



Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control Winter 2018/2019

Lecturer: Sebastian Korczak, PhD, Eng.

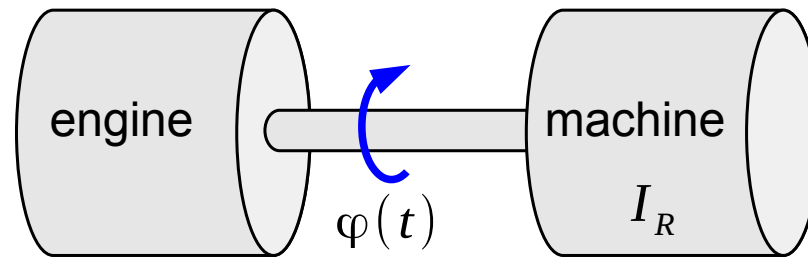
Lecture 7

Non-uniformity of machine motion. Introduction to automatic control.

Materials license: only for educational purposes of Warsaw University of Technology students.

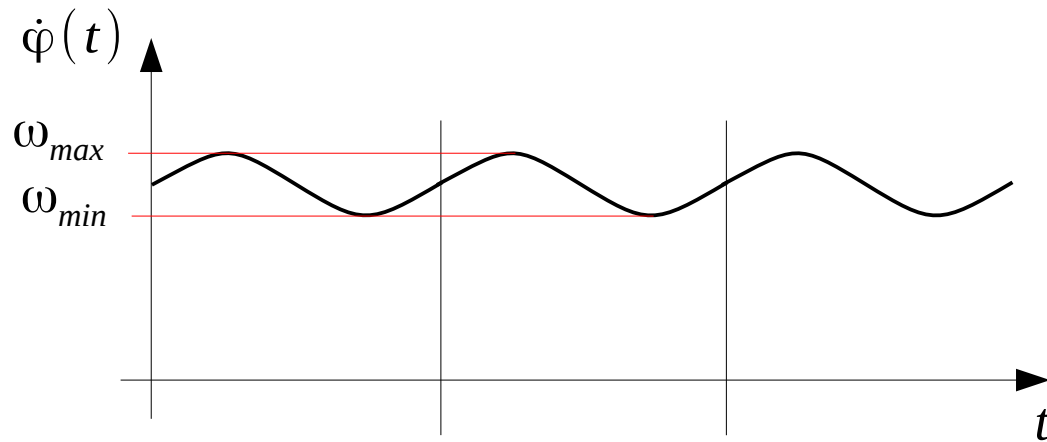
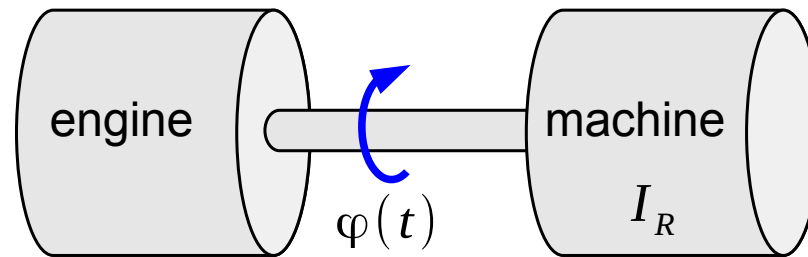
Non-uniformity of machine motion

Steady-state motion



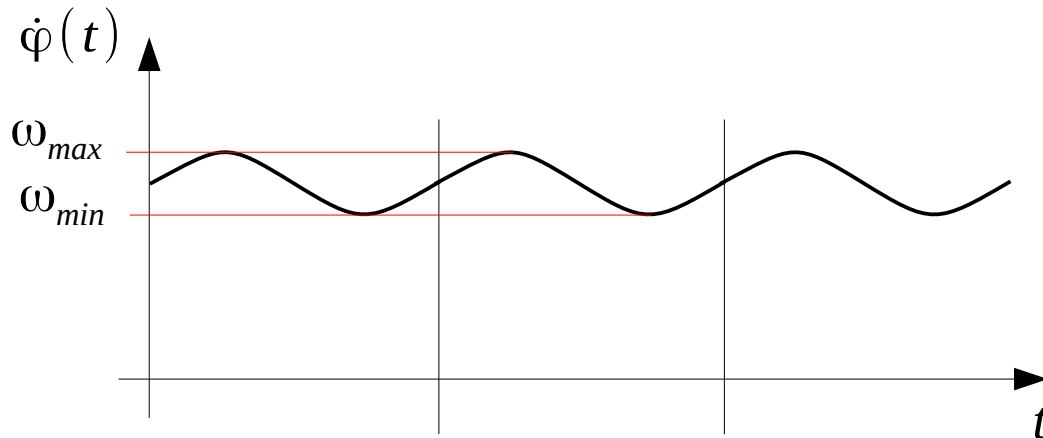
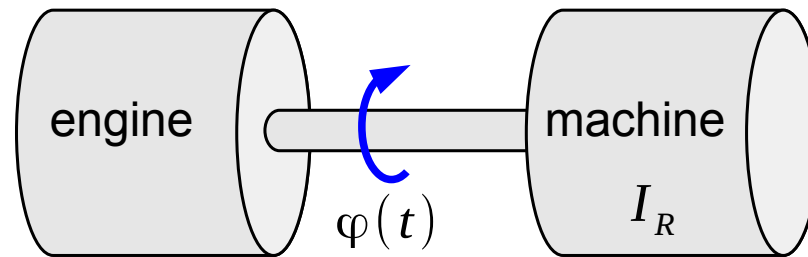
Non-uniformity of machine motion

Steady-state motion



Non-uniformity of machine motion

Steady-state motion

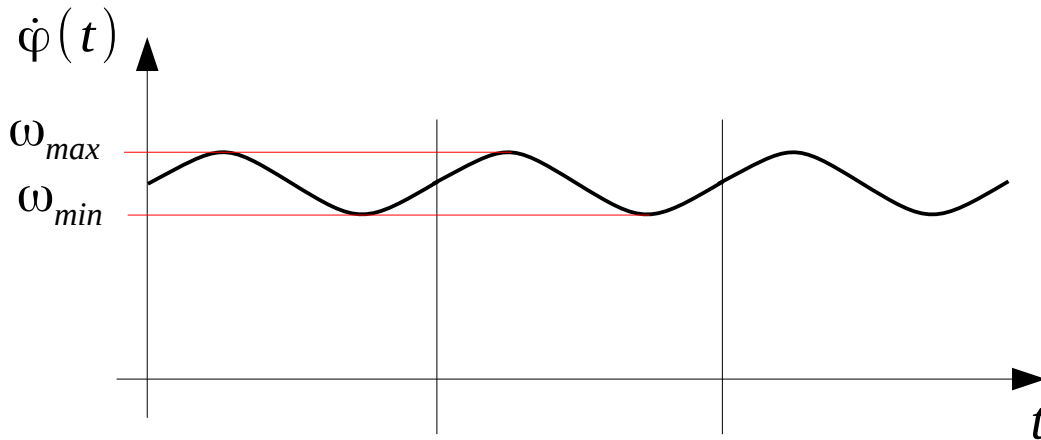
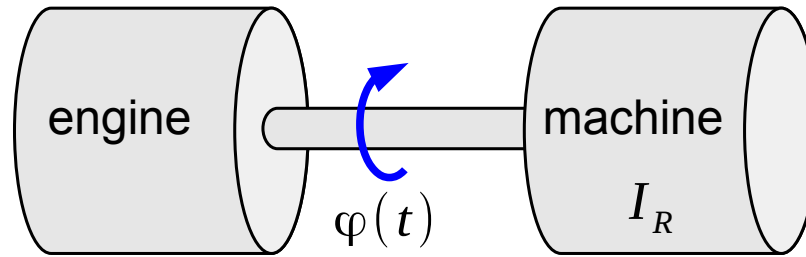


Non-uniformity of machine motion

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \quad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

Non-uniformity of machine motion

Steady-state motion



Non-uniformity of machine motion

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \quad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

pumps

$$\delta = 1/5 \div 1/30$$

combustion engines

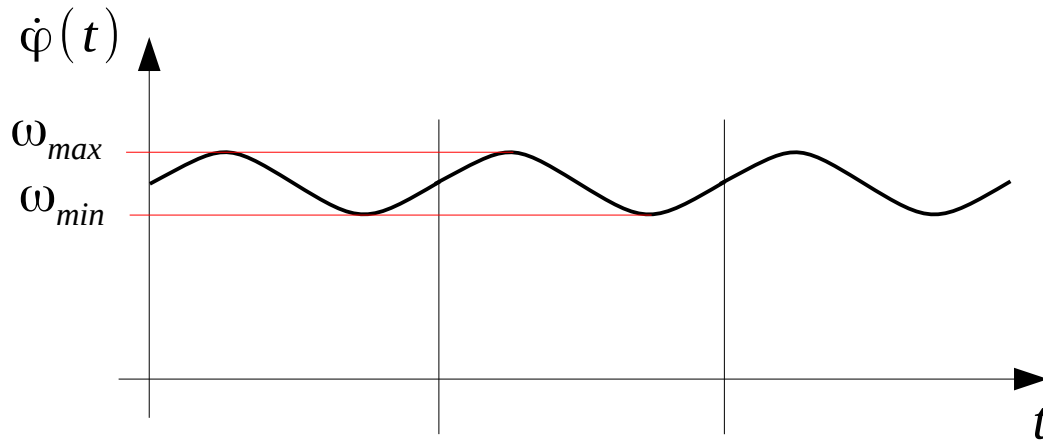
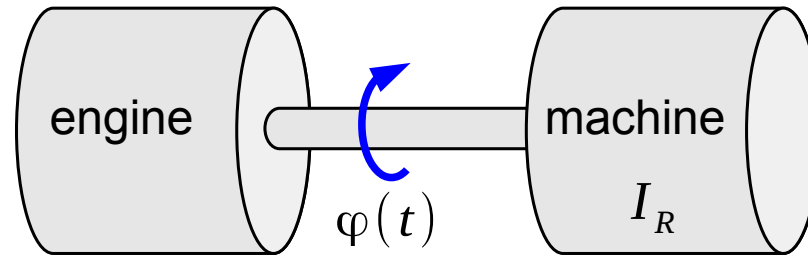
$$\delta = 1/50 \div 1/150$$

generators

$$\delta = 1/200 \div 1/300$$

Non-uniformity of machine motion

Steady-state motion



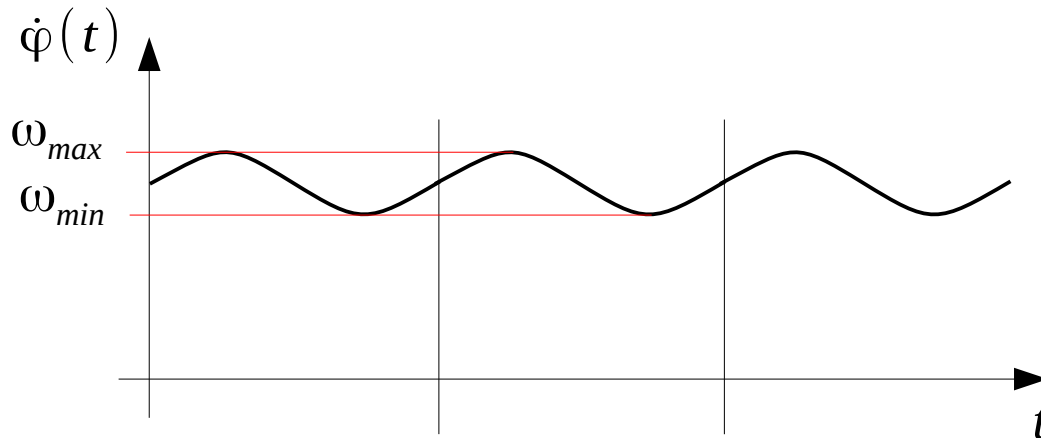
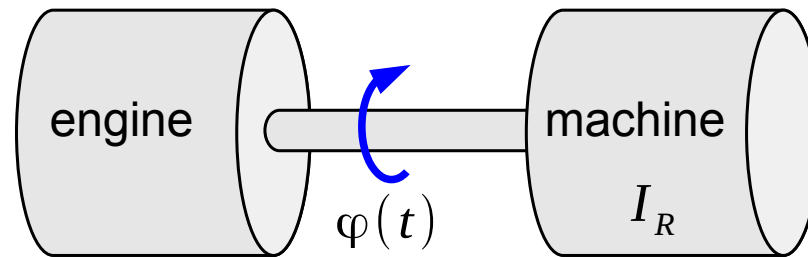
Non-uniformity of machine motion

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \quad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

$$T_{max} = \frac{1}{2} I_R \omega_{max}^2 \quad T_{min} = \frac{1}{2} I_R \omega_{min}^2$$

Non-uniformity of machine motion

Steady-state motion



Non-uniformity of machine motion

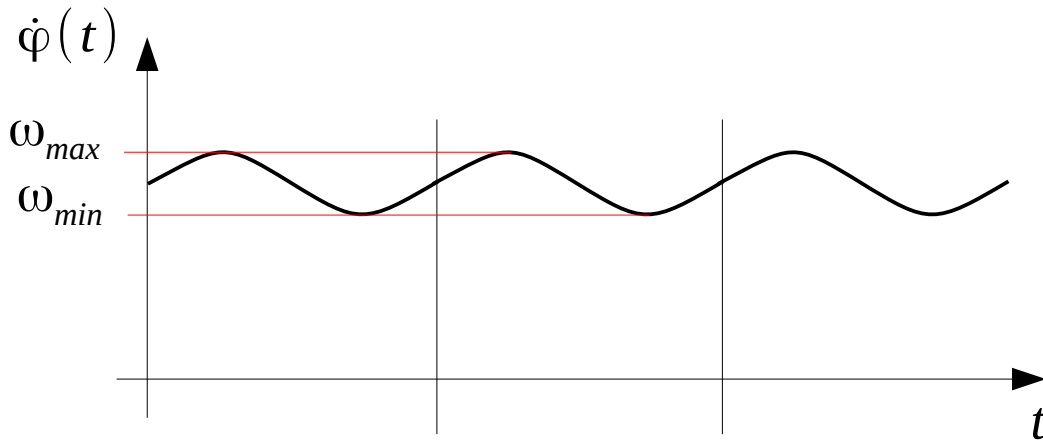
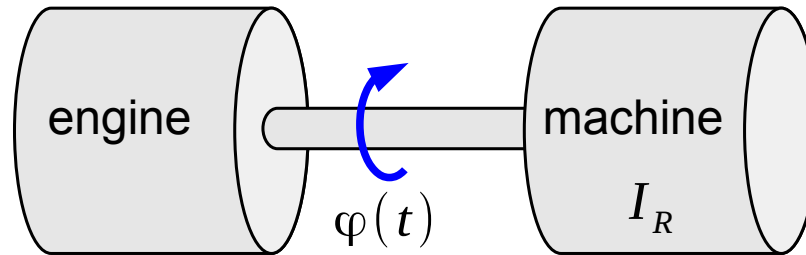
$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \quad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

$$T_{max} = \frac{1}{2} I_R \omega_{max}^2 \quad T_{min} = \frac{1}{2} I_R \omega_{min}^2$$

$$W = T_{max} - T_{min}$$

Non-uniformity of machine motion

Steady-state motion



Non-uniformity of machine motion

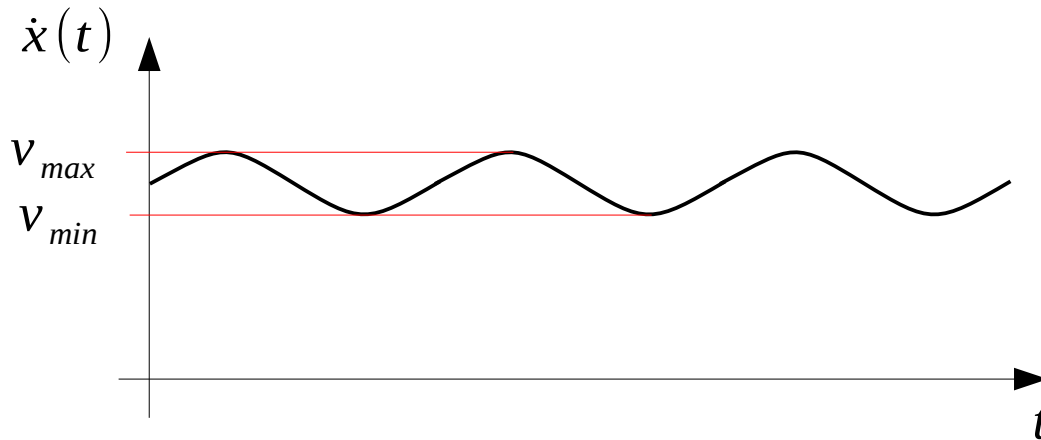
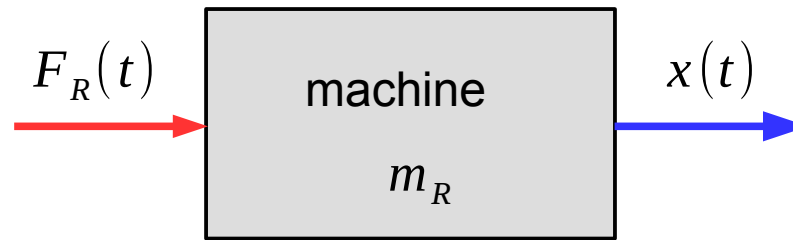
$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} \quad \omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2}$$

$$T_{max} = \frac{1}{2} I_R \omega_{max}^2 \quad T_{min} = \frac{1}{2} I_R \omega_{min}^2$$

$$W = T_{max} - T_{min} = \delta I_R \omega_{mean}^2$$

Non-uniformity of machine motion

Steady-state motion



Non-uniformity of machine motion

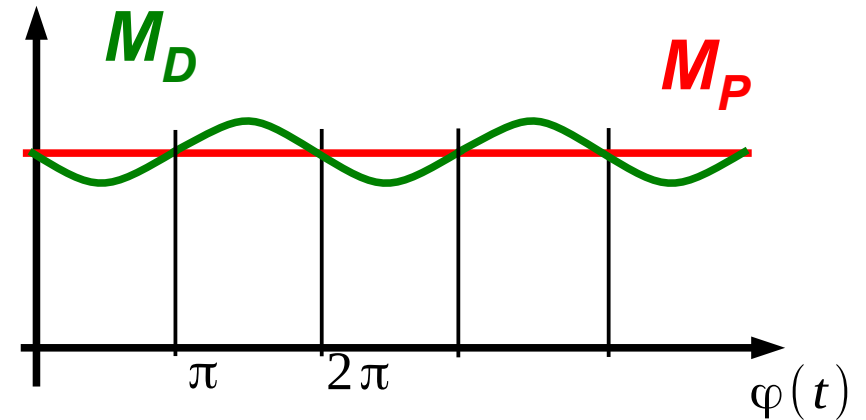
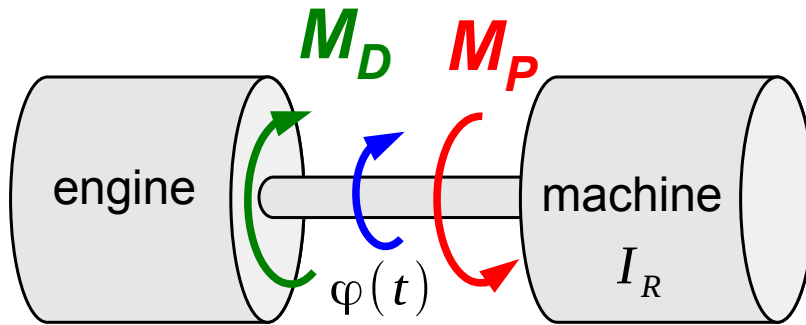
$$\delta = \frac{v_{max} - v_{min}}{v_{mean}} \quad v_{mean} = \frac{v_{max} + v_{min}}{2}$$

