Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control

Winter 2019/2020

Lecturer: Sebastian Korczak, PhD Eng.

Lecture 11

Transfer function analysis – example.
Block diagram algebra.
Control and controllers.

Example

Calculate and sketch step response and Bode Plots for a system with transfer function H(s) = 3/s.

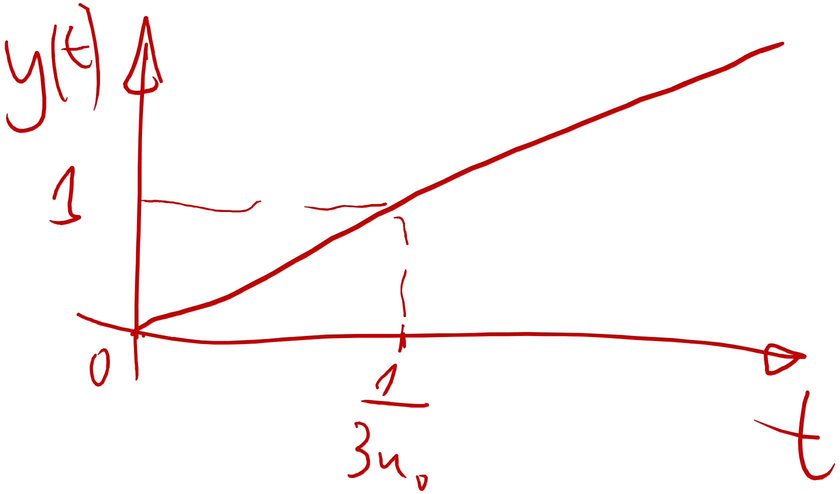
$f(t)$	$F(s)$
t	$\frac{1}{s^2}$

1. STEP RESPONSE

$$u(t) = u_0 \mathbb{1}(t) \;;\; U(s) = u_0 \frac{1}{s}$$

$$Y(s) = U(s) \cdot H(s) = u_0 \frac{1}{s} \cdot \frac{3}{s} = \frac{3u_0}{s^2}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = 3u_0 t$$



Example

Calculate and sketch step response and Bode Plots for a system with transfer function H(s) = 3/s.

2. Bode plot

$$H(j\omega) = 3/j\omega = \frac{3}{j\omega} \cdot \frac{j}{j} = \frac{3j}{j^2\omega} = -\frac{3j}{\omega}$$

$$P(\omega) = 0 \; ; \quad Q(\omega) = -\frac{3}{\omega}$$

$$A(\omega) = \sqrt{P^2 + Q^2} = \left|\frac{3}{\omega}\right|$$

$$L(\omega)\,[dB] = 20 \log A(\omega) = 20 \log \left|\frac{3}{\omega}\right|$$

$$\varphi(\omega)\,[rad] = atan \frac{Q}{P} = atan\left(\frac{-\frac{3}{\omega}}{0}\right) = atan(-\infty) = -\frac{\pi}{2}$$

Example

Calculate and sketch step response and Bode Plots for a system with transfer function $H(s) = 3/s$.

Example

Calculate and sketch step response and Bode Plots for a system with transfer function H(s) = 3/s.

$$L(\omega)\,[\mathrm{dB}] = 20\log\left|\frac{3}{\omega}\right|$$

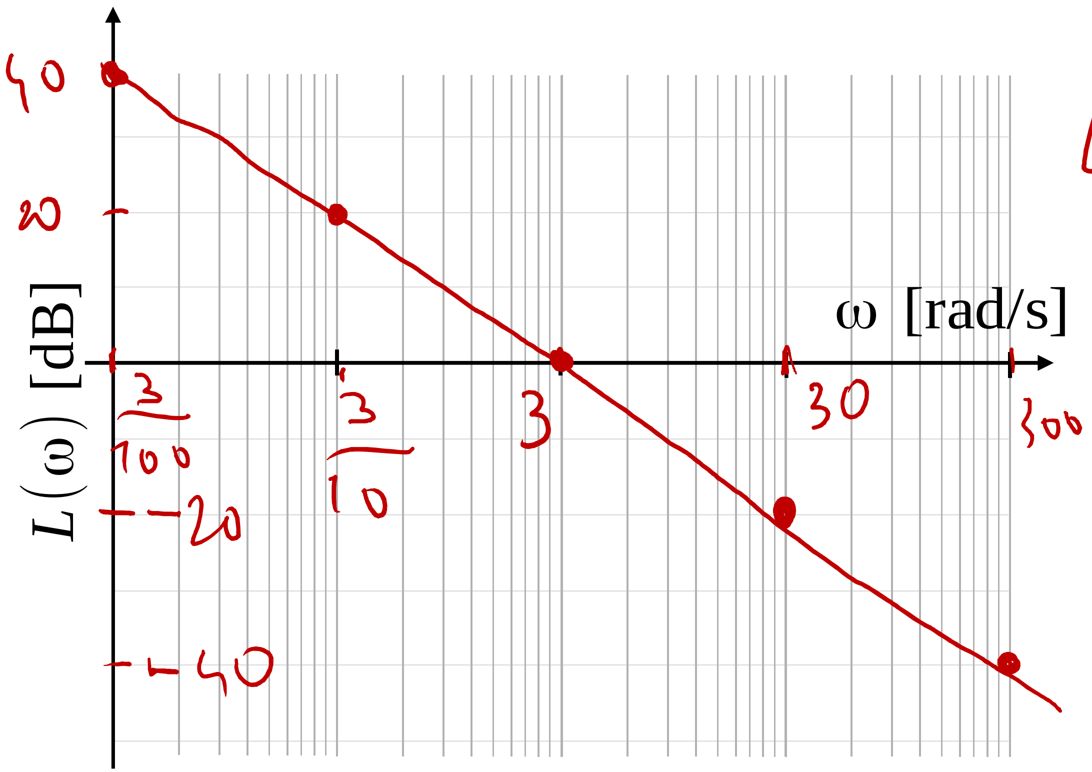
$$L(\omega=3) = 20\log\left|\frac{3}{3}\right| = 0$$

$$L(\omega=30) = 20\log\left|\frac{3}{30}\right| = -20$$

$$L(\omega=300) = 20\log\left|\frac{3}{300}\right| = -40$$

$$L\left(\omega=\frac{3}{10}\right) = 20\log 10 = 20$$

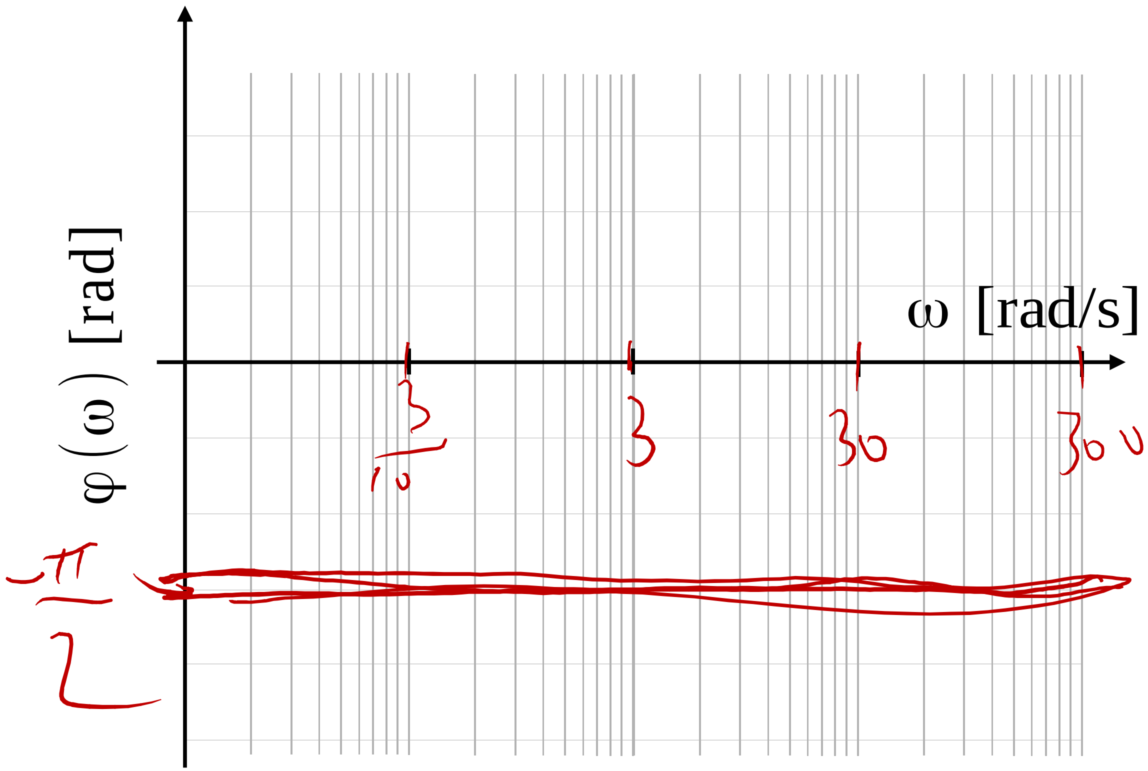
$$L\left(\omega=\frac{3}{100}\right) = 20\log 100 = 40$$



Example

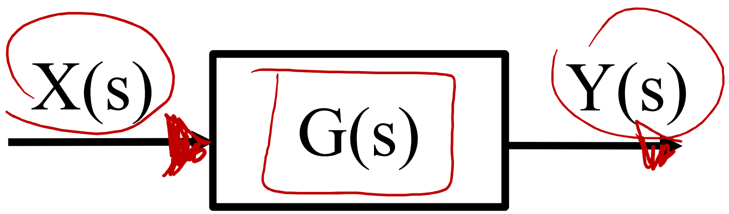
Calculate and sketch step response and Bode Plots for a system with transfer function H(s) = 3/s.

$\varphi(\omega) = -\frac{\pi}{2}$

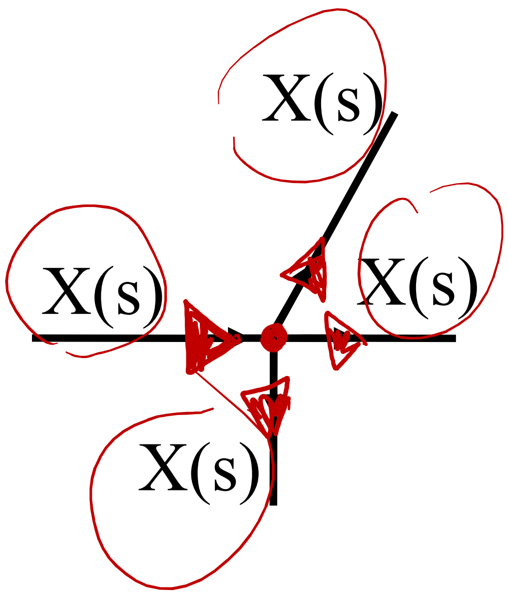


BLOCK DIAGRAM ALGEBRA

BLOCK DIAGRAM ALGEBRA
Transfer function (SISO system)

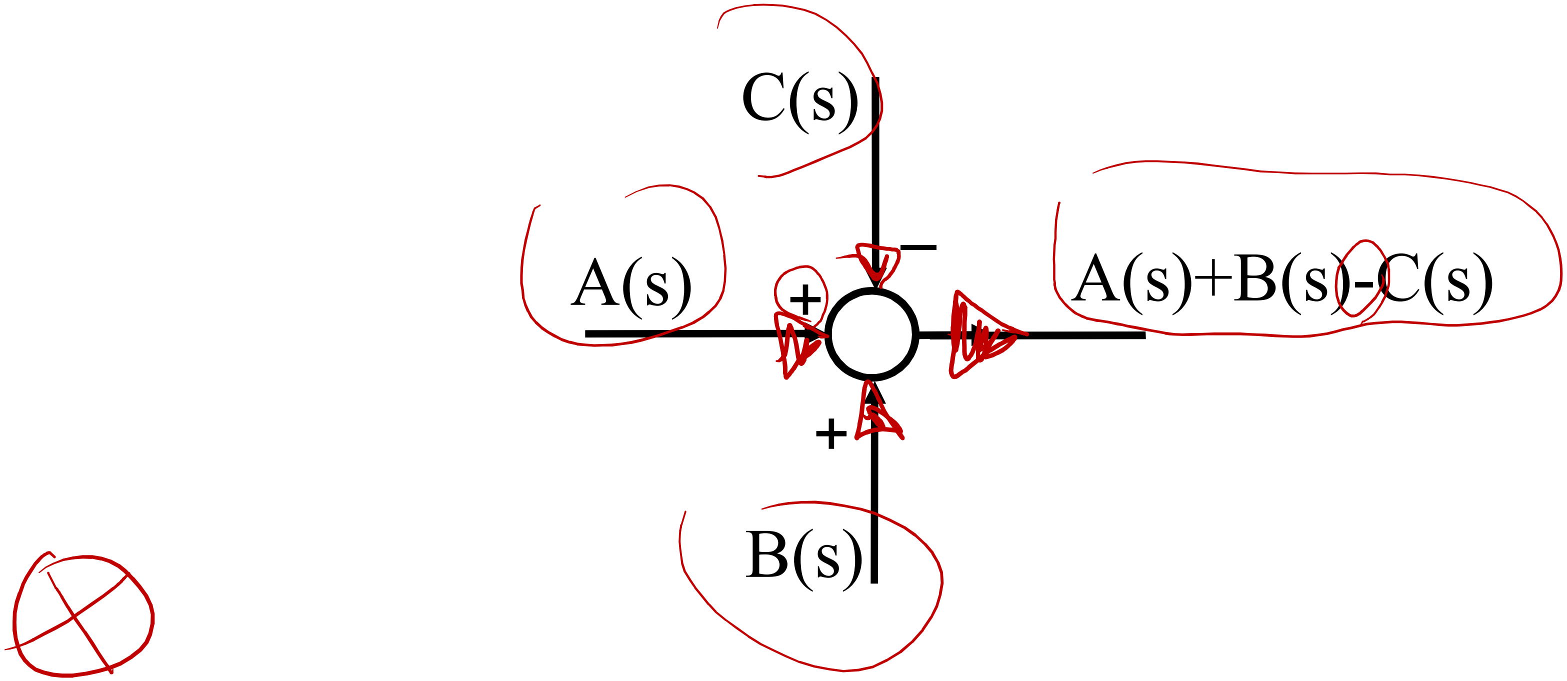


BLOCK DIAGRAM ALGEBRA
information node



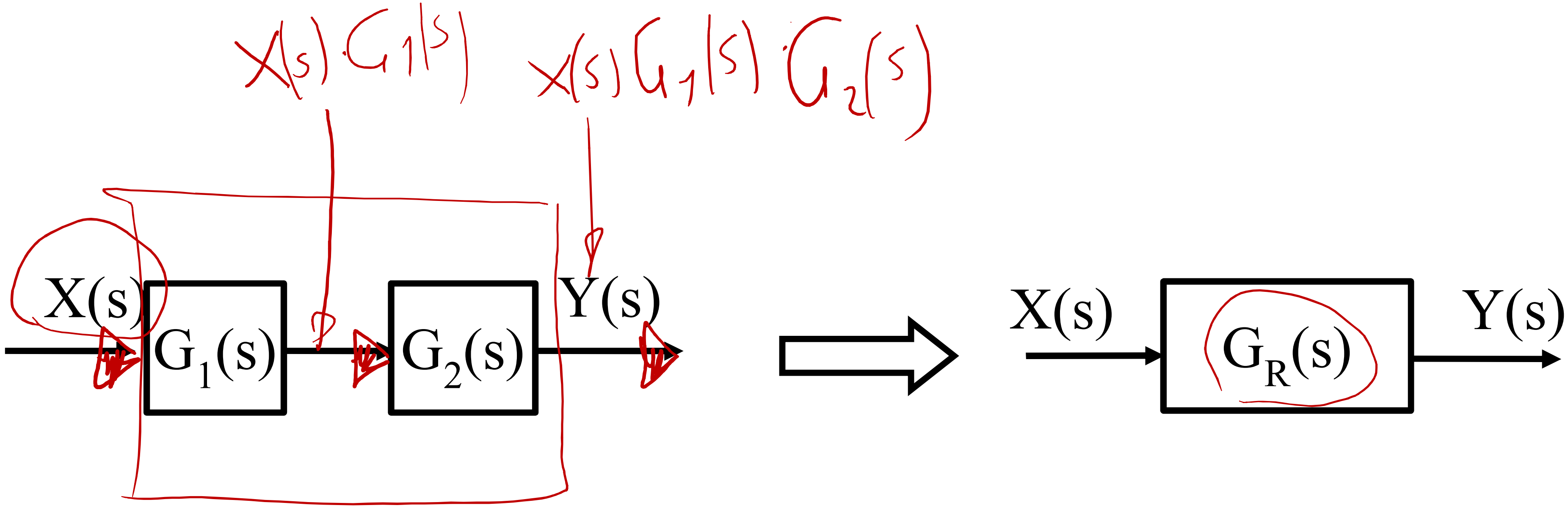
one input,
a few outputs,

BLOCK DIAGRAM ALGEBRA
sum node



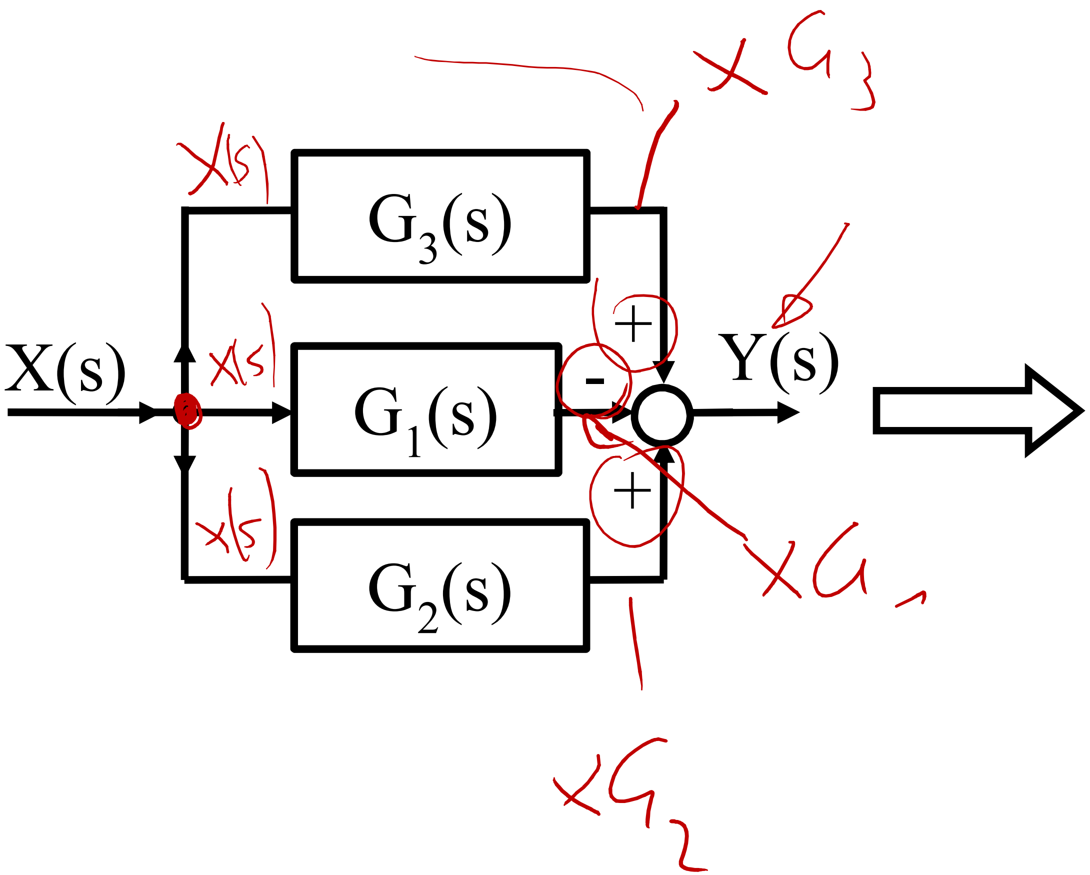
a few inputs,
one output,

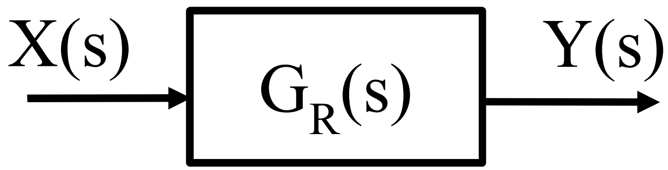
BLOCK DIAGRAM ALGEBRA
serial connection



$$G_R(s)=G_1(s)\,G_2(s)$$

BLOCK DIAGRAM ALGEBRA
parallel connection

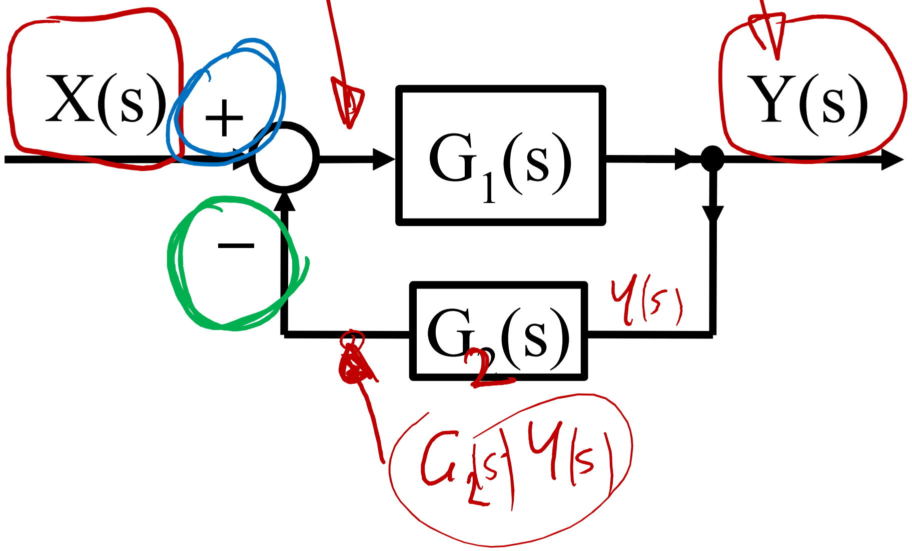


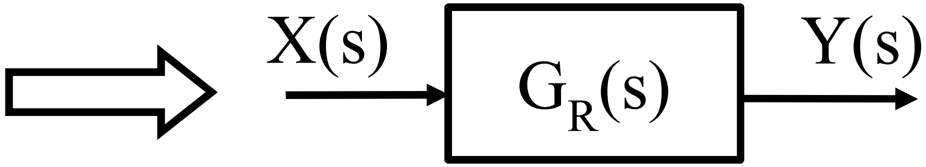


$$G_R(s) = -G_1(s) + G_2(s) + G_3(s)$$

BLOCK DIAGRAM ALGEBRA

feedback





$$G_R = \frac{G_1}{1 + G_1 G_2}$$

$X(s) - G_2 Y(s)$

$Y(s) = (X - G_2 Y) \cdot G_1$

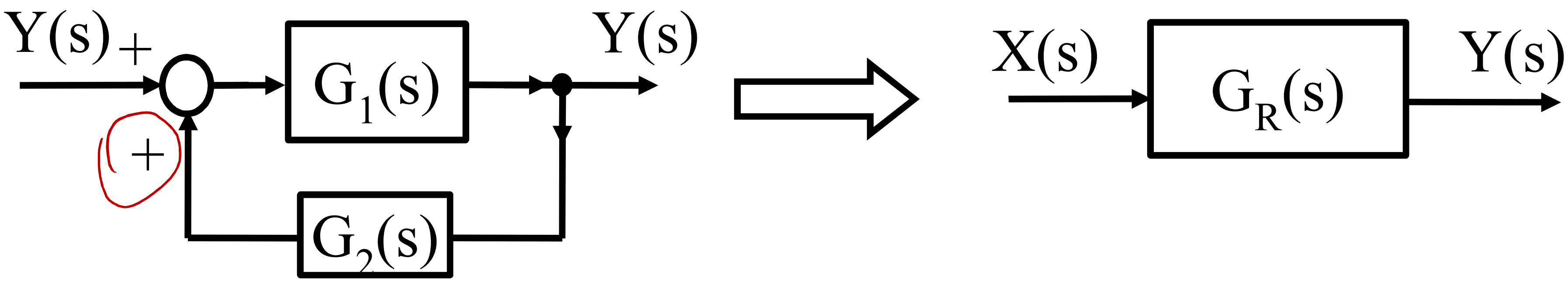
$G_2(s)\,Y(s)$

$Y = X G_1 - G_1 G_2 Y$

$Y(1 + G_1 G_2) = X G_1$

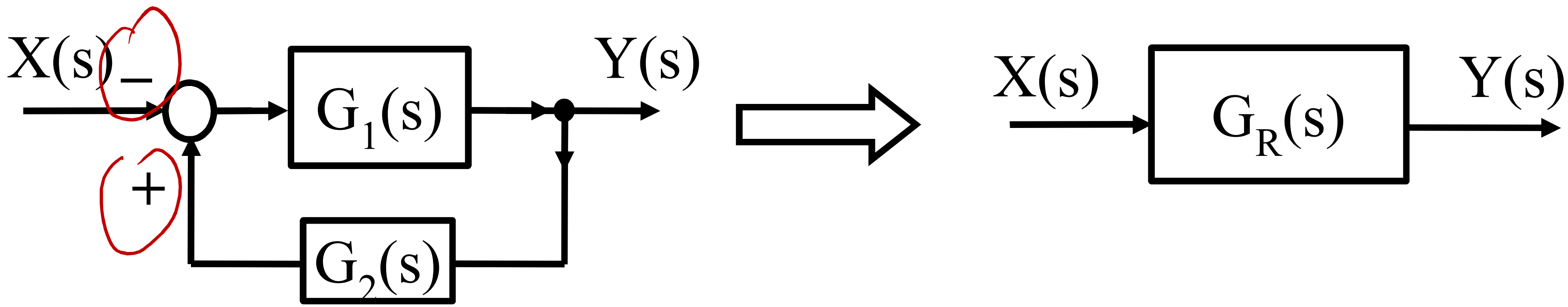
$\frac{Y}{X} = \frac{G_1}{1 + G_1 G_2}$

BLOCK DIAGRAM ALGEBRA
feedback



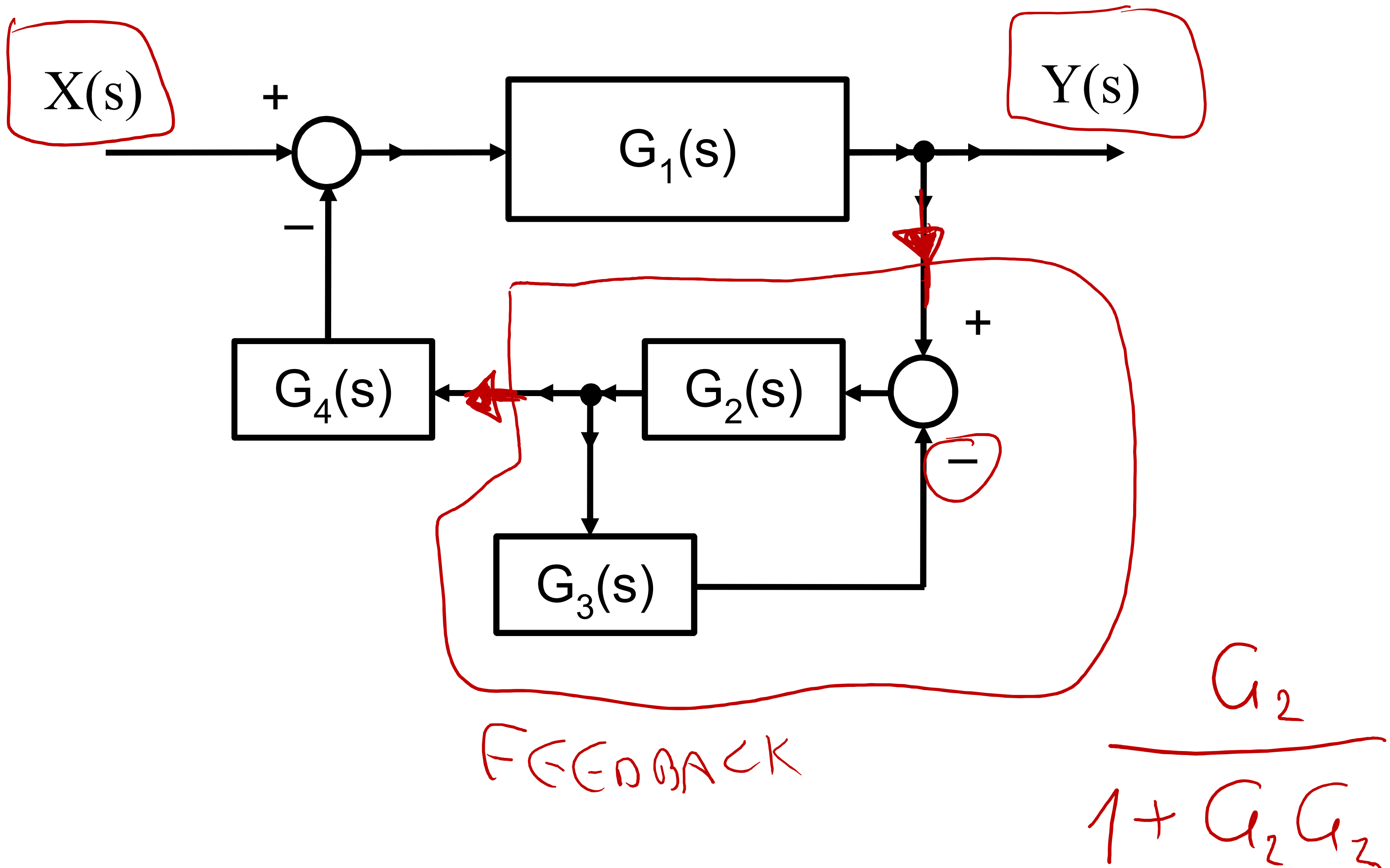
$$G_R = \frac{G_1}{1 - G_1 G_2}$$

BLOCK DIAGRAM ALGEBRA
feedback



$$G_R = \frac{-G_1}{1 - G_1 G_2}$$

EXAMPLE 1



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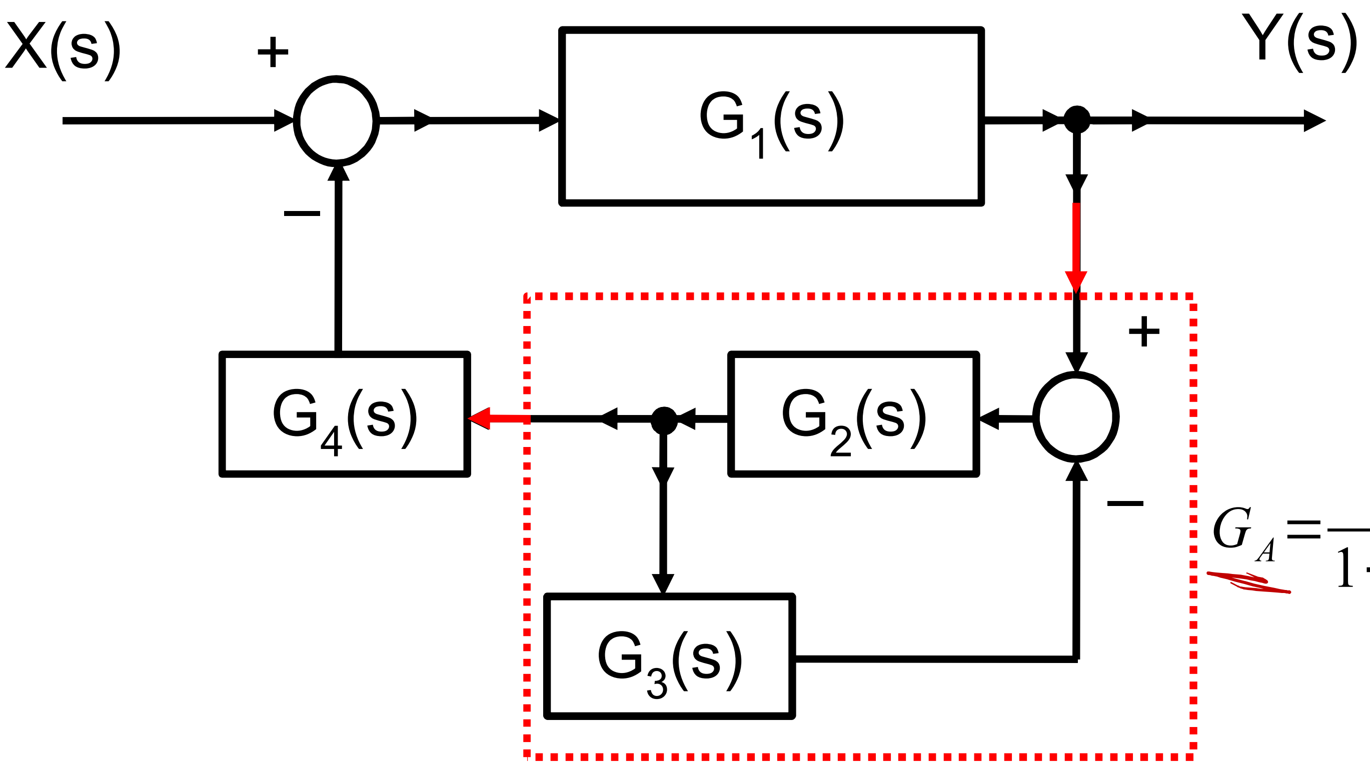

$$G_A = \frac{G_2}{1 + G_2 G_3}$$

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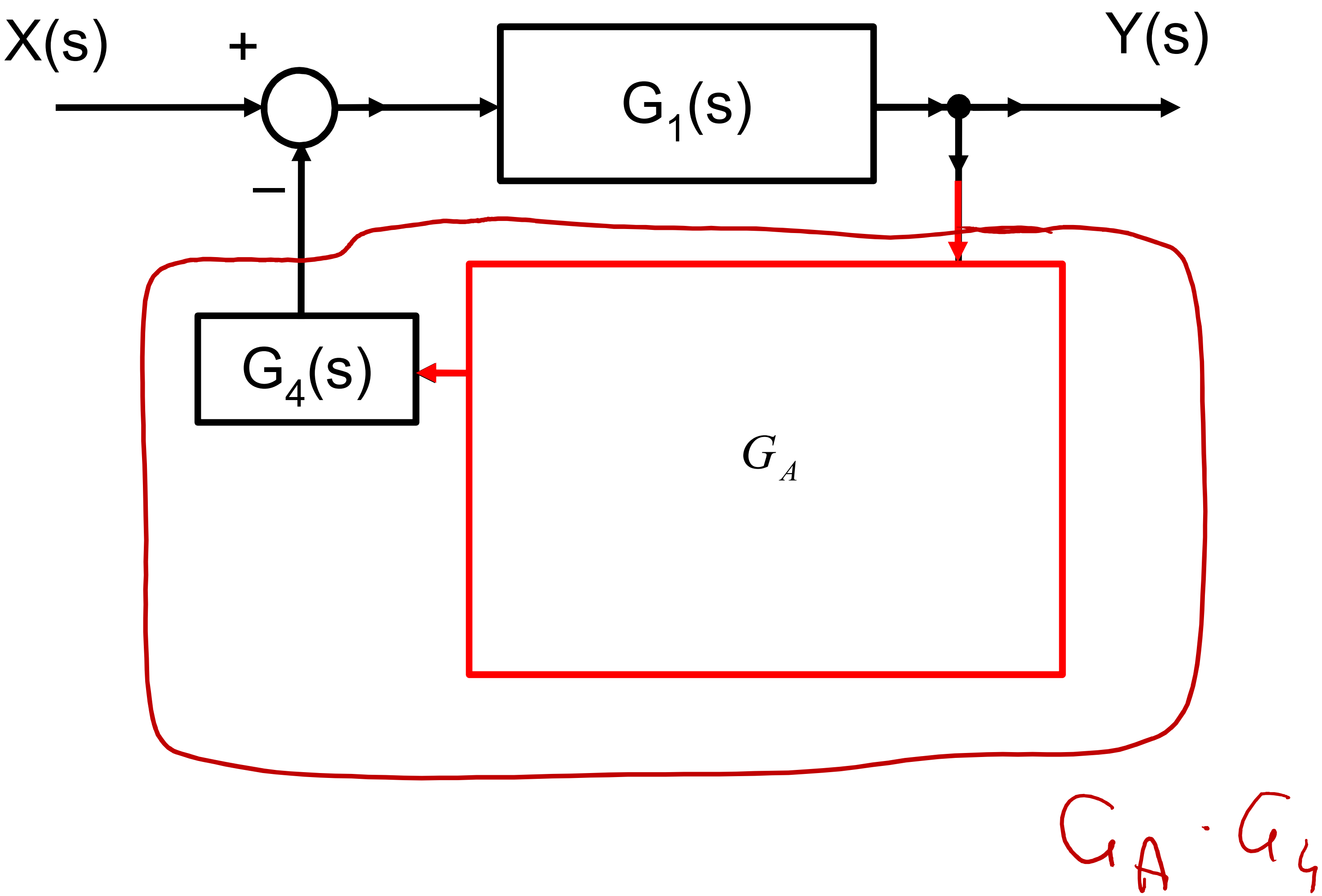
EXAMPLE 1



