



# Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

## *Theory of Machines and Automatic Control* Winter 2018/2019

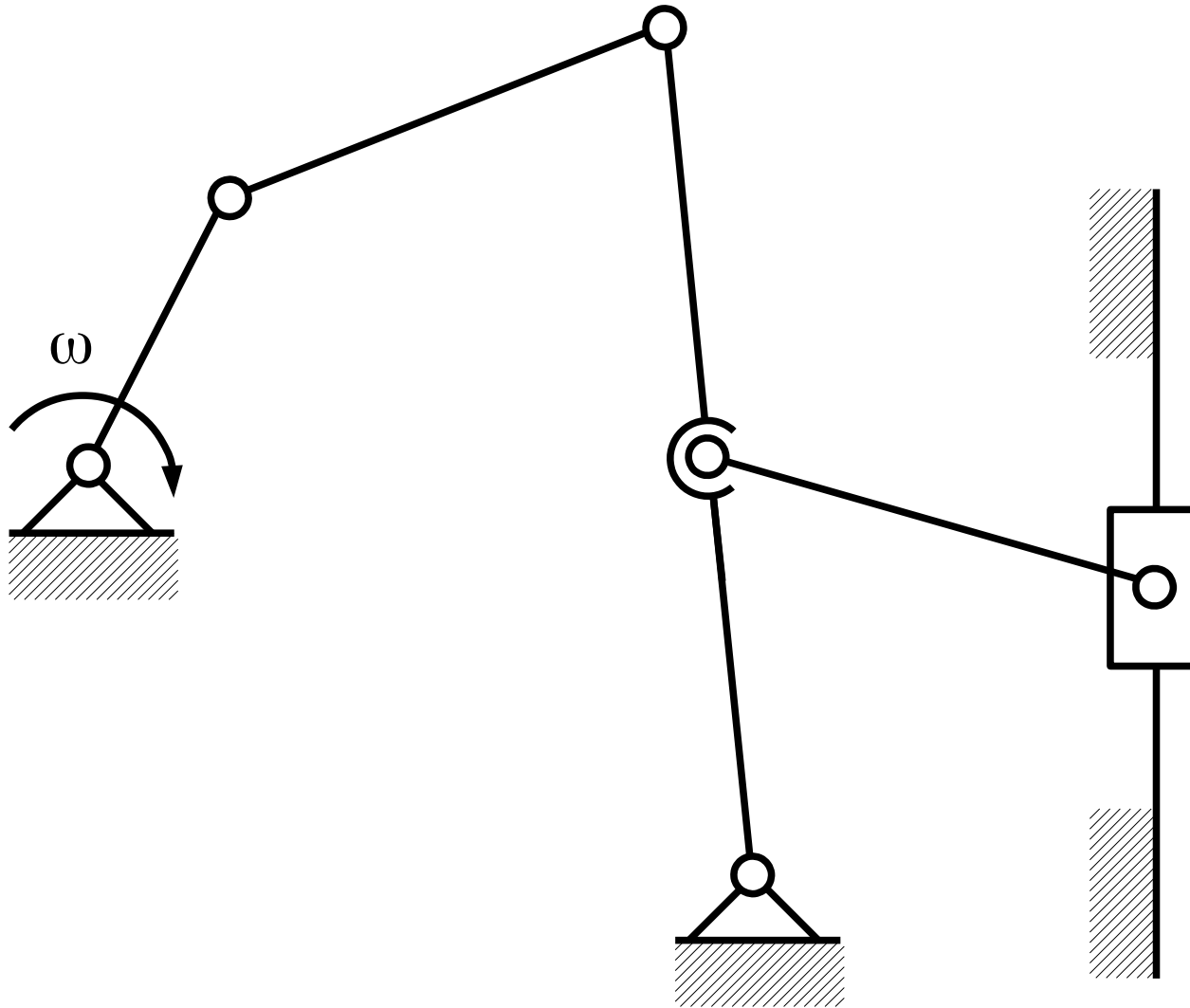
**Lecturer: Sebastian Korczak, PhD, Eng.**