$$W = \delta m_R v_{mean}^2$$

Non-uniformity of machine motion

Steady-state motion

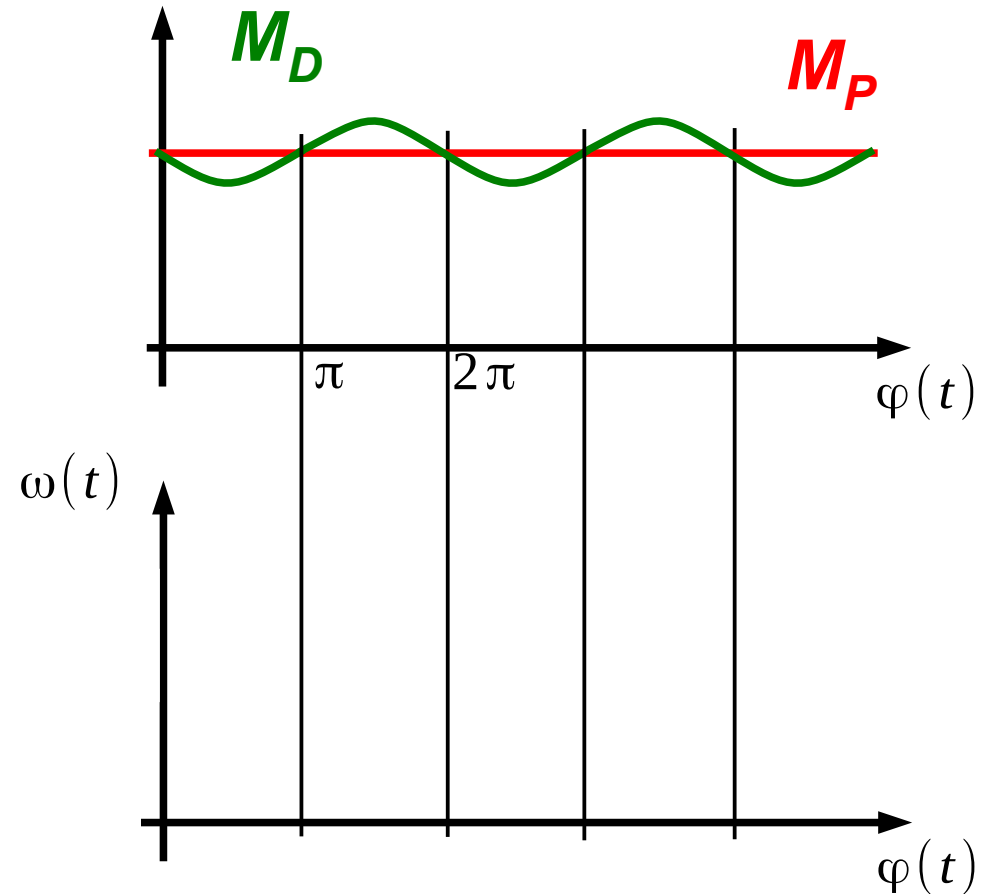
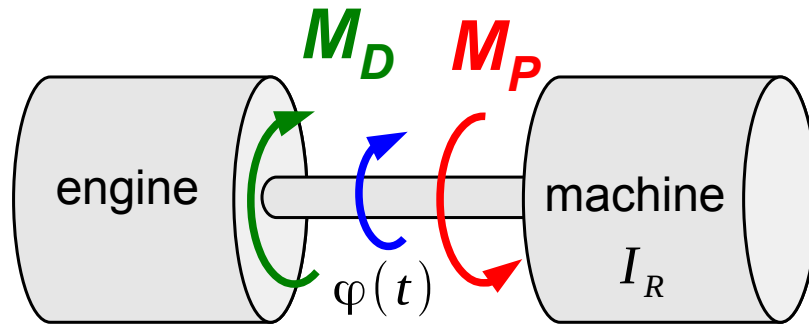
Example



Non-uniformity of machine motion

Steady-state motion

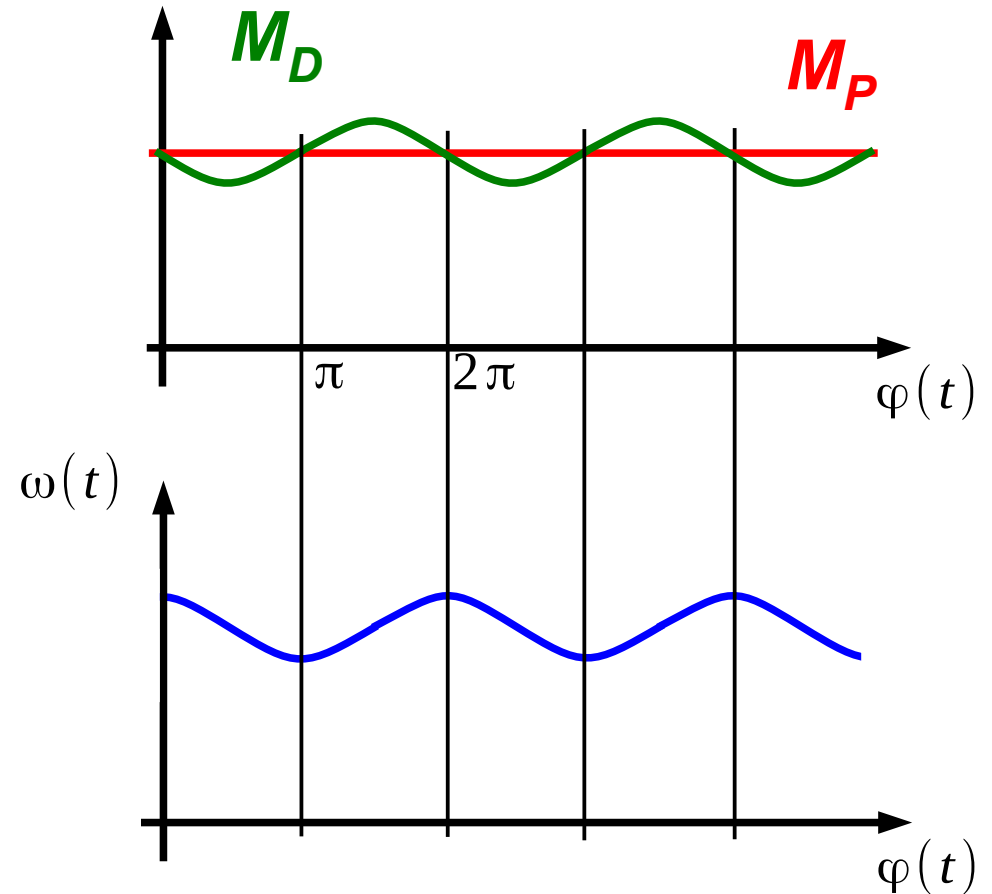
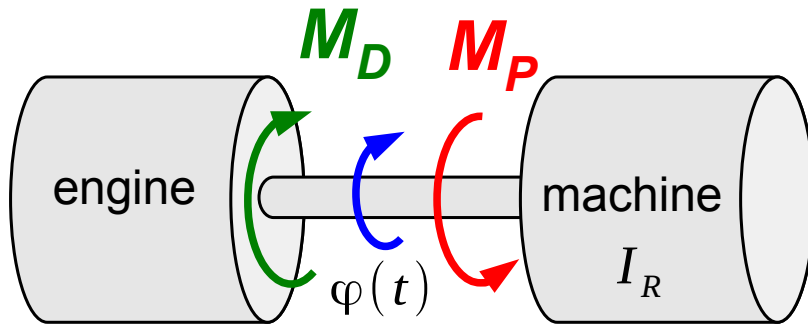
Example



Non-uniformity of machine motion

Steady-state motion

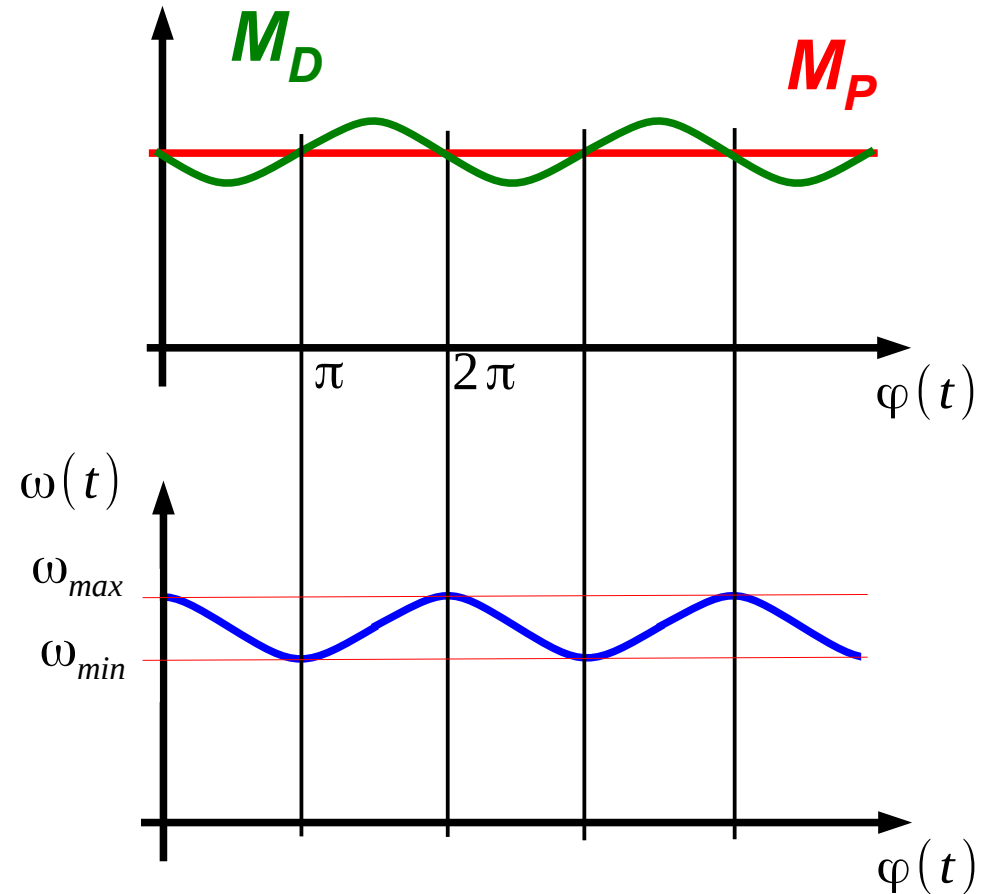
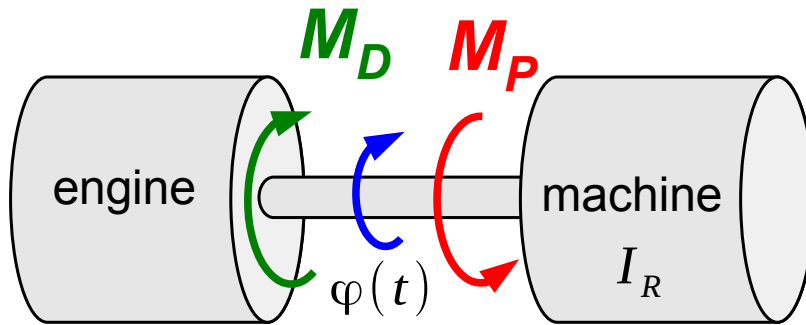
Example



Non-uniformity of machine motion

Steady-state motion

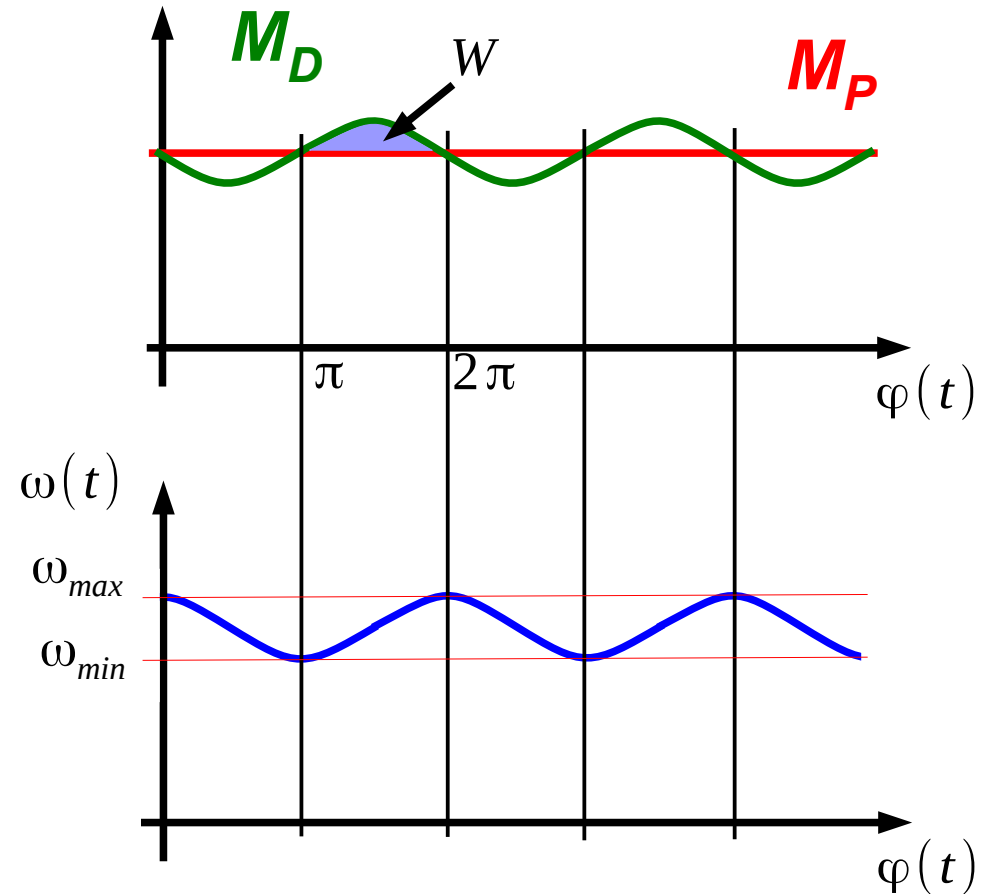
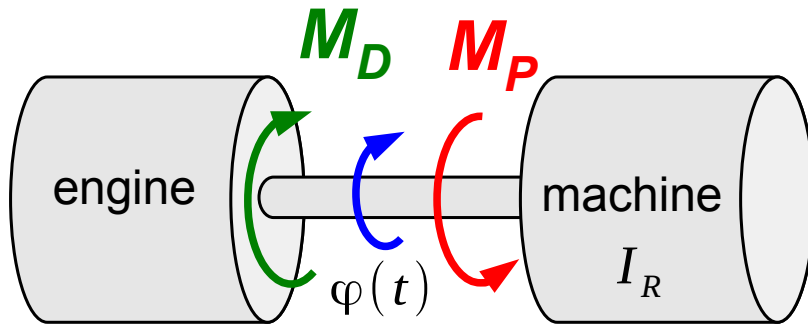
Example



Non-uniformity of machine motion

Steady-state motion

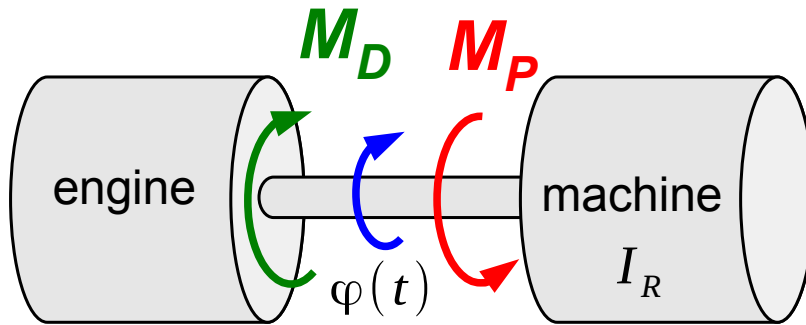
Example



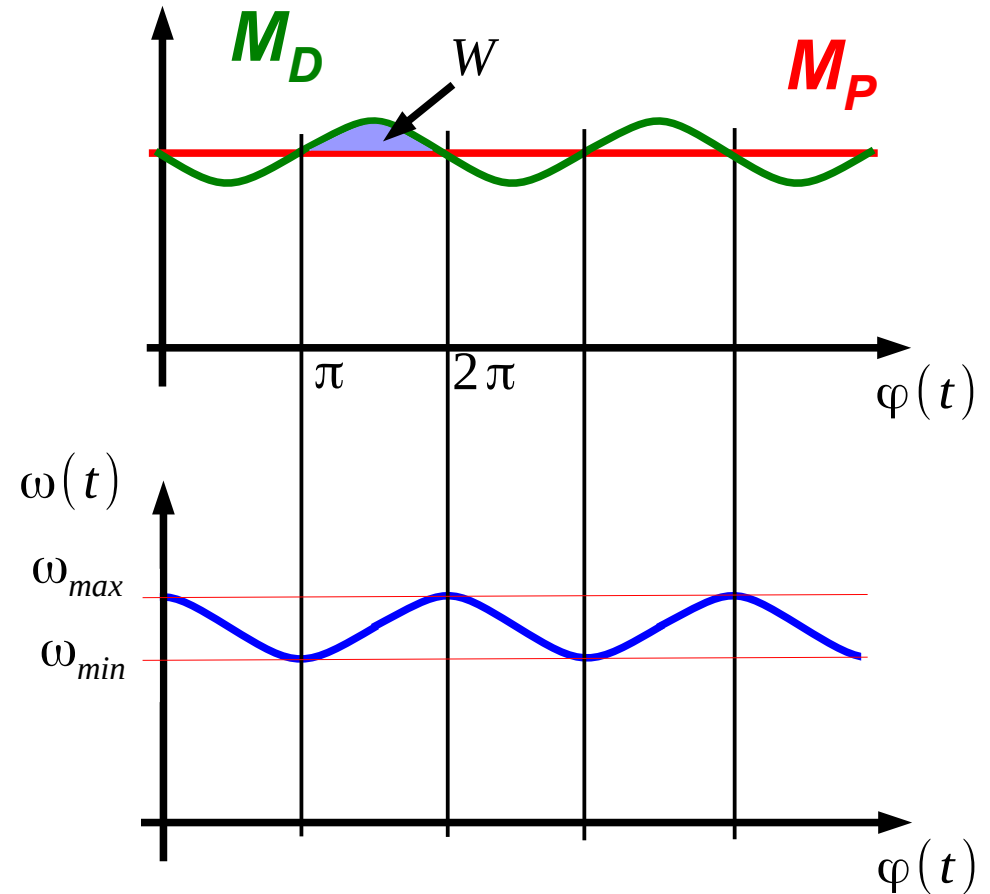
Non-uniformity of machine motion

Steady-state motion

Example



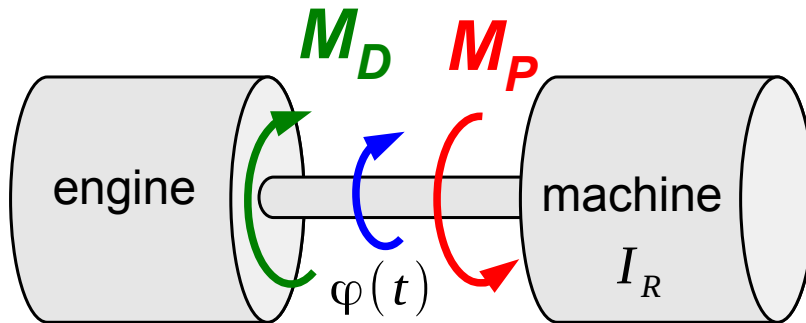
$$W = \int_{\varphi_{min}}^{\varphi_{max}} (M_D - M_P) d\varphi$$



