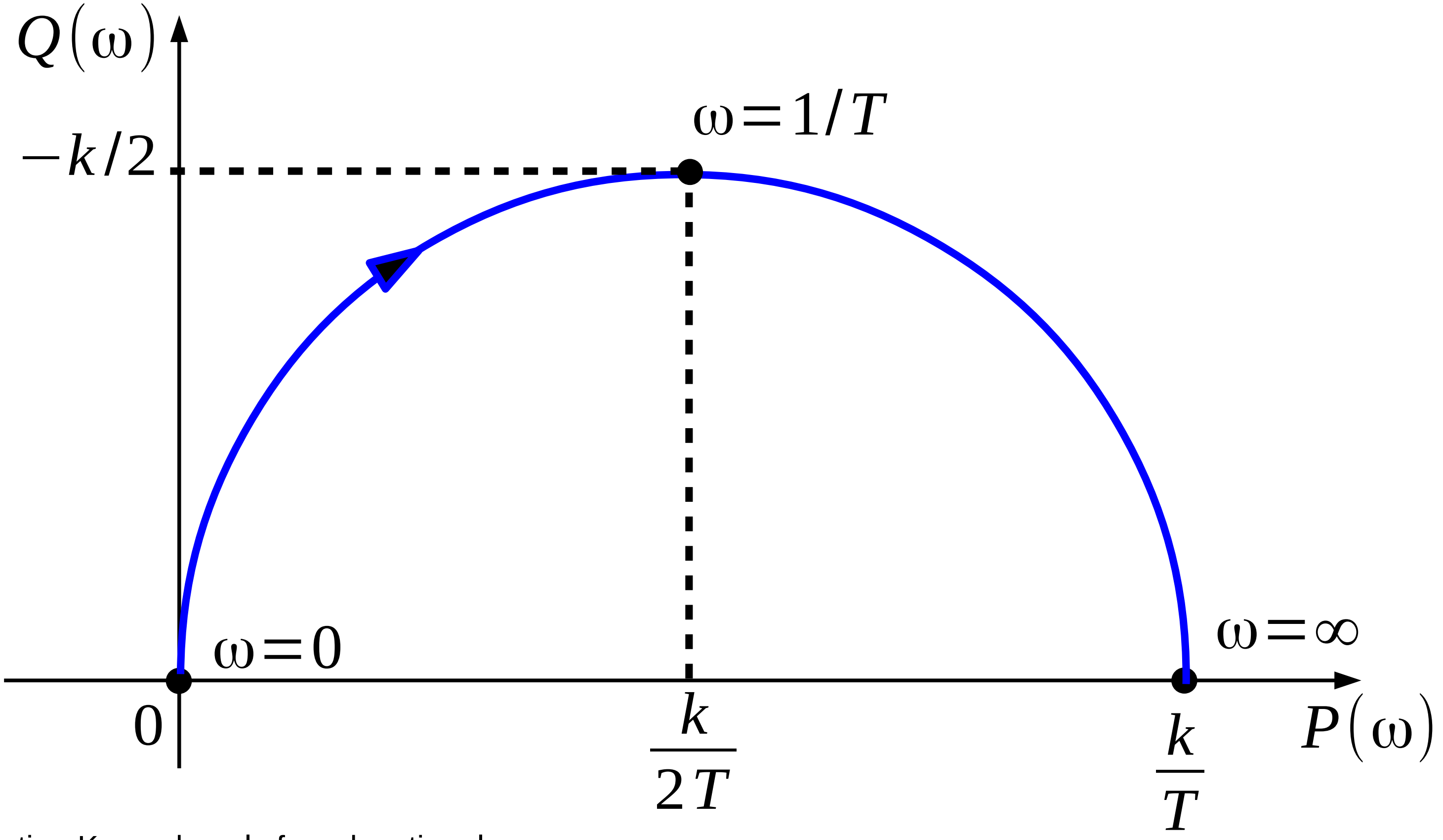
Real differentiator (derivative+1st order)

5. Frequency response: $H(j\omega)=\dfrac{k\,j\omega}{Tj\omega+1}$

$$P(\omega)=\frac{kT\omega^2}{T^2\omega^2+1},\quad Q(\omega)=\frac{k\omega}{T^2\omega^2+1}$$

6. Nyquist plot:

for $k>0$

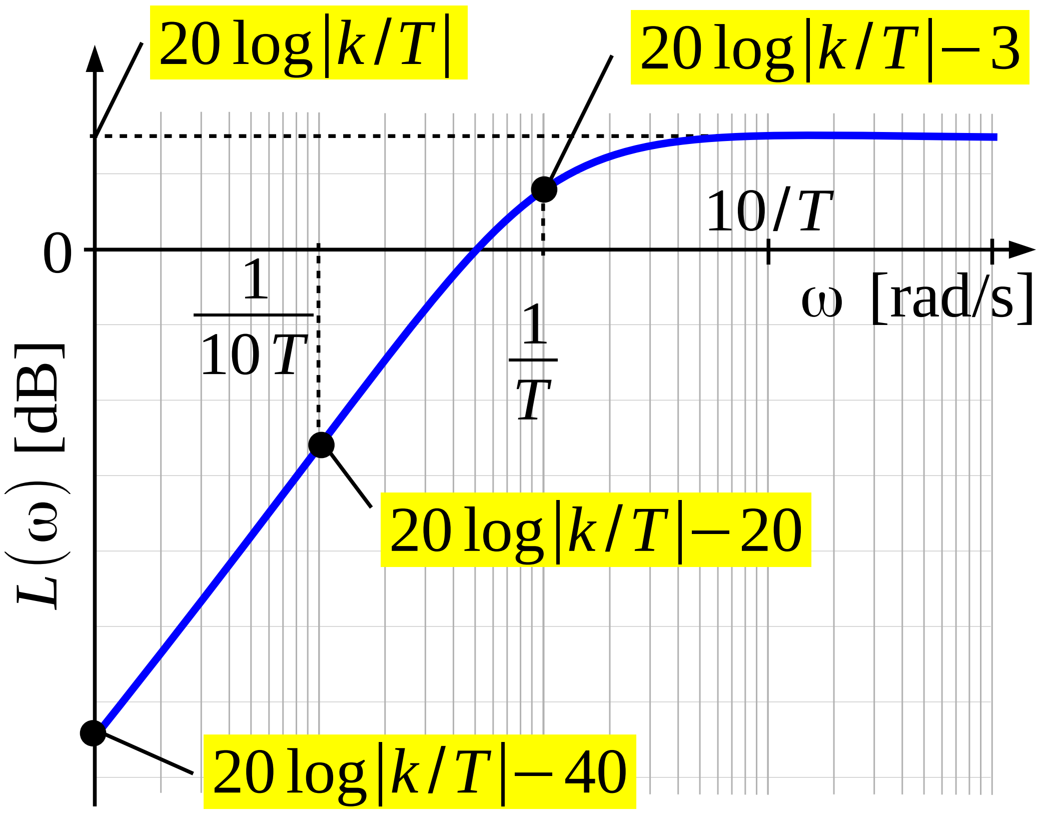


Real differentiator (derivative+1st order)

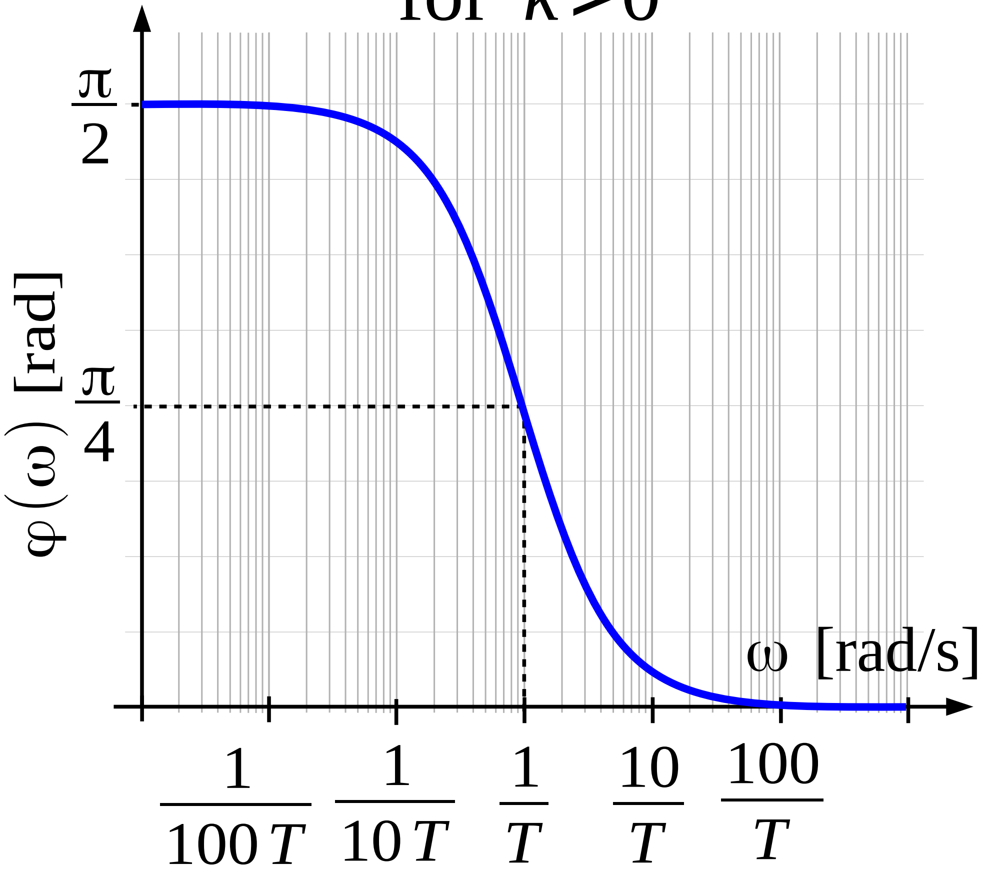
7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=|k\omega|/\sqrt{T^2\omega^2+1}$

$$L(\omega)=20\log A(\omega)=20\log|k\omega|-20\log\sqrt{T^2\omega^2+1}$$

$$\varphi(\omega)=\arctan\frac{Q}{P}=\arctan\left(\frac{1}{T\omega}\right)$$



for $k>0$

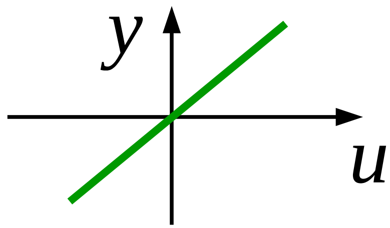


Delay

1. Element equation: $y(t)=u(t-\tau)$

$u(t)$ - input
$y(t)$ - output

2. Static characteristic (steady state): $y=u$ for $\frac{dy}{dt}=0 \wedge \frac{du}{dt}=0$



3. Transfer function: $H(s)=e^{-\tau s}$

Delay

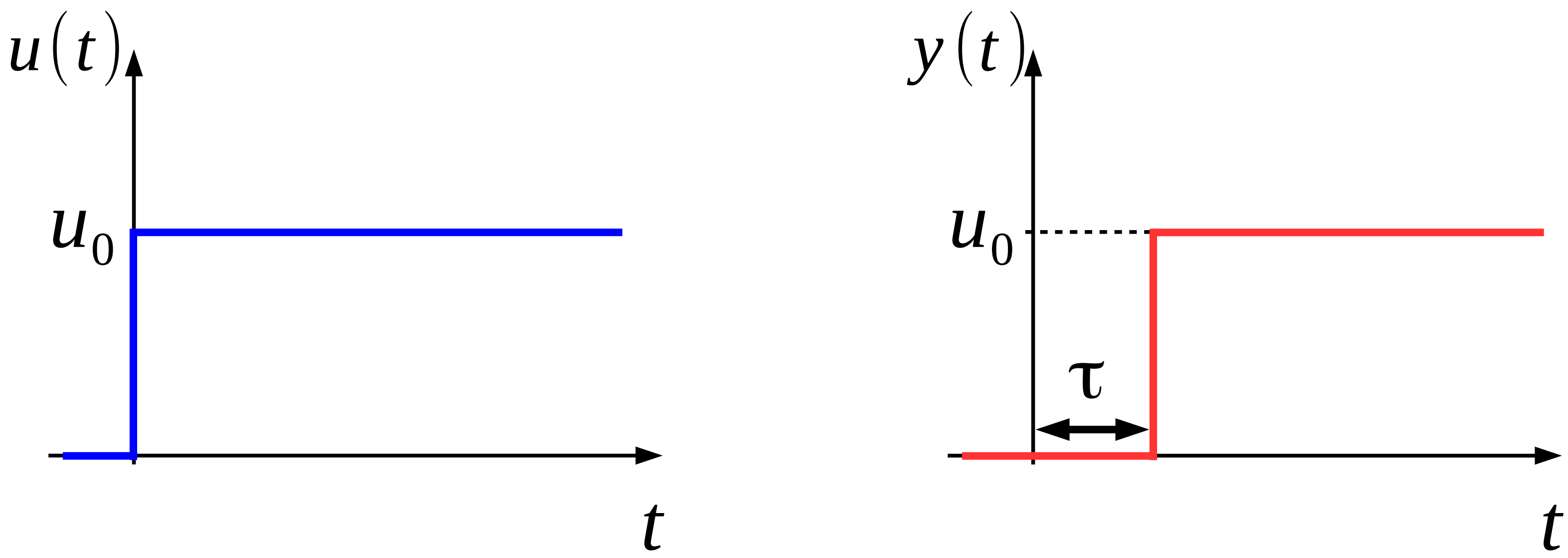
4. Step response:

input: $u(t) = u_0 1(t)$

Laplace of input: $U(s) = u_0 \frac{1}{s}$

Laplace of output: $Y(s) = H(s)U(s) = \frac{u_0}{s} e^{-\tau s}$

output: $y(t) = L^{-1}\{Y(s)\} = u_0 1(t - \tau)$



Delay

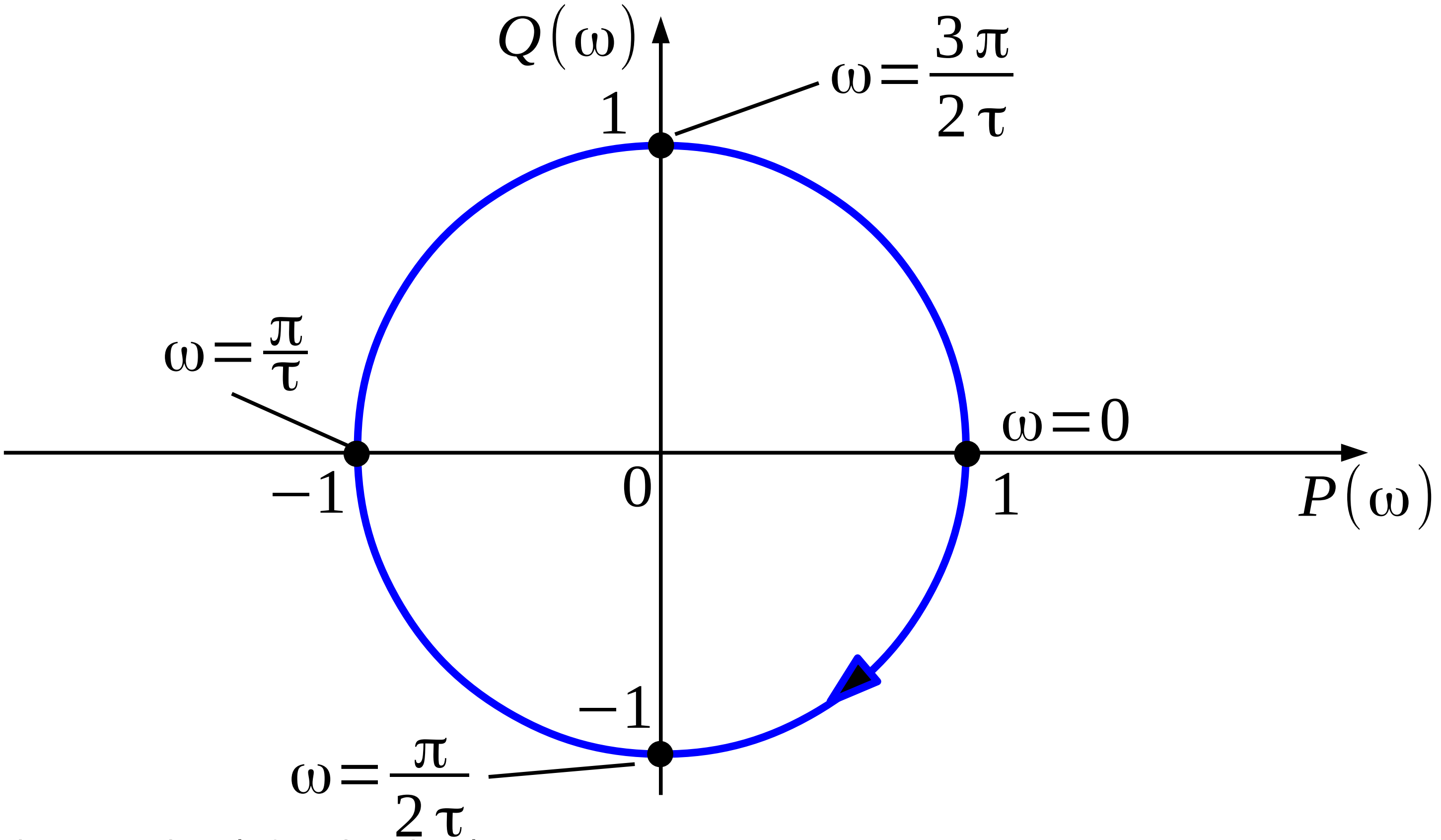
5. Frequency response: $H(j\omega) = e^{-\tau j\omega}$

$$e^{-jx} = \cos x - j\sin x$$

$$P(\omega) = \cos(\tau\omega), \quad Q(\omega) = -\sin(\tau\omega)$$

6. Nyquist plot:

for $k > 0$

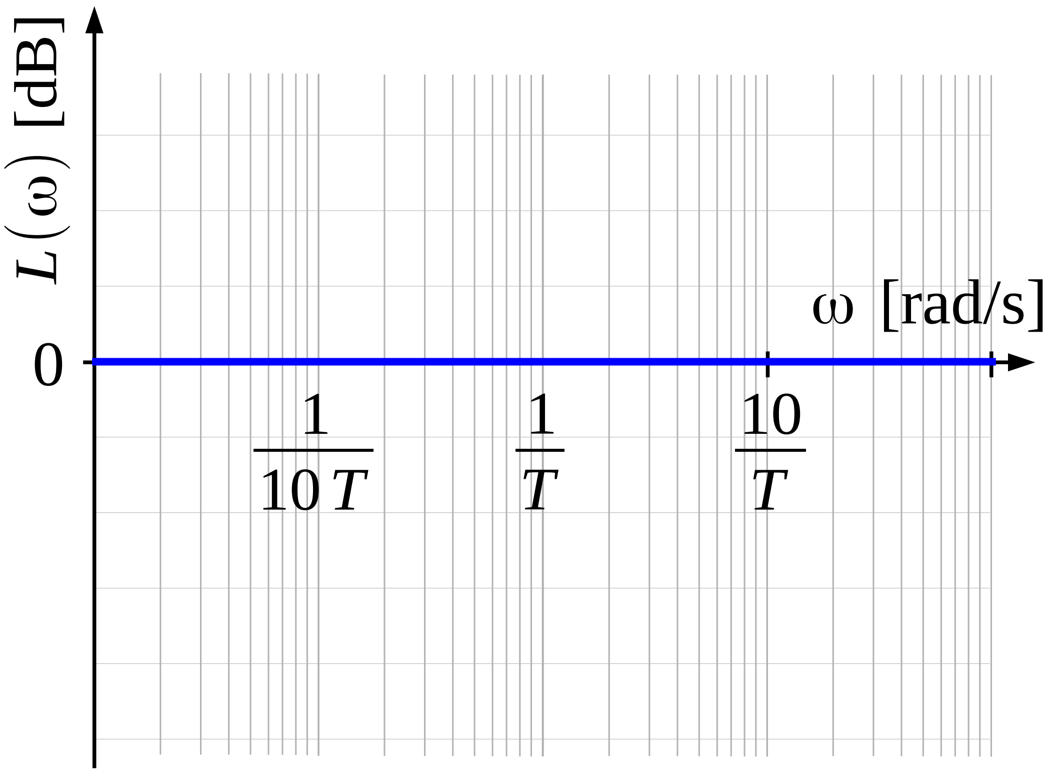


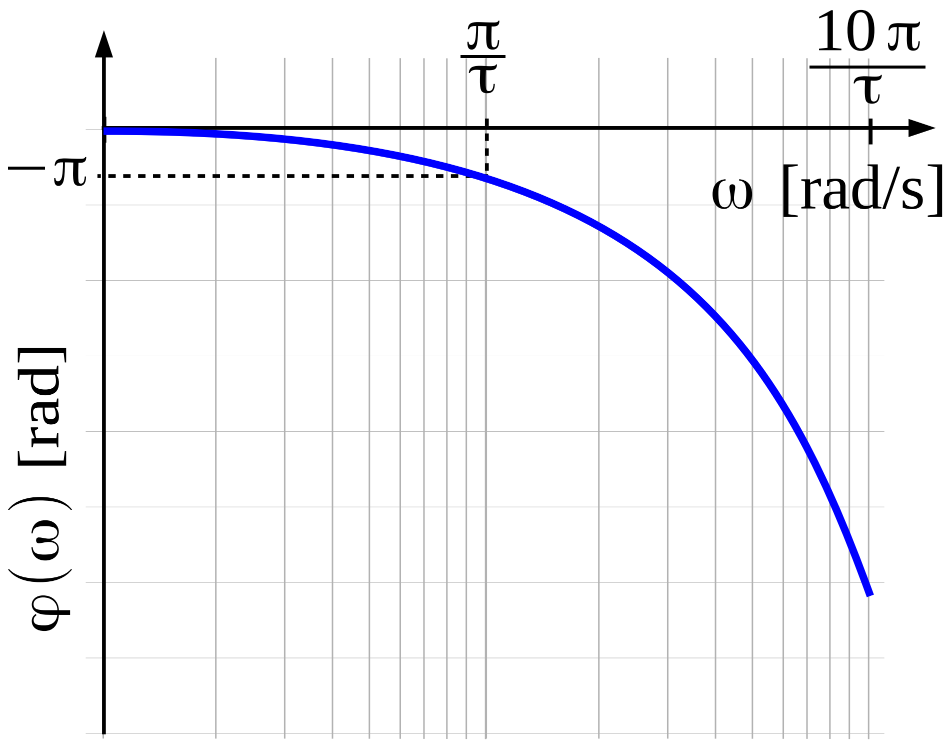
Delay

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}=1$

$$L(\omega)=20\log A(\omega)=20\log 1=0$$

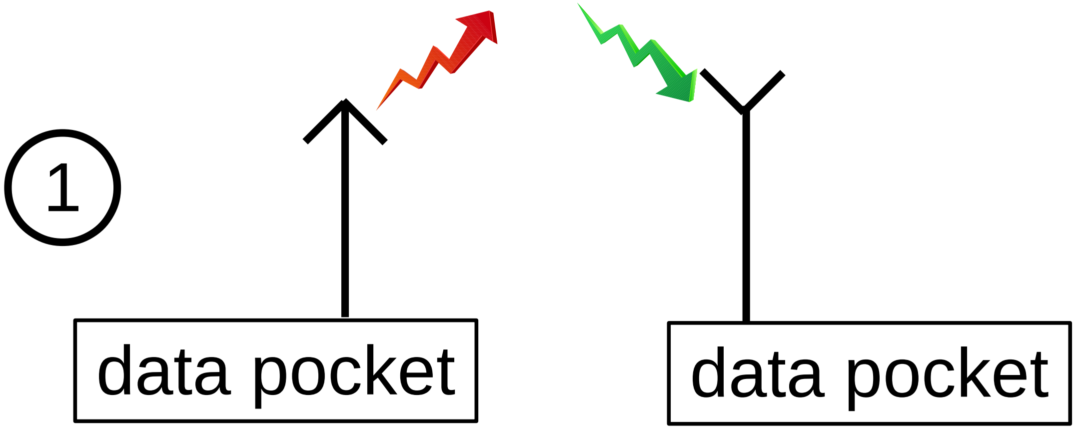
$$\varphi(\omega)=\arctan\frac{Q}{P}=\arctan\left(-\tan(\tau\omega)\right)=-\tau\omega$$





