



# Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

## *Theory of Machines and Automatic Control* Winter 2018/2019

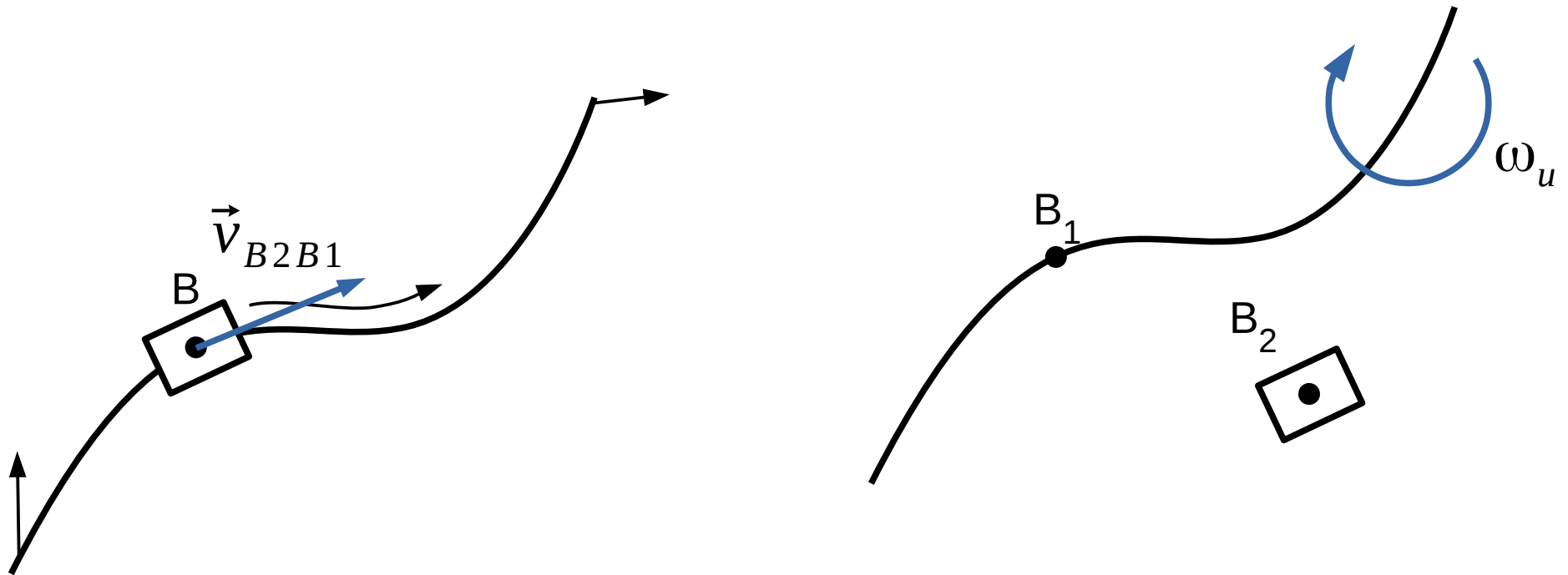
**Lecturer: Sebastian Korczak, PhD, Eng.**

# Lecture 3 cont.

## Accelerations in planar mechanisms.

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# Accelerations in relative motion



$$\vec{a}_{B2} = \vec{a}_{B1}^u + \vec{a}_{B2B1}^w + \vec{a}^c$$

absolute acceleration  
of point B2

Transportation acceleration  
(absolute acceleration of  
point B1)

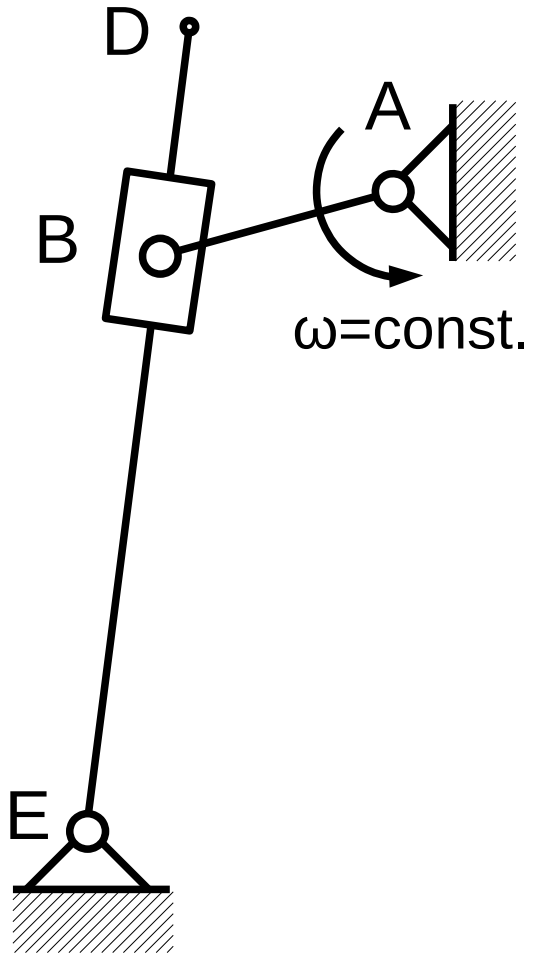
Relative  
acceleration

Coriolis  
acceleration

$$\vec{a}^c = 2 \vec{\omega}_u \times \vec{v}_{B2B1}$$

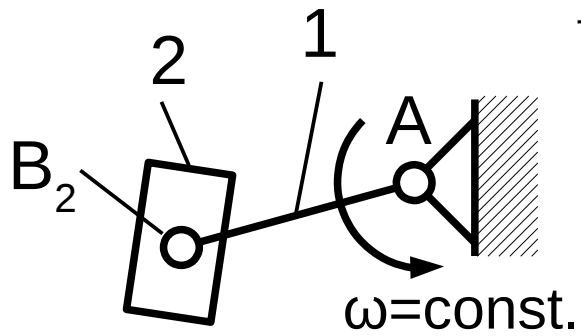
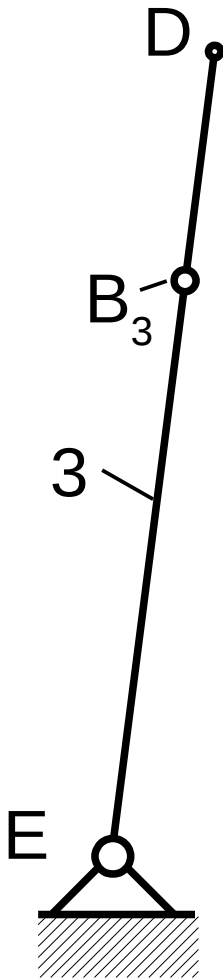
# Accelerations in relative motion

## Example



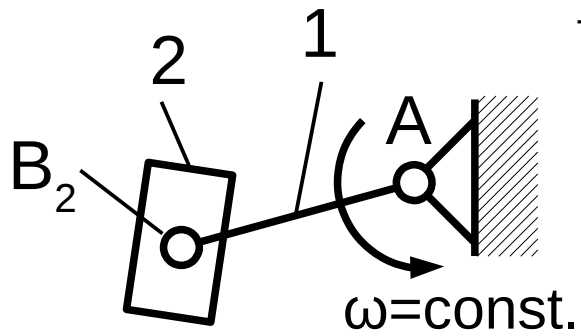
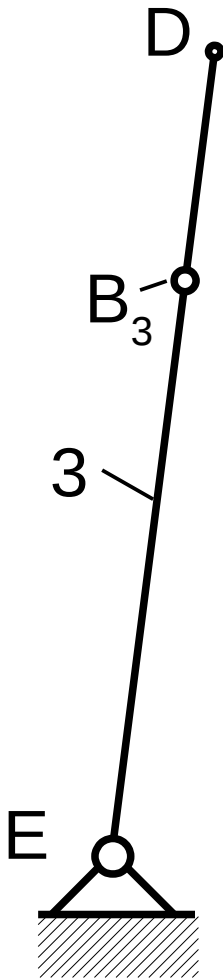
# Accelerations in relative motion

## Example



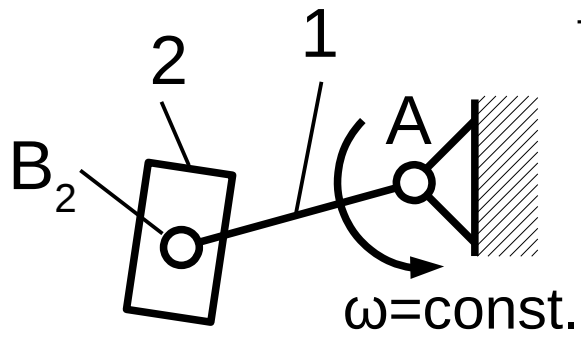
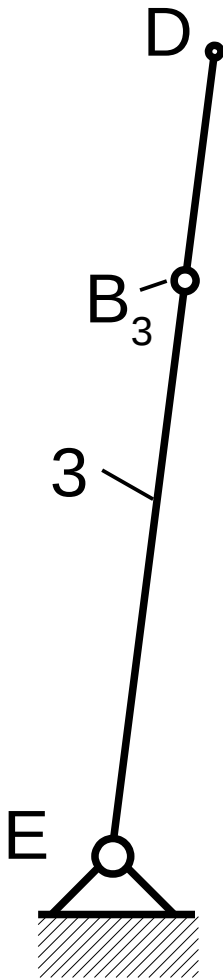
# Accelerations in relative motion

## Example



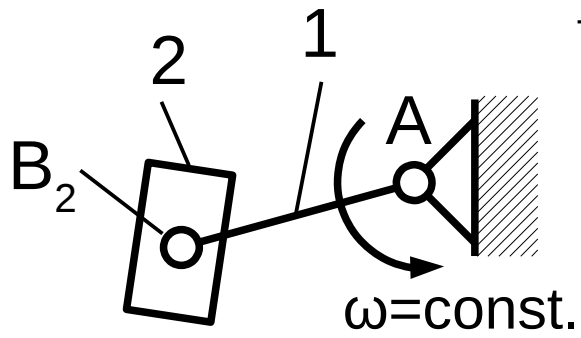
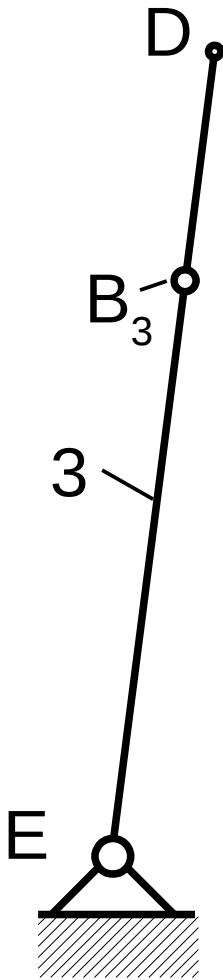
# Accelerations in relative motion

## Example



# Accelerations in relative motion

## Example





# Lecture 4

## Analytical method. Cam mechanisms.

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# Methods of velocities and accelerations determination

## Graphical methods

- velocity projection method,
- instantaneous center of rotation method,
- instantaneous center of acceleration method,
- method of rotated velocities,
- velocity decomposition method,
- acceleration decomposition method,
- velocity scheme (diagram) method,
- accelerations scheme (diagram) method.

## Analytical method

# Procedure of analytical determination of velocities and accelerations in planar mechanisms.

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4. Define “directed angles” for all vectors defined in the same manner. Assume that this angles are created by the positive x axis counter-clockwise rotation.

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2. Substitute the mechanism's members with set of vectors. All vectors can move with mechanism's elements, change their size, location and orientation.
3. Vectors must to create closed polygons.
4. Define “directed angles” for all vectors defined in the same manner. Assume that this angles are created by the positive x axis counter-clockwise rotation.
5. For each of polygon write down sum of vectors, e.g.:

$$\sum_{i=1}^{i=n} \vec{l}_i = 0$$



# Procedure of analytical determination of velocities and accelerations in planar mechanisms.

6a. Write down projections of each polygon onto coordinate system's axes:

$$x: \sum_{i=1}^{i=n} l_i \cos \varphi_i = 0 \qquad y: \sum_{i=1}^{i=n} l_i \sin \varphi_i = 0$$

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(we do not need to analyze signs because of „directed angles” setup procedure)

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6b. Define which vectors' lengths and angles are given and/or constant (related to geometry), and which are variable in time and unknown.

(for a proper defined system number of unknown variables is equal to the number of equations)

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6b. Define which vectors' lengths and angles are given and/or constant (related to geometry), and which are variable in time and unknown.

(for a proper defined system number of unknown variables is equal to the number of equations)

7. Solve the equations. The resulting functions describes motion of the mechanism.

# Procedure of analytical determination of velocities and accelerations in planar mechanisms.

8. Differentiate functions achieved in p.7 to obtain velocities. Differentiate once again to obtain accelerations.

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9. If desired informations was not obtained in p.8, differentiate equations from p.6. Sometimes rotation of the coordinate system is useful here.

# Analytical method – example: crank-slider mechanism

Given:

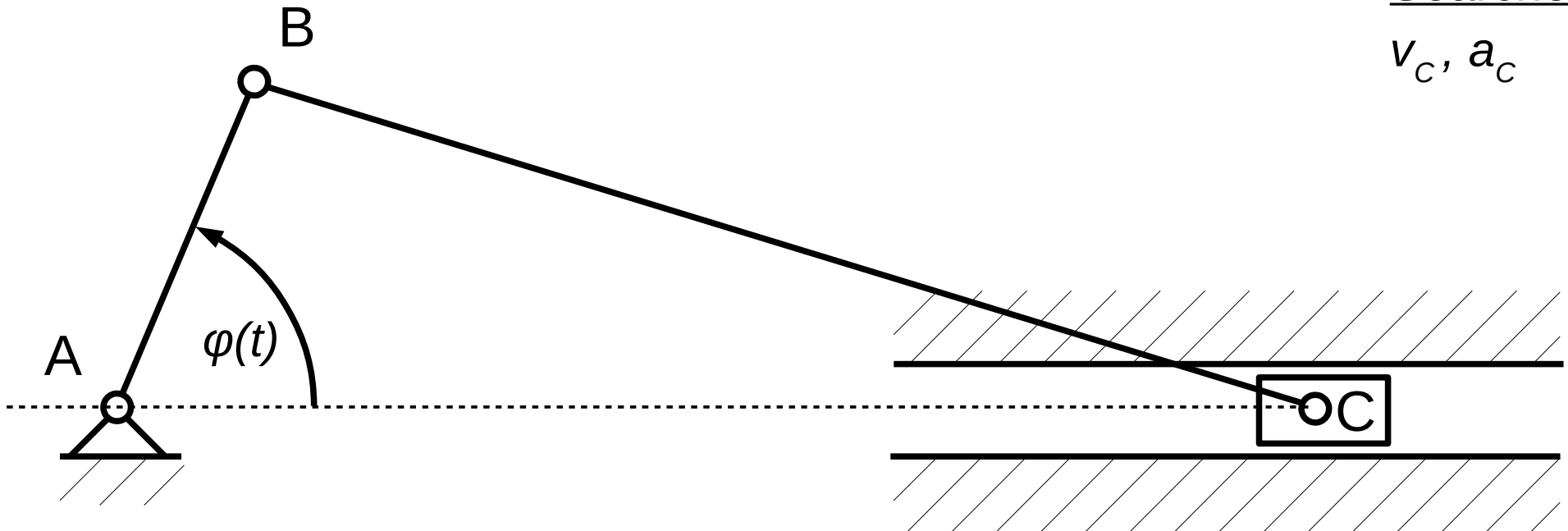
$$|AB| = r$$

$$|BC| = l$$

$$\varphi(t)$$

Searched:

$$v_C, a_C$$



# Analytical method – example: crank-slider mechanism

Given:

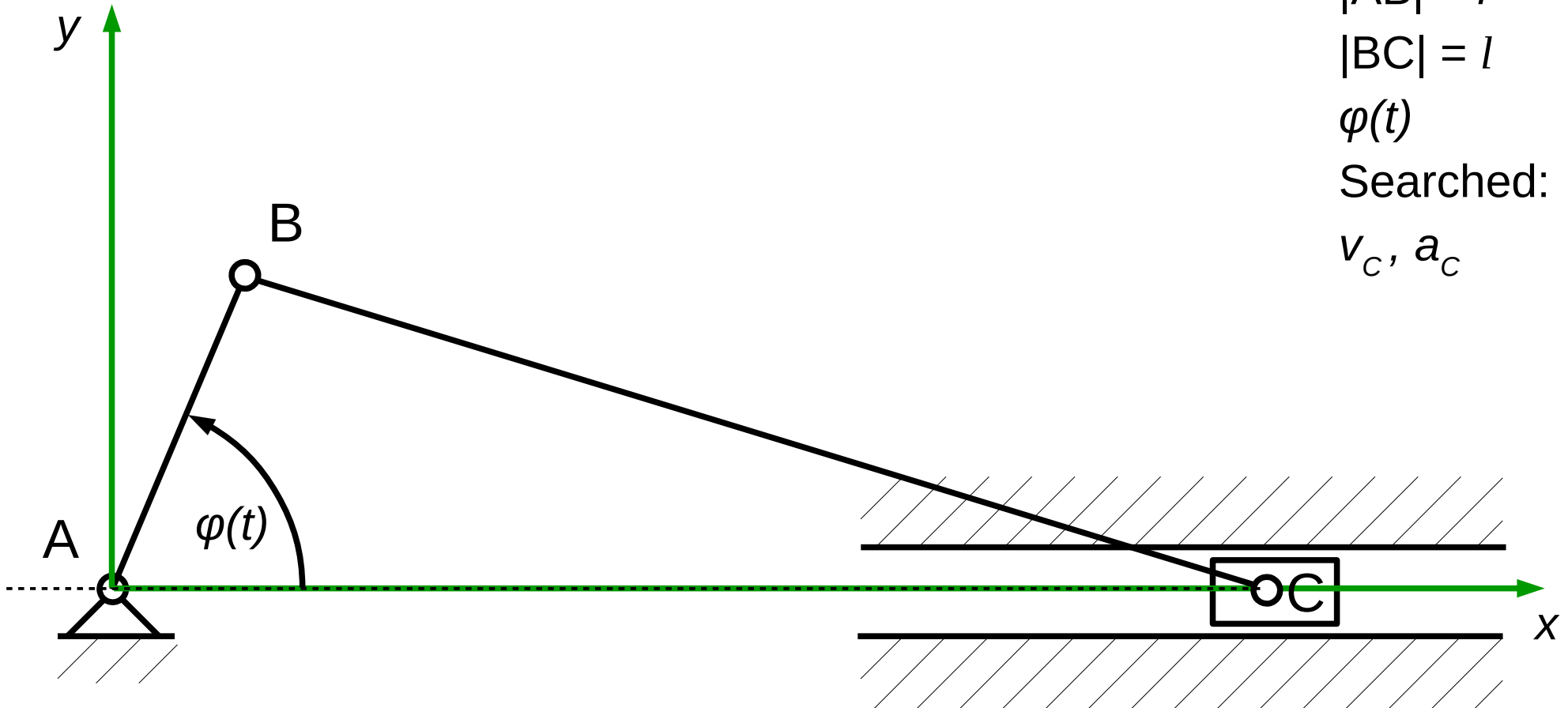
$$|AB| = r$$

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Searched:

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# Analytical method – example: crank-slider mechanism

Given:

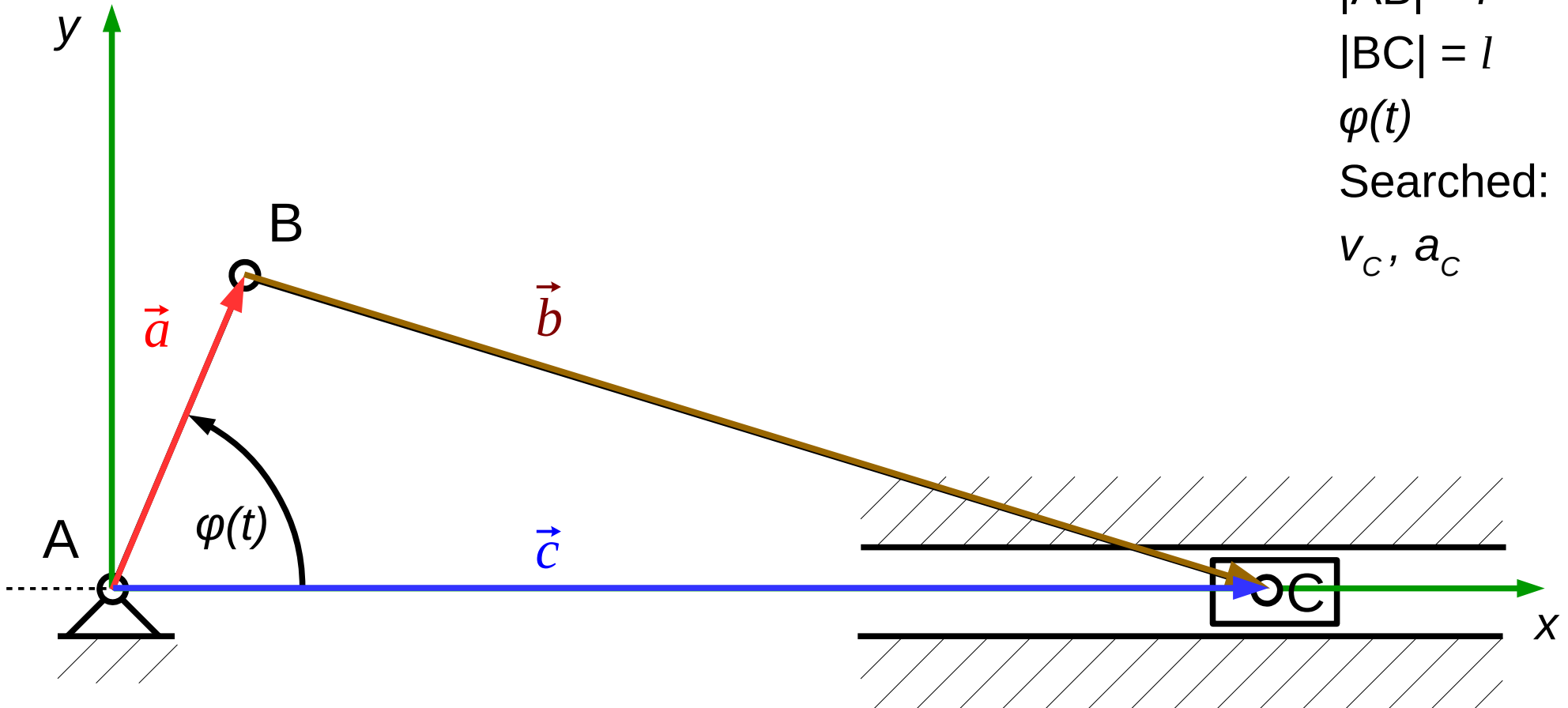
$$|AB| = r$$

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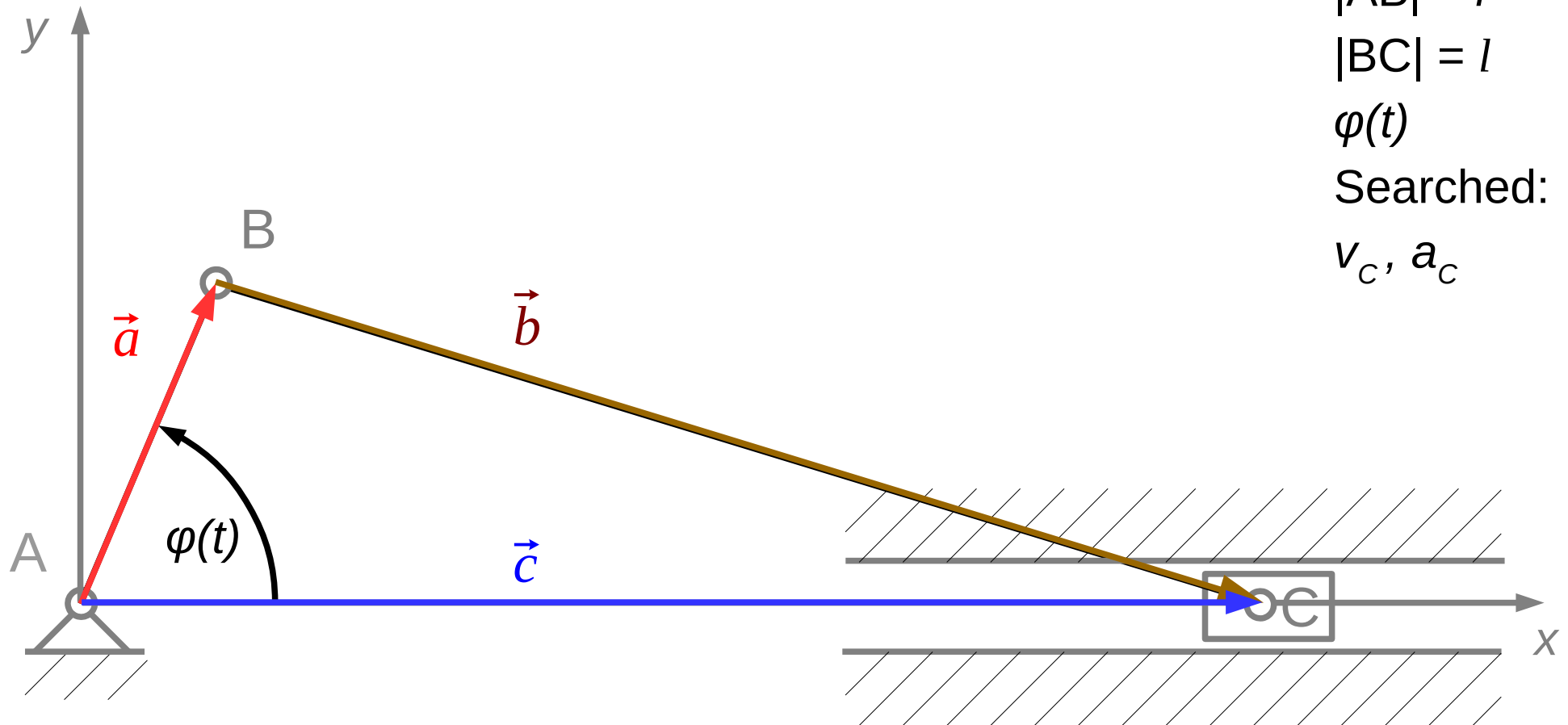
$$|AB| = r$$

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# Analytical method – example: crank-slider mechanism

Given:

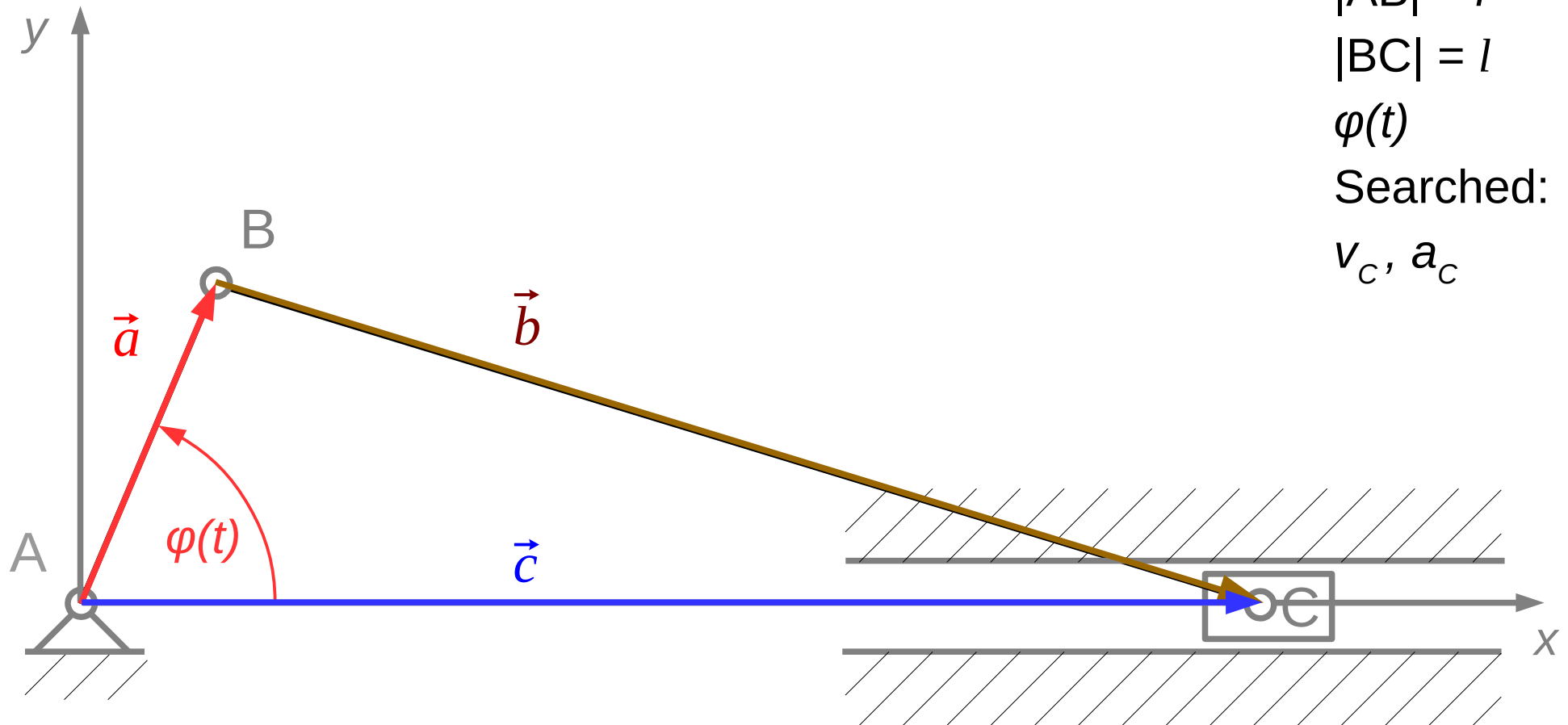
$$|AB| = r$$

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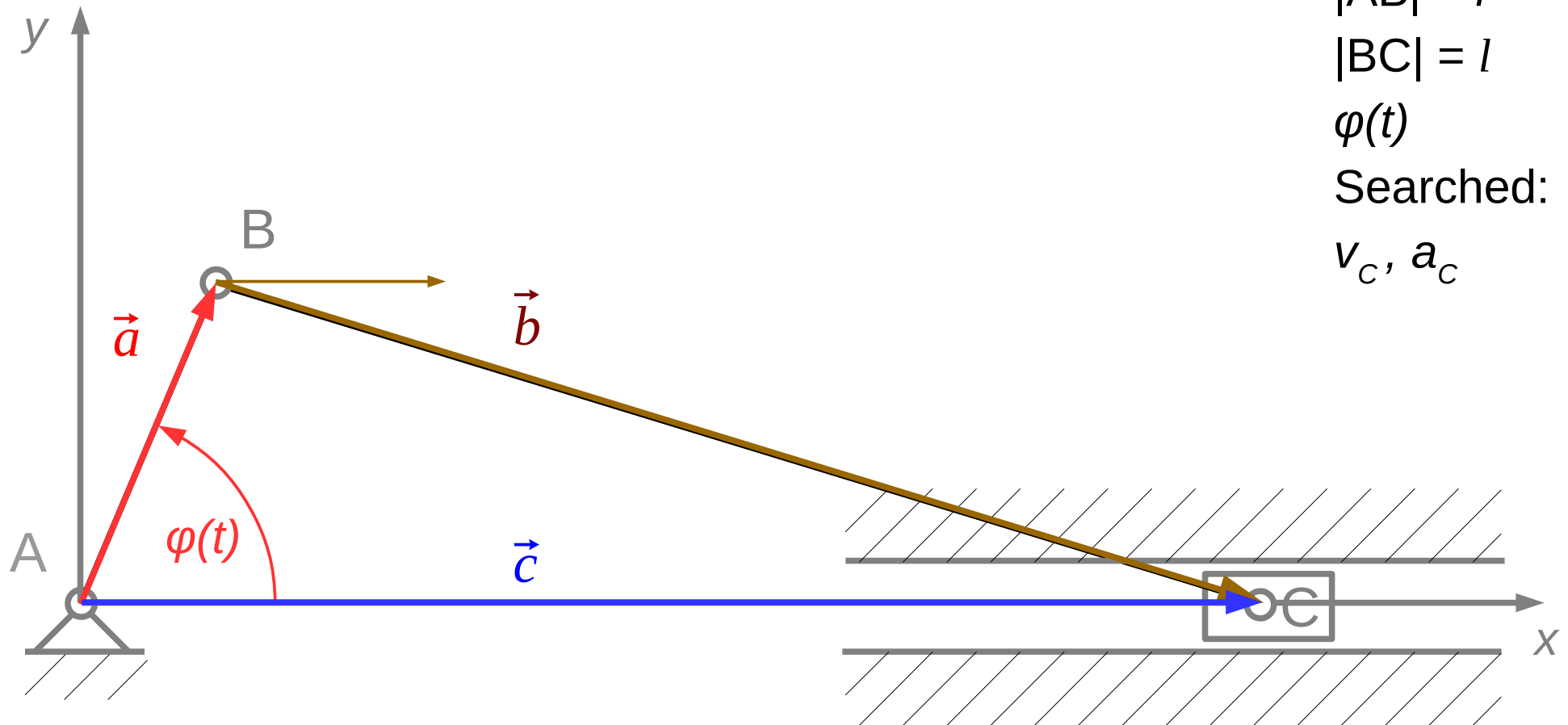
$$\varphi(t)$$

Searched:

$$v_C, a_C$$



# Analytical method – example: crank-slider mechanism



Given:

$$|AB| = r$$

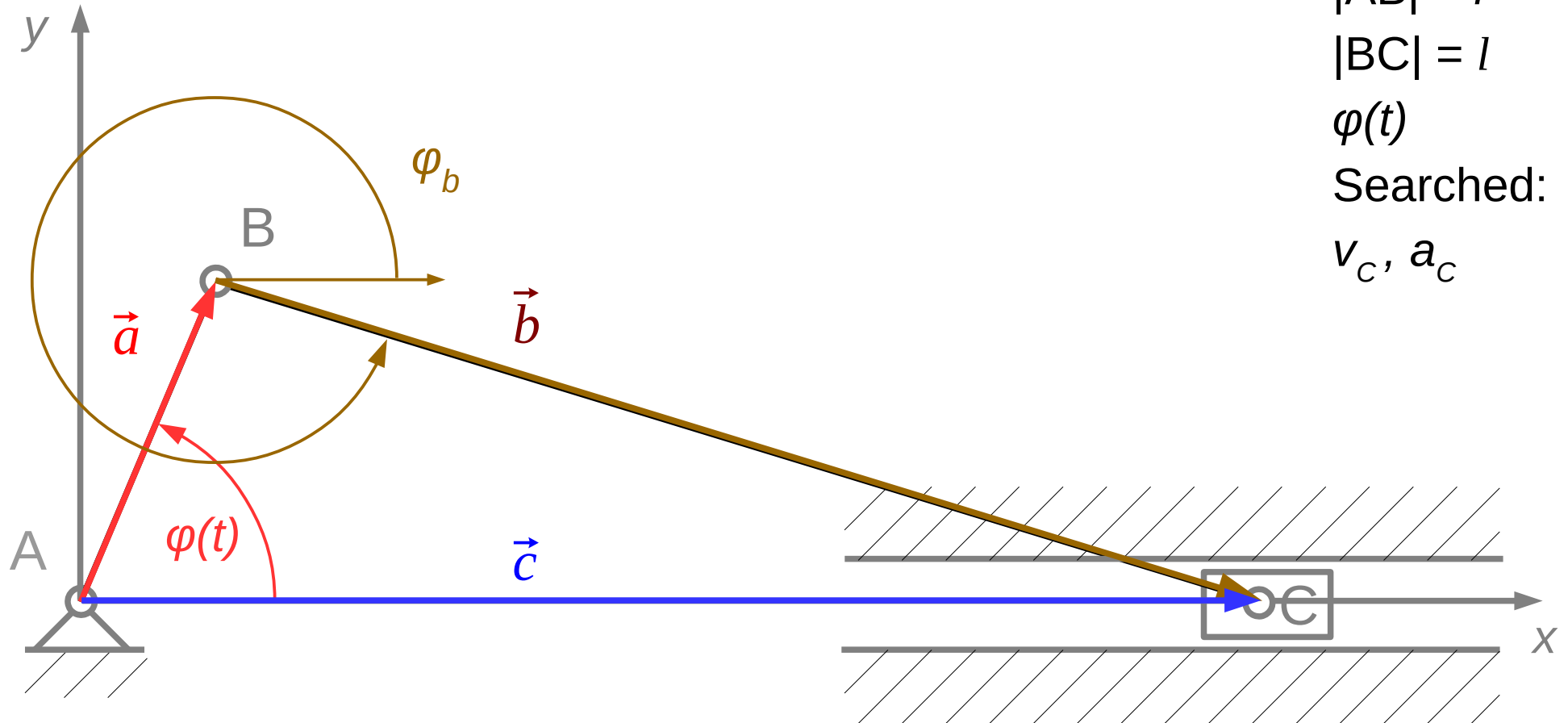
$$|BC| = l$$

$$\varphi(t)$$

Searched:

$$v_C, a_C$$

# Analytical method – example: crank-slider mechanism



Given:

$$|AB| = r$$

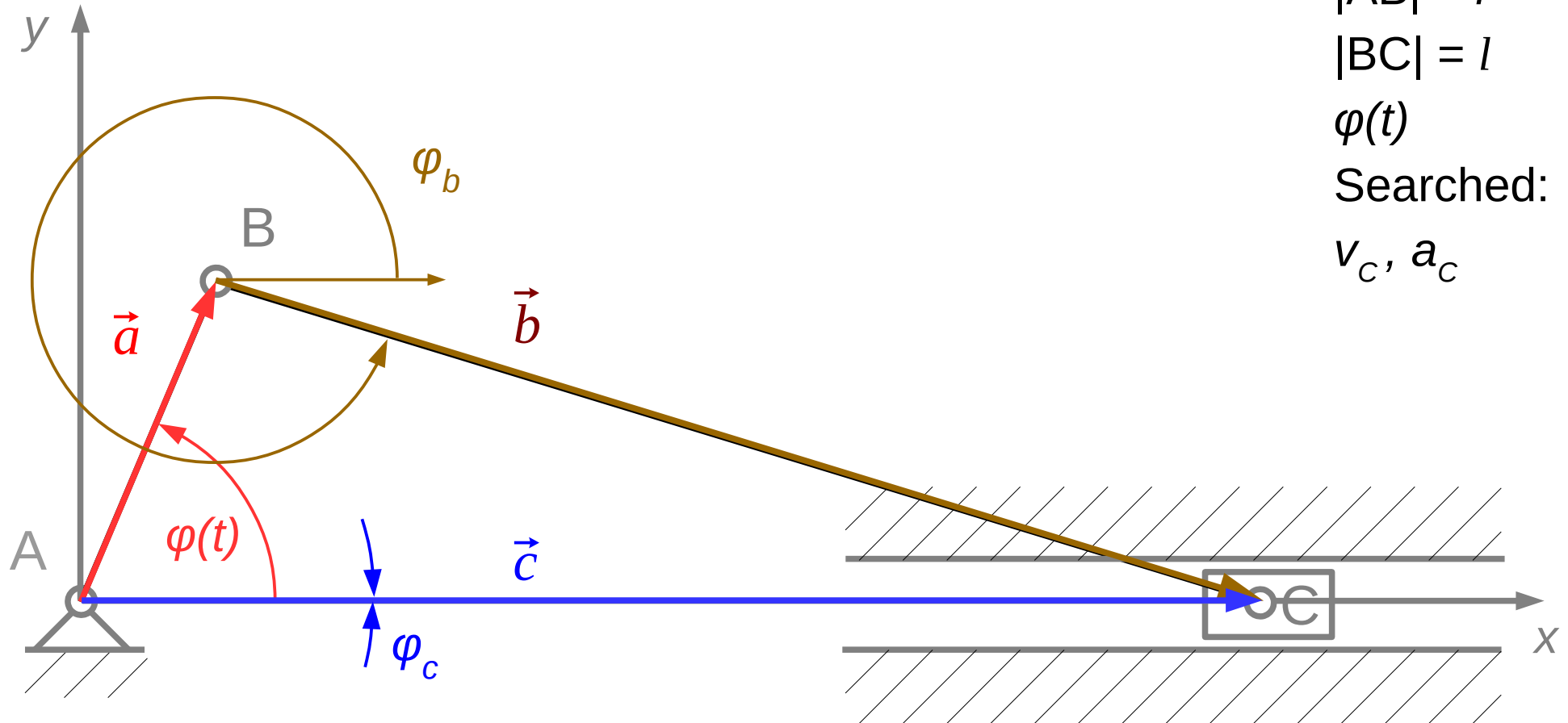
$$|BC| = l$$

$$\varphi(t)$$

Searched:

$$v_C, a_C$$

# Analytical method – example: crank-slider mechanism



Given:

$$|AB| = r$$

$$|BC| = l$$

$$\varphi(t)$$

Searched:

$$v_c, a_c$$

# Analytical method – example: crank-slider mechanism

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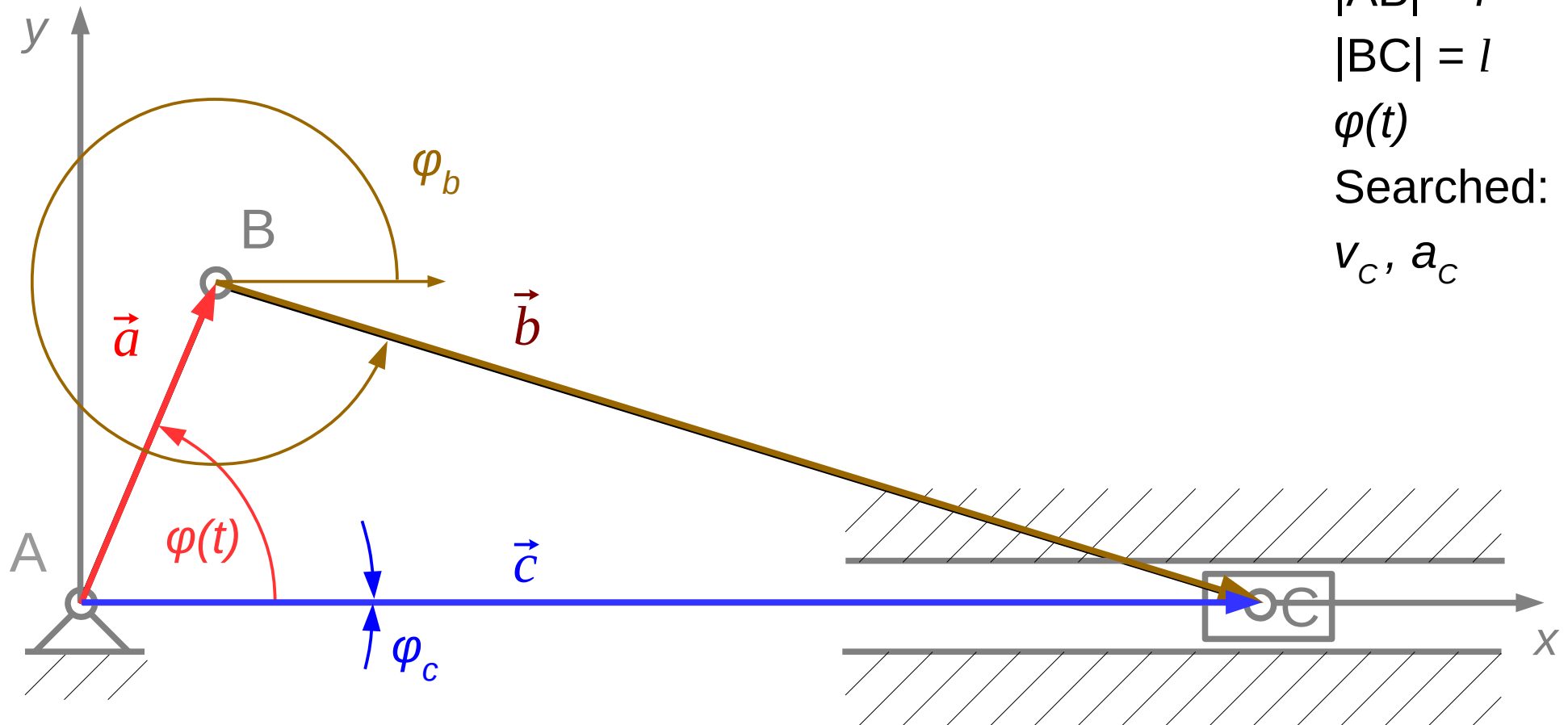
$$|AB| = r$$

$$|BC| = l$$

$$\varphi(t)$$

Searched:

$$v_C, a_C$$

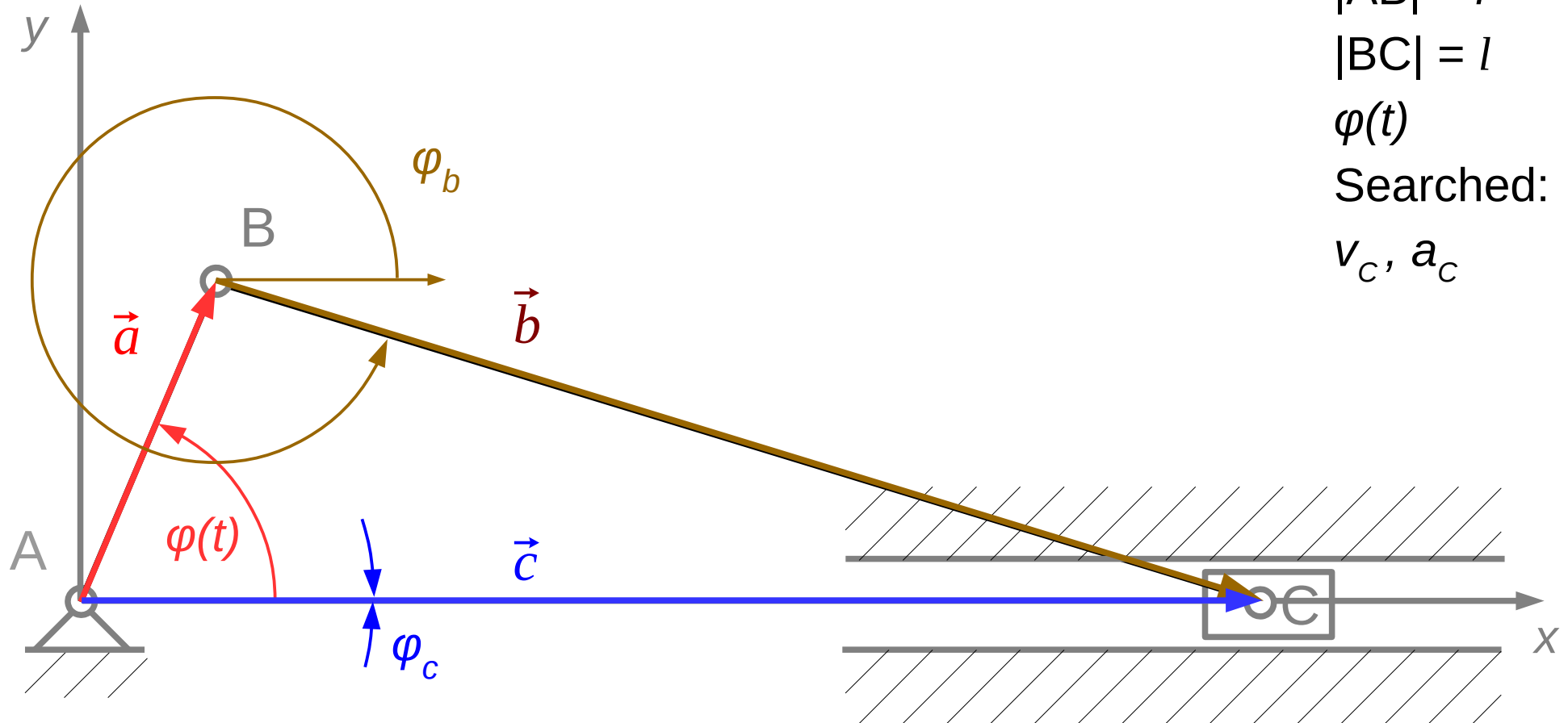


$$|\vec{a}| = r \quad \varphi(t)$$

$$|\vec{b}| = l \quad \varphi_b(t)$$

$$|\vec{c}| = c(t) \quad \varphi_c = 0$$

# Analytical method – example: crank-slider mechanism

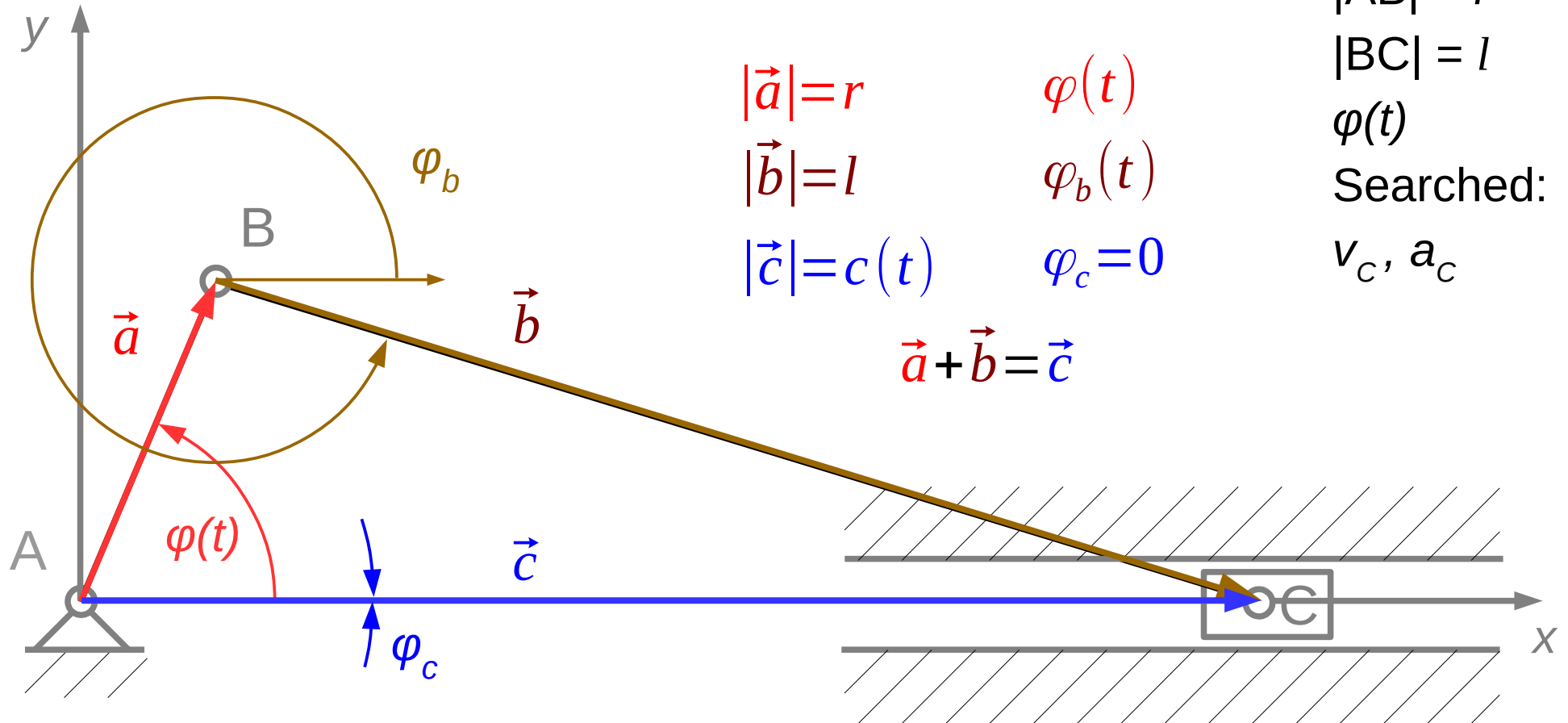


Given:  
 $|AB| = r$   
 $|BC| = l$   
 $\varphi(t)$   
 Searched:  
 $v_c, a_c$

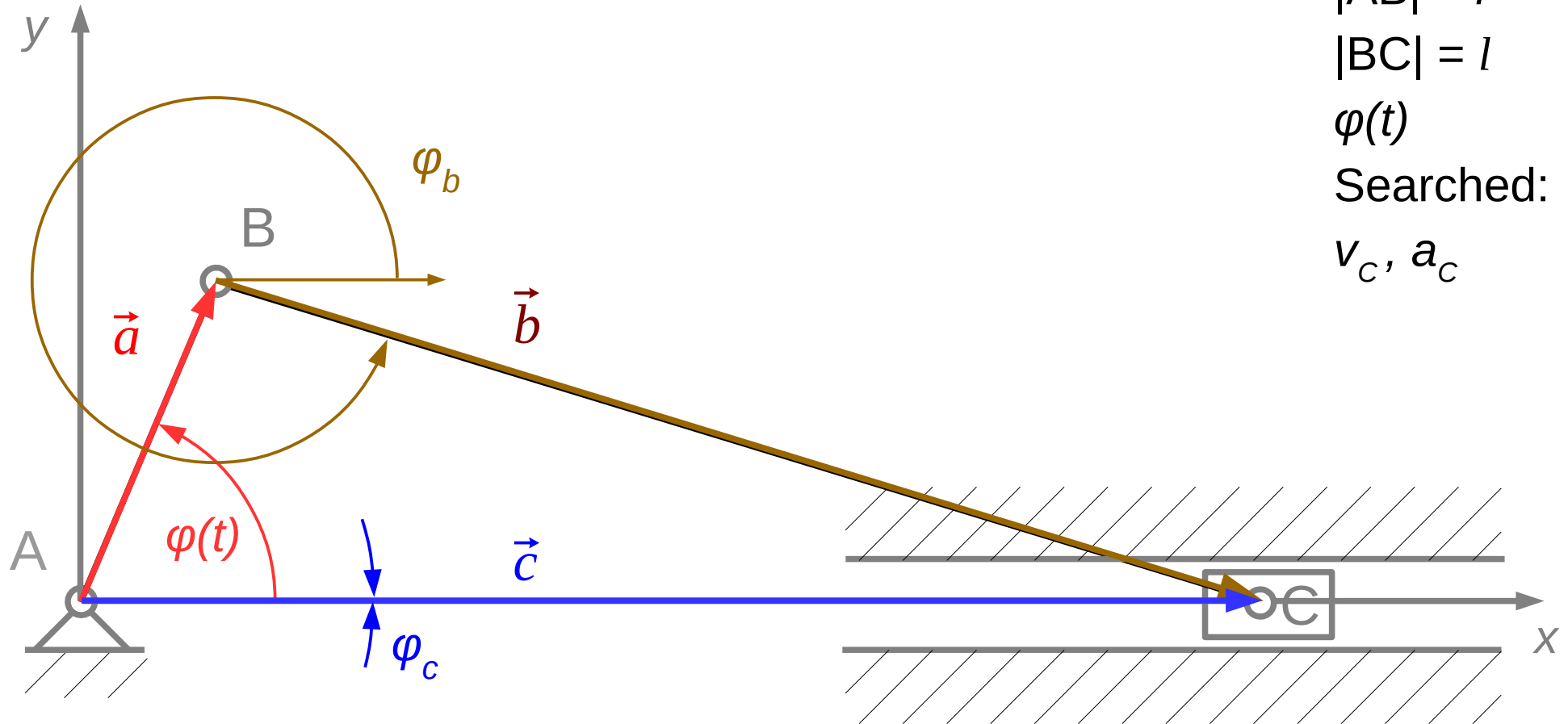
$$\begin{array}{ll}
 |\vec{a}| = r & \varphi(t) \\
 |\vec{b}| = l & \varphi_b(t) \\
 |\vec{c}| = c(t) & \varphi_c = 0
 \end{array}
 \quad \vec{a} + \vec{b} = \vec{c}$$



# Analytical method – example: crank-slider mechanism



# Analytical method – example: crank-slider mechanism



Given:  
 $|AB| = r$   
 $|BC| = l$   
 $\varphi(t)$   
 Searched:  
 $v_c, a_c$

$$|\vec{a}| = r$$

$$\varphi(t)$$

$$|\vec{b}| = l$$

$$\varphi_b(t)$$

$$|\vec{c}| = c(t)$$

$$\varphi_c = 0$$

$$\vec{a} + \vec{b} = \vec{c}$$

$$x: r \cos \varphi(t) + l \cos \varphi_b(t) = c(t) \cos 0$$

$$y: r \sin \varphi(t) + l \sin \varphi_b(t) = c(t) \sin 0$$

# Analytical method – example: crank-slider mechanism

$$r \cos \varphi(t) + l \cos \varphi_b(t) = c(t) \cos 0$$

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*2 unknowns*

## Analytical method – example: crank-slider mechanism

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$$r \sin \varphi(t) + l \sin \varphi_b(t) = 0$$

$$\sin \varphi_b(t) = -\frac{r}{l} \sin \varphi(t) = -\lambda \sin \varphi(t)$$

## Analytical method – example: crank-slider mechanism

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$$\sin^2 \varphi_b(t) + \cos^2 \varphi_b(t) = 1$$

$$\cos \varphi_b(t) = \pm \sqrt{1 - \sin^2 \varphi_b(t)}$$

## Analytical method – example: crank-slider mechanism

$$r \cos \varphi(t) + l \cos \varphi_b(t) = c(t) \cos 0$$

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## Analytical method – example: crank-slider mechanism