# **Lecture 2 cont.**

## velocities in planar mechanisms

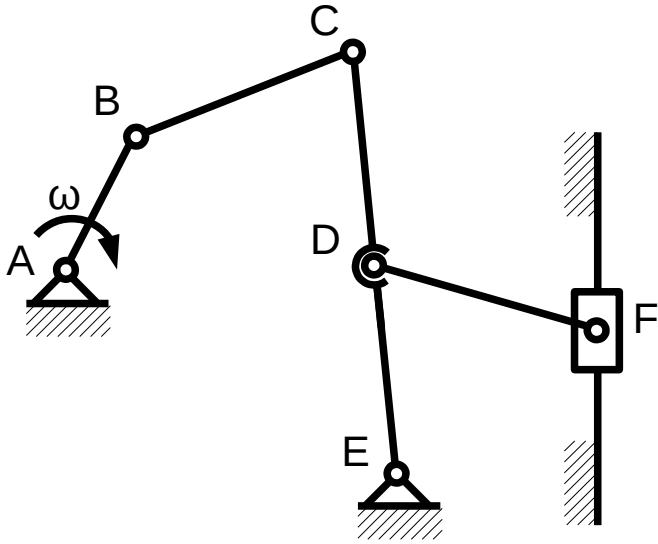
# Velocity scheme method

## Example



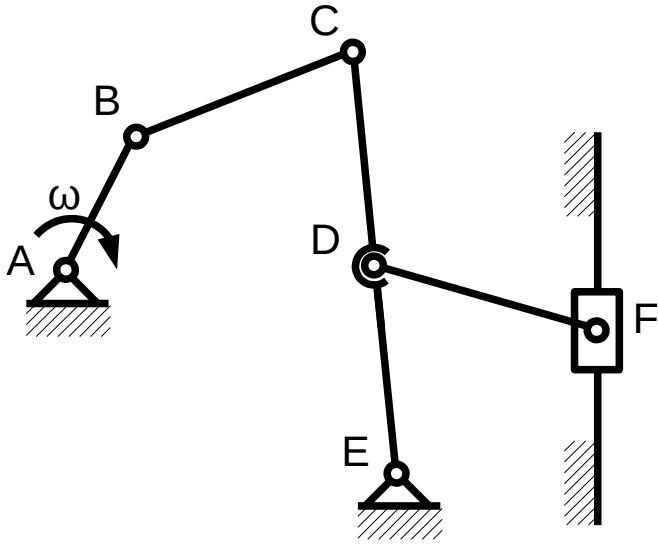
# Velocity scheme method

## Example

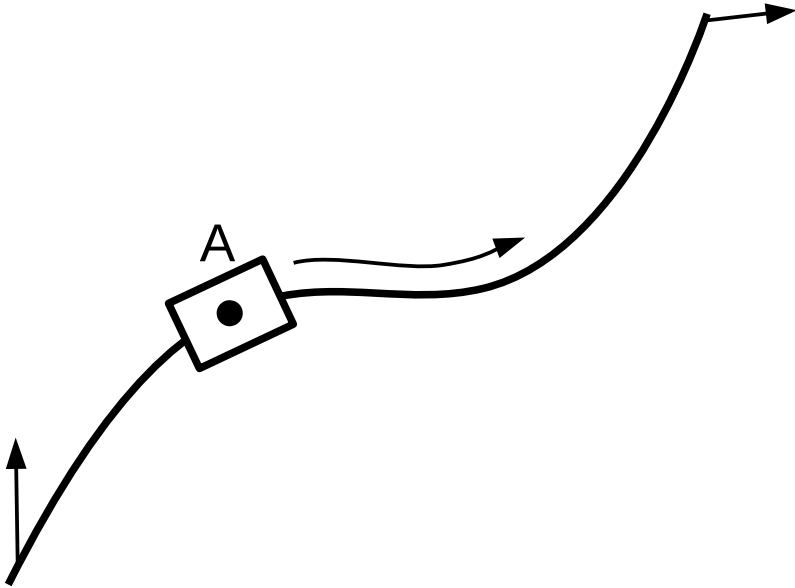


# Velocity scheme method

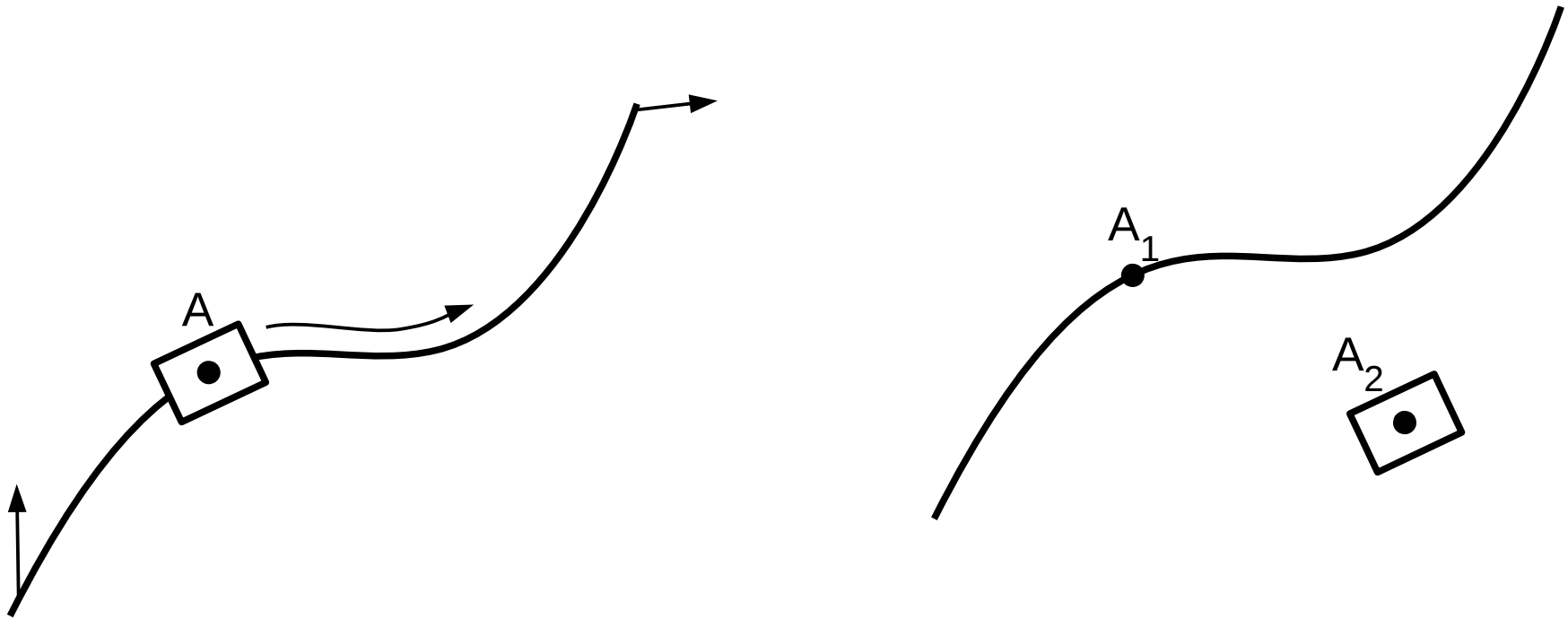
## Example



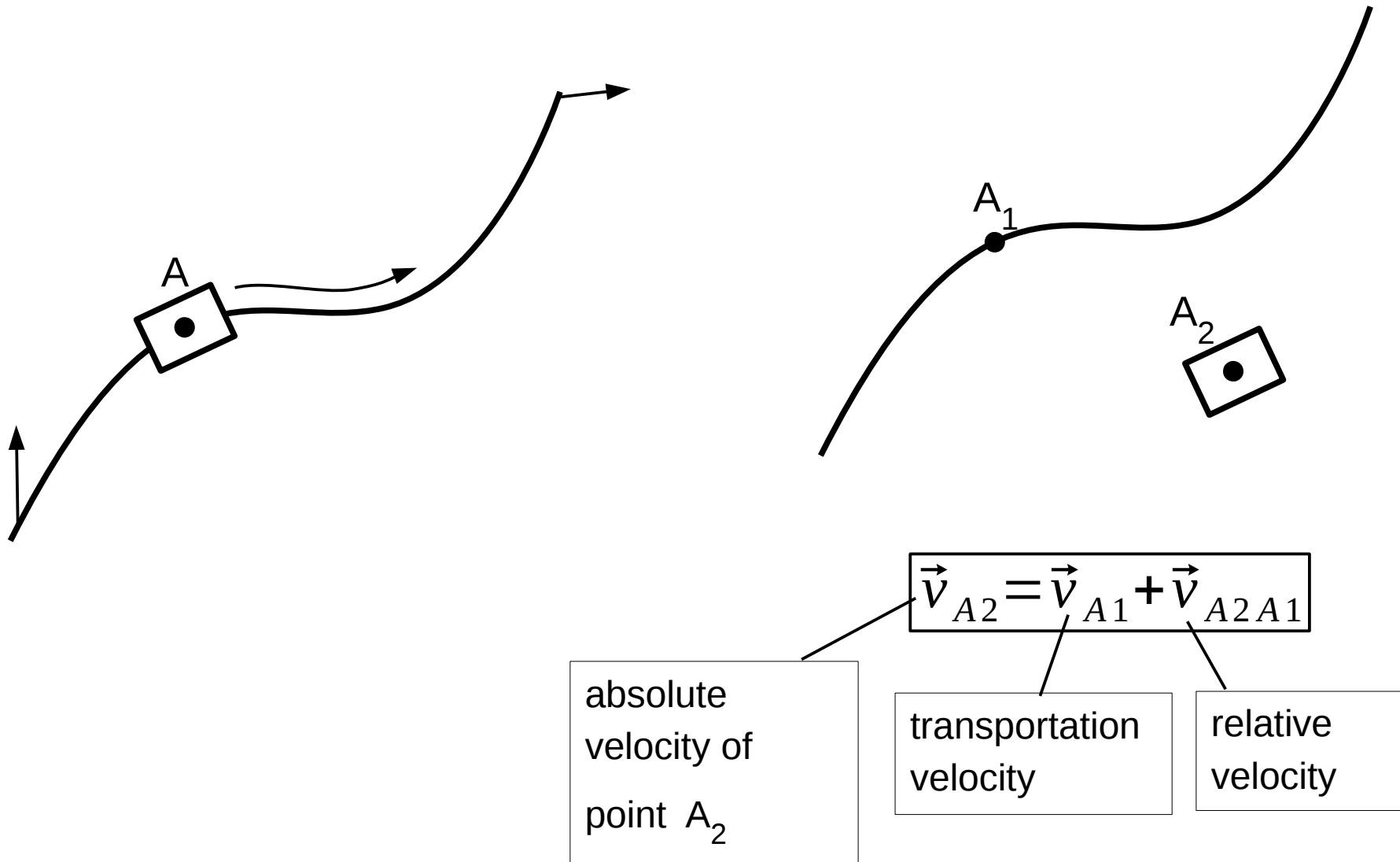
# Velocities in relative motion



# Velocities in relative motion



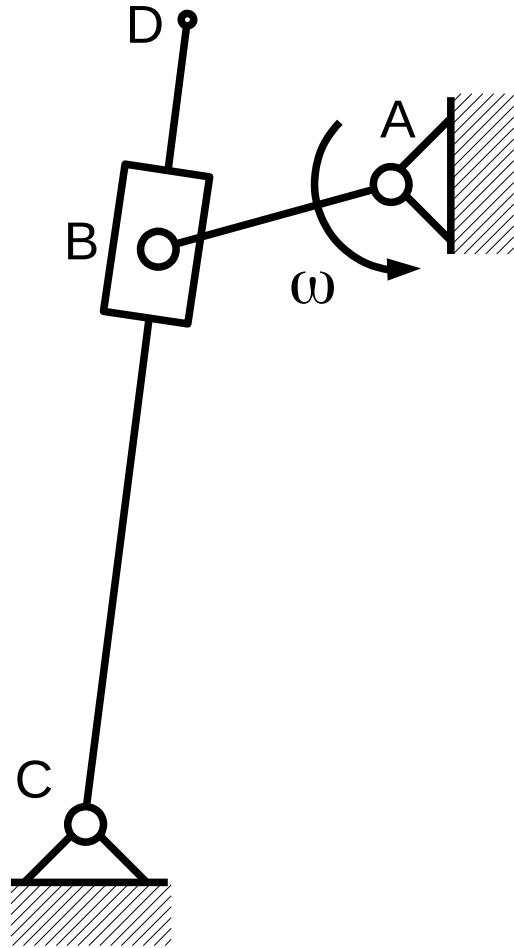
# Velocities in relative motion





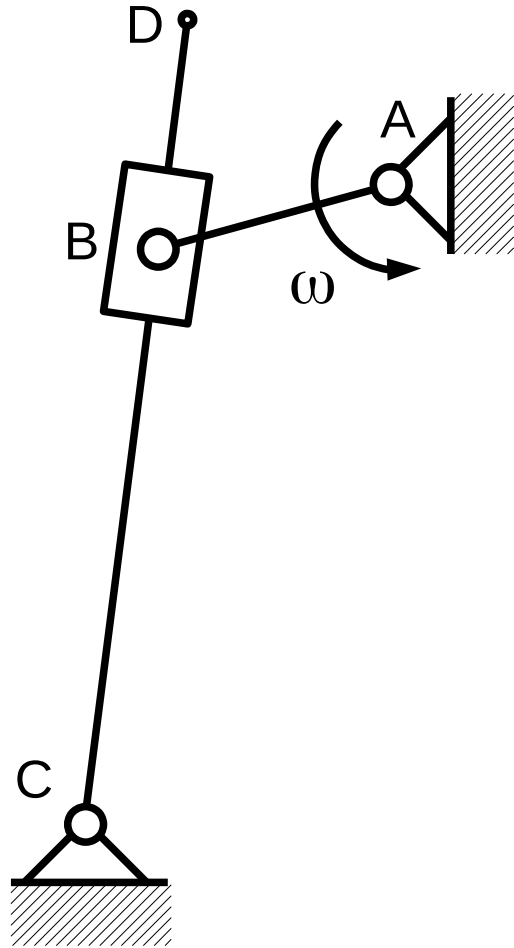
# Velocities in relative motion

## Example



# Velocities in relative motion

## Example



# Lecture 3

## Accelerations in planar mechanisms.

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# Kinematics of mechanisms