Delay

Examples

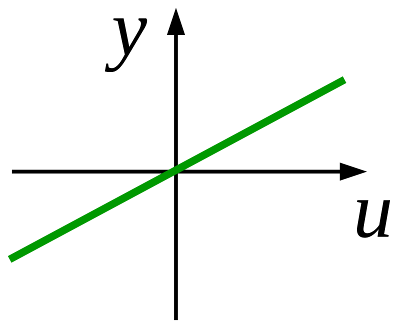


WIRELESS TRANSMISSION:
input – sent data
output – received data

Second-order inertial element

1. Element equation: $$T_1^2 \frac{d^2 y(t)}{dt^2} + T_2 \frac{dy(t)}{dt} + y(t) = k\, u(t)$$

2. Static characteristic (steady state): $y = ku$ for $\frac{dy}{dt} = 0 \wedge \frac{du}{dt} = 0$



3. Transfer function: $$H(s) = \frac{k}{T_1^2 s^2 + T_2 s + 1}$$

Second-order inertial element

4. Step response:

input: $u(t)=u_0 1(t)$

Laplace of input: $U(s)=u_0\frac{1}{s}$

Laplace of output: $Y(s)=H(s)U(s)=\frac{k u_0}{s\left(T_1^2 s^2+T_2 s+1\right)}$

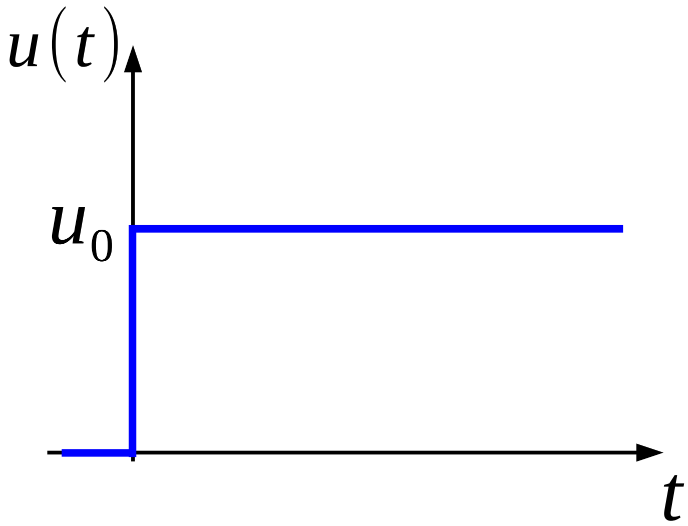
output: $y(t)=L^{-1}\{Y(s)\}=$

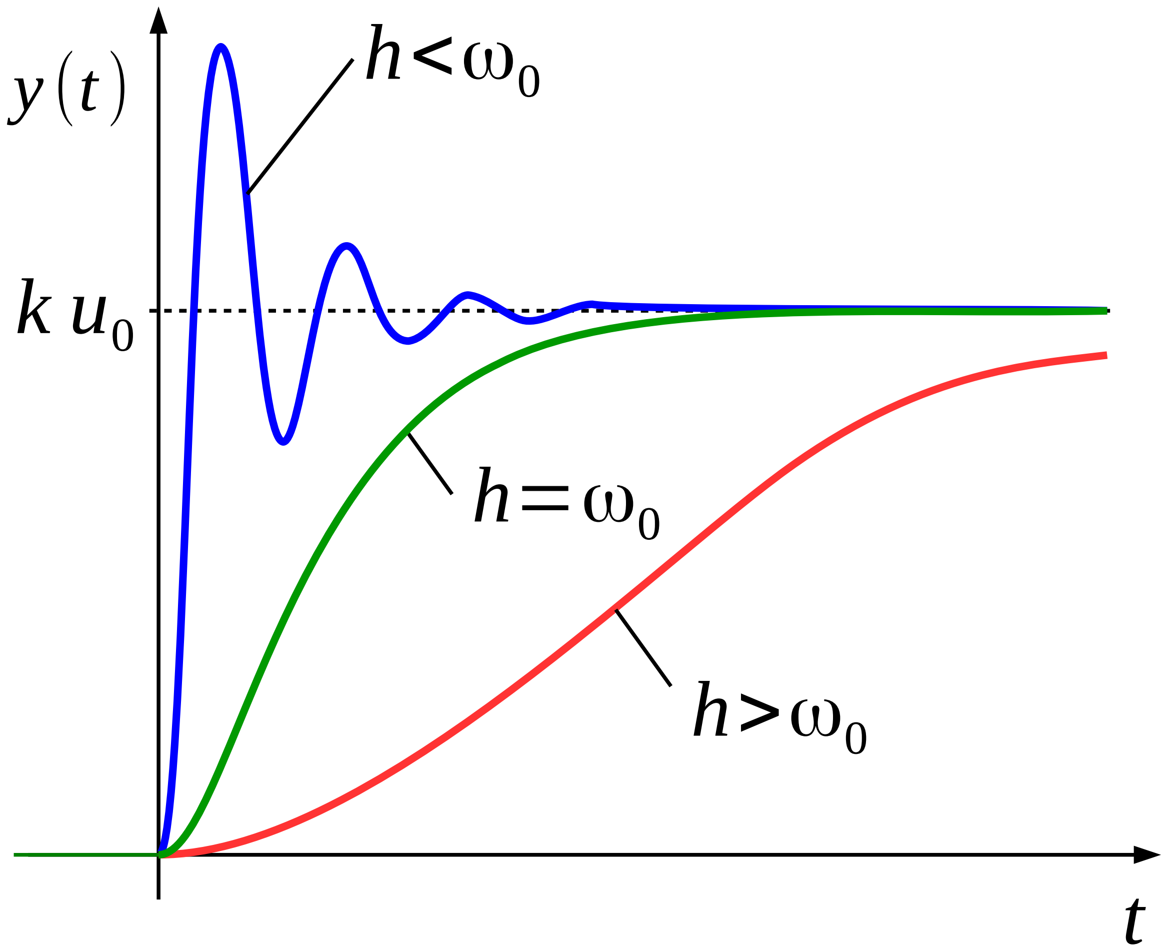
$$=\begin{cases} k u_0 \omega_0^2\left(1-e^{-ht}\left(\cos\omega t+\frac{h}{\omega}\sin\omega t\right)\right), \text{ for } h\le\omega_0 \\ k u_0 \omega_0^2\left(1+e^{-ht}\left(\left(\frac{h+w}{2w}-1\right)e^{-wt}-\frac{h+w}{2w}e^{wt}\right)\right), \text{ for } h\ge\omega_0 \end{cases}$$

where: $h=\frac{T_2}{2T_1^2}$, $\omega_0=\frac{1}{T_1}$, $\omega=\sqrt{\omega_0^2-h^2}$, $w=\sqrt{h^2-\omega_0^2}$

Second-order inertial element

4. Step response:





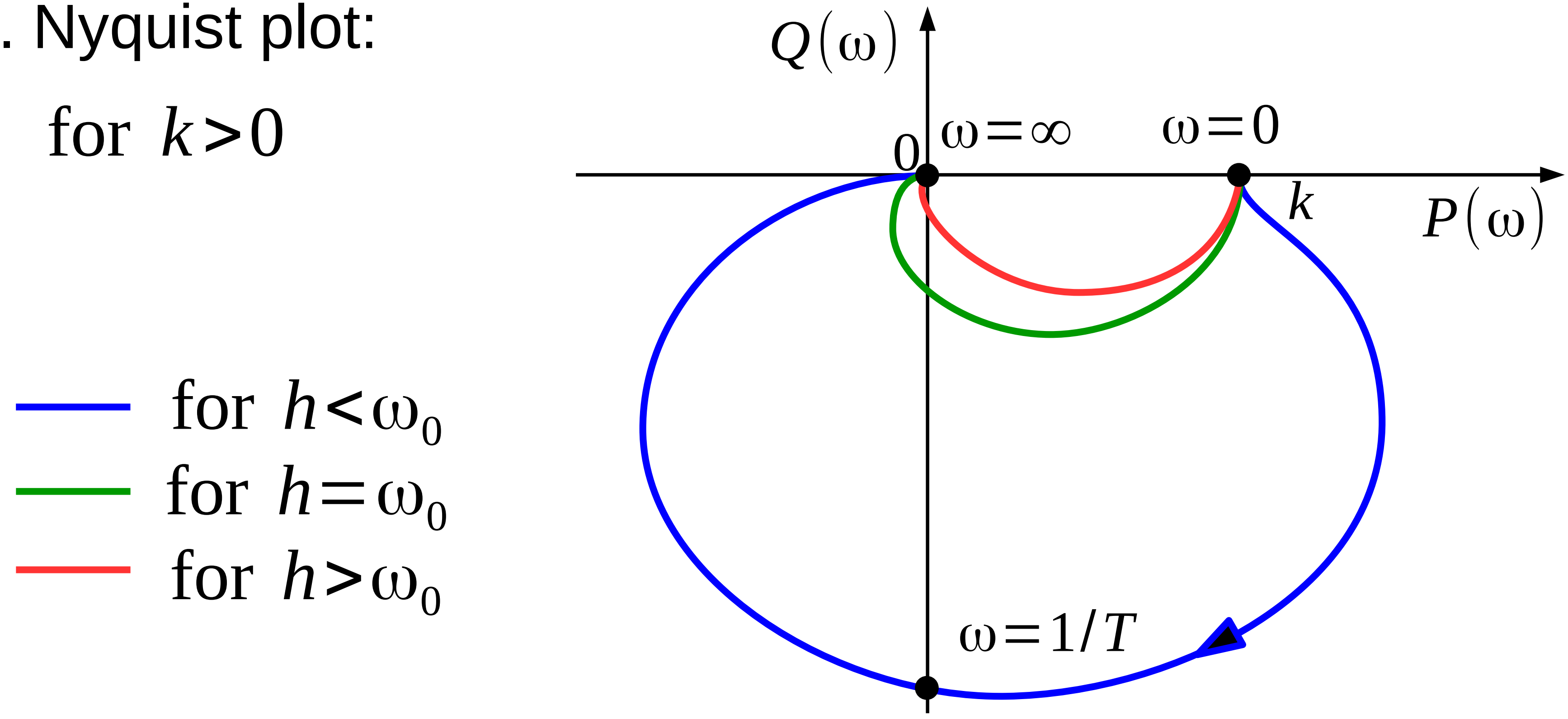
Second-order inertial element

5. Frequency response: $H(j\omega)=\dfrac{k}{-T_1^2\omega^2+T_2 j\omega+1}$

$$P(\omega)=\frac{k(1-T_1^2\omega^2)}{(1-T_1^2\omega^2)^2+T_2^2\omega^2},\quad Q(\omega)=\frac{-kT_2\omega}{(1-T_1^2\omega^2)^2+T_2^2\omega^2}$$

6. Nyquist plot:

for $k>0$

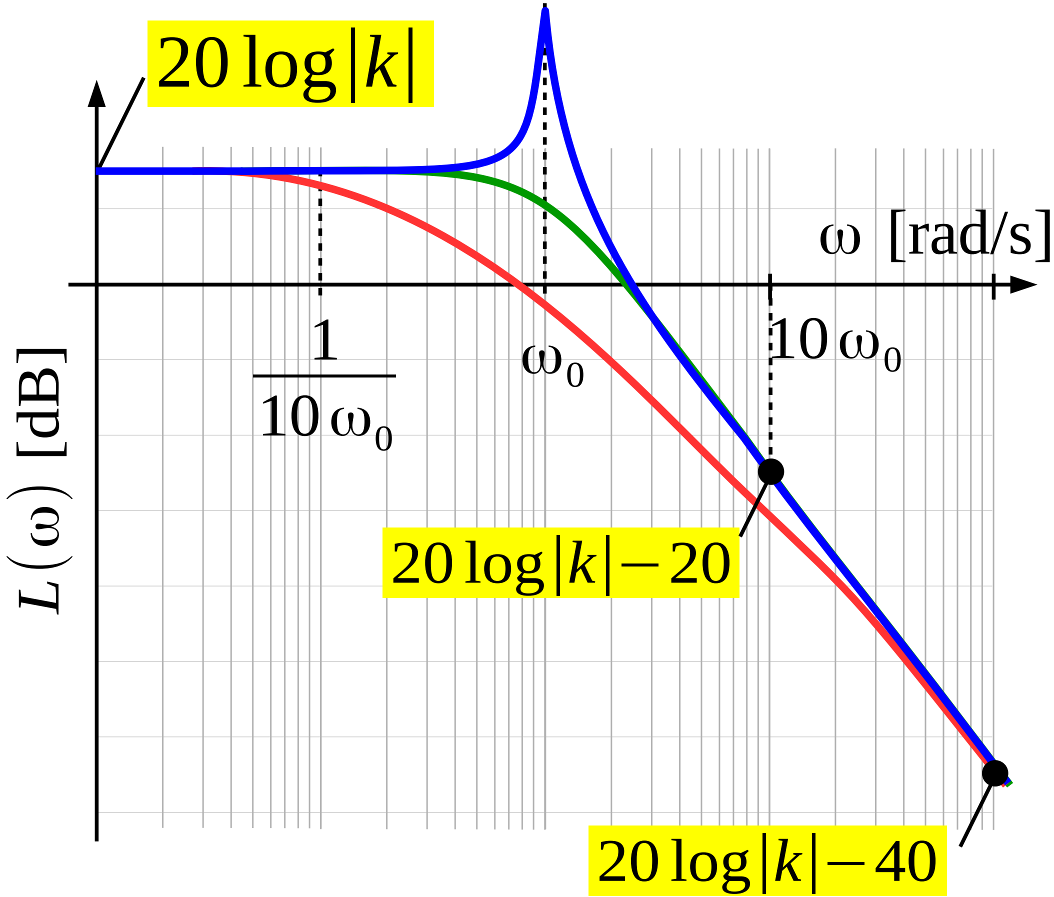


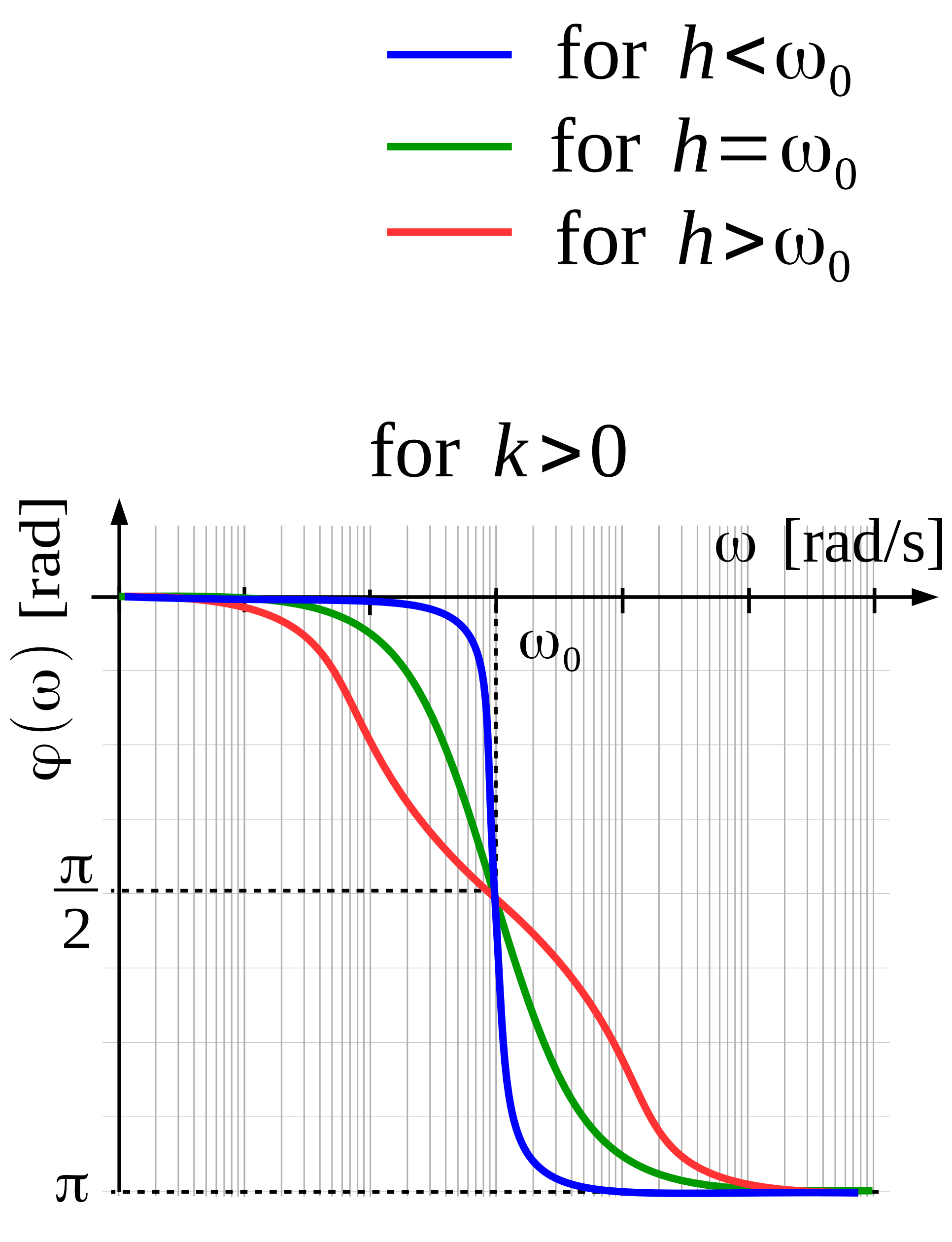
Second-order inertial element

7. Bode plot: $A(\omega)=\sqrt{P^2+Q^2}$

$$L(\omega)=20\log A(\omega)$$

$$\varphi(\omega)=\arctan\frac{Q}{P}$$

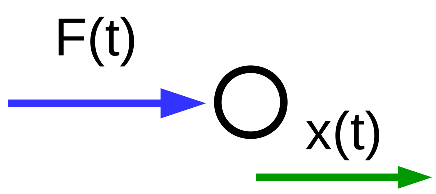




Second-order inertial element

Examples

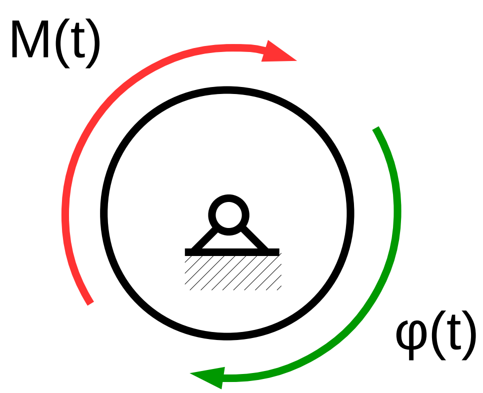
2



LINEAR MOTION OF A MATERIAL POINT WITH LINEAR DAMPING:
input – force F(t)
output – displacement x(t)

example: car driving on a flat surface with air resistance proportional to velocity, described using machine equation of motion, with assumption of constant reduced mass.

3

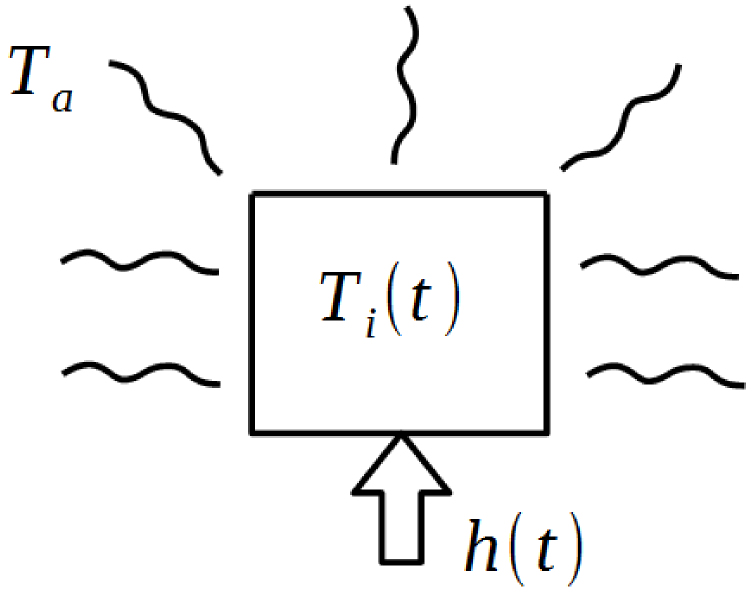


ANGULAR MOTION OF A RIGID BODY WITH LINEAR DAMPING:
input – torque M(t)
output – angle φ(t)

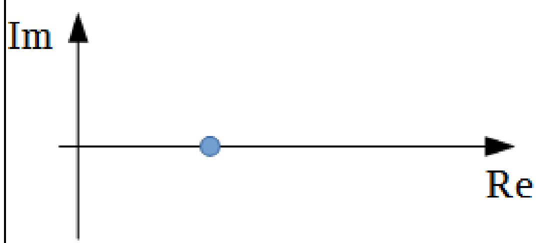
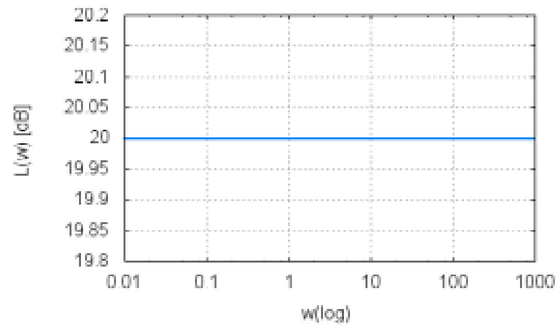
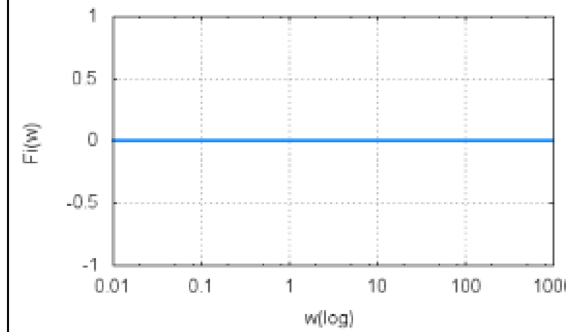
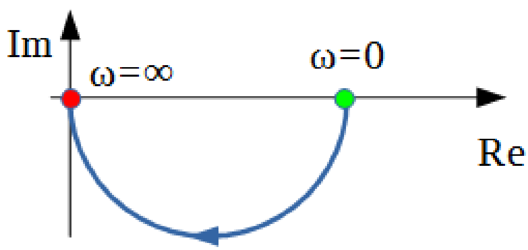
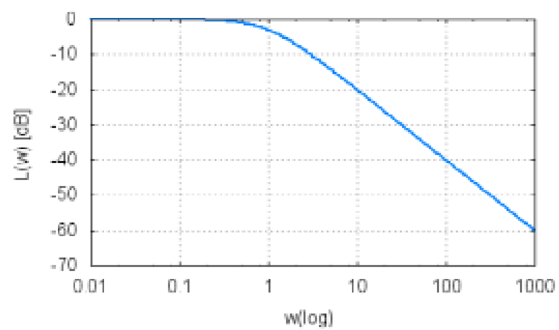
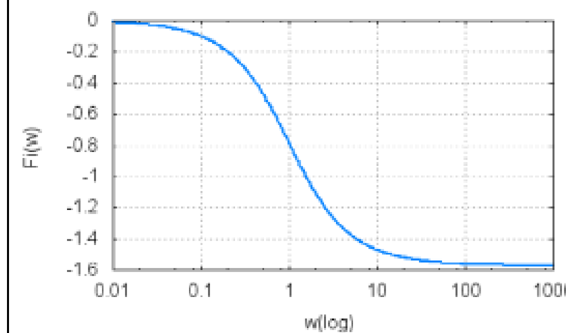
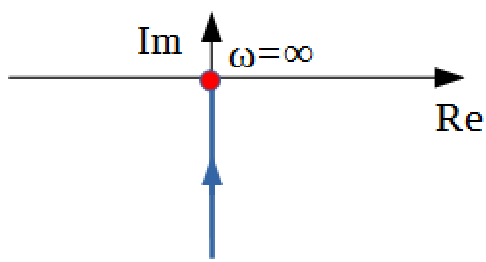
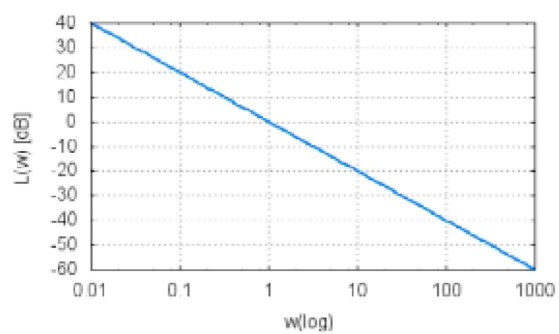
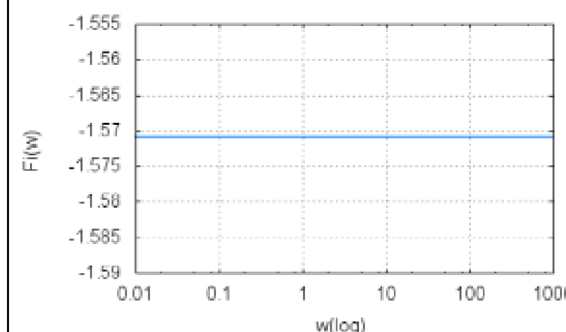
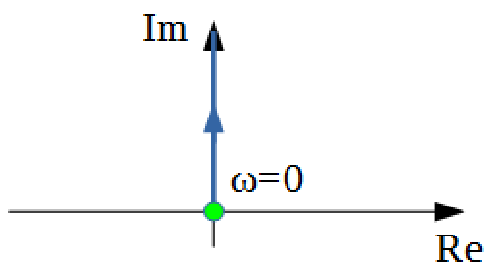
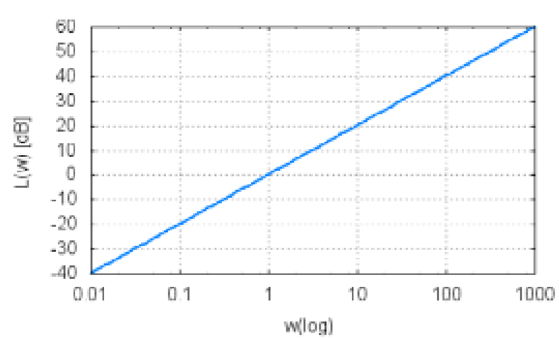
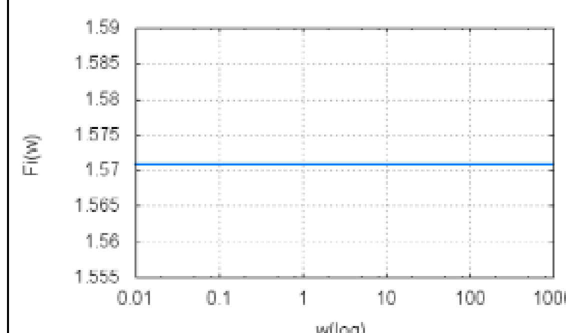
Second-order inertial element

Examples

4



HEATED OBJECT WITH HIGH INERTIA:
input – heater power h(t)
output – object temperature $T_i(t)$

Nyquist plot	Bode Plot (example)	
		
		
		
		

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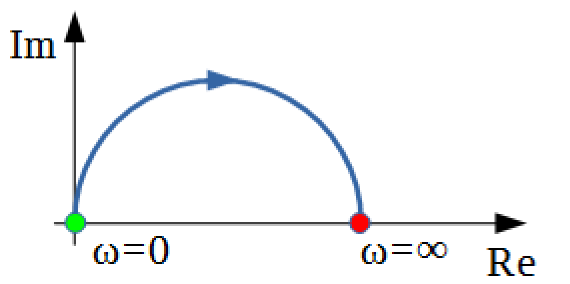
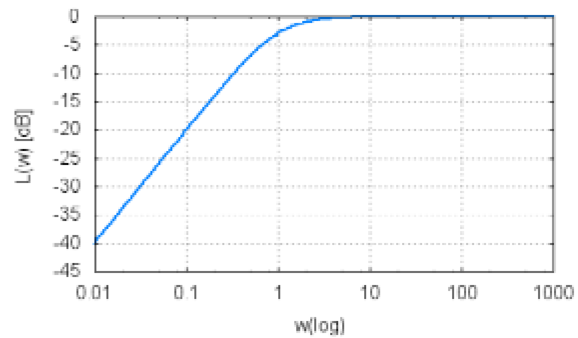
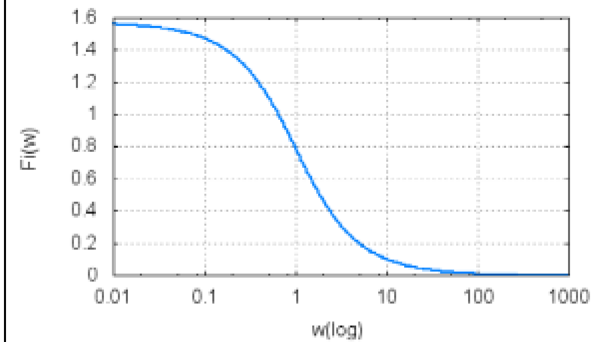
<tr><th>Nyquist plot</th><th colspan="2">Bode Plot (example)</th></tr>
<tr><td>
</td><td>
</td><td>
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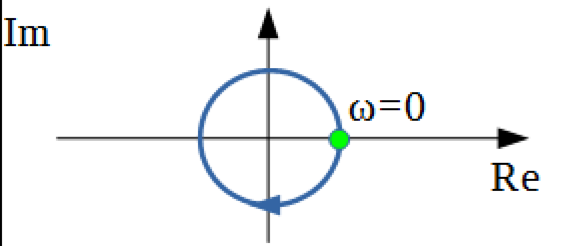
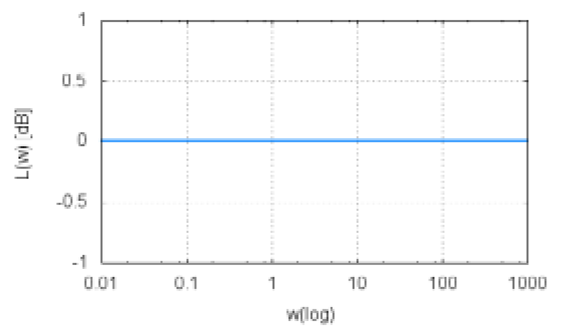
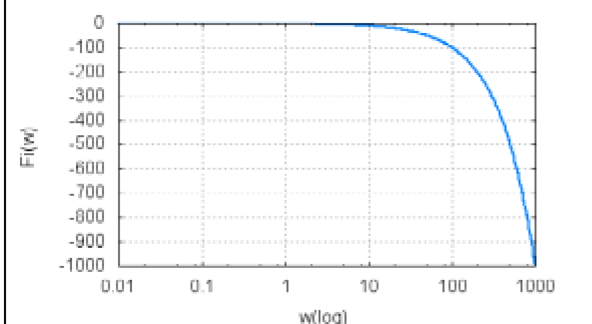
<tr><td>
</td><td>
</td><td>
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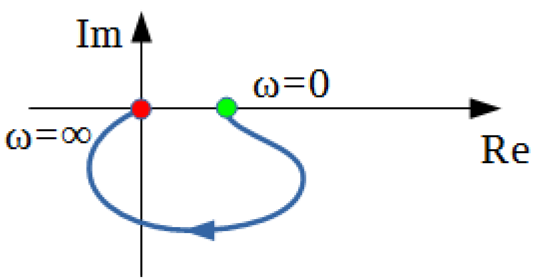
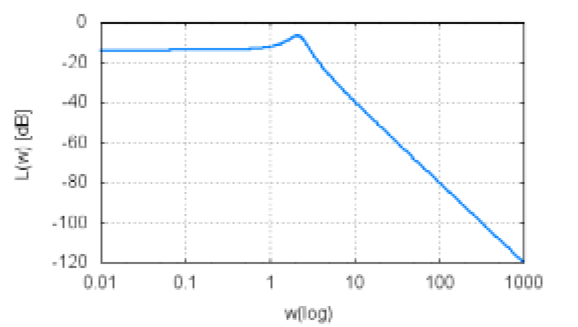
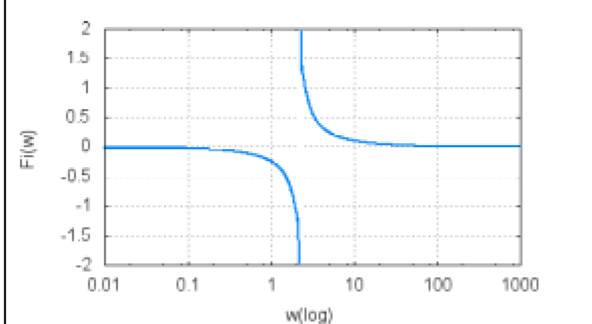
<tr><td>
</td><td>
</td><td>
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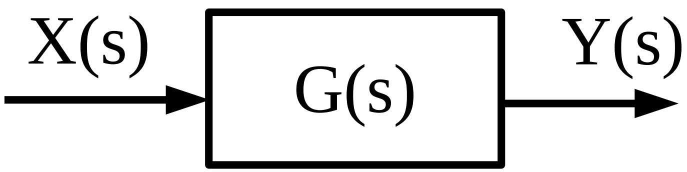
Lecture 11

Transfer function analysis – example.
Block diagram algebra.
Control and controllers.