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$$c(t) = r \cos \varphi(t) \pm l \sqrt{1 - \lambda^2 \sin^2 \varphi(t)}$$

# Analytical method – example: crank-slider mechanism

$$r \cos \varphi(t) + l \cos \varphi_b(t) = c(t) \cos 0$$

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2 unknowns

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$$\text{for } \varphi(t) = 0$$

$$c(t) = r + l$$

# Analytical method – example: crank-slider mechanism

$$r \cos \varphi(t) + l \cos \varphi_b(t) = c(t) \cos 0$$

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$$c(t) = r \cos \varphi(t) + l \sqrt{1 - \lambda^2 \sin^2 \varphi(t)}$$



for  $\varphi(t) = 0$

$$c(t) = r + l$$

# Analytical method – example: crank-slider mechanism

*slider movement*

$$c(t) = r \cos \varphi(t) + l \sqrt{1 - \lambda^2 \sin^2 \varphi(t)}$$

$$v_C(t) = \frac{dc(t)}{dt} = -r \frac{d\varphi(t)}{dt} \sin \varphi(t) - \frac{-2l\lambda^2 \frac{d\varphi(t)}{dt} \sin \varphi(t) \cos \varphi(t)}{2\sqrt{1 - \lambda^2 \sin^2 \varphi(t)}}$$

$$a_C(t) = \frac{dv_C(t)}{dt} = \dots$$

# Analytical method – example: crank-slider mechanism

*calculations with wxmaxima*

```
(%i14) c: r*cos(%phi(t))+l*sqrt(1-%lambda^2*(sin(%phi(t)))^2);
v: diff(c,t,1);
a: diff(v,t,1);
```

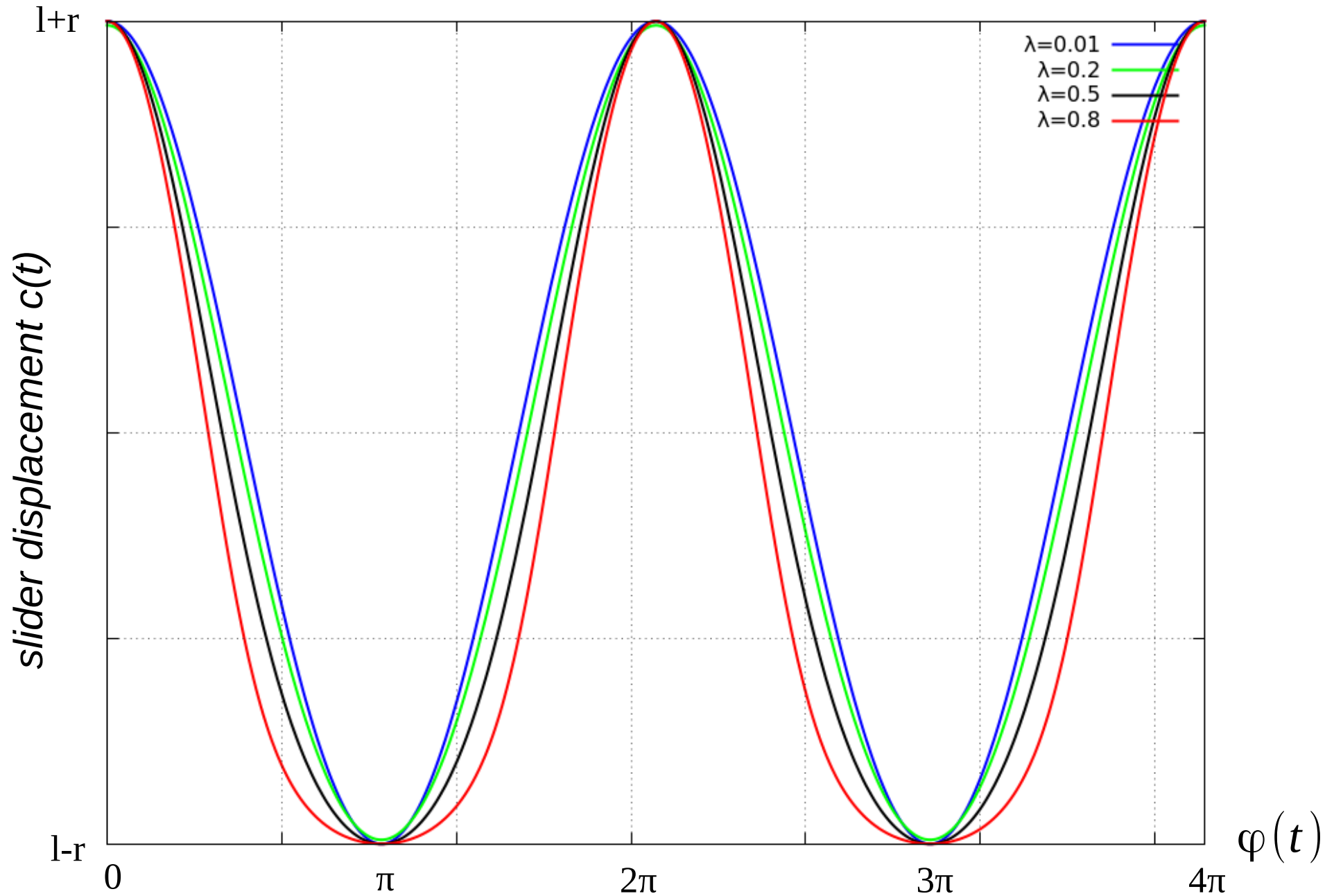
$$(c) \quad l \sqrt{1 - \lambda^2 \sin(\varphi(t))^2} + r \cos(\varphi(t))$$

$$(v) \quad - \frac{\lambda^2 l \cos(\varphi(t)) \sin(\varphi(t)) \left( \frac{d}{dt} \varphi(t) \right)}{\sqrt{1 - \lambda^2 \sin(\varphi(t))^2}} - r \sin(\varphi(t)) \left( \frac{d}{dt} \varphi(t) \right)$$

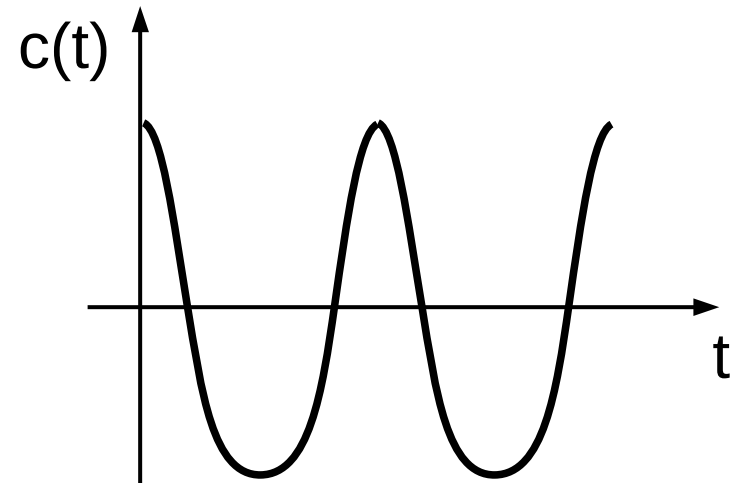
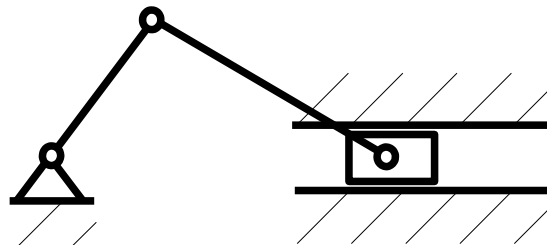
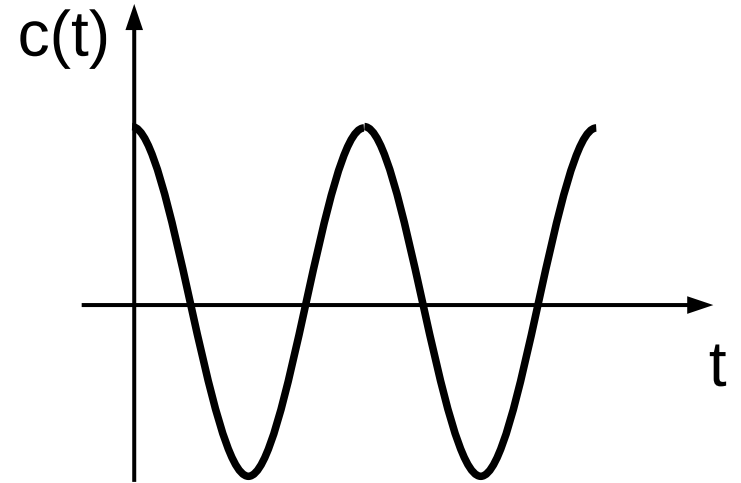
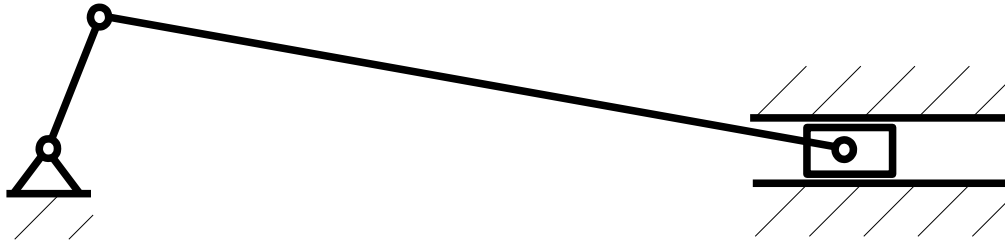
$$(a) \quad - \frac{\lambda^2 l \cos(\varphi(t)) \sin(\varphi(t)) \left( \frac{d^2}{dt^2} \varphi(t) \right)}{\sqrt{1 - \lambda^2 \sin(\varphi(t))^2}} - r \sin(\varphi(t)) \left( \frac{d^2}{dt^2} \varphi(t) \right) + \frac{\lambda^2 l \sin(\varphi(t))^2 \left( \frac{d}{dt} \varphi(t) \right)^2}{\sqrt{1 - \lambda^2 \sin(\varphi(t))^2}}$$

$$\frac{\lambda^2 l \cos(\varphi(t))^2 \left( \frac{d}{dt} \varphi(t) \right)^2}{\sqrt{1 - \lambda^2 \sin(\varphi(t))^2}} - \frac{\lambda^4 l \cos(\varphi(t))^2 \sin(\varphi(t))^2 \left( \frac{d}{dt} \varphi(t) \right)^2}{(1 - \lambda^2 \sin(\varphi(t))^2)^{3/2}} - r \cos(\varphi(t)) \left( \frac{d}{dt} \varphi(t) \right)^2$$

# Analytical method – example: crank-slider mechanism



# Analytical method – example: crank-slider mechanism



Interesting reading: <http://www.enginebuildermag.com/2016/08/understanding-rod-ratios/>

# Analytical method – example: crank-slider mechanism

*connecting rod motion*

$$\varphi_b(t) = -\arcsin(\lambda \sin \varphi(t))$$

$$\omega_b(t) = \frac{d\varphi_b(t)}{dt} = \frac{-\lambda \dot{\varphi}(t) \cos \varphi(t)}{\sqrt{1 - \lambda^2 \sin^2 \varphi(t)}}$$

$$\varepsilon_b(t) = \frac{d\omega_b(t)}{dt} = -\frac{\lambda \cos(\varphi(t)) \left( \frac{d^2}{dt^2} \varphi(t) \right)}{\sqrt{1 - \lambda^2 \sin(\varphi(t))^2}} + \frac{\lambda \sin(\varphi(t)) \left( \frac{d}{dt} \varphi(t) \right)^2}{\sqrt{1 - \lambda^2 \sin(\varphi(t))^2}} - \frac{\lambda^3 \cos(\varphi(t))^2 \sin(\varphi(t)) \left( \frac{d}{dt} \varphi(t) \right)^2}{\left( 1 - \lambda^2 \sin(\varphi(t))^2 \right)^{3/2}}$$



# Analytical method – example: slider-yoke

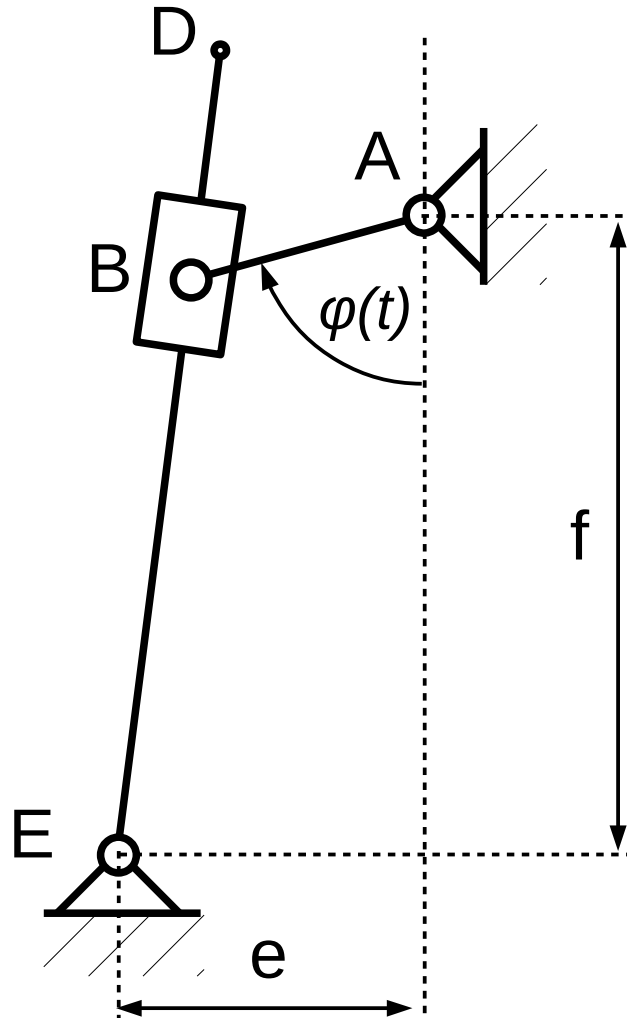
Given:

$$|AB| = r$$

$e, f, \varphi(t)$

Searched:

*angular velocity*  
 $\omega_2$  *and*  
*acceleration*  $\varepsilon_2$   
*of ED element*



# Analytical method – example: slider-yoke

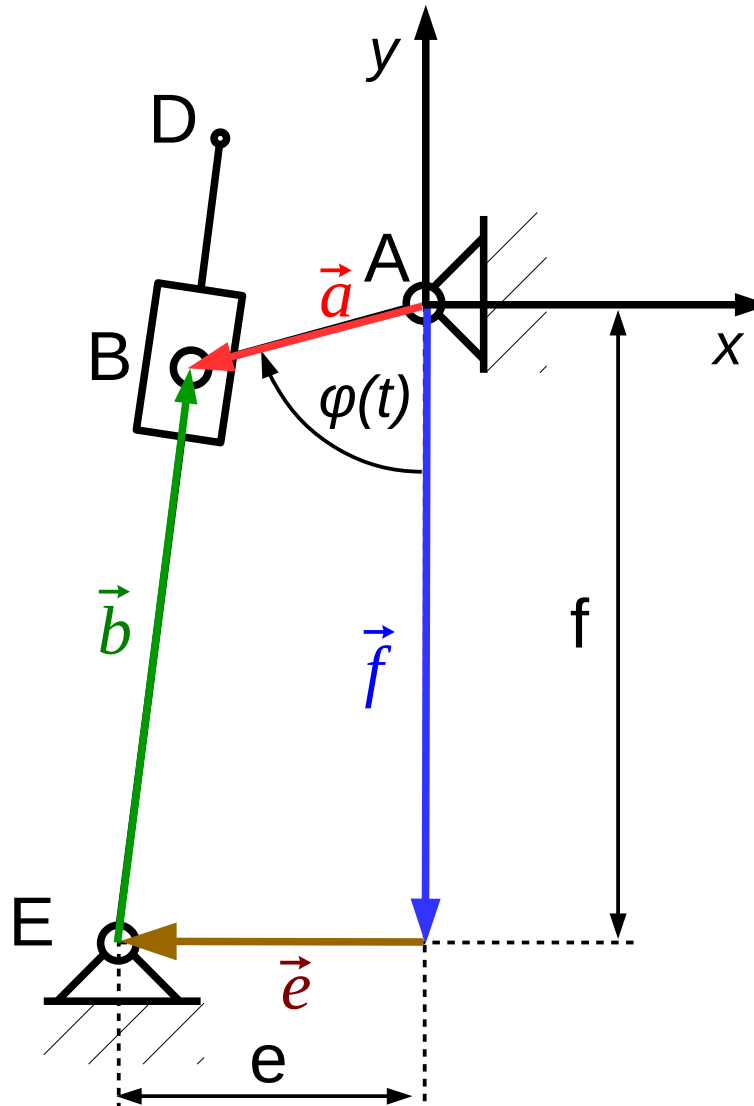
Given:

$$|AB| = r$$

$e, f, \varphi(t)$

Searched:

*angular velocity*  
 $\omega_2$  *and*  
*acceleration*  $\varepsilon_2$   
*of ED element*



# Analytical method – example: slider-yoke

Given:

$$|AB| = r$$

$e, f, \varphi(t)$

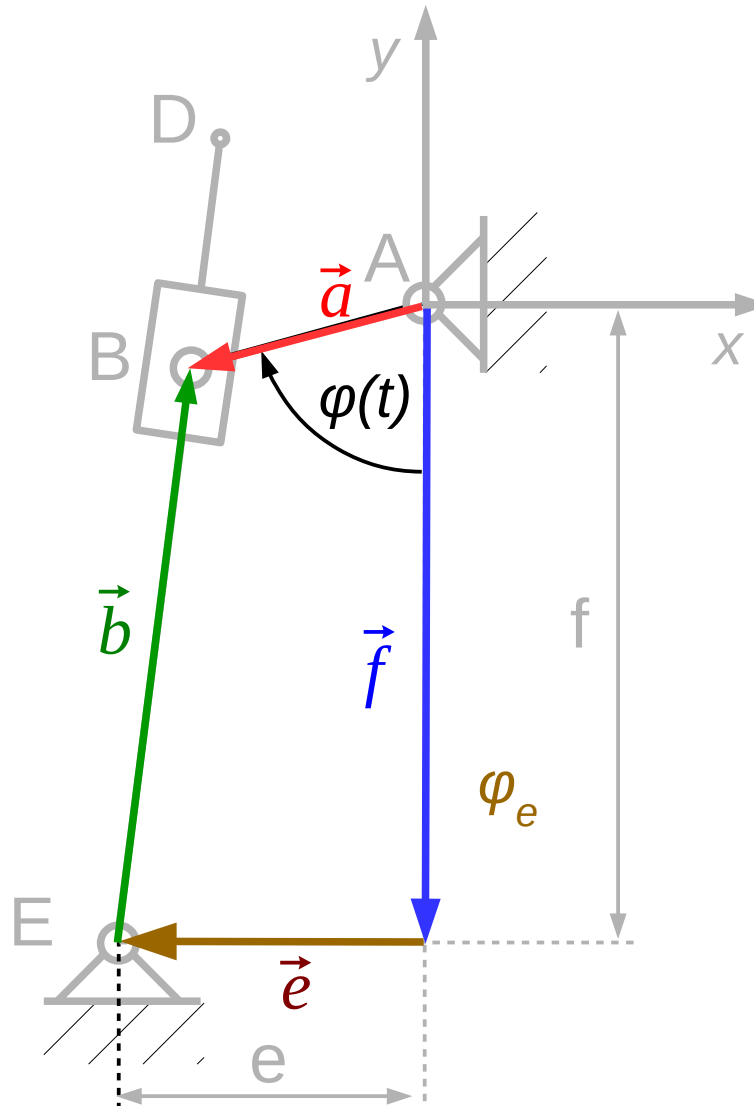
Searched:

*angular velocity*

$\omega_2$  *and*

*acceleration*  $\varepsilon_2$

*of ED element*



# Analytical method – example: slider-yoke

Given:

$$|AB| = r$$

$e, f, \varphi(t)$

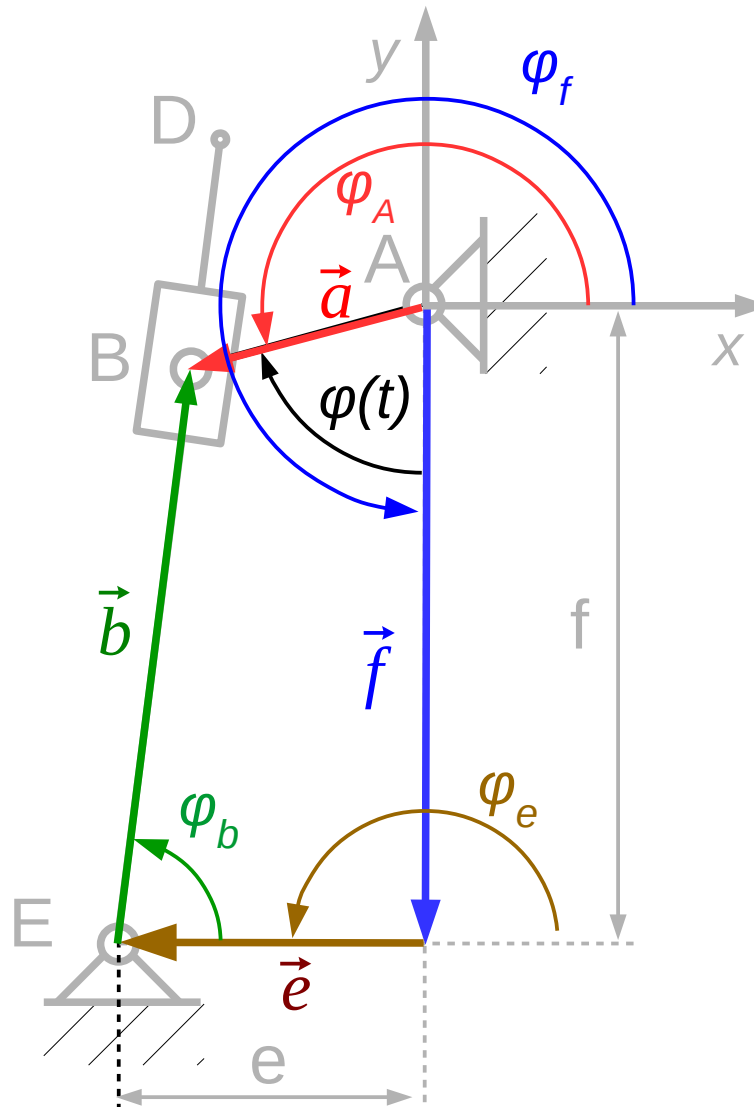
Searched:

*angular velocity*

$\omega_2$  *and*

*acceleration*  $\varepsilon_2$

*of ED element*



# Analytical method – example: slider-yoke

Given:

$$|AB| = r$$

$e, f, \varphi(t)$

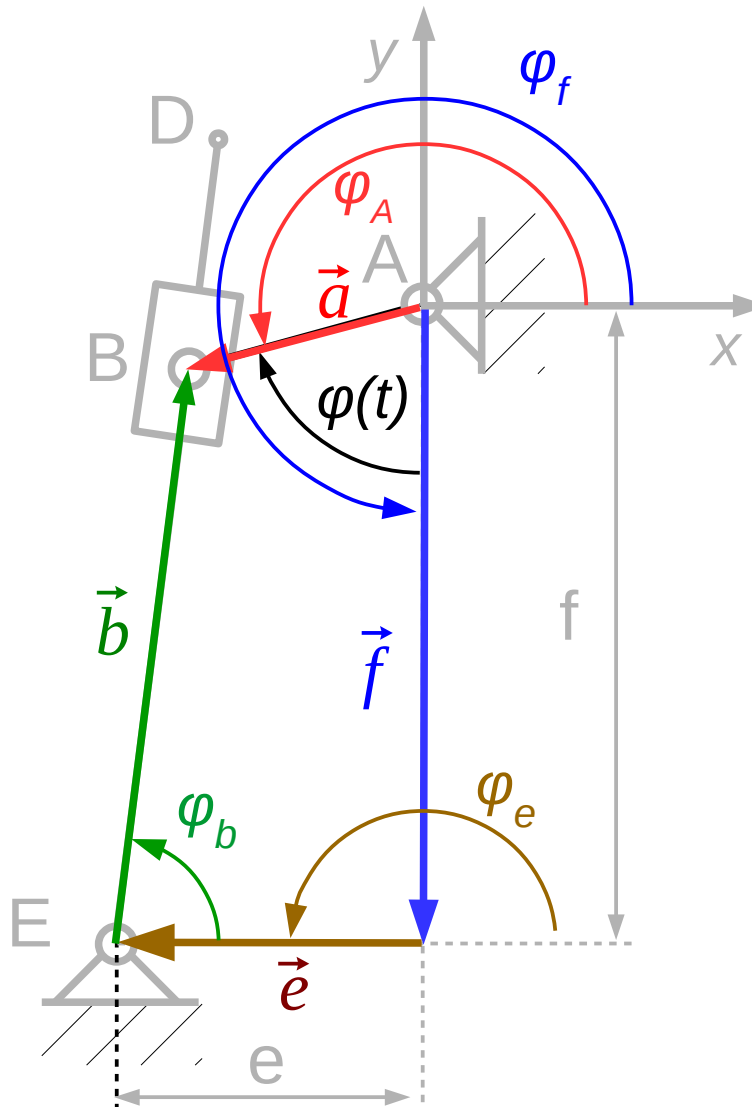
Searched:

angular velocity

$\omega_2$  and

acceleration  $\varepsilon_2$

of ED element



$$|\vec{a}| = r$$

$$\varphi_a(t) = 270^\circ - \varphi(t)$$

$$|\vec{b}| = b(t)$$

$$\varphi_b(t)$$

$$|\vec{e}| = e$$

$$\varphi_e = 180^\circ$$

$$|\vec{f}| = f$$

$$\varphi_f = 270^\circ$$

# Analytical method – example: slider-yoke

Given:

$$|AB| = r$$

$e, f, \varphi(t)$

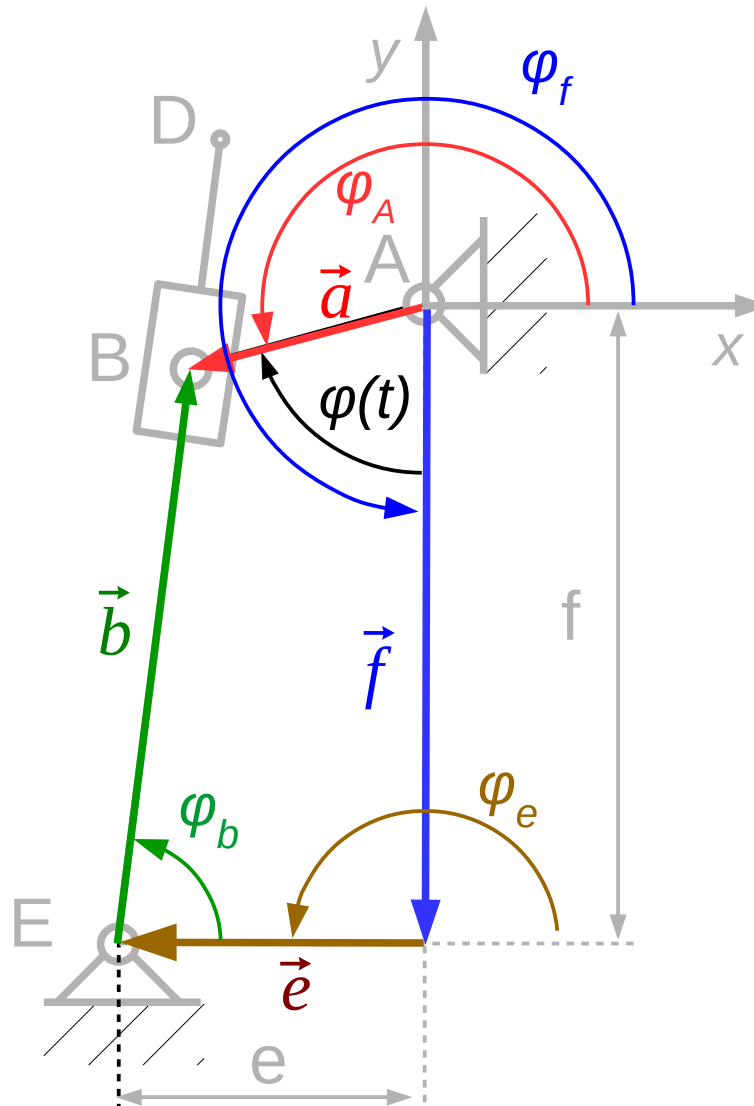
Searched:

angular velocity

$\omega_2$  and

acceleration  $\varepsilon_2$

of ED element



$$|\vec{a}| = r$$

$$\varphi_a(t) = 270^\circ - \varphi(t)$$

$$|\vec{b}| = b(t)$$

$$\varphi_b(t)$$

$$|\vec{e}| = e$$

$$\varphi_e = 180^\circ$$

$$|\vec{f}| = f$$

$$\varphi_f = 270^\circ$$

$$\vec{a} = \vec{b} + \vec{e} + \vec{f}$$

# Analytical method – example: slider-yoke

Given:

$$|AB| = r$$

$e, f, \varphi(t)$

Searched:

*angular velocity*

$\omega_2$  and

*acceleration  $\varepsilon_2$*

*of ED element*

$$|\vec{a}| = r$$

$$\varphi_a(t) = 270^\circ - \varphi(t)$$

$$|\vec{b}| = b(t)$$

$$\varphi_b(t)$$

$$|\vec{e}| = e$$

$$\varphi_e = 180^\circ$$

$$|\vec{f}| = f$$

$$\varphi_f = 270^\circ$$

$$\vec{a} = \vec{b} + \vec{e} + \vec{f}$$

$$x: r \cos(270^\circ - \varphi(t)) = b(t) \cos \varphi_b(t) + e \cos 180^\circ + f \cos 270^\circ$$

$$y: r \sin(270^\circ - \varphi(t)) = b(t) \sin \varphi_b(t) + e \sin 180^\circ + f \sin 270^\circ$$

# Analytical method – example: slider-yoke

Given:

$$|AB| = r$$

$e, f, \varphi(t)$

Searched:

*angular velocity*

$\omega_2$  and

*acceleration  $\varepsilon_2$*

*of ED element*

$$|\vec{a}| = r$$

$$\varphi_a(t) = 270^\circ - \varphi(t)$$

$$|\vec{b}| = b(t)$$

$$\varphi_b(t)$$

$$|\vec{e}| = e$$

$$\varphi_e = 180^\circ$$

$$|\vec{f}| = f$$

$$\varphi_f = 270^\circ$$

$$\vec{a} = \vec{b} + \vec{e} + \vec{f}$$

$$x: r \cos(270^\circ - \varphi(t)) = b(t) \cos \varphi_b(t) + e \cos 180^\circ + f \cos 270^\circ$$

$$y: r \sin(270^\circ - \varphi(t)) = b(t) \sin \varphi_b(t) + e \sin 180^\circ + f \sin 270^\circ$$

$$x: -r \sin \varphi(t) = b(t) \cos \varphi_b(t) - e$$

$$y: -r \cos \varphi(t) = b(t) \sin \varphi_b(t) - f$$



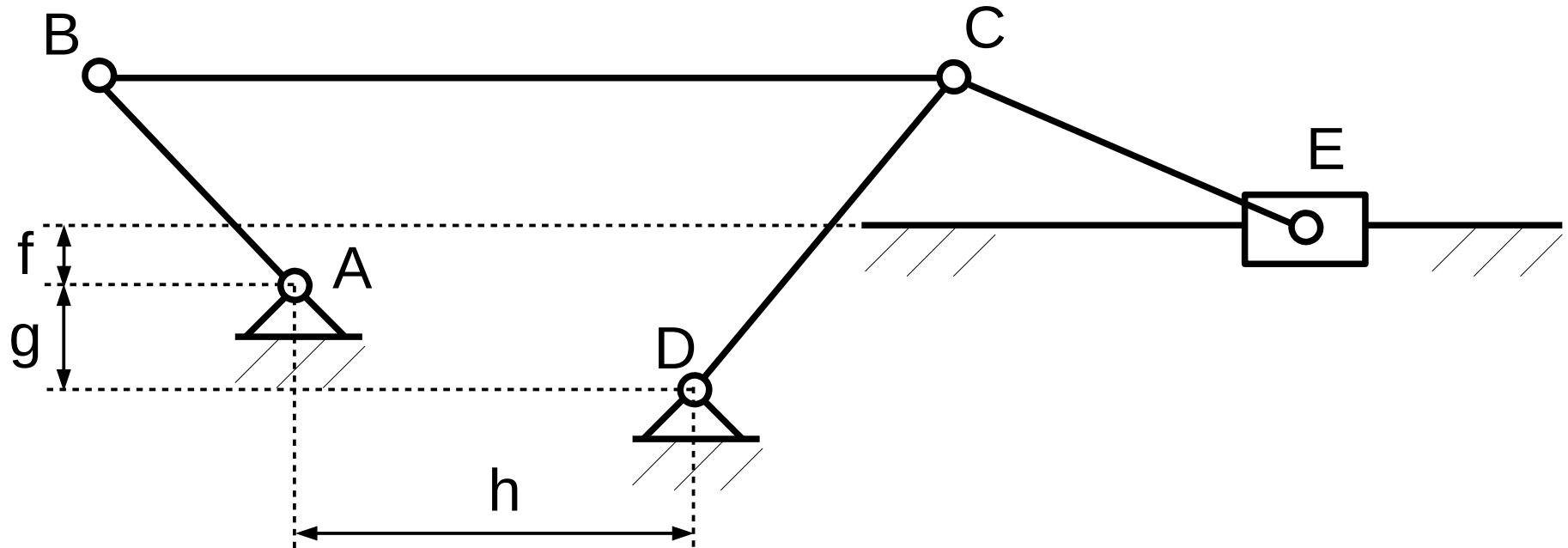
## Analytical method – example: slider-yoke

$$\begin{aligned} e - r \sin \varphi(t) &= b(t) \cos \varphi_b(t) \\ f - r \cos \varphi(t) &= b(t) \sin \varphi_b(t) \end{aligned}$$

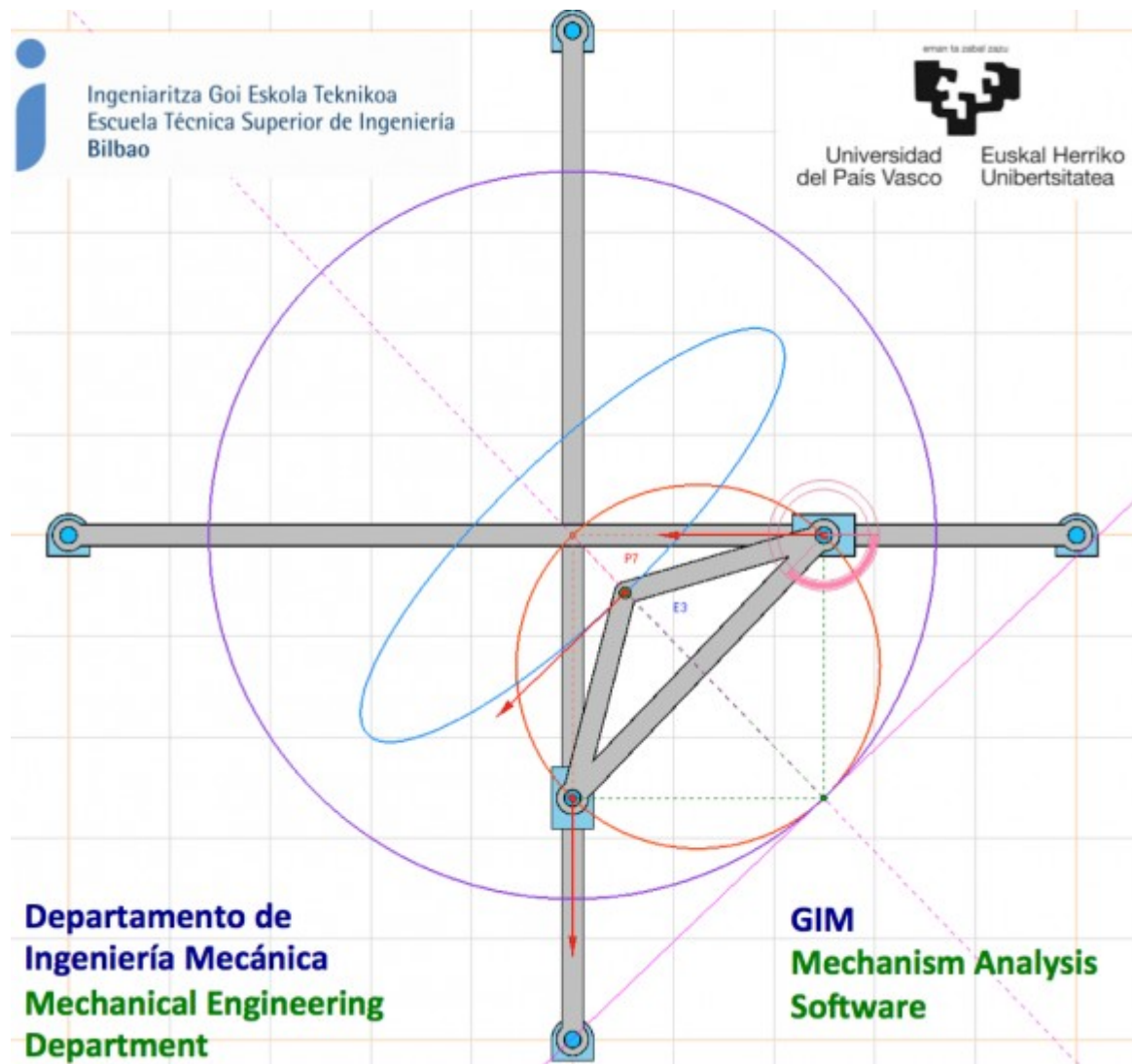
## Analytical method – example: slider-yoke

$$\begin{aligned} e - r \sin \varphi(t) &= b(t) \cos \varphi_b(t) \\ f - r \cos \varphi(t) &= b(t) \sin \varphi_b(t) \end{aligned}$$