Non-uniformity of machine motion

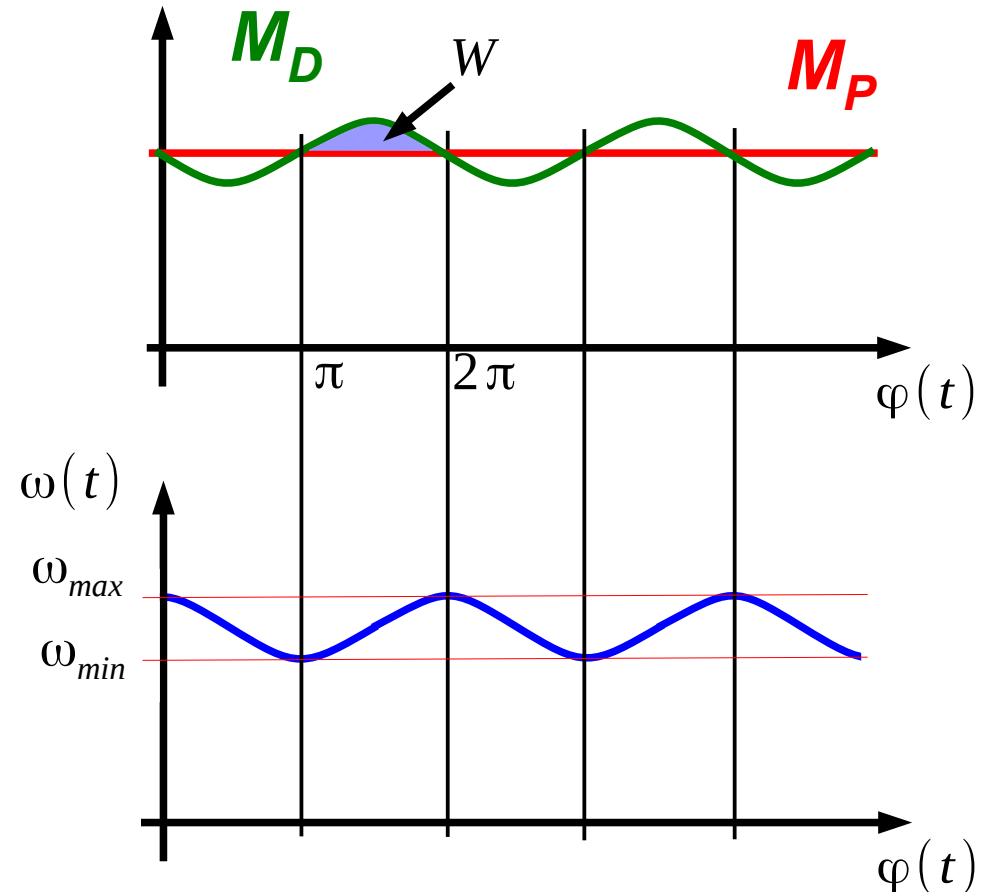
Steady-state motion

Example



$$W = \int_{\varphi_{min}}^{\varphi_{max}} (M_D - M_P) d\varphi$$

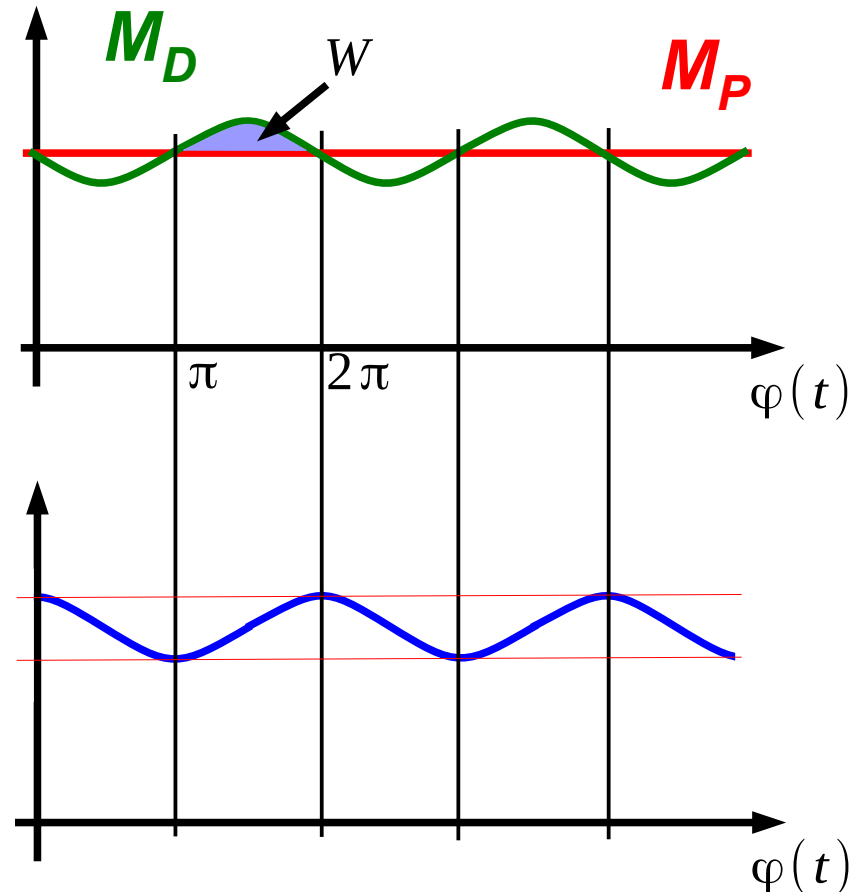
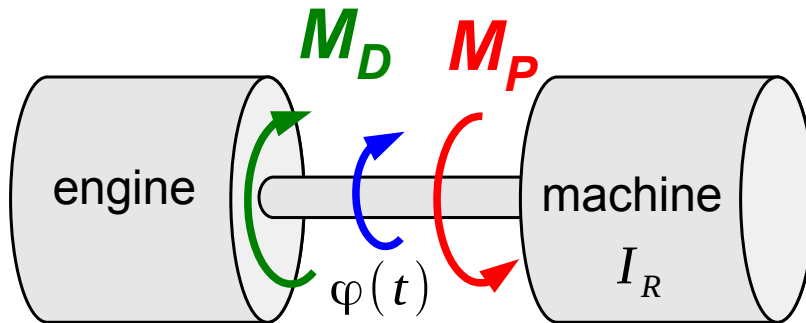
$$W = T_{max} - T_{min} = \delta I_R \omega_{mean}^2$$



Non-uniformity of machine motion

Steady-state motion

Example



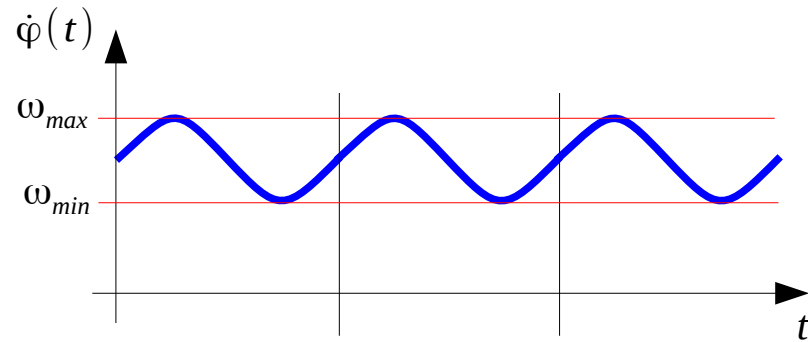
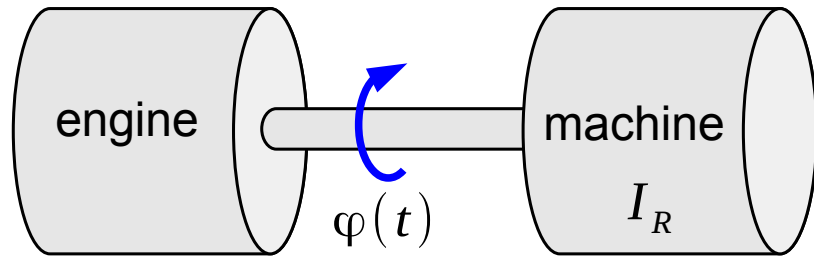
$$W = \int_{\varphi_{min}}^{\varphi_{max}} (M_D - M_P) d\varphi$$

$$W = T_{max} - T_{min} = \delta I_R \omega_{mean}^2$$

$$\delta = \frac{W}{I_R \omega_{mean}^2}$$

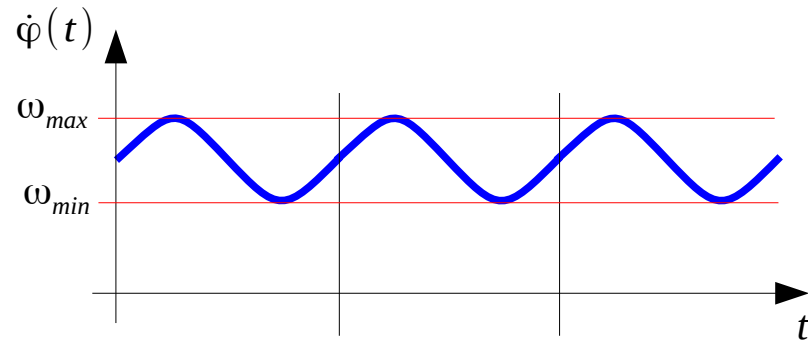
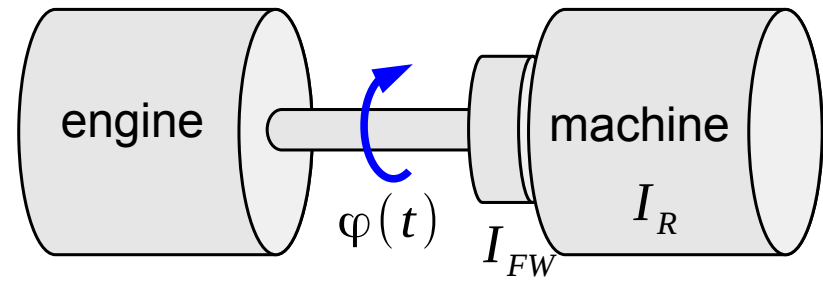
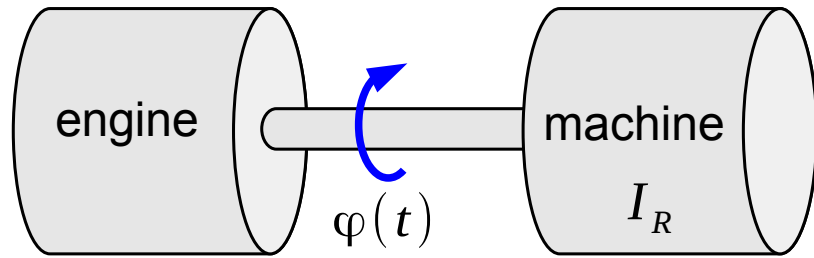
Flywheel

Steady-state motion



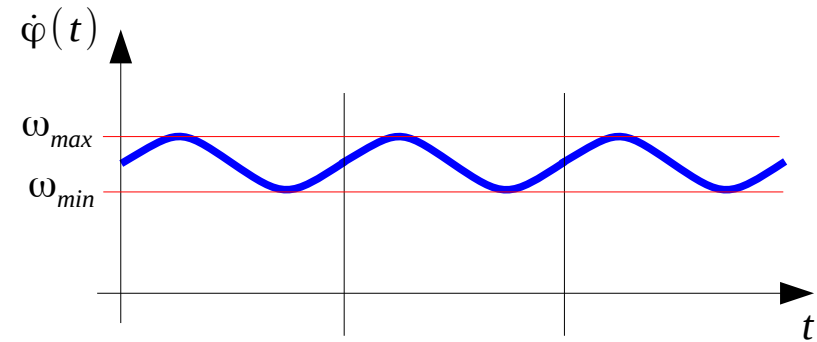
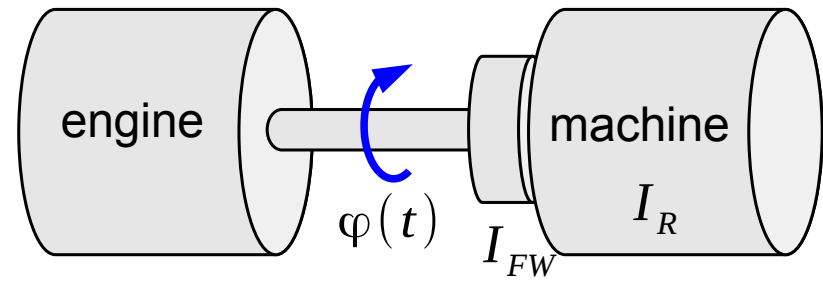
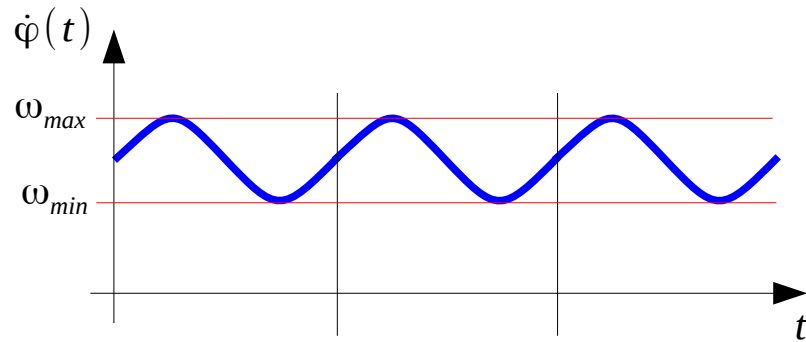
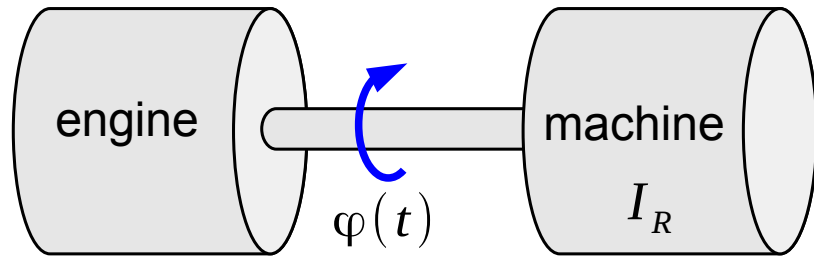
Flywheel

Steady-state motion



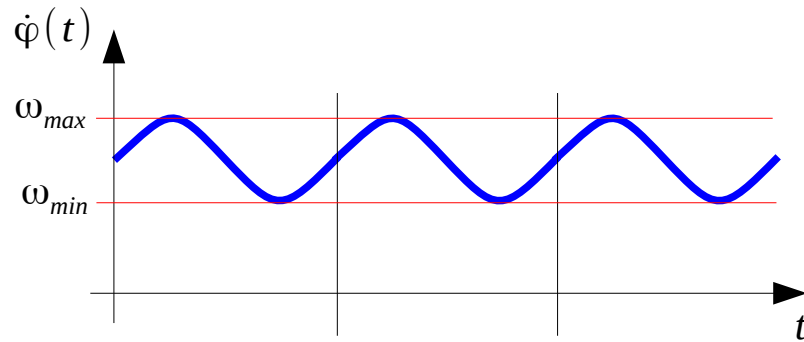
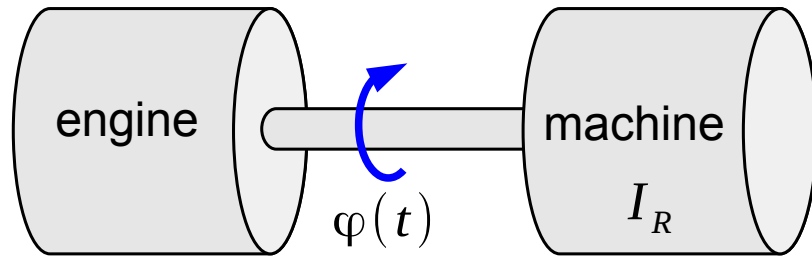
Flywheel

Steady-state motion

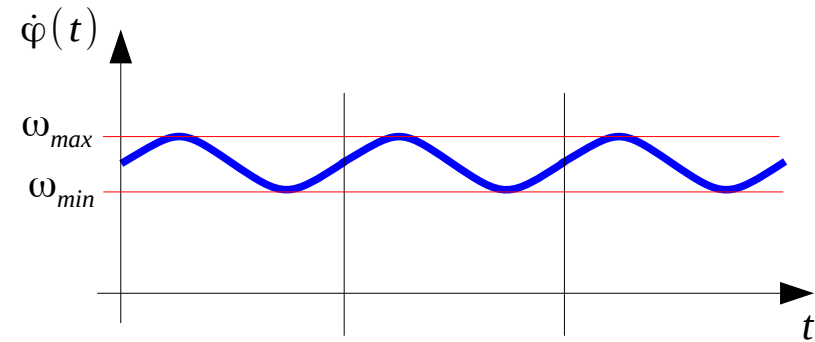
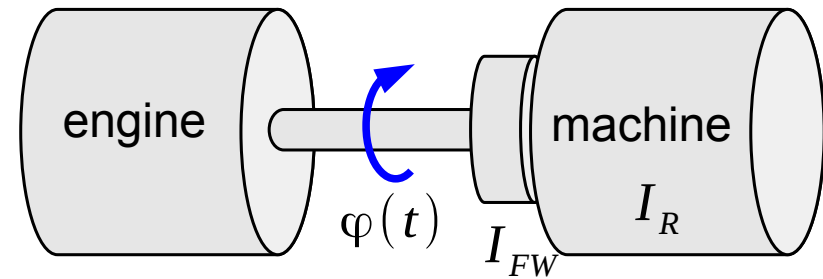