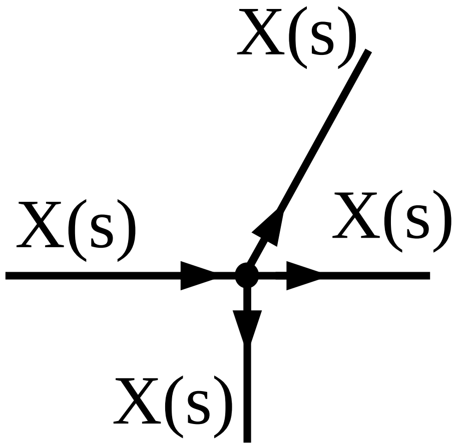
Example

Calculate and sketch step response and Bode Plots for a system with transfer function H(s) = ……..

BLOCK DIAGRAM ALGEBRA
Transfer function (SISO system)

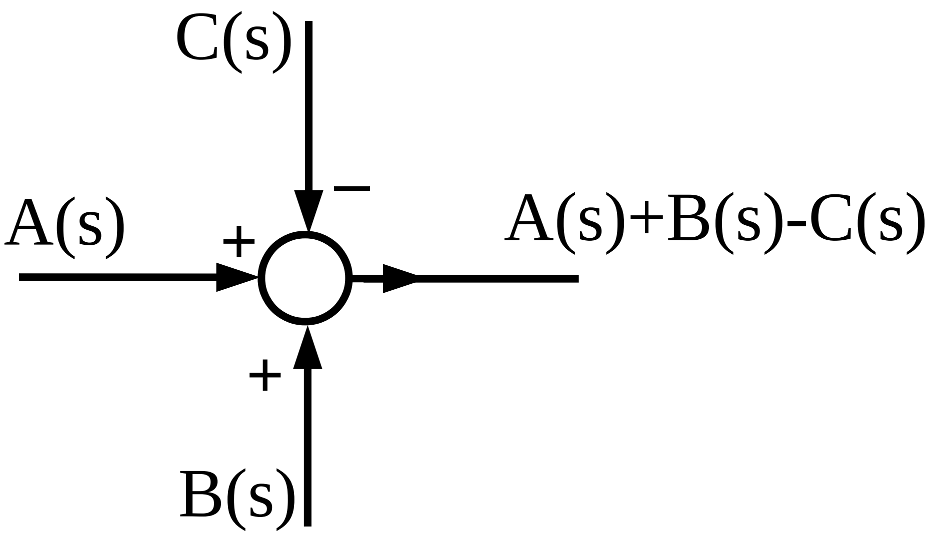


BLOCK DIAGRAM ALGEBRA
information node



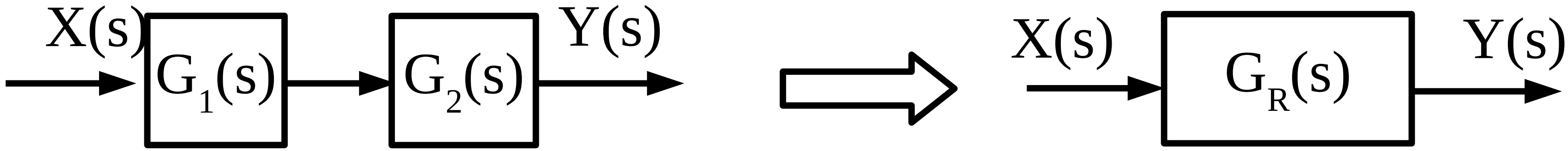
one input,
a few outputs,

BLOCK DIAGRAM ALGEBRA
sum node



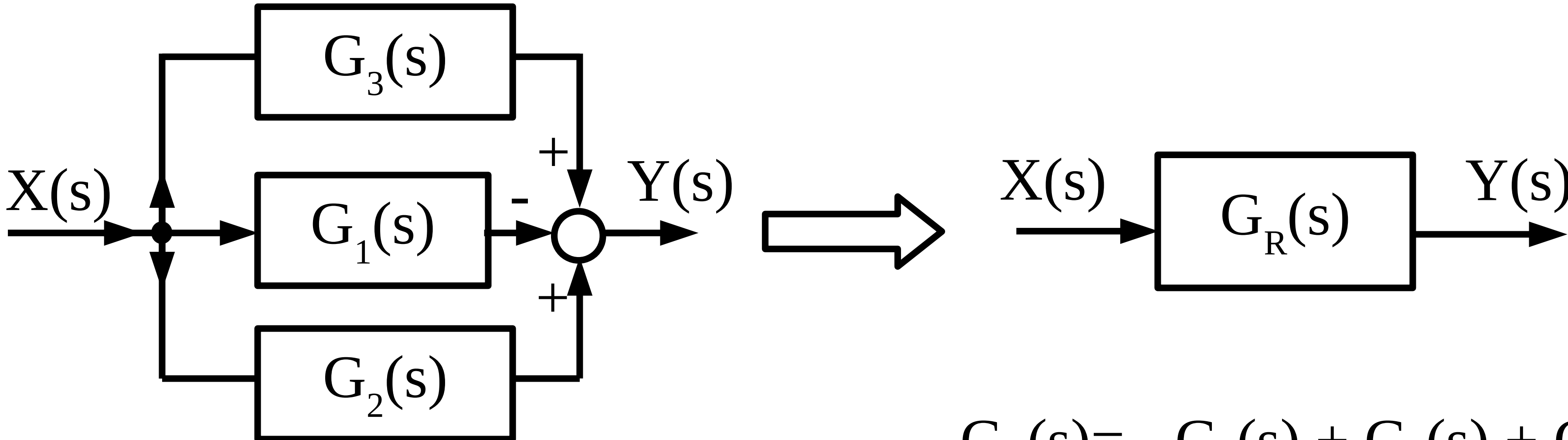
a few inputs,
one output,

BLOCK DIAGRAM ALGEBRA
serial connection



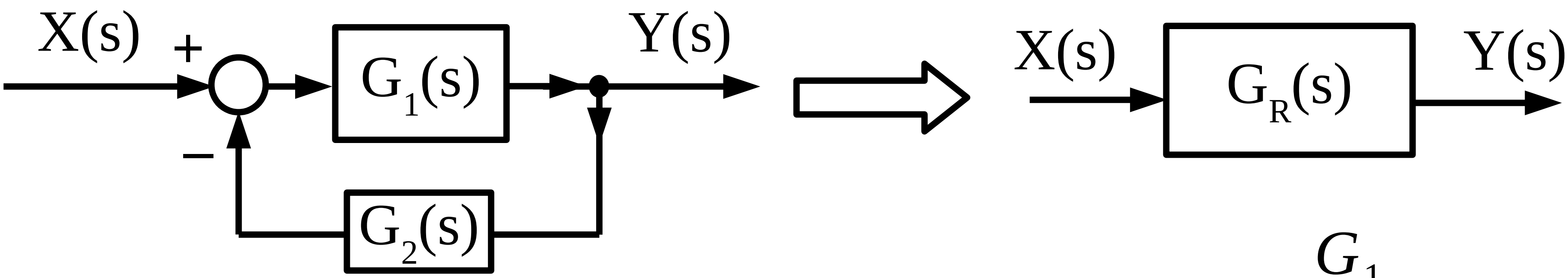
$$G_R(s)=G_1(s)\,G_2(s)$$

BLOCK DIAGRAM ALGEBRA
parallel connection



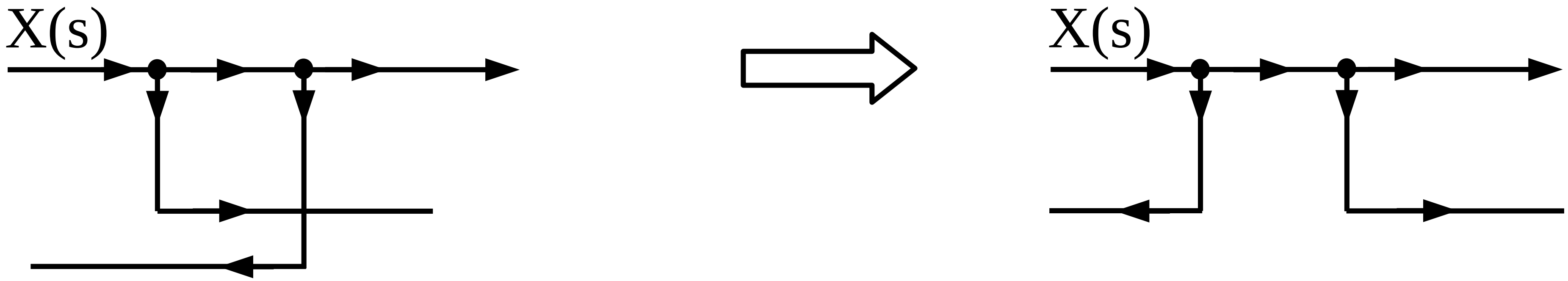
$$G_R(s) = -G_1(s) + G_2(s) + G_3(s)$$

BLOCK DIAGRAM ALGEBRA
feedback

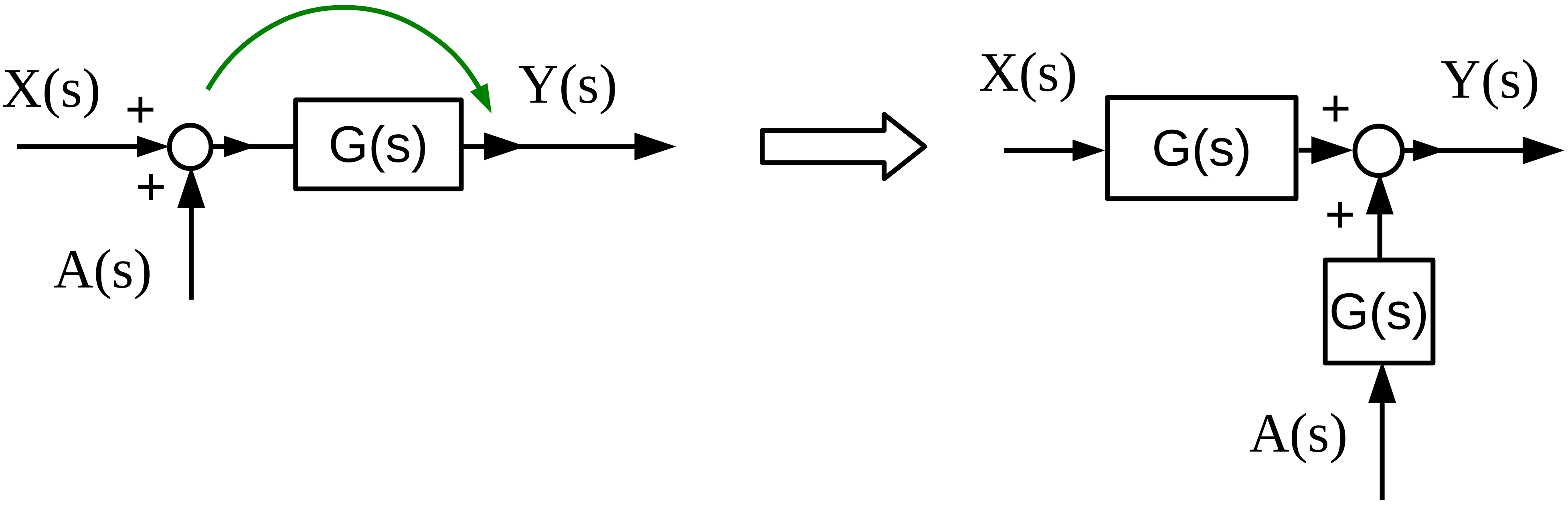


$$G_R = \frac{G_1}{1 + G_1 G_2}$$

BLOCK DIAGRAM ALGEBRA
change of information points order



BLOCK DIAGRAM ALGEBRA
order change of sum node and block

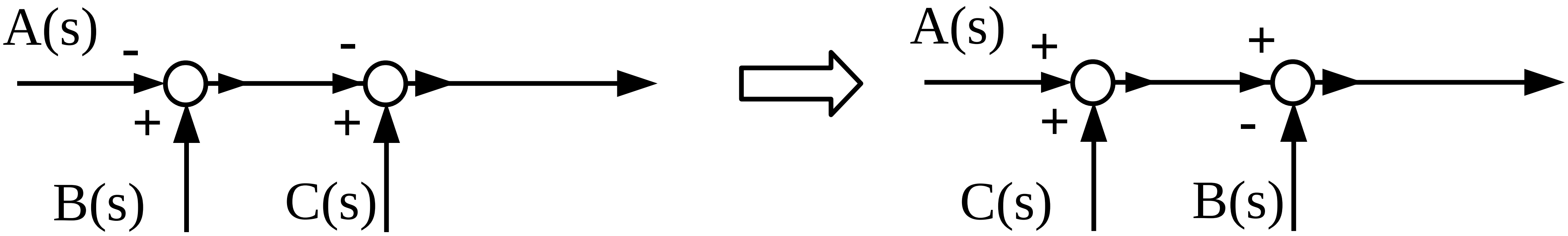


Y=(X+A)G

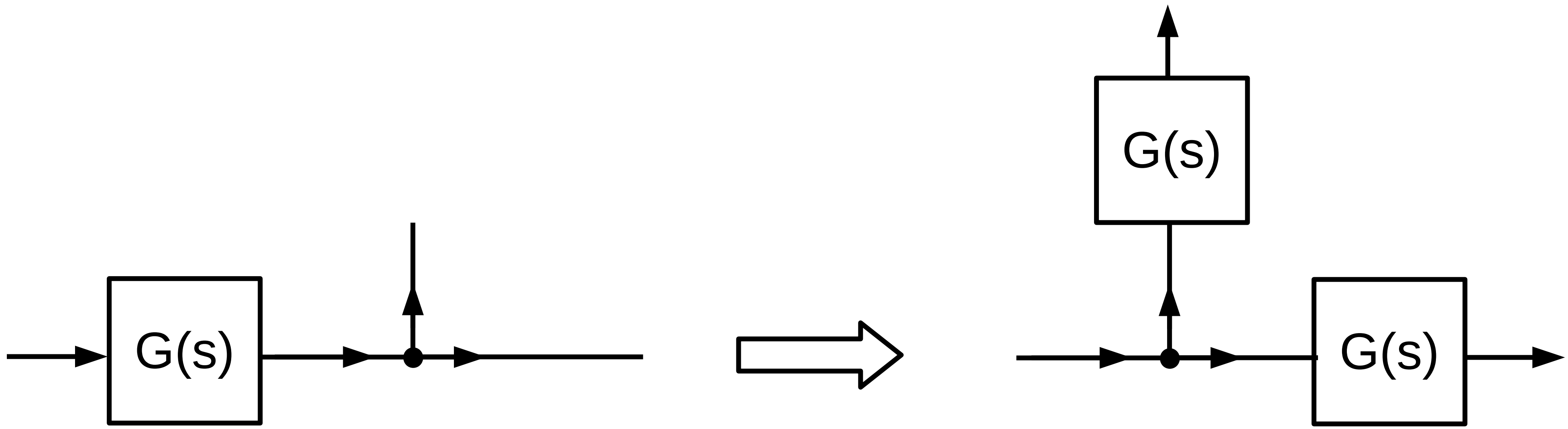
Y=XG+AG

BLOCK DIAGRAM ALGEBRA
order change of sum nodes

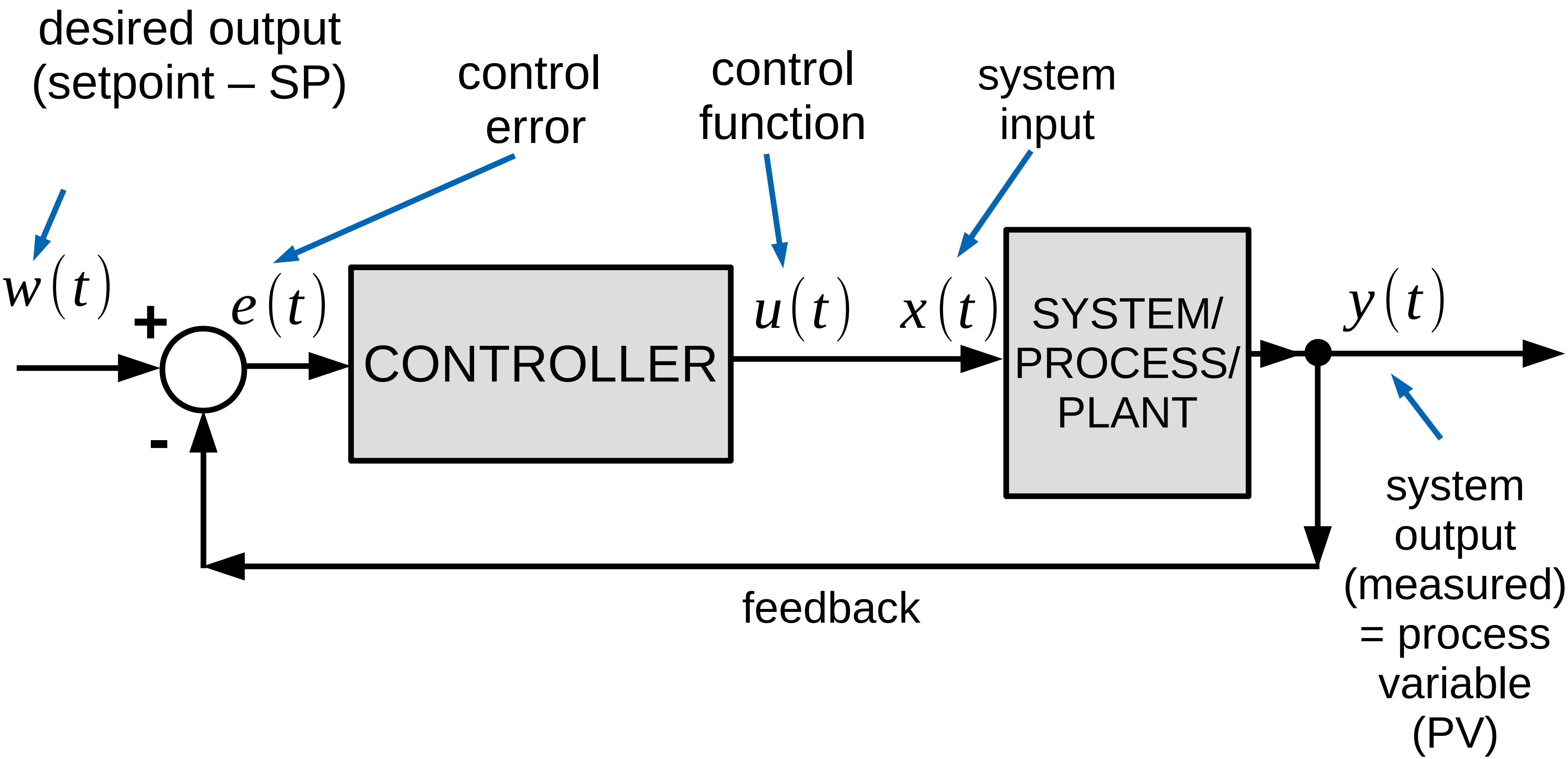
Example 2 – attention to signs!