Kinematic analysis of a mechanism – determination of velocities and accelerations of selected mechanism members' points at considered configuration. Mechanism structure must be given (geometry of members, kinematic pairs) and drive method must be known.

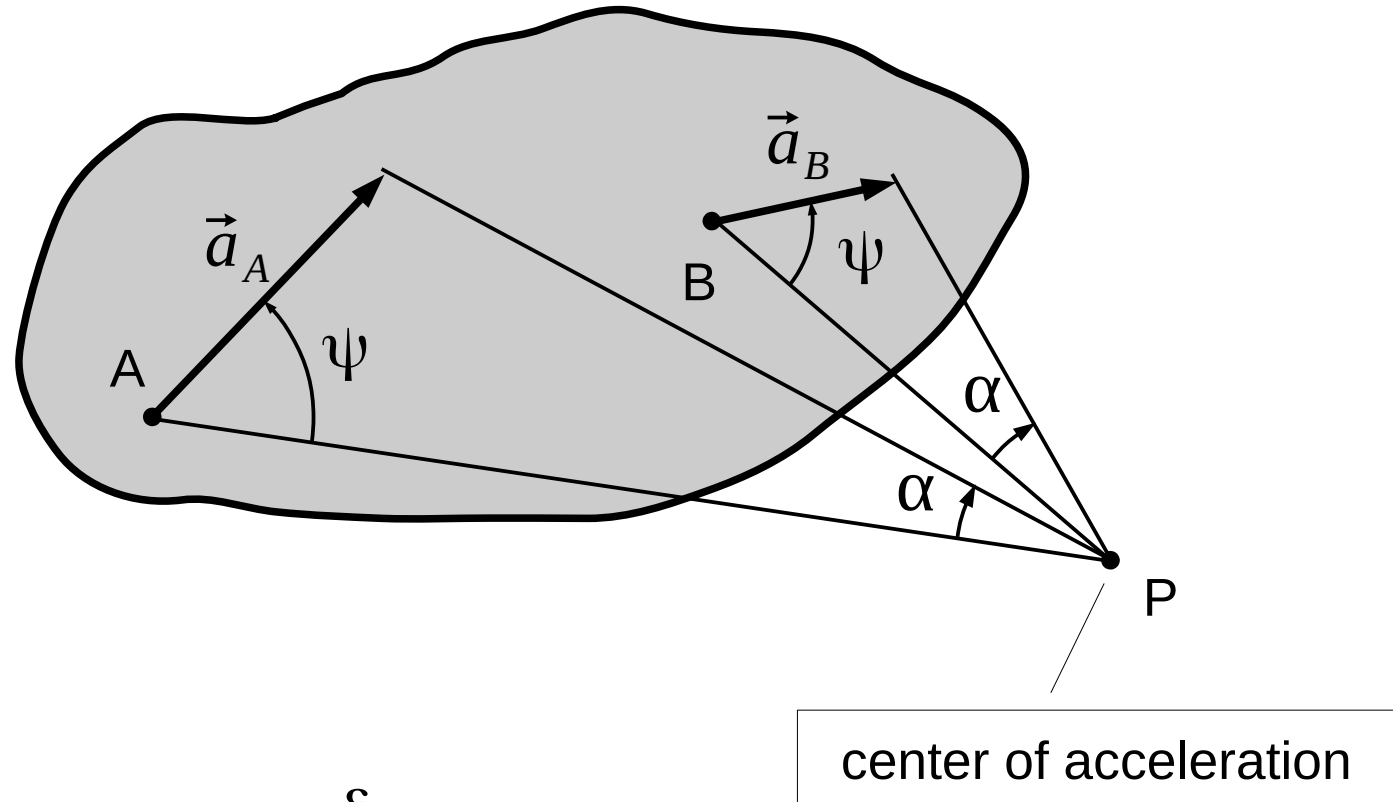
# Methods of velocities and acceleration determination

## Graphical methods

- velocity projection method,
- instantaneous center of rotation method,
- instantaneous center of acceleration method,
- method of rotated velocities,
- velocity decomposition method,
- acceleration decomposition method,
- velocity scheme (diagram) method,
- accelerations scheme (diagram) method.