Flywheel

Steady-state motion



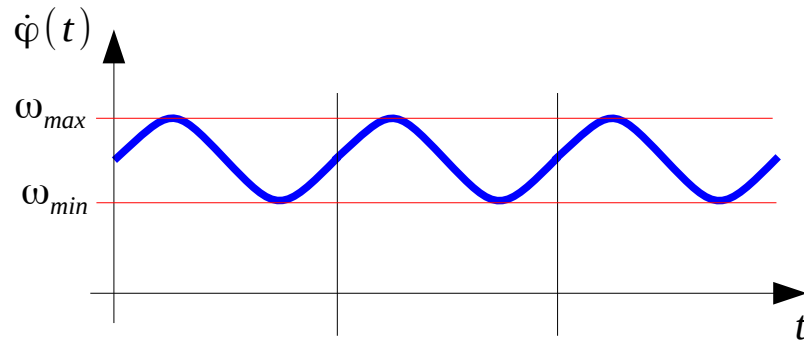
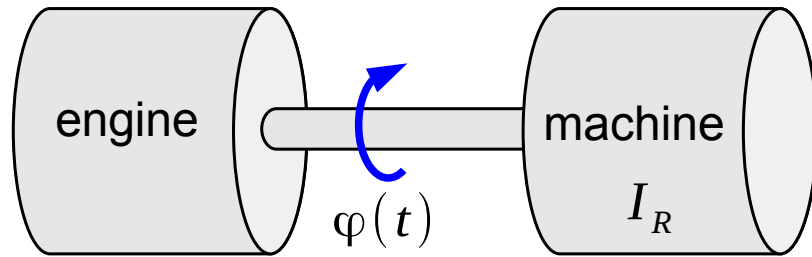
$$W = \delta_1 I_R \omega_{mean}^2$$



assume
 $I_R \approx const.$

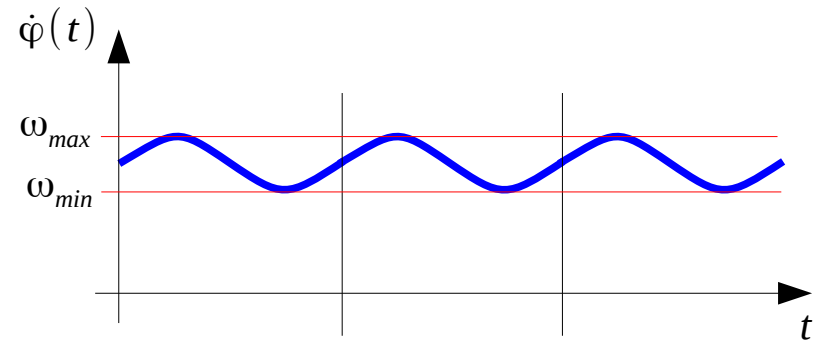
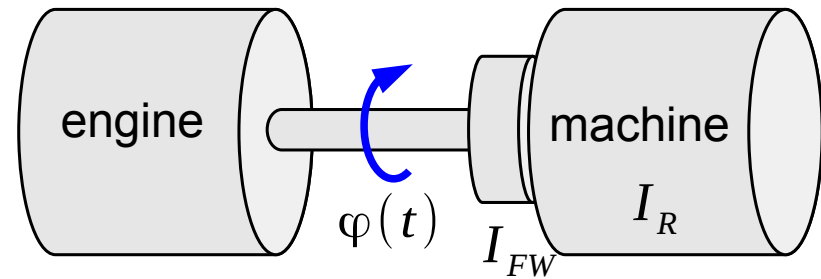
Flywheel

Steady-state motion



$$W = \delta_1 I_R \omega_{mean}^2$$

assume
 $I_R \approx const.$

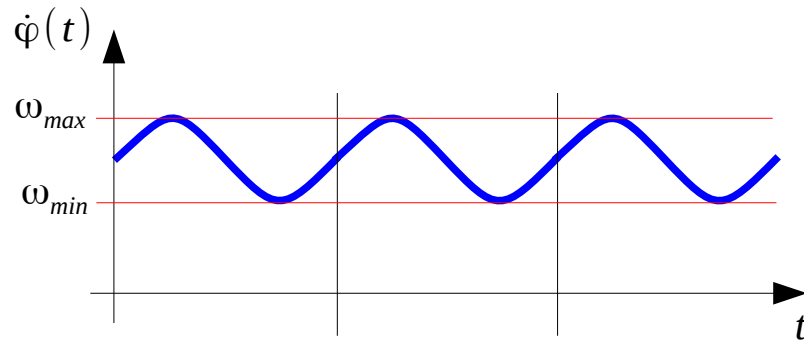
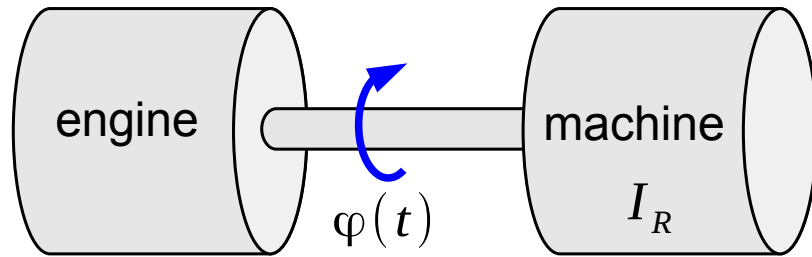


$$W = \delta_2 (I_R + I_{FW}) \omega_{mean}^2$$

(if velocity of a flywheel
same as analyzed velocity)

Flywheel

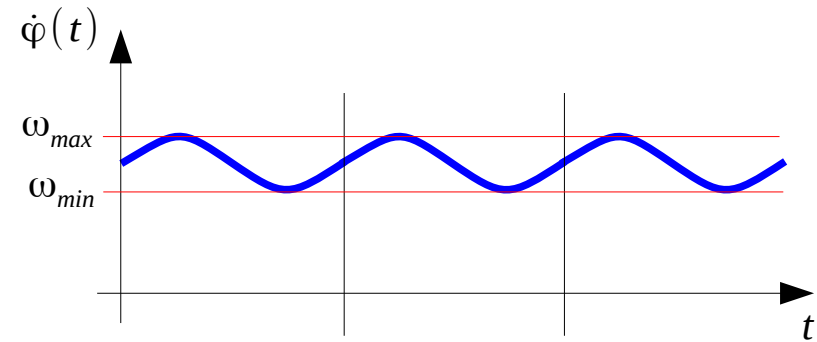
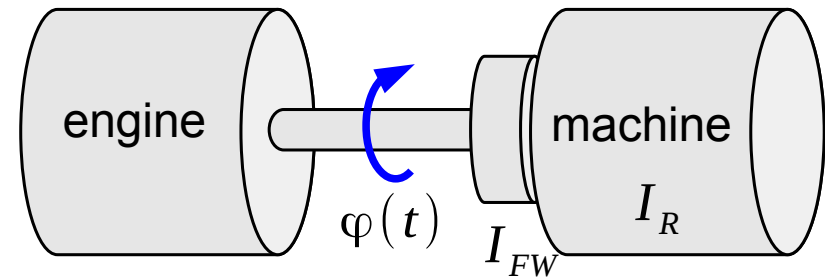
Steady-state motion



$$W = \delta_1 I_R \omega_{mean}^2$$

assume

$$I_R \approx const.$$

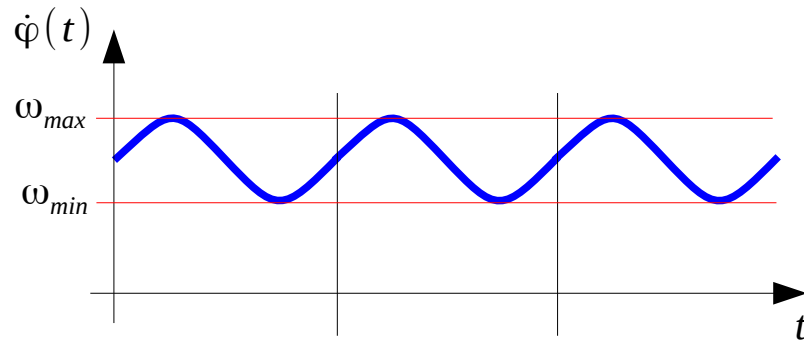
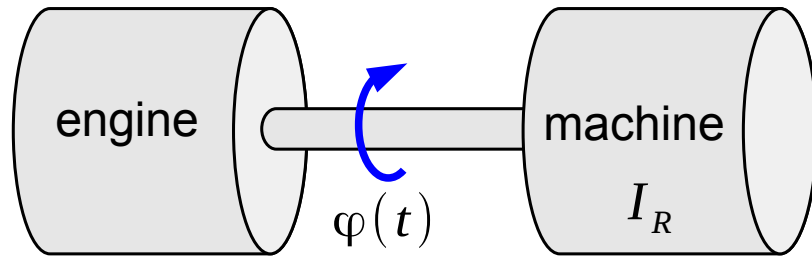


$$W = \delta_2 (I_R + I_{FW}) \omega_{mean}^2$$

$$\delta_1 I_R \omega_{mean}^2 = \delta_2 (I_R + I_{FW}) \omega_{mean}^2$$

Flywheel

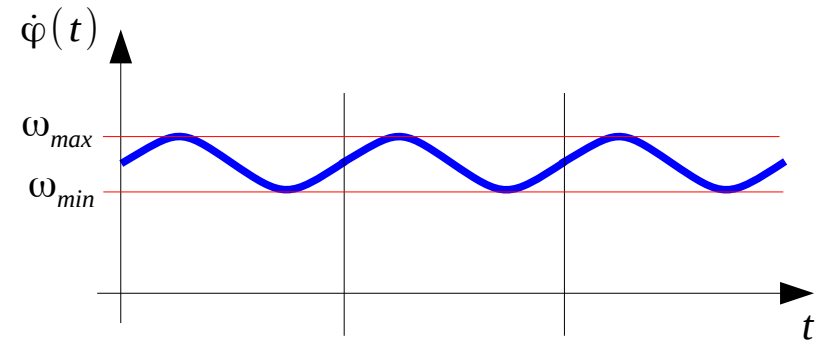
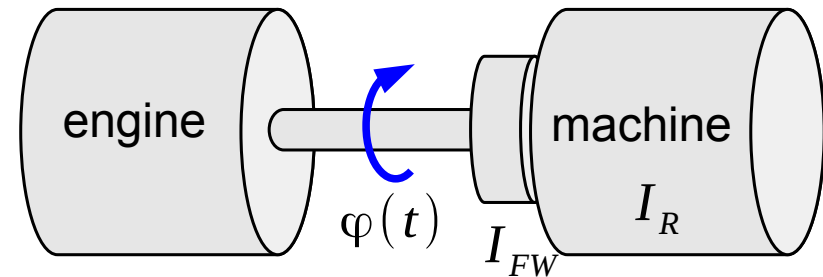
Steady-state motion



$$W = \delta_1 I_R \omega_{mean}^2$$

assume

$$I_R \approx const.$$



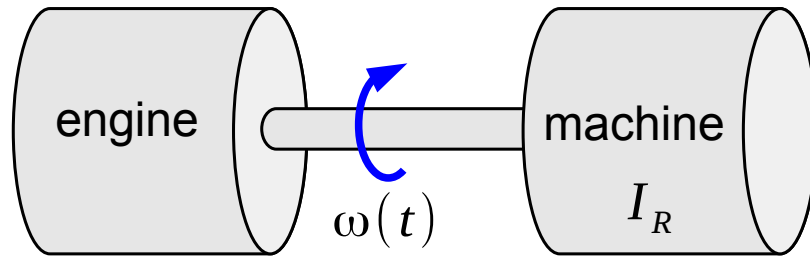
$$W = \delta_2 (I_R + I_{FW}) \omega_{mean}^2$$

$$\delta_1 I_R \omega_{mean}^2 = \delta_2 (I_R + I_{FW}) \omega_{mean}^2$$

$$I_{FW} = \left(\frac{\delta_1}{\delta_2} - 1 \right) I_R$$

Non-uniformity of machine motion

Example 1



Given:

$$\omega_{max}(t) = 1000 \text{ rpm}$$

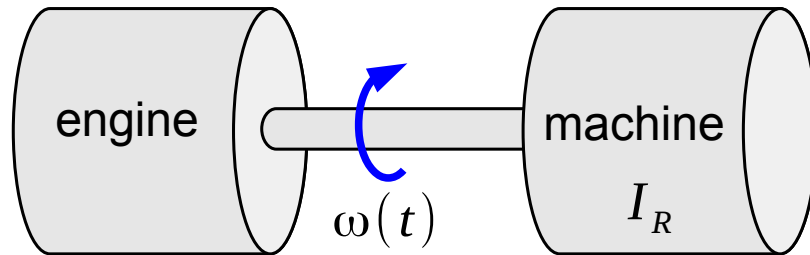
$$\omega_{min}(t) = 950 \text{ rpm}$$

$$I_R = 10 \text{ kgm}^2$$

Design flywheel to
obtain 10rpm
velocity variations.

Non-uniformity of machine motion

Example 1



Given:

$$\omega_{max}(t) = 1000 \text{ rpm}$$

$$\omega_{min}(t) = 950 \text{ rpm}$$

$$I_R = 10 \text{ kgm}^2$$

Design flywheel to obtain 10rpm velocity variations.

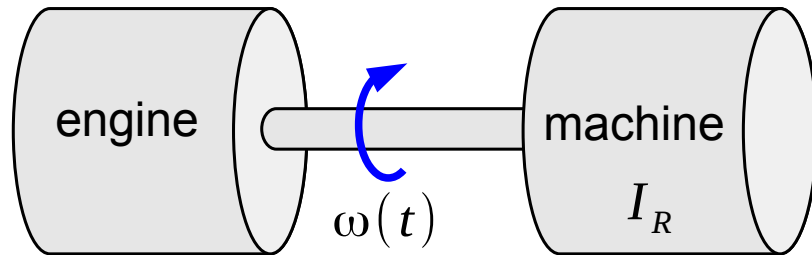
Non-uniformity without a flywheel:

Desired non-uniformity with a flywheel:

Mass moment of inertia of a flywheel:

Non-uniformity of machine motion

Example 1



Given:

$$\omega_{max}(t) = 1000 \text{ rpm}$$

$$\omega_{min}(t) = 950 \text{ rpm}$$

$$I_R = 10 \text{ kgm}^2$$

Design flywheel to obtain 10rpm velocity variations.

Non-uniformity without a flywheel:

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}} = \frac{50}{975} = 0,0513$$

Desired non-uniformity with a flywheel:

$$\delta_{FW} = \frac{10}{975} = 0,010256$$

Mass moment of inertia of a flywheel:

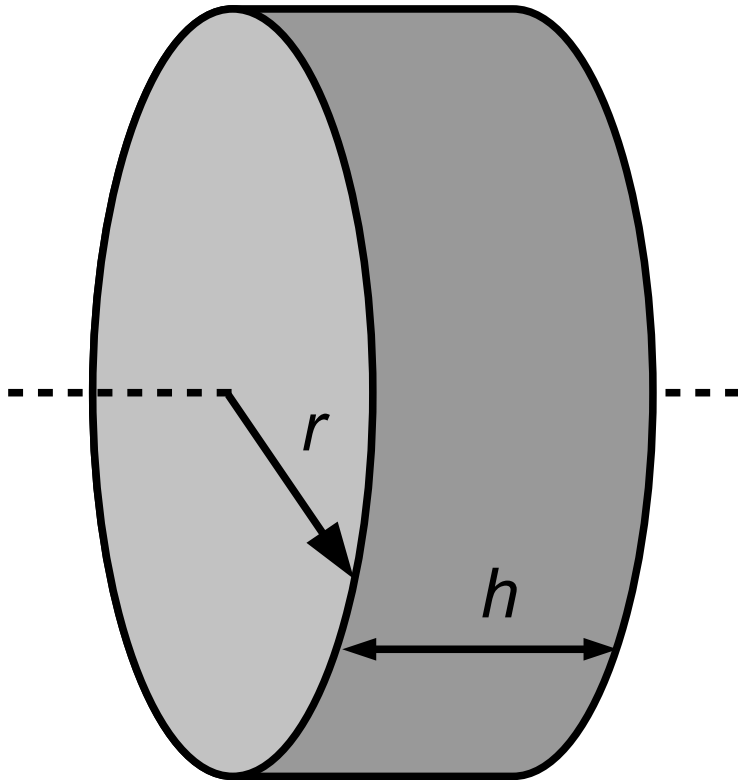
$$I_{FW} = \left(\frac{\delta}{\delta_{FW}} - 1 \right) I_R = 40 \text{ kg m}^2$$