BLOCK DIAGRAM ALGEBRA
order change of block and information node



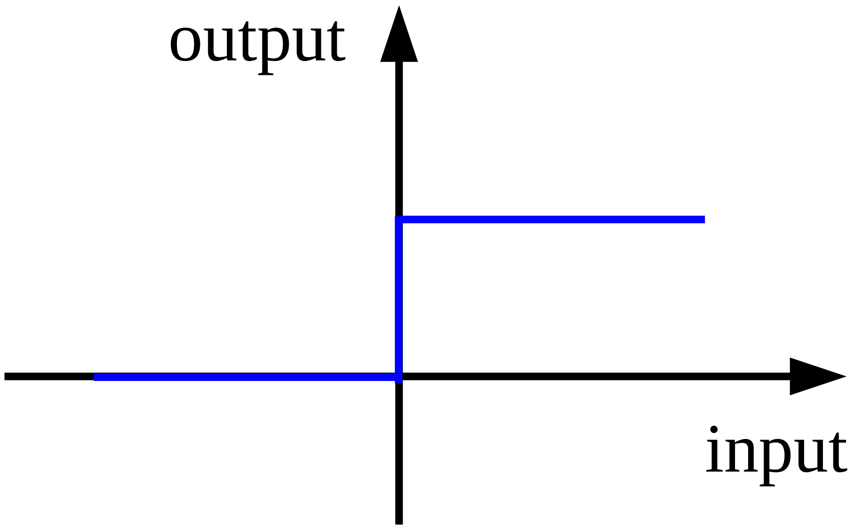
Closed loop control

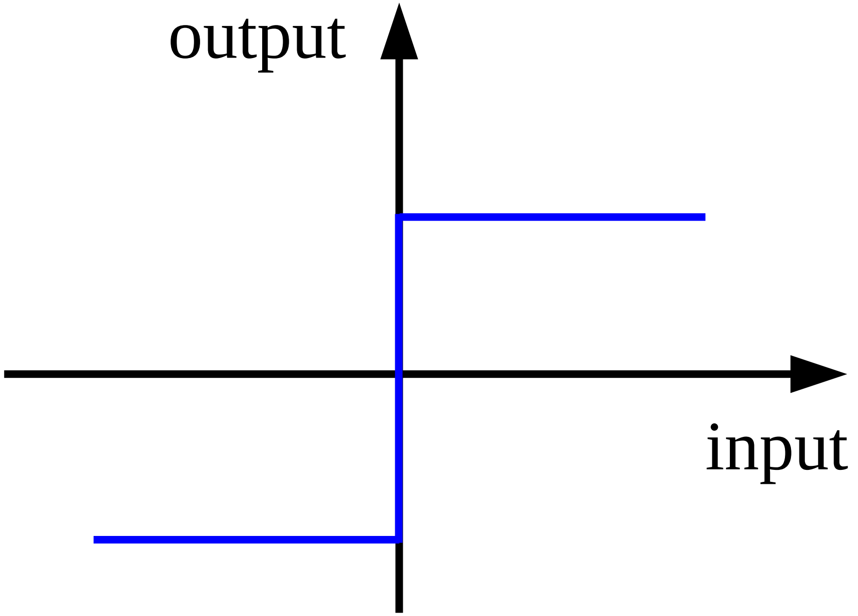


Types of controllers

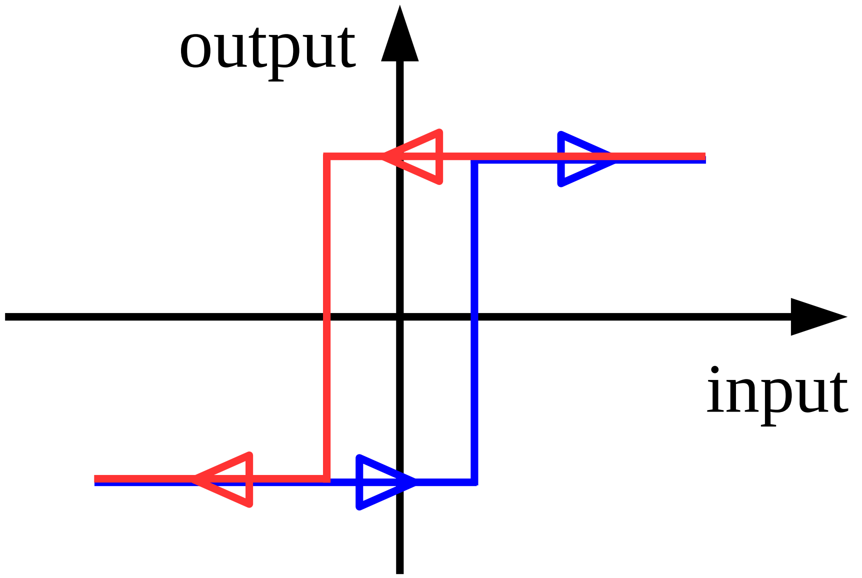
- two state (ON/OFF)
- three state
- Proportional (P)
- Integrator (I)
- Differentiator (D)
- Proportional-inegral-derivative (PID)

ON-OFF CONTROLLER
(RELAY / TWO STATE / BANG-BANG)

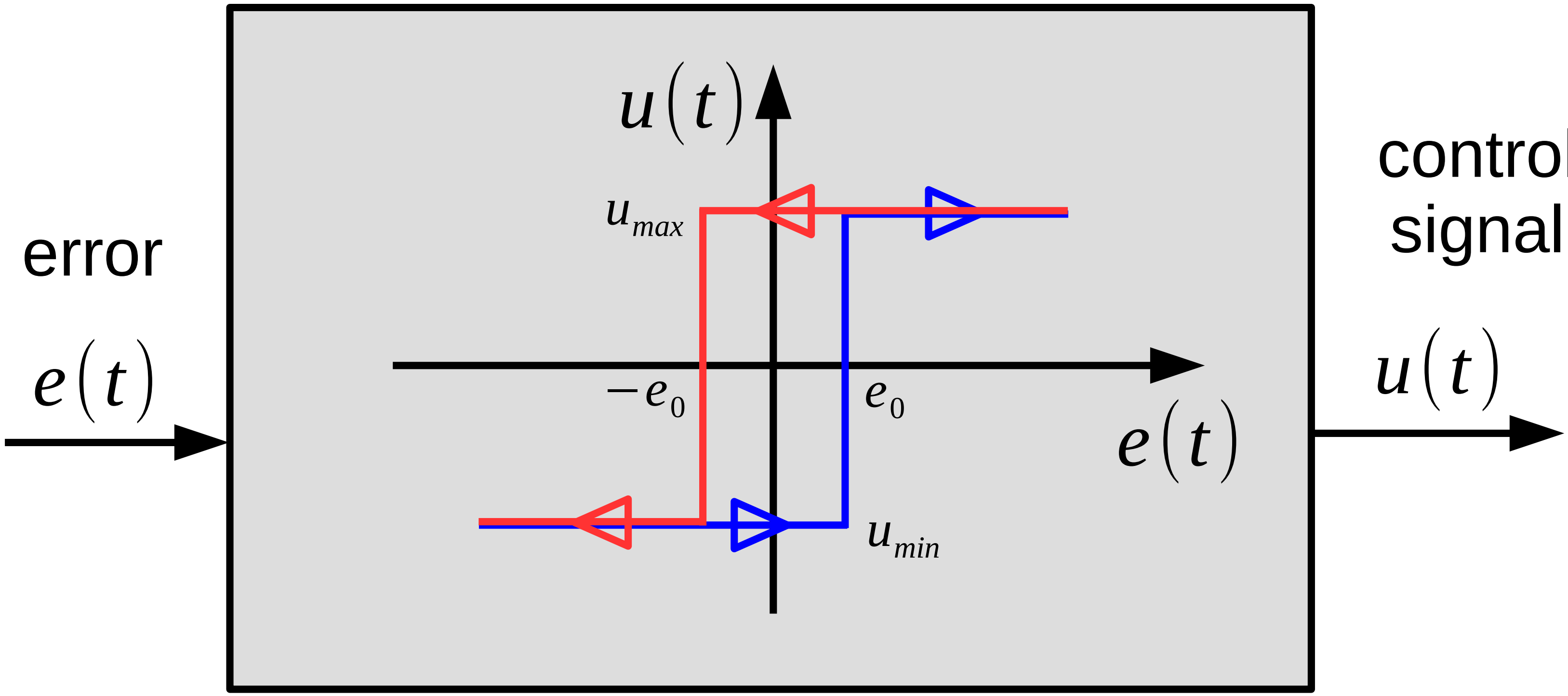




real
(with hysteresis)



ON-OFF CONTROLLER (RELAY / TWO STATE / BANG-BANG)

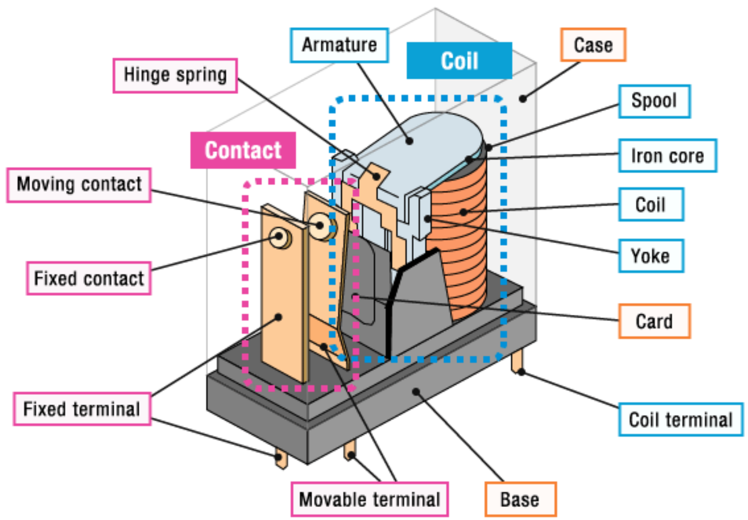


$$u(t)=\begin{cases} u_{max}, \text{ if } e>e_0 \\ u_{min}, \text{ if } e<-e_0 \\ \text{no change, in other situations} \end{cases}$$

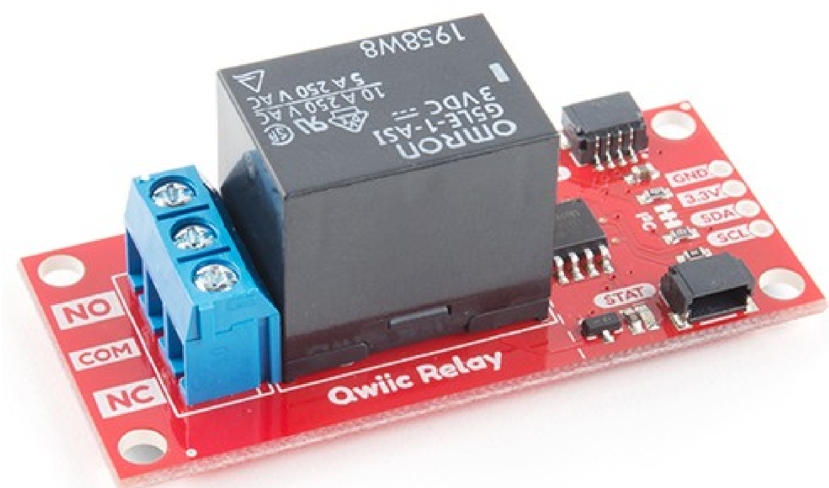
e_o - mechanical or programmed hysteresis

ON-OFF CONTROLLER
(RELAY / TWO STATE / BANG-BANG)

mechanical relay

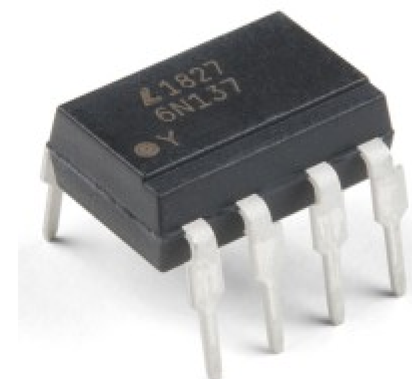


Source: https://www.components.omron.com/relay-basics/basic

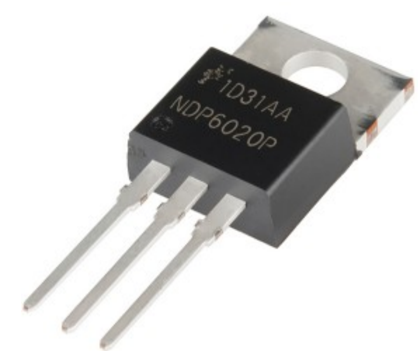


Source: https://www.sparkfun.com/products/15093

electronic relay (optoisolators, MOSFETs)

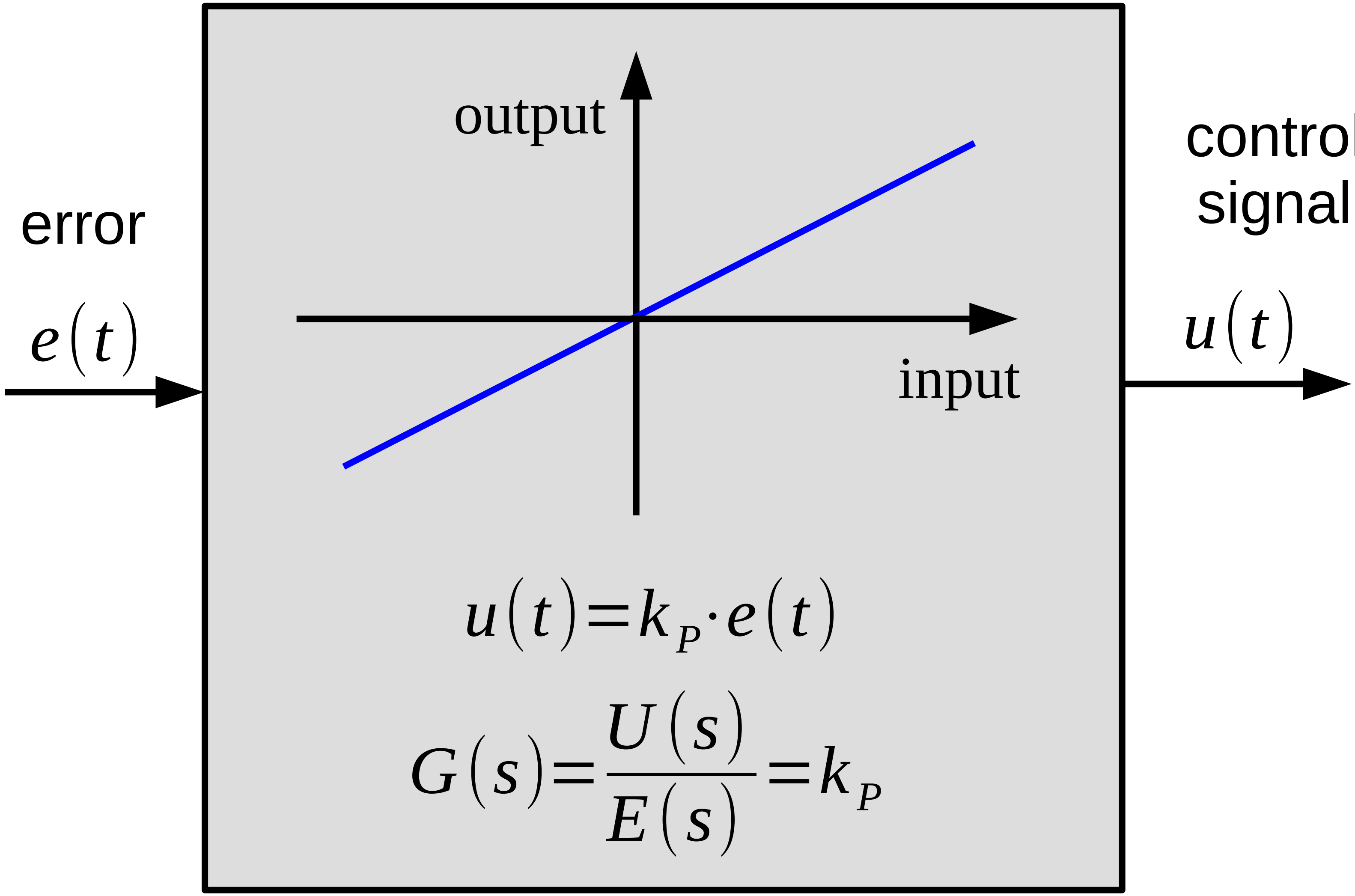


source: https://www.sparkfun.com/products/15105



source: https://www.sparkfun.com/products/12901

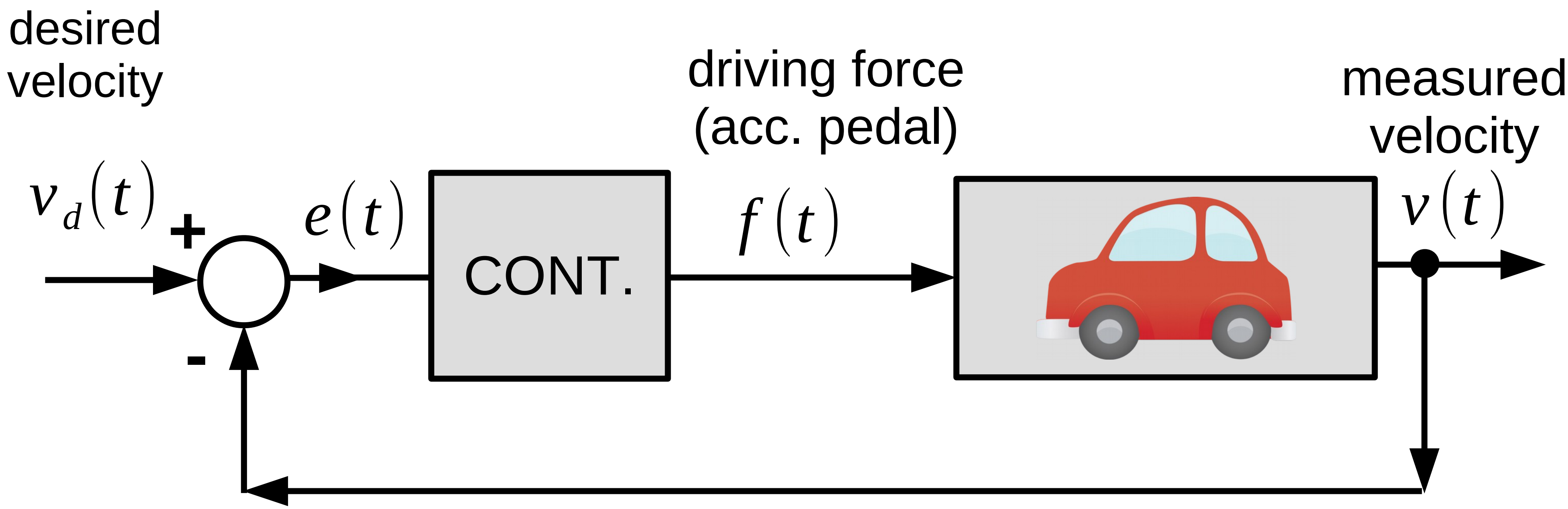
Proportional (P) CONTROLLER



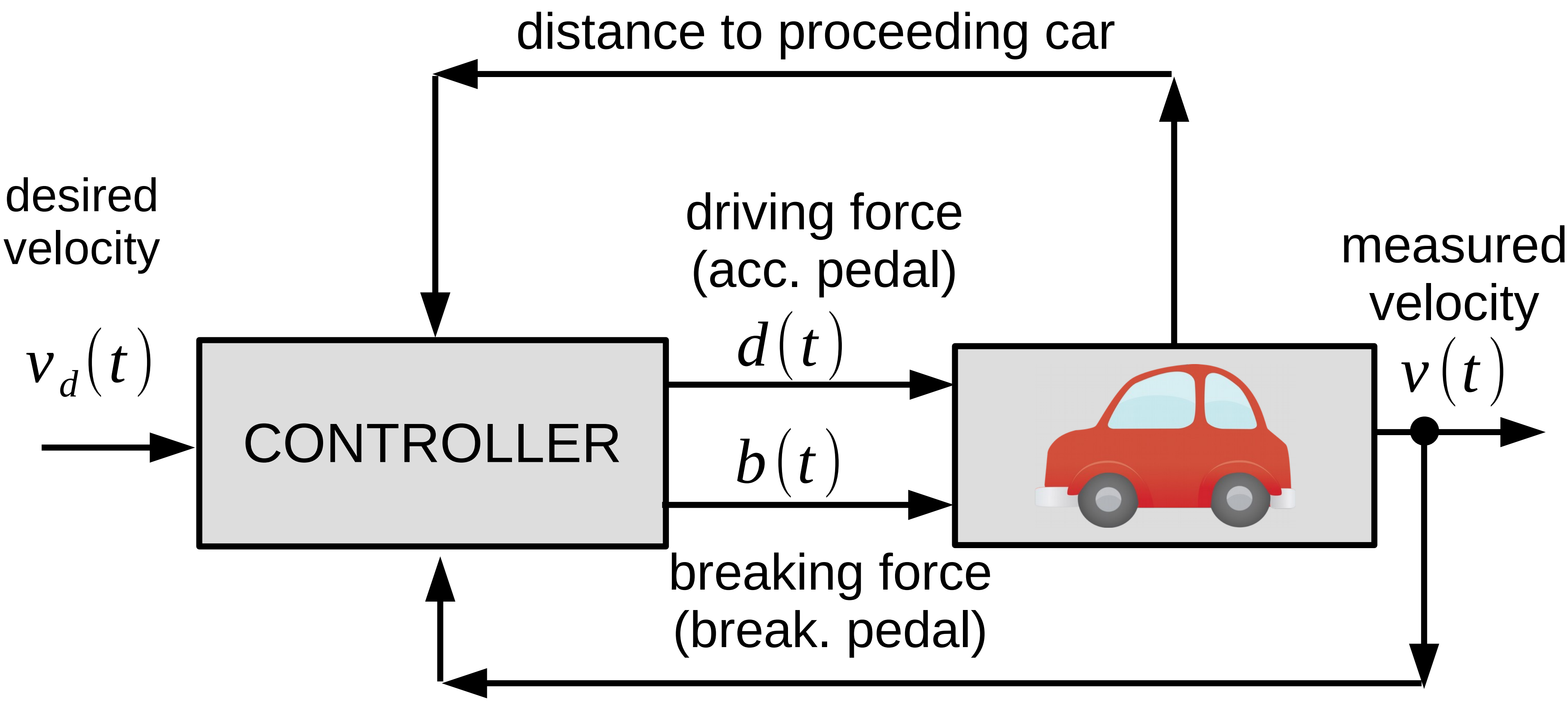
$$u(t) = k_P \cdot e(t)$$

$$G(s) = \frac{U(s)}{E(s)} = k_P$$

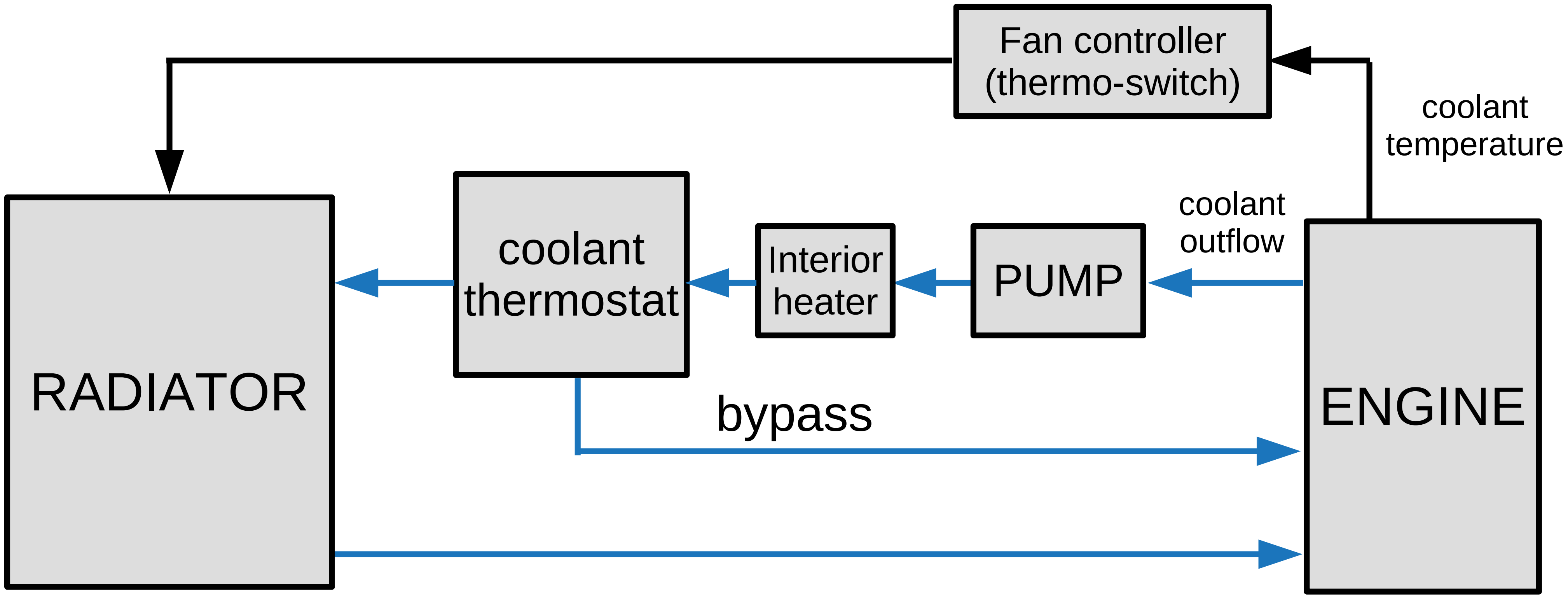
Speed control
(cruise control, autocruise, tempomat)



Speed control
(adaptive cruise control)



Engine's temperature control



Lecture 12

PID controller.
Stability.

P, I and D controllers transfer functions

Controller	Transfer function
Proportional (P)	k_P
Integral (I)	$\frac{1}{T_i s}$
Ideal derivative (D)	$T_d s$
Real derivative (D)	$\frac{T_d s}{T s + 1}$

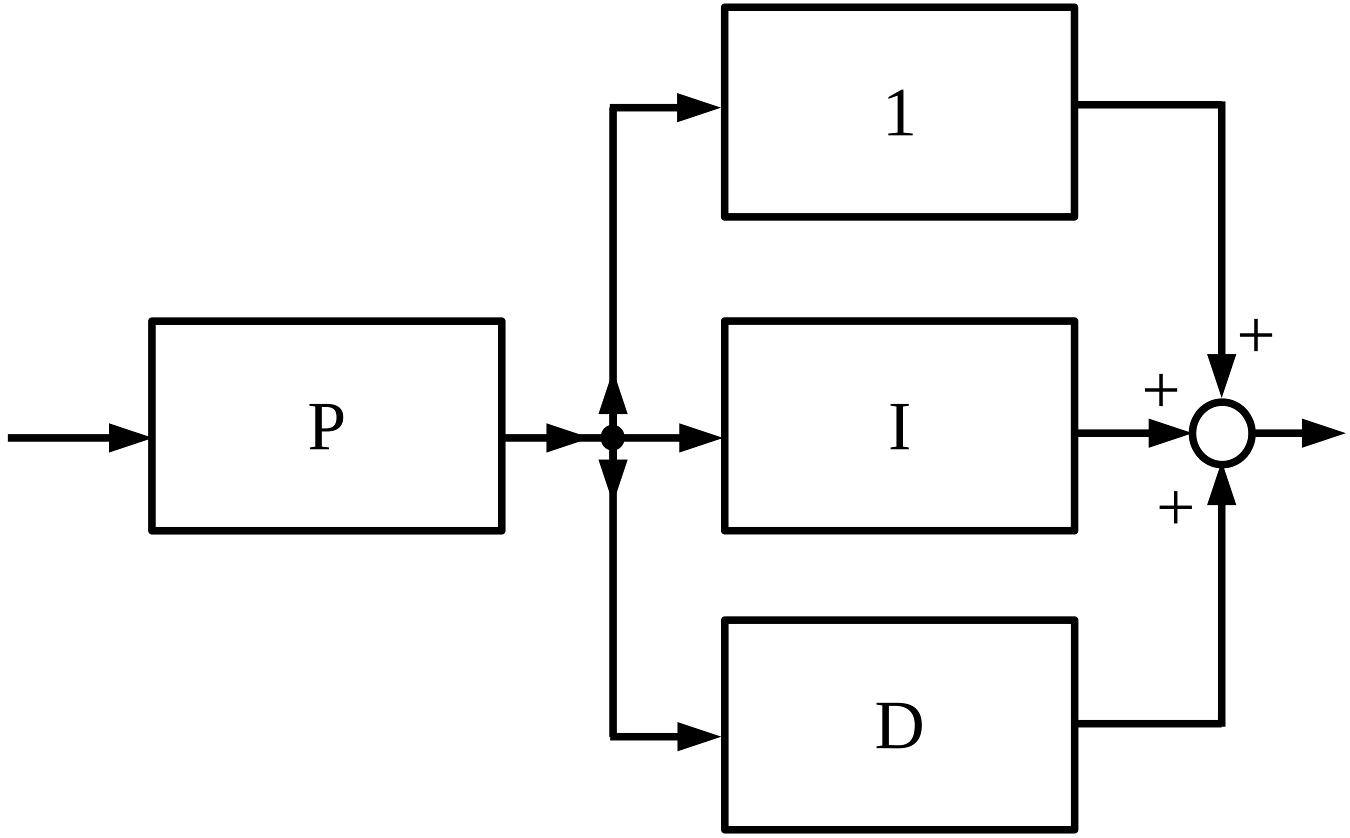
PID controllers transfer functions

Controller	**Transfer function**
Proportional-integral-derivative (PID) in standard form with ideal derivative	$k_P\left(1+\frac{1}{T_i s}+T_d s\right)$
Proportional-integral-derivative (PID) in parallel form with ideal derivative	$k_P+k_i\frac{1}{s}+k_d s$