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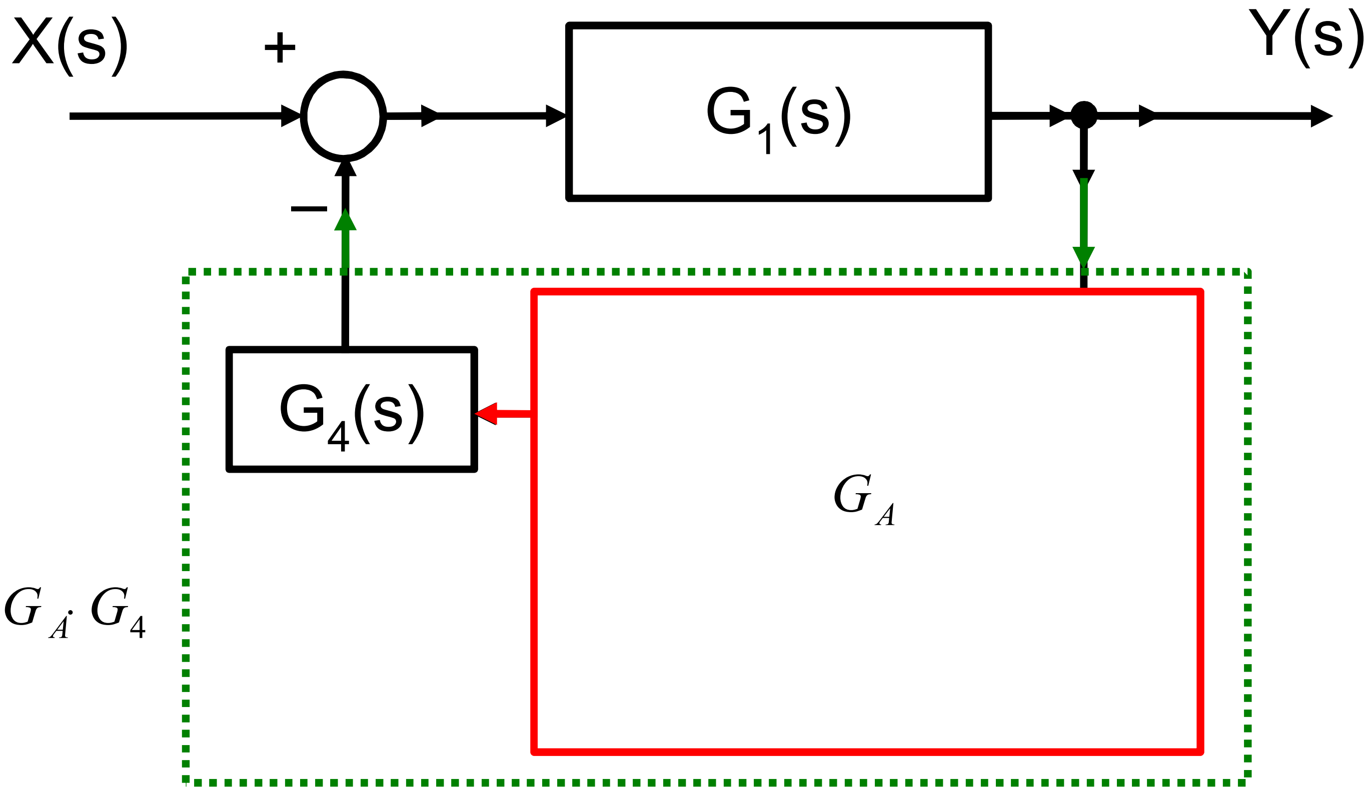
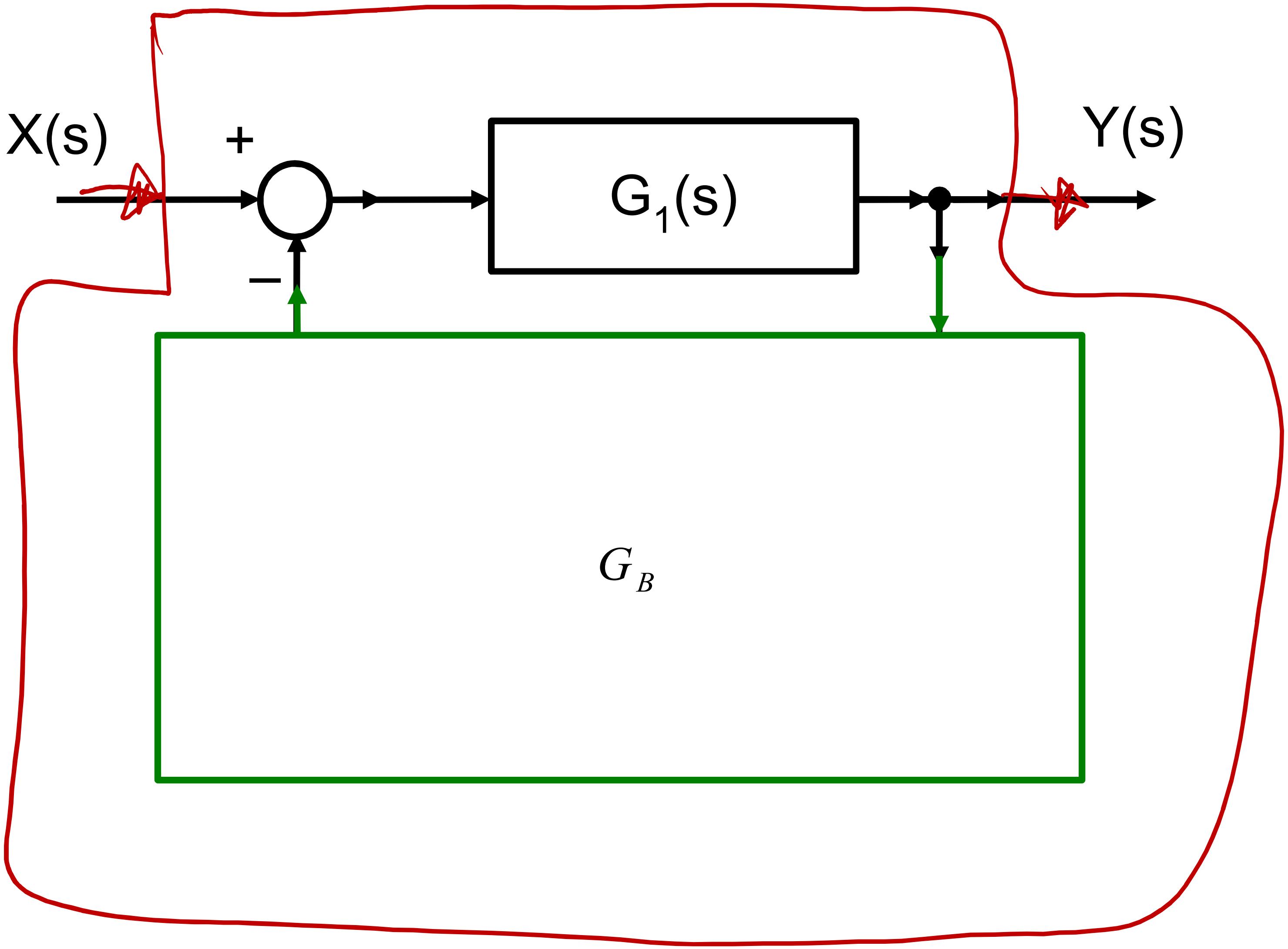


$$G_B = G_A \cdot G_4$$

EXAMPLE 1

$$G_A = \frac{G_2}{1 + G_2 G_3}$$

$$G_B = G_A \cdot G_4$$

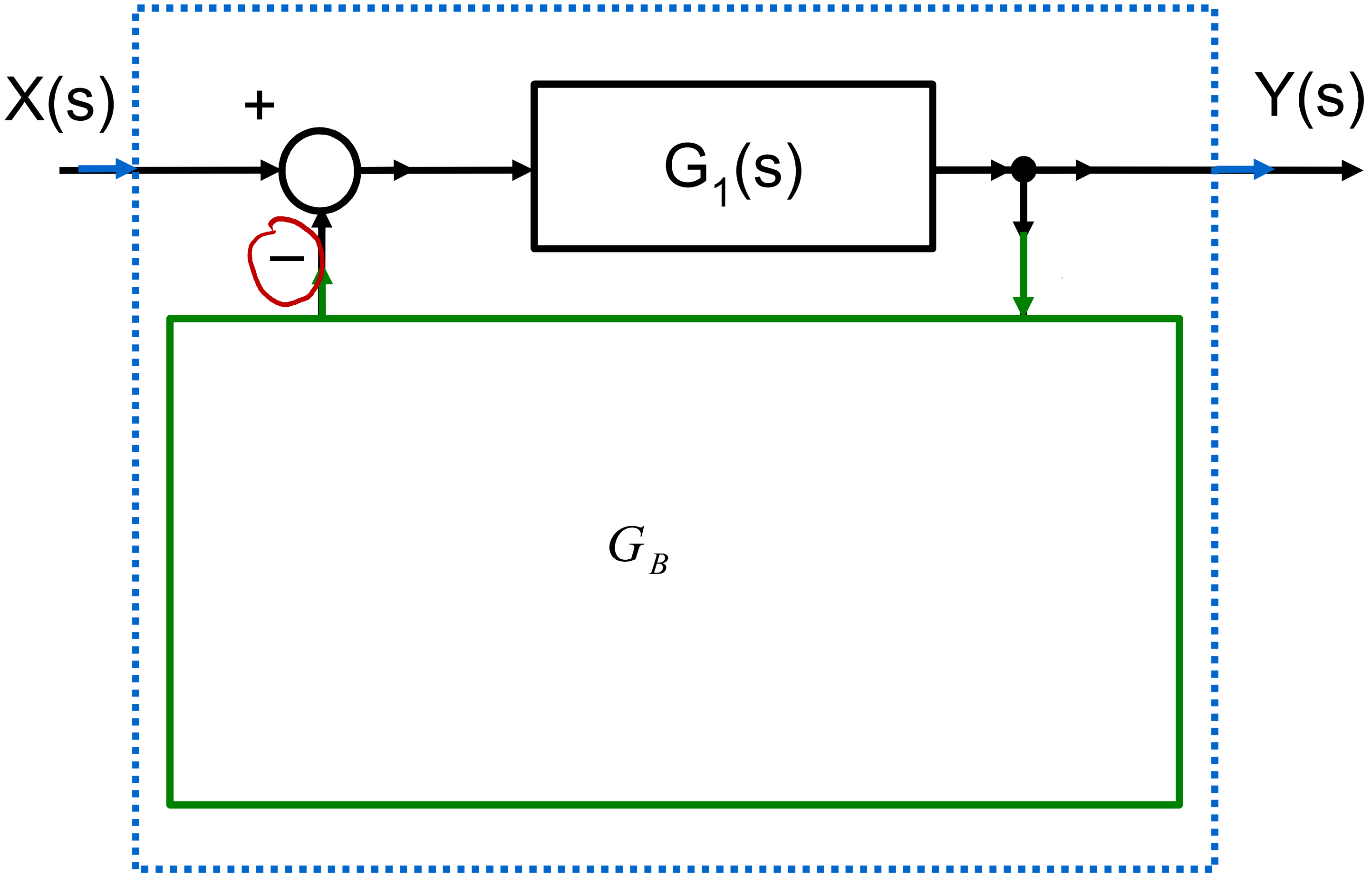


EXAMPLE 1

$$G_A = \frac{G_2}{1+G_2 G_3}$$

$$G_B = G_A \cdot G_4$$

$$\frac{G_1}{1+G_1 G_B}$$

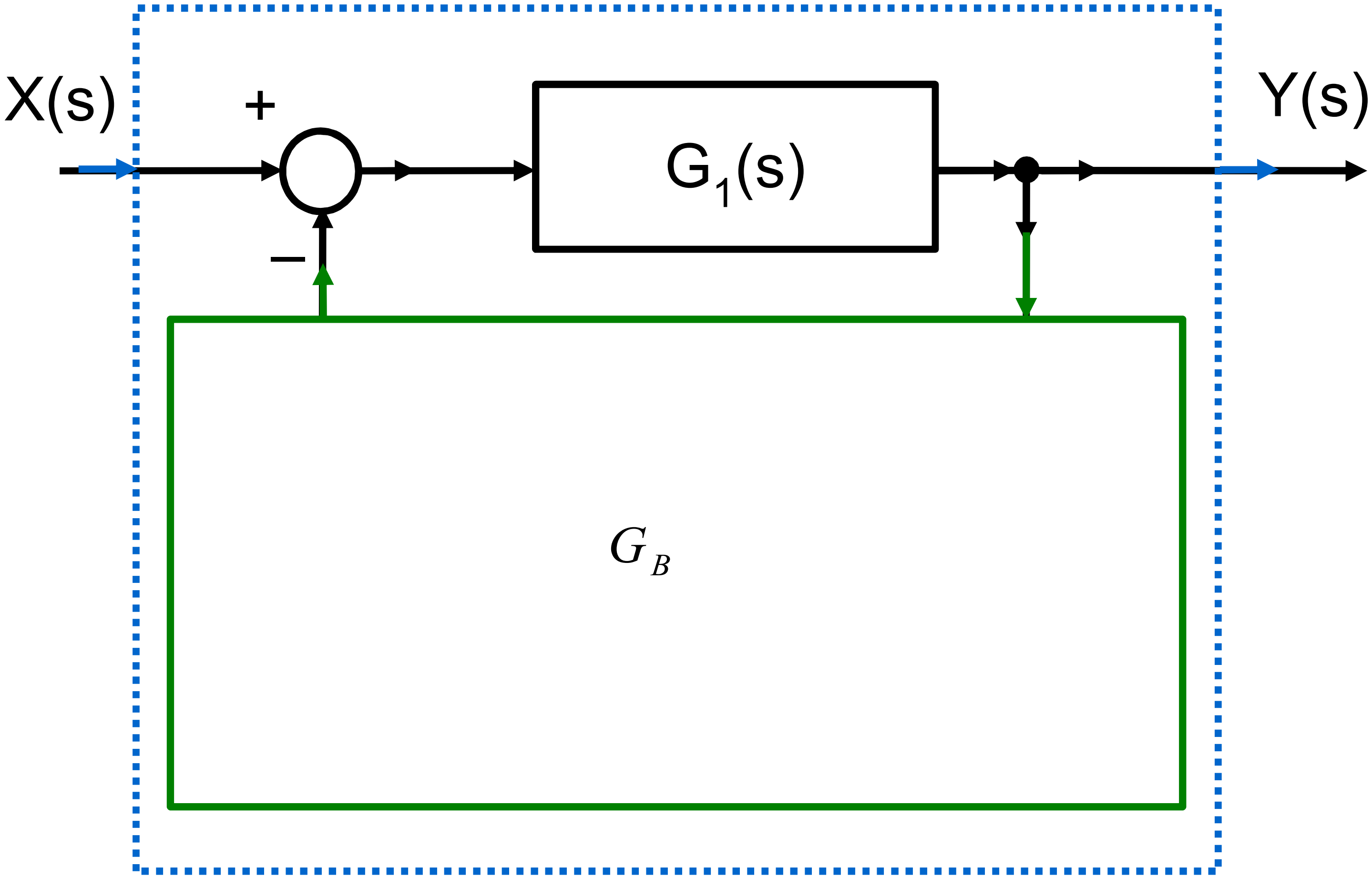


$$G_A = \frac{G_2}{1+G_2 G_3}$$

$$G_B = G_A \cdot G_4$$

EXAMPLE 1

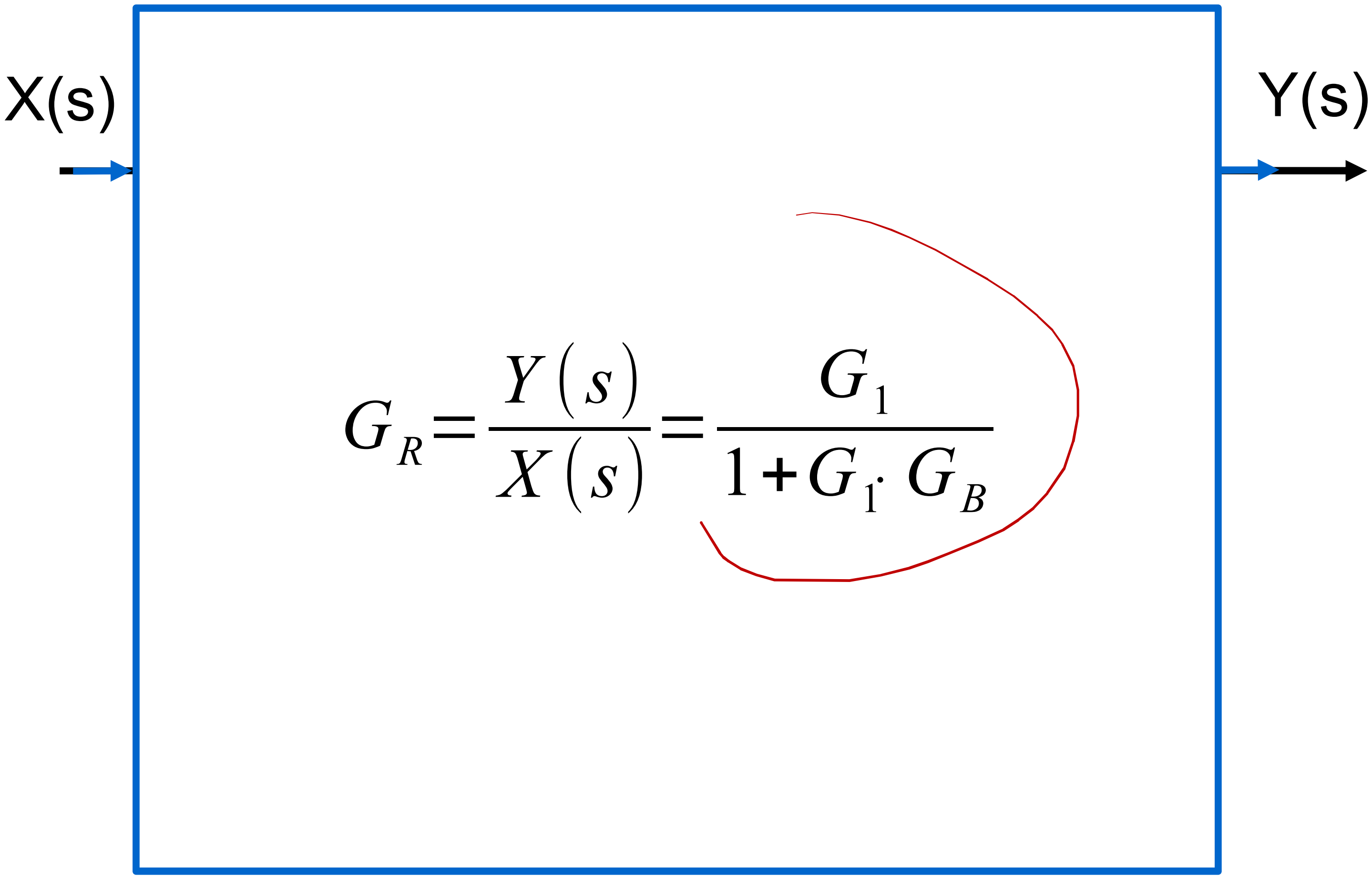
$$G_R = \frac{Y(s)}{X(s)} = \frac{G_1}{1+G_1 \cdot G_B}$$



EXAMPLE 1

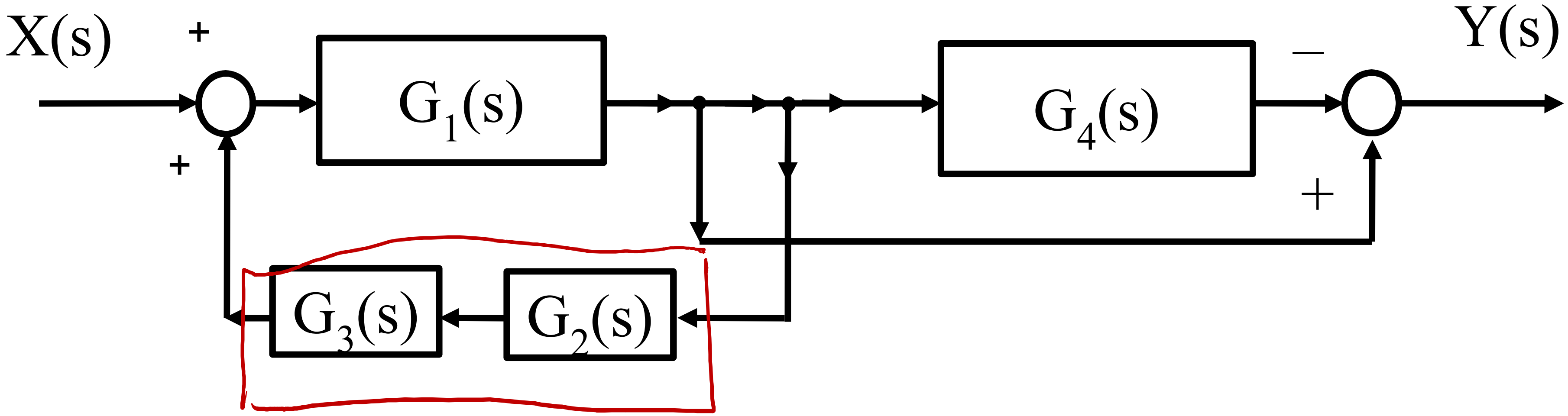
$$G_A = \frac{G_2}{1 + G_2 G_3}$$

$$G_B = G_A \cdot G_4$$

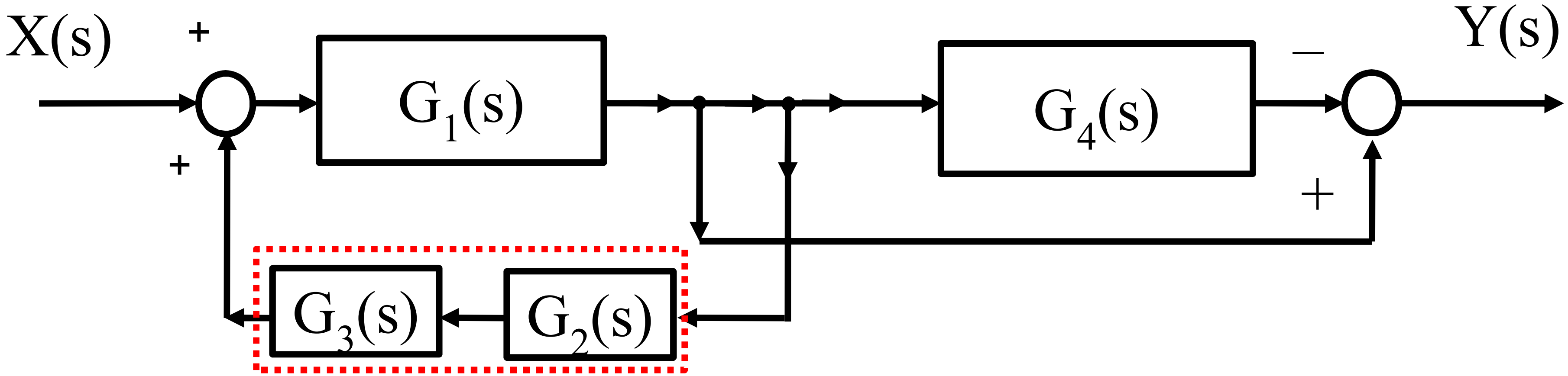


$$G_R = \frac{Y(s)}{X(s)} = \frac{G_1}{1 + G_1 \cdot G_B}$$

EXAMPLE 2



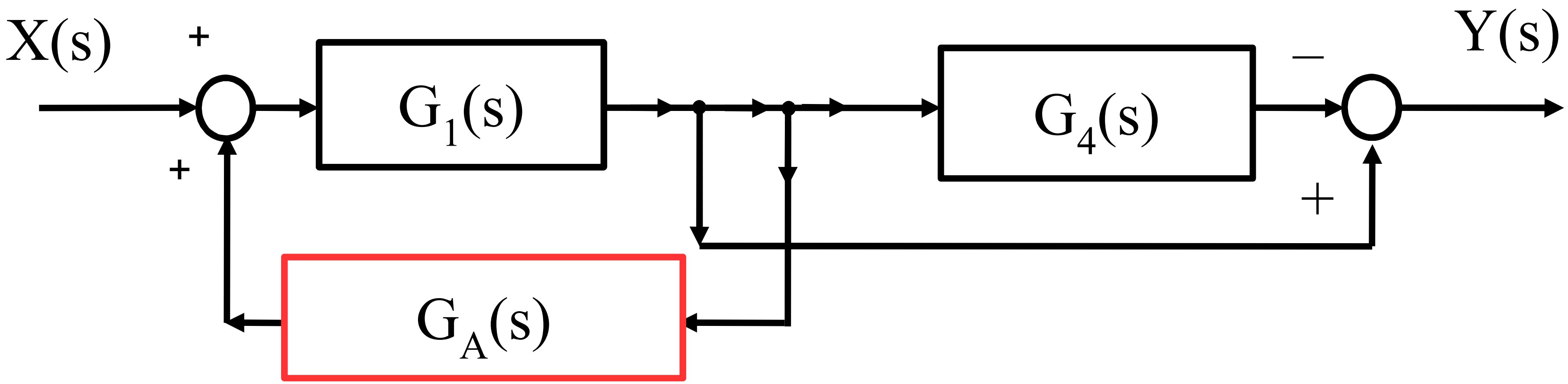
EXAMPLE 2



$$G_A = G_2 G_3$$

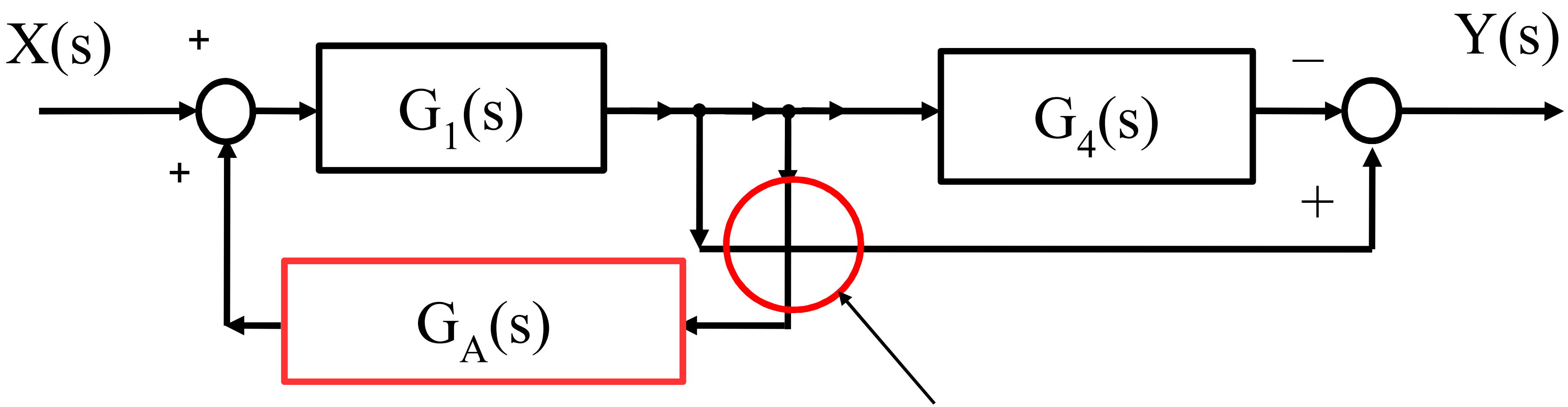
$G_A = G_2 G_3$

EXAMPLE 2



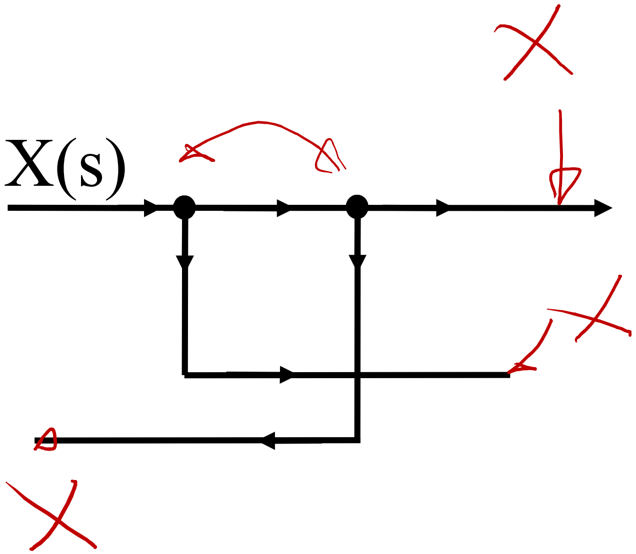
$G_A = G_2 G_3$

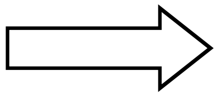
EXAMPLE 2



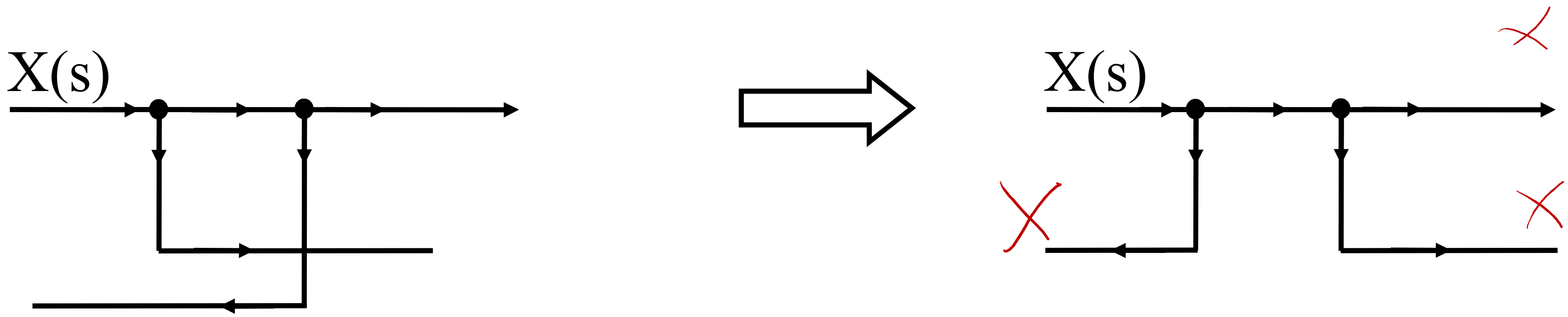
problem with crossed lines (no connection)