## Analytical method

# Instantaneous center of acceleration



$$\psi = \text{atan} \frac{\varepsilon}{\omega^2}$$

$\varepsilon$  - angular acceleration

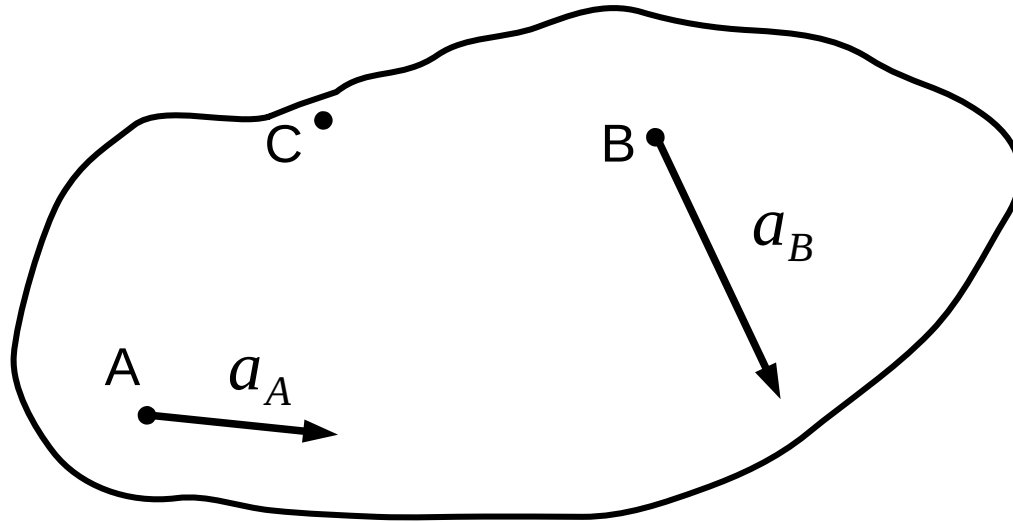
$\omega$  - angular velocity

# Instantaneous center of acceleration method

## Example

Given:  $\bar{a}_A$  and  $\bar{a}_B$

Searched:  $\bar{a}_C$



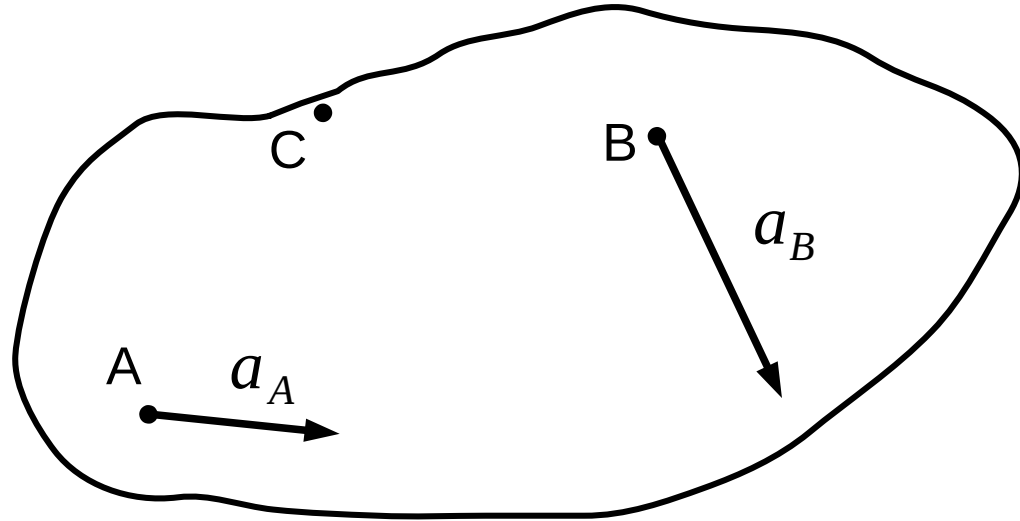
# Instantaneous center of acceleration method

## Example

Given:  $a_A$  and  $a_B$

Searched:  $a_C$

1. STEP:  
construction of  $\psi$





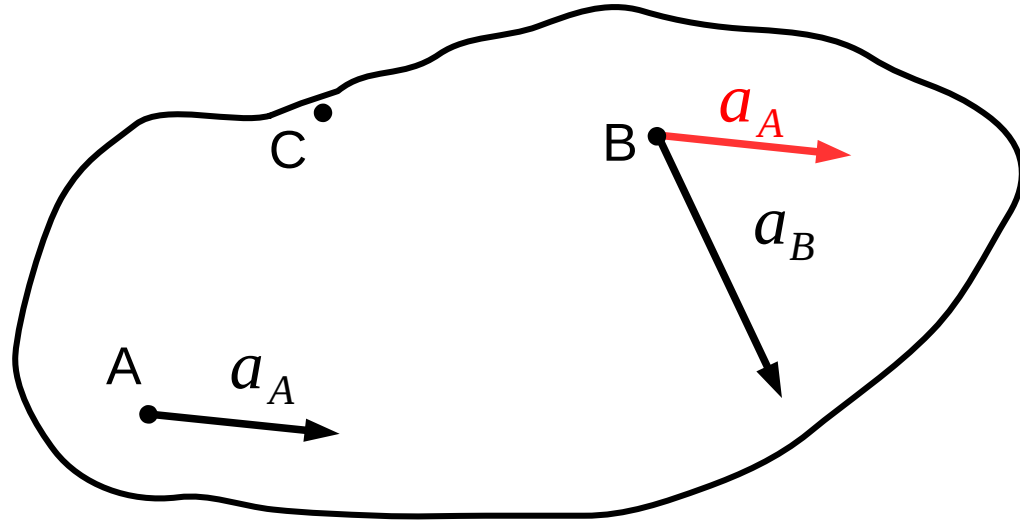
# Instantaneous center of acceleration method