# Analytical method – example



# Software

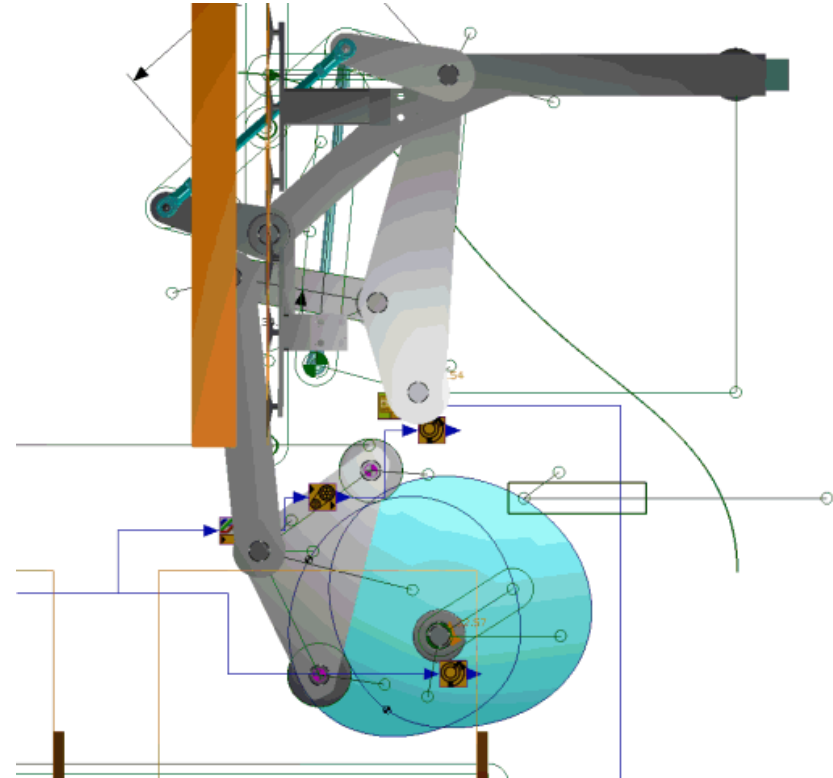
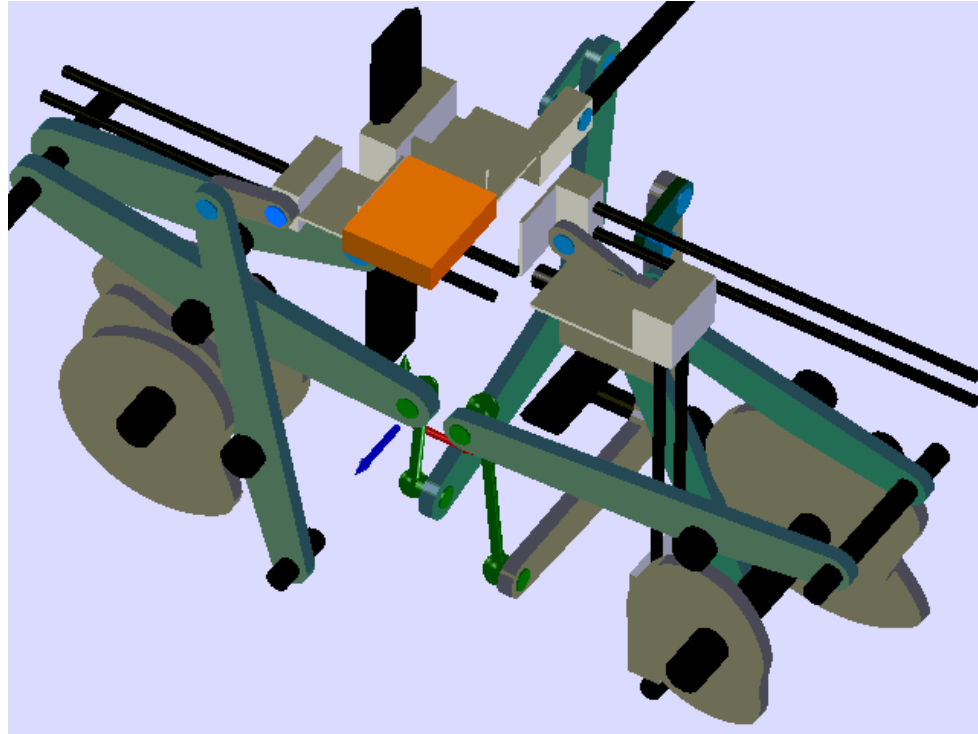


<http://www.ehu.es/compmech/software/>

# Cam-follower mechanisms

# Cam-follower

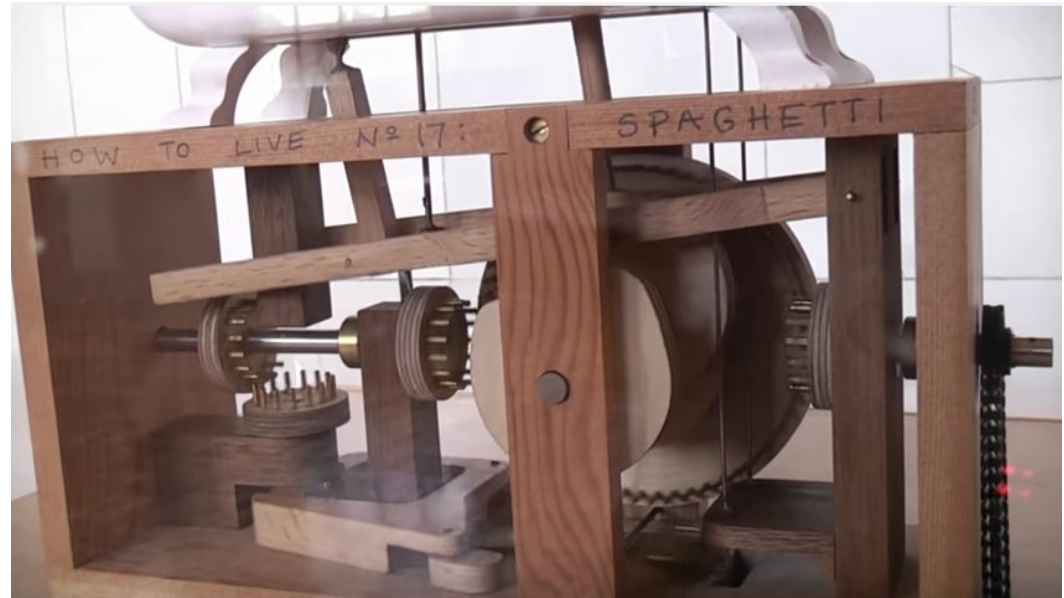
## Inspirations



*source: psmotion.com*

# Cam-follower

## art inspiration



Mechanics Alive! Cabaret Mechanical  
Theatre Automata Exhibition  
<https://www.youtube.com/watch?v=kv1CpJi60xQ>

The "Draughtsman-Writer" automaton by Henri Maillardet  
[https://en.wikipedia.org/wiki/File:Maillardet  
%27s\\_automaton\\_at\\_the\\_Franklin\\_Institute.webm](https://en.wikipedia.org/wiki/File:Maillardet_%27s_automaton_at_the_Franklin_Institute.webm)

# Cam-follower

Cam-follower mechanism – mechanism build of a cam and a follower (tappet) connected as a IV class kinematic pair.

Cam is rotating (sometimes is translating)

follower is reciprocating (sometimes is swinging/oscillating)

## advantages

- simple to construction,
- simple to create,
- any dimensions,
- simple to create advanced motions.

## disadvantages

- no adaptation possible.



# Cam-follower

## Classification

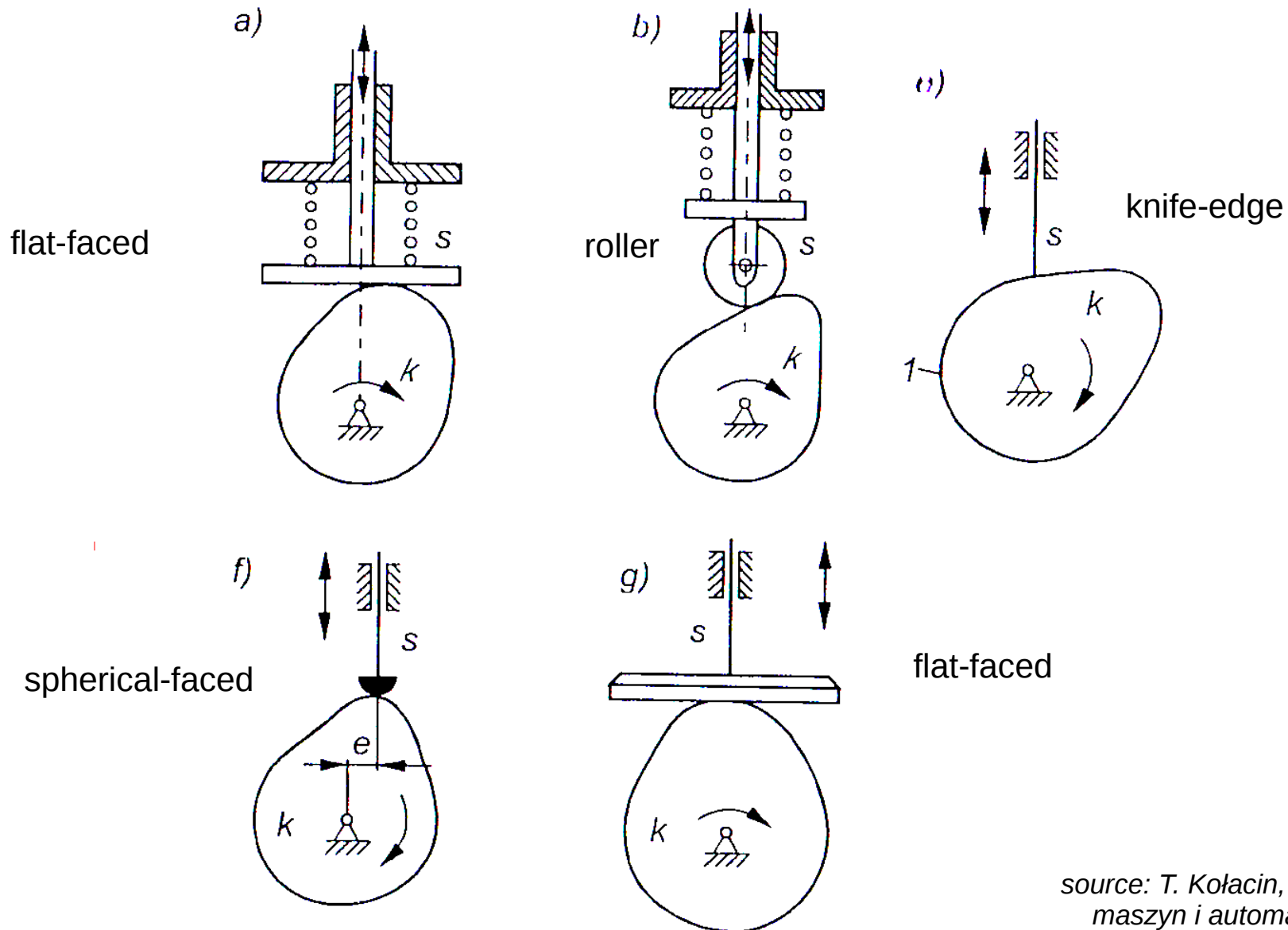
flat / spatial

with in-line (central) follower / with offset (eccentric) follower

closed with geometry / closed with force

# Cam-follower

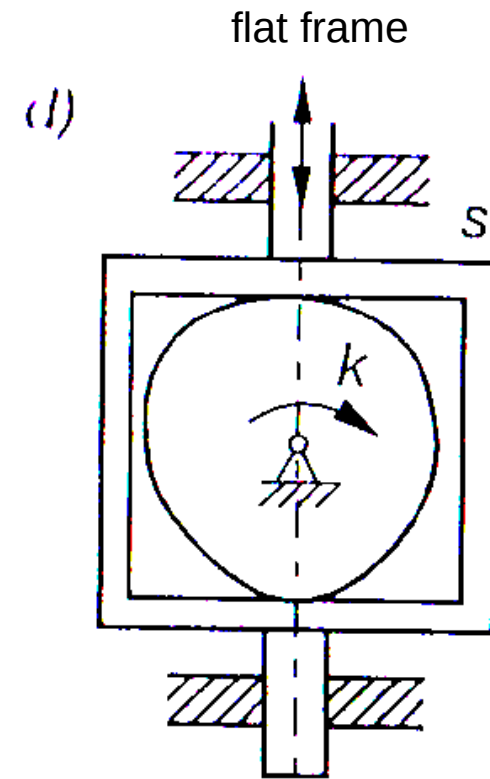
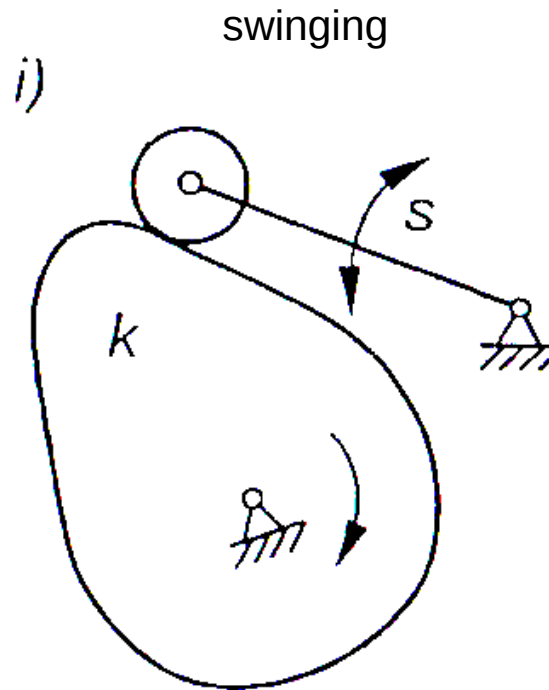
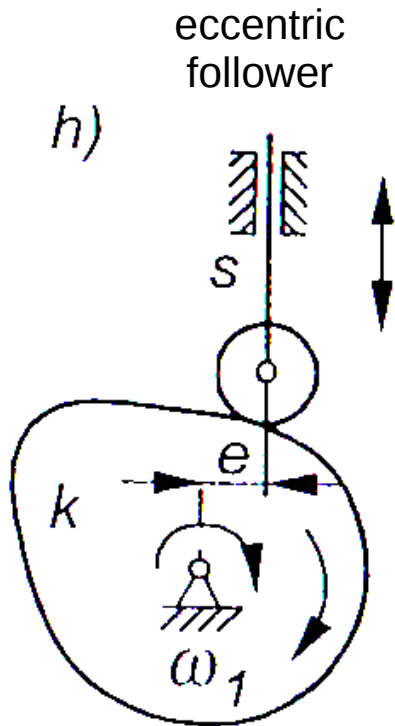
## Followers



source: T. Kołacin, „Podstawy teorii maszyn i automatyki”, OW PW

# Cam-follower

## Followers

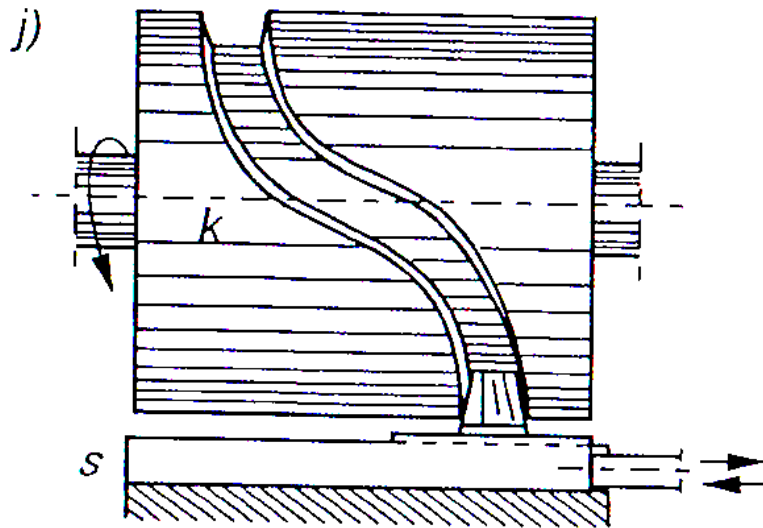


source: T. Kołacin, „Podstawy teorii maszyn i automatyki”, OW PW

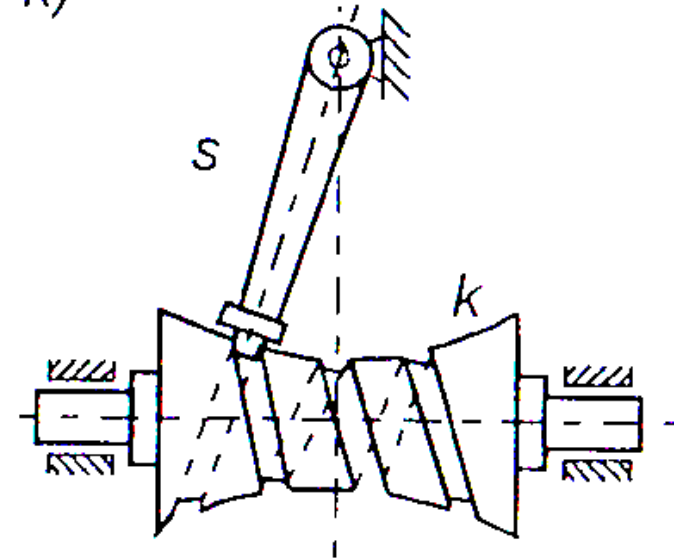
# Cam-follower

## examples

cylindrical cam

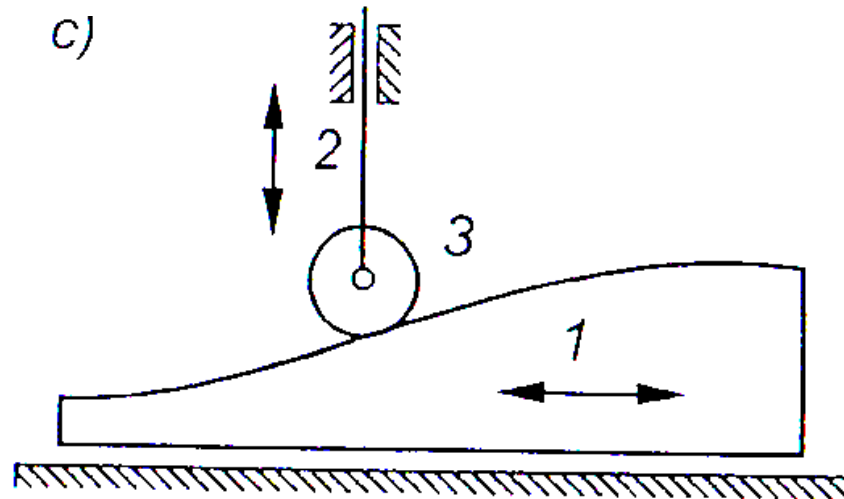


k)