Non-uniformity of machine motion

Example 1

Solid cylinder

$$I_{FW} = \frac{1}{2} m r^2 = \frac{1}{2} \rho \pi h r^4$$

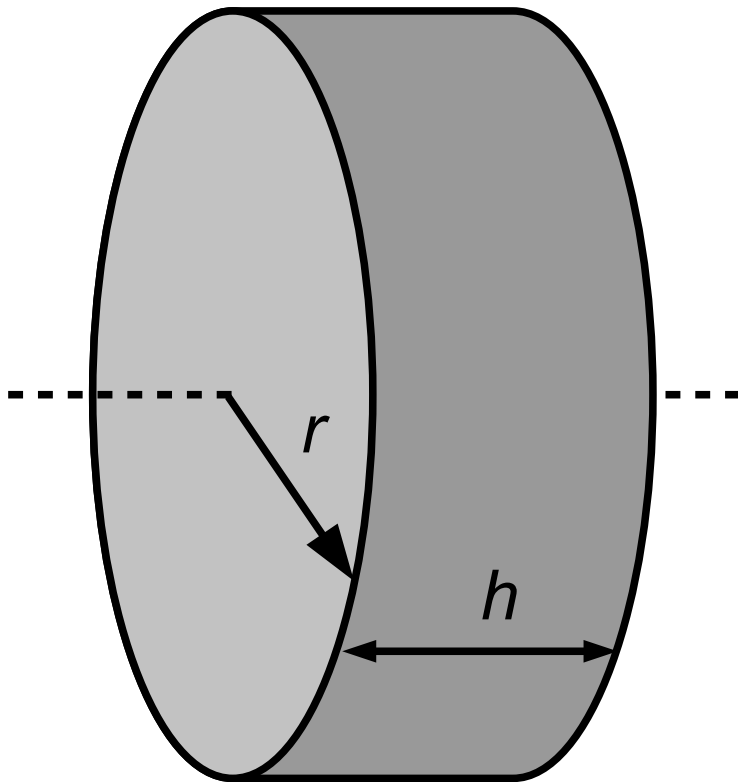


Non-uniformity of machine motion

Example 1

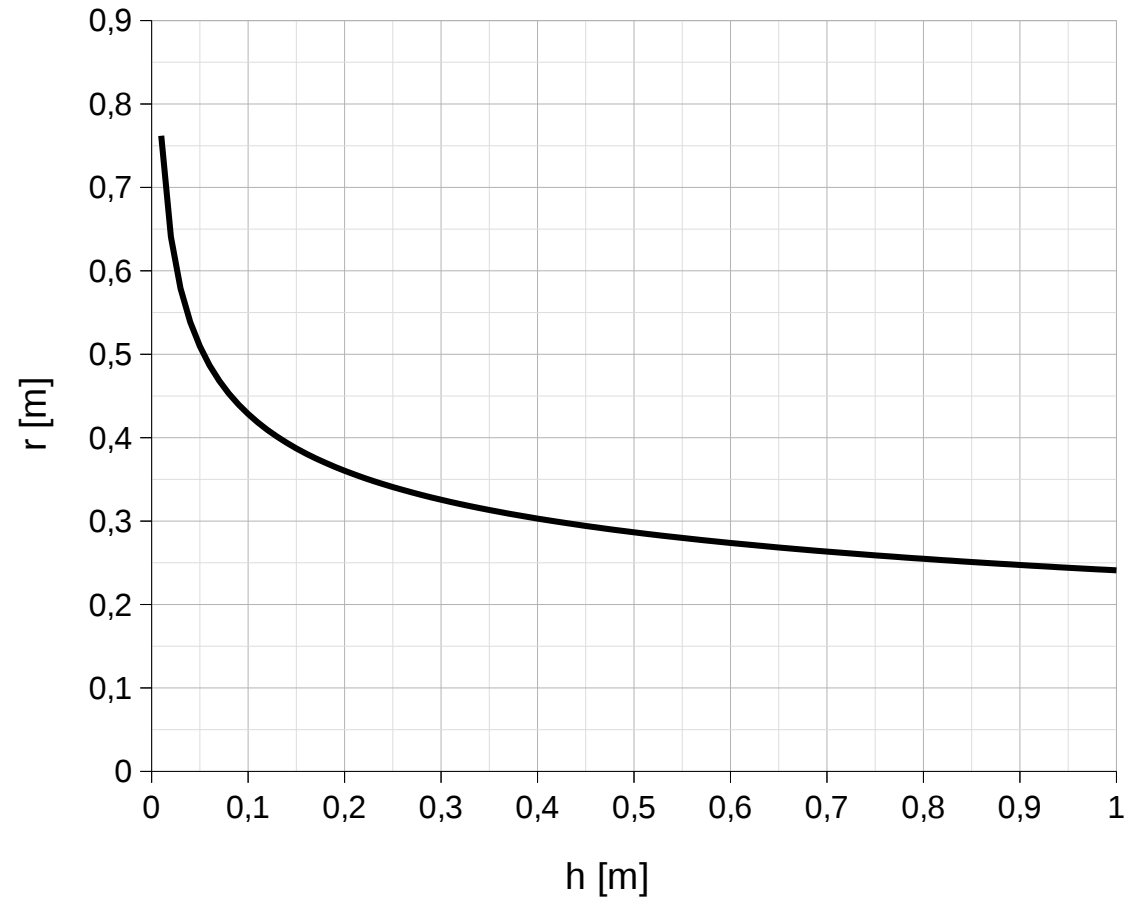
Solid cylinder

$$I_{FW} = \frac{1}{2} m r^2 = \frac{1}{2} \rho \pi h r^4$$



$$I_{FW} = 40 \text{ kgm}^2$$

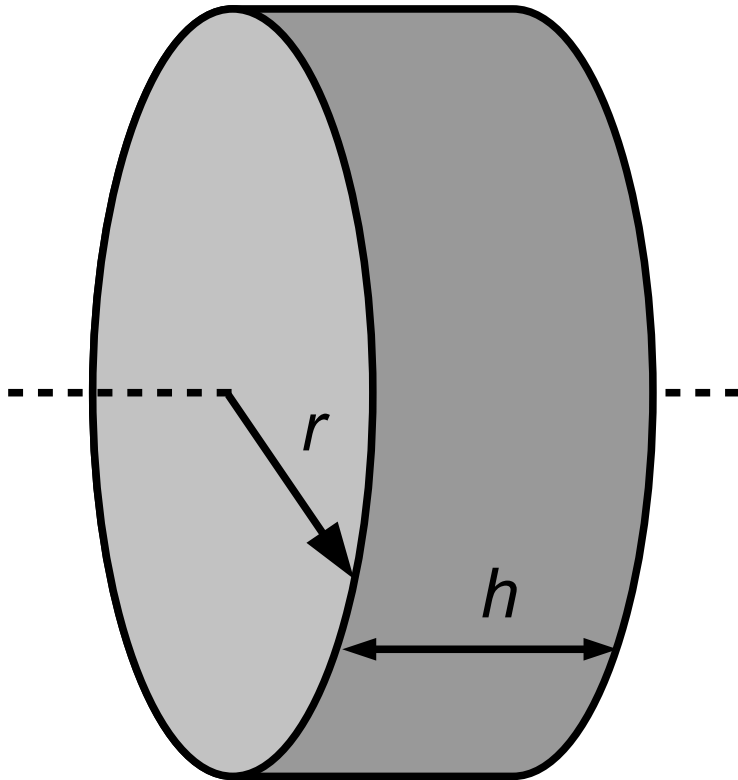
$$\rho_{\text{steel}} = 7800 \text{ kg/m}^3$$