BLOCK DIAGRAM ALGEBRA
change of information points order



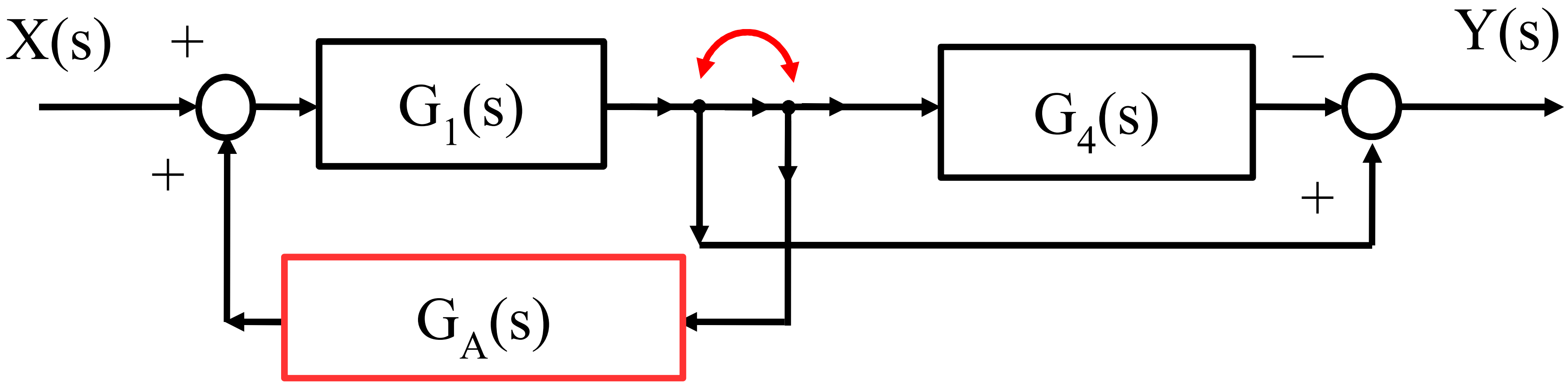


BLOCK DIAGRAM ALGEBRA
change of information points order



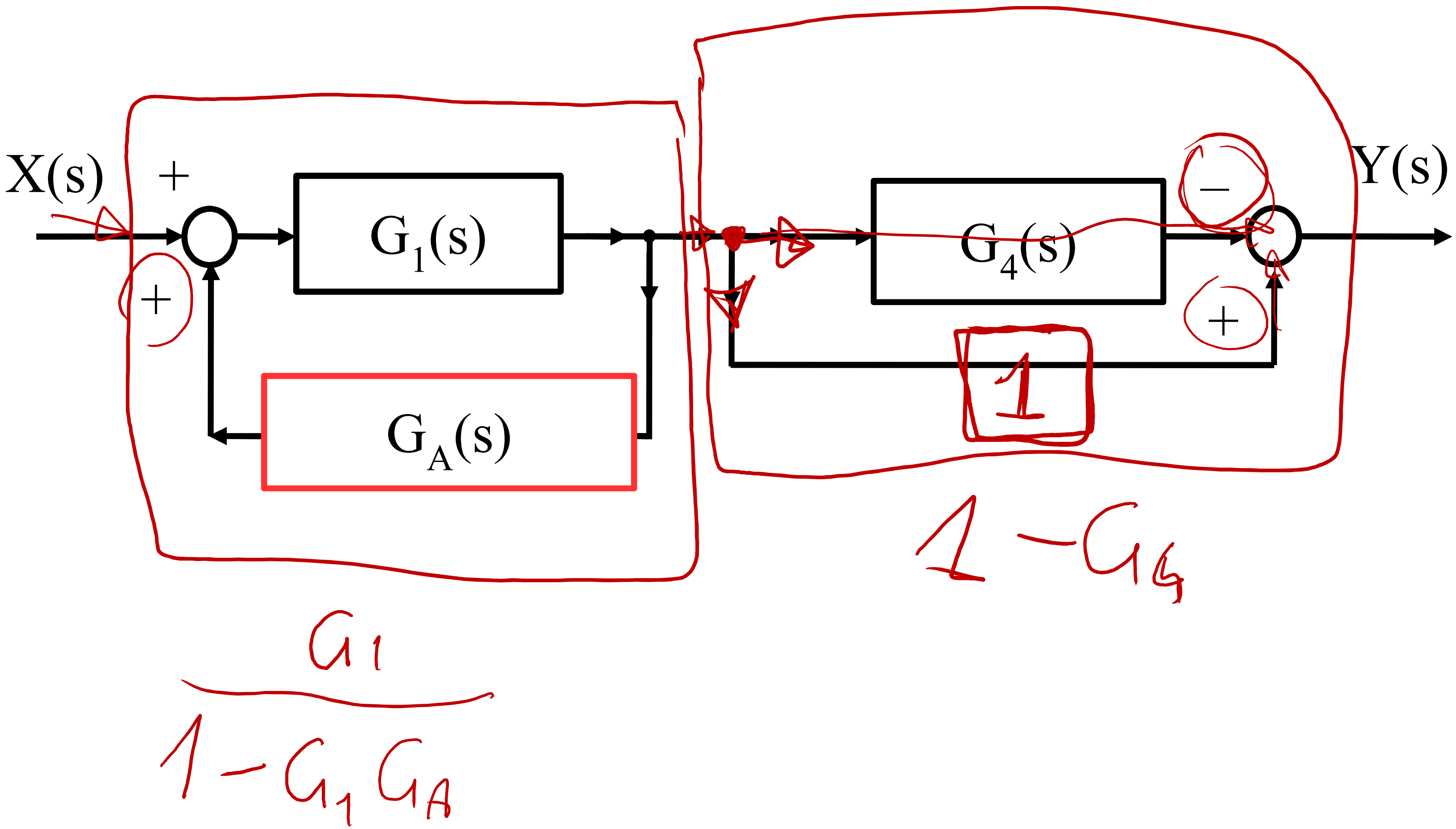
$$G_A = G_2 G_3$$

EXAMPLE 2



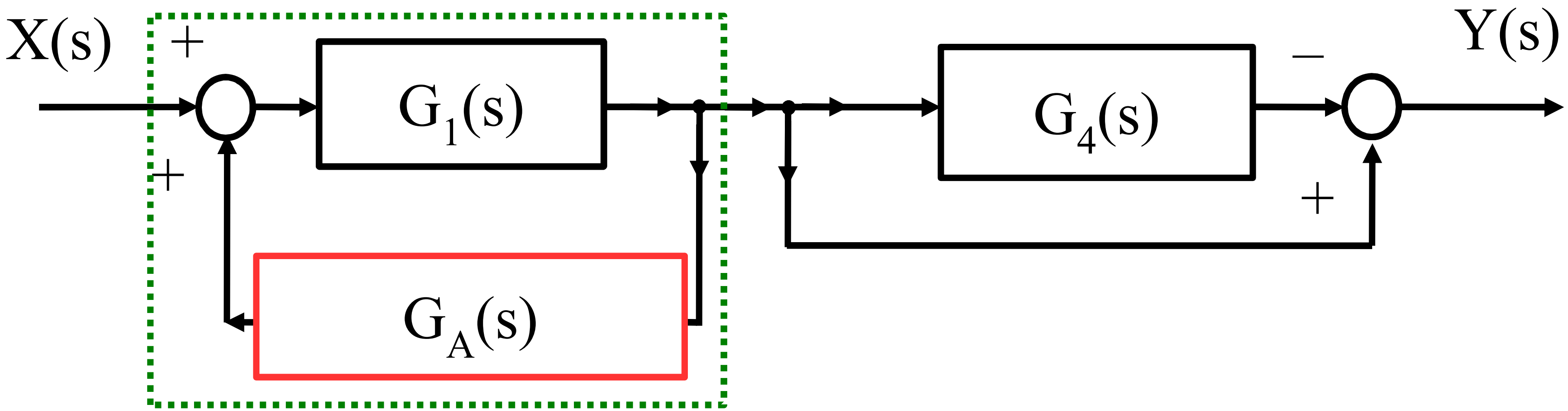
$G_A = G_2 G_3$

EXAMPLE 2



$$G_A = G_2 G_3$$

EXAMPLE 2

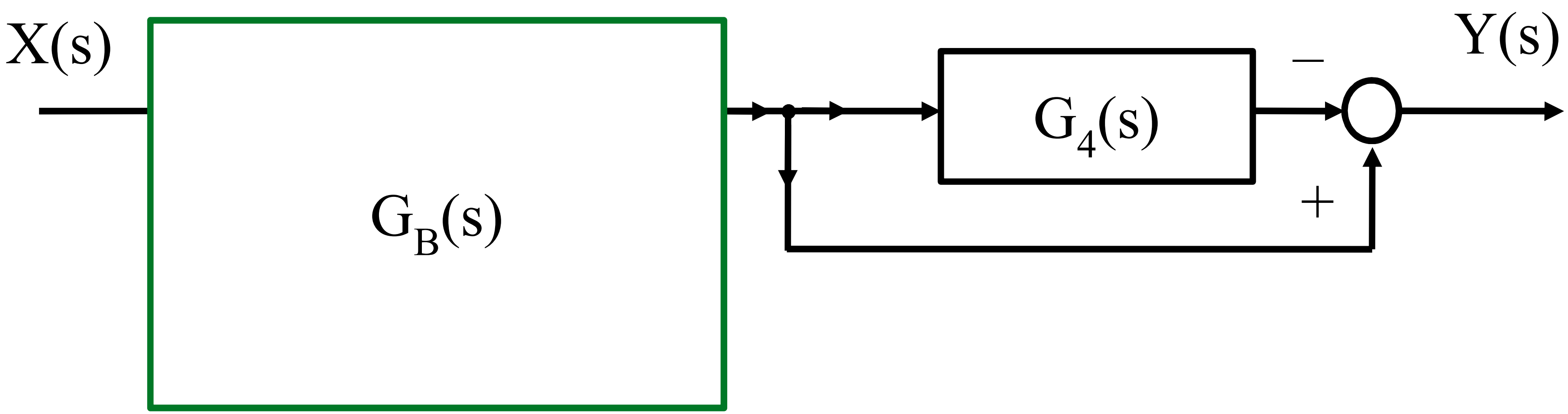


$$G_B = \frac{G_1}{1 - G_1 G_A}$$

$$G_A = G_2 G_3$$

EXAMPLE 2

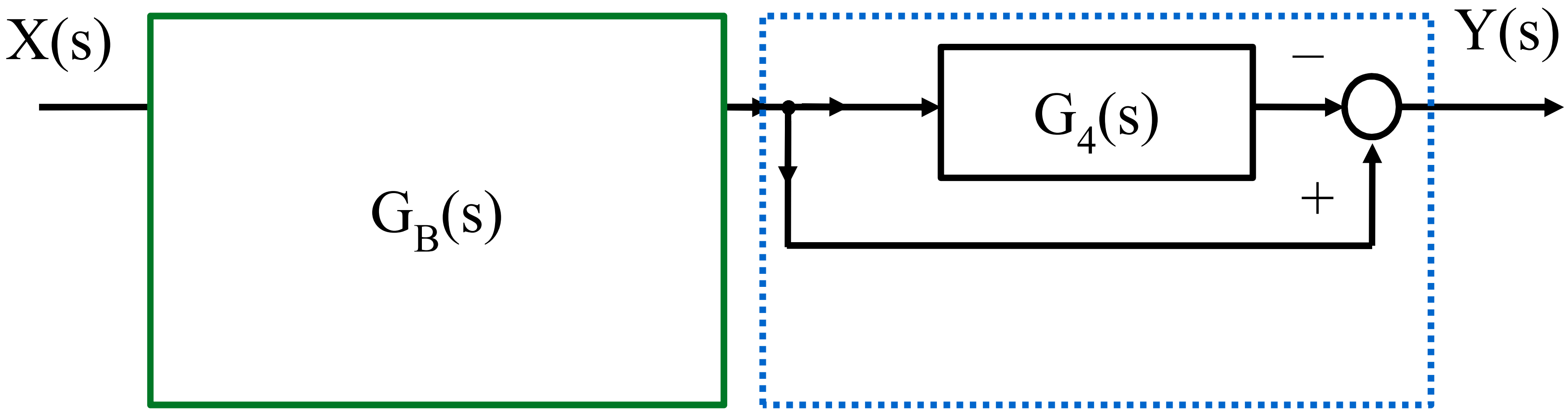
$$G_B = \frac{G_1}{1 - G_1 G_A}$$



EXAMPLE 2

$$G_A = G_2 G_3$$

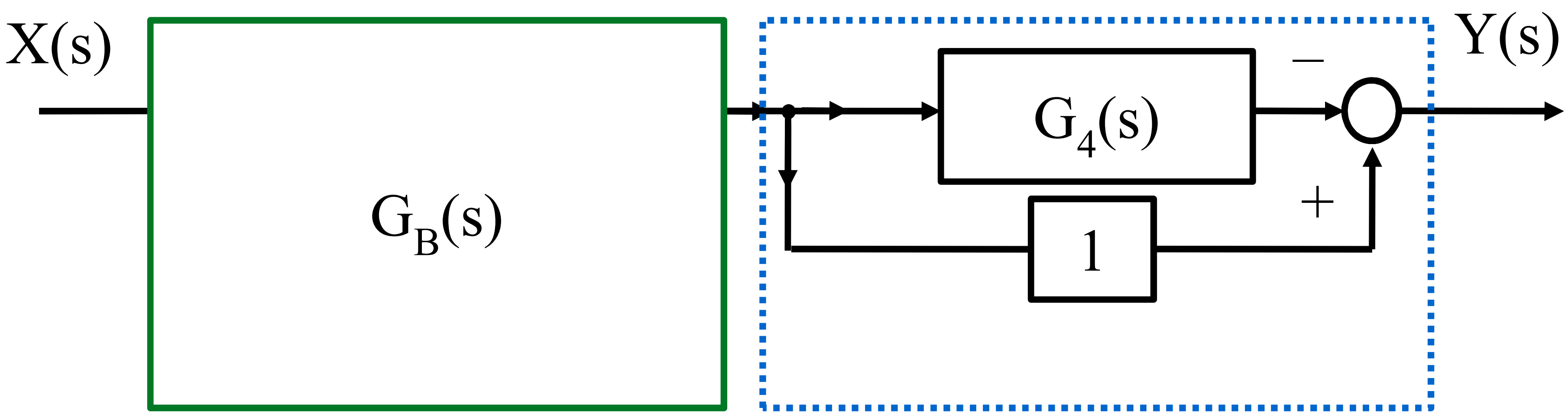
$$G_B = \frac{G_1}{1 - G_1 G_A}$$



$$G_A = G_2 G_3$$

EXAMPLE 2

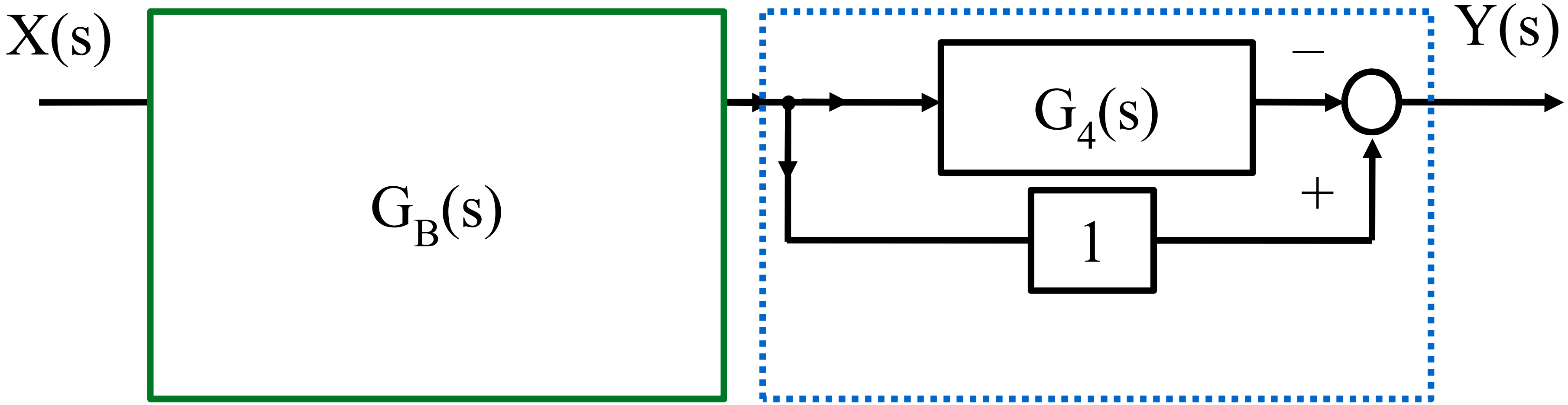
$$G_B = \frac{G_1}{1 - G_1 G_A}$$



$$G_A = G_2 G_3$$

$$G_B = \frac{G_1}{1 - G_1 G_A}$$

EXAMPLE 2



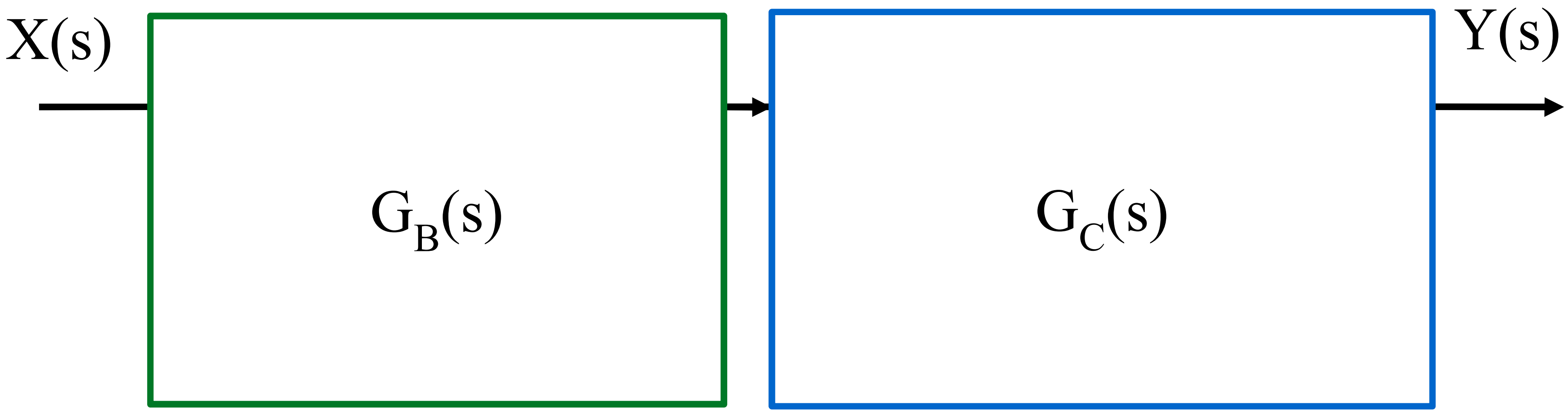
$$G_C = -G_4 + 1$$

EXAMPLE 2

$$G_A = G_2 G_3$$

$$G_B = \frac{G_1}{1 - G_1 G_A}$$

$$G_C = -G_4 + 1$$



$$G_A = G_2 G_3$$

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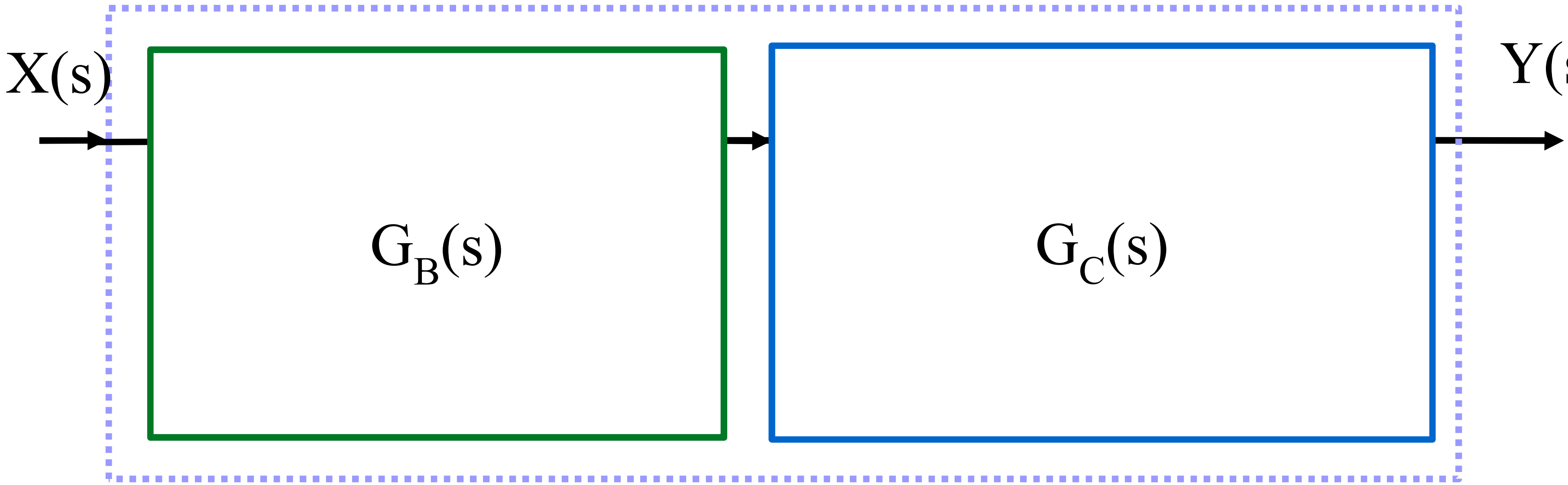
$$G_C = - G_4 + 1$$

EXAMPLE 2



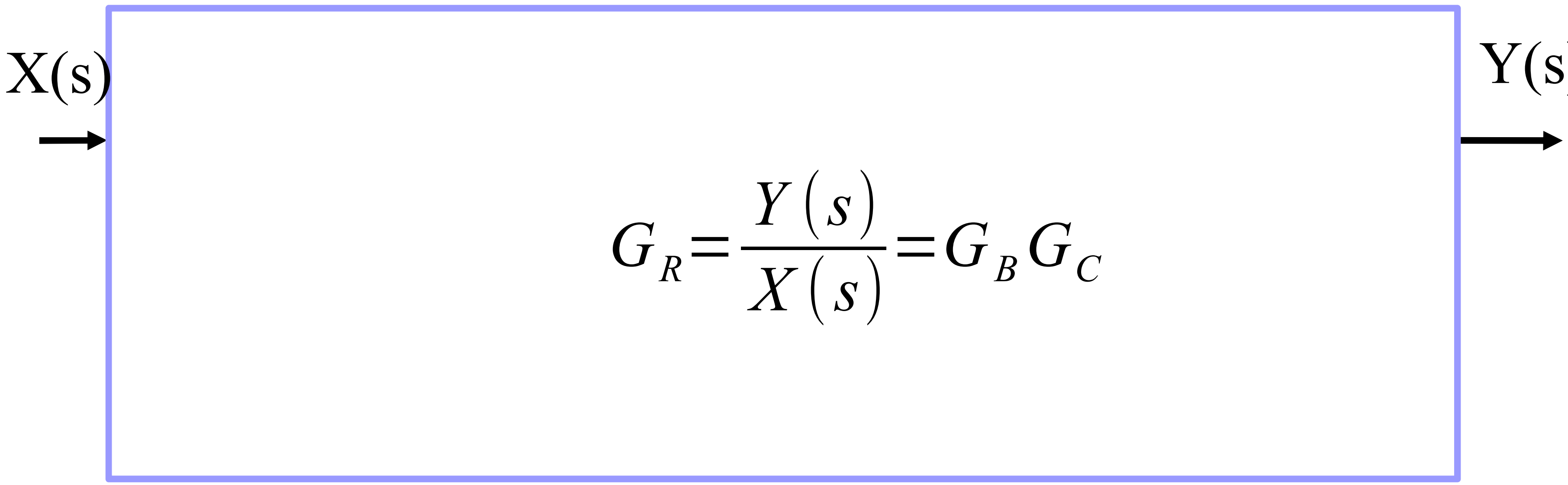
$$G_R = \frac{Y(s)}{X(s)} = G_B G_C$$

EXAMPLE 2

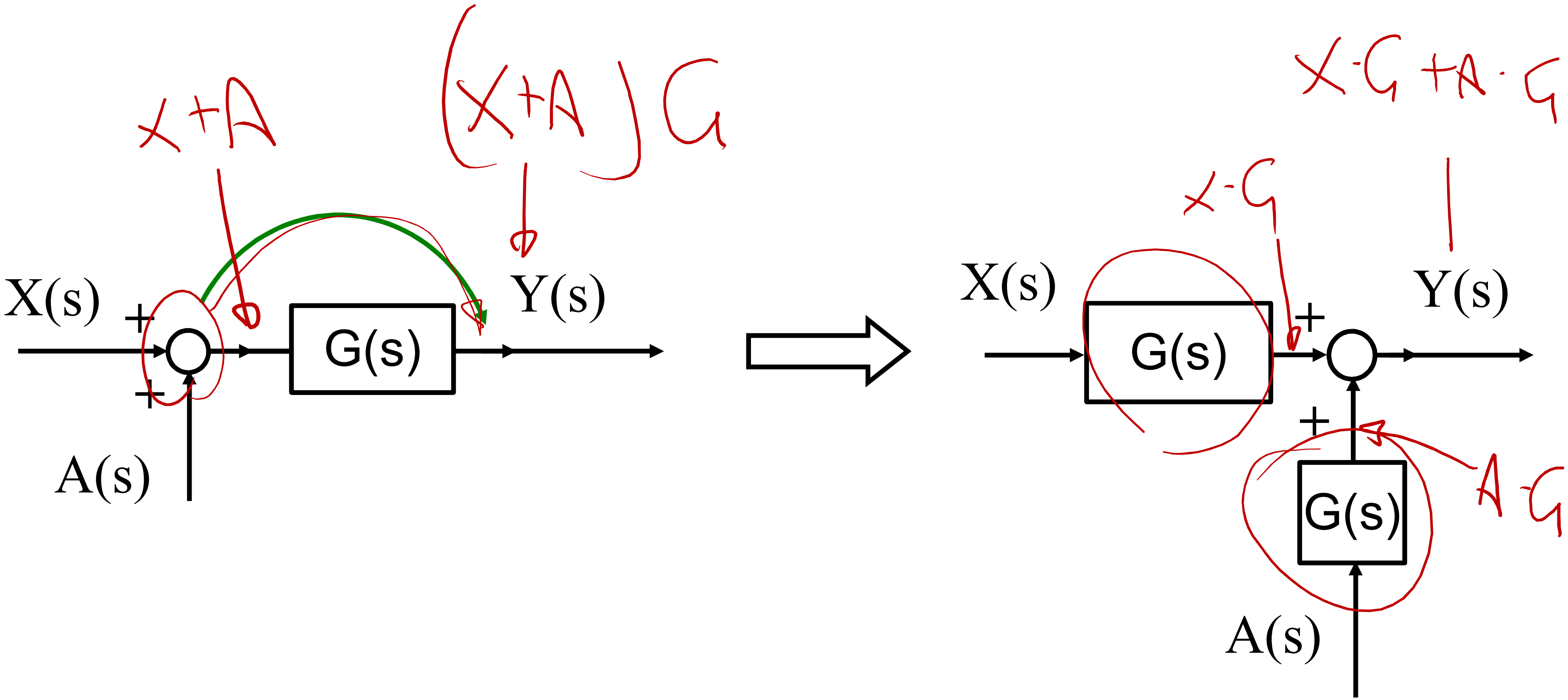
$$G_A = G_2 G_3$$

$$G_B = \frac{G_1}{1 - G_1 G_A}$$

$$G_C = -G_4 + 1$$



BLOCK DIAGRAM ALGEBRA
order change of sum node and block

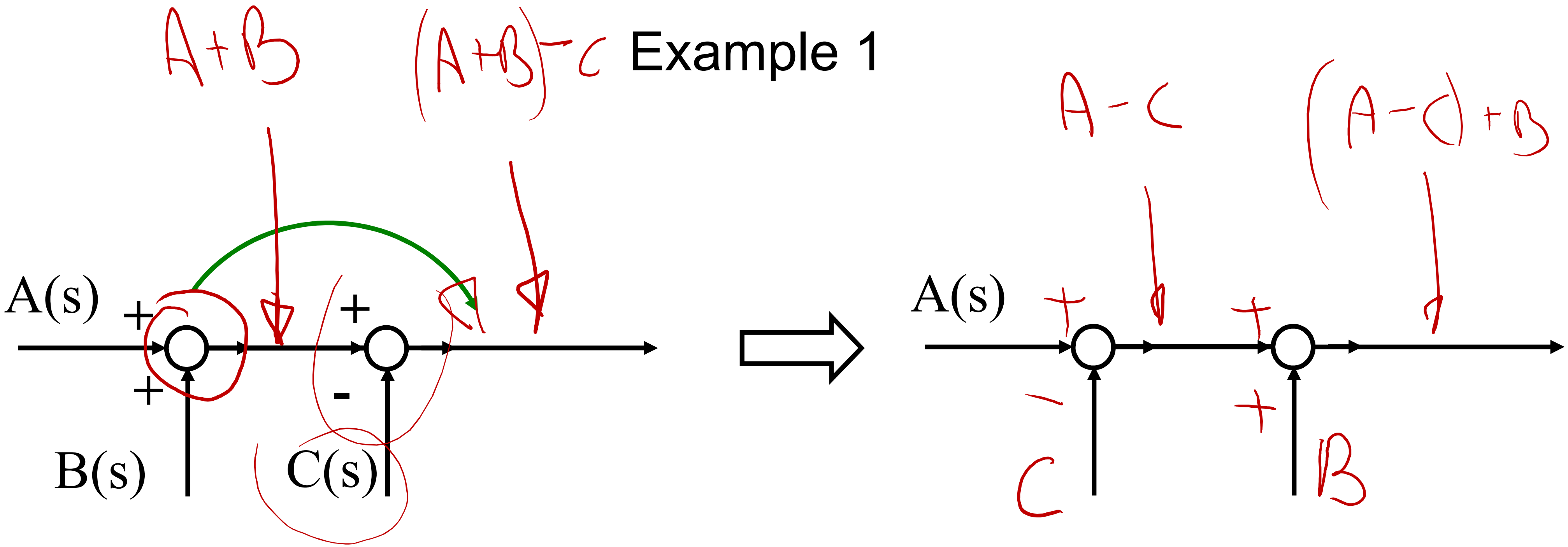


Y=(X+A)G

Y=XG+AG

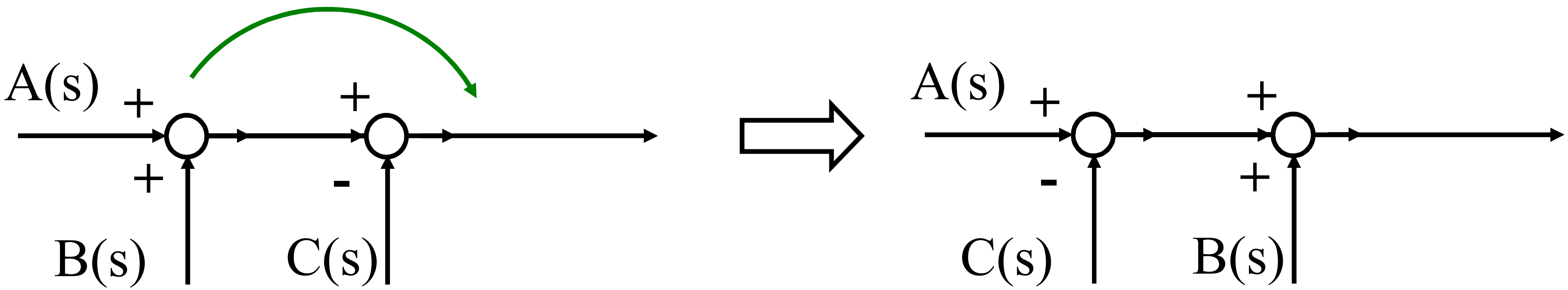
BLOCK DIAGRAM ALGEBRA
order change of sum nodes

Example 1



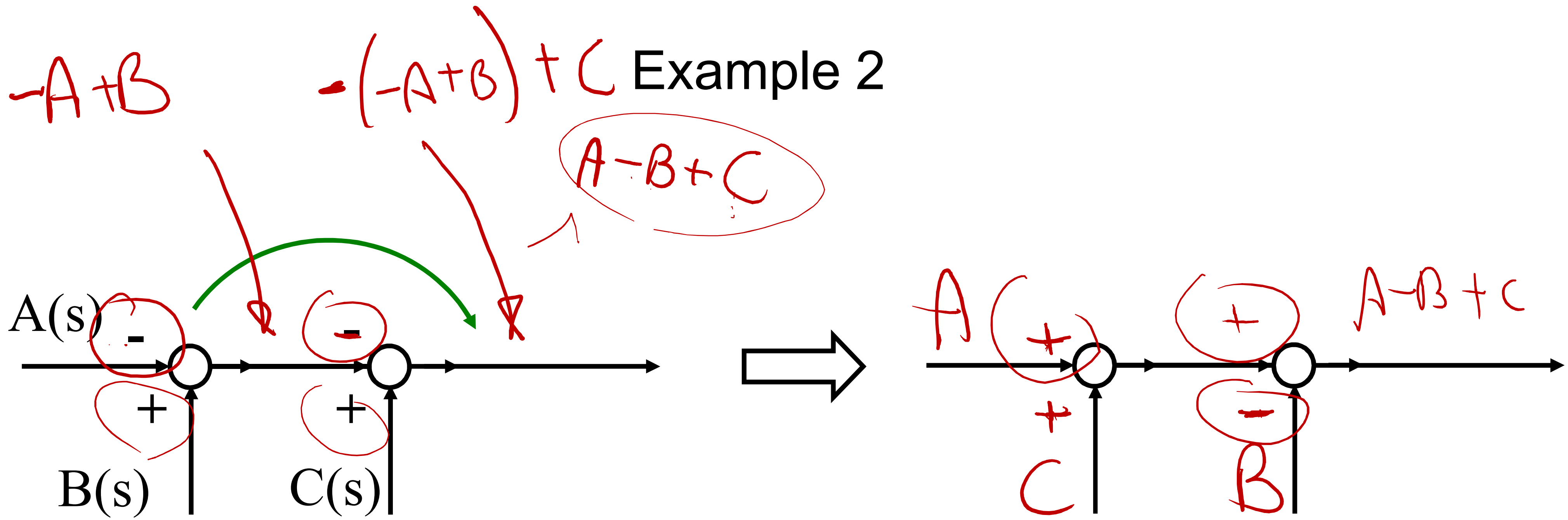
BLOCK DIAGRAM ALGEBRA
order change of sum nodes

Example 1



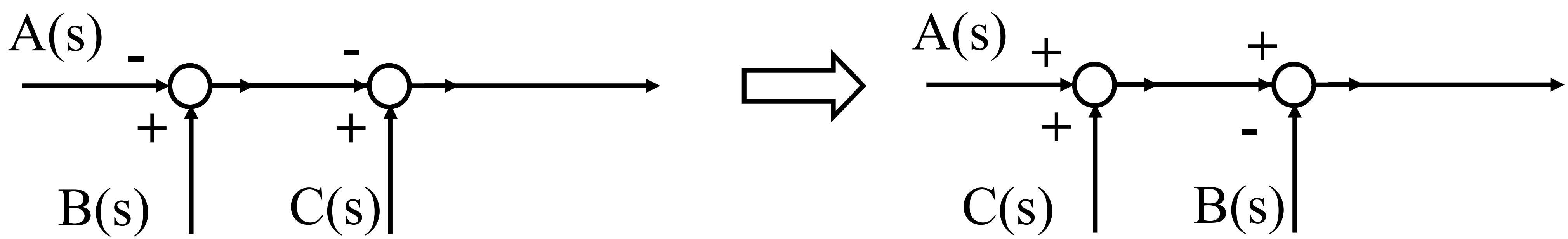
BLOCK DIAGRAM ALGEBRA
order change of sum nodes

Example 2

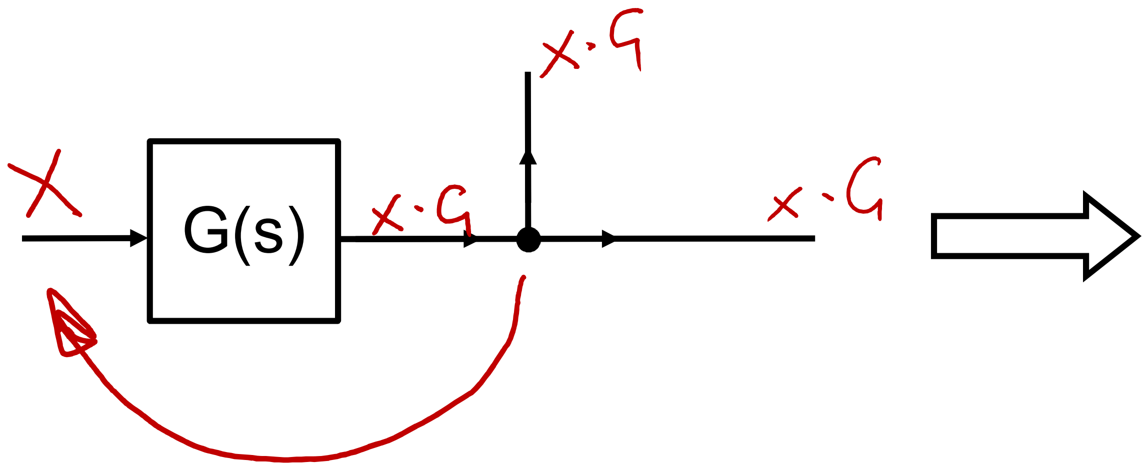


BLOCK DIAGRAM ALGEBRA
order change of sum nodes

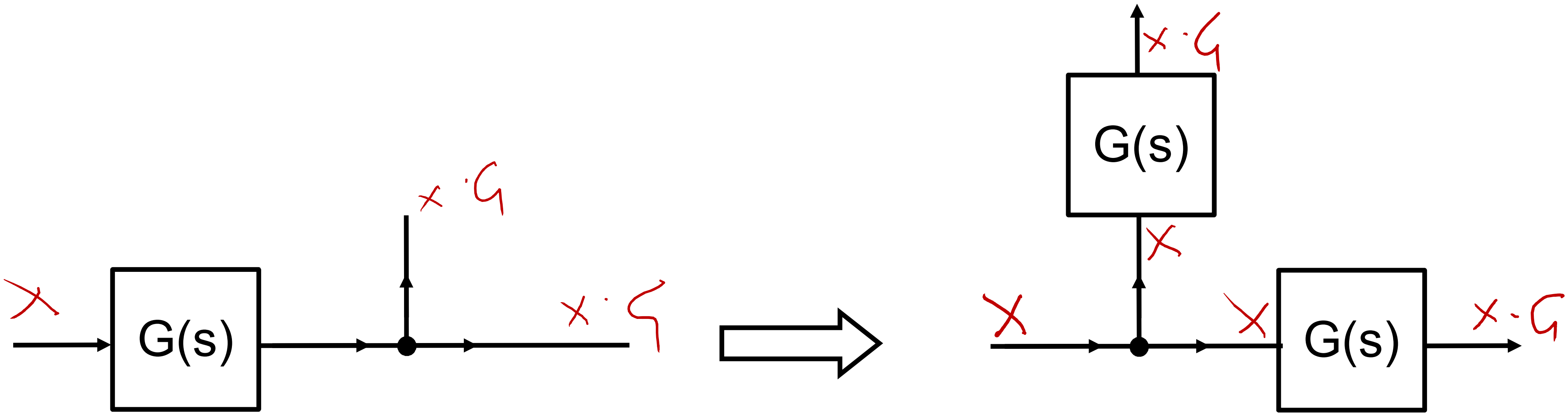
Example 2 – attention to signs!