globoidal

translating cam



source: T. Kołacin, „Podstawy teorii maszyn i automatyki”, OW PW

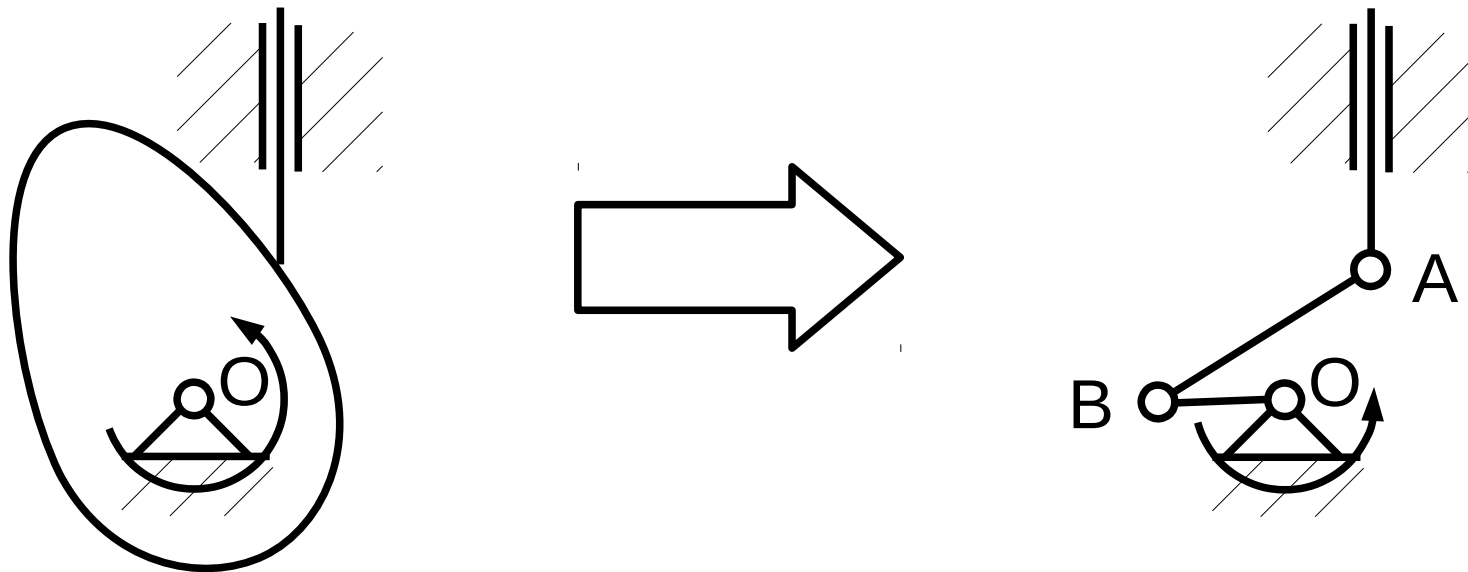
# Analysis and synthesis of cam-follower mechanism

Analysis – calculation of displacement, velocity and acceleration functions for a follower motion with respect to a cam's rotations angle for arbitrary given geometry.

Synthesis – calculation of a cam geometry needed to obtain given displacement/velocity/acceleration functions. Limitations must be included, i.e. some maximum values, geometry limitations and jerk values (third derivative).

# Analysis of a cam-follower mechanism

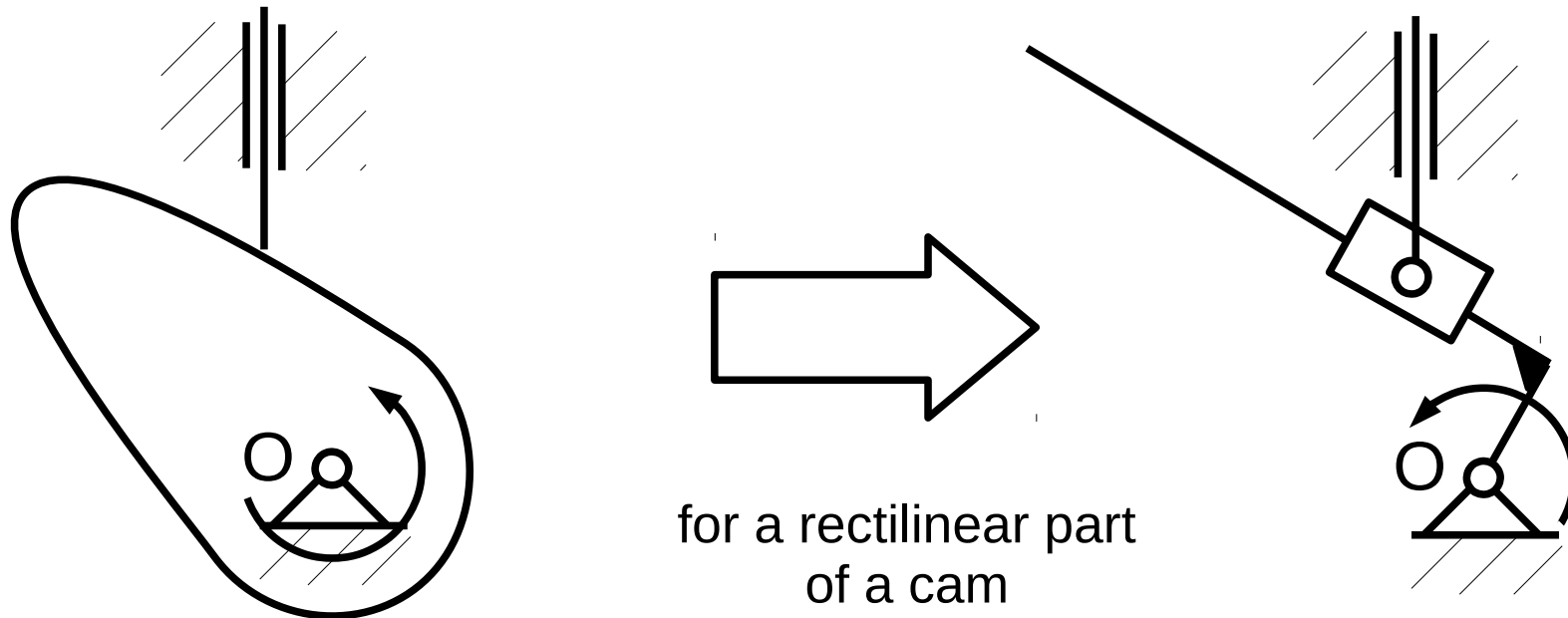
Graphical method with IV. class kinematic pair substitution with V. class kinematic pairs.



AB – radius of curvature in the contact point

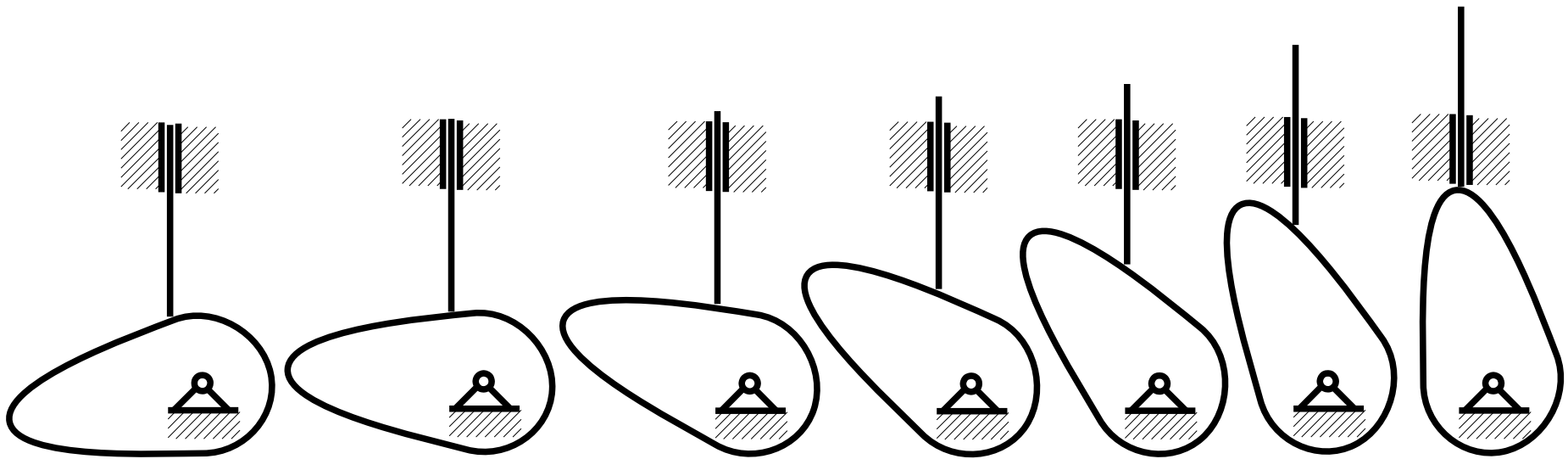
# Analysis of a cam-follower mechanism

Graphical method with IV. class kinematic pair substitution with V. class kinematic pairs.



# Analysis of a cam-follower mechanism

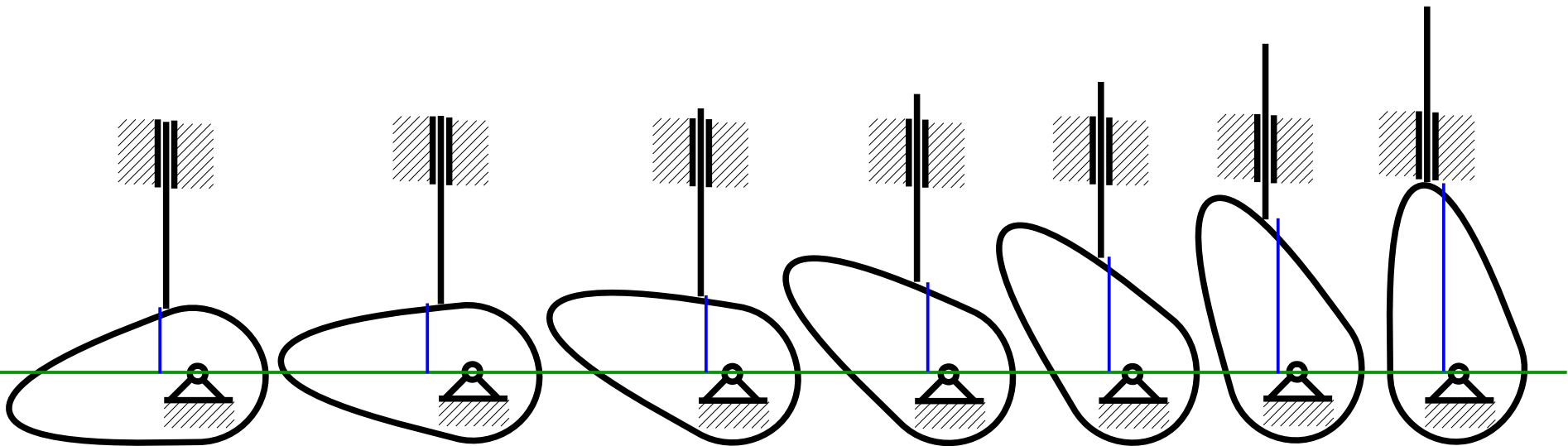
## Graphical determination of a follower movement





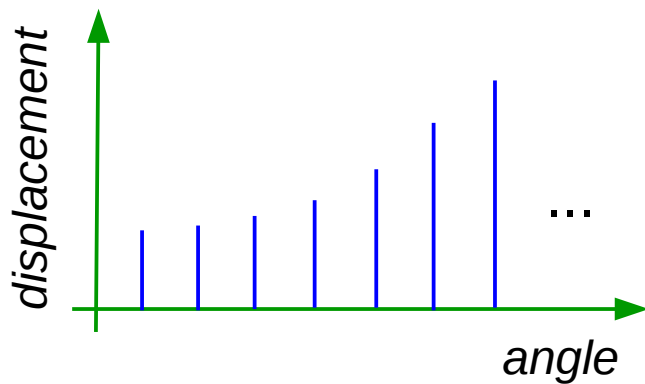
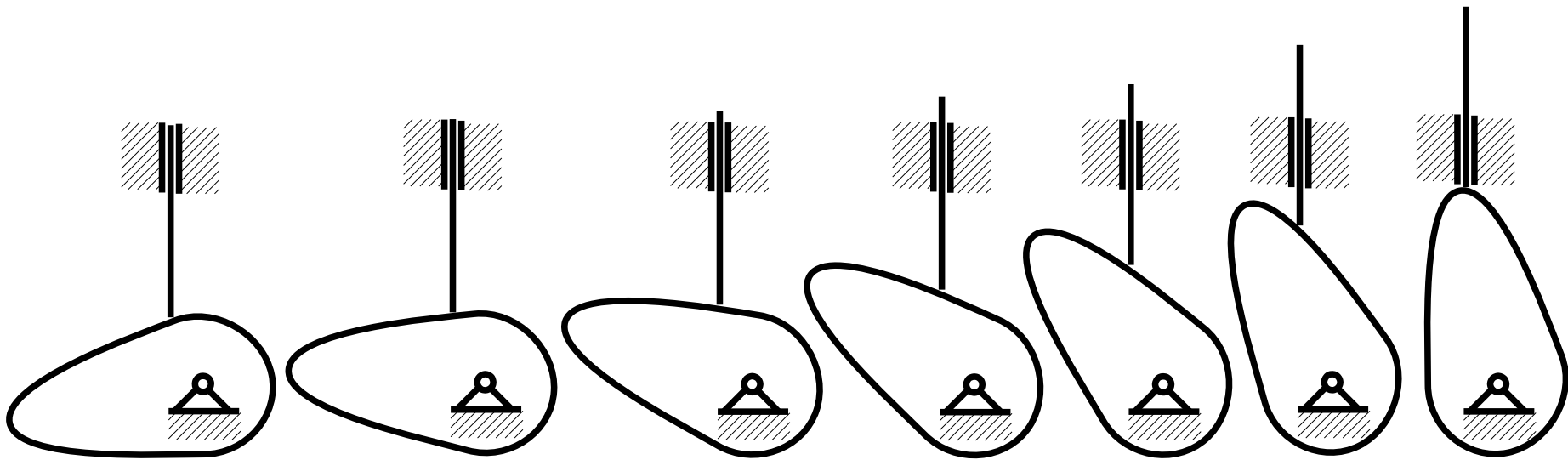
# Analysis of a cam-follower mechanism

## Graphical determination of a follower movement



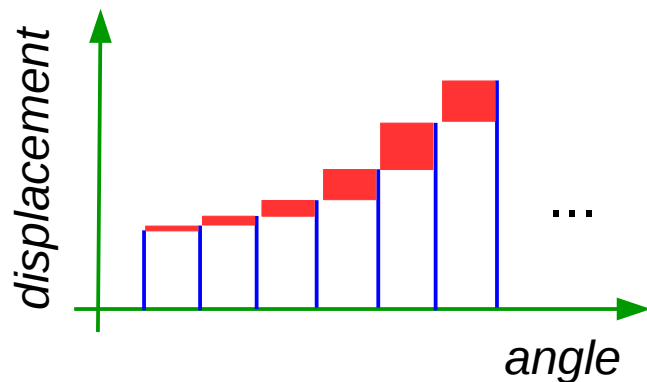
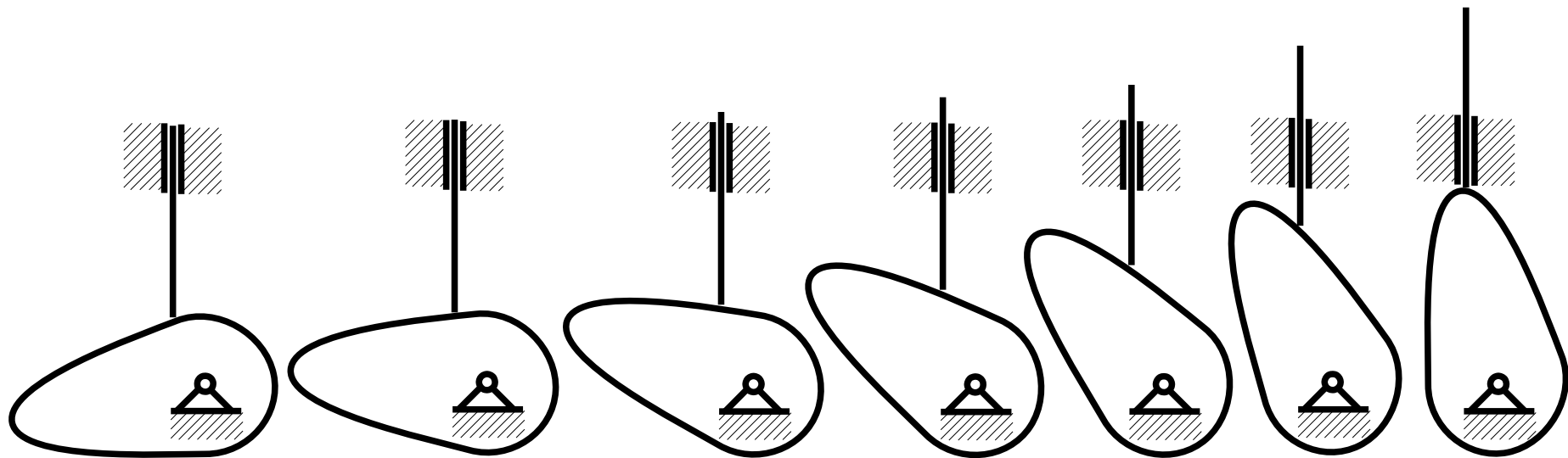
# Analysis of a cam-follower mechanism