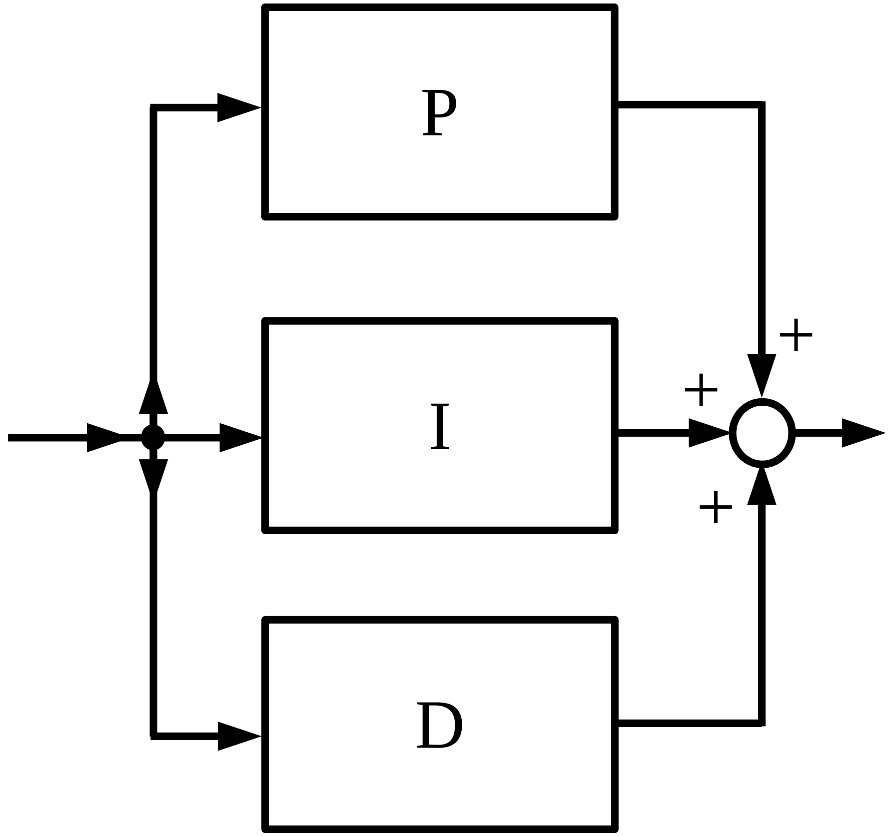
PID controllers transfer functions

Controller	**Transfer function**
Proportional-integral-derivative (PID) <u>in standard form</u> with real derivative	$k_P\left(1+\frac{1}{T_i s}+\frac{T_d s}{Ts+1}\right)$
Proportional-integral-derivative (PID) <u>in parallel form</u> with real derivative	$k_P+k_i\frac{1}{s}+k_d\frac{s}{Ts+1}$