## Example

Given:  $a_A$  and  $a_B$

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1. STEP:  
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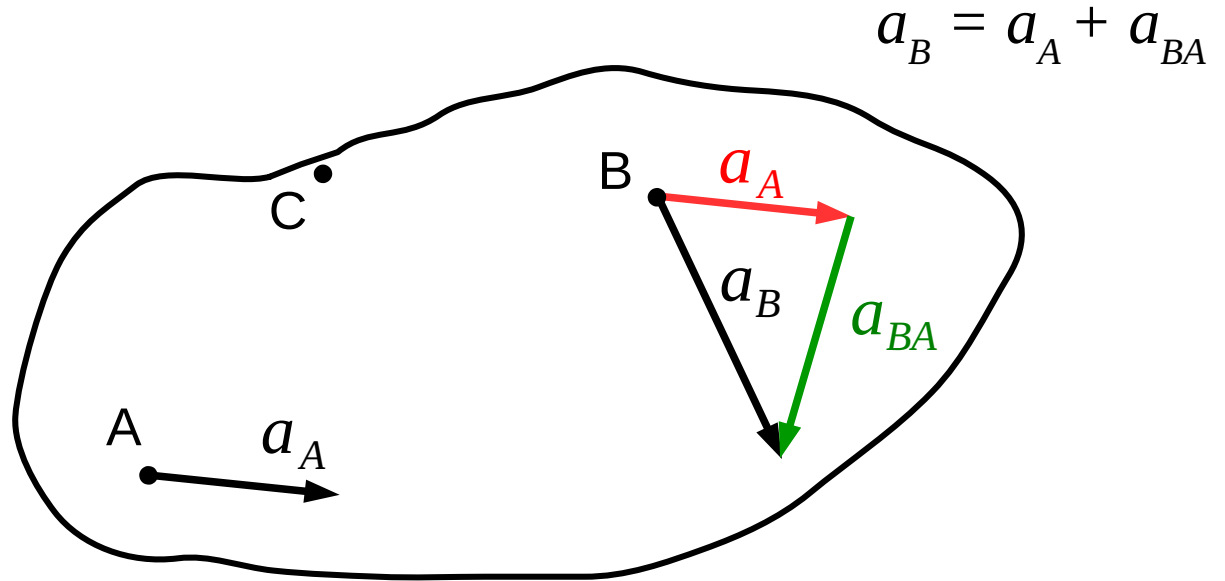
# Instantaneous center of acceleration method

## Example

Given:  $a_A$  and  $a_B$

Searched:  $a_C$

1. STEP:  
construction of  $\psi$



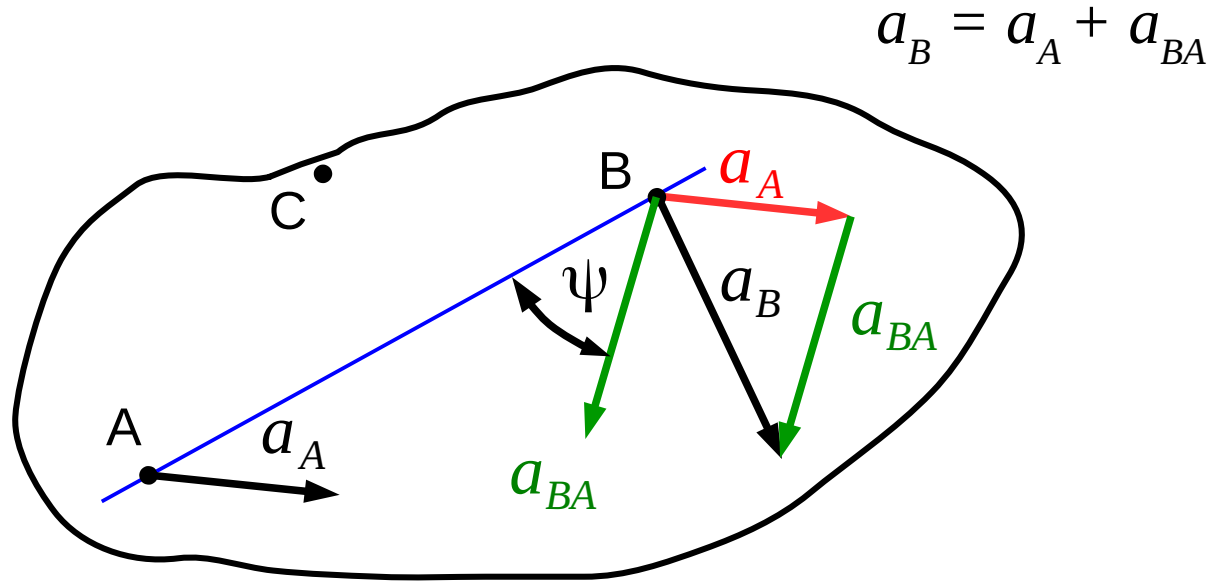
# Instantaneous center of acceleration method

## Example

Given:  $a_A$  and  $a_B$

Searched:  $a_C$

1. STEP:  
construction of  $\psi$



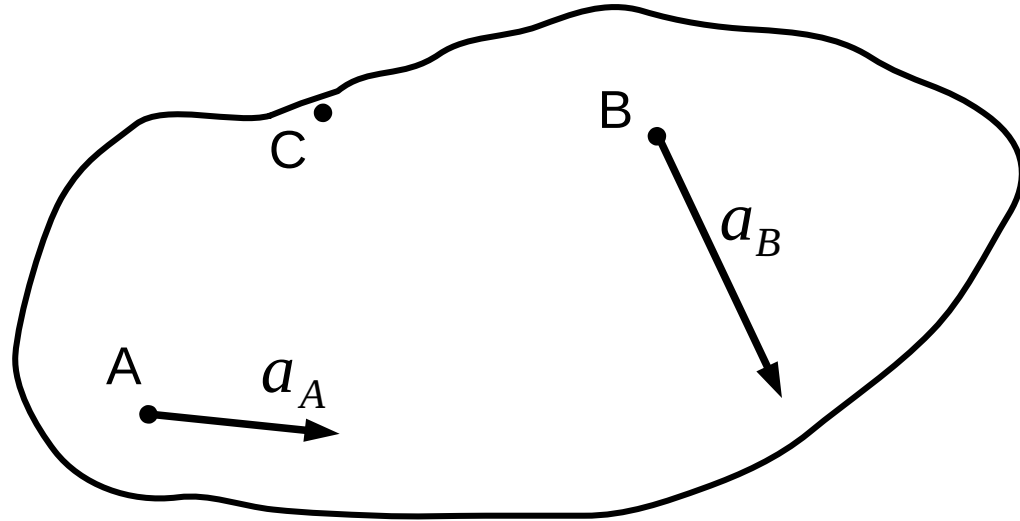
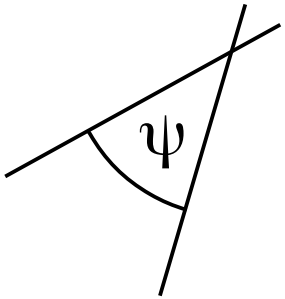
# Instantaneous center of acceleration method

## Example

Given:  $a_A$  and  $a_B$

Searched:  $a_C$

1. STEP:  
construction of  $\psi$



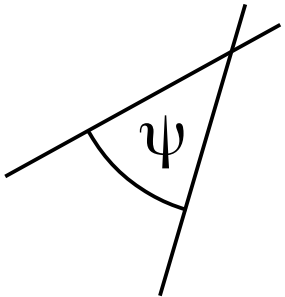
# Instantaneous center of acceleration method