## Graphical determination of a follower movement



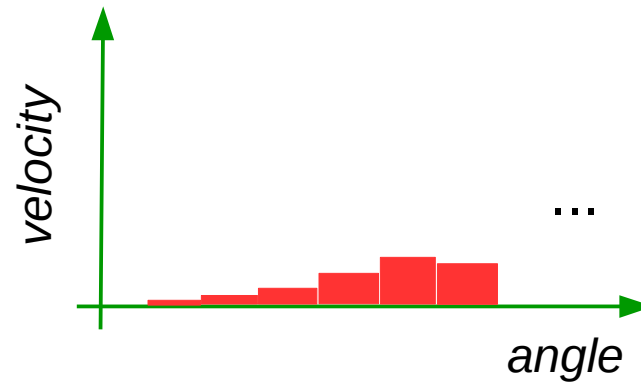
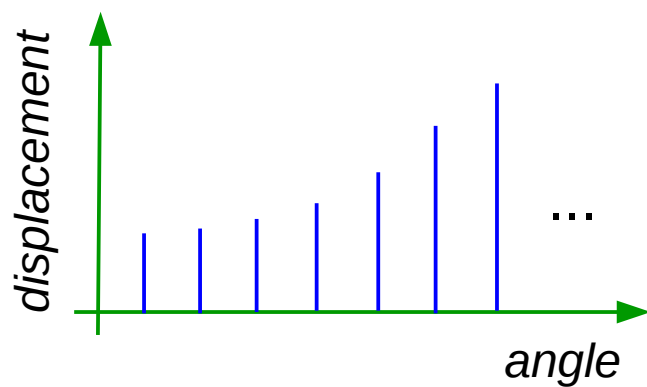
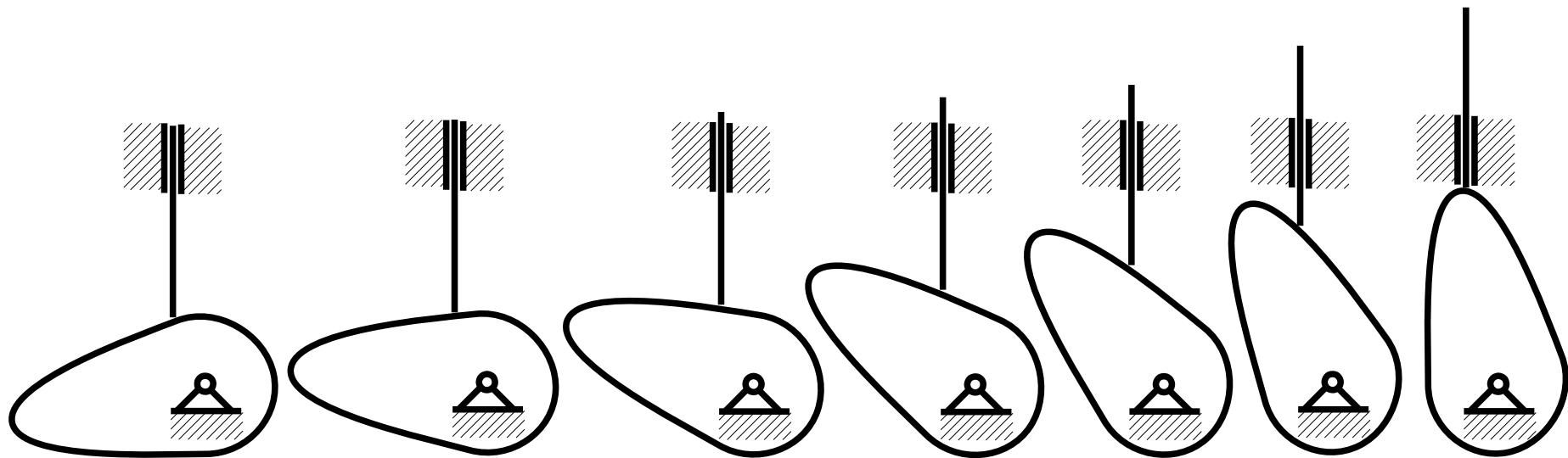
# Analysis of a cam-follower mechanism

## Graphical determination of a follower movement



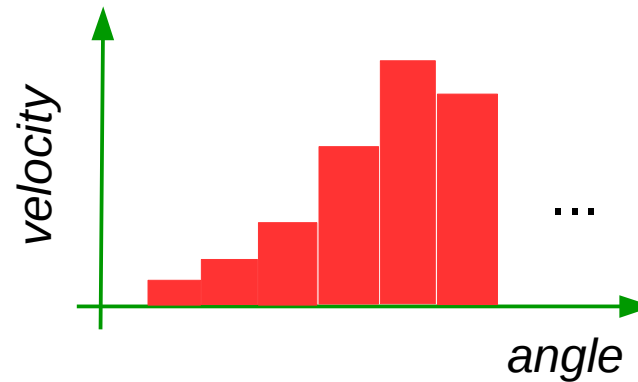
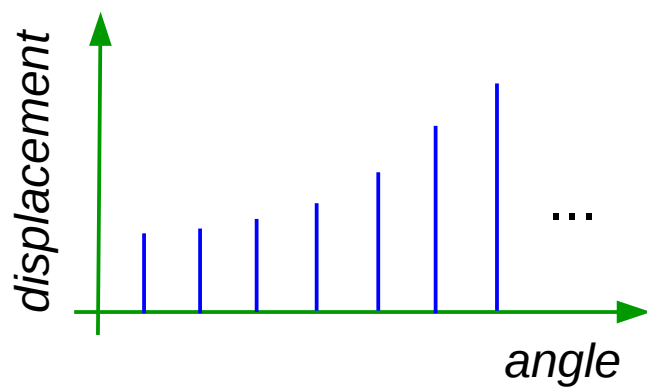
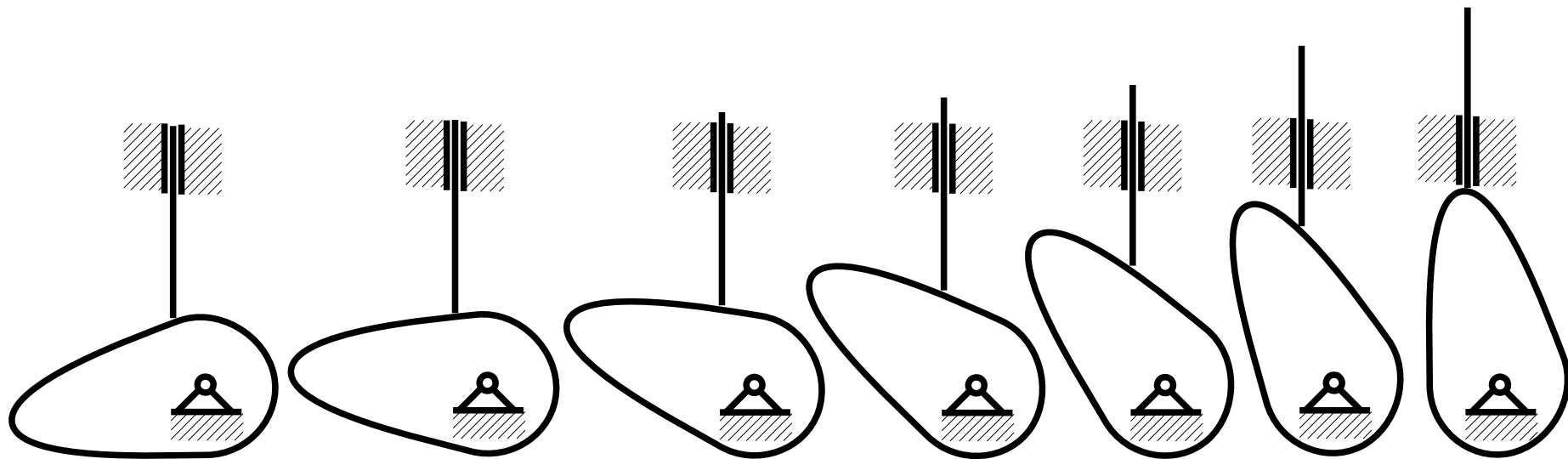
# Analysis of a cam-follower mechanism

## Graphical determination of a follower movement



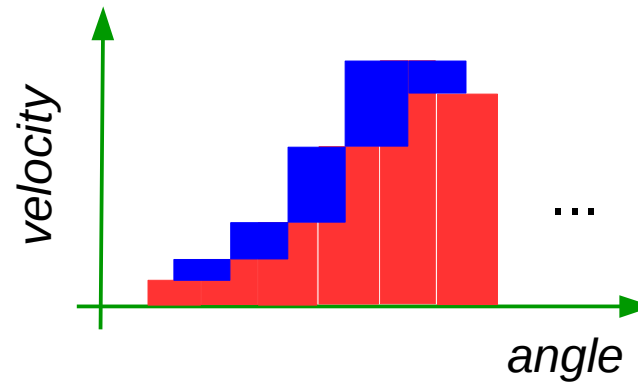
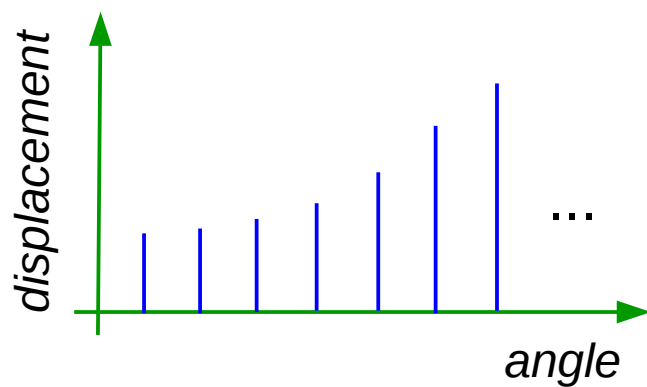
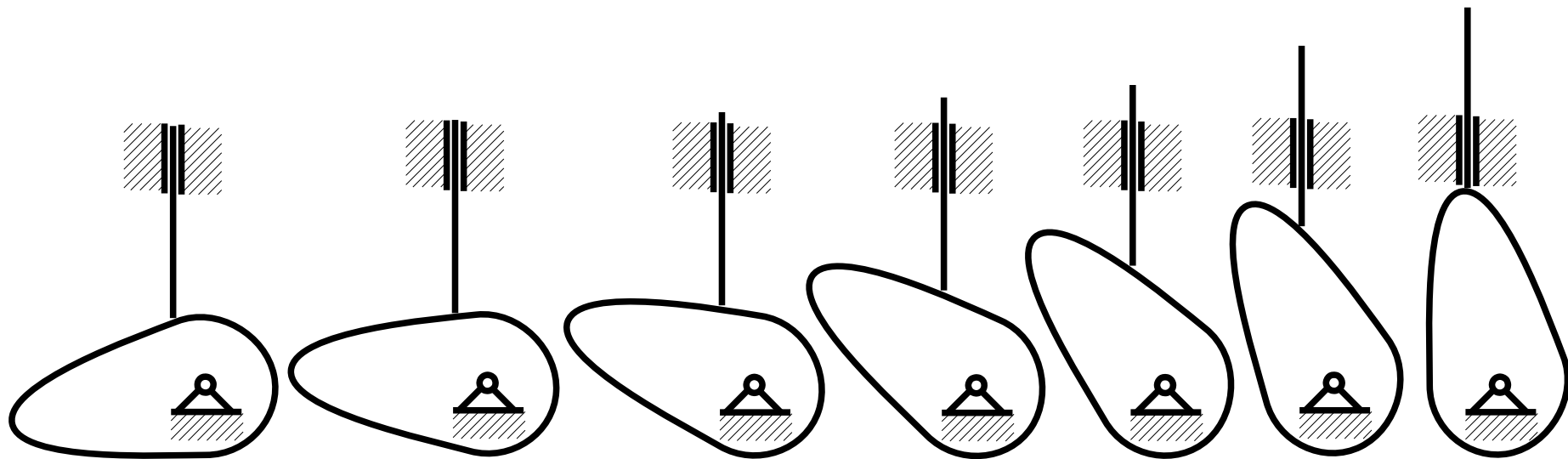
# Analysis of a cam-follower mechanism

## Graphical determination of a follower movement



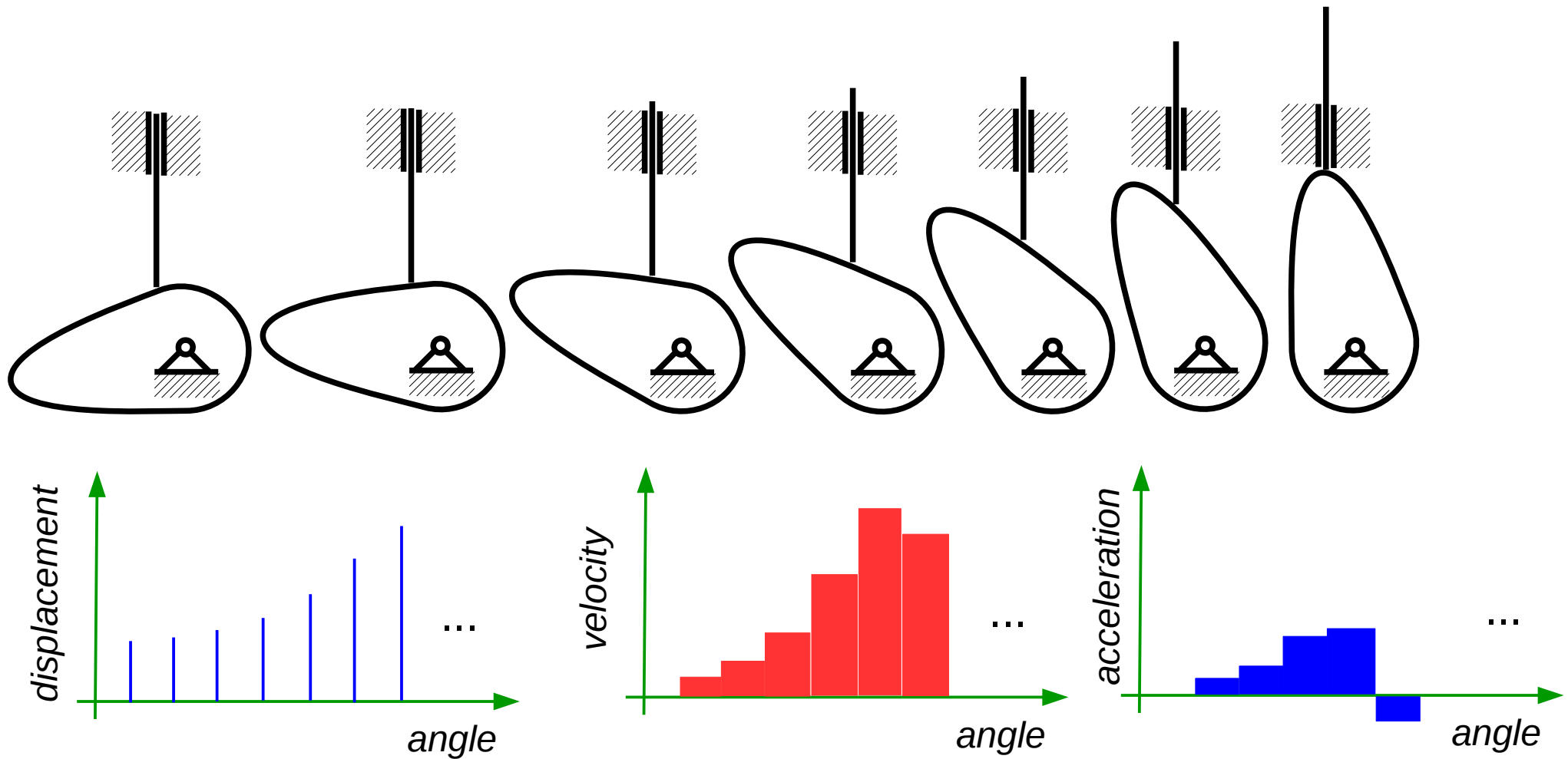
# Analysis of a cam-follower mechanism

## Graphical determination of a follower movement



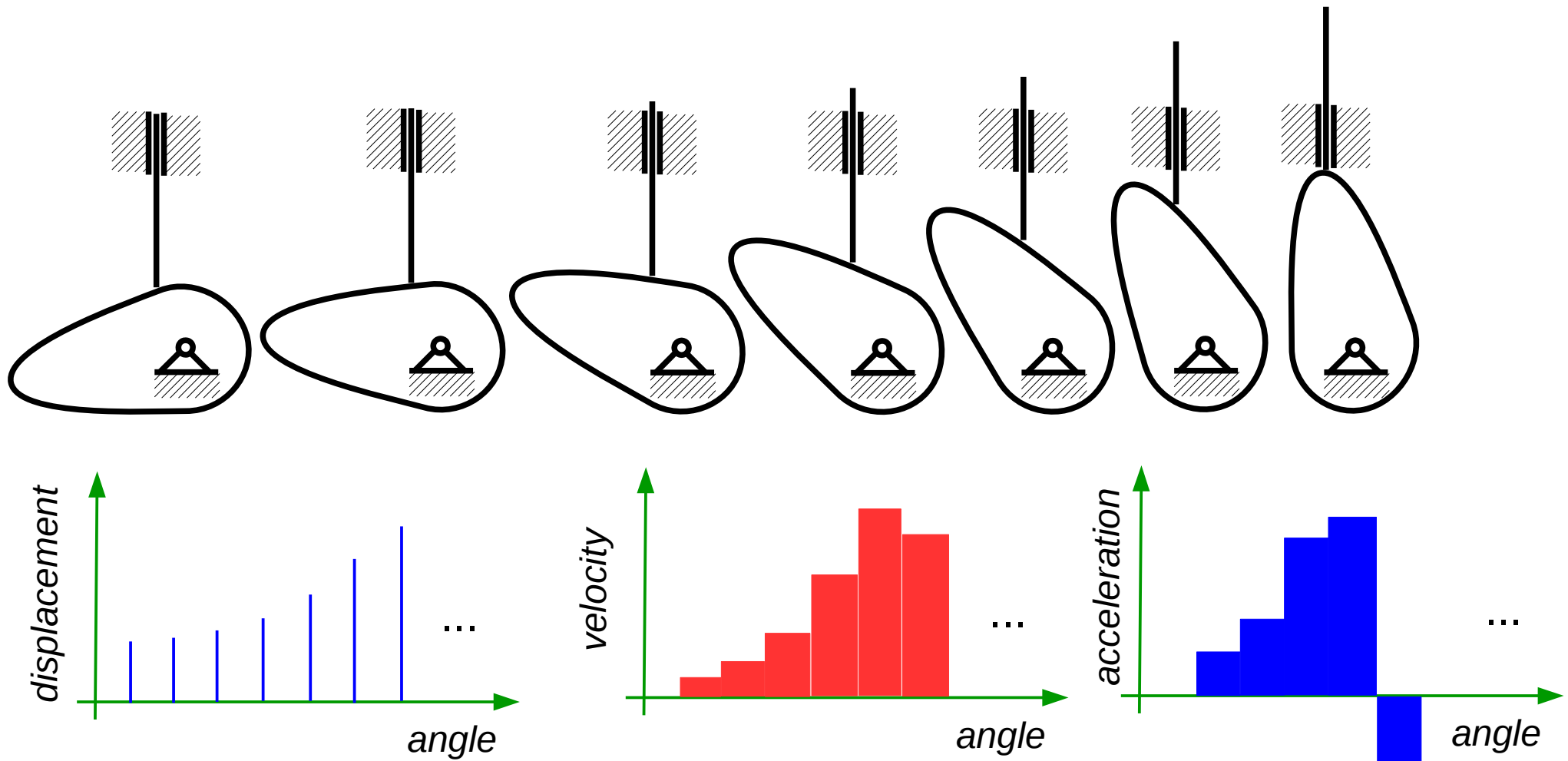
# Analysis of a cam-follower mechanism

## Graphical determination of a follower movement



# Analysis of a cam-follower mechanism

## Graphical determination of a follower movement





# Analysis of a cam-follower mechanism

## Analytical method - example