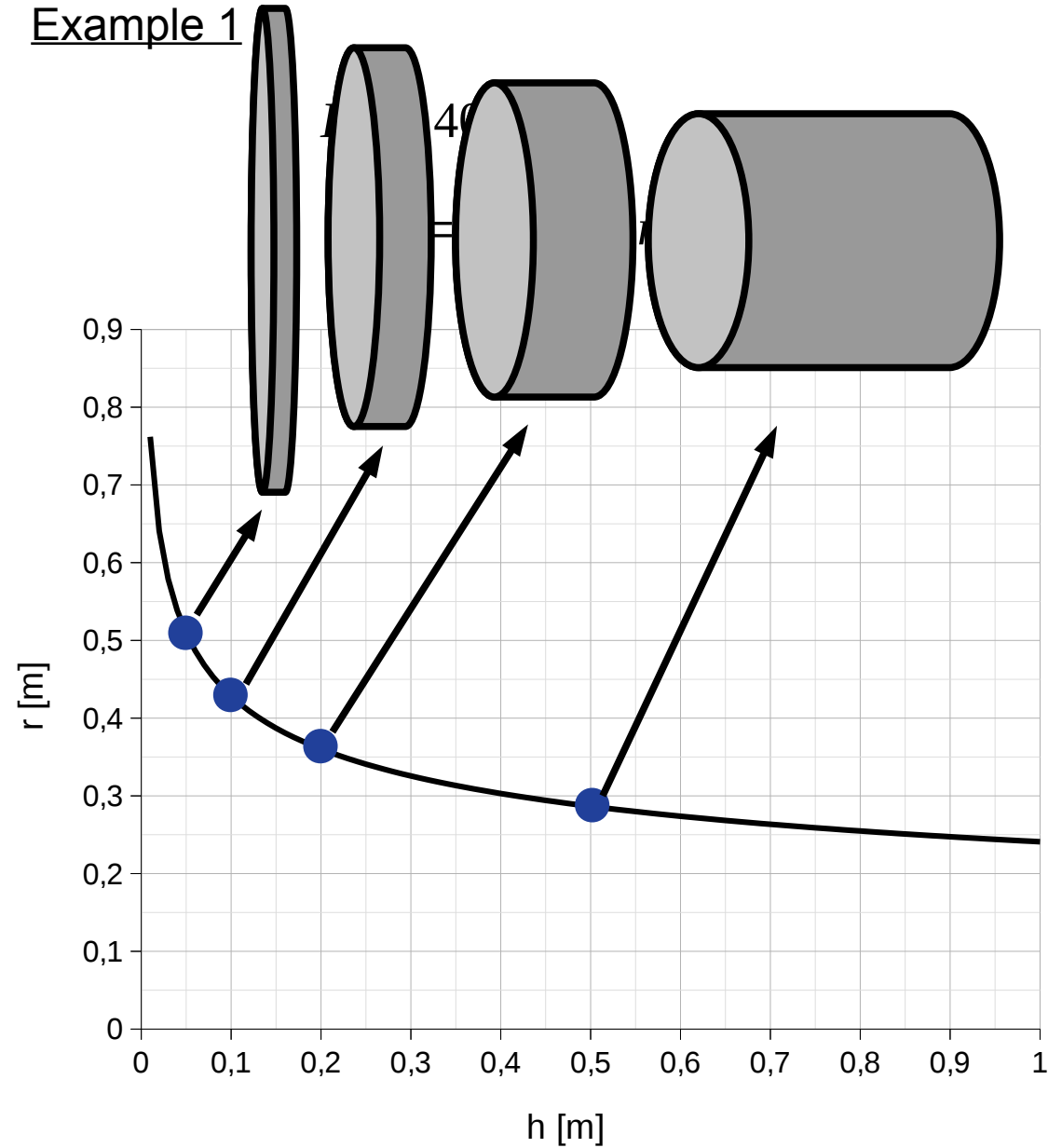
Non-uniformity of machine motion

Solid cylinder

$$I_{FW} = \frac{1}{2} m r^2 = \frac{1}{2} \rho \pi h r^4$$



Example 1

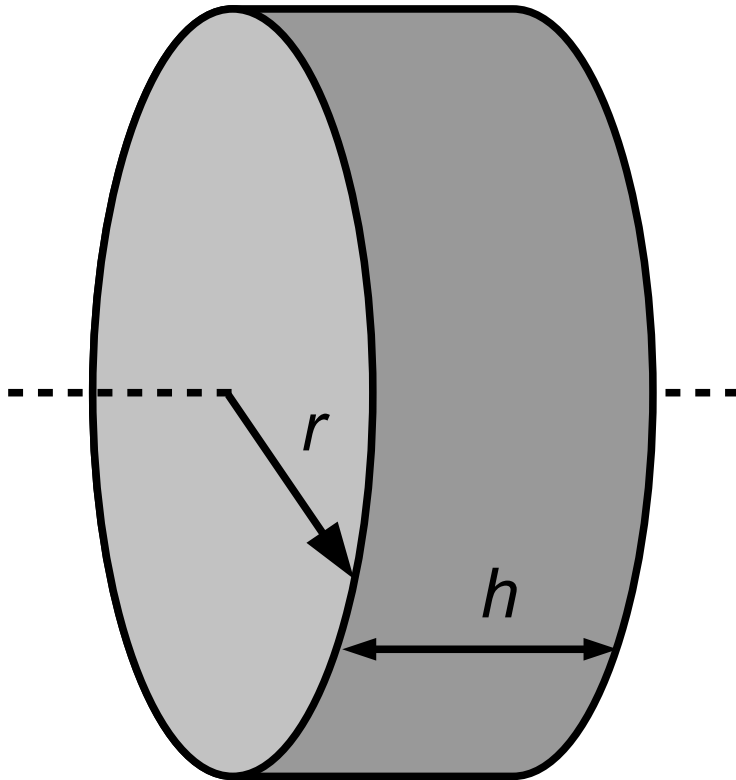


Non-uniformity of machine motion

Example 1

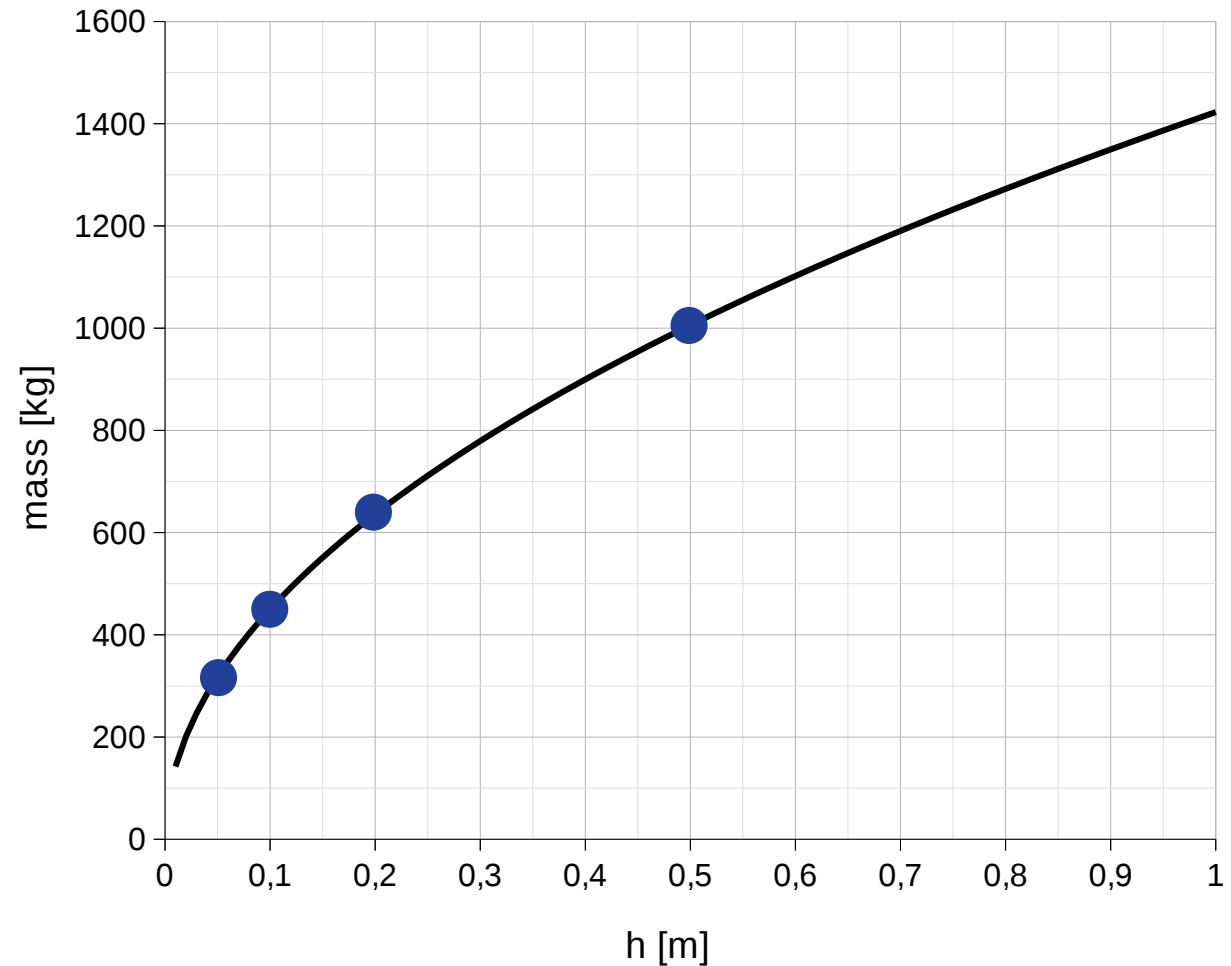
Solid cylinder

$$I_{FW} = \frac{1}{2} m r^2 = \frac{1}{2} \rho \pi h r^4$$



$$I_{FW} = 40 \text{ kgm}^2$$

$$\rho_{\text{steel}} = 7800 \text{ kg/m}^3$$

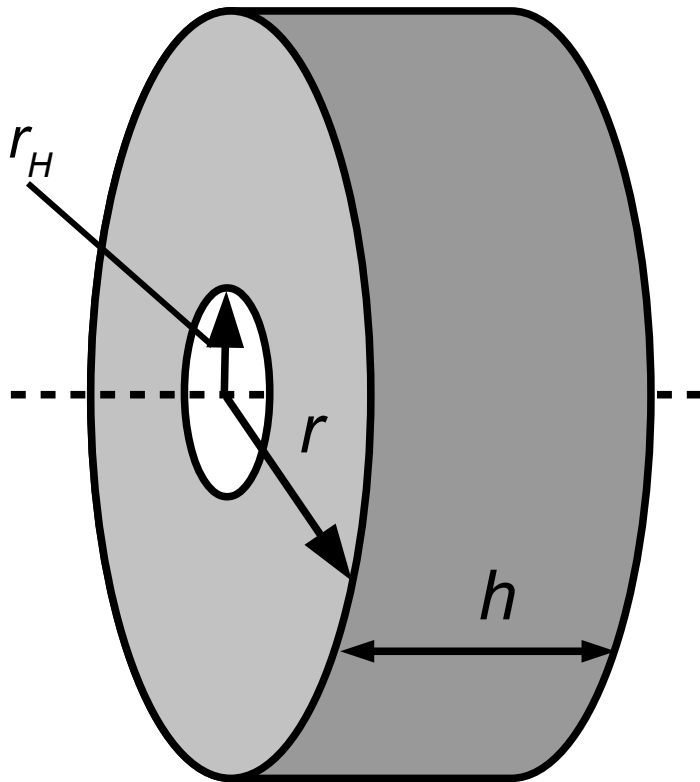


Non-uniformity of machine motion

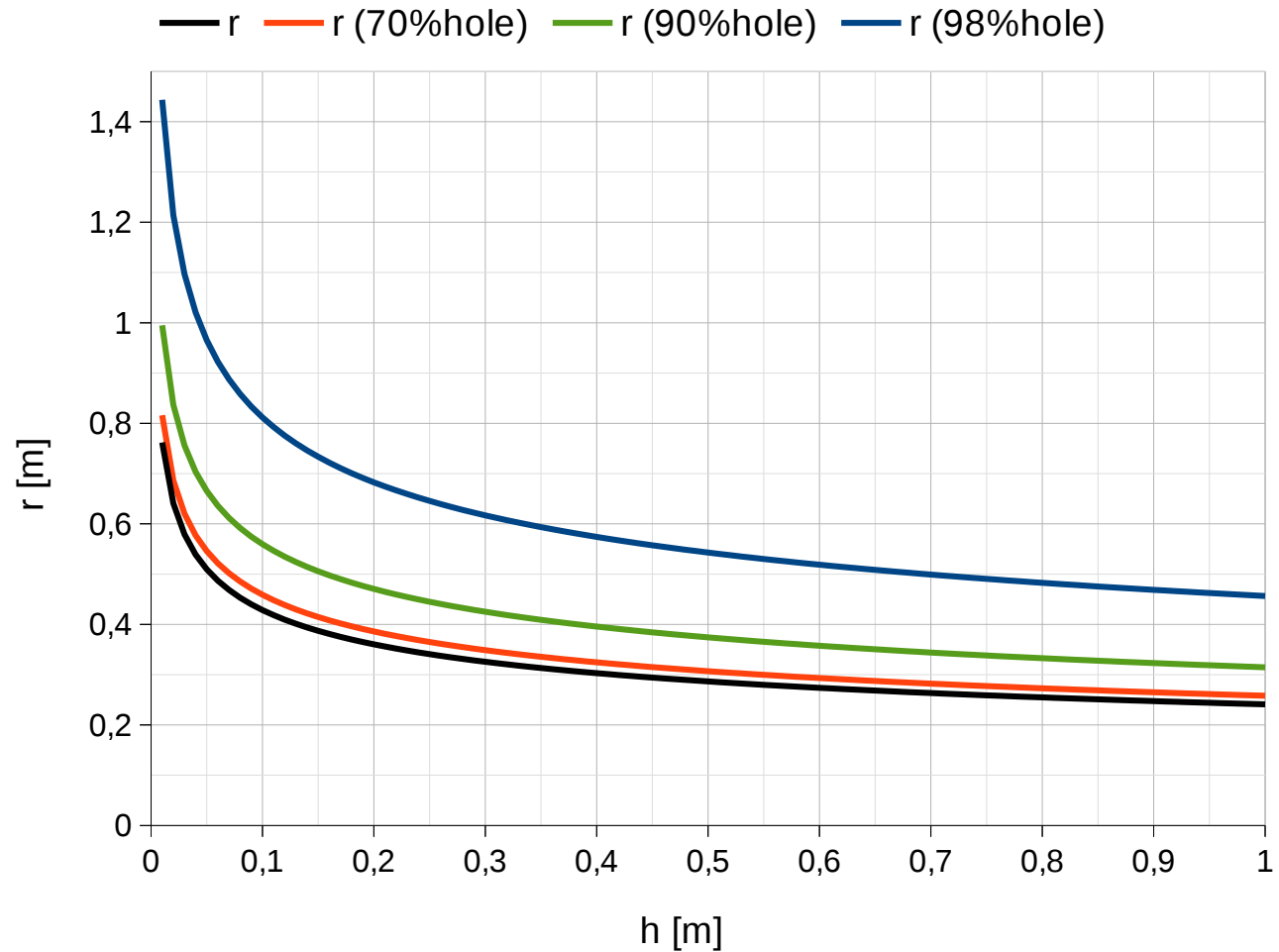
Example 1

Cylinder with a hole

$$I_{FW} = \frac{1}{2} \rho \pi h r^4 - \frac{1}{2} \rho \pi h r_H^4$$



$$I_{FW} = 40 \text{ kgm}^2$$



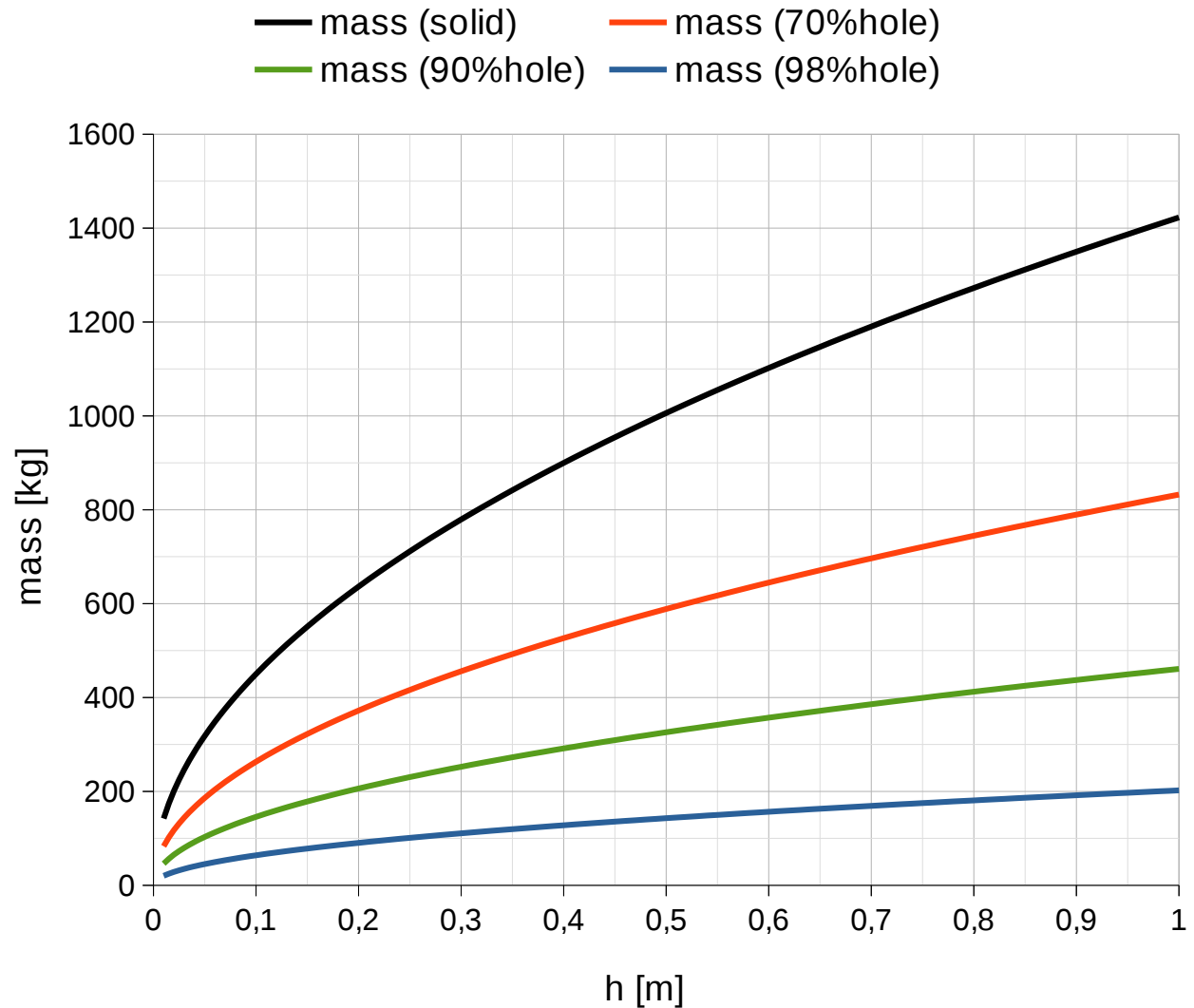
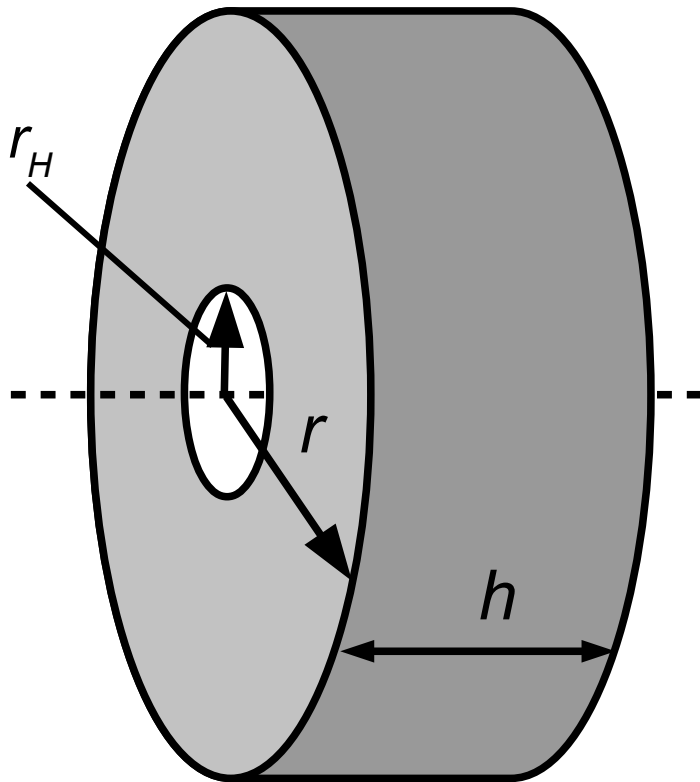
Non-uniformity of machine motion

Example 1

Cylinder with a hole

$$I_{FW} = 40 \text{ kgm}^2$$

$$I_{FW} = \frac{1}{2} \rho \pi h r^4 - \frac{1}{2} \rho \pi h r_H^4$$



Non-uniformity of machine motion

$$I_{FW} = 40 \text{ kgm}^2$$

Example 1

	Solid cylinder	Cylinder 90% hole	Solid cylinder	Cylinder 98% hole
h=	10 cm	10 cm	5 cm	5 cm
r=	43 cm	56 cm	50 cm	96 cm
r _H =	--	50.4 cm	--	94 cm
m=	442.8 kg	143.5 kg	313 kg	44.5 kg

Non-uniformity of machine motion

$$I_{FW} = 40 \text{ kgm}^2$$

Example 1

	Solid cylinder	Cylinder 90% hole	Solid cylinder	Cylinder 98% hole
h=	10 cm	10 cm	5 cm	5 cm
r=	43 cm	56 cm	50 cm	96 cm
r _H =	--	50.4 cm	--	94 cm
m=	442.8 kg	143.5 kg	313 kg	44.5 kg

Non-uniformity of machine motion

To minimize flywheel's mass moment of inertia:

- you should mount a flywheel on a shaft that rotates with the highest angular velocity
- You can add extra transmission to increase angular velocity of a flywheel

Automatic control

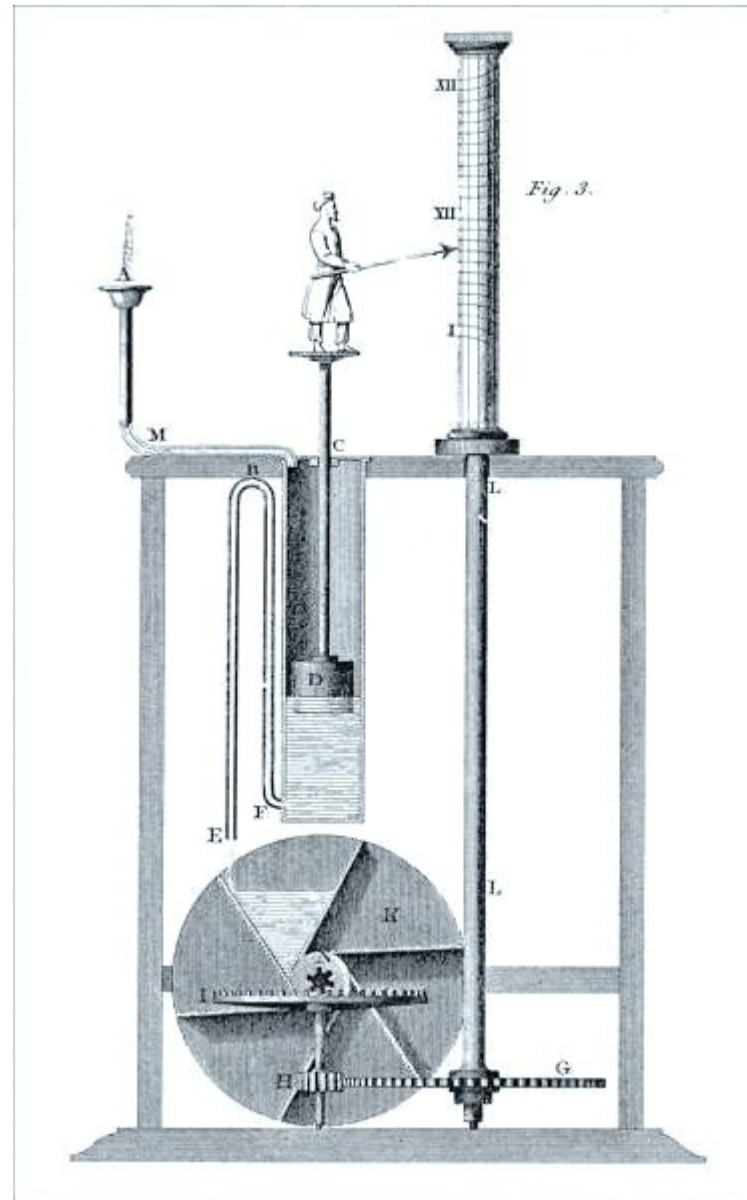
Automatic control

“Automatic control in engineering and technology is a wide generic term covering the application of mechanisms to the operation and regulation of processes without continuous direct human intervention.” - *wikipedia*

Control theory – branch of mathematics and cybernetics that deals with analysis and mathematical modeling of objects and processes treated as dynamical systems with **feedback**.

Automatic control history

Ancient Greece, Arabs



**water clocks,
automatic wine metering,
door opening in temples**

Ctesibius's clepsydra
(3rd century BC).

Source-wikipedia: Abraham Rees (1819) "Clepsydra" in Cyclopædia: or, a New Universal Dictionary of Arts and Sciences The image is the JPEG reproduction published 2007-02-01 by the Horological Foundation.

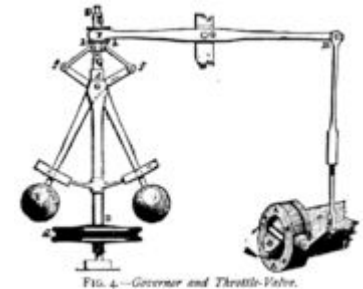
Automatic control history

XVII-XVIII

Temperature regulators for fireplaces and boilers
pressure regulators for pressure cookers

XVIII-XIX

float regulators for water distribution and steam engines
velocity and force regulators for grain mills
Watt's regulator for steam engines (centrifugal governor)



XIX-XX

Laplace transform and Z-transform
Lyapunov stability theory
Routh stability criterion
Hurwitz stability criterion
Nyquist stability criterion and frequency domain analysis
Bode & Nichols frequency domain analysis
Evans root locus analysis

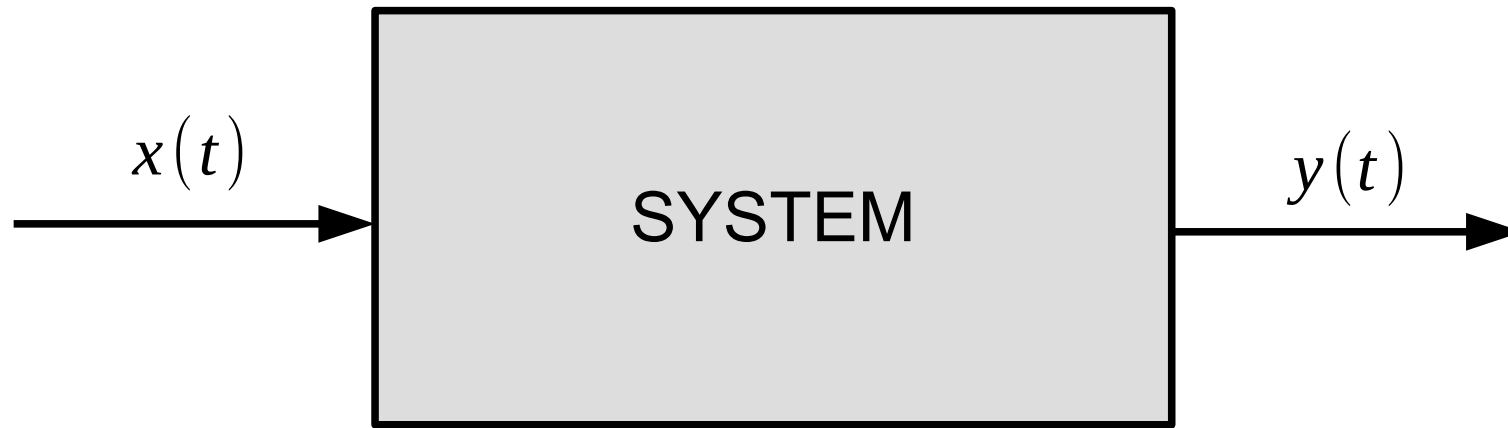
Automatic control

Classical control theory	
single input, single output (SISO)	
usually linear systems	
time independent systems	
description by a transfer functions	
time and frequency domain analysis	
system response is the most important	

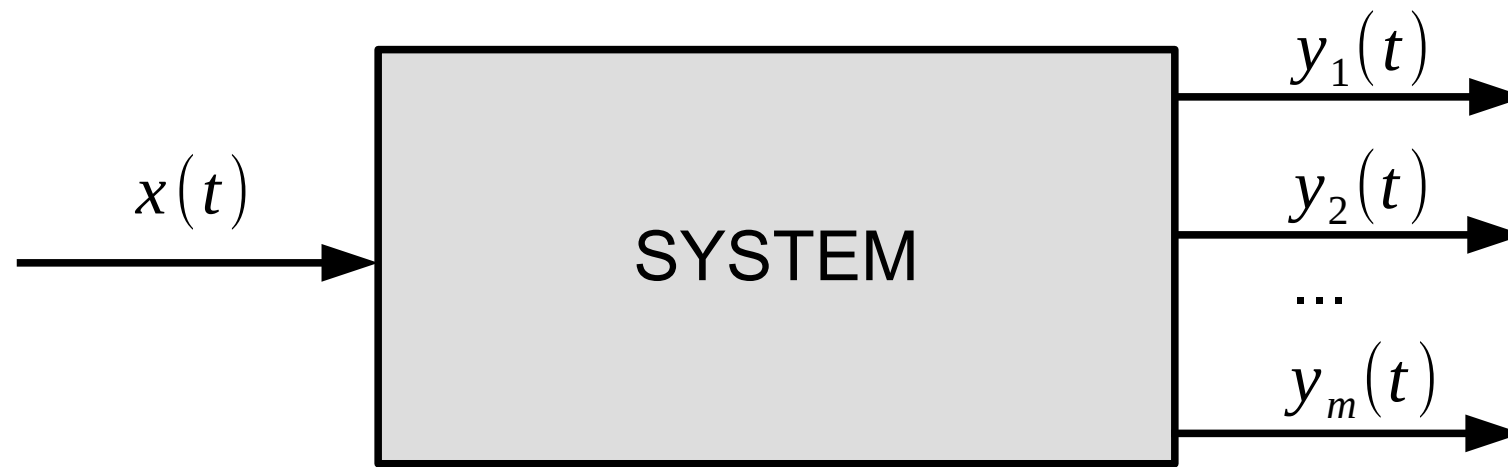
Automatic control

Classical control theory	modern control theory (1950-now)
single input, single output (SISO)	multiple input, multiple output (MIMO)
usually linear systems	often nonlinear systems
time independent systems	time dependent systems
description by a transfer functions	description by a state equations
time and frequency domain analysis	time domain analysis
system response is the most important	system state is the most important