## Example

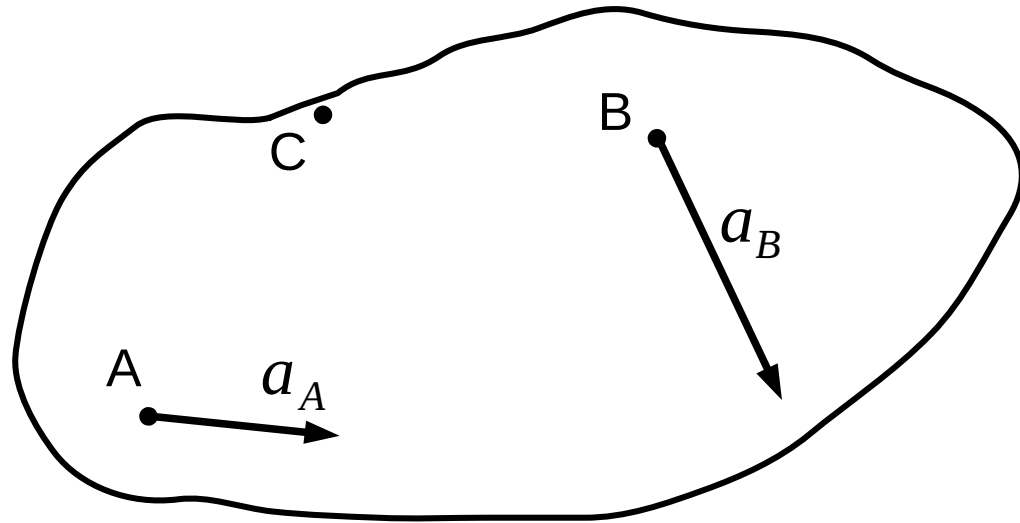
Given:  $a_A$  and  $a_B$

Searched:  $a_C$

1. STEP:  
construction of  $\psi$



2. STEP: find out  
the center of  
acceleration



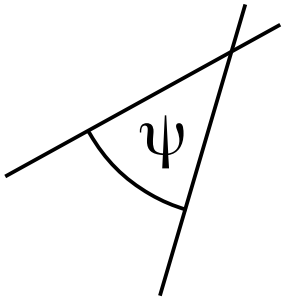
# Instantaneous center of acceleration method

## Example

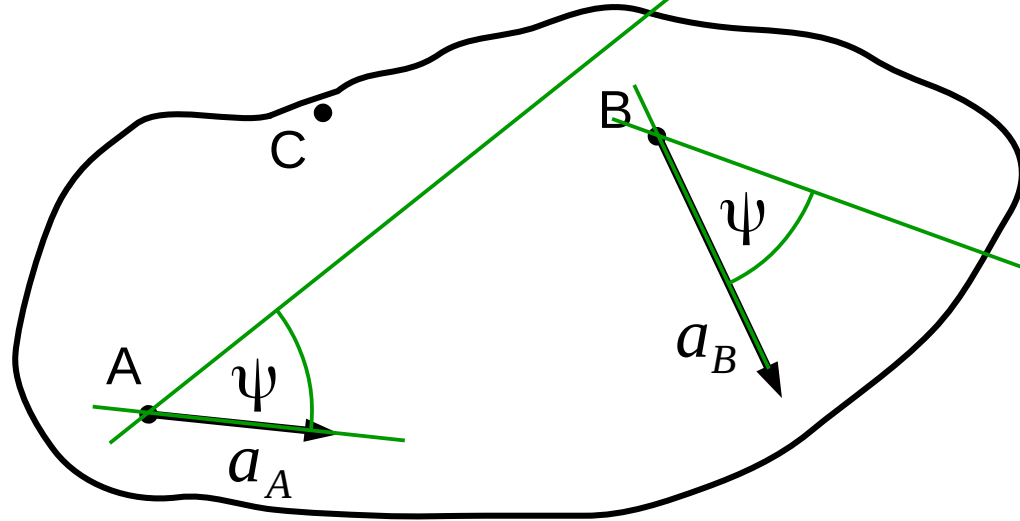
Given:  $a_A$  and  $a_B$

Searched:  $a_C$

1. STEP:  
construction of  $\psi$



2. STEP: find out  
the center of  
acceleration



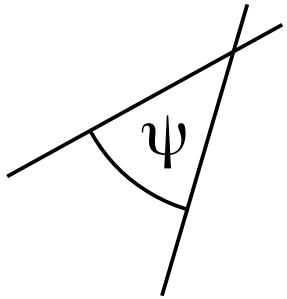
# Instantaneous center of acceleration method

## Example

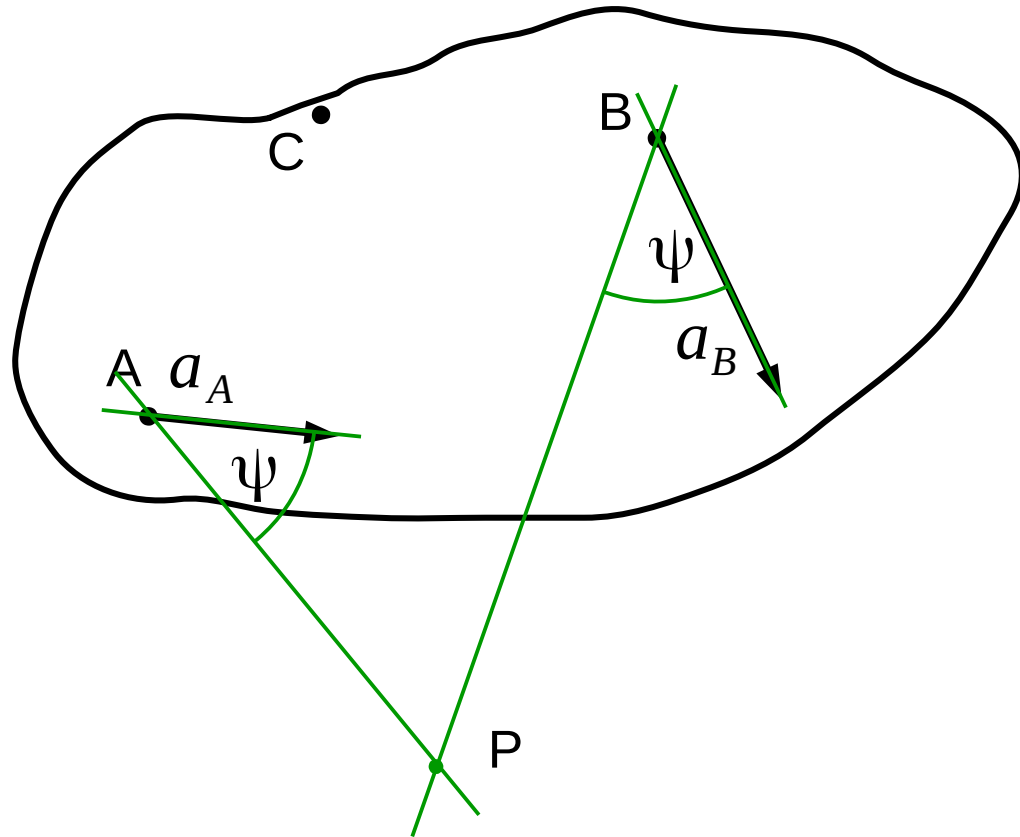
Given:  $a_A$  and  $a_B$

Searched:  $a_C$

1. STEP:  
construction of  $\psi$



2. STEP: find out  
the center of  
acceleration



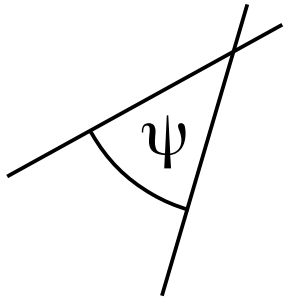
# Instantaneous center of acceleration method