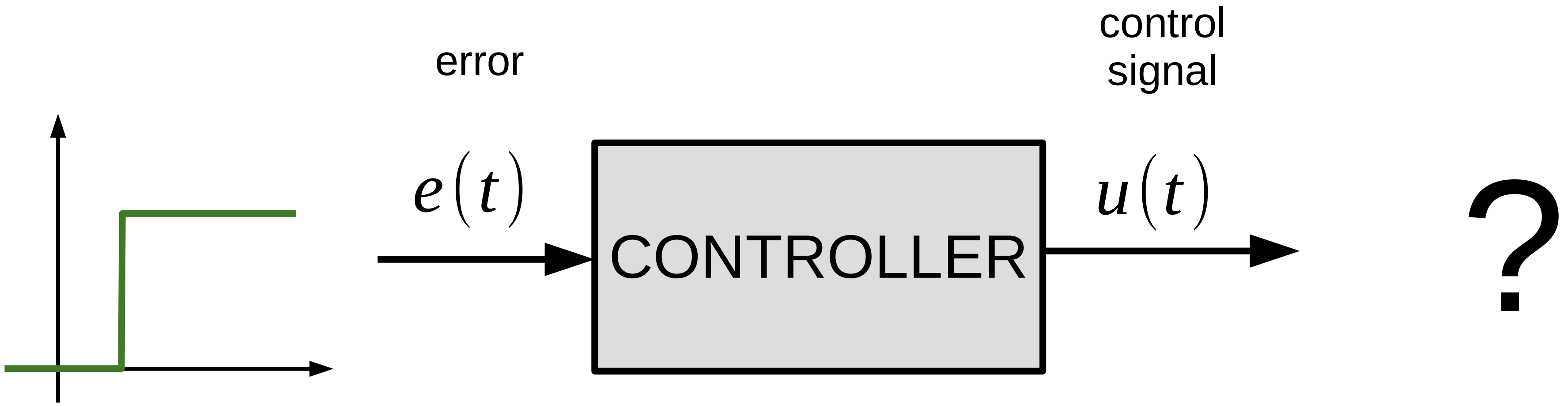
PID CONTROLLER
standard form



PID CONTROLLER
parallel form

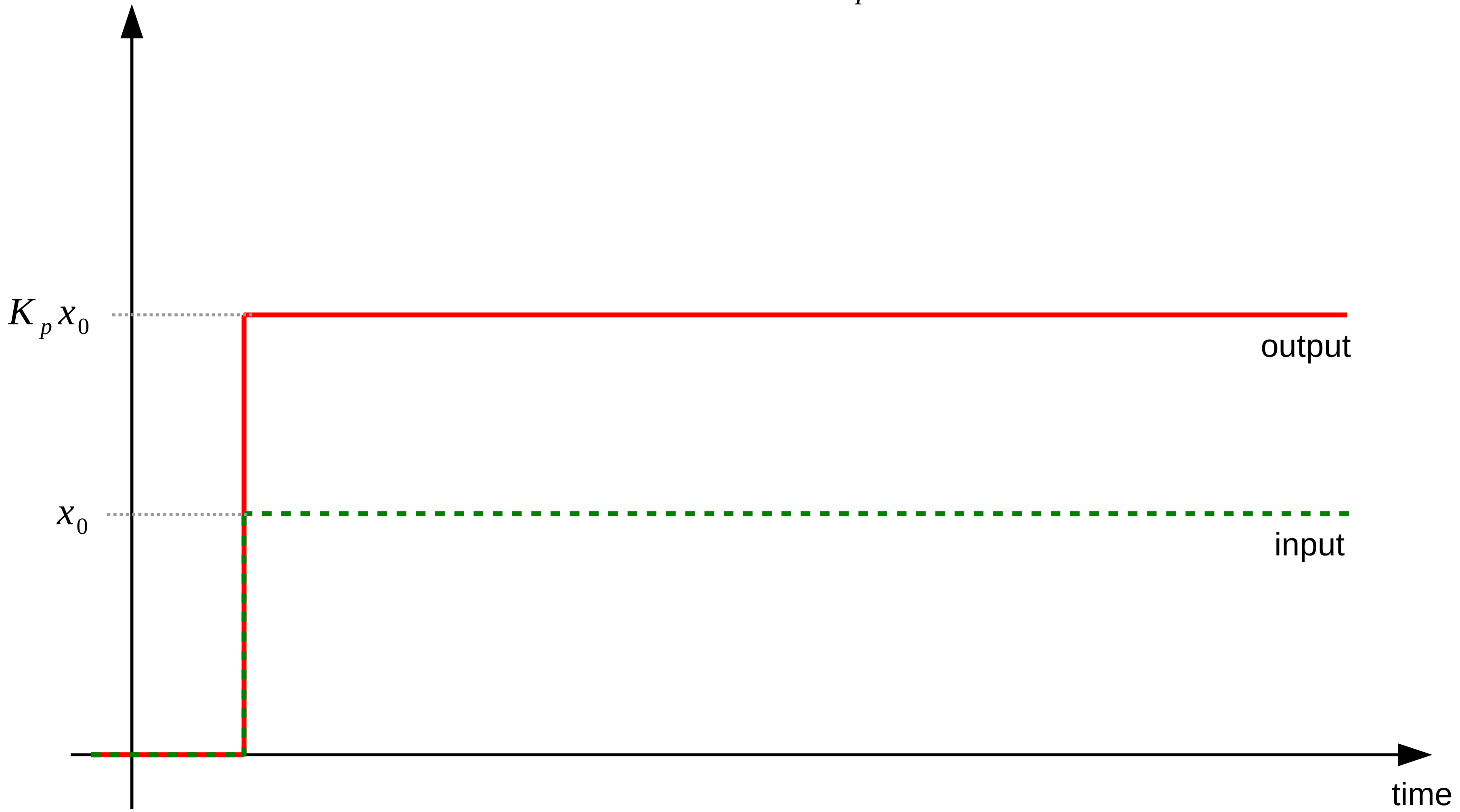


PID CONTROLLER
step responses



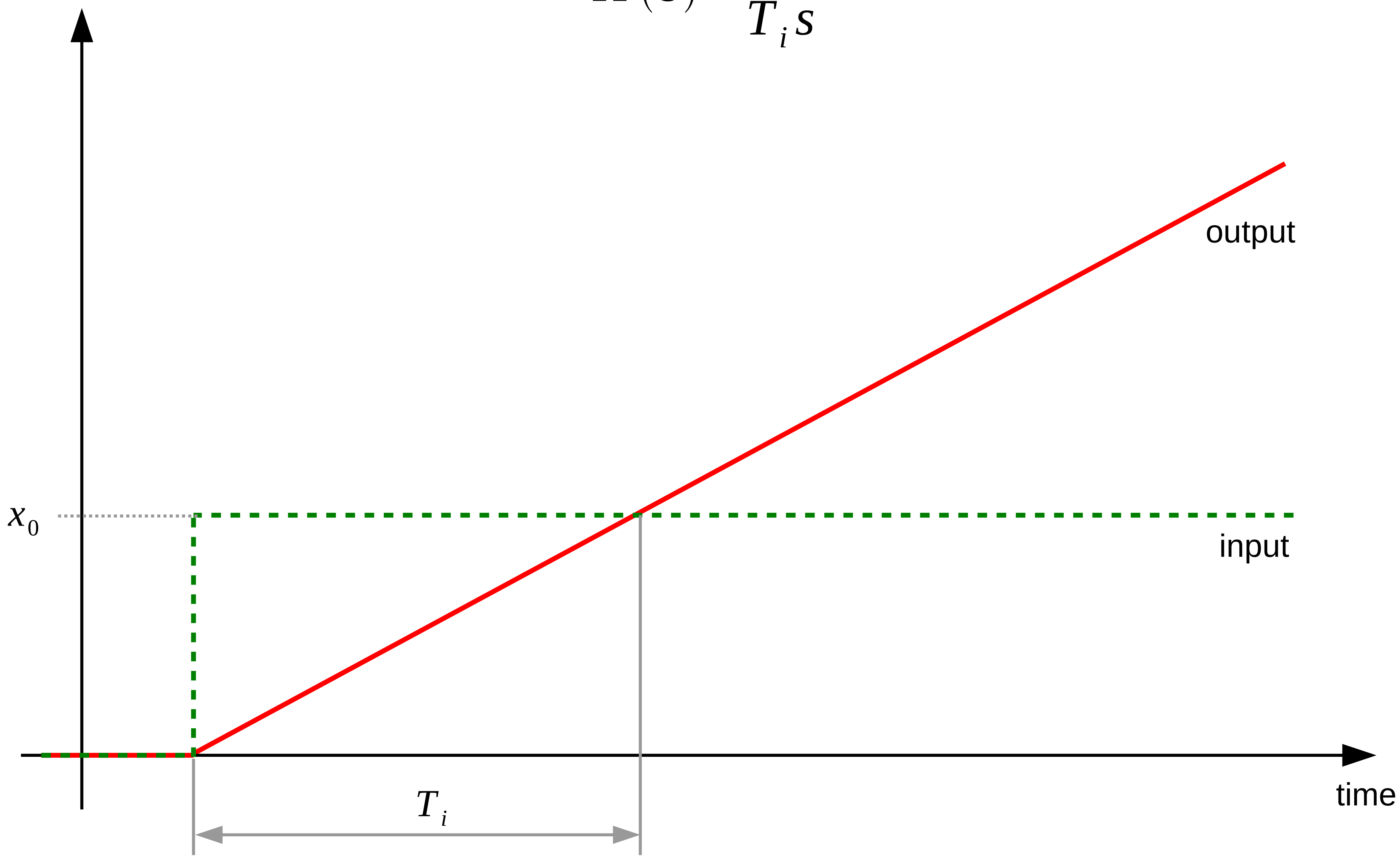
P - CONTROLLER

$$H(s) = K_p$$



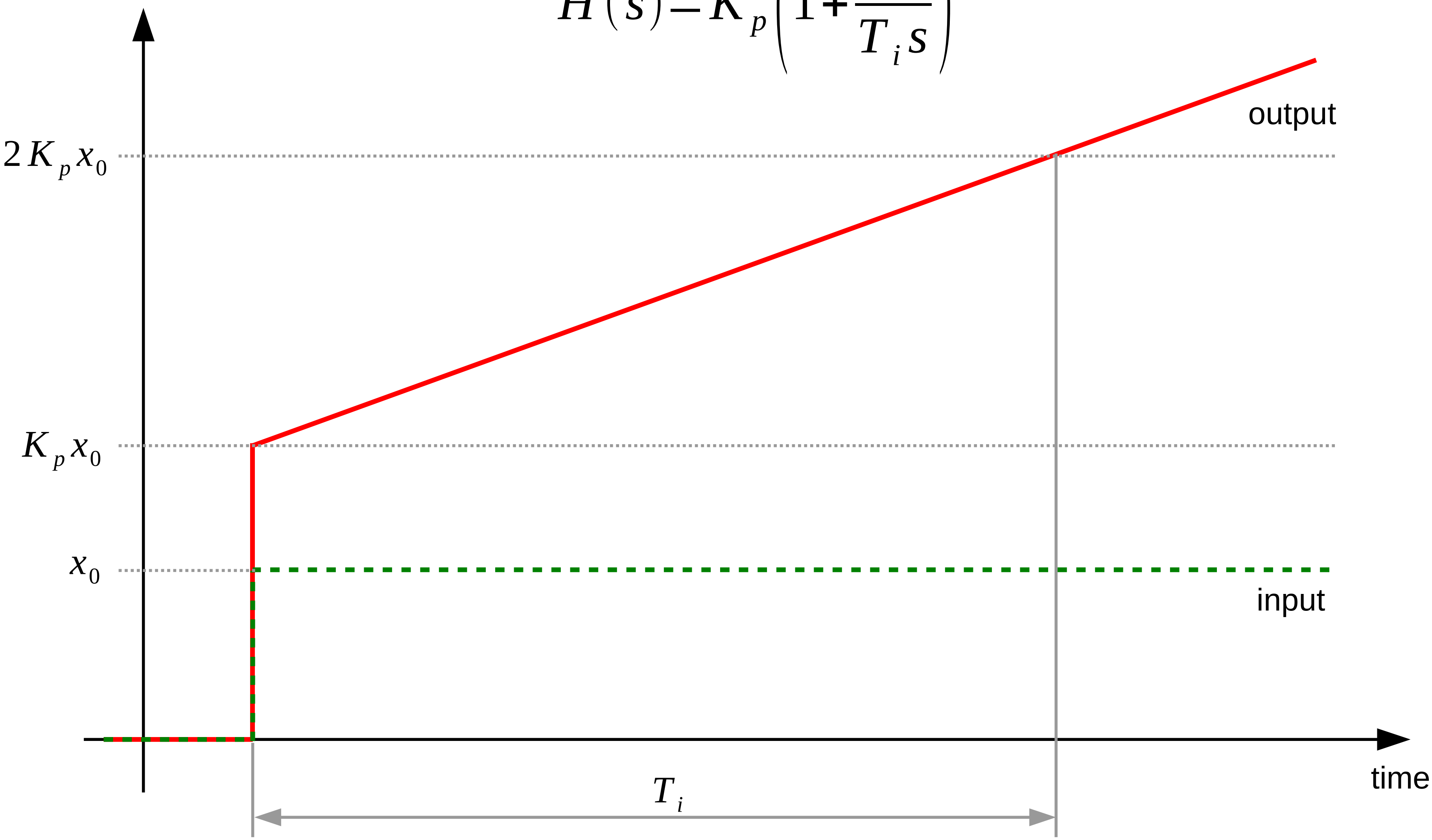
I - CONTROLLER

$$H(s)=\frac{1}{T_i s}$$



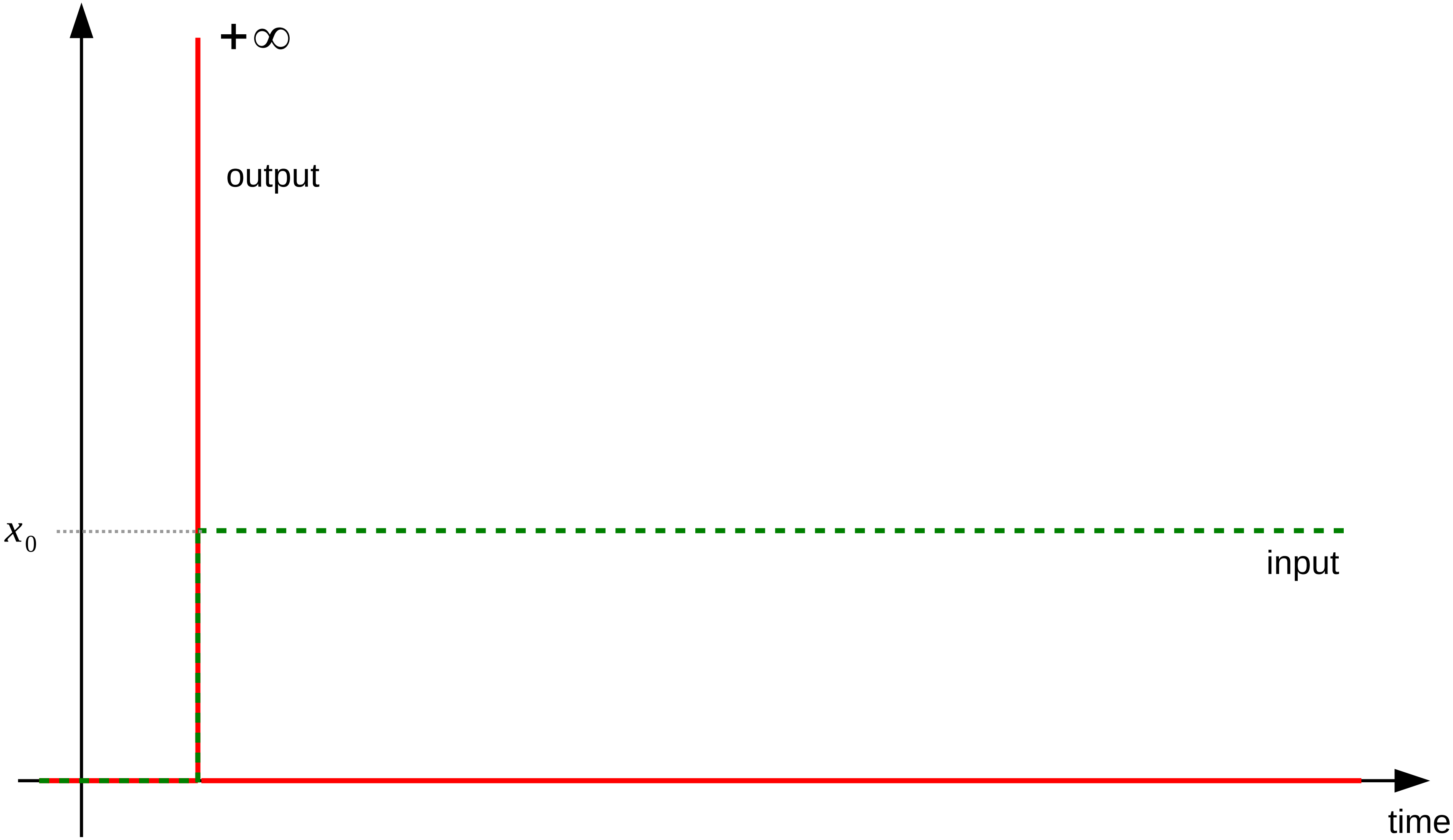
PI - CONTROLLER

$$H(s)=K_p\left(1+\frac{1}{T_i s}\right)$$



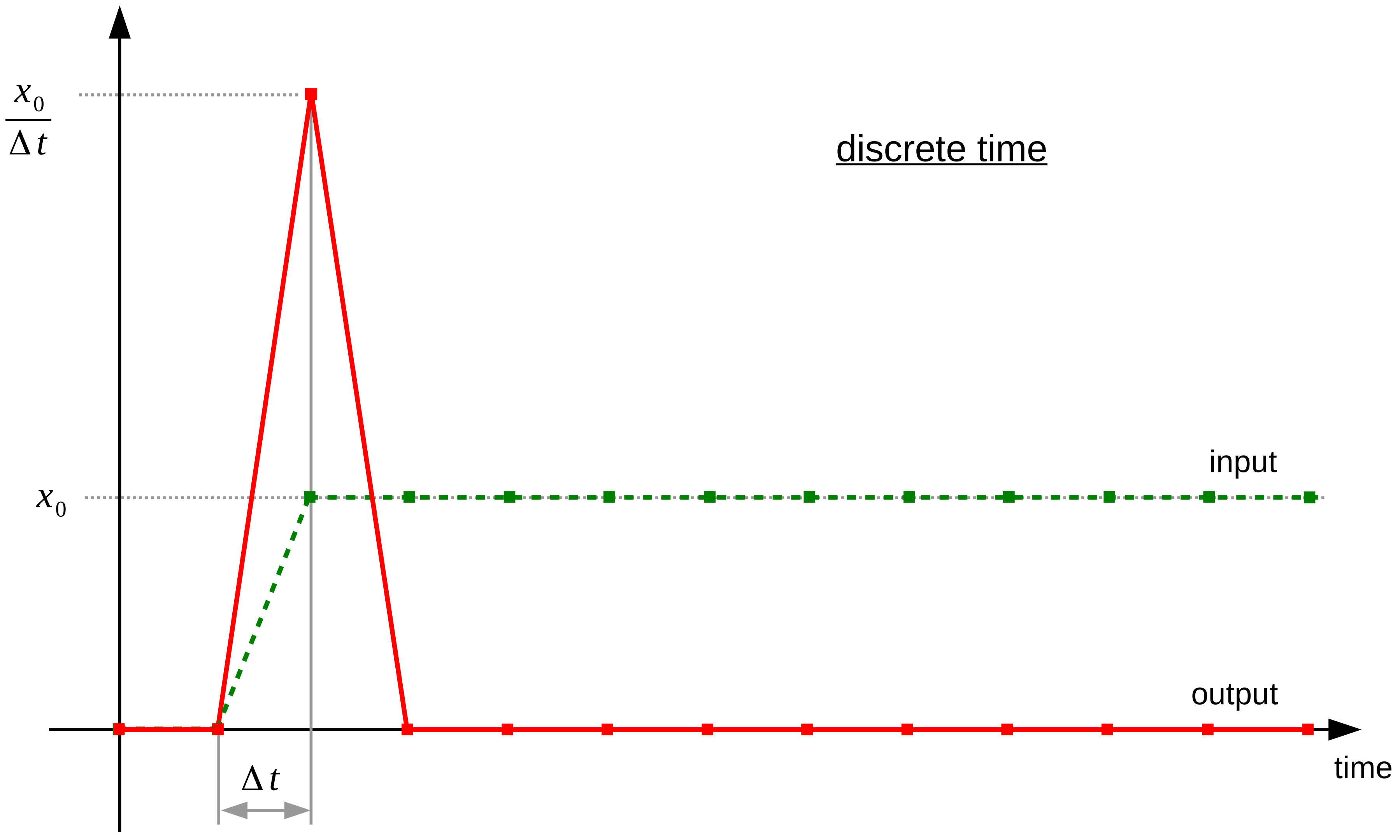
D - CONTROLLER

$$H(s) = T_d s$$



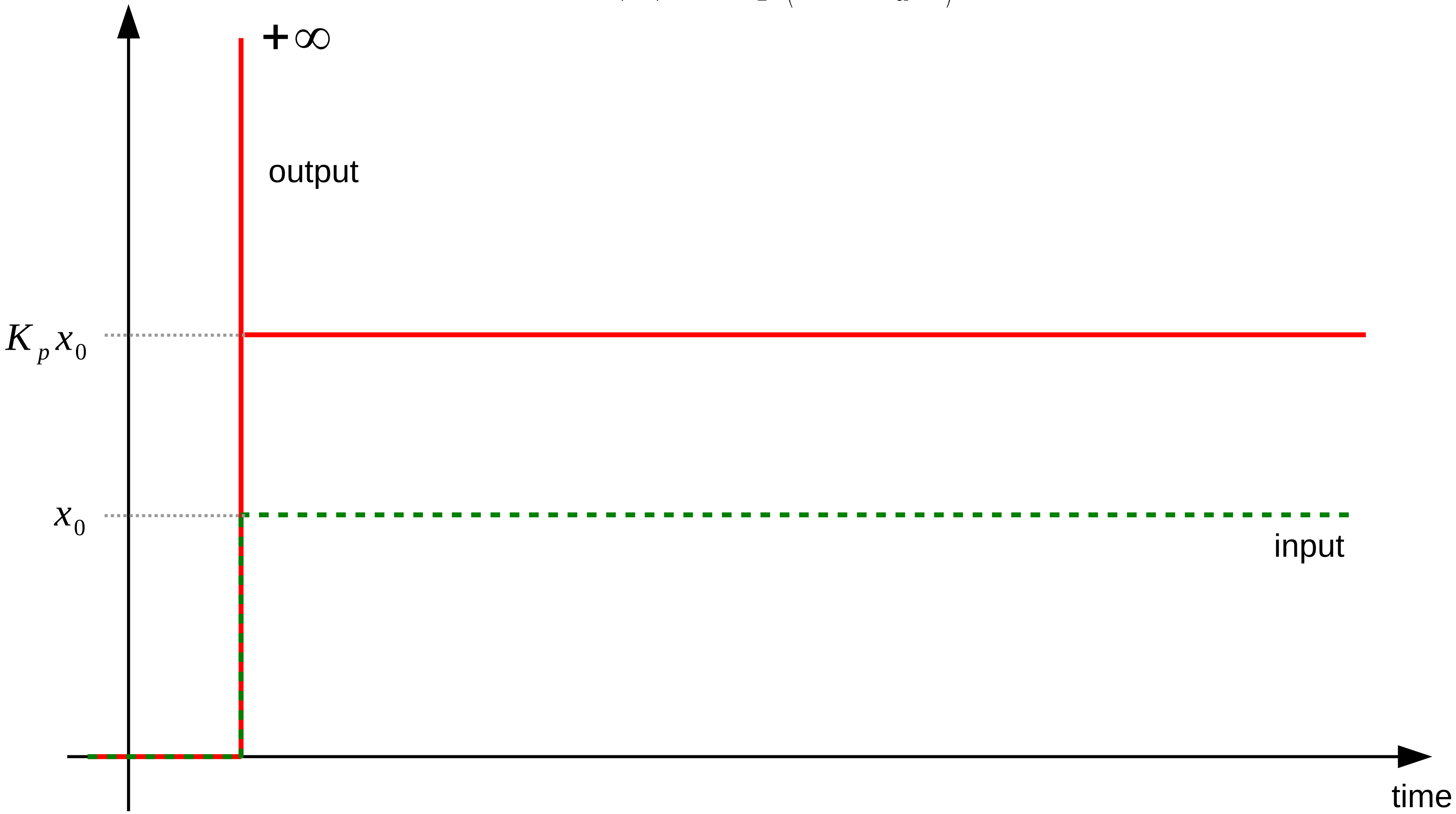
D - CONTROLLER

$$H(s) = T_d\, s$$



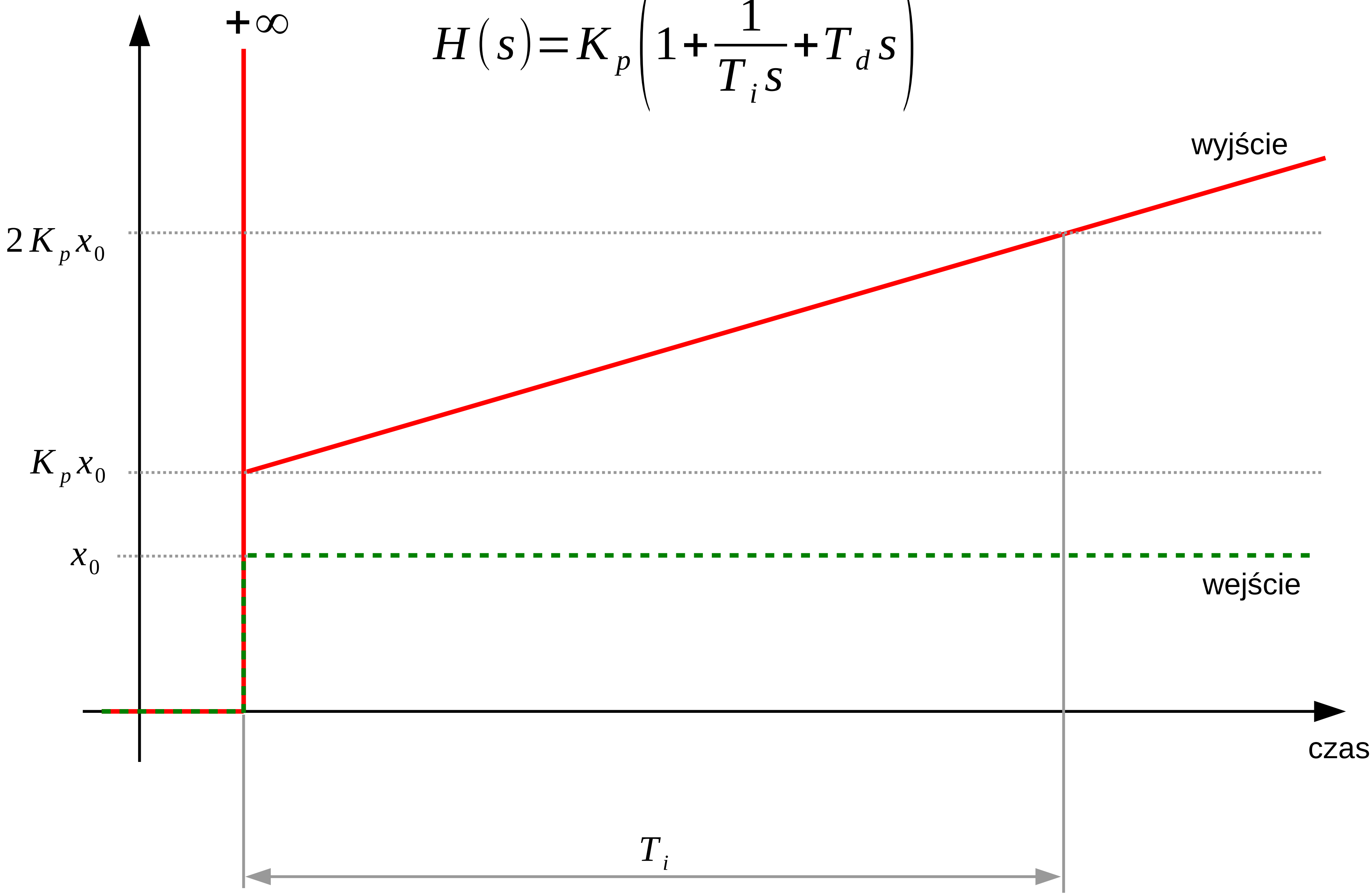
PD - CONTROLLER

$$H(s) = K_P(1 + T_d s)$$



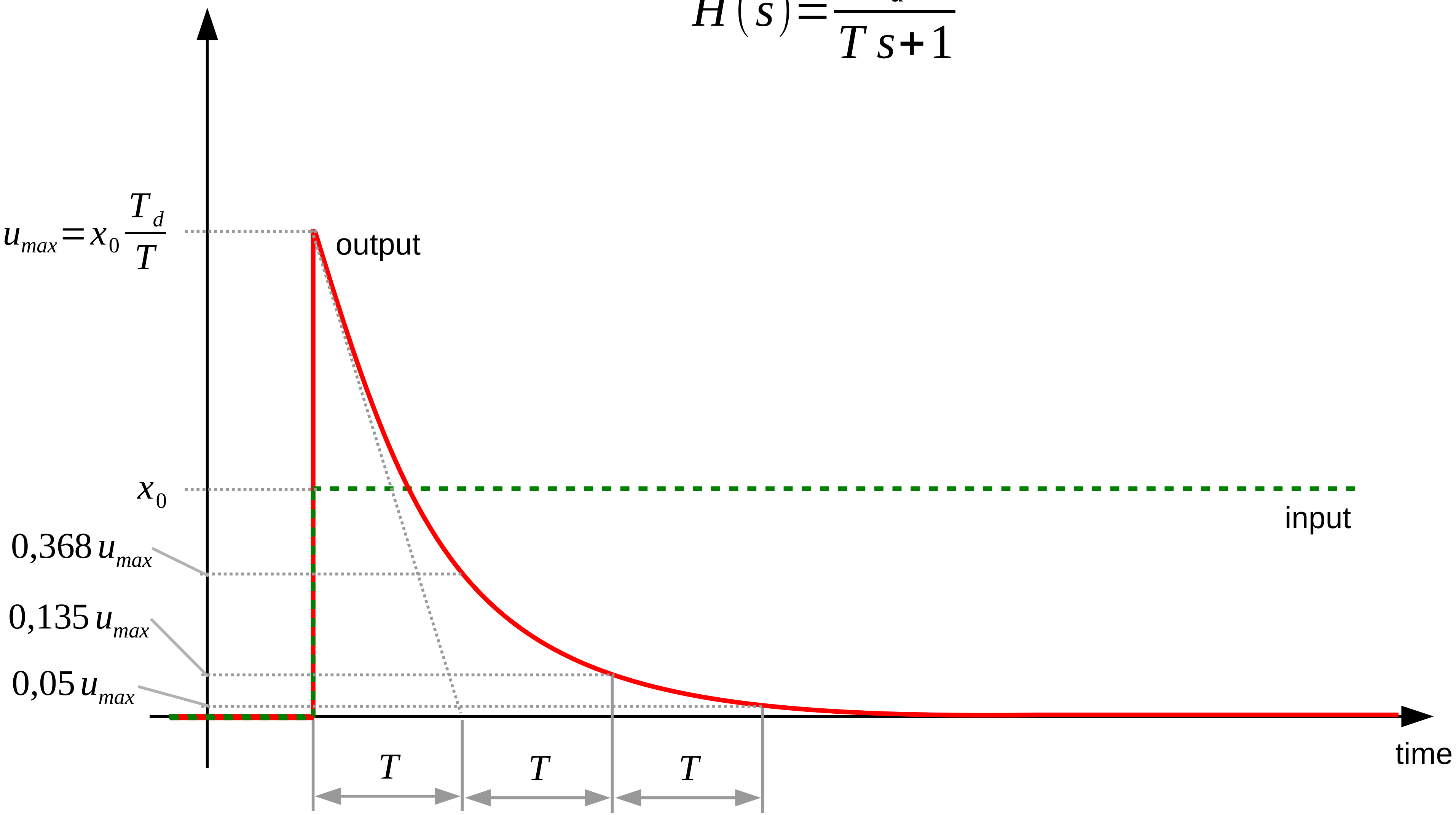
PID – CONTROLLER
standard form, ideal derivative

$$H(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$



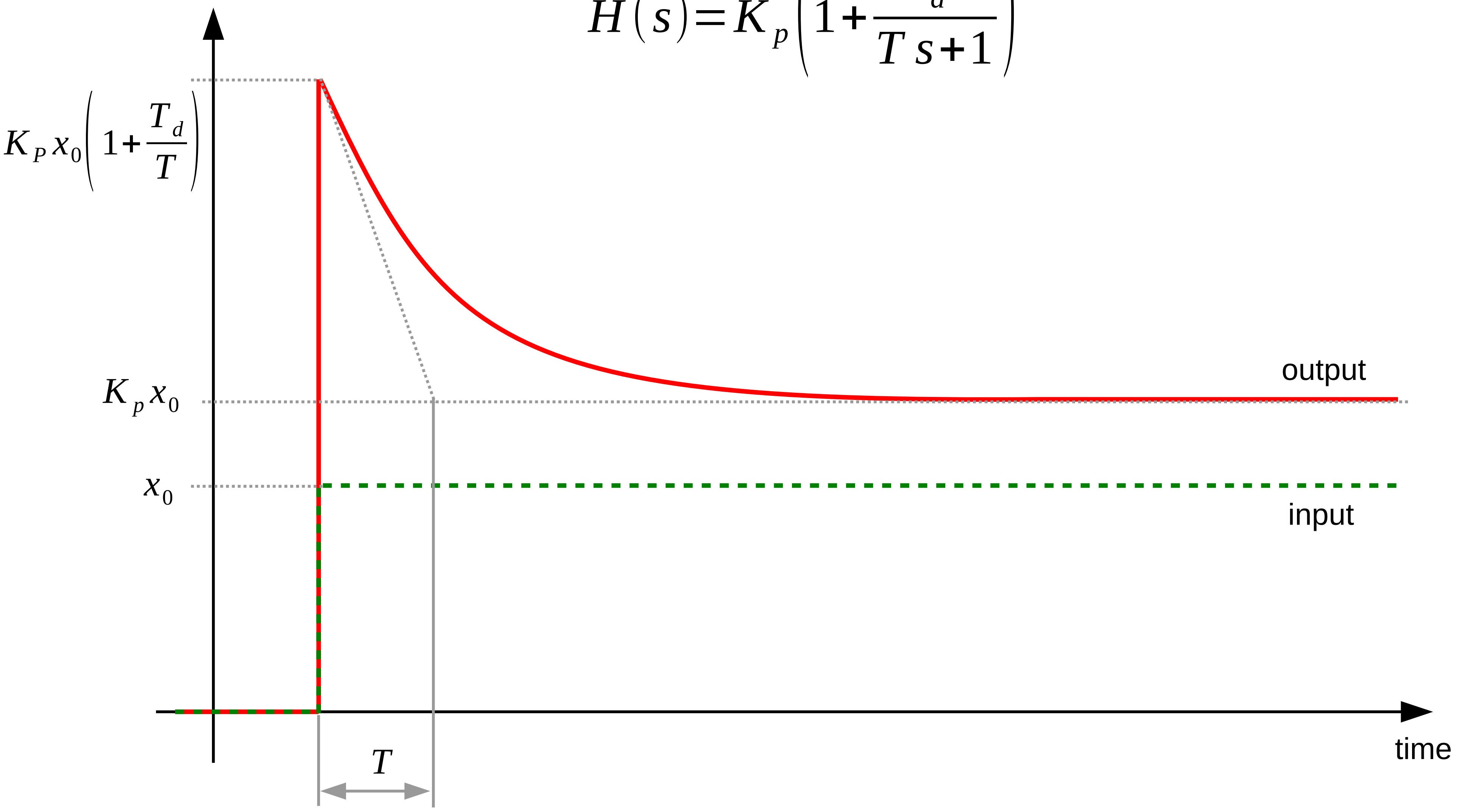
D - CONTROLLER

$$H(s)=\frac{T_d s}{T s+1}$$



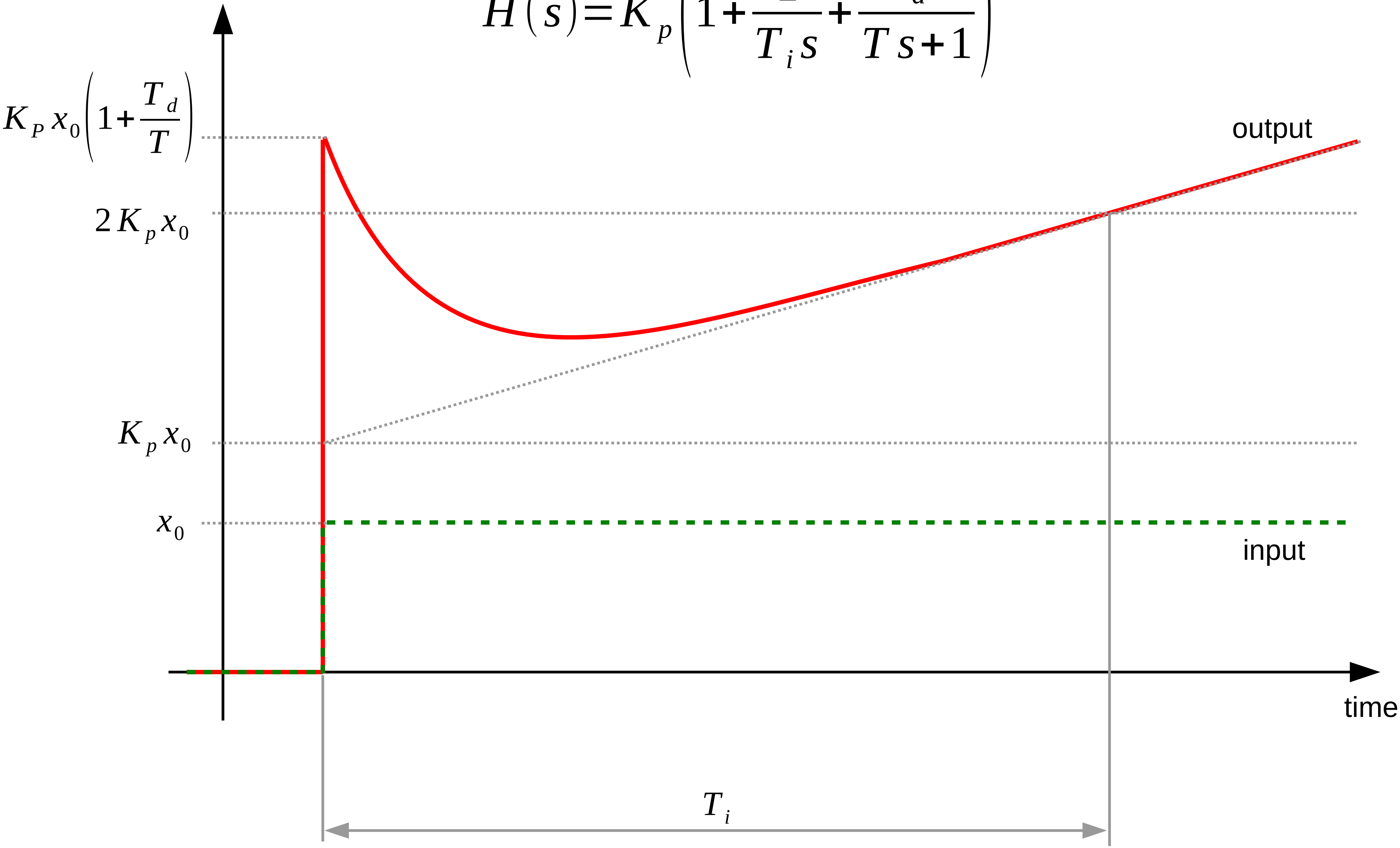
PD - CONTROLLER

$$H(s) = K_p \left(1 + \frac{T_d s}{T s + 1} \right)$$



PID – CONTROLLER
standard form, real derivative

$$H(s)=K_p\left(1+\frac{1}{T_i s}+\frac{T_d s}{T s+1}\right)$$



PID CONTROLLER

important notes

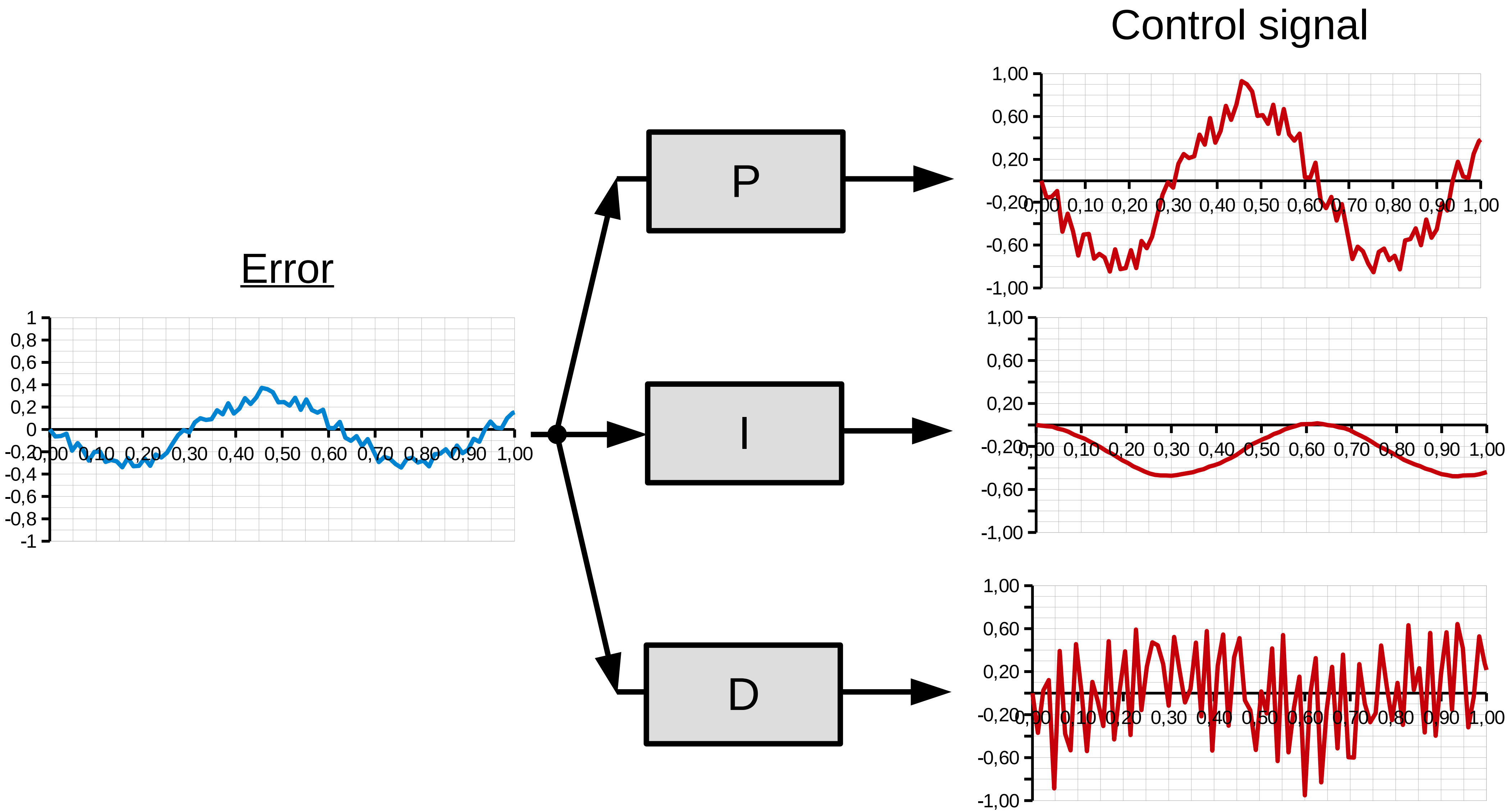
Proportional term – necessary part of the controller, creates a main part of control signal that bring output of the system closer to desired value; higher K_P coefficient gives lover errors; control signal is based on present error;

Integral term – this part of the controller accumulates error; for nonzero error control signal increases that helps to achieve zero error; control signal is based on past error values; “integral windup” problem;

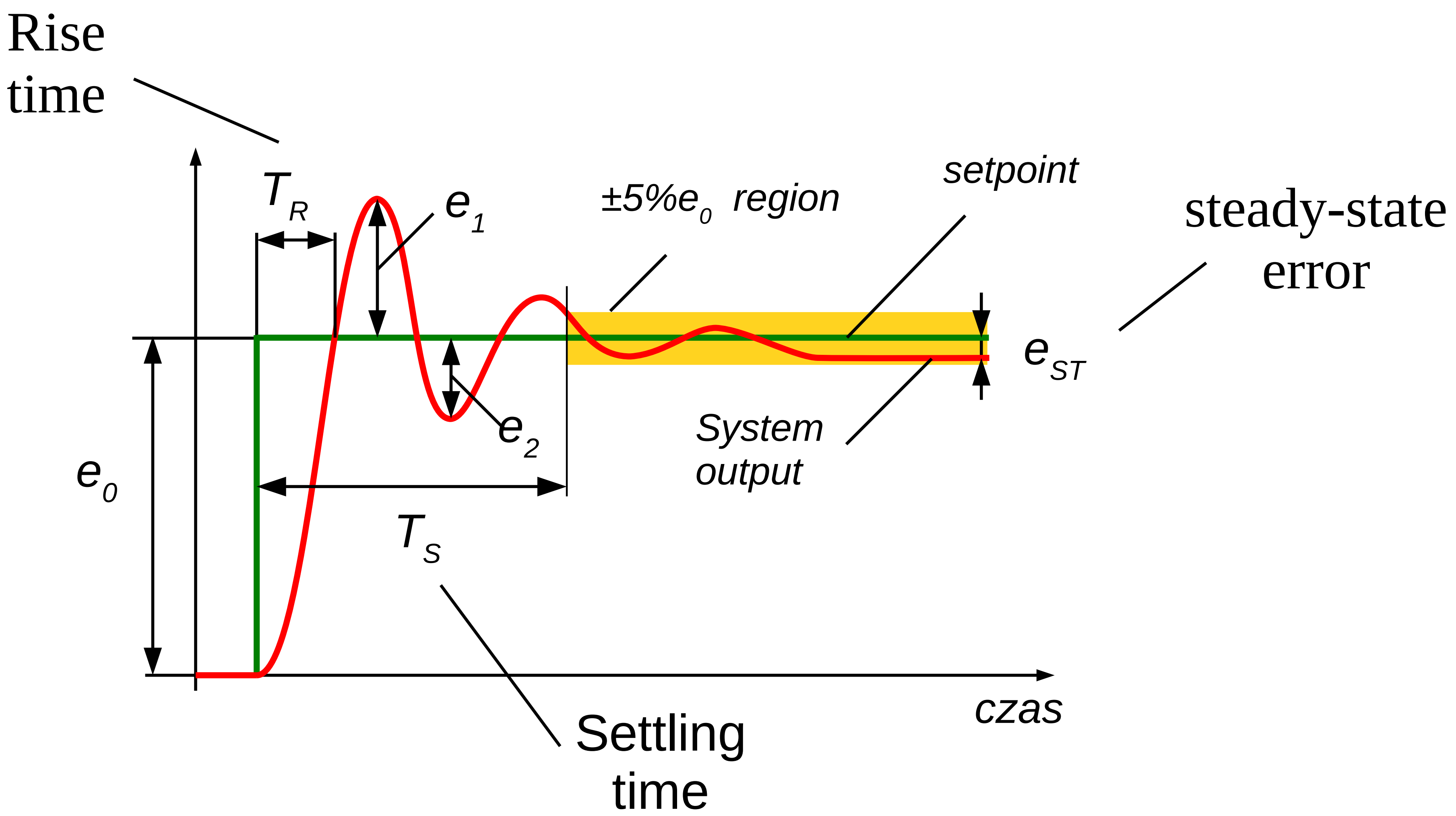
Derivative term – this part of the controller reacts on error changes; for constant error control signal is zero; control signal is based on the trend of future error; this term is very sensitive to noise;

PID CONTROLLER

Influence of errors onto control signal



Quality of the control process



Overshoot: $w = \frac{e_1}{e_0} 100\,\%$

Damping: $d = \frac{e_2}{e_1} 100\,\%$

PID CONTROLLER

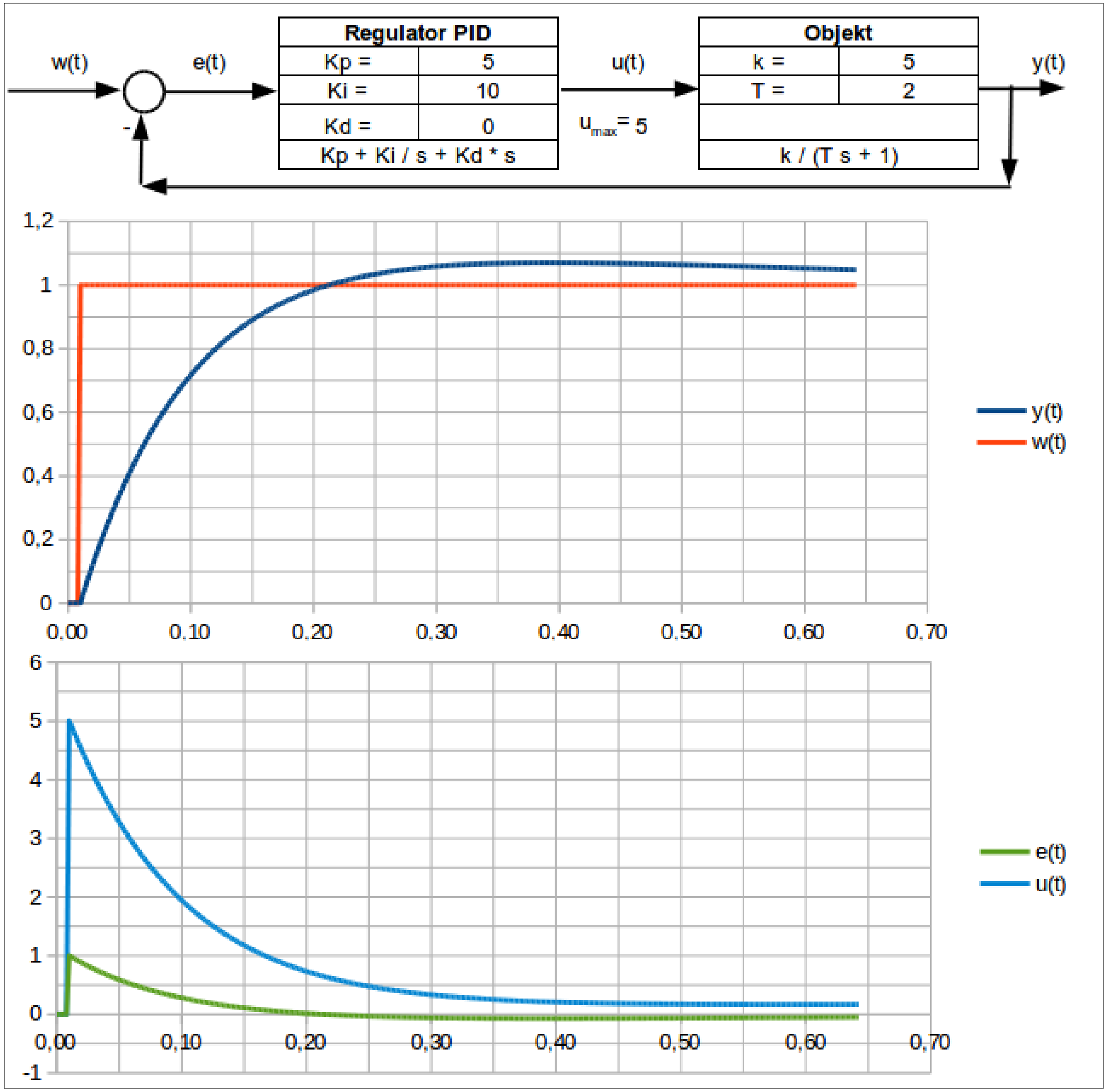
tuning methods

Analytical	With a simulation	Experimental
1st step: calculation of the system's reduced transfer function 2nd step: calculation of the system's step response 3rd step: tuning of the Kp, Ki and Kd coefficients to obtain desired shape of step response	1st step: calculation of the system's reduced transfer function 2nd step: numerical implementation of the system's reduced transfer function 3rd step: tuning of the Kp, Ki and Kd coefficients to obtain desired shape of the system's simulated outputs	Manual tuning or methods: • Ziegler-Nichols • Pessen • Cohen-Coon • Åström–Hägglund

PID CONTROLLER

interactive simulation and tuning

Download spreadsheet file from the website



PID CONTROLLER

Ziegler-Nichols tuning method (PID in standard form)

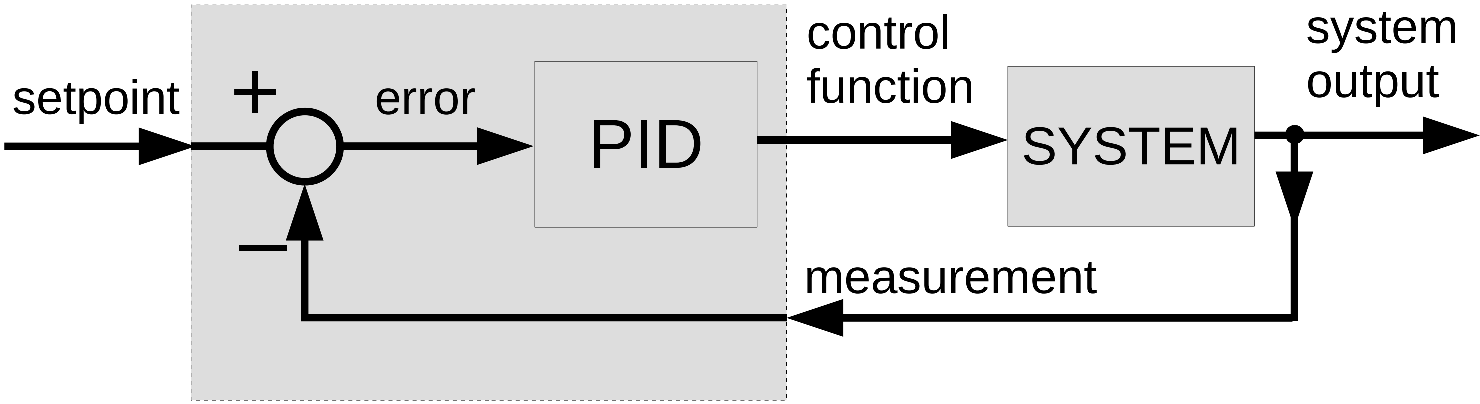
1. Disable integral and derivative terms of the controller. Set proportional gain to small value.
2. Observe a step response of the output of control loop. Go to point 3, if you observe stable and consistent oscillations. If not, increase proportional gain and repeat step 2.