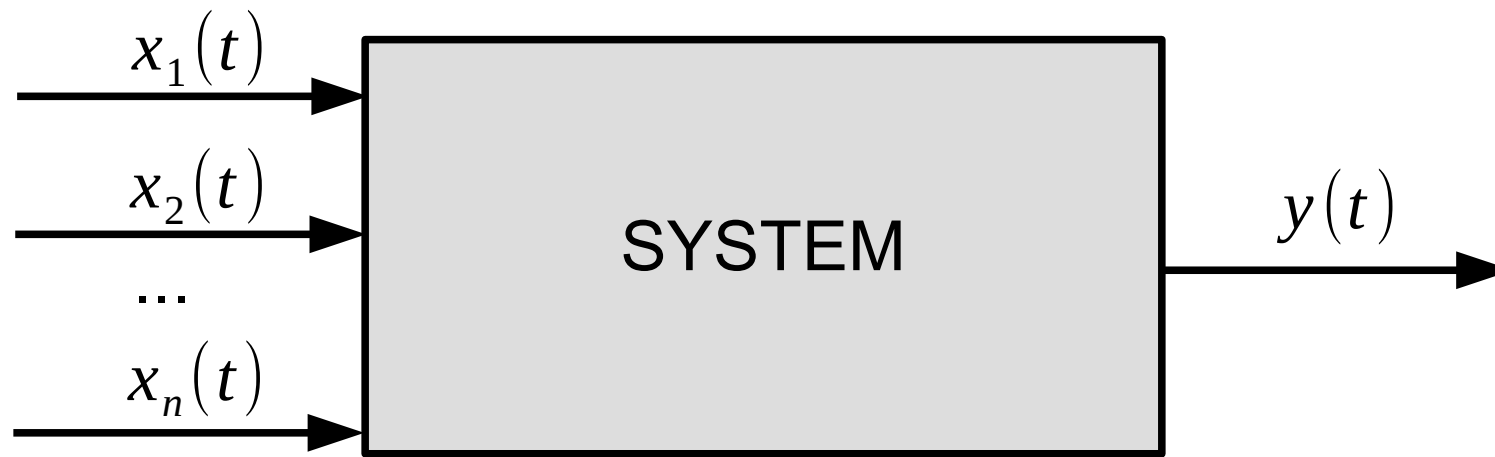
Single Input Single Output (SISO) system



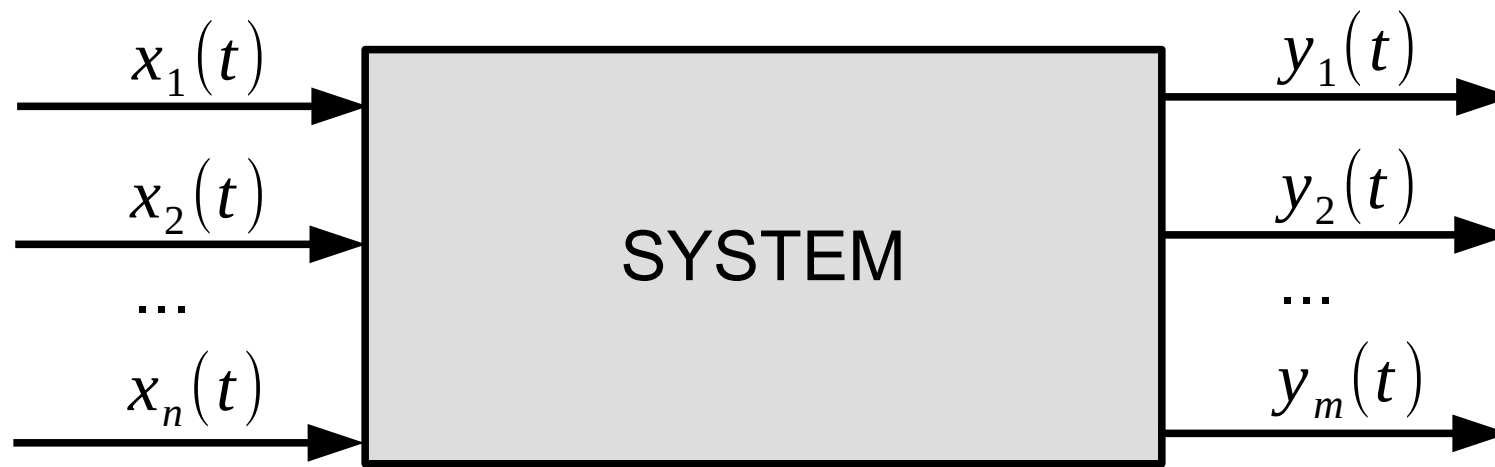
Single Input Multiple Output (SIMO) system



Multiple Input Single Output (MISO) system



Multiple Input Multiple Output (MIMO) system



Linear time-invariant (LTI) system

Linear system

$x(t)$ - input, $y(t) = h(x(t))$ - output

$h(\alpha x(t)) = \alpha h(x(t)) = \alpha y(t)$ scaling

$h(x_1(t) + x_2(t)) = h(x_1(t)) + h(x_2(t))$ superposition

Linear time-invariant (LTI) system

Time-invariant system

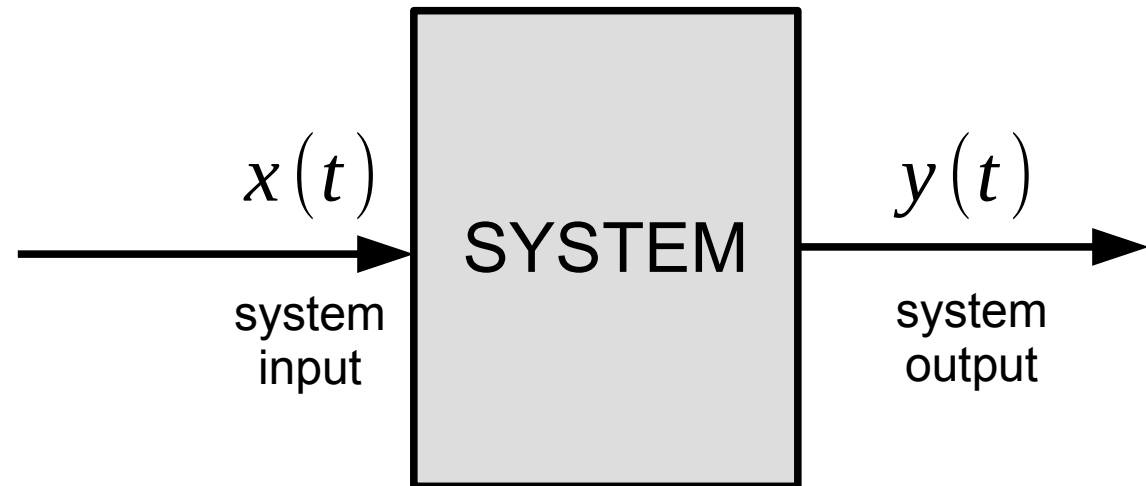
output does not depend explicitly on time

if $y(t) = h(x(t))$ then $y(t - \tau) = h(x(t - \tau))$

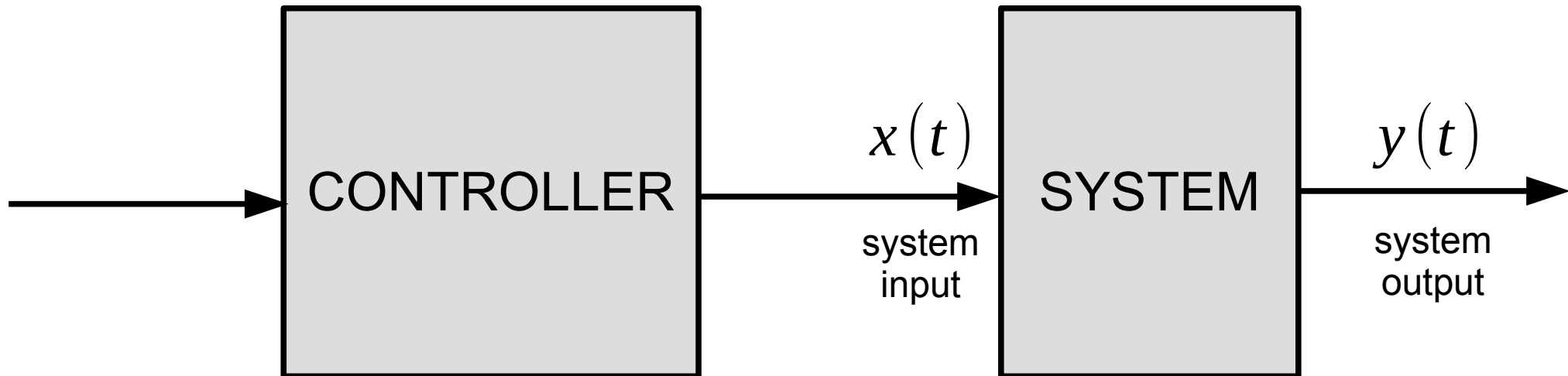
Time-varying system

if $y(t) = h(x(t))$ then $y(t - \tau) \neq h(x(t - \tau))$

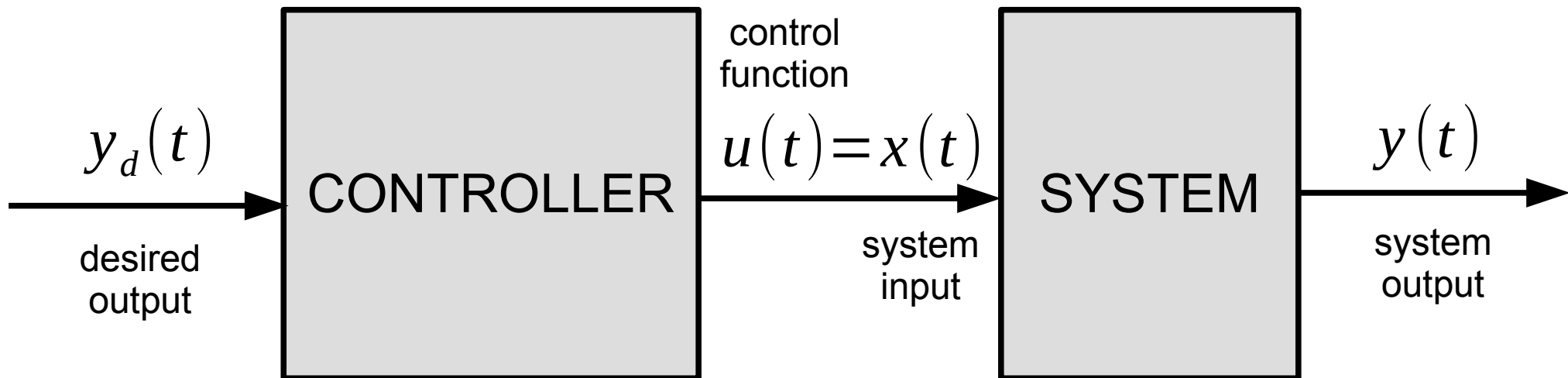
Open loop control



Open loop control



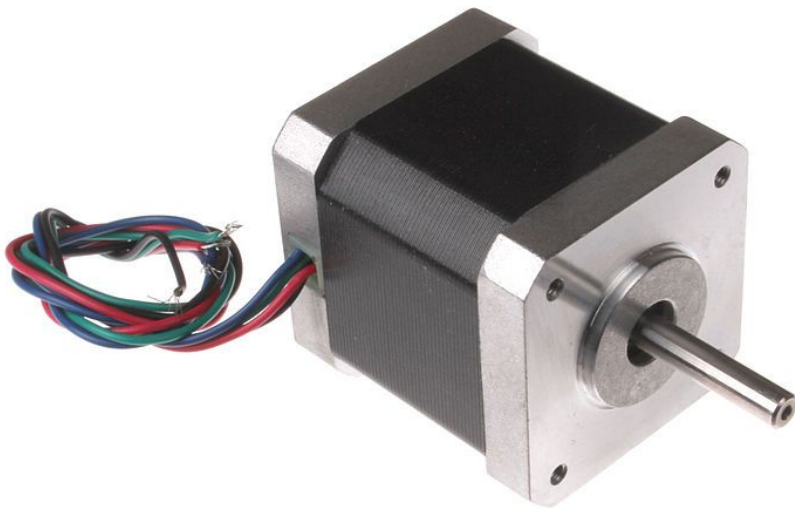
Open loop control



Open loop control

Example usages

stepper motor



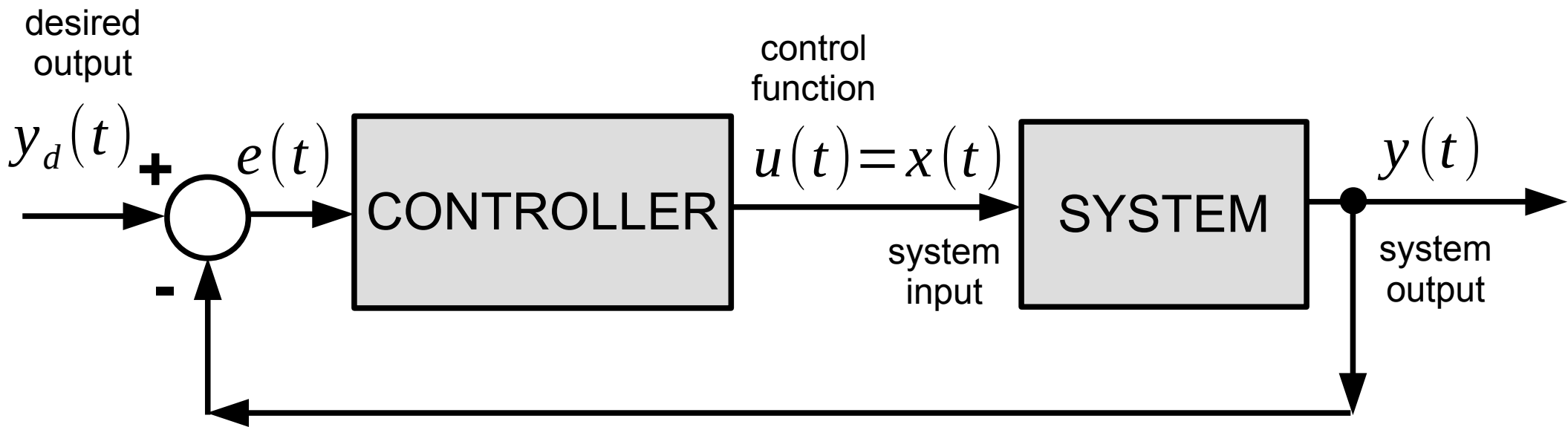
source: wikimedia.org; author: oomlout

two wheeled platform
(flat surface, no slip)

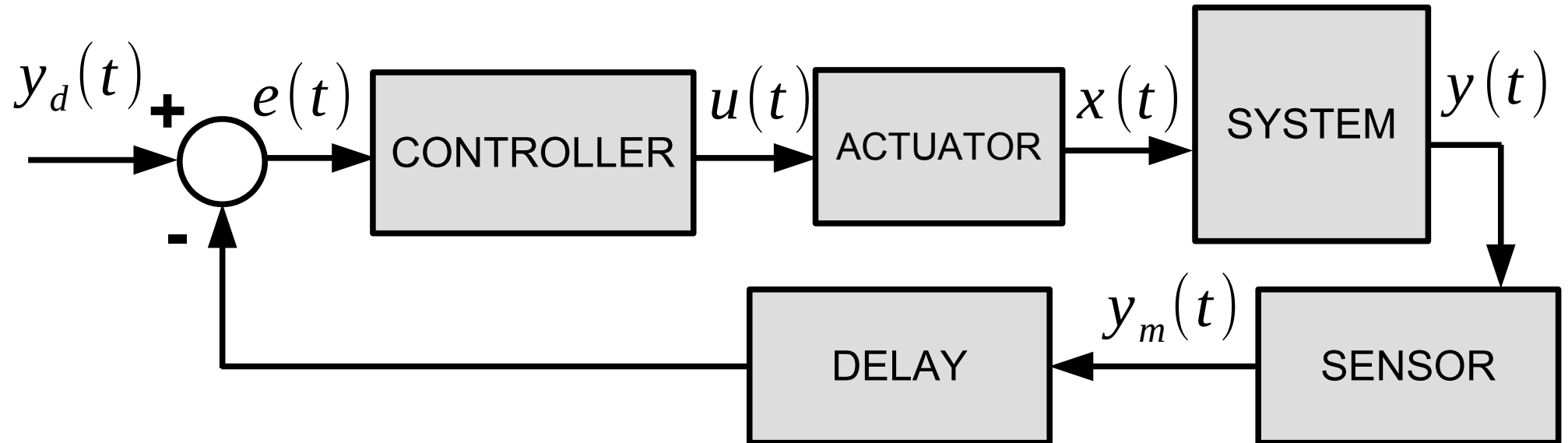


source: <http://www.robotliving.com>

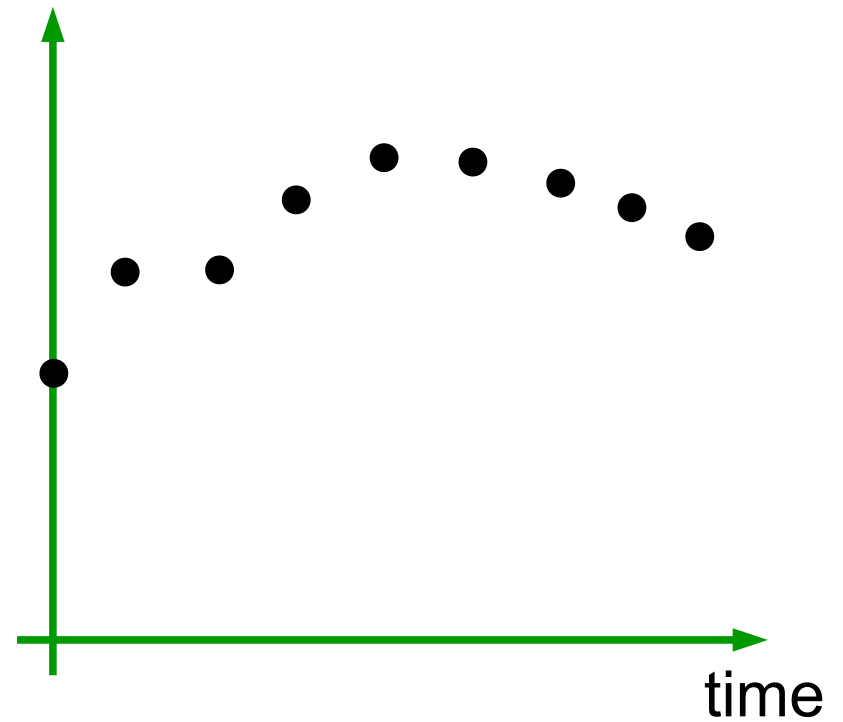
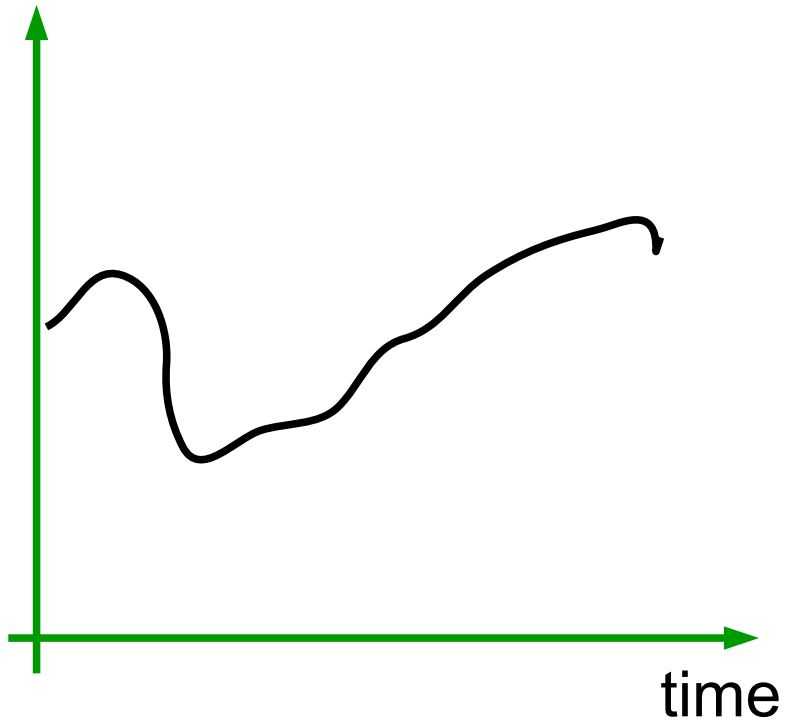
Closed loop control