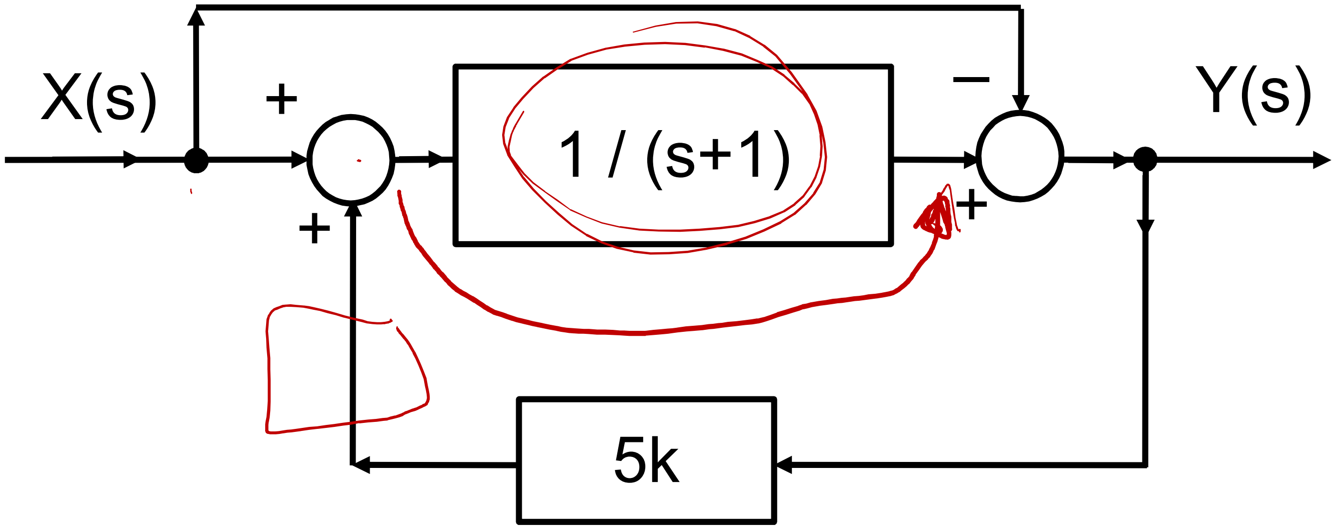
BLOCK DIAGRAM ALGEBRA
order change of block and information node



BLOCK DIAGRAM ALGEBRA
order change of block and information node



EXAMPLE 3



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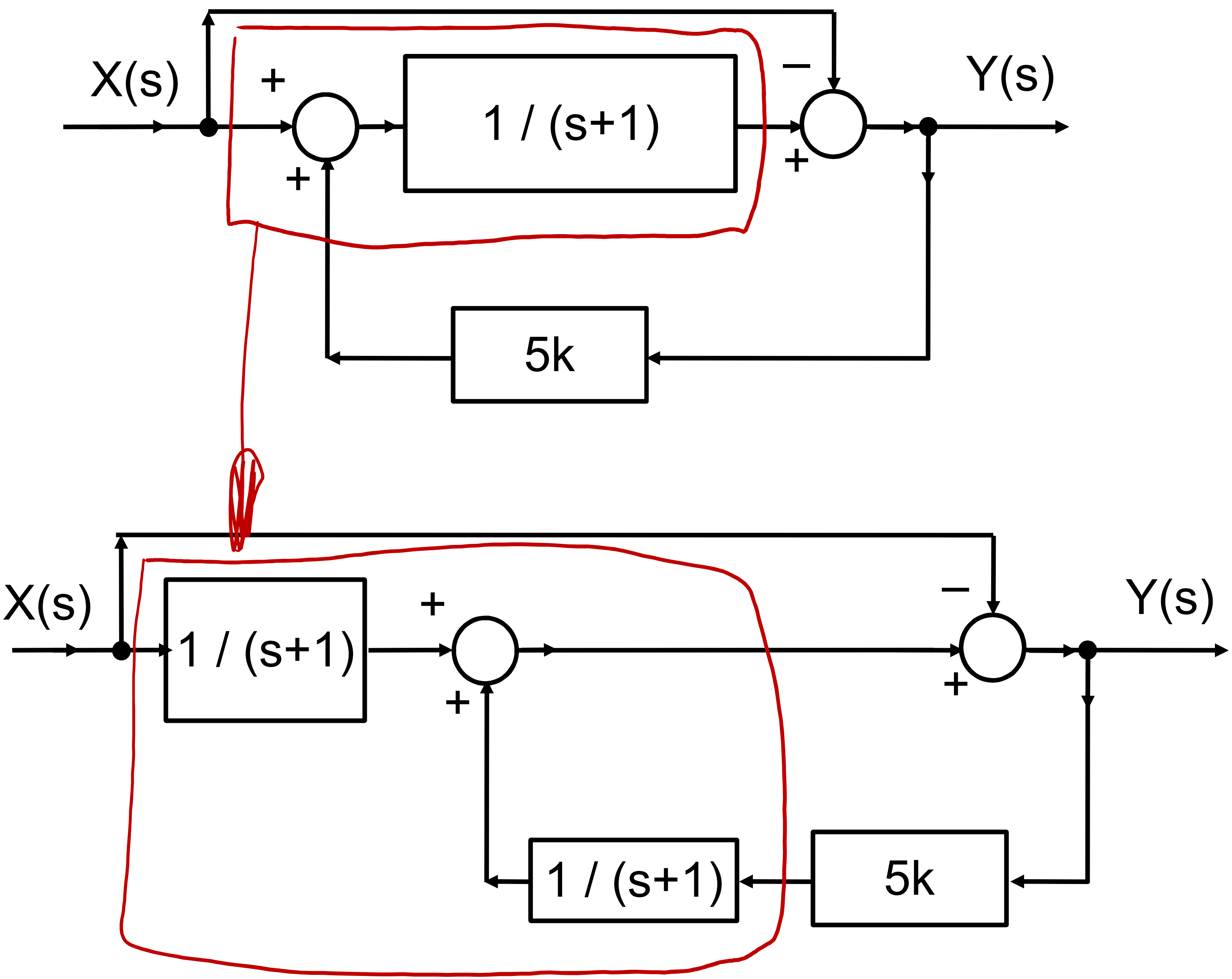

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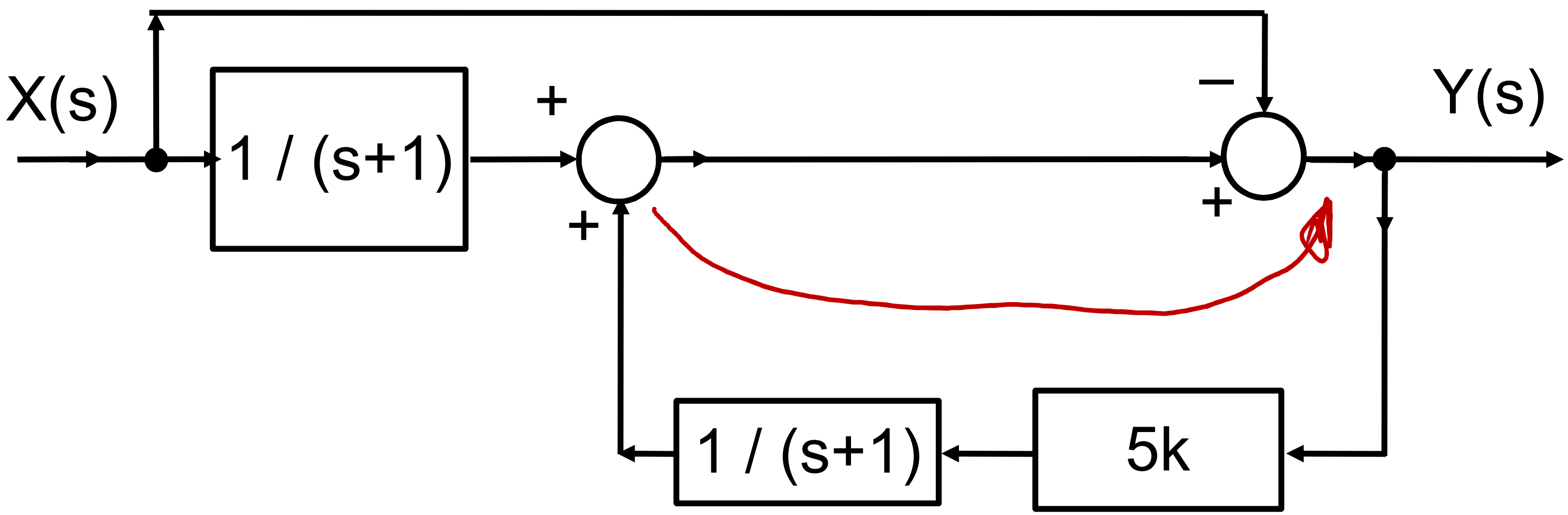

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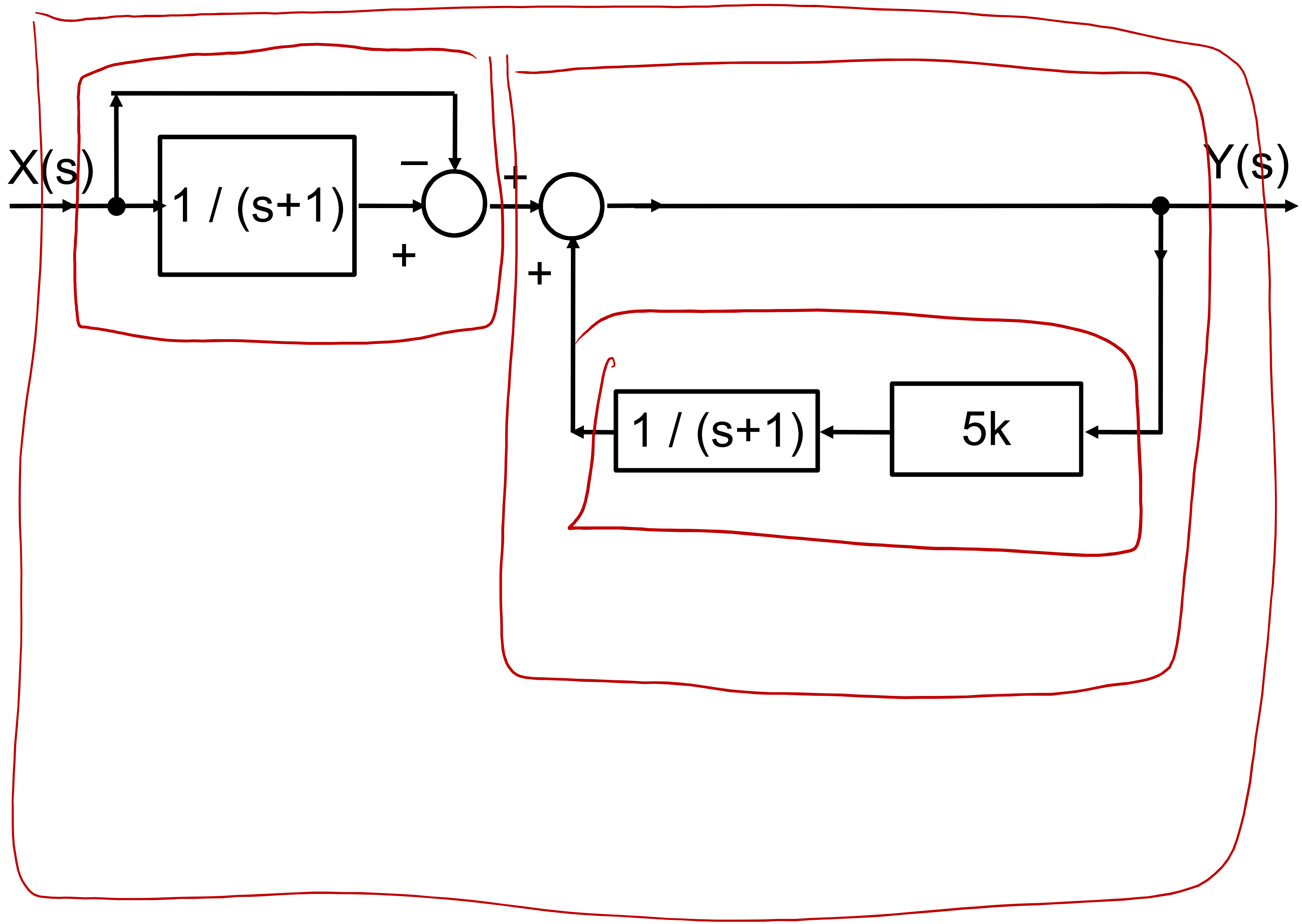

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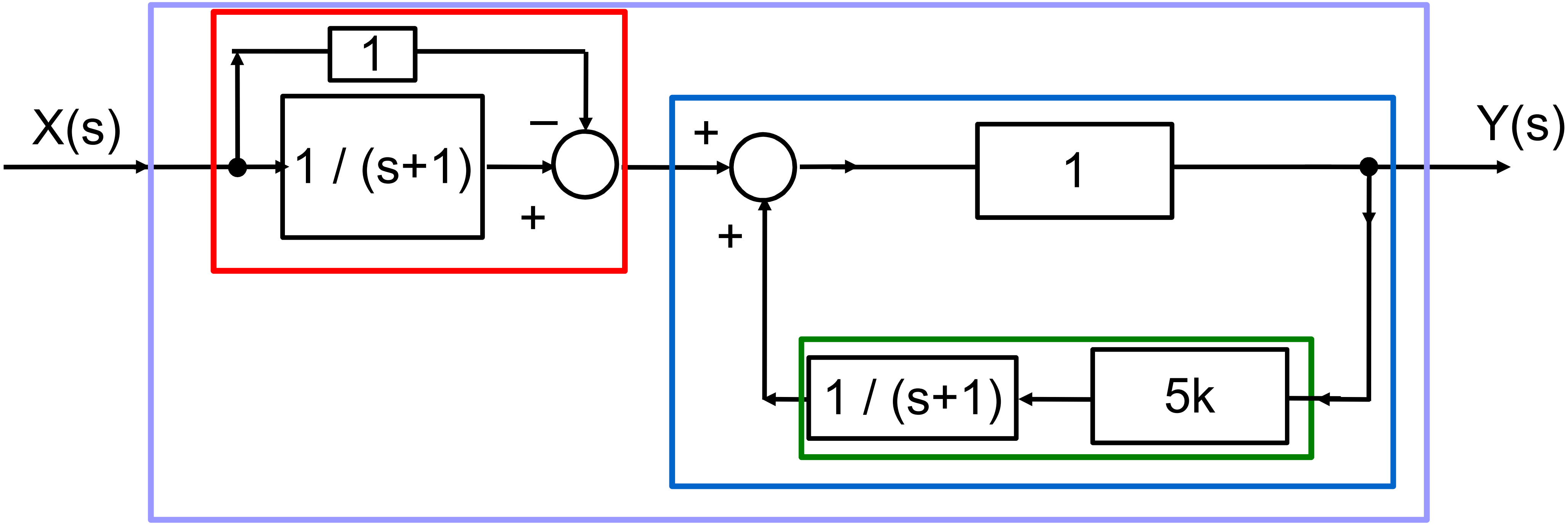
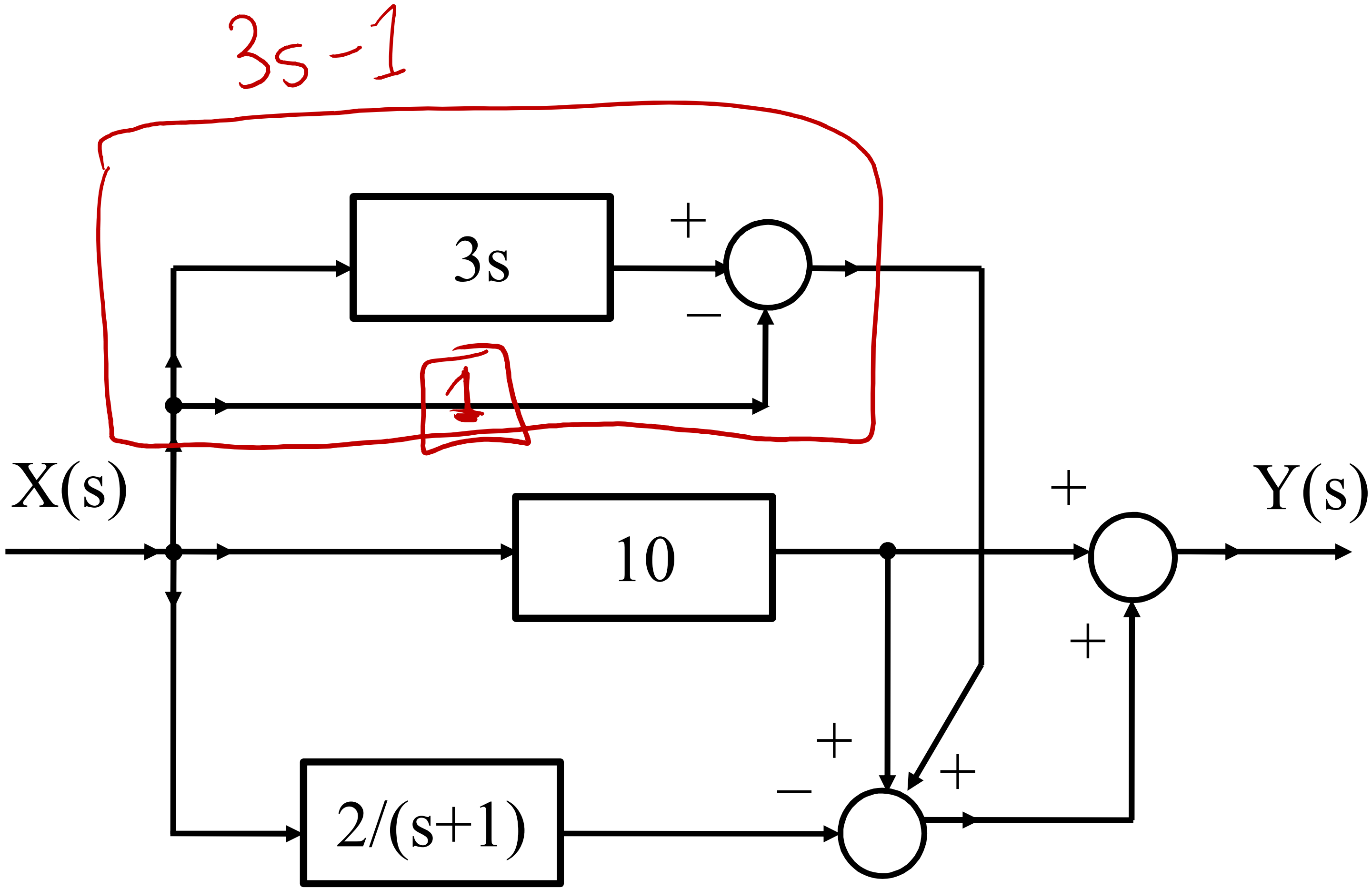
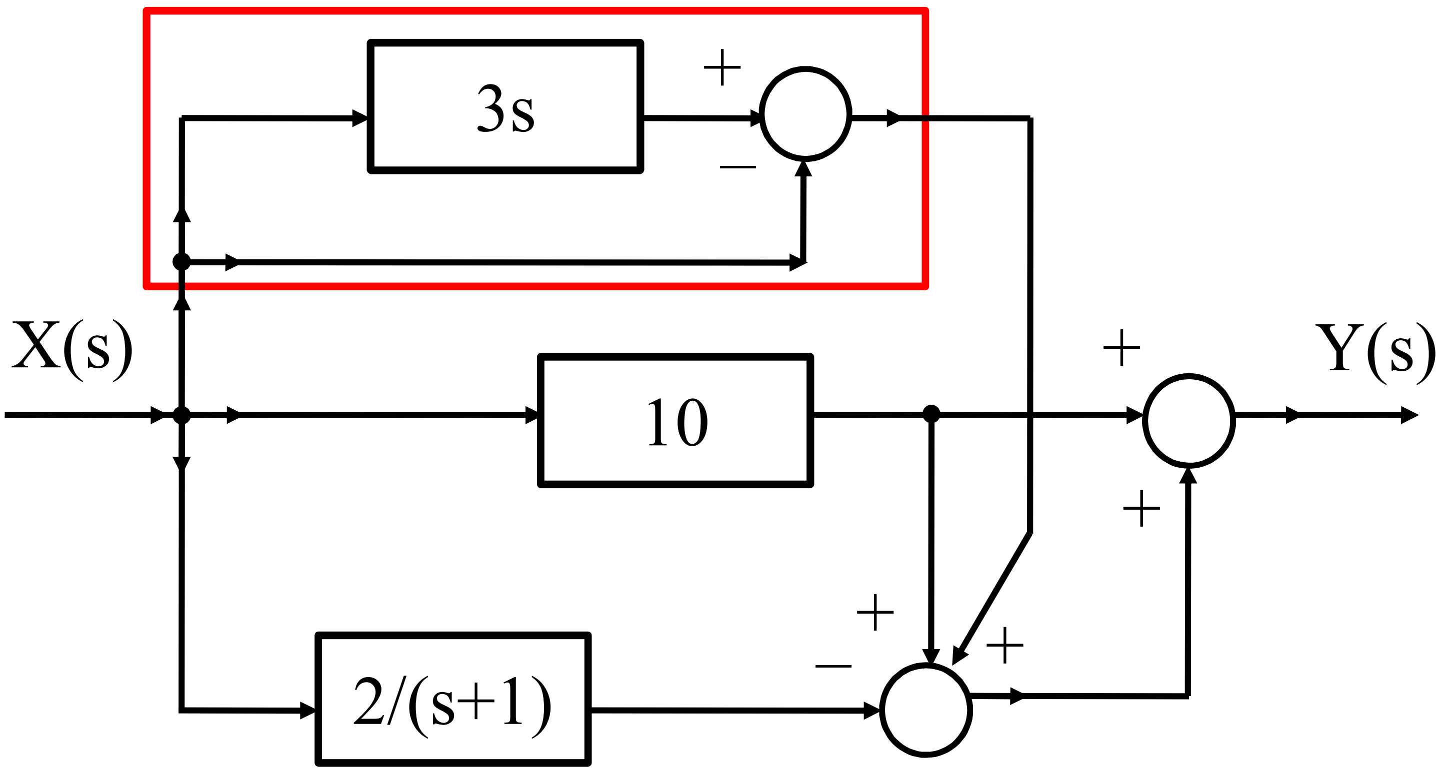


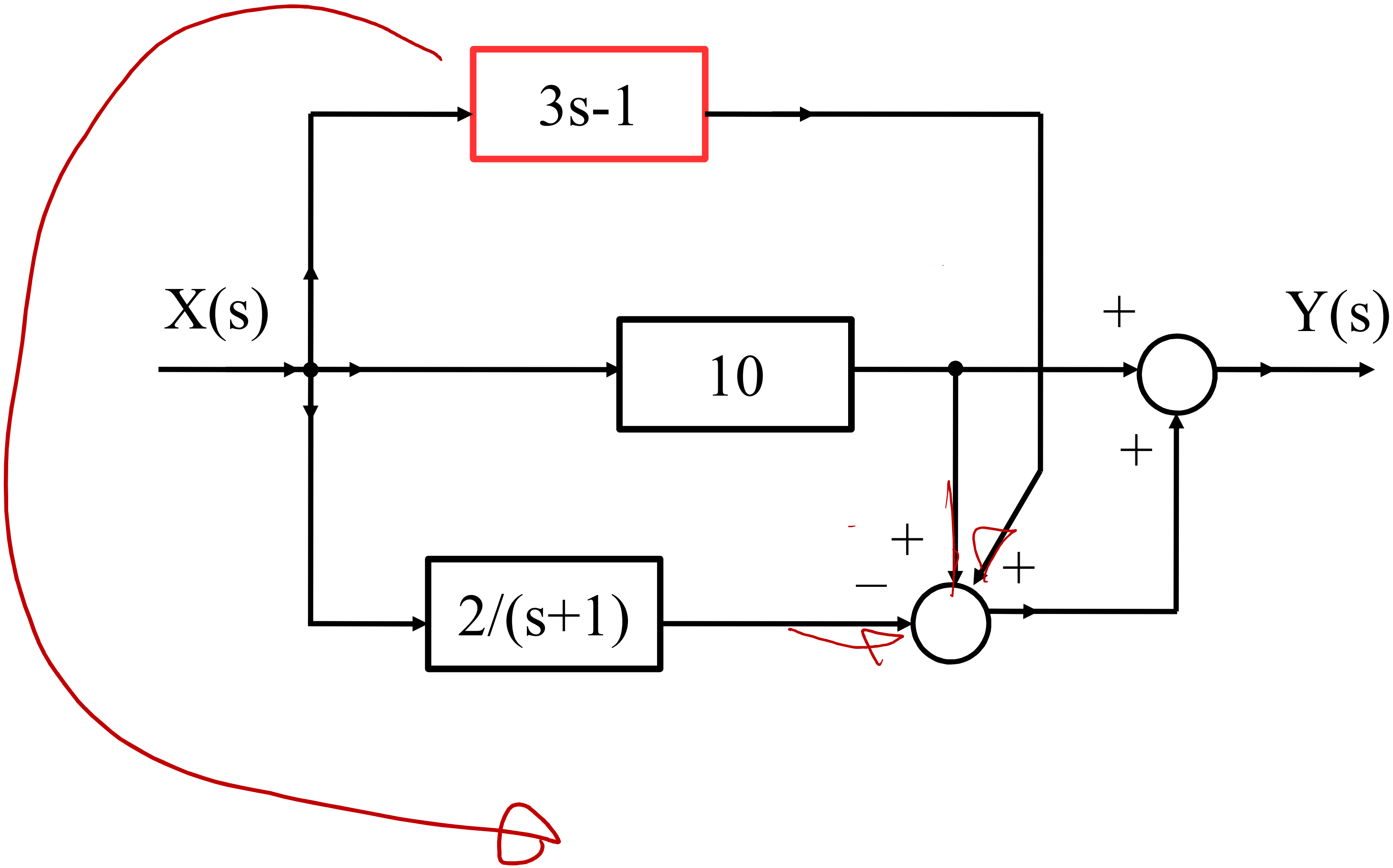
EXAMPLE 4



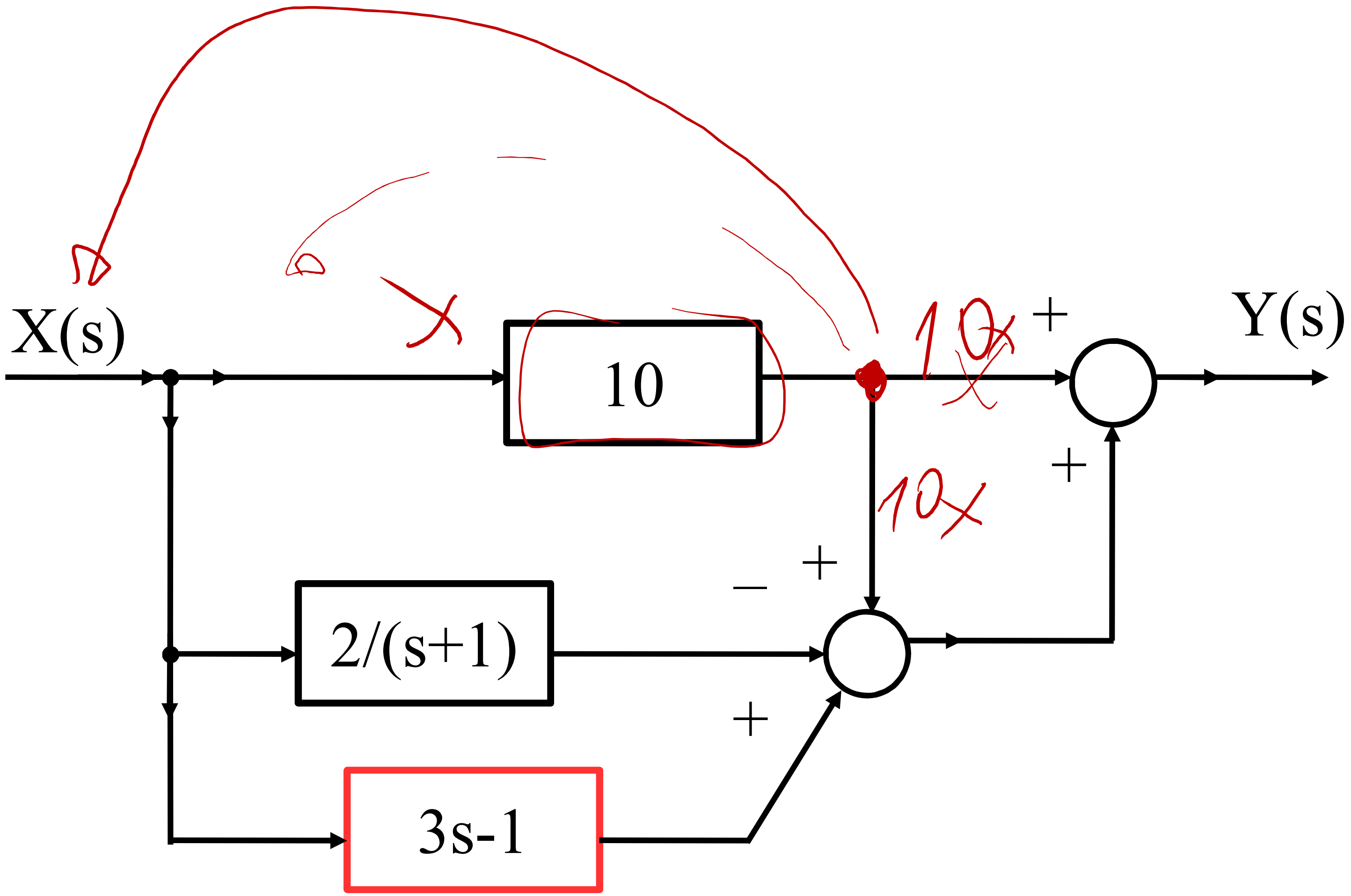
EXAMPLE 4



EXAMPLE 4



EXAMPLE 4



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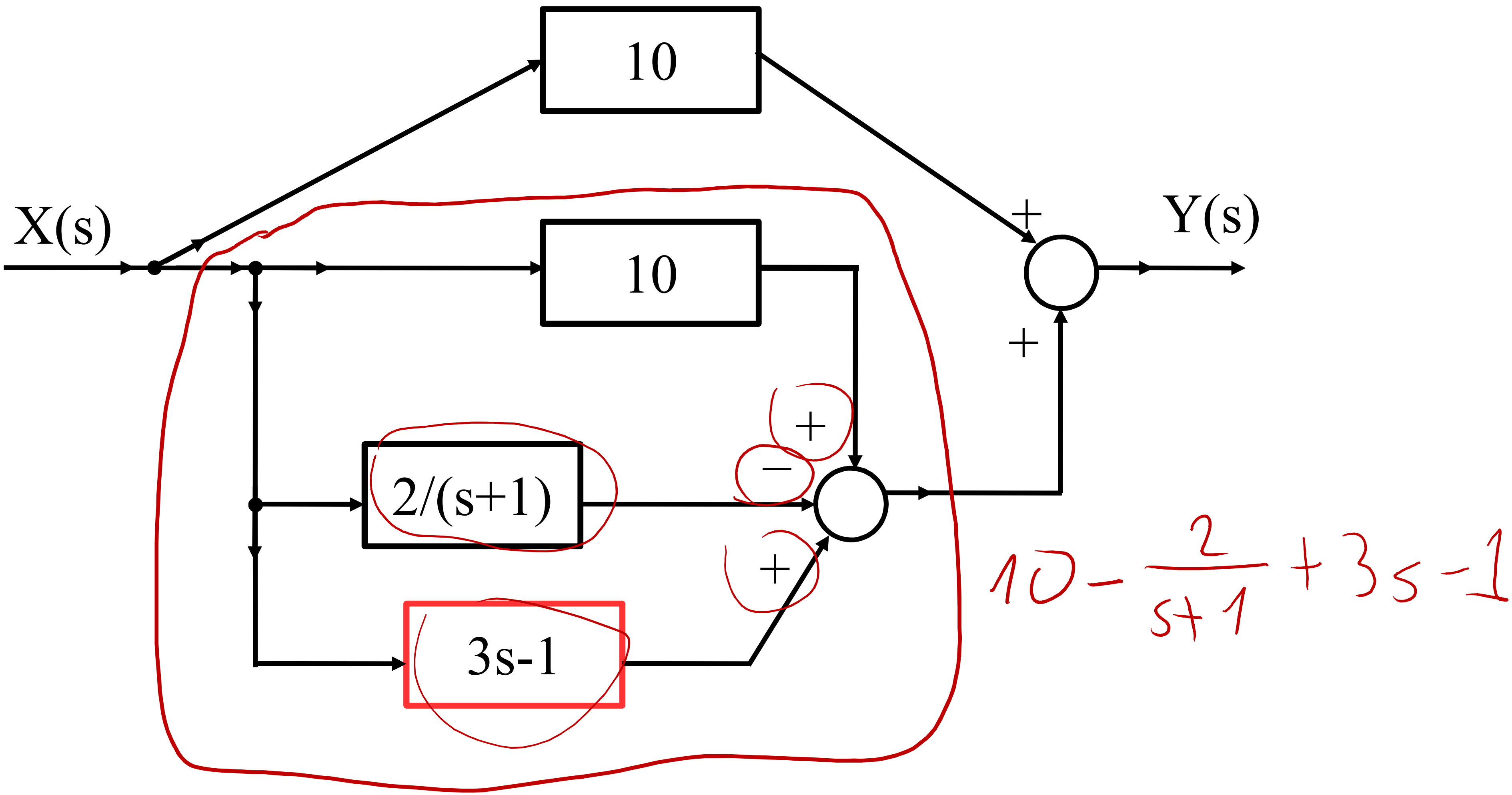