## Example

Given:  $a_A$  and  $a_B$

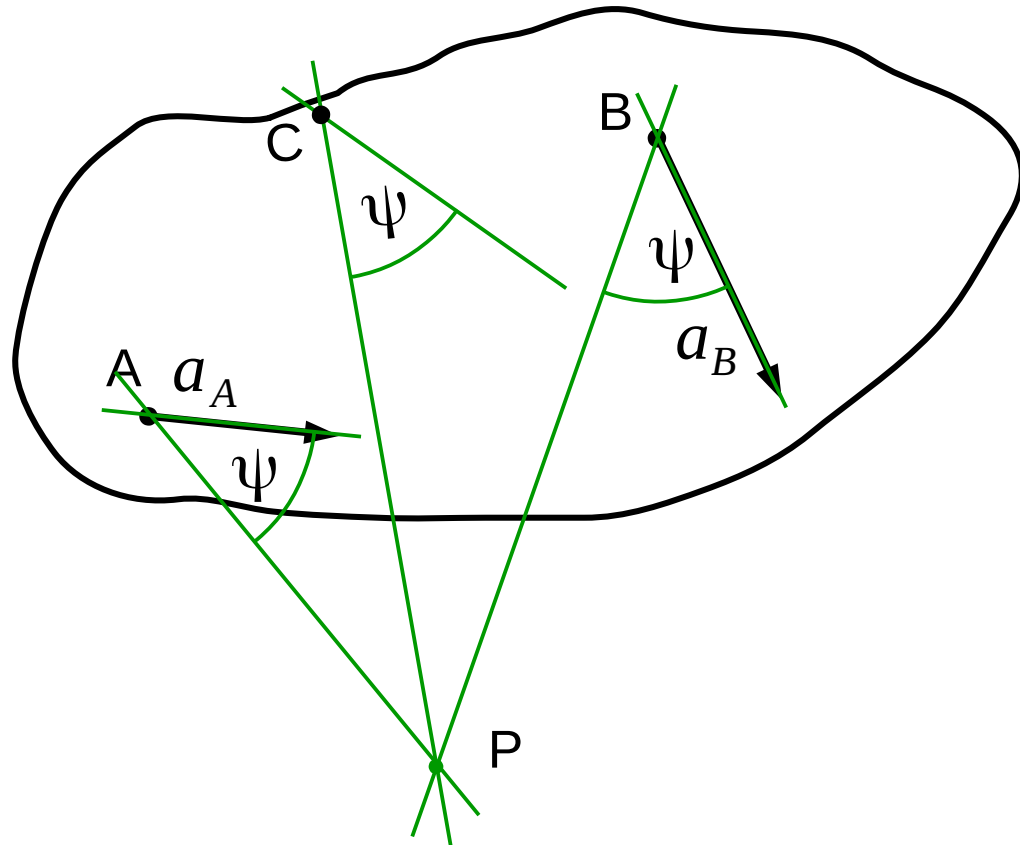
Searched:  $a_C$

1. STEP:  
construction of  $\psi$



2. STEP: find out the  
center of acceleration

3. STEP: find out  $a_C$





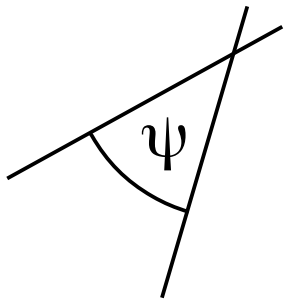
# Instantaneous center of acceleration method