3. For the ultimate gain K_u from step 2 and oscillation period T_u calculate parameters of the controller according to the table:

	k_p	T_i	T_d
Classic Ziegler-Nichols	$0.6\ K_u$	$0.5\ T_u$	$0.125\ T_u$
Pessen	$0.7\ K_u$	$0.4\ T_u$	$0.15\ T_u$
no overshoot	$0.2\ K_u$	$0.5\ T_u$	$0.333\ T_u$

PID CONTROLLER

programming

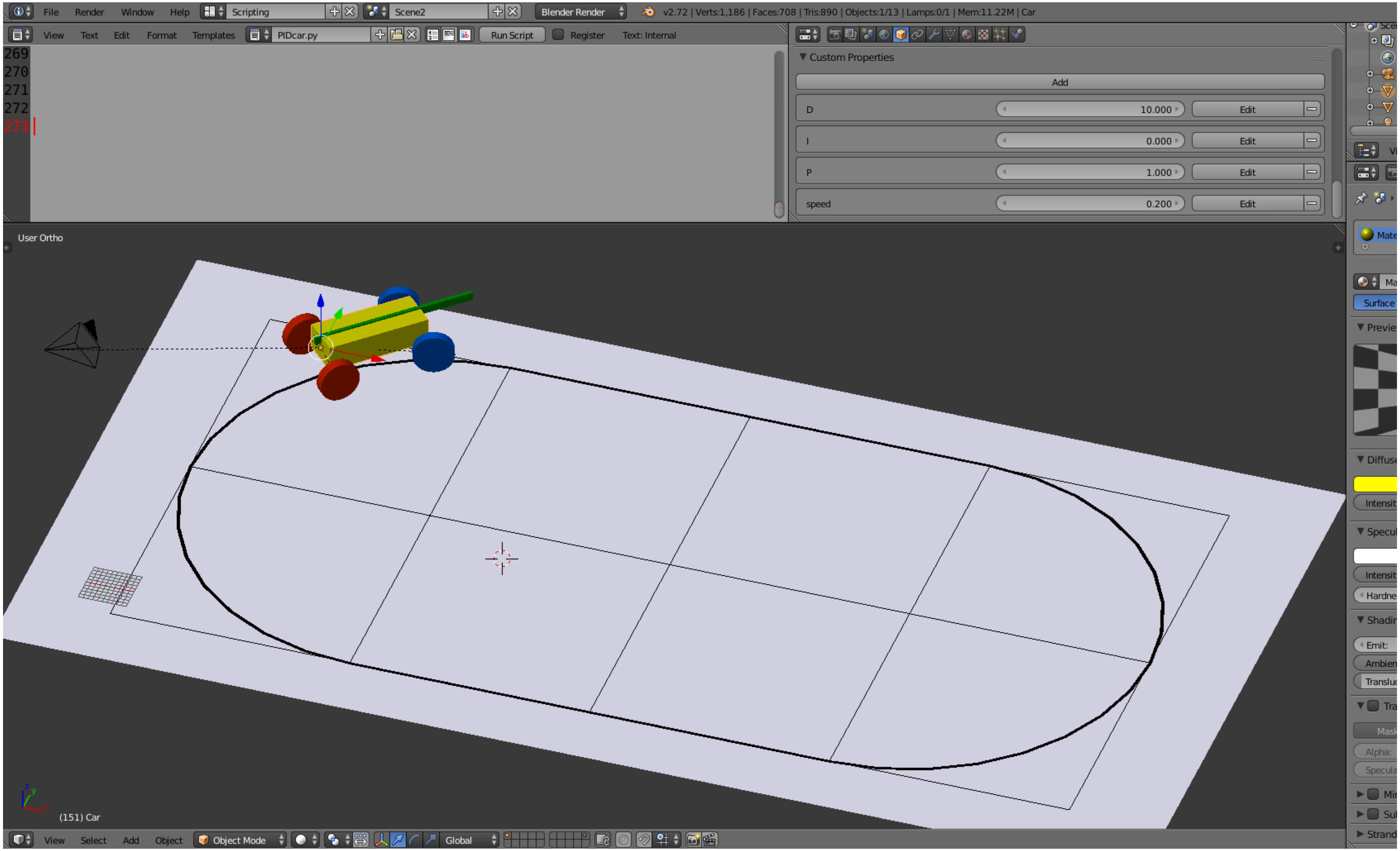



```
dt = 0.1
p_error = 0.
sum = 0.
Kp = 2.
Ki = 0.5
Kd = 0.01
start:
  setpoint = …
  measurement = …
  error = setpoint – measurement
  sum = sum + error * dt
  derivative = (error – p_error) / dt
  output = Kp*error + Ki*sum + Kd*derivative
  p_error = error
  wait(dt)
  goto start
```

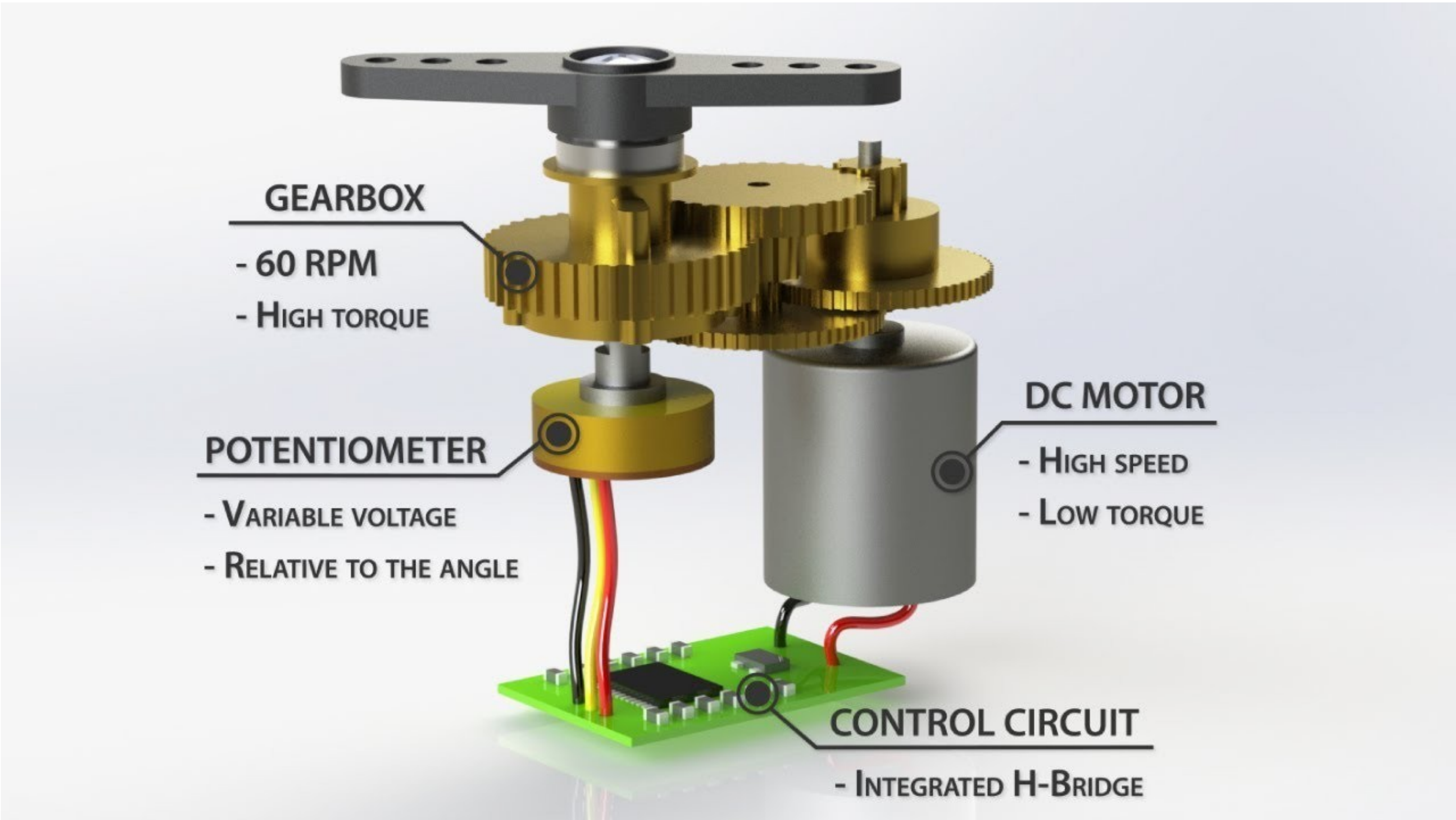
PID CONTROLLER

interactive simulation

PID for a car position control – real-time simulation

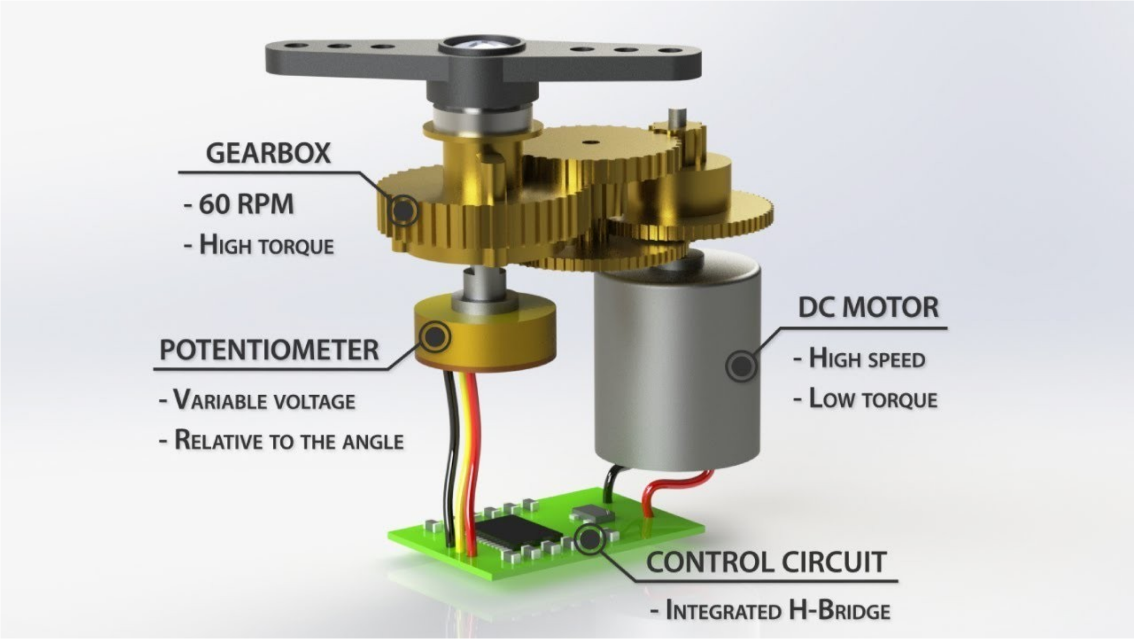


Position control (servomotor)



Source: https://howtomechatronics.com/how-it-works/how-servo-motors-work-how-to-control-servos-using-arduino/

Position control (servomotor)



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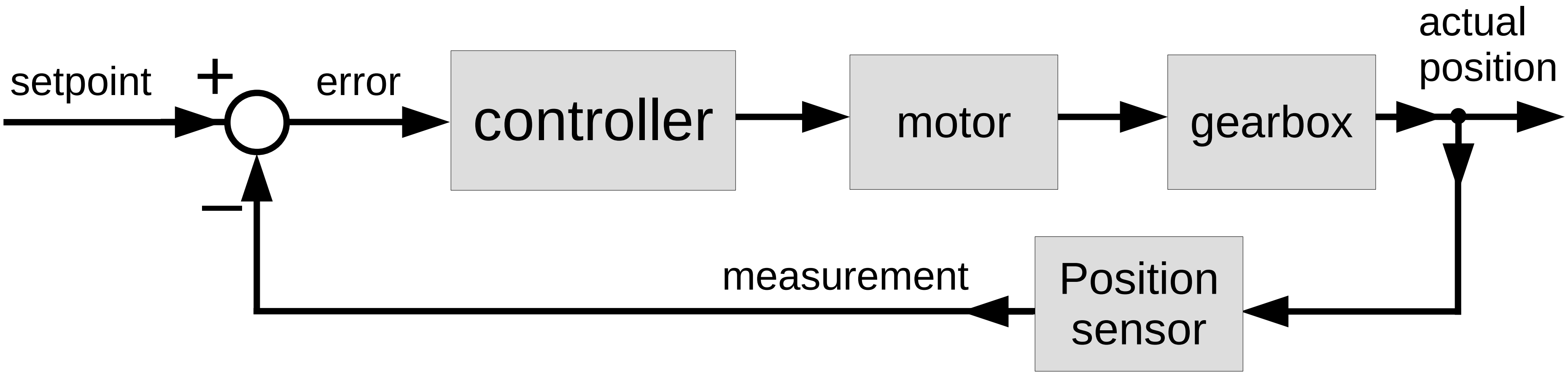


Stability

Stability theory (math) - study of the stability of differential equations' and dynamical systems' trajectories under small perturbations of initial conditions

- Lyapunov stability
- asymptotic stability
- orbital stability
- structural stability

STABILITY CRITERIA

General stability criterion

Hurwitz criterion

Nyquist stability criterion

Lecture 13

Stability criteria.
Gain margin and phase margin.
System correction.
Summing of Bode plots.

General stability criterion – definition

LTI SISO system is asymptotically stable if real part of every pole of the system's transfer function is less than zero.

$$H(s)=\frac{(s-z_1)(s-z_2)\dots(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_n)}$$

$$\mathrm{Re}\, p_1<0 \ \wedge \ \mathrm{Re}\, p_2<0 \ \wedge\dots\wedge \ \mathrm{Re}\, p_n<0$$

Hurwitz criterion – definition

LTI SISO system with a transfer function

$$H(s)=\frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

is stable if:

1. $a_n > 0, \quad a_{n-1} > 0, \quad \ldots, \quad a_1 > 0, \quad a_0 > 0$

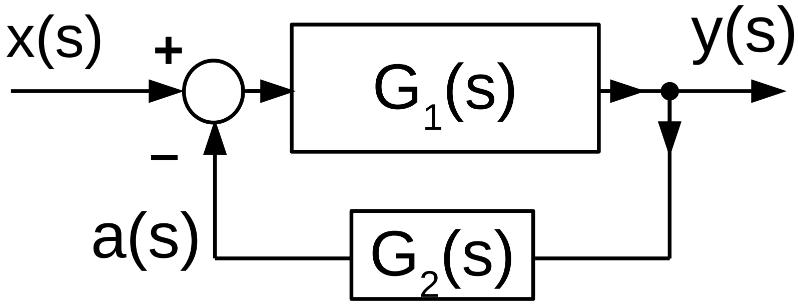
2. $\det \Delta_2 > 0$
   $\det \Delta_3 > 0$
   ...
   $\det \Delta_{n-1} > 0$

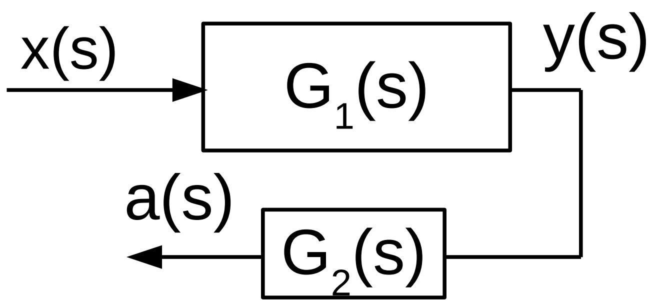
Δ_2 Δ_3 Δ_{n-1}

$$M_n = \begin{bmatrix} a_{n-1} & a_n & 0 & 0 & 0 & 0 \\ a_{n-3} & a_{n-2} & a_{n-1} & a_n & 0 & 0 \\ a_{n-5} & a_{n-4} & a_{n-3} & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & a_0 & a_1 & a_2 \\ 0 & 0 & 0 & 0 & 0 & a_0 \end{bmatrix}$$

Δ_i *- leading principal minor of order i*

Nyquist stability criterion – idea

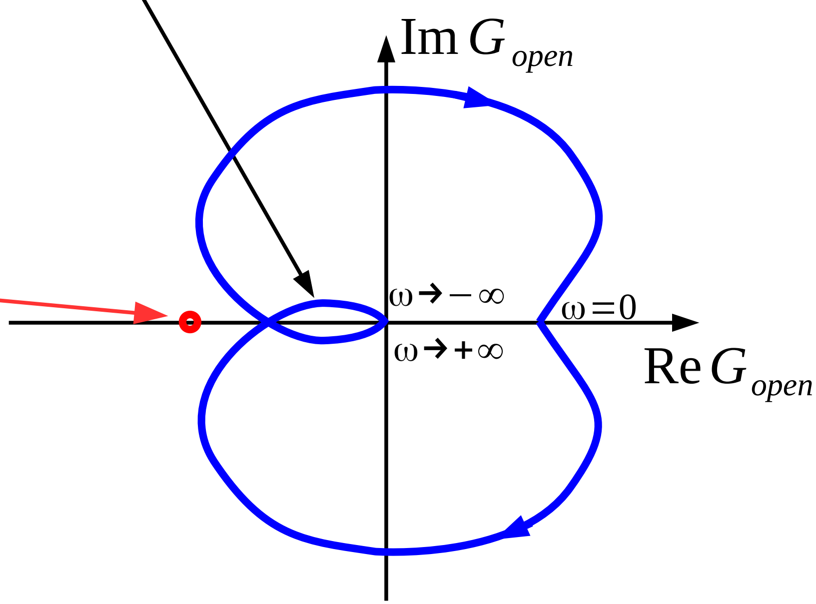




$$G_z(s)=\frac{y(s)}{x(s)}=\frac{G_1(s)}{1+G_1(s)G_2(s)}$$

$$G_{open}(s)=\frac{a(s)}{x(s)}=G_1(s)G_2(s)$$

Unstable if: $G_1 G_2 = -1$



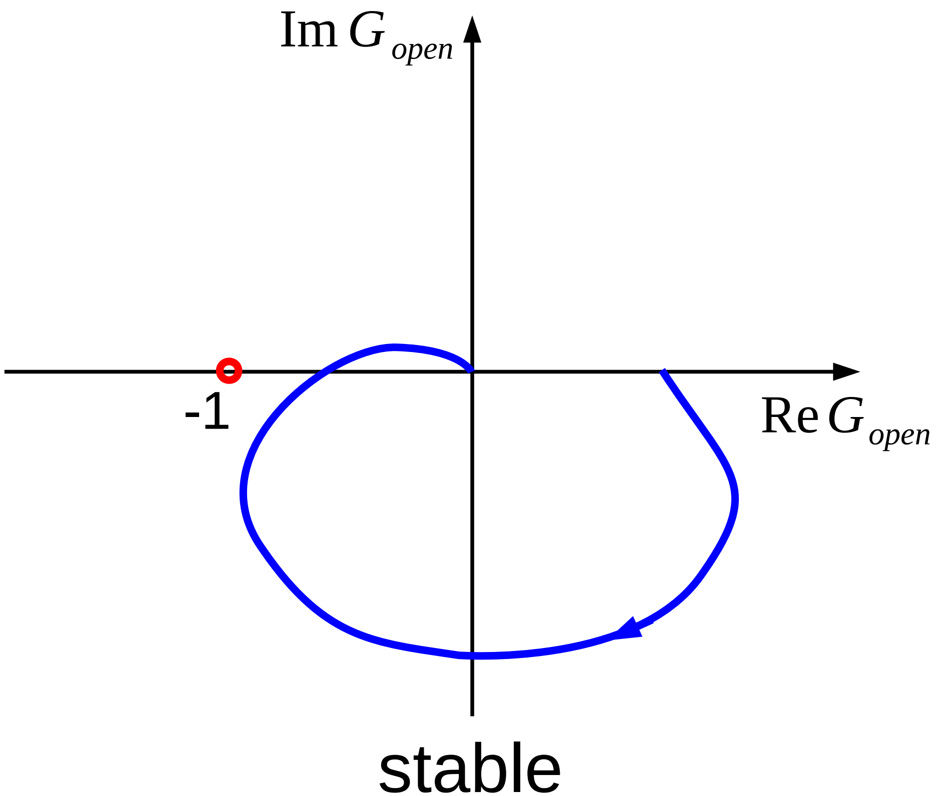
Particular Nyquist stability criterion – definition

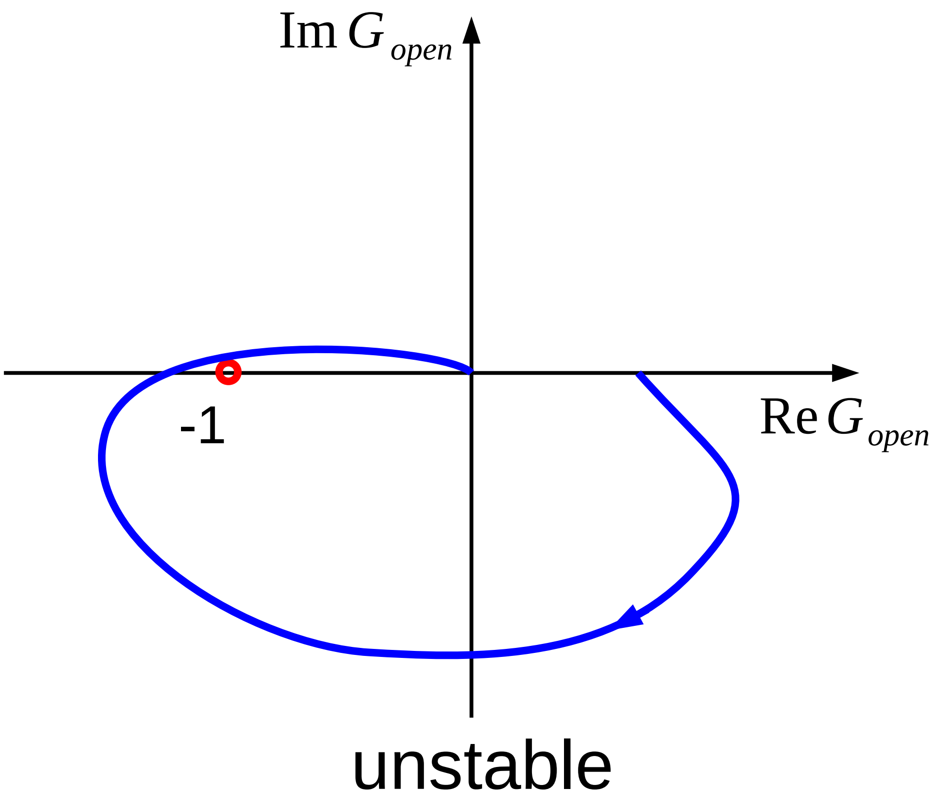
The closed-loop system with feedback is stable if:

1) open-loop transfer function is stable

AND

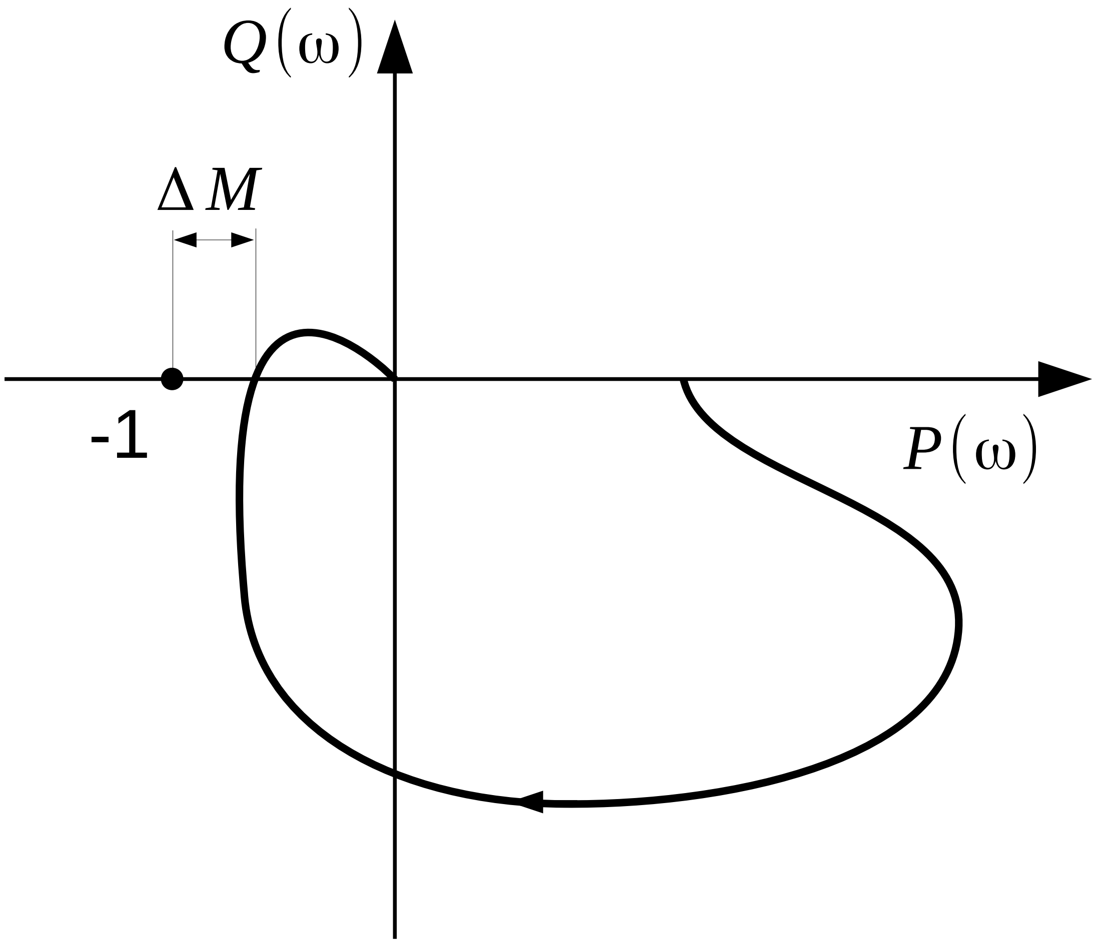
2) open-loop transfer function not enclosing the point (-1,j0).