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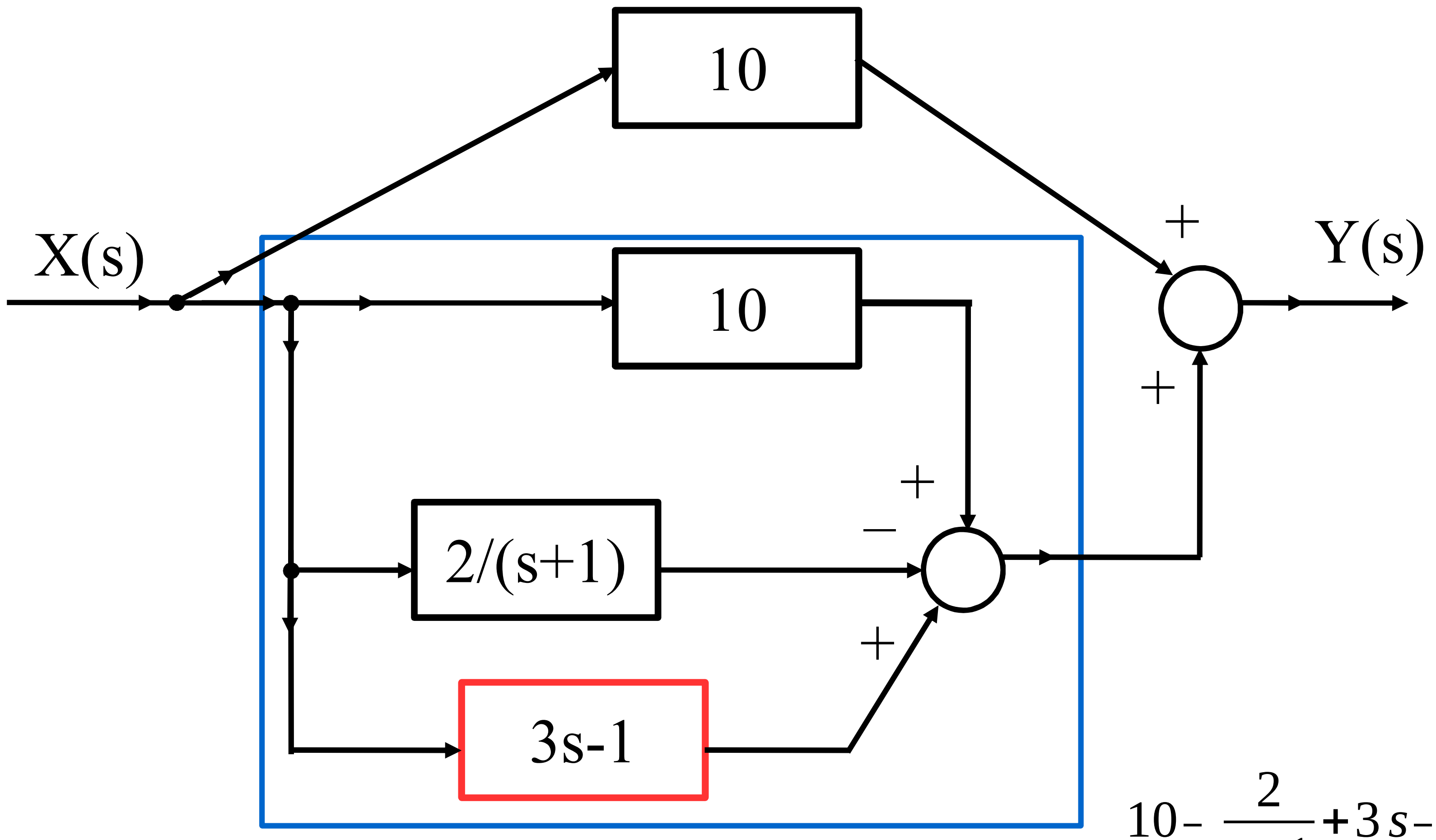
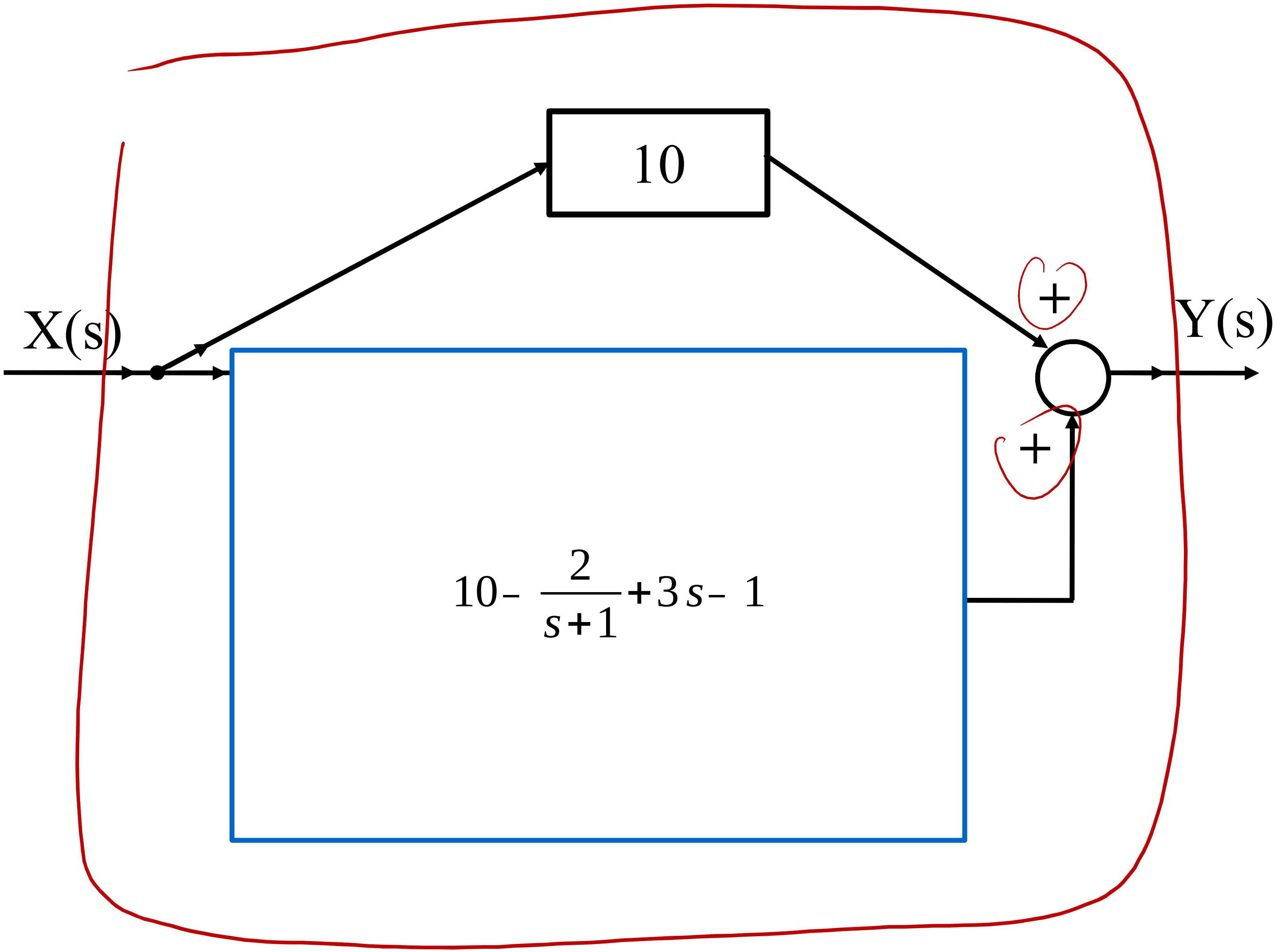
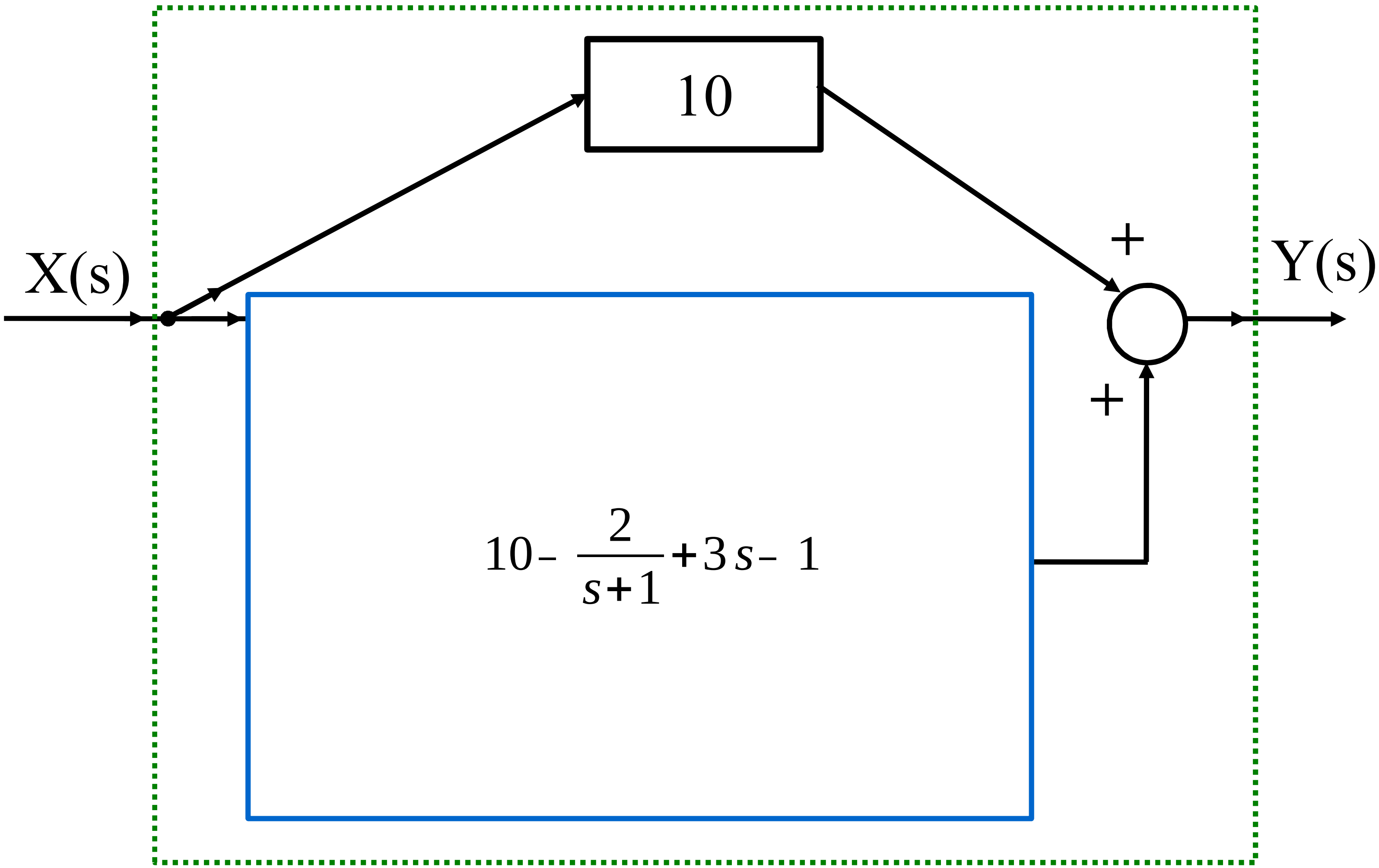


$$10-\frac{2}{s+1}+3s-1$$

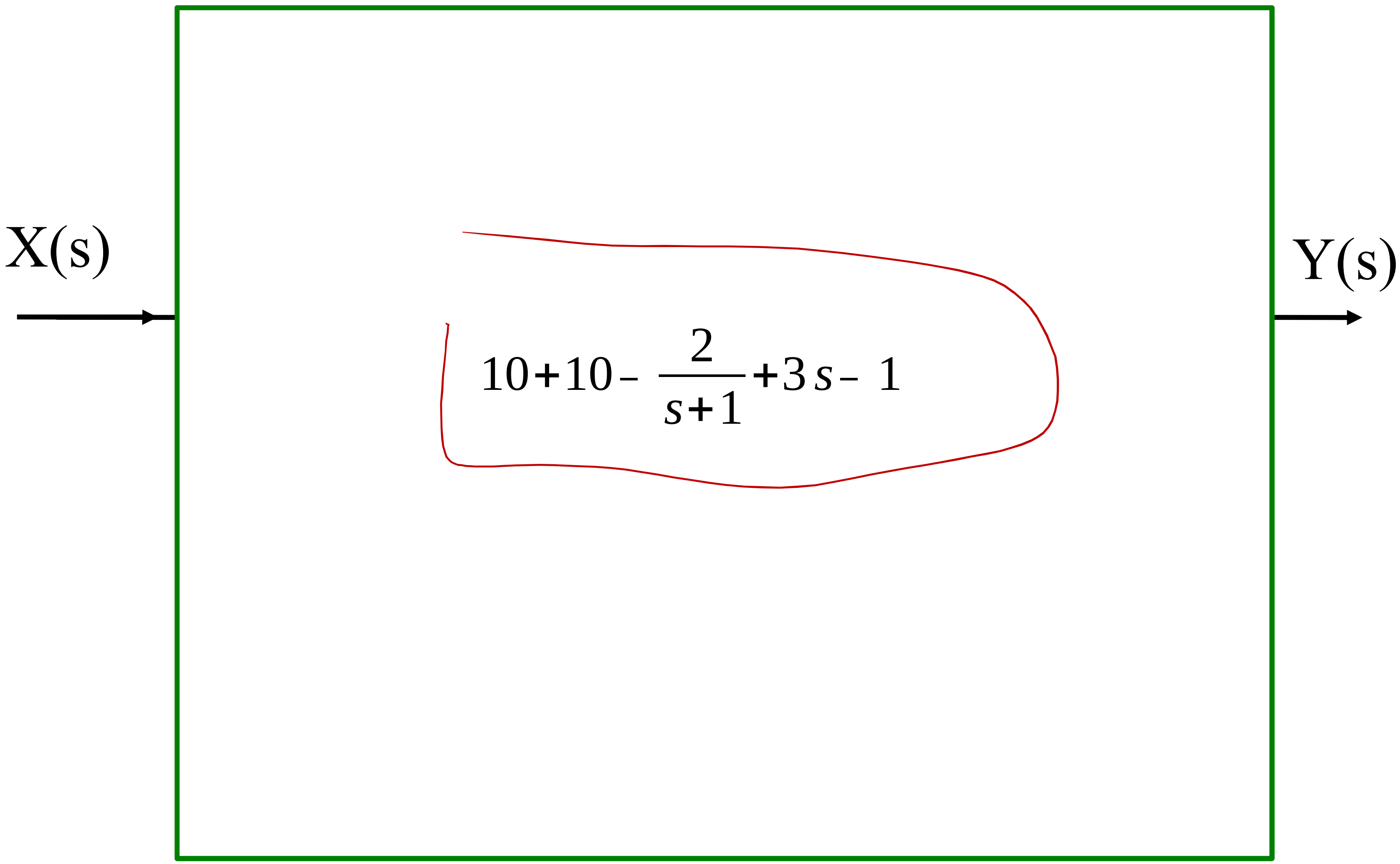
EXAMPLE 4



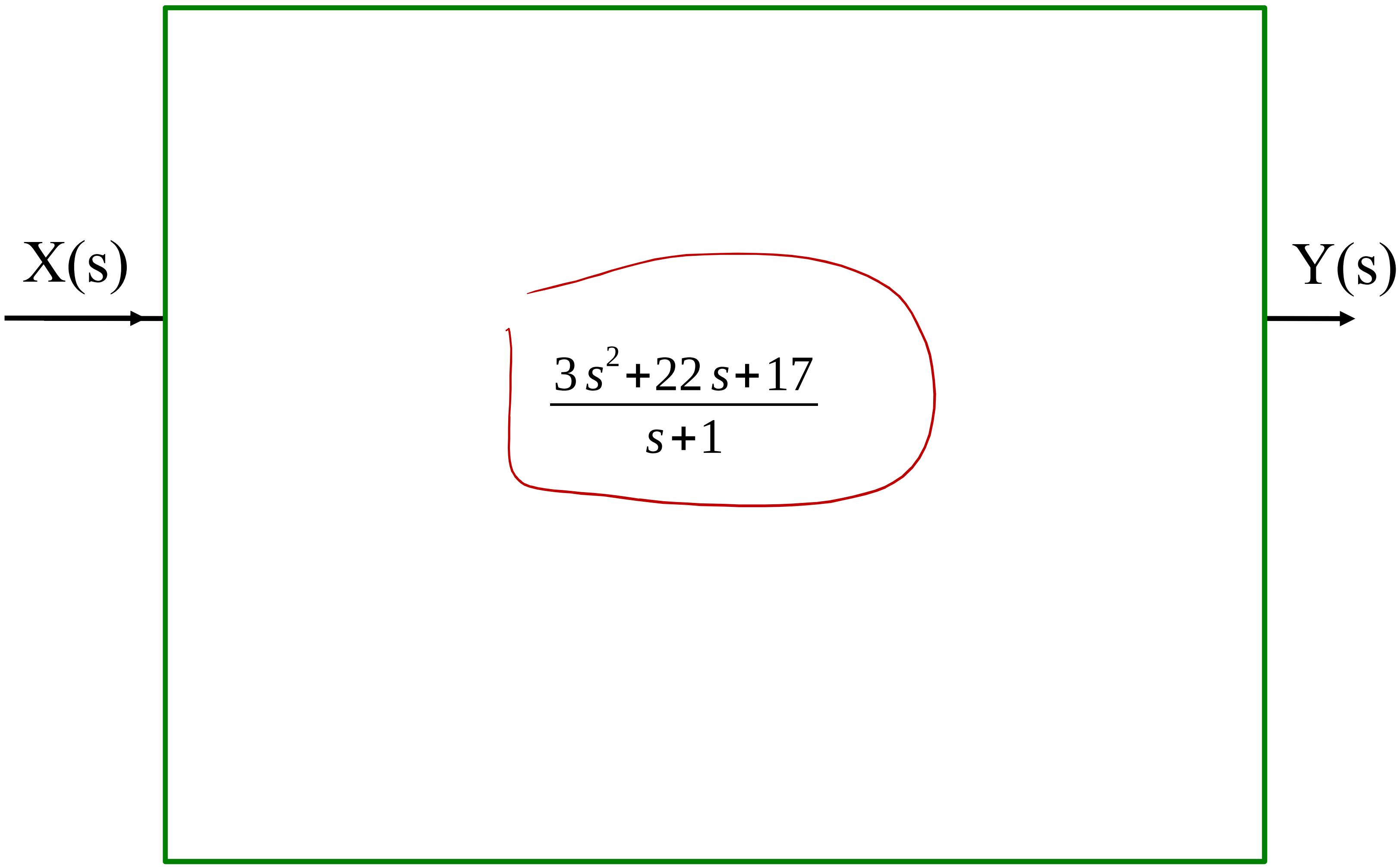
EXAMPLE 4



EXAMPLE 4

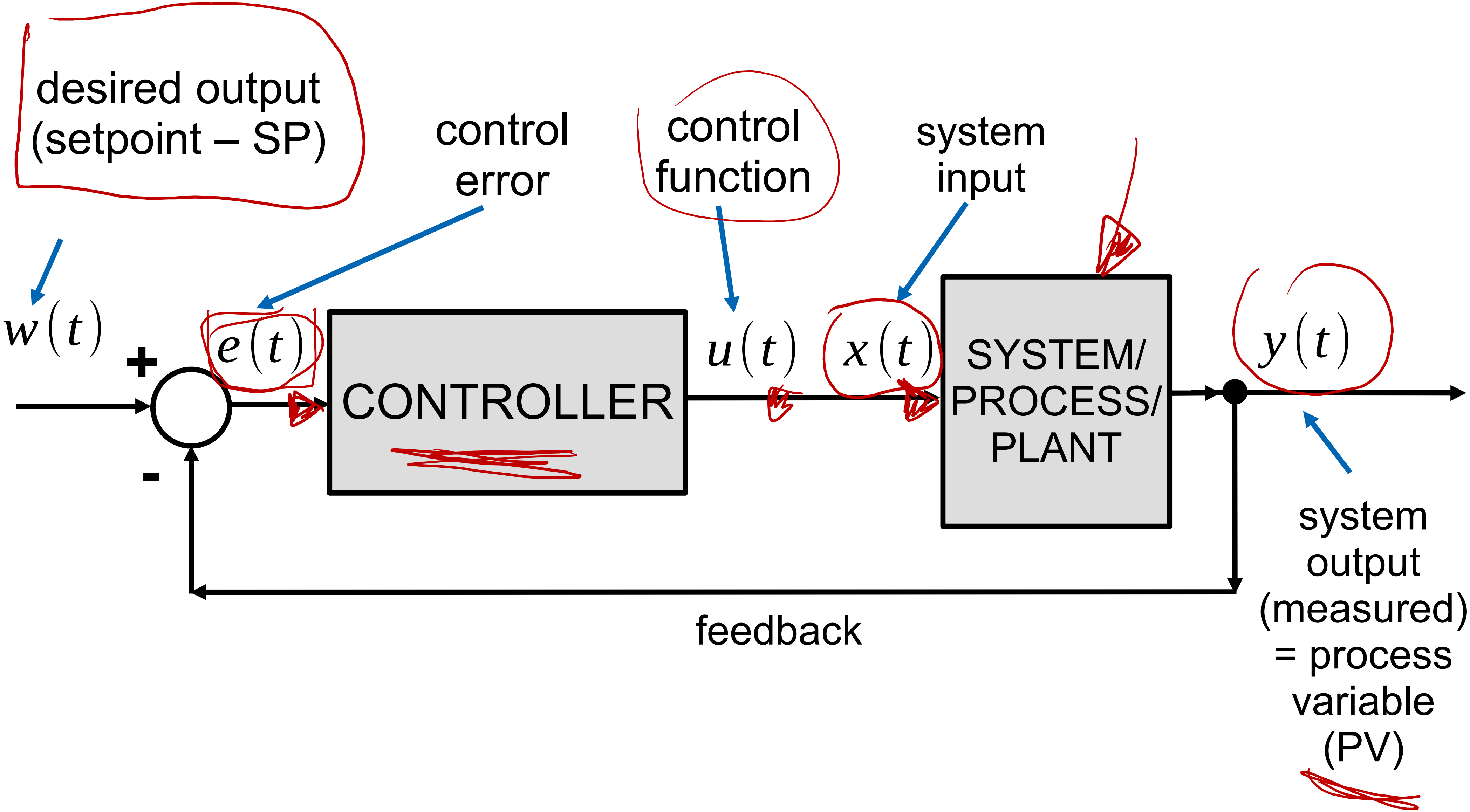


EXAMPLE 4



CONTROLLERS

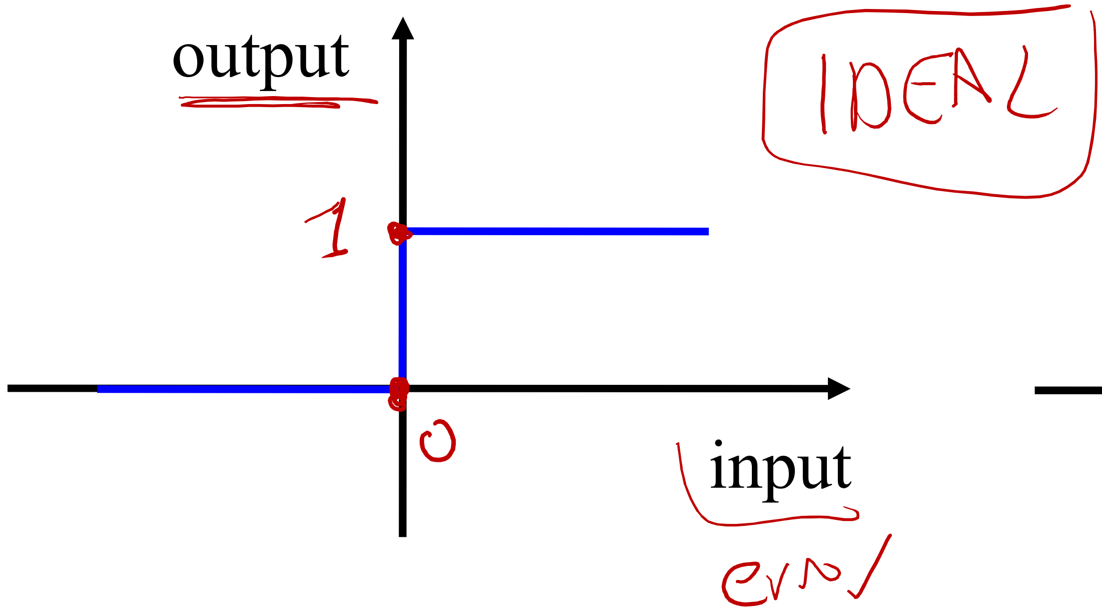
Closed loop control

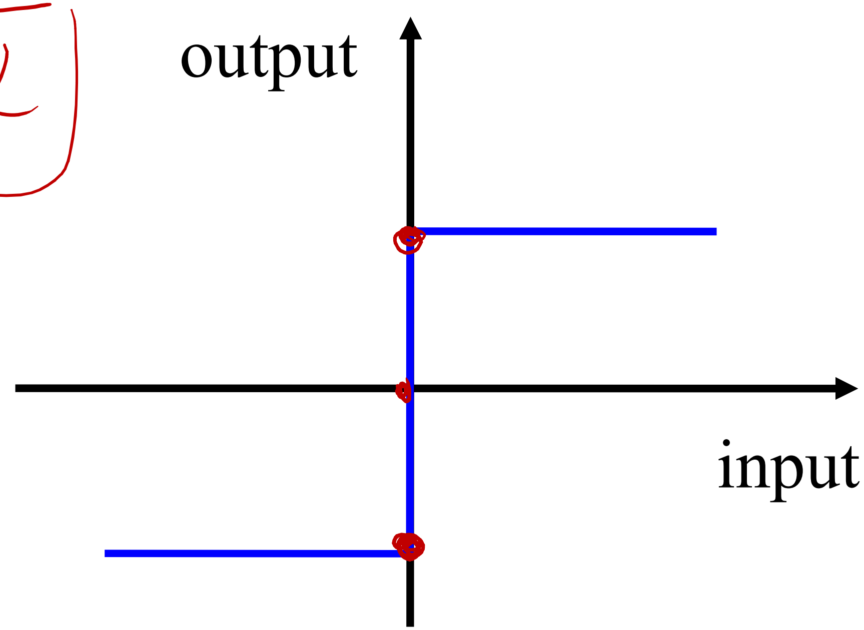


Types of controllers

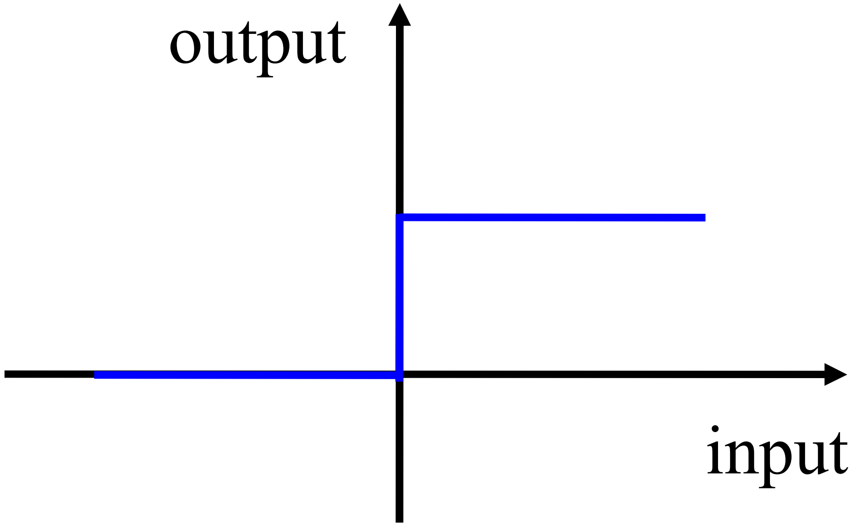
- two state (ON/OFF)
- three state
- Proportional (P)
- Integrator (I)
- Differentiator (D)
- Proportional-inegral-derivative (PID)

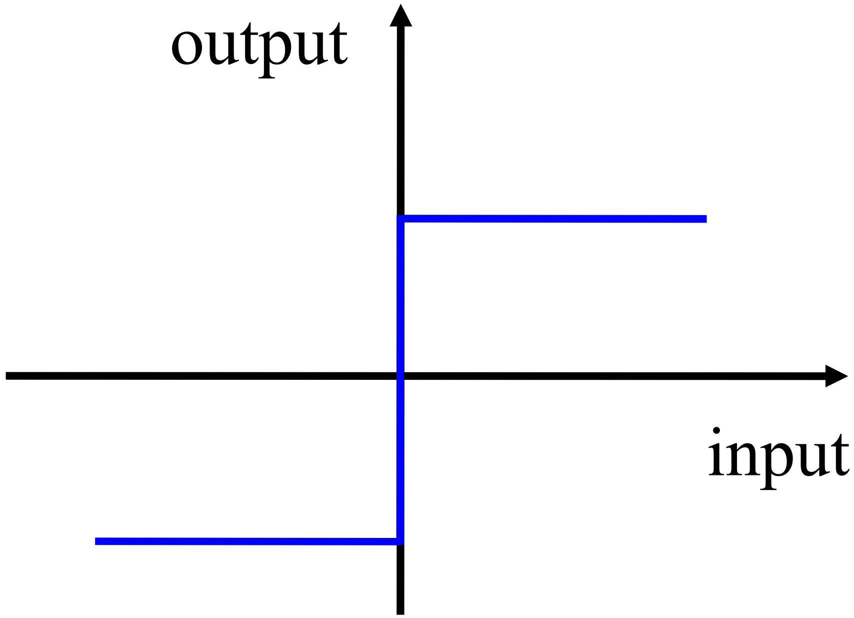
ON-OFF CONTROLLER
(RELAY / TWO STATE / BANG-BANG)



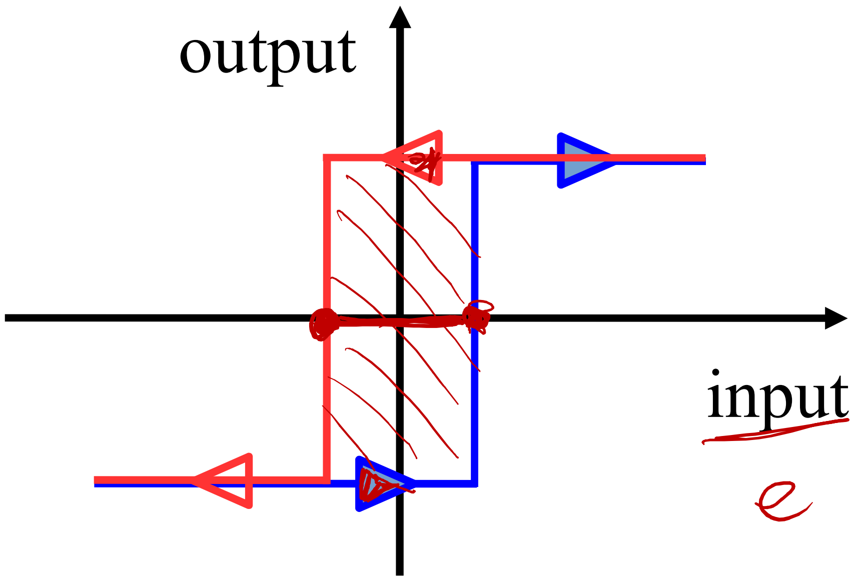


ON-OFF CONTROLLER
(RELAY / TWO STATE / BANG-BANG)

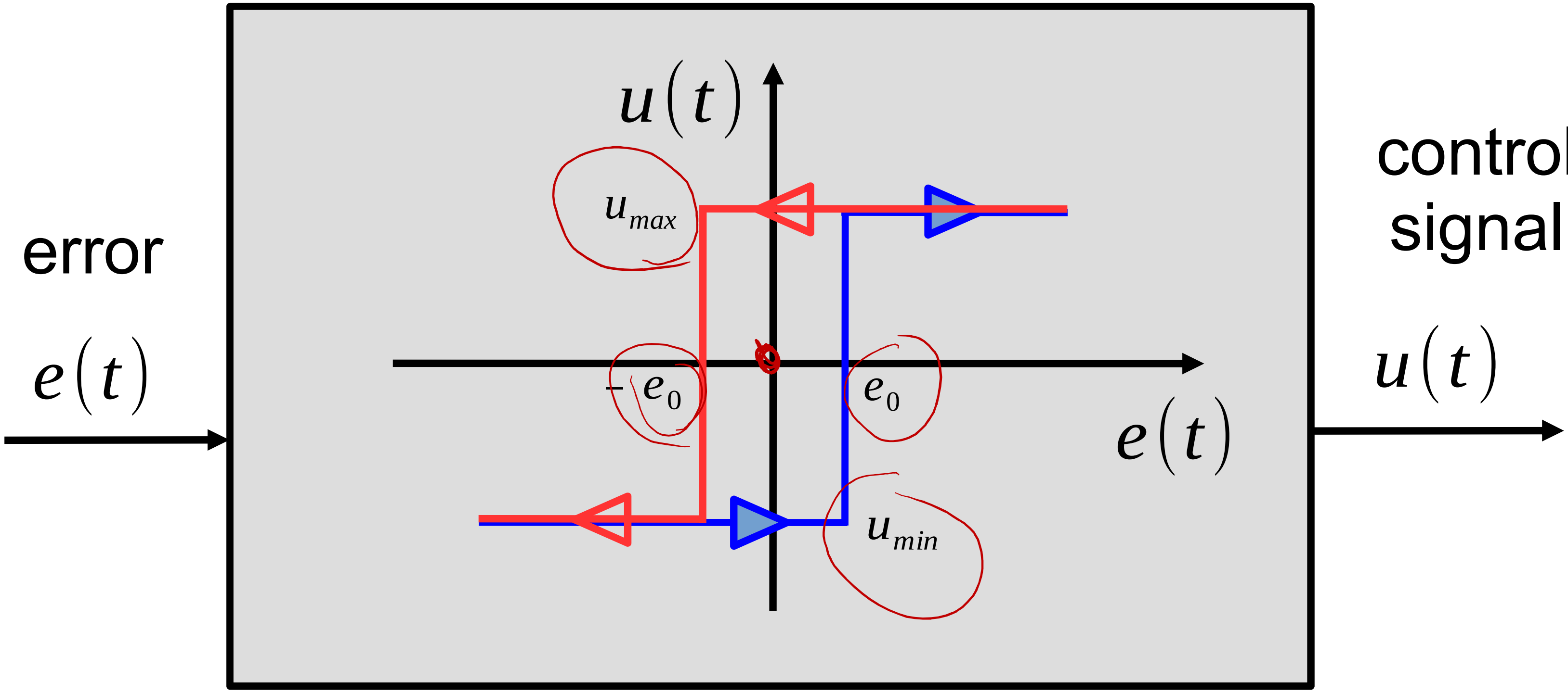




real
(with hysteresis)



ON-OFF CONTROLLER (RELAY / TWO STATE / BANG-BANG)

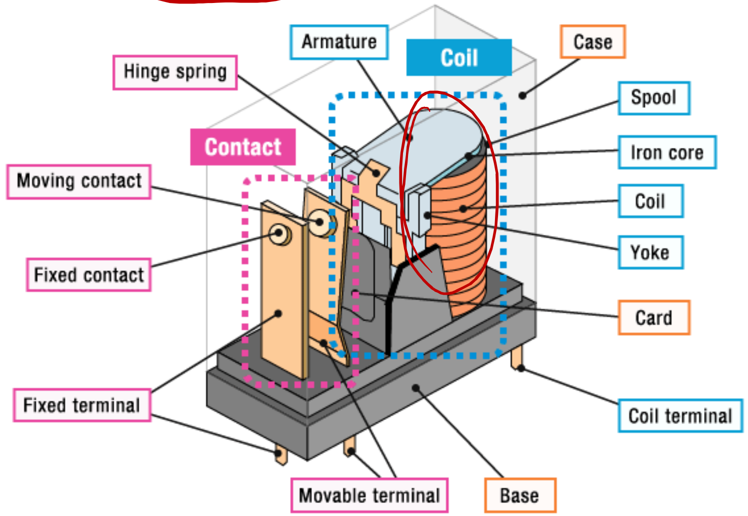


$$u(t)=\begin{cases} u_{max}, \text{ if } e>e_0 \\ u_{min}, \text{ if } e<-e_0 \\ \text{no change, in other situations} \end{cases}$$

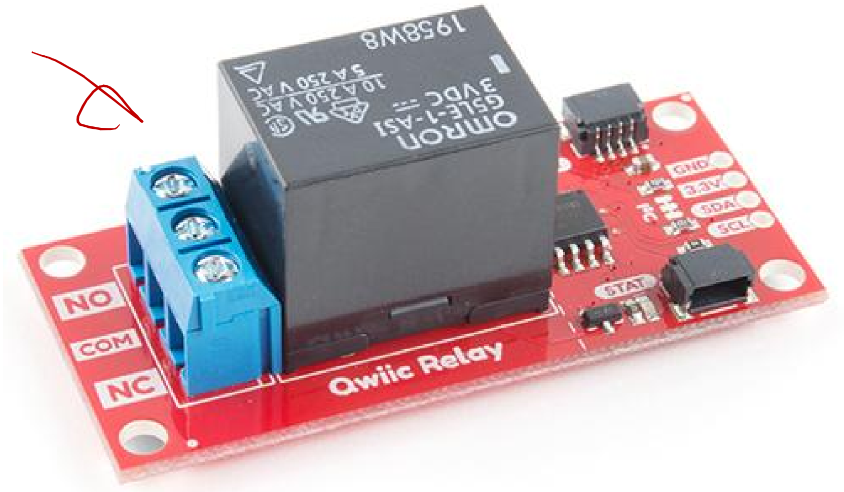
e_o - mechanical or programmed hysteresis

ON-OFF CONTROLLER
(RELAY / TWO STATE / BANG-BANG)

mechanical relay

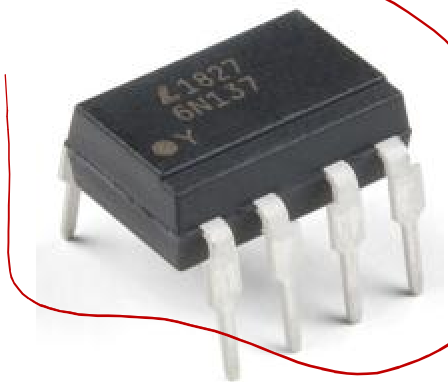


Source: https://www.components.omron.com/relay-basics/basic

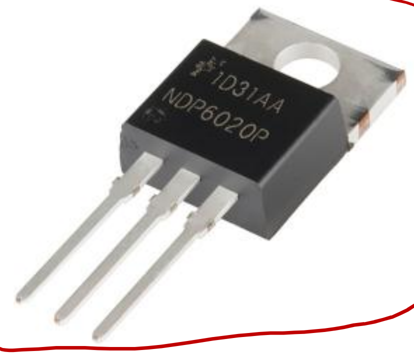


Source: https://www.sparkfun.com/products/15093

electronic relay
(optoisolators, MOSFETs)