## Example

Given:  $\vec{a}_A$  and  $\vec{a}_B$

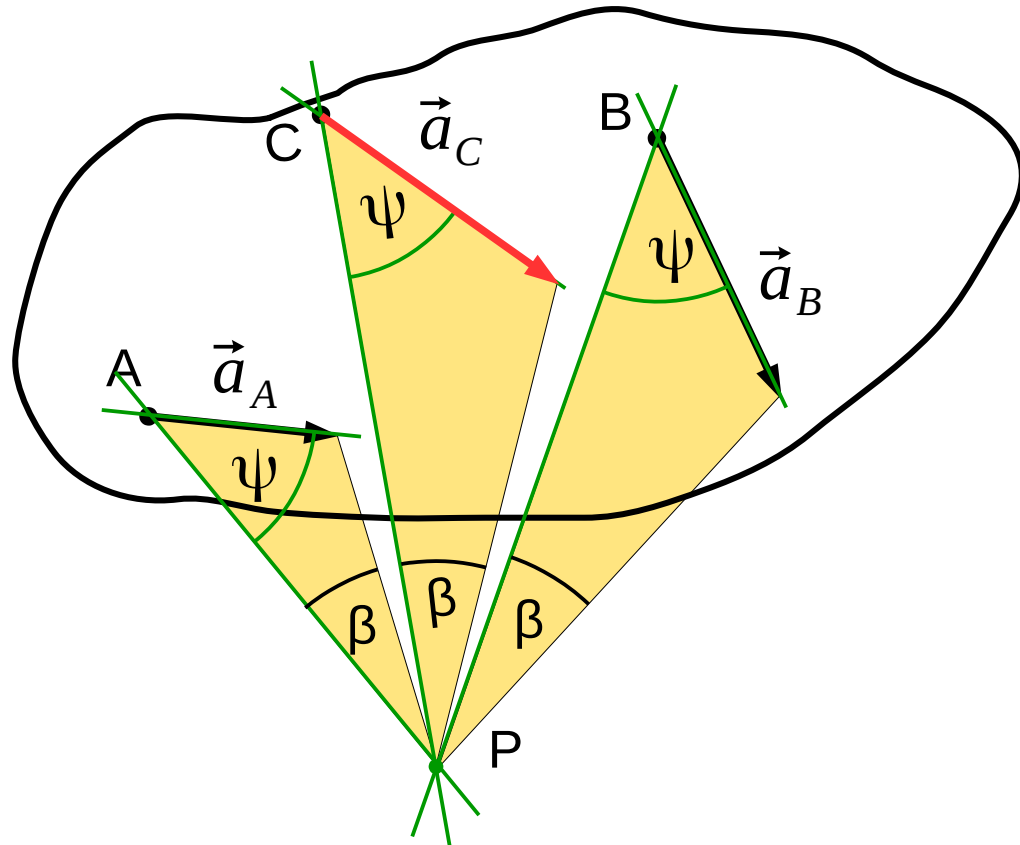
Searched:  $\vec{a}_C$

1. STEP:  
construction of  $\psi$



2. STEP: find out the  
center of acceleration

3. STEP: find out  $\vec{a}_C$

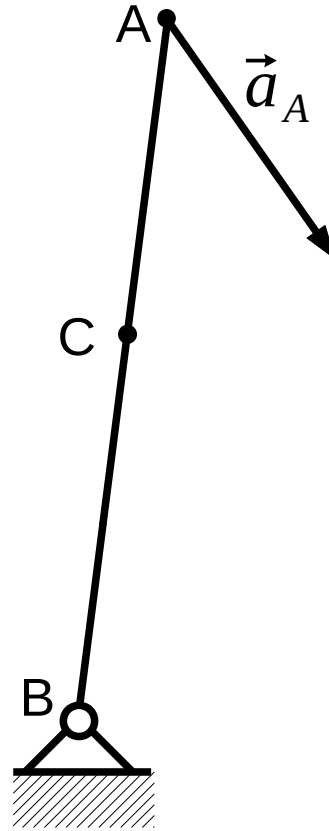


# Instantaneous center of acceleration method

## Example 2

Given:  $\bar{a}_A$

Searched:  $\bar{a}_C$

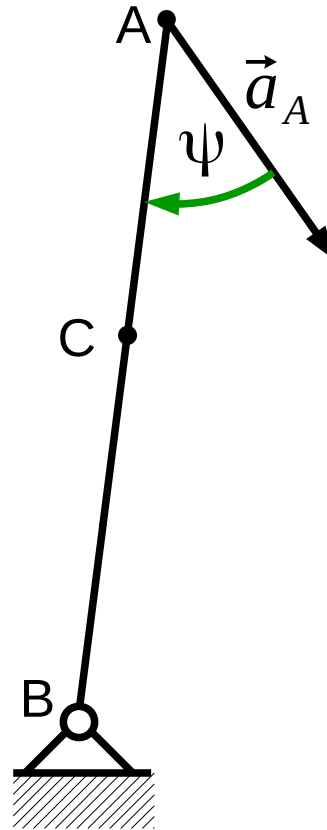


# Instantaneous center of acceleration method

## Example 2

Given:  $\bar{a}_A$

Searched:  $\bar{a}_C$

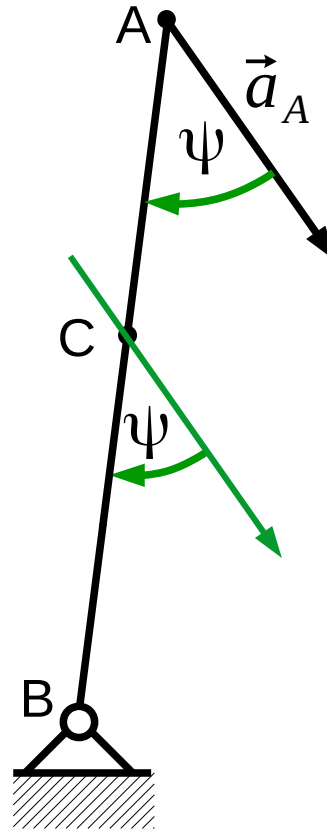


# Instantaneous center of acceleration method

## Example 2

Given:  $\vec{a}_A$

Searched:  $\vec{a}_C$

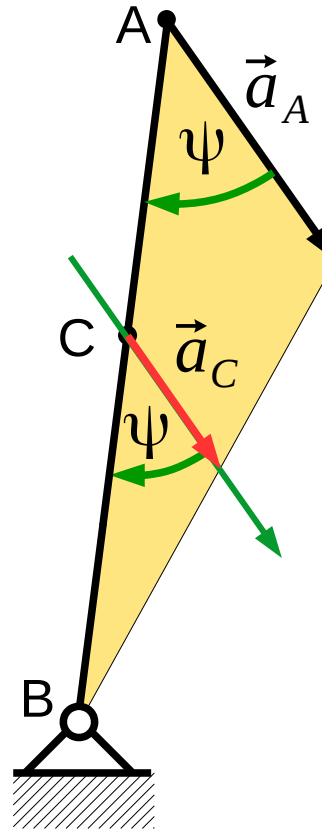


# Instantaneous center of acceleration method

## Example 2

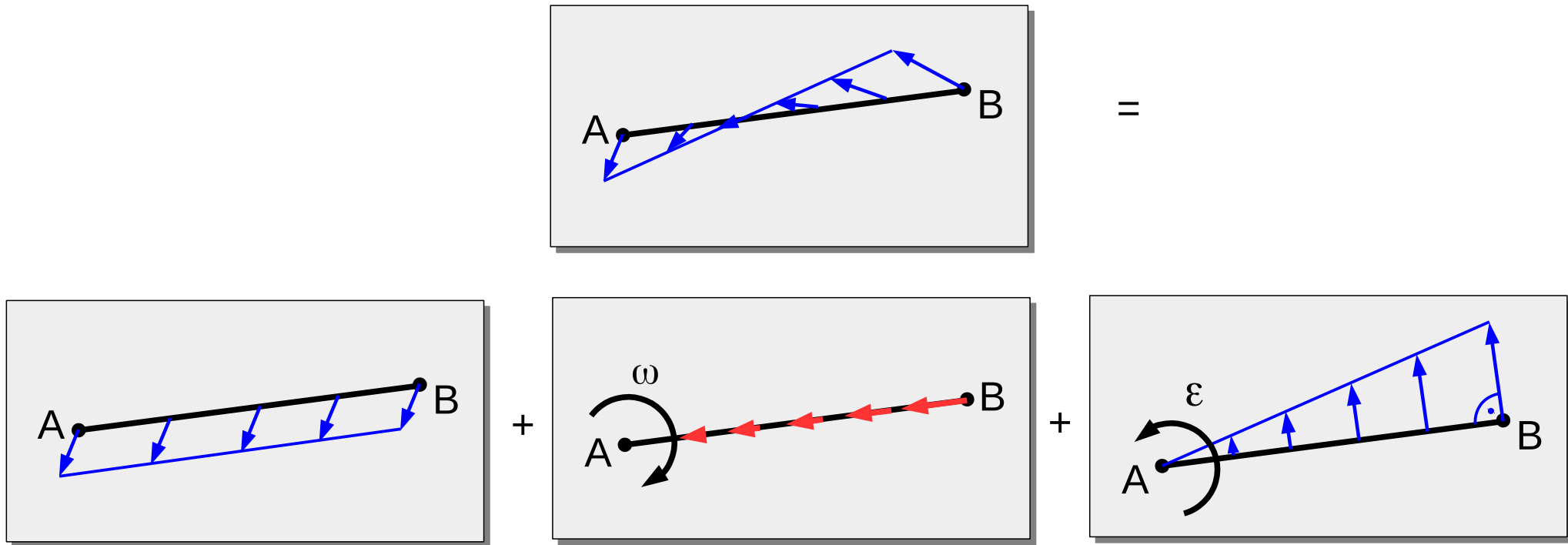
Given:  $\vec{a}_A$

Searched:  $\vec{a}_C$



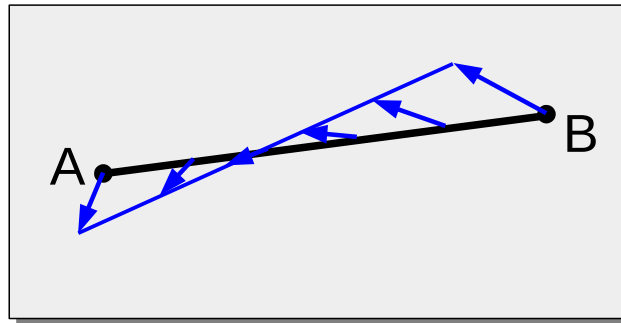
# Acceleration decomposition method

## Example

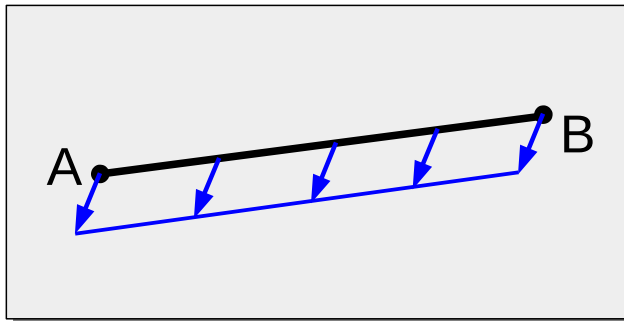


# Acceleration decomposition method