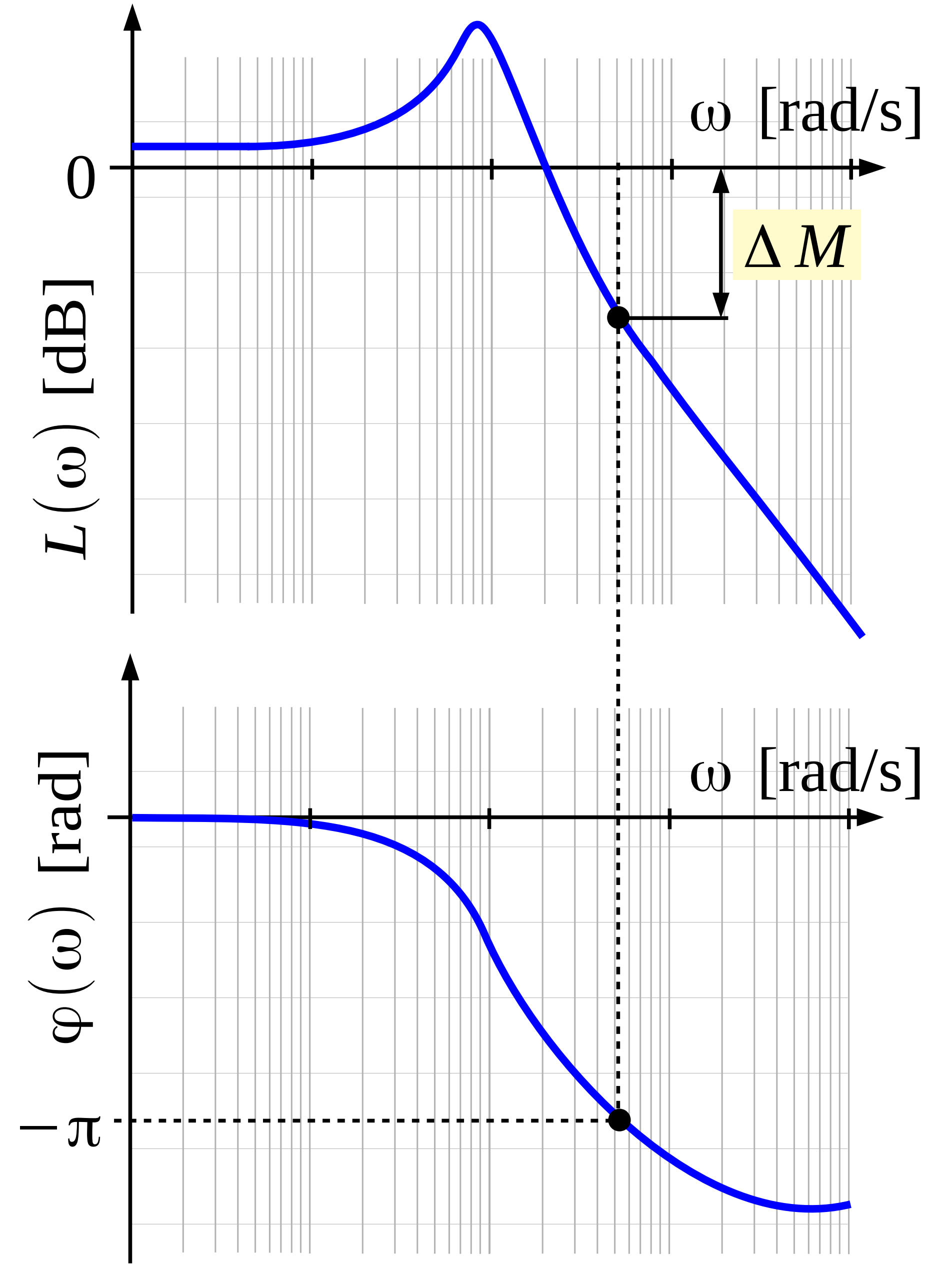




Gain margin

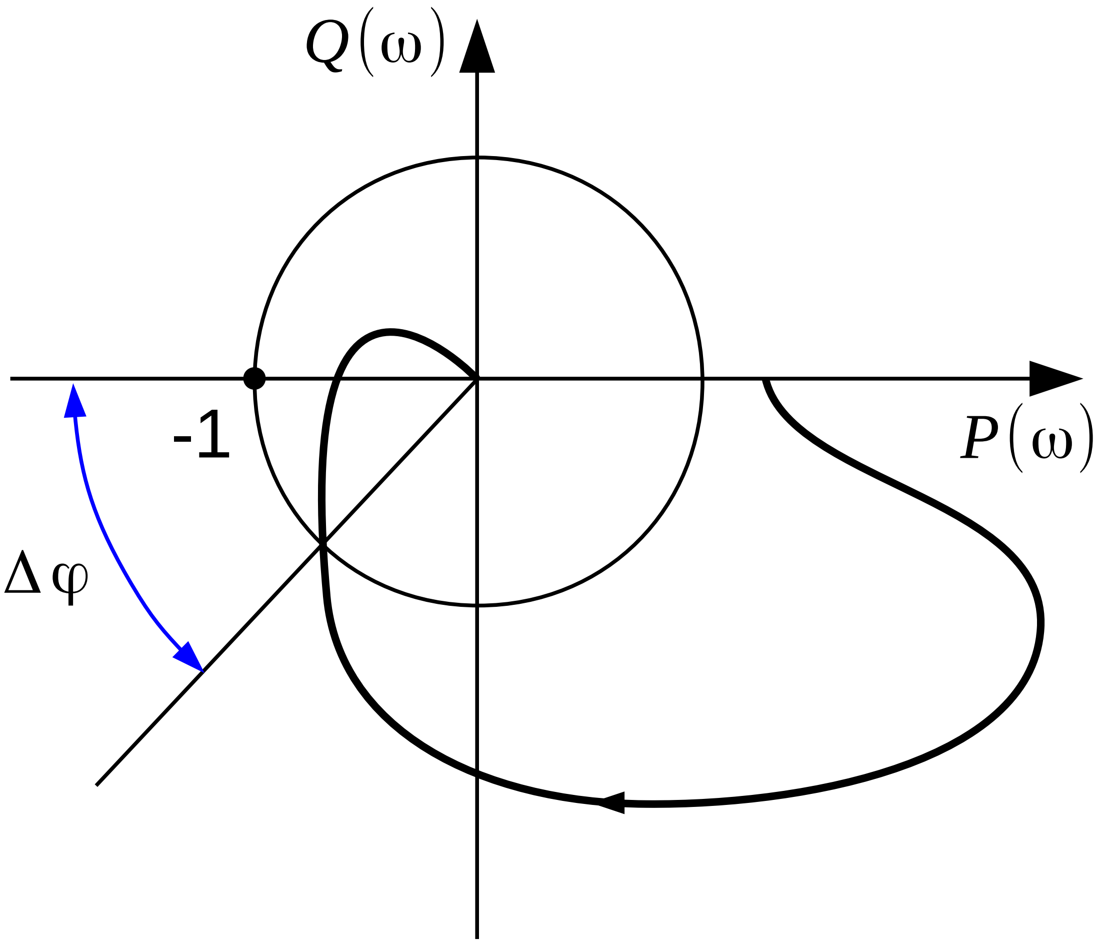
Closed-loop system will loose its stability if we add additional gain (in serial) greater or equals to gain margin.

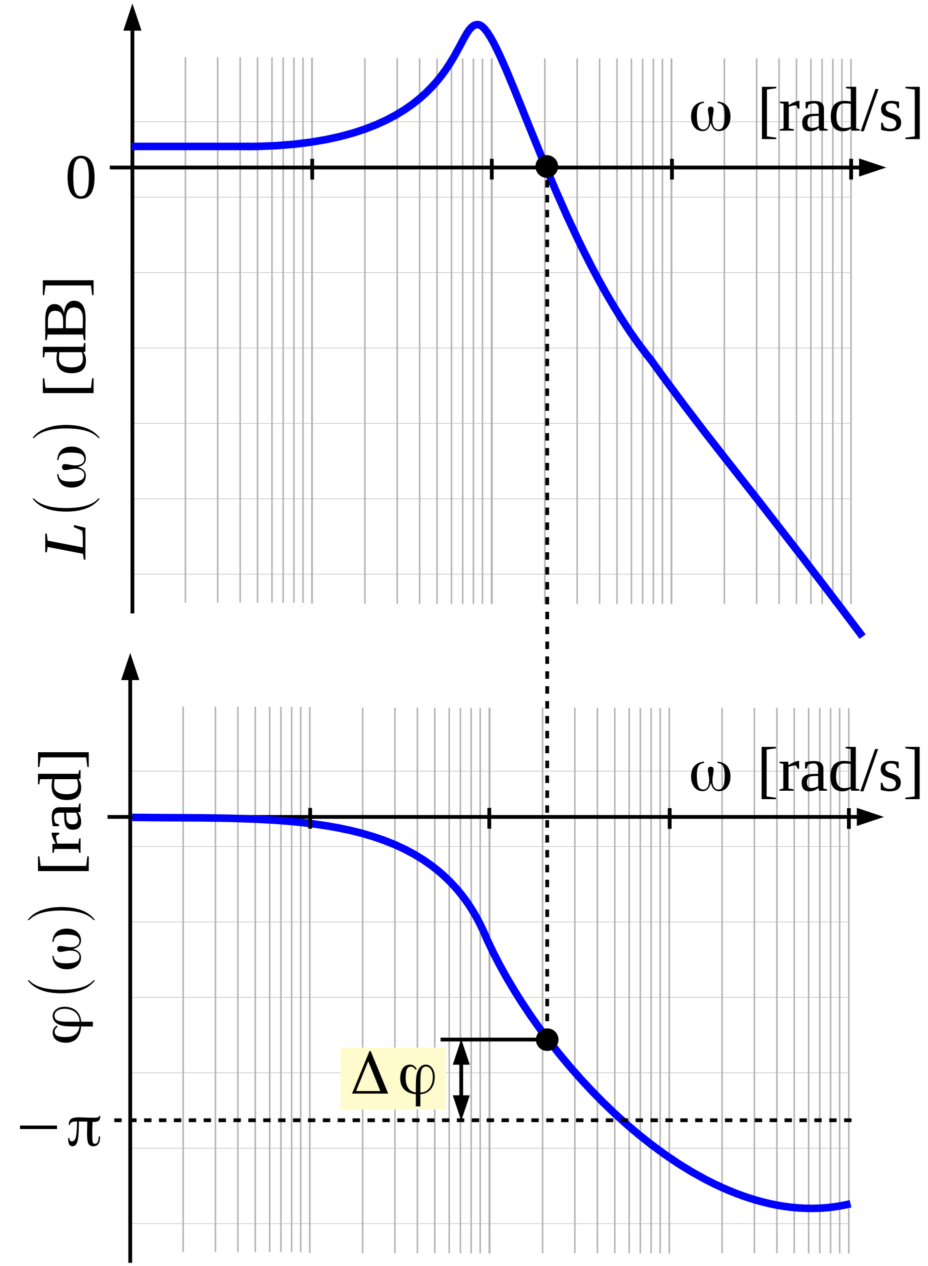




Phase margin

Closed-loop system will loose its stability if we add additional delay (in serial) greater or equals to phase margin.





State-space representation

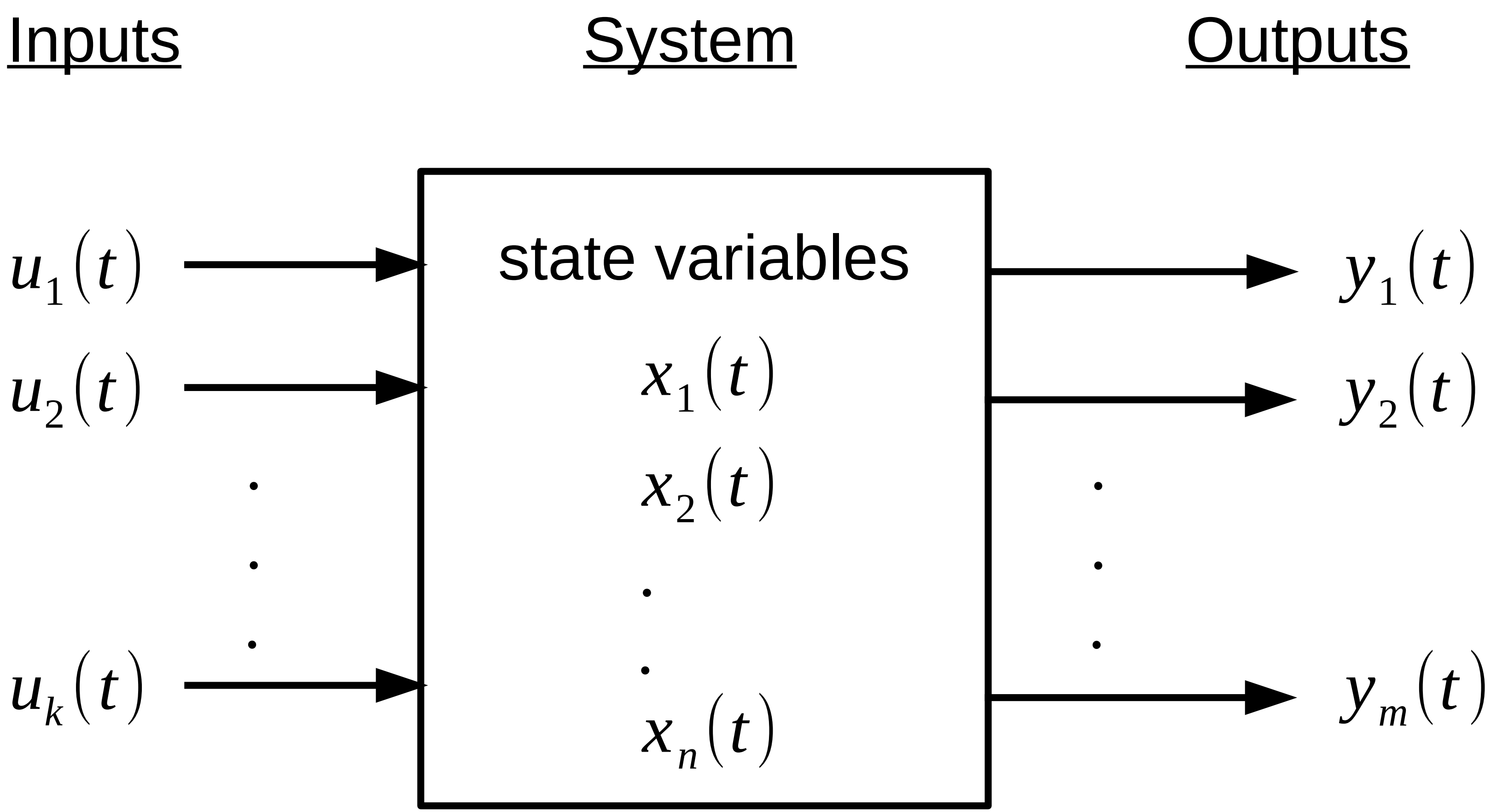
State-space representation is an alternative to transfer function form for writing system models.

State variable or set of state variables – representation of status of a system at any time.

Typical state variables: position, velocity, temperature, pressure, volume flow, current, voltage.

There are many different state variable representations for the same system, but input-output relation does not depend on its' selection.

State-space representation



State-space representation

For continouse, linear and time-invariant system

State-space equation: $\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\,\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)$

$\boldsymbol{x}_{n\,x\,1}(t)$ - state variables vector

$\boldsymbol{A}_{n\,x\,n}$ - state matrix (system matrix)

$\boldsymbol{B}_{n\,x\,k}$ - input matrix

$\boldsymbol{u}_{k\,x\,1}(t)$ - control inputs

External outputs equation: $\boldsymbol{y}(t)=\boldsymbol{C}\,\boldsymbol{x}(t)+\boldsymbol{D}\,\boldsymbol{u}(t)$

$\boldsymbol{y}_{m\,x\,1}(t)$ - outputs vector

$\boldsymbol{C}_{m\,x\,n}$ - output matrix

$\boldsymbol{D}_{m\,x\,k}$ - transmition matrix (direct feedthrough matrix)

$\boldsymbol{u}_{k\,x\,1}(t)$ - control inputs

State-space representation

Example for n = 2, k = 4, m = 3

State-space equation: $\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\,\boldsymbol{x}(t)+\boldsymbol{B}\,\boldsymbol{u}(t)$

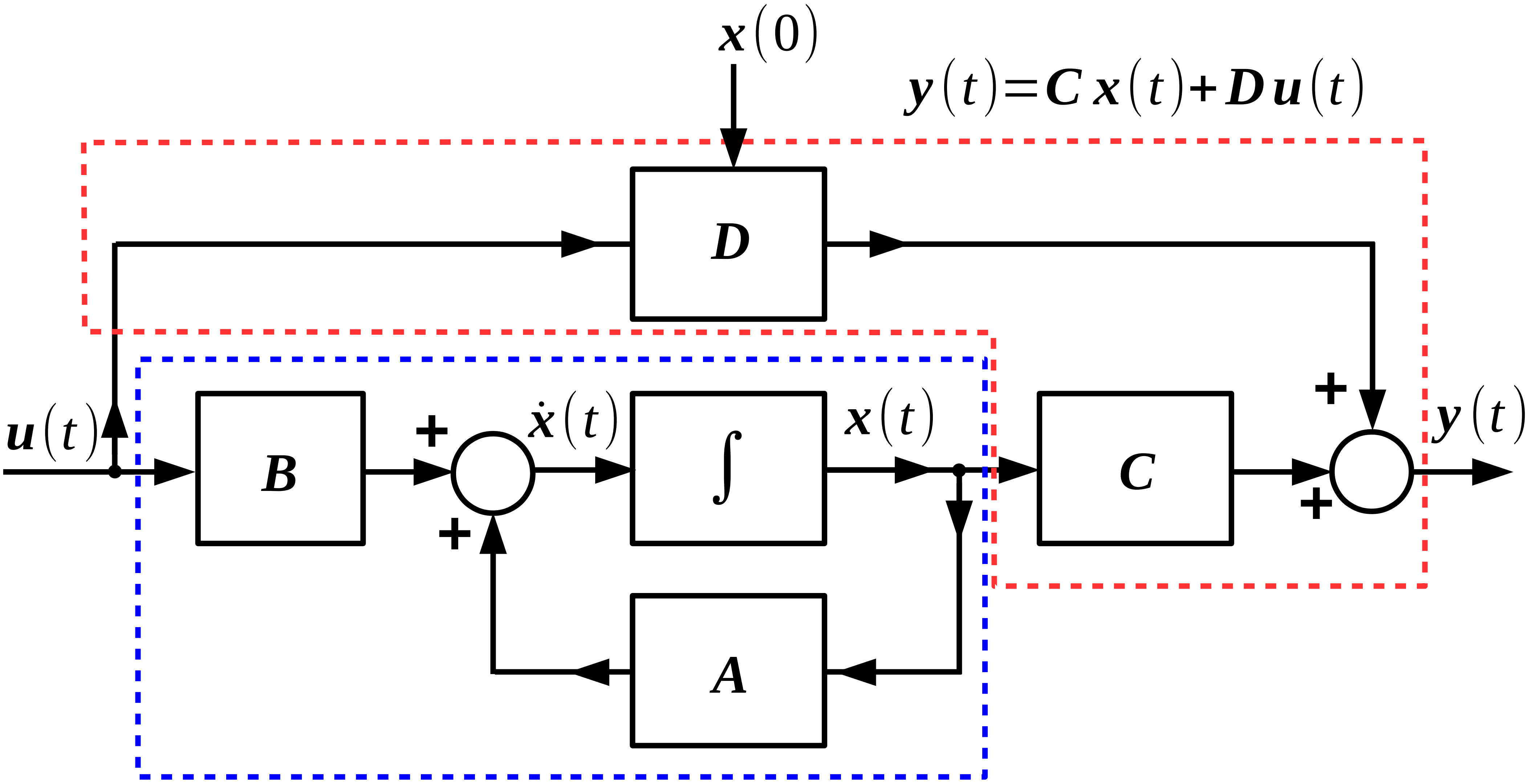
$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}B_{11} & B_{12} & B_{13} & B_{14}\\ B_{21} & B_{22} & B_{23} & B_{24}\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\\ u_3(t)\\ u_4(t)\end{bmatrix}$$

Outputs equation: $\boldsymbol{y}(t)=\boldsymbol{C}\,\boldsymbol{x}(t)+\boldsymbol{D}\,\boldsymbol{u}(t)$

$$\begin{bmatrix}y_1(t)\\ y_2(t)\\ y_3(t)\end{bmatrix}=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\\ C_{31} & C_{32}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}D_{11} & D_{12} & D_{13} & D_{14}\\ D_{21} & D_{22} & D_{23} & D_{24}\\ D_{31} & D_{32} & D_{33} & D_{34}\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\\ u_3(t)\\ u_4(t)\end{bmatrix}$$

State-space representation

Time-domain block diagram representation



$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\,\boldsymbol{x}(t) + \boldsymbol{B}\,\boldsymbol{u}(t)$$

State-space representation to transfer function conversion

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\,\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t) \qquad \boldsymbol{y}(t)=\boldsymbol{C}\,\boldsymbol{x}(t)+\boldsymbol{D}\boldsymbol{u}(t)$$

$\mathcal{L}$ + zero IC

$$s\,\boldsymbol{X}(s)=\boldsymbol{A}\,\boldsymbol{X}(s)+\boldsymbol{B}\boldsymbol{U}(s)$$

$$s\,\boldsymbol{X}(s)-\boldsymbol{A}\,\boldsymbol{X}(s)=\boldsymbol{B}\boldsymbol{U}(s)$$

$$(s\boldsymbol{I}-\boldsymbol{A})\boldsymbol{X}(s)=\boldsymbol{B}\boldsymbol{U}(s)$$

for $\det(s\boldsymbol{I}-\boldsymbol{A})\neq 0$

$$\boldsymbol{X}(s)=(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}\boldsymbol{U}(s)$$

$\mathcal{L}$ + zero IC

$$\boldsymbol{Y}(s)=\boldsymbol{C}\,\boldsymbol{X}(s)+\boldsymbol{D}\boldsymbol{U}(s)$$

$$\boldsymbol{Y}(s)=\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}\boldsymbol{U}(s)+\boldsymbol{D}\boldsymbol{U}(s)$$

$$\boldsymbol{Y}(s)=\left(\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D}\right)\boldsymbol{U}(s)$$

$$\boldsymbol{H}(s)=\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D}$$

List of important exam topics

- Mechanism mobility
- Graphical method
- Procedure of analytical method
- Equation of machine motion
- Nonuniformity and flywheel
- Transfer function of LTI SISO
- Step response & Bode Plot of a given system
- Block diagram algebra
- PID controller & tuning
- General stability criterion
- Hurwitz criterion
- particular Nyquist criterion
- state-space representation

Materials for exam – lectures from 1 to 14

Lecture 15 – modern control theory overview,
experiment with control system,
Consultations.

Exam: Wednesday, 5th February, 12:00-13:30, room 2.19
Wednesday, 12th February, 12:00-13:30, room 2.19

EXAM – IMPORTANT NOTES

- You have to pass the project class to attend the exam.
- Student card or erasmus paper is needed on the exam.
- Please write the exam clearly on the A4 paper.
- Everyone must to return the exam.
- You can not use any electronic devices during the exam (mobile phones, smart watches, calculators).
- Table of Laplce transform will be displayed on the screen.
- Additional persons are delegated to help during the exam.
- Any cheating behaviors will cause exam failure.
- Topics will be distributed in printed form or displayed.

EXAM – IMPORTANT NOTES

- Your answers will be rated with points.
- Exam mark will be based on the total number of points achieved with the rules:
  - < 50% - mark 2 (exam failed)
  - 51%-60% - mark 3,0
  - 61%-70% - mark 3,5
  - 71%-80% - mark 4,0
  - 81%-90% - mark 4,5
  - >90% - mark 5,0
- If marks from project class and exam are positive, then

  Final_mark = 0.5 * project_mark + 0.5 * exam_mark

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consultations: Tuesdays at 11:00-12:00 and Fridays at 13:00-14:00

website with presentations: http://myinventions.pl/students/