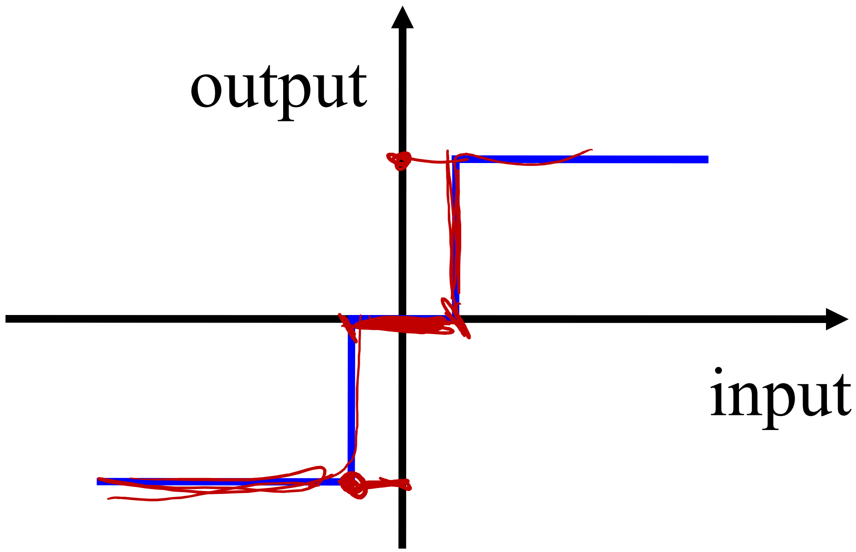


source: https://www.sparkfun.com/products/15105



source: https://www.sparkfun.com/products/12901

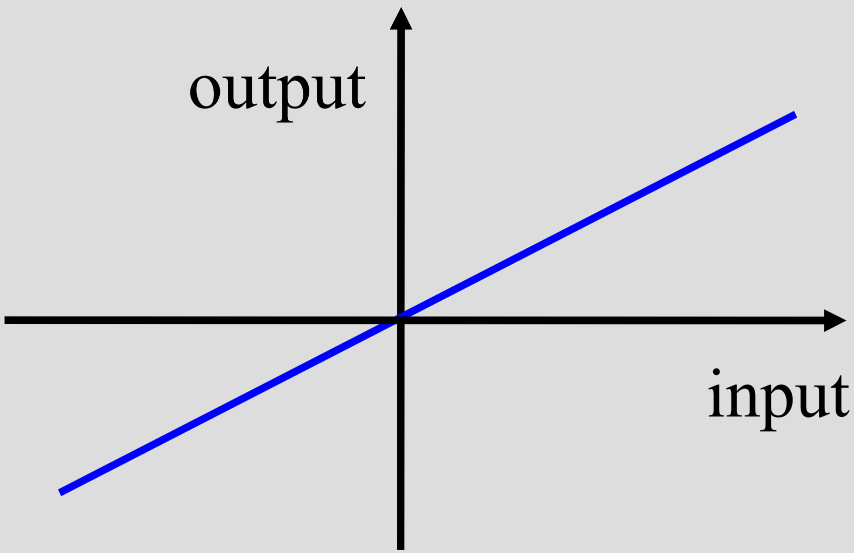
THREE STATE CONTROLLER



Proportional (P) CONTROLLER

error

$e(t)$



$$u(t) = k_P \cdot e(t)$$

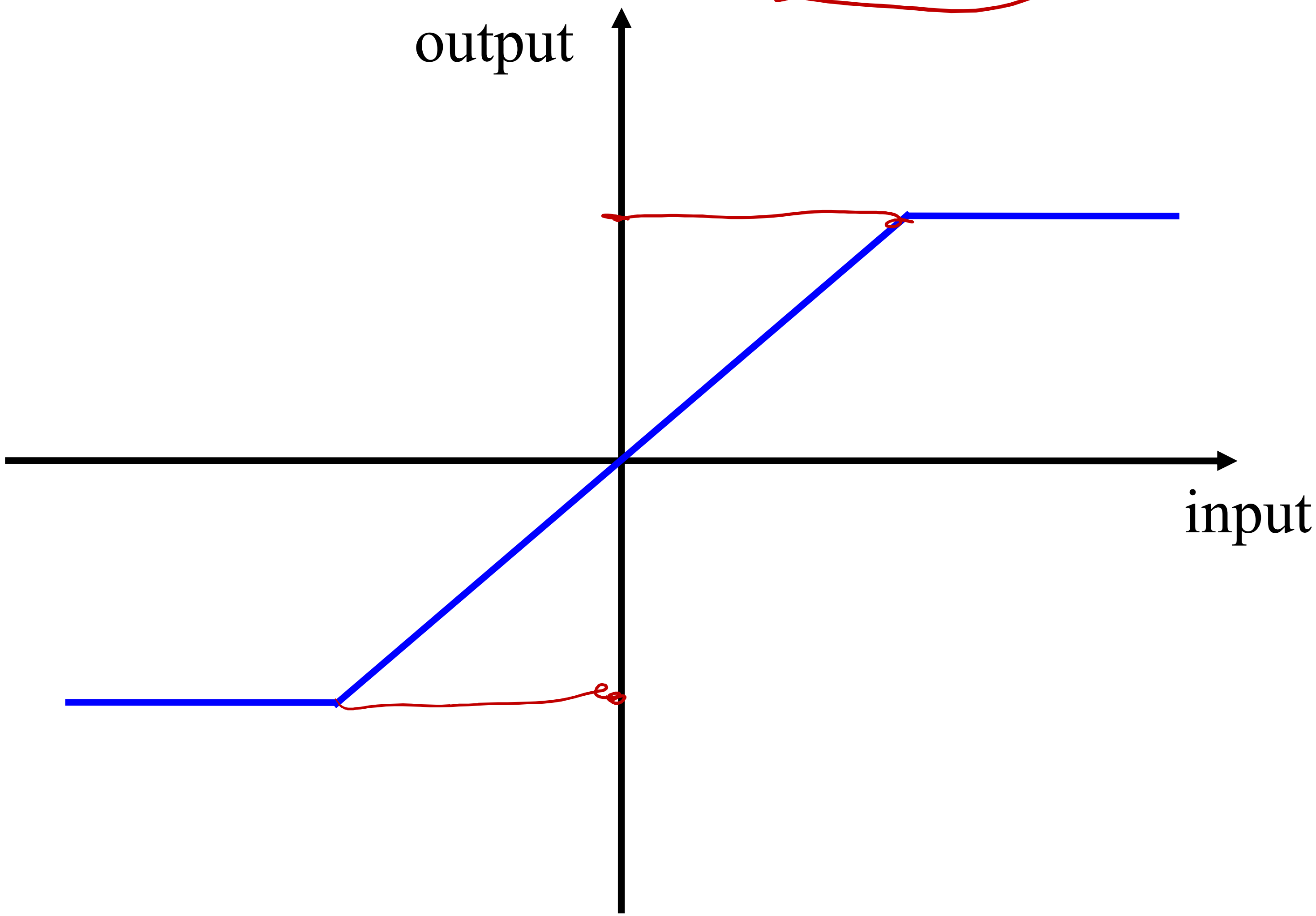
$$G(s) = \frac{U(s)}{E(s)} = k_P$$

control signal

$u(t)$

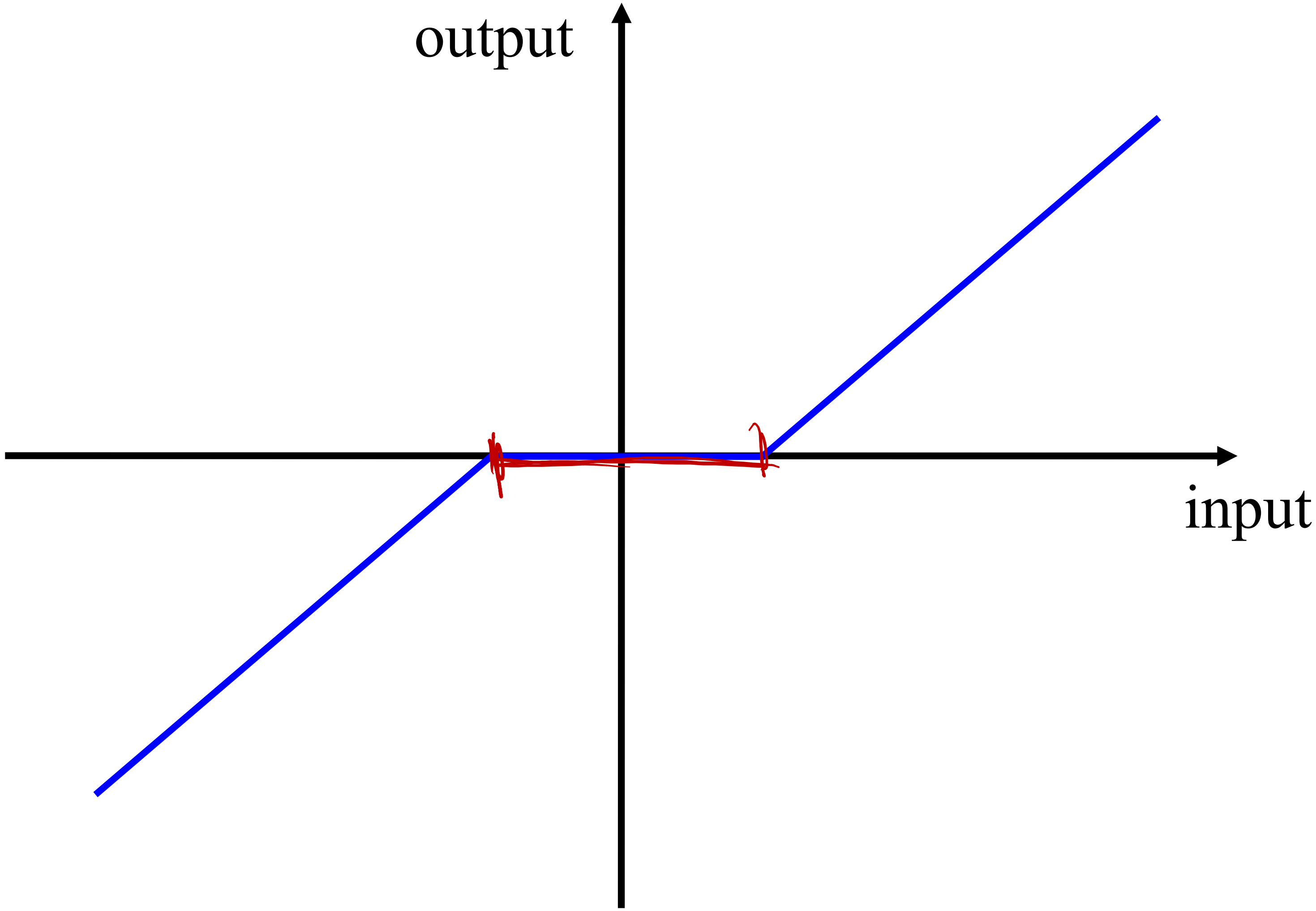
NONLINEARITIES

Symmetric hard limiting saturation

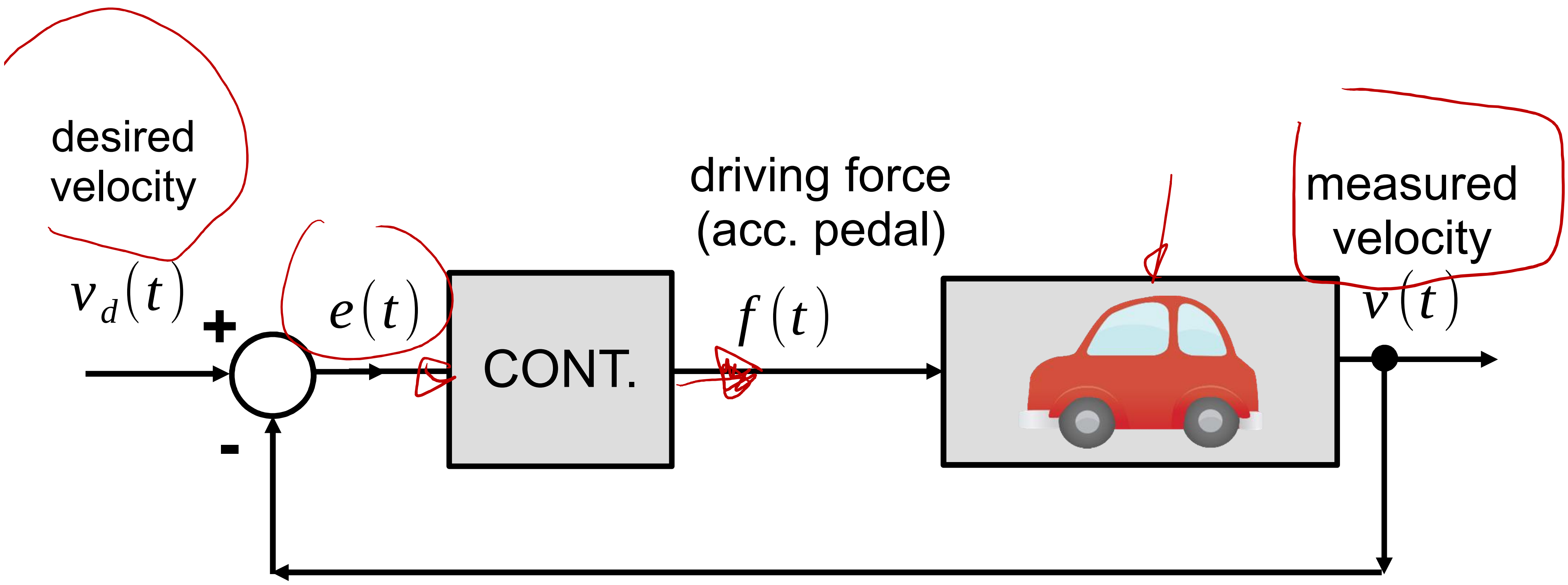


NONLINEARITIES

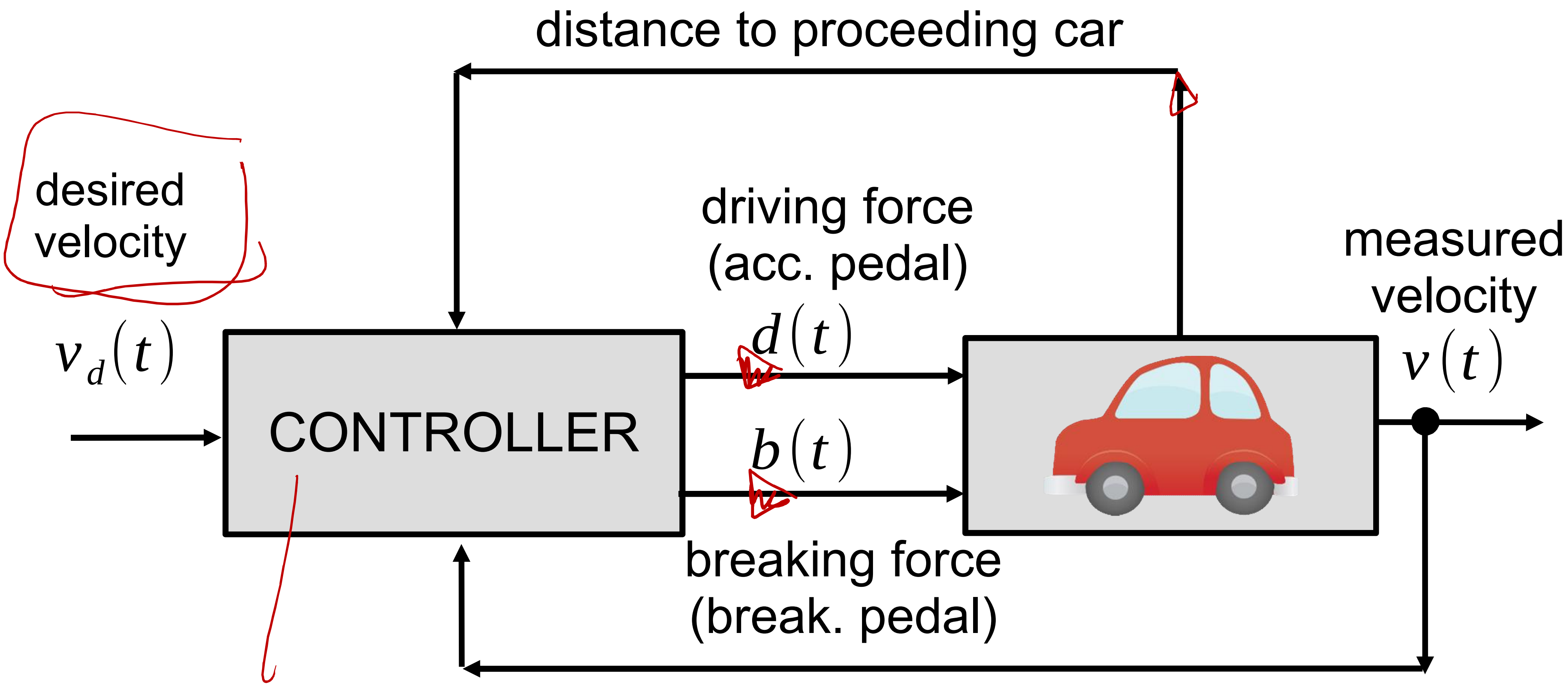
Dead zone



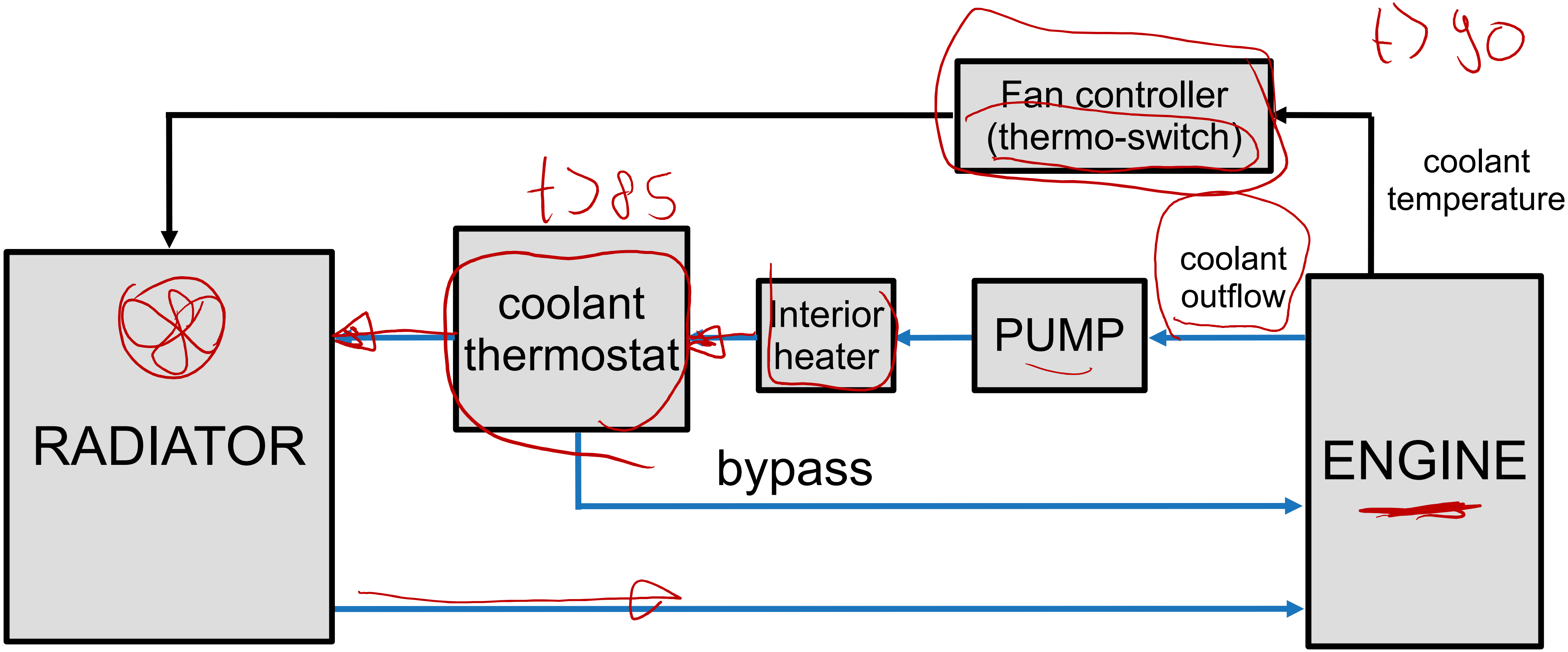
Speed control
(cruise control, autocruise, tempomat)



Speed control
(adaptive cruise control)



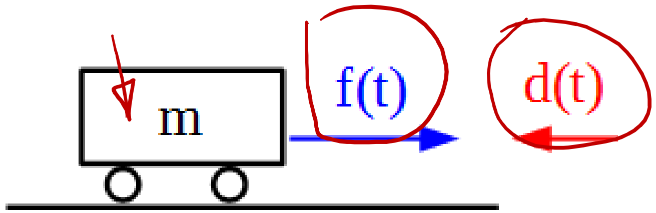
Engine's temperature control



EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

Car on a flat surface
m – mass,
f(t) – driving force,
d(t)=c*v(t) – air resistance,
v(t) – velocity



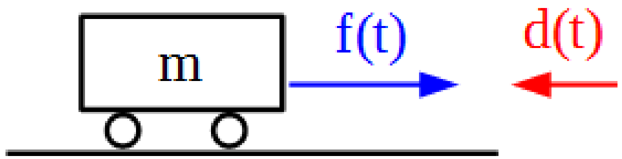
$$m\frac{dv(t)}{dt} = f(t) - d(t)$$

$$H(s) = \frac{V(s)}{F(s)} = \frac{1}{ms + c}$$

EXAMPLE 1

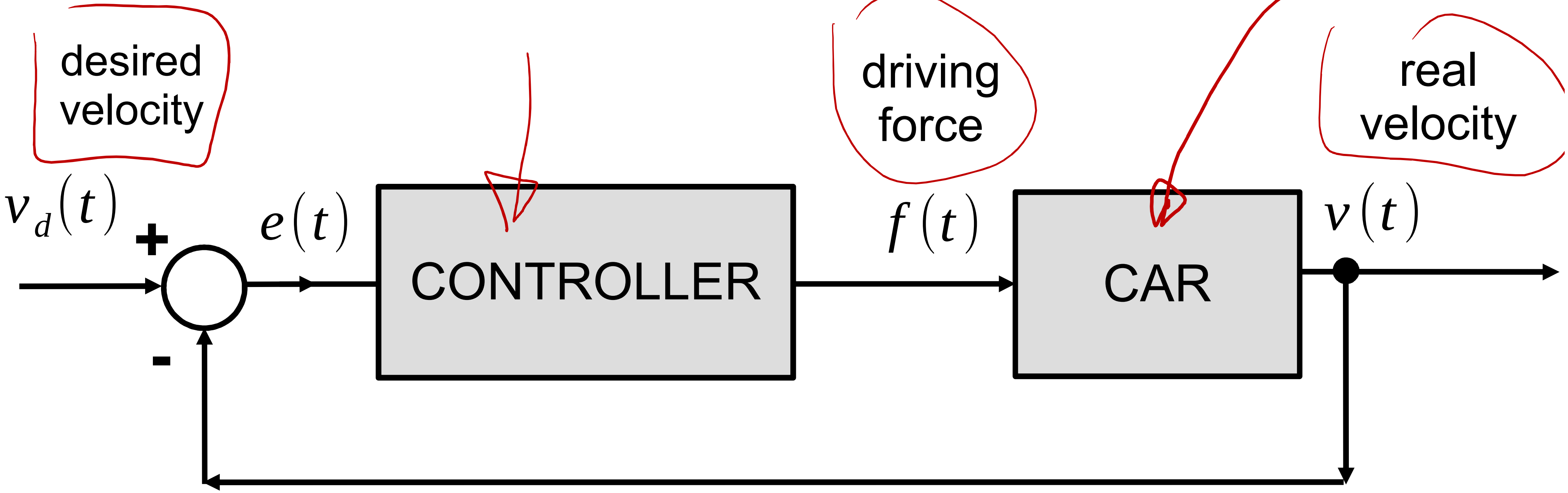
Speed control (cruise control, autocruise, tempomat)

Car on a flat surface
m – mass,
f(t) – driving force,
d(t)=c*v(t) – air resistance,
v(t) – velocity



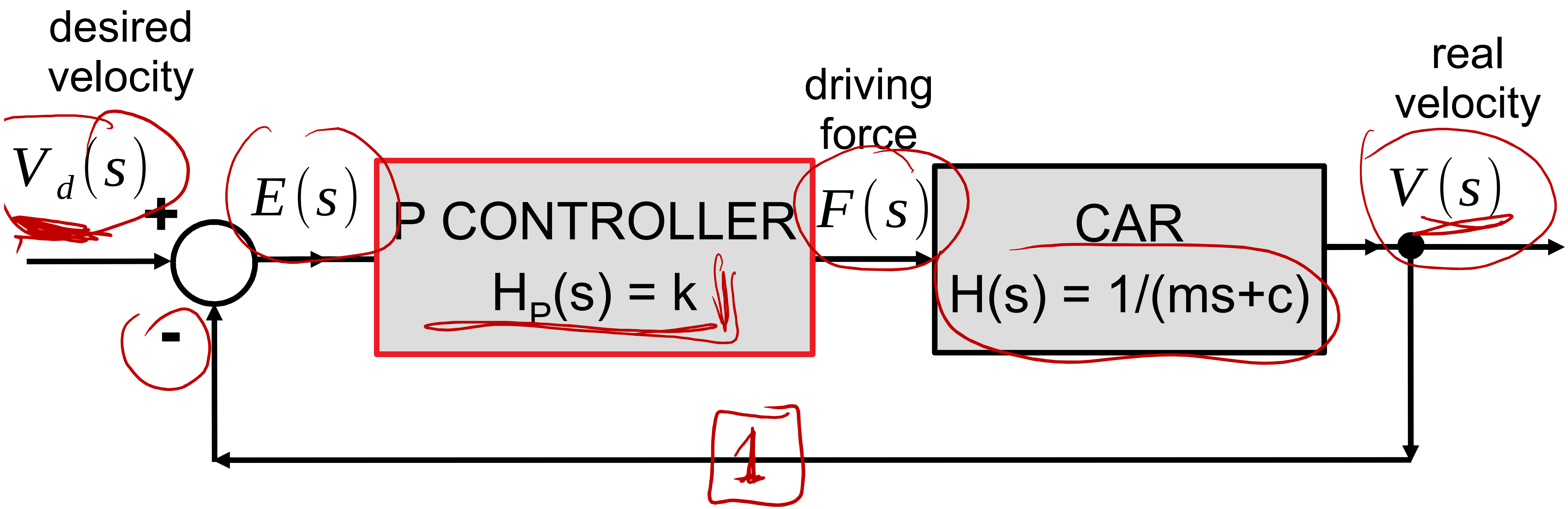
$$m\frac{dv(t)}{dt}=f(t)-d(t)$$

$$H(s)=\frac{V(s)}{F(s)}=\frac{1}{ms+c}$$



EXAMPLE 1

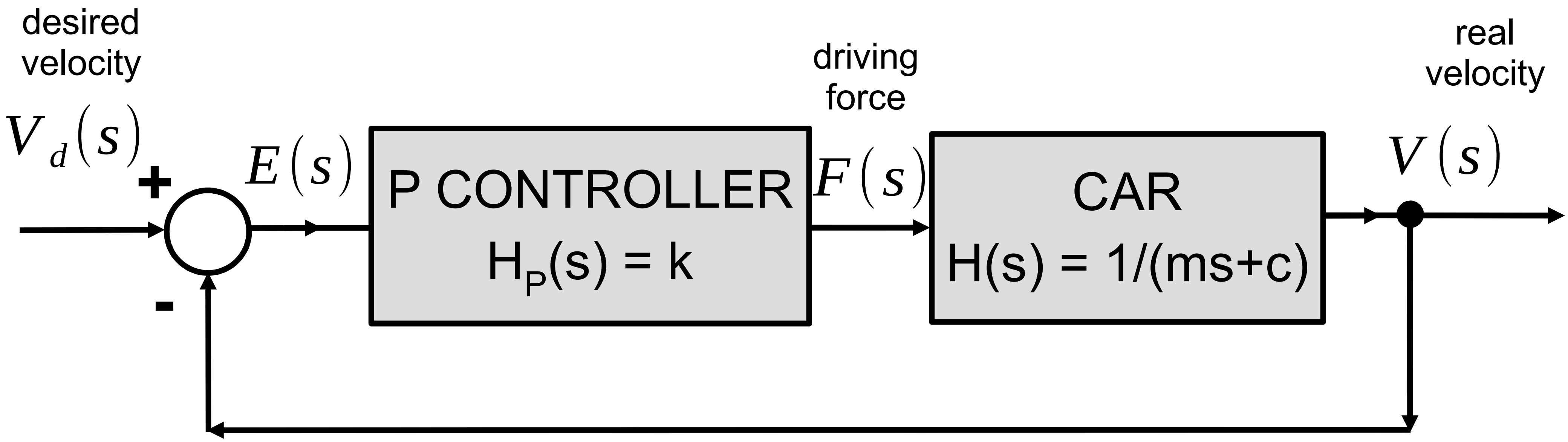
Speed control (cruise control, autocruise, tempomat)



$$H_R(s) = \frac{H_p \cdot H}{1 + H_p \cdot H \cdot 1} = \frac{k \cdot \frac{1}{ms+c}}{1 + k \frac{1}{ms+c}} = \frac{V(s)}{V_d(s)} = \frac{k}{ms+c+k}$$

EXAMPLE 1

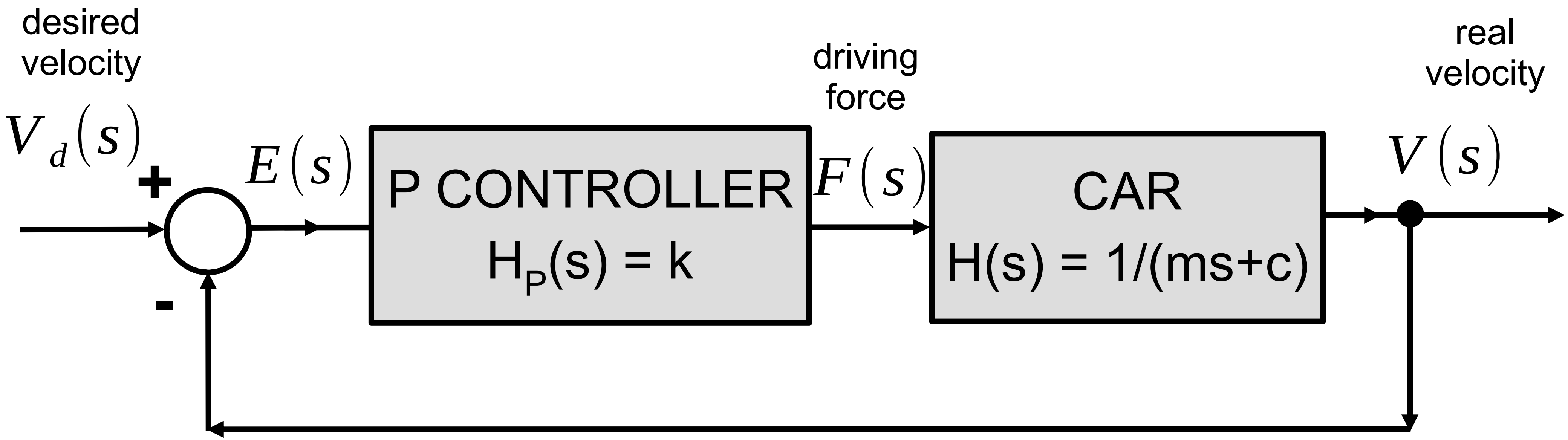
Speed control (cruise control, autocruise, tempomat)



$$H_R(s) = \frac{H_P(s)H(s)}{1 + H_P(s)H(s)}$$

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

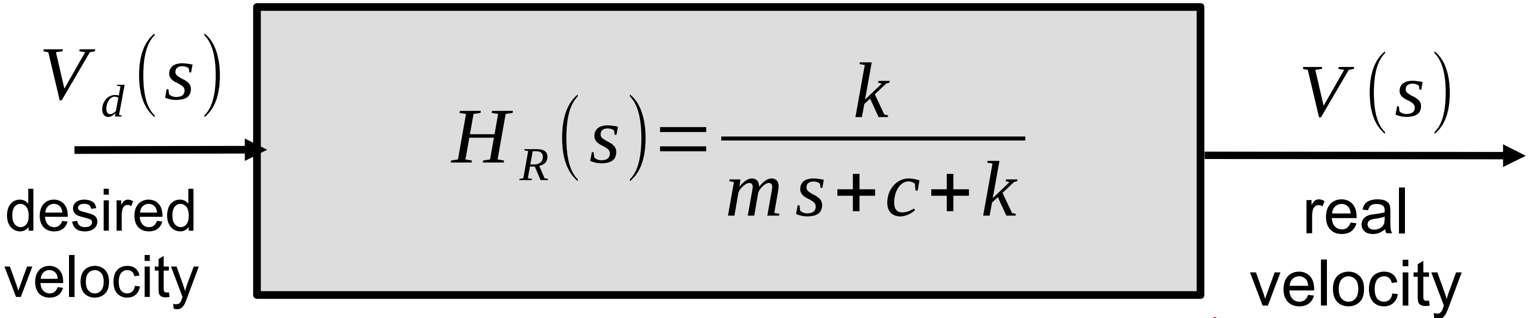


$$H_R(s) = \frac{H_P(s)H(s)}{1+H_P(s)H(s)}$$

$$H_R(s) = \frac{k}{ms+c+k}$$

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)



Input function: $v_d(t)=v_0 1(t)$

$$V_d(s)=V_0\frac{1}{s}$$

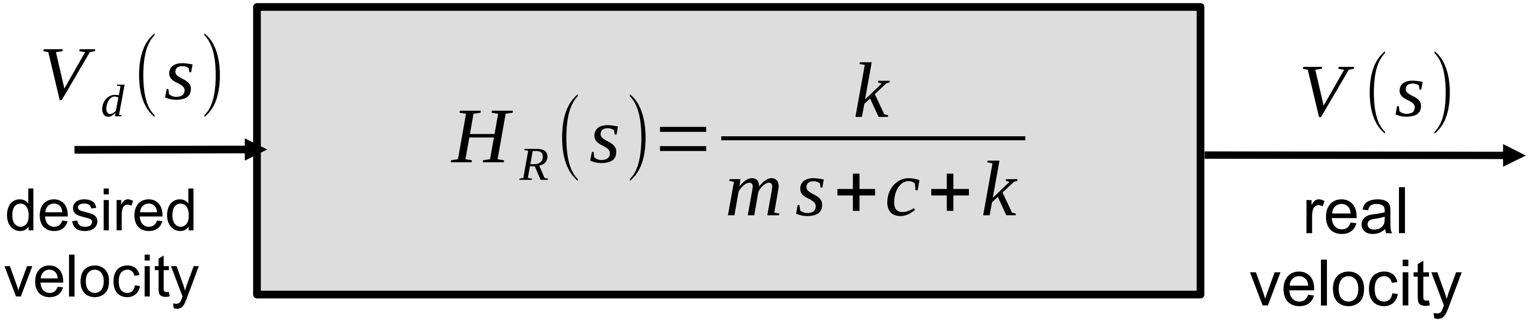
$$V(s)=H_R(s)\cdot V_d(s)=\frac{k}{ms+c+k}\cdot\frac{V_0}{s}=kV_0\frac{1}{s(ms+c+k)}$$

$$=kV_0\frac{\frac{1}{m}}{s\left(s+\frac{c+k}{m}\right)}=\frac{kV_0}{c+k}\,\frac{\frac{c+k}{m}}{s\left(s+\frac{c+k}{m}\right)}$$

$$v(t)=\mathcal{L}^{-1}\{V(s)\}=\frac{kV_0}{c+k}\left(1-e^{-\frac{c+k}{m}t}\right)$$

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)