Closed loop control



Continuous/discrete signals



Mathematical modeling of systems

ordinary differential equations (ODEs)

partial differential equations (PDEs)

table representation

recurrence equations

stochastic representation

neural networks

logical

arithmetic

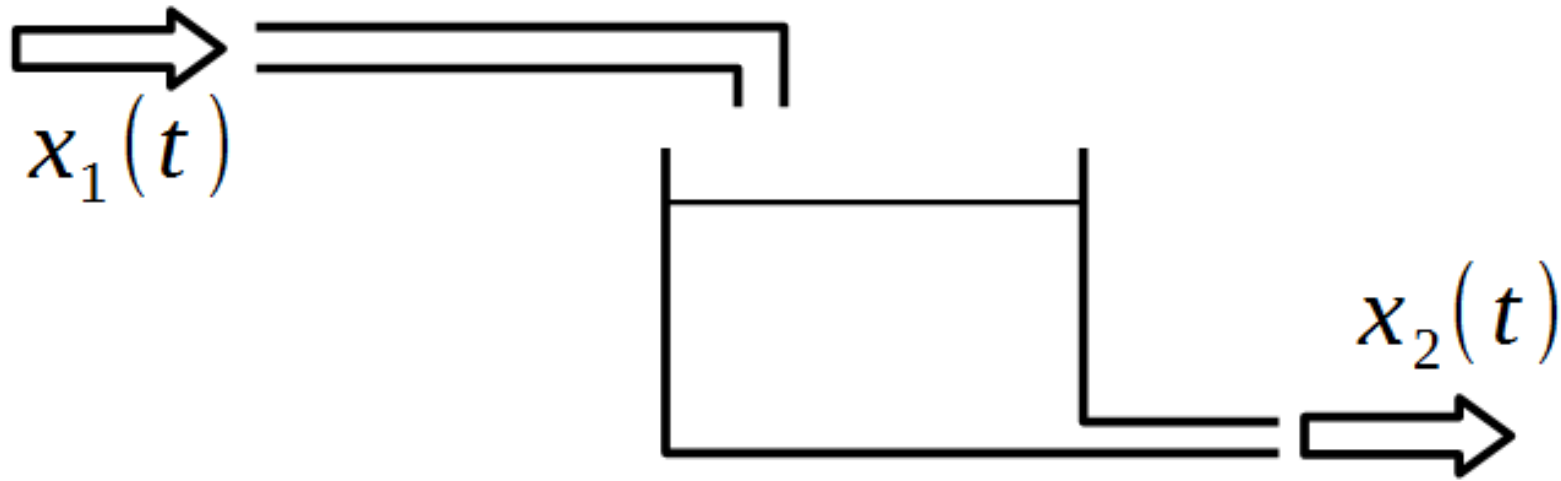
integral equations

combination of above

...

Mathematical modeling of systems

Example 1 – cylindrical tank



$x_1(t)[m^3/s]$ - inflow of a liquid

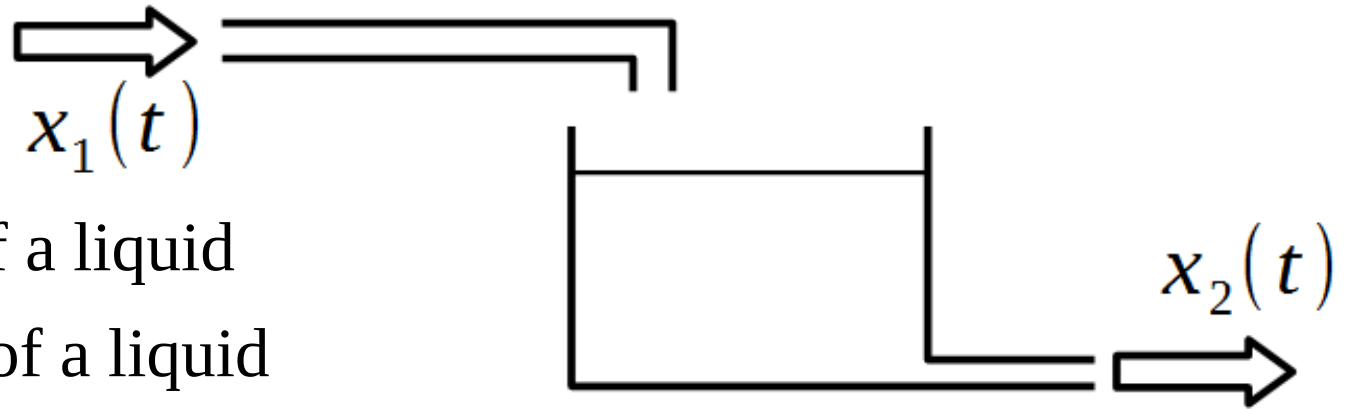
$x_2(t)[m^3/s]$ - outflow of a liquid

$v(t)[m^3]$ - volume of a liquid in a tank

Question: find out a relation between inflow, outflow and volume.

Mathematical modeling of systems

Example 1 – cylindrical tank



$x_1(t)[m^3/s]$ - inflow of a liquid

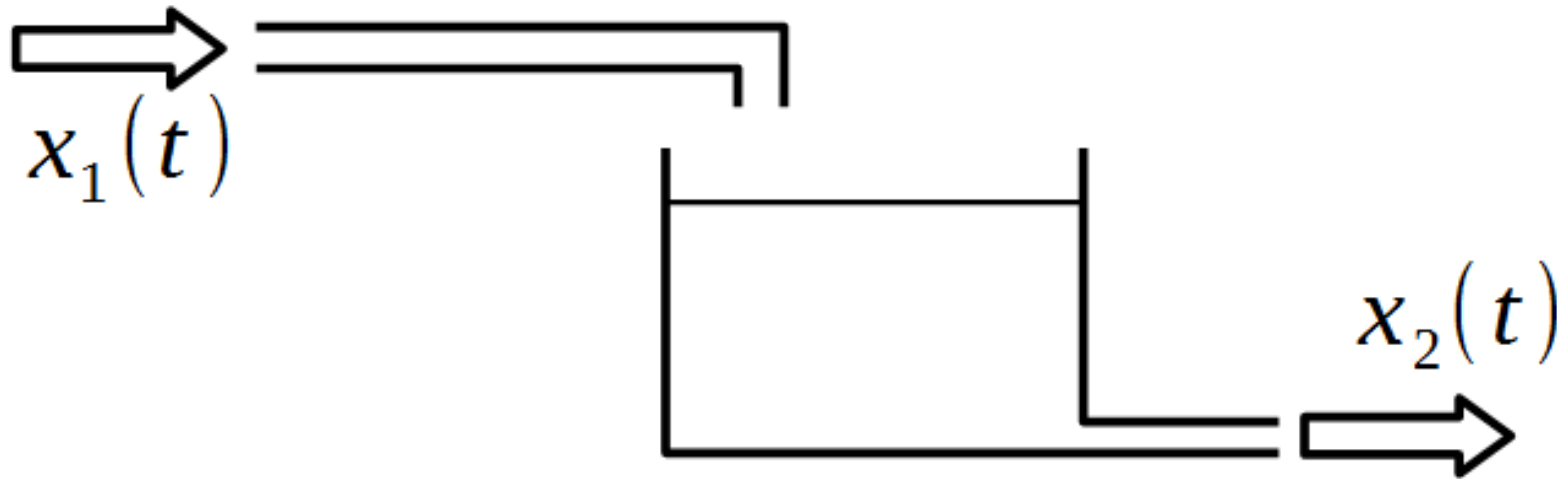
$x_2(t)[m^3/s]$ - outflow of a liquid

$v(t)[m^3]$ - volume of a liquid in a tank

Question: find out a relation between inflow, outflow and volume.

Mathematical modeling of systems

Example 1



$x_1(t)$ [m^3/s] - inflow of a liquid

$x_2(t)$ [m^3/s] - outflow of a liquid

$v(t)$ [m^3] - volume of a liquid in a tank

Question: find out a relation between inflow, outflow and volume.

Answer:

$$t_2 = t_1 + \Delta$$

$$v(t_2) \approx v(t_1) + \Delta (x_1(t_2) - x_2(t_2))$$

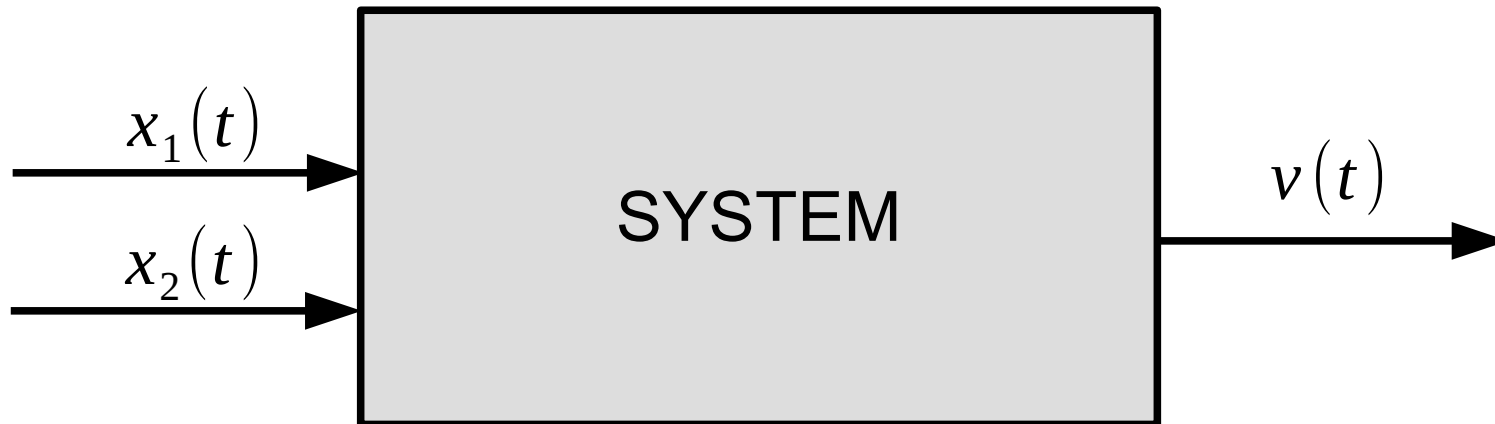
$$\frac{v(t_2) - v(t_1)}{\Delta} \approx x_1(t_2) - x_2(t_2)$$

$$\frac{dv(t)}{dt} = x_1(t) - x_2(t)$$

Mathematical modeling of systems

Example 1

$$\frac{dv(t)}{dt} = x_1(t) - x_2(t)$$



Mathematical modeling of systems

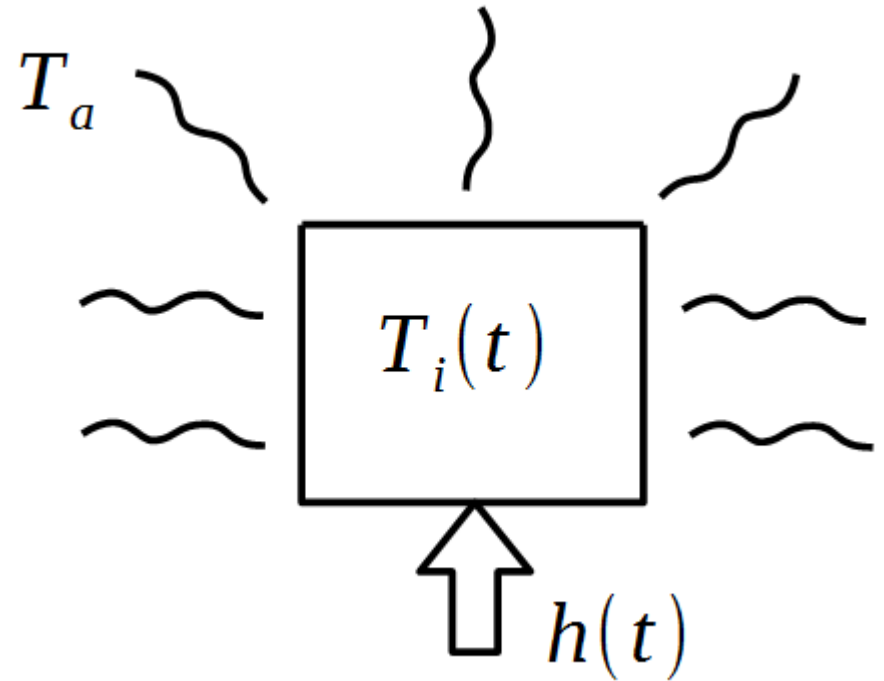
Example 2

$h(t)[W]$ - heater power

$T_a(t)[K]$ - ambient temperature

$T_i(t)[K]$ - object temperature

Question: how to obtain a relation between heater power (input) and object temperature (output)?
Assume energy loss only by convection.



Mathematical modeling of systems

Example 2

Answer:

rate of change of heat = heat gain – heat loss

Mathematical modeling of systems

Example 2

Answer:

rate of change of heat = heat gain – heat loss

$$\frac{dQ(t)}{dt} = Q_H - Q_L$$

Mathematical modeling of systems

Example 2

Answer:

rate of change of heat = heat gain – heat loss

$$\frac{dQ(t)}{dt} = Q_H - Q_L$$

$Q[J] = c_p m T_i$ - heat energy inside the object

$c_p[J/kg K]$ - specific heat coefficient, $m[kg]$ - mass of the object

Mathematical modeling of systems

Example 2

Answer:

rate of change of heat = heat gain – heat loss

$$\frac{dQ(t)}{dt} = Q_H - Q_L$$

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$Q_H[W] = h(t)$ - increase of heat energy caused by the heater

Mathematical modeling of systems

Example 2

Answer:

rate of change of heat = heat gain – heat loss

$$\frac{dQ(t)}{dt} = Q_H - Q_L$$

$Q[J] = c_p m T_i$ - heat energy inside the object

$c_p[J/kg K]$ - specific heat coefficient, $m[kg]$ - mass of the object

$Q_H[W] = h(t)$ - increase of heat energy caused by the heater

$Q_L[W] = \alpha(T_i - T_a)$ - heat energy loss caused by convection

$\alpha[W/K]$ - convection coefficient (assume constant)

Mathematical modeling of systems

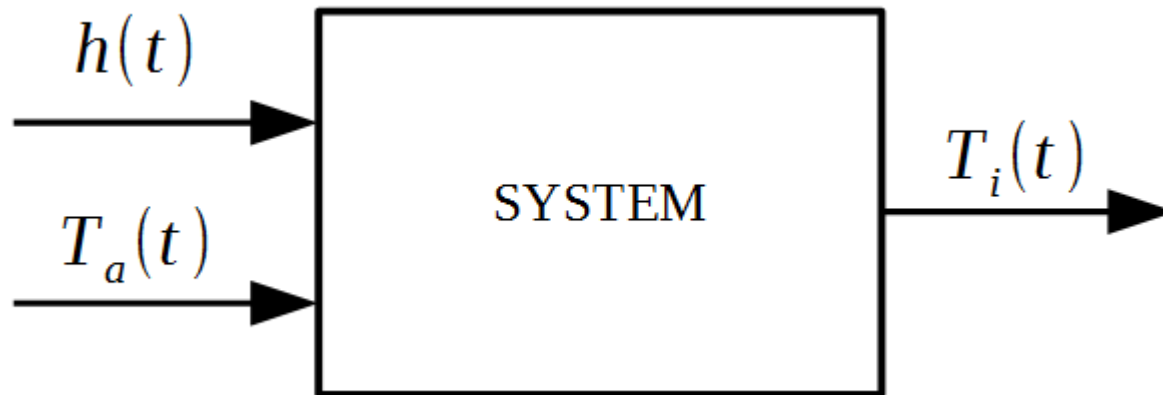
Example 2

$$c_p m \frac{dT_i(t)}{dt} = h(t) - \alpha (T_i(t) - T_a(t))$$

Mathematical modeling of systems

Example 2

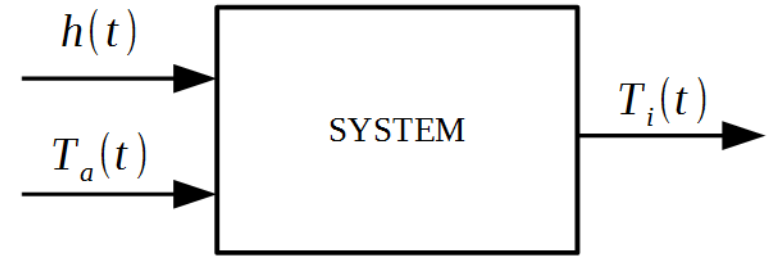
$$c_p m \frac{dT_i(t)}{dt} = h(t) - \alpha(T_i(t) - T_a(t))$$



Mathematical modeling of systems

Example 2

$$c_p m \frac{dT_i(t)}{dt} = h(t) - \alpha(T_i(t) - T_a(t))$$



Question: Can we convert this MISO model into SISO model?