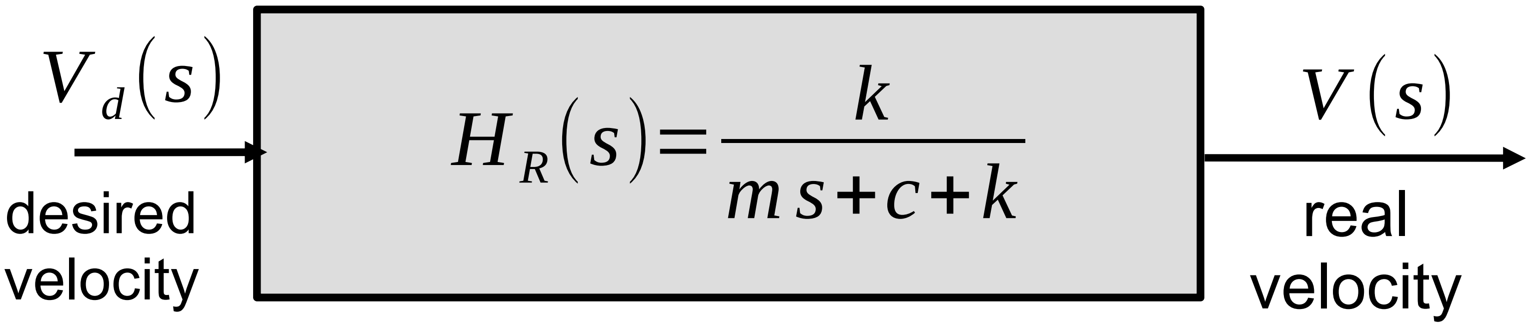
Input function: $v_d(t)=v_0 1(t)$ Laplace of input: $V_d(s)=v_0\frac{1}{s}$

Laplace of output:

$$V(s)=V_d(s)H_R(s)=\frac{v_0 k}{s(ms+c+k)}=\frac{v_0 k}{c+k}\frac{\frac{c+k}{m}}{s\left(s+\frac{c+k}{m}\right)}$$

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)



Input function: $v_d(t)=v_0 1(t)$ Laplace of input: $V_d(s)=v_0\frac{1}{s}$

Laplace of output:

$$V(s)=V_d(s)H_R(s)=\frac{v_0 k}{s(ms+c+k)}=\frac{v_0 k}{c+k}\frac{\frac{c+k}{m}}{s\left(s+\frac{c+k}{m}\right)}$$

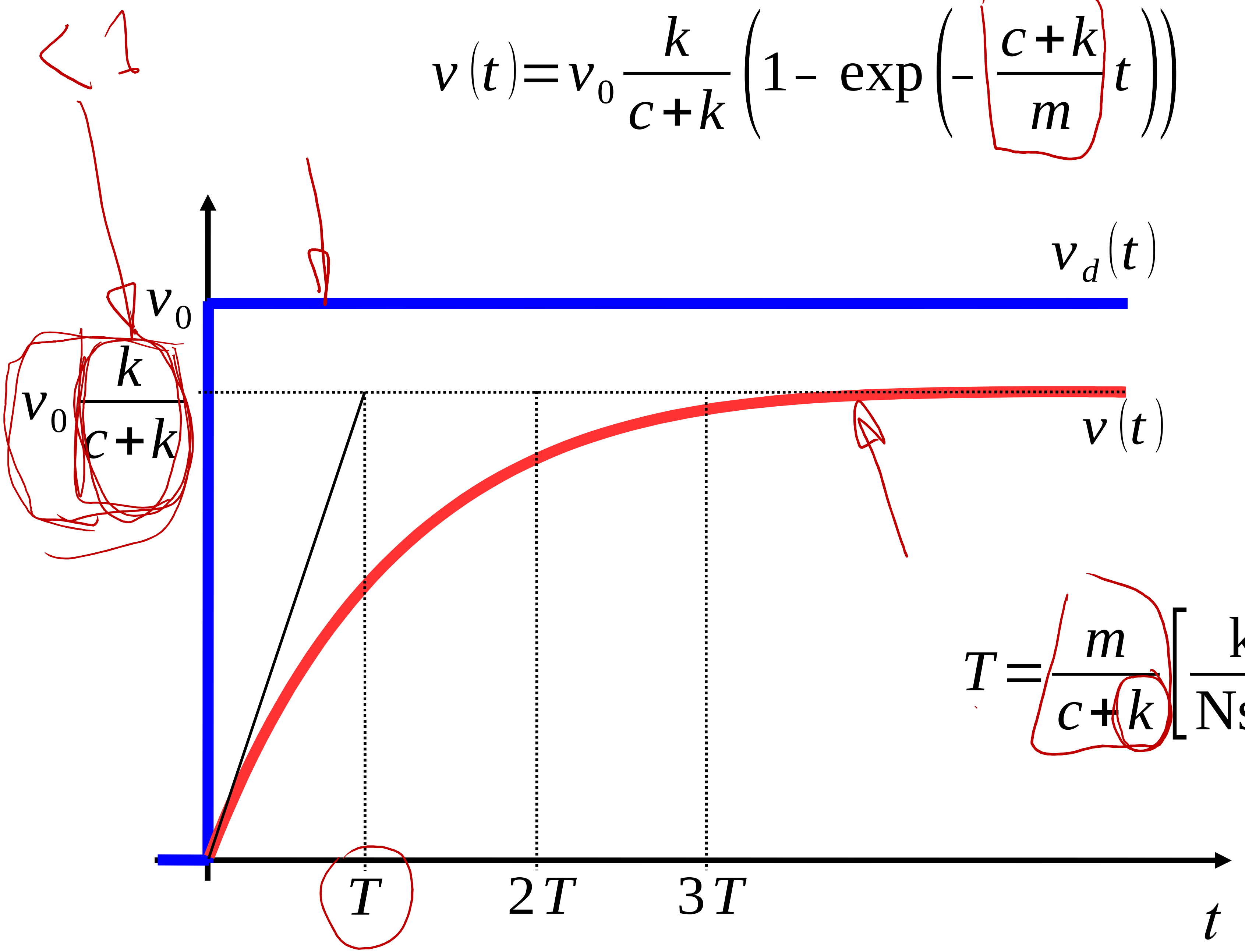
Output:

$$v(t)=\frac{v_0 k}{c+k}\left(1-\exp\left(-\frac{c+k}{m}t\right)\right)$$

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

$$v(t) = v_0 \frac{k}{c+k}\left(1 - \exp\left(-\frac{c+k}{m}t\right)\right)$$

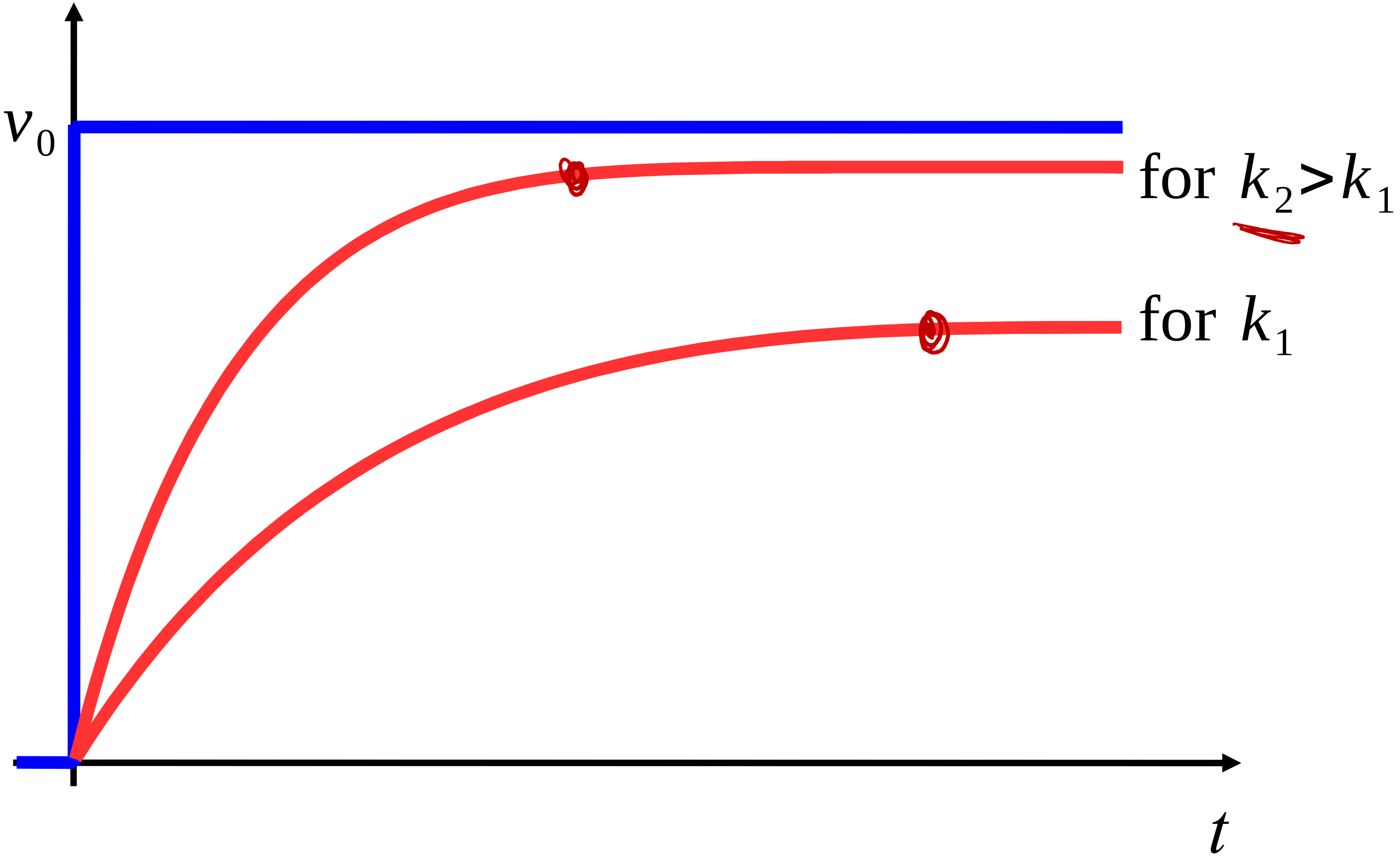


$$T = \frac{m}{c+k}\left[\frac{\text{kg}}{\text{Ns/m}} = \text{s}\right]$$

EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

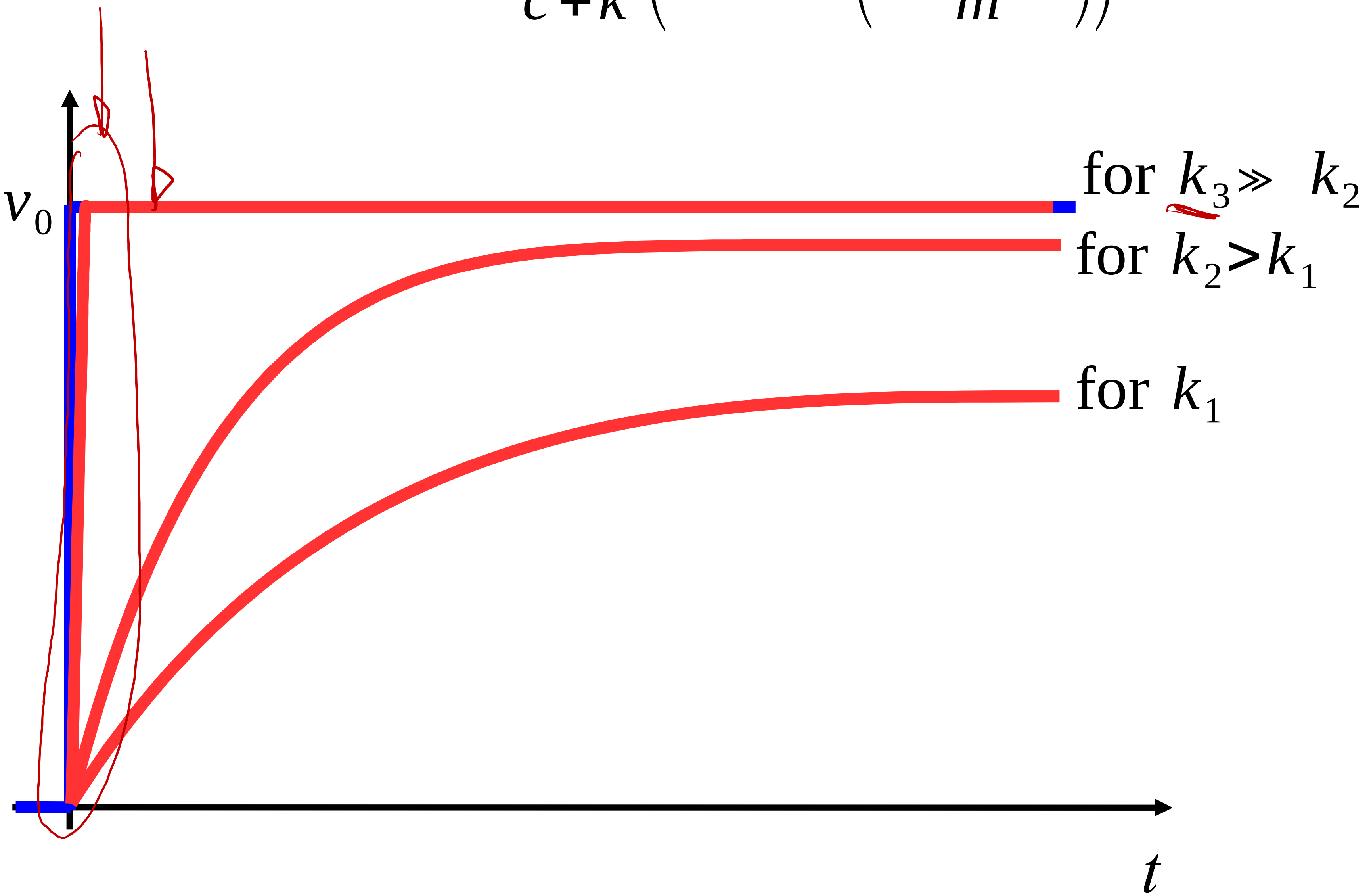
$$v(t) = v_0 \frac{k}{c+k}\left(1 - \exp\left(-\frac{c+k}{m}t\right)\right)$$



EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

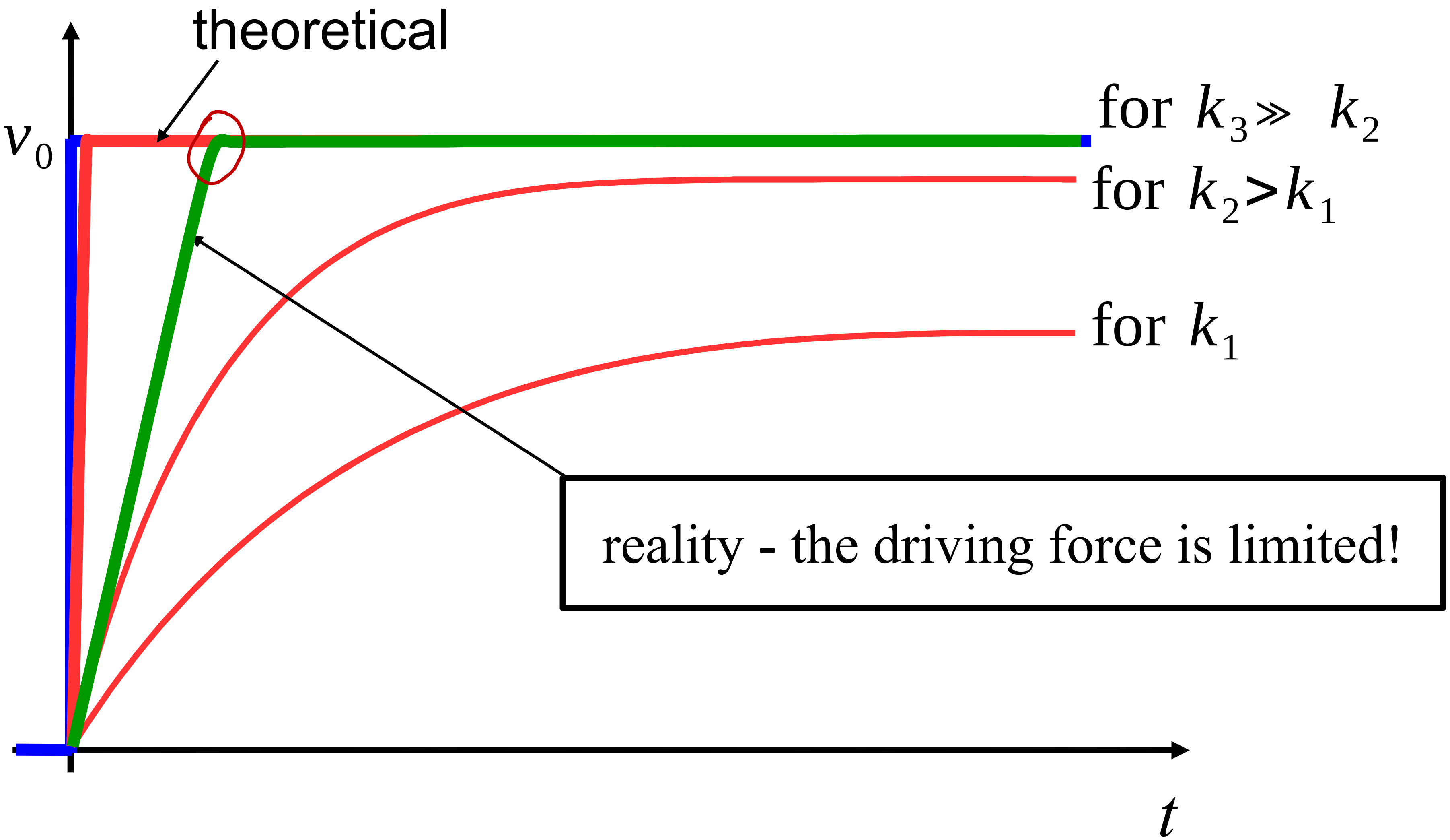
$$v(t) = v_0 \frac{k}{c+k}\left(1 - \exp\left(-\frac{c+k}{m}t\right)\right)$$



EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

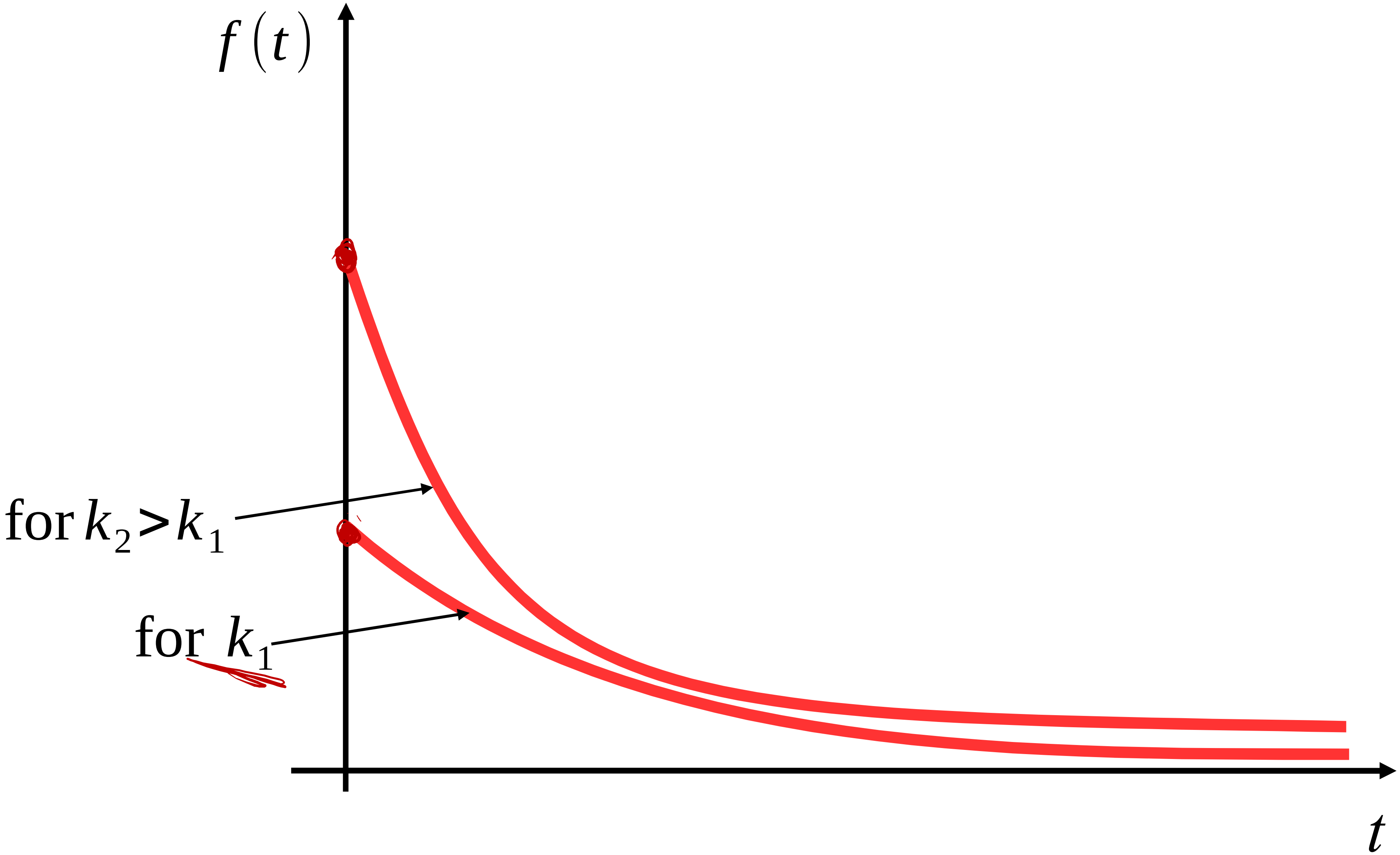
$$v(t) = v_0 \frac{k}{c+k}\left(1 - \exp\left(-\frac{c+k}{m}t\right)\right)$$



EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

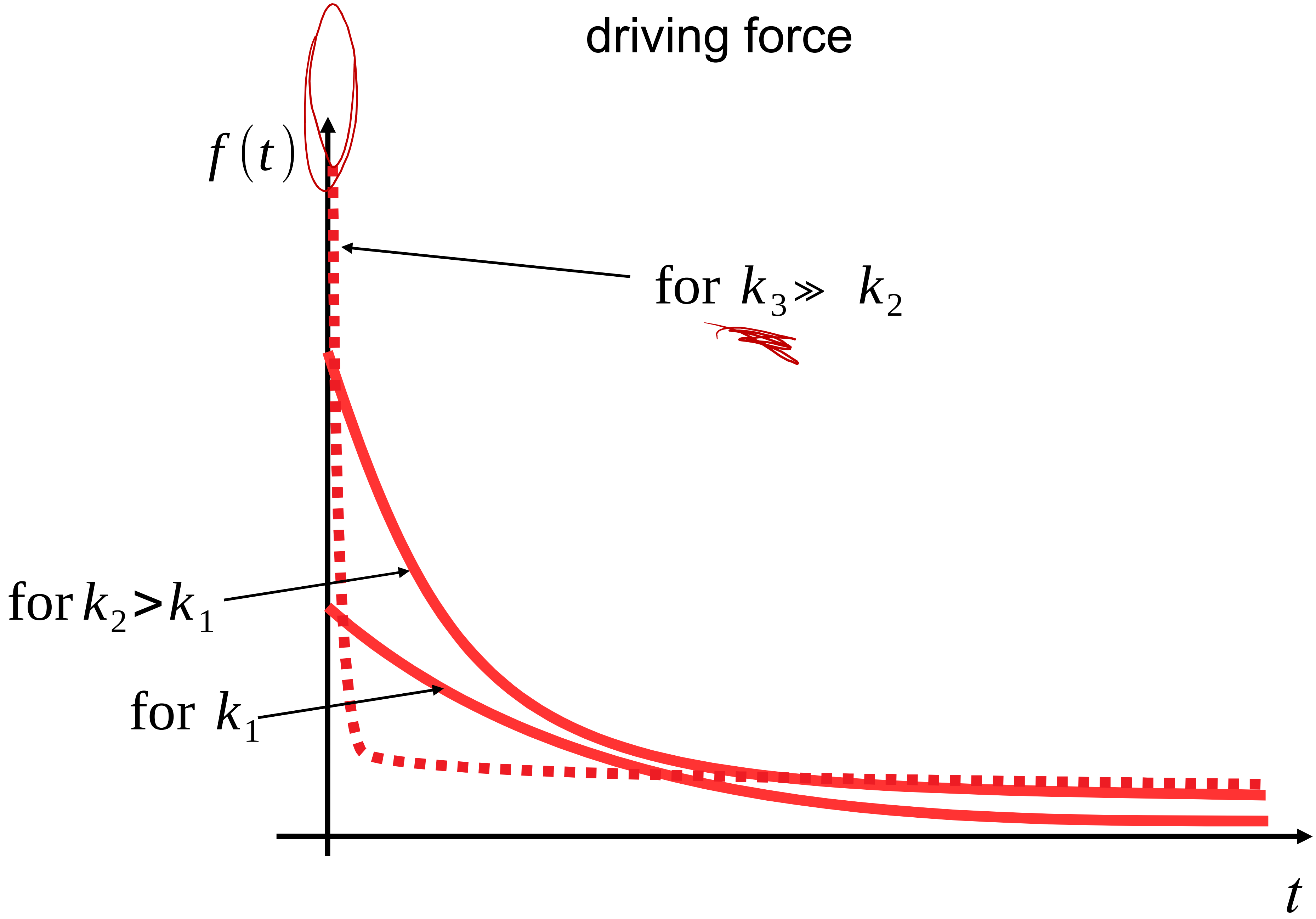
driving force



EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

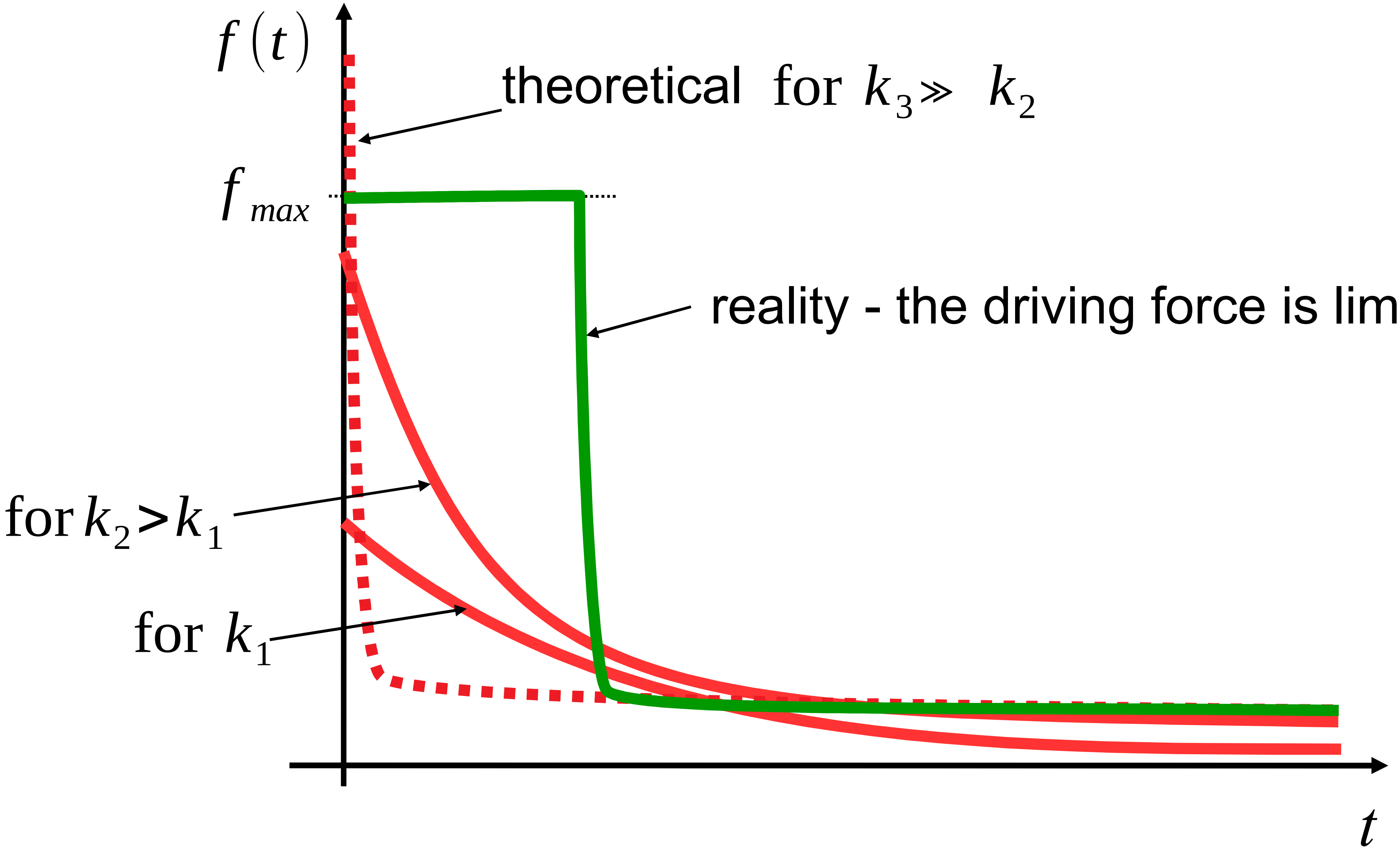
driving force



EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

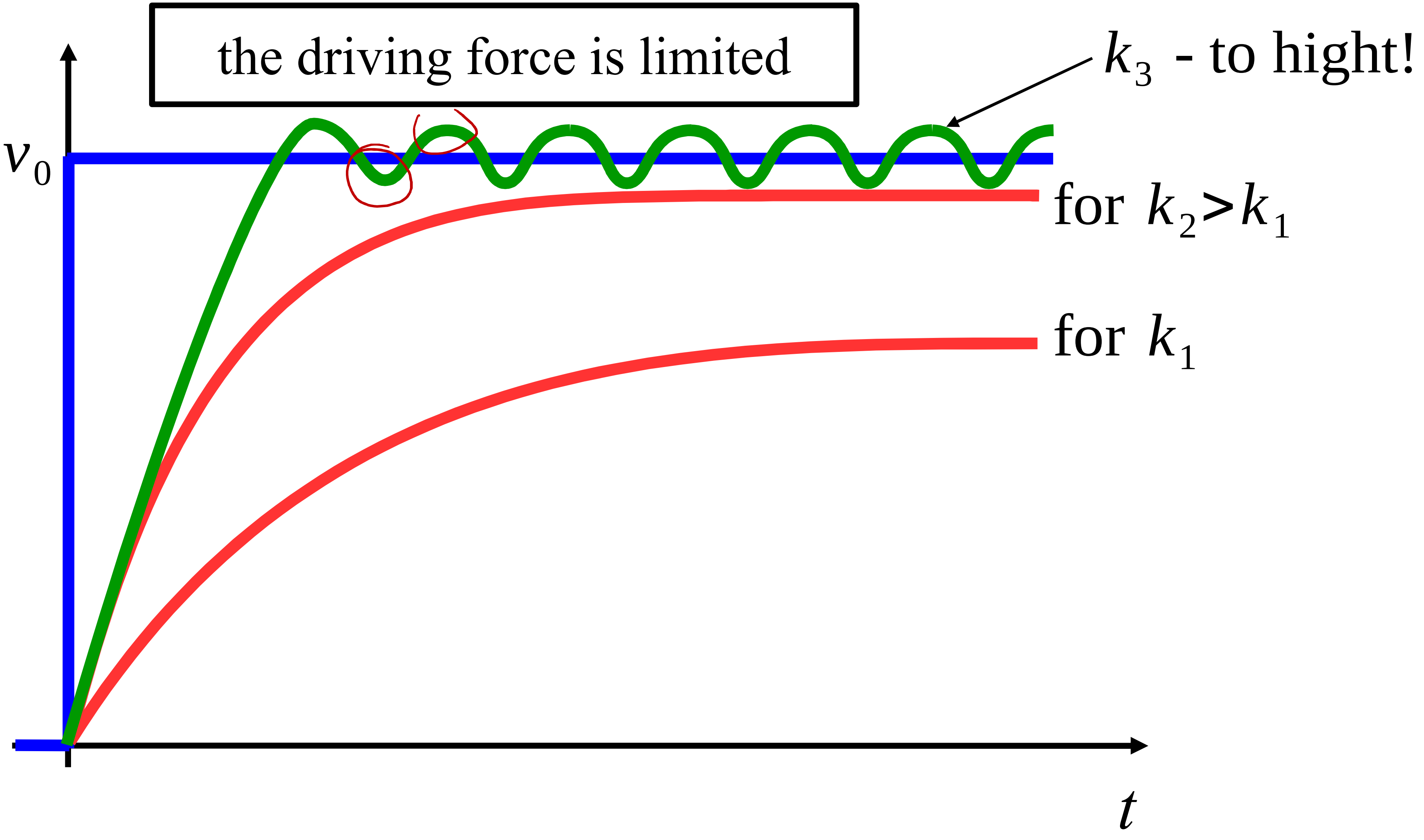
driving force



EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

$$v(t) = v_0 \frac{k}{c+k}\left(1 - \exp\left(-\frac{c+k}{m}t\right)\right)$$



NOTE

Control signal limitation

=

System is nonlinear

=

Linear model (transfer function) is not valid in all situations

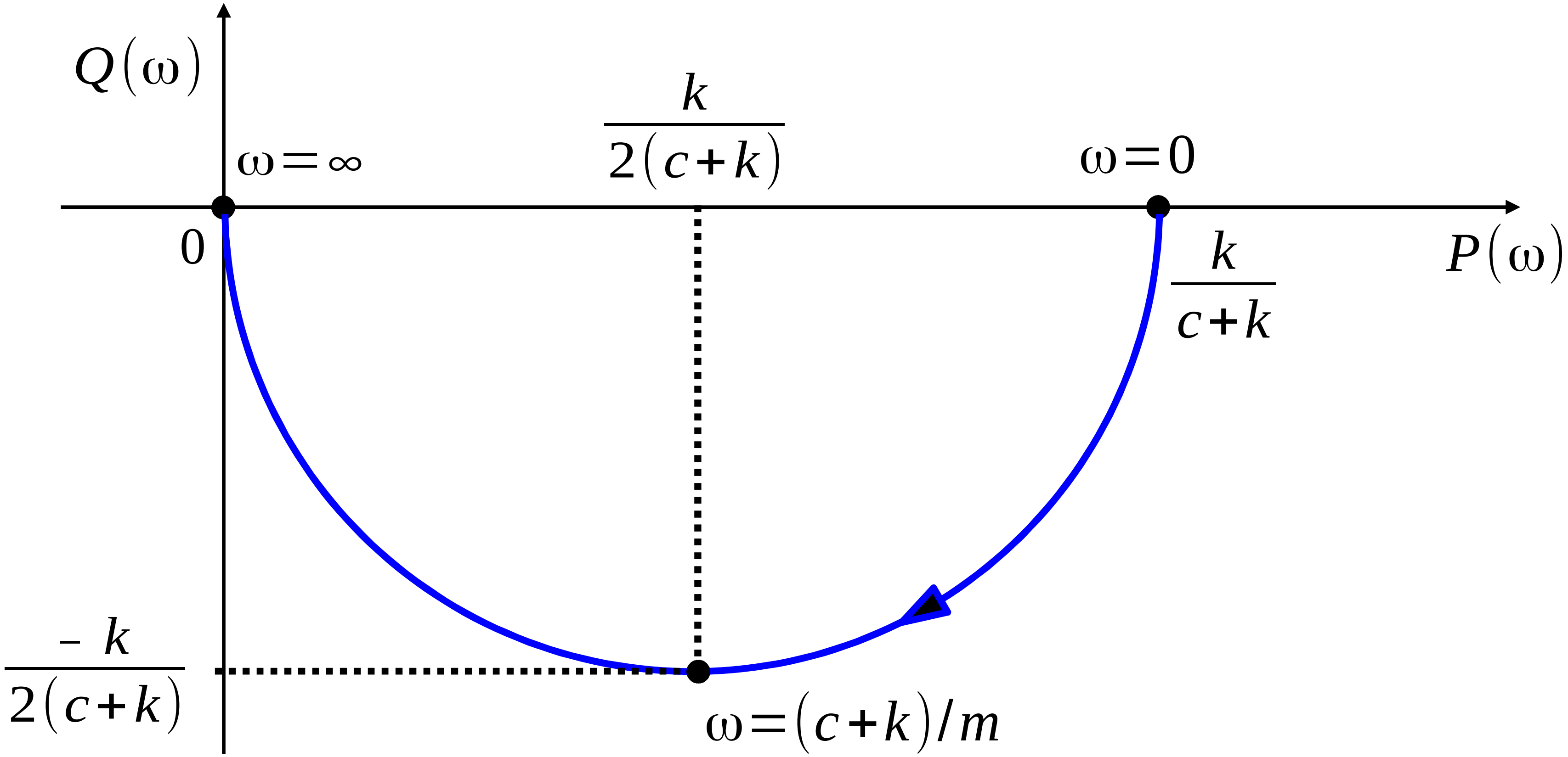
EXAMPLE 1

Speed control (cruise control, autocruise, tempomat)

$$H_R(s)=\frac{k}{ms+c+k},\quad H(j\omega)=\frac{k}{mj\omega+c+k}$$

$$P(\omega)=\frac{k(c+k)}{m^2\omega^2+(c+k)^2},\quad Q(\omega)=\frac{-km\omega}{m^2\omega^2+(c+k)^2}$$

for $k>0$



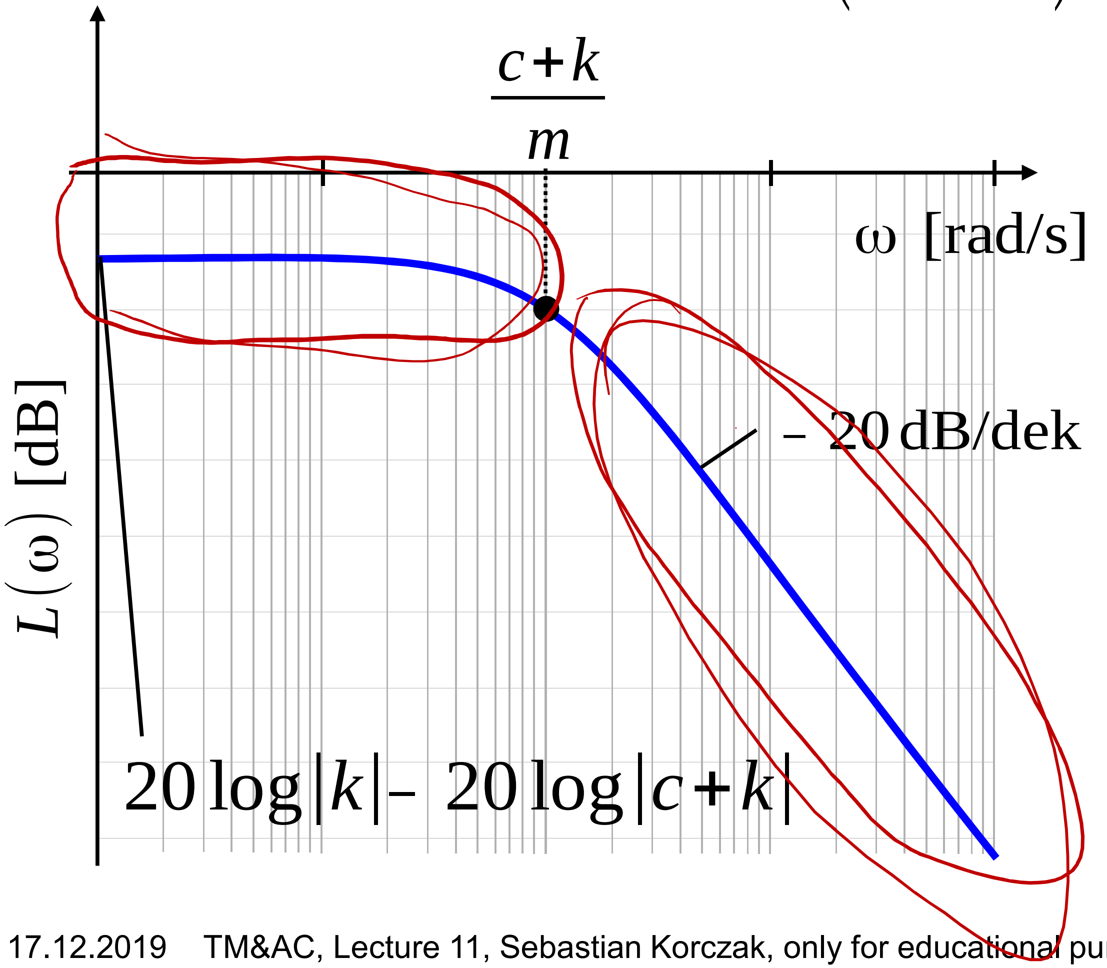
EXAMPLE 1

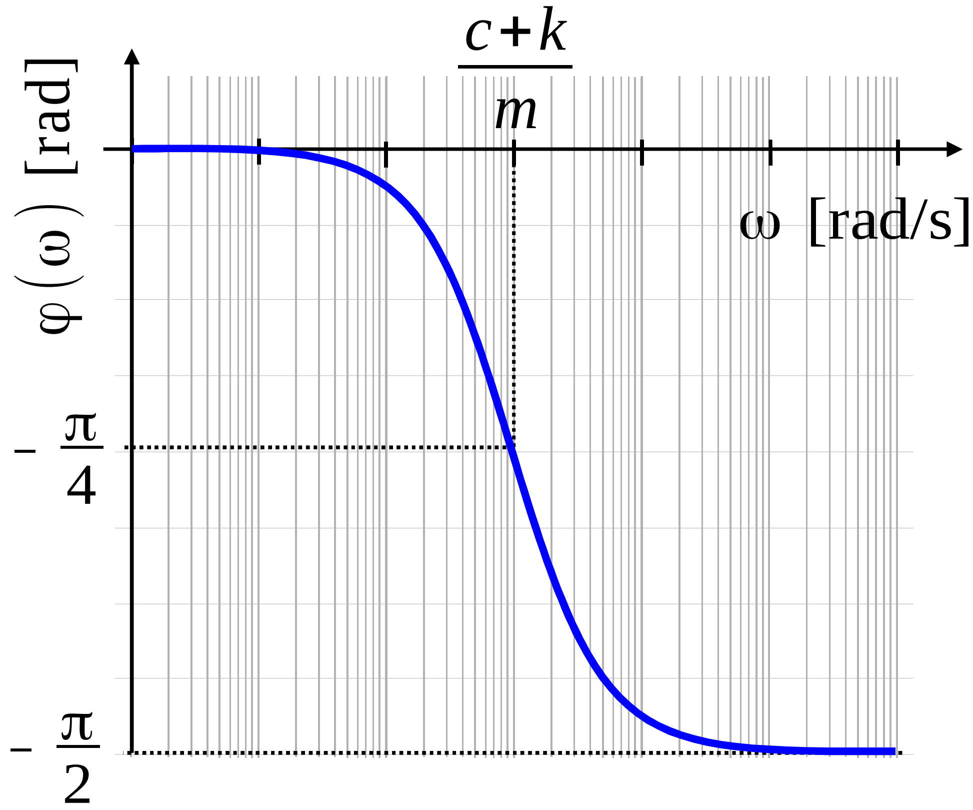
Speed control (cruise control, autocruise, tempomat)

$$A(\omega)=\sqrt{P^2+Q^2}=|k|/\sqrt{m^2\omega^2+c+k}$$

$$L(\omega)=20\log A(\omega)=20\log|k|-20\log\sqrt{m^2\omega^2+(c+k)^2}$$

$$\varphi(\omega)=\arctan\frac{Q}{P}=\arctan\left(-\frac{m\omega}{c+k}\right)$$





EXAMPLE 1

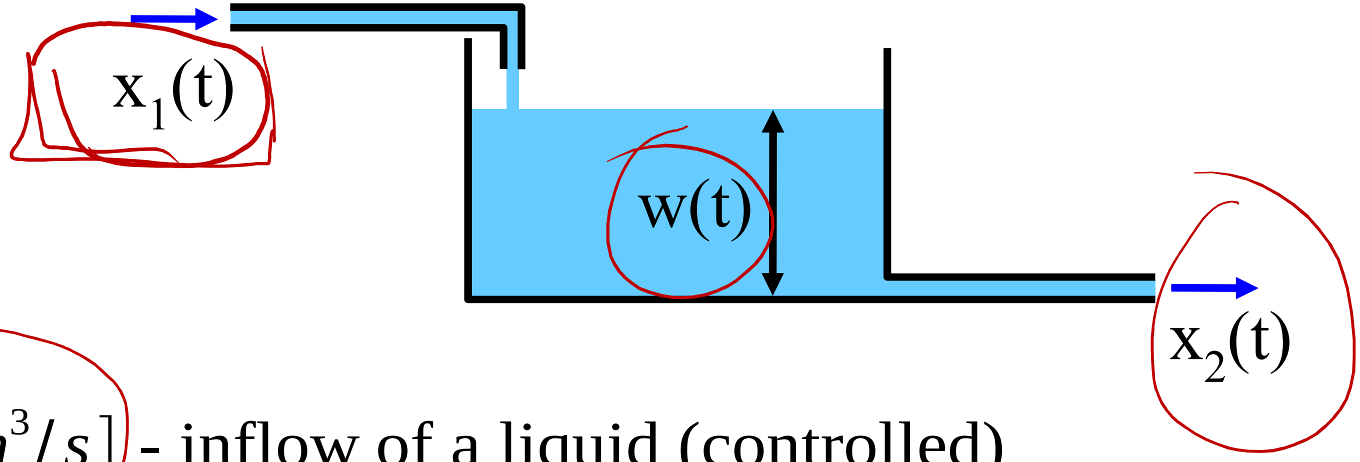
Speed control (cruise control, autocruise, tempomat)

<u>Conclusions for "P" controller + first order system</u>

- constant error in steady state
- P gain increasing = rise time decreasing + error decreasing
- control signal limitations = rise time limitations
- control signal limitations = nonlinear system

EXAMPLE 2

Water level control



$x_1(t)[m^3/s]$ - inflow of a liquid (controlled)

$x_2(t)[m^3/s]$ - outflow of a liquid (not controlled)

$w(t)[m]$ - level of a liquid in a tank

$A[m^2]$ - constant surface area

$$A\frac{dw(t)}{dt} = x_1(t) - x_2(t)$$

$$\frac{dV(t)}{dt} = x_1 - x_2$$

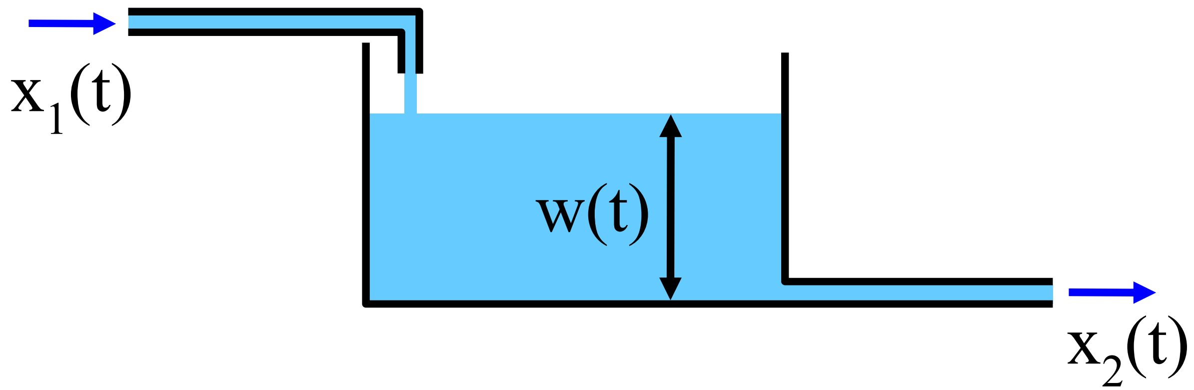
$$\mathcal{L}_0$$

$$AsW(s) = X_1(s) - X_2(s)$$

$$V = w \cdot A$$

EXAMPLE 2

Water level control



$x_1(t)[m^3/s]$ - inflow of a liquid (controlled)

$x_2(t)[m^3/s]$ - outflow of a liquid (not controlled)

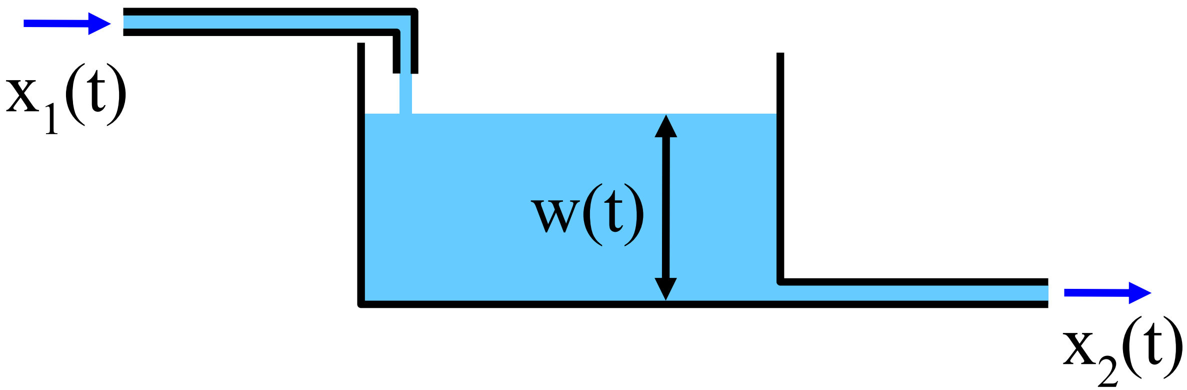
$w(t)[m]$ - level of a liquid in a tank

$A[m^2]$ - constant surface area

$$\frac{dv(t)}{dt} = x_1(t) - x_2(t)$$

EXAMPLE 2

Water level control



$x_1(t)[m^3/s]$ - inflow of a liquid (controlled)

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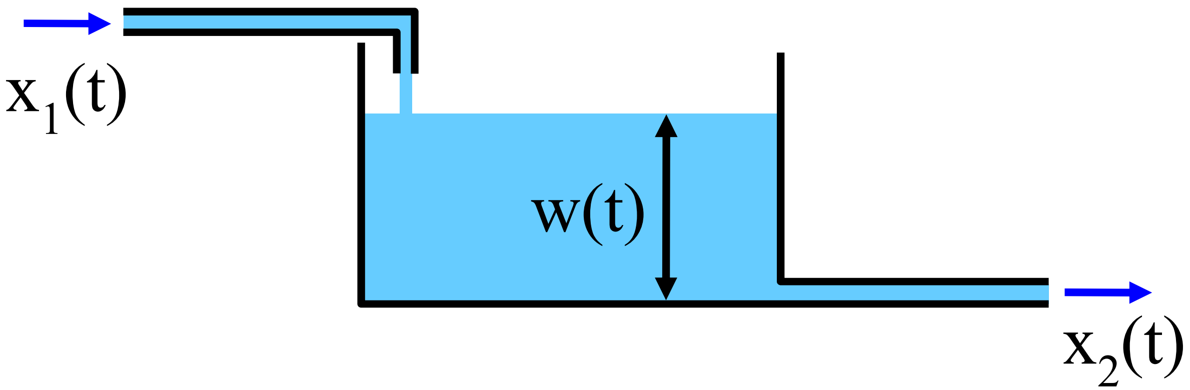
$A[m^2]$ - constant surface area

$$\frac{dv(t)}{dt} = x_1(t) - x_2(t)$$

$$A\frac{dw(t)}{dt} = x_1(t) - x_2(t)$$

EXAMPLE 2

Water level control



$x_1(t)[m^3/s]$ - inflow of a liquid (controlled)

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$w(t)[m]$ - level of a liquid in a tank

$A[m^2]$ - constant surface area

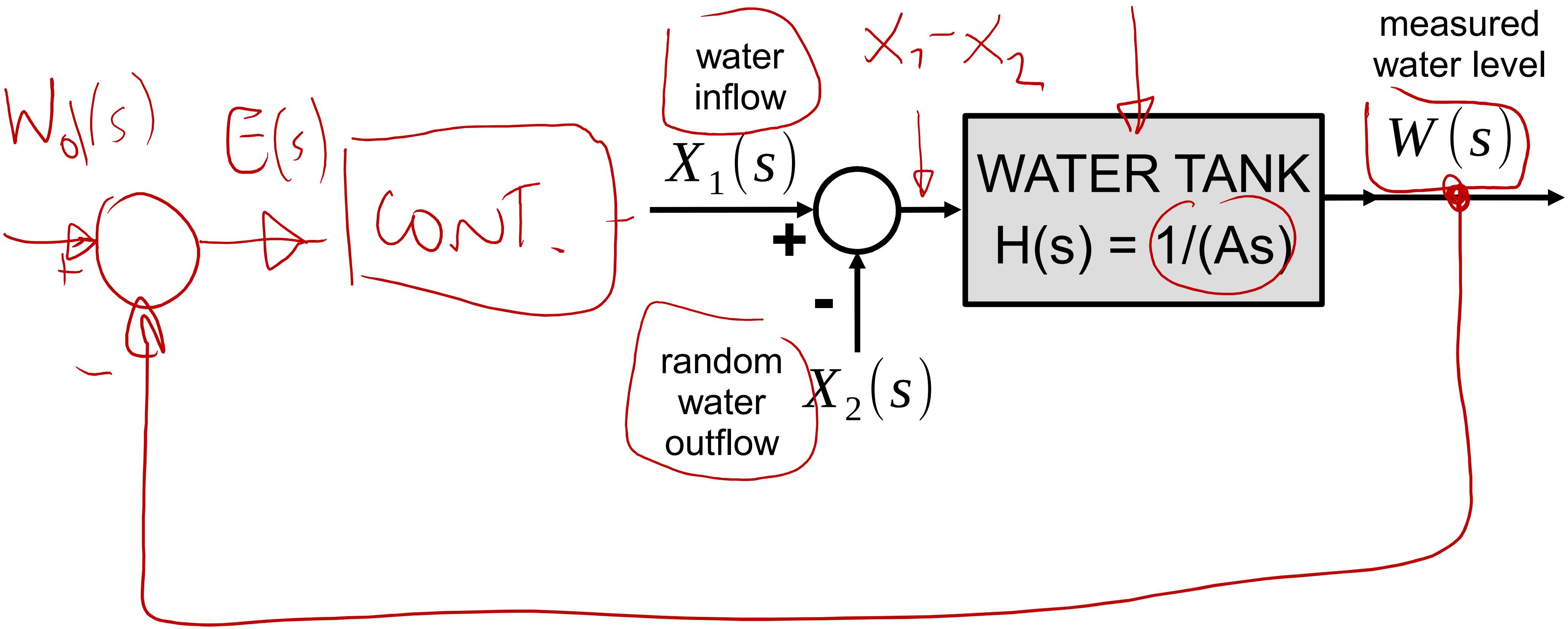
$$\frac{dv(t)}{dt} = x_1(t) - x_2(t)$$

$$A\frac{dw(t)}{dt} = x_1(t) - x_2(t)$$

$$H(s) = \frac{W(s)}{X_1(s) - X_2(s)} = \frac{1}{As}$$

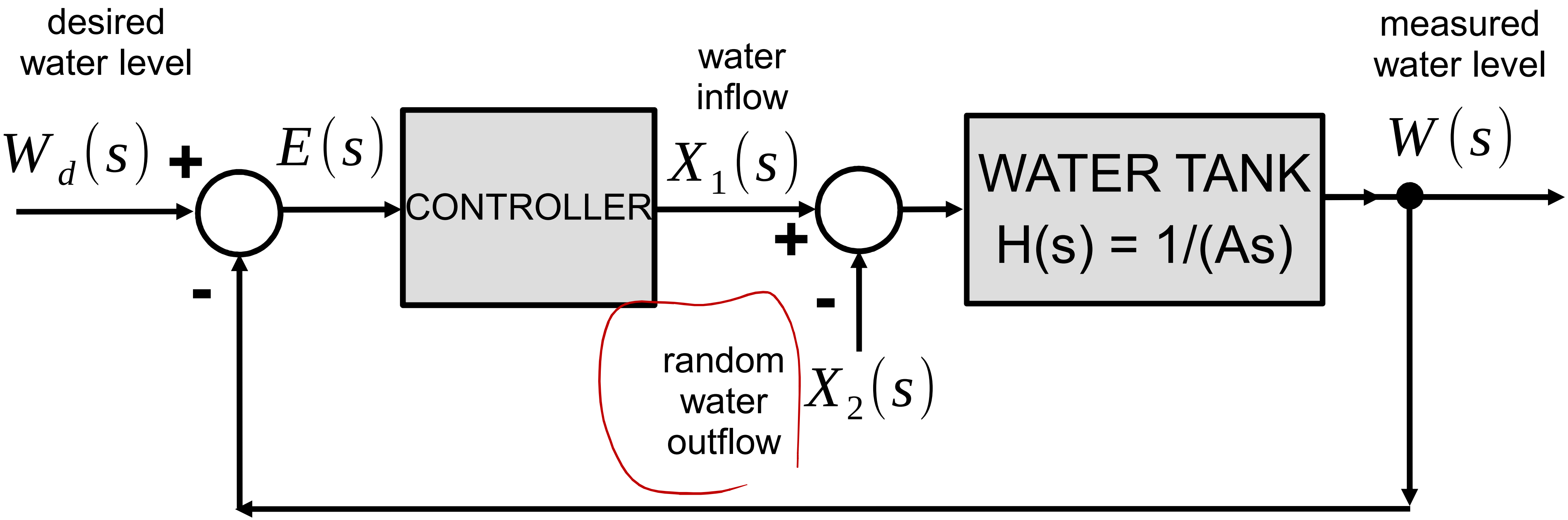
EXAMPLE 2

Water level control



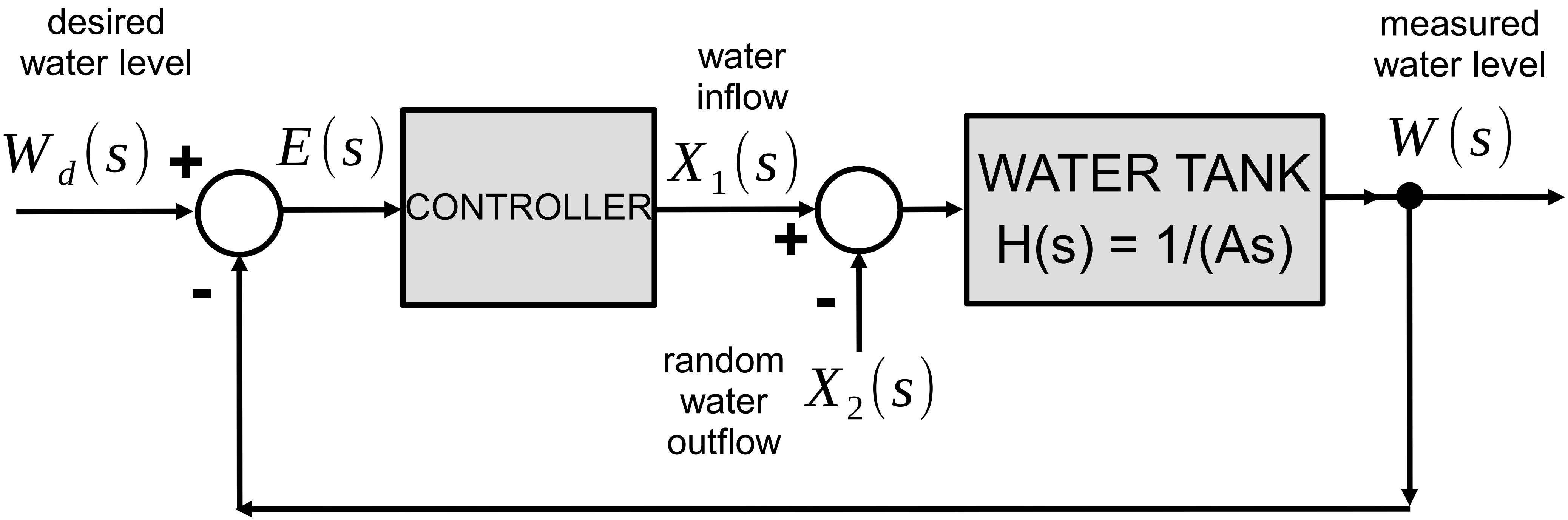
EXAMPLE 2

Water level control



EXAMPLE 2

Water level control



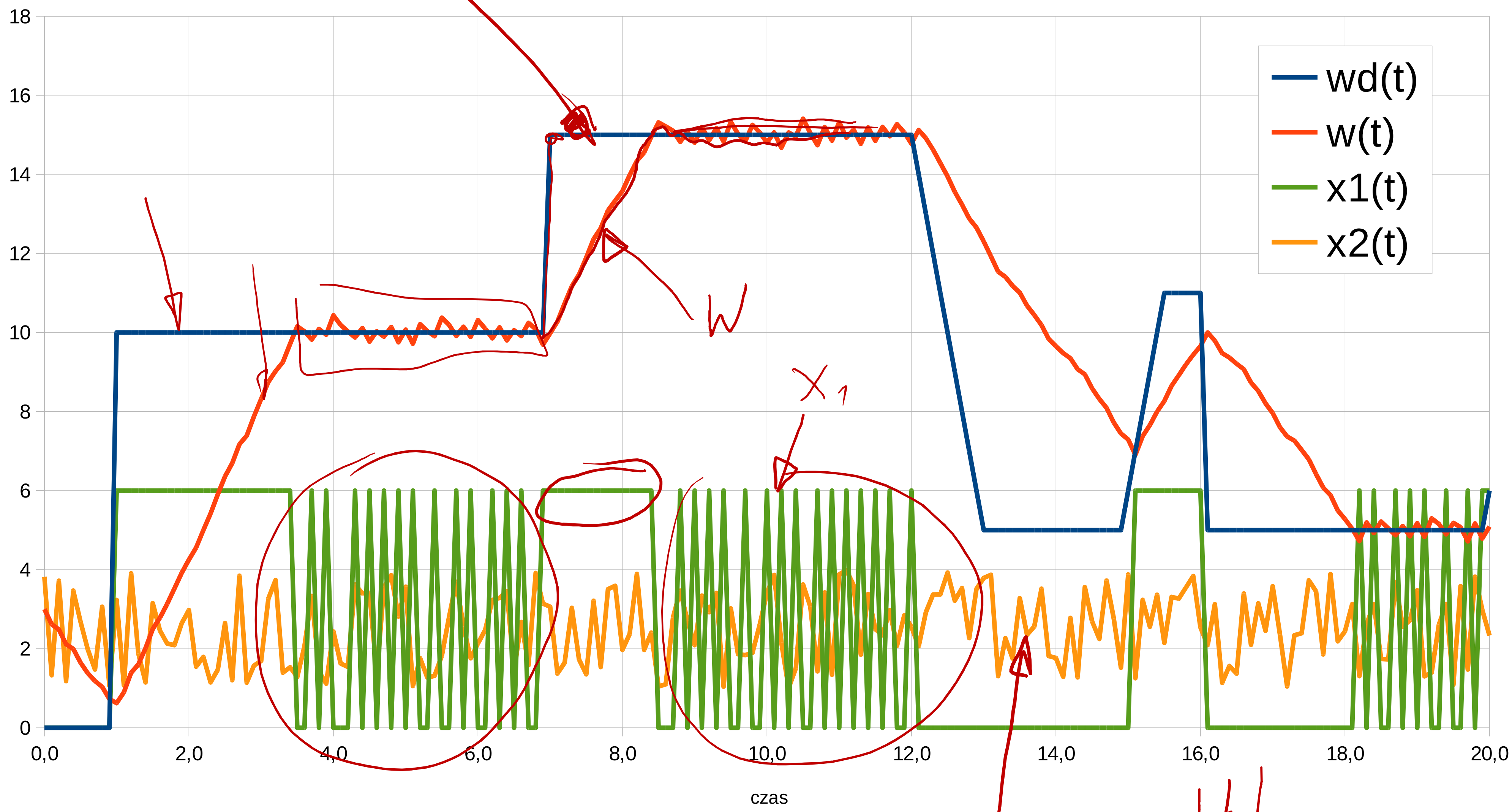
Proposed controllers:

- ideal on/off controller
- on/off controller with hysteresis
- proportional controller

EXAMPLE 2

Water level control

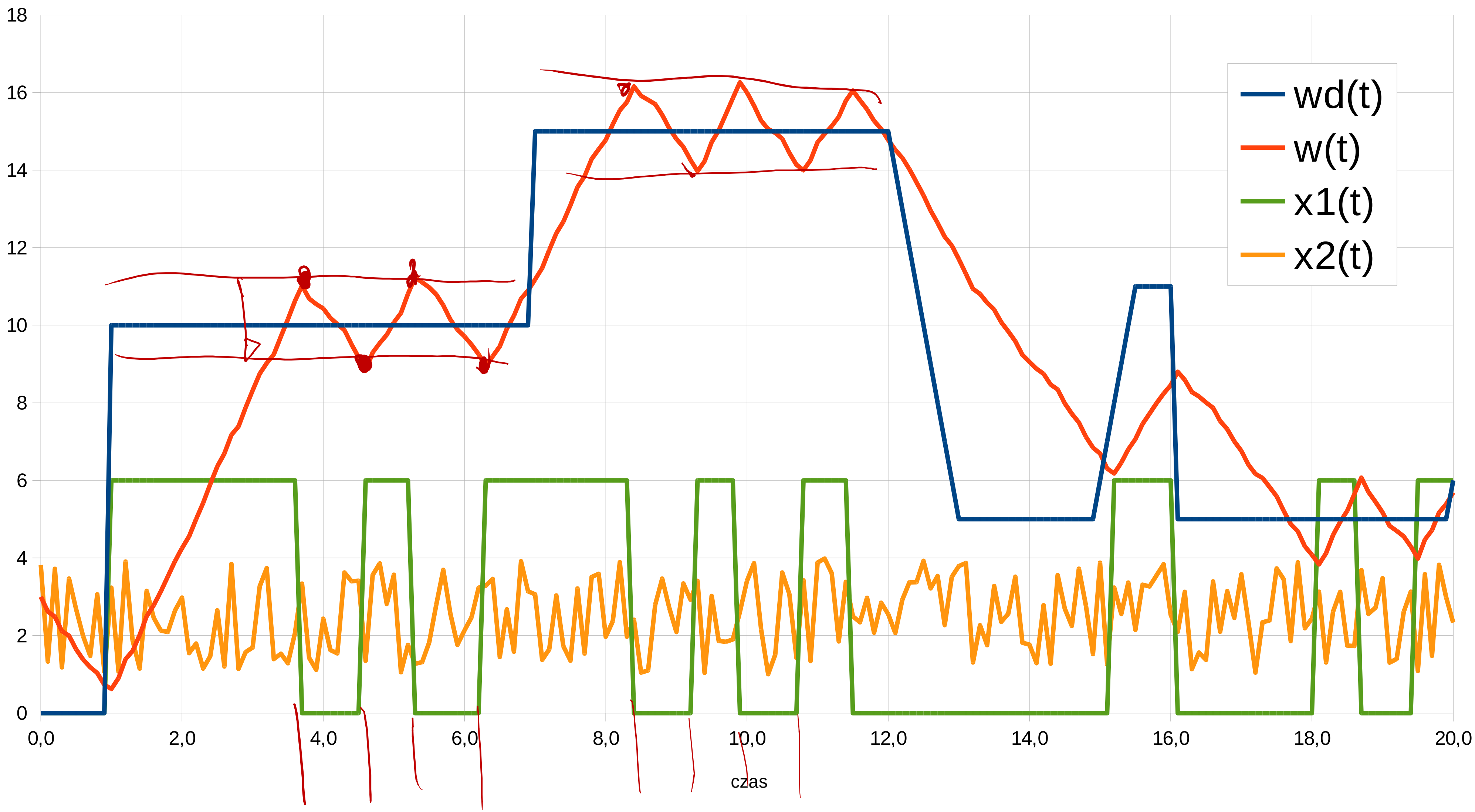
ideal on/off controller



EXAMPLE 2

Water level control

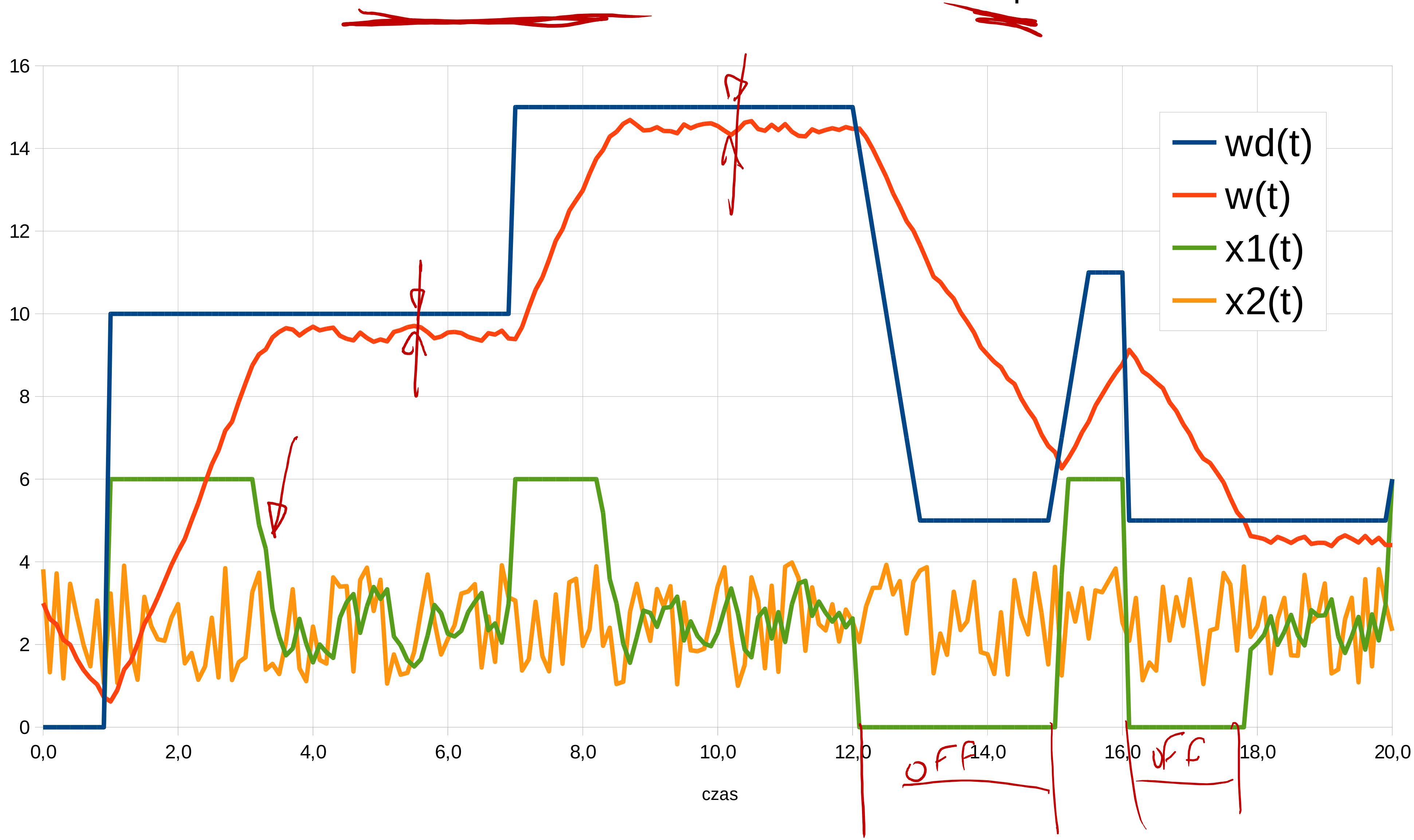
on/off controller with hysteresis



EXAMPLE 2

Water level control

proportional controller (small k_P)



EXAMPLE 2

Water level control

proportional controller (high k_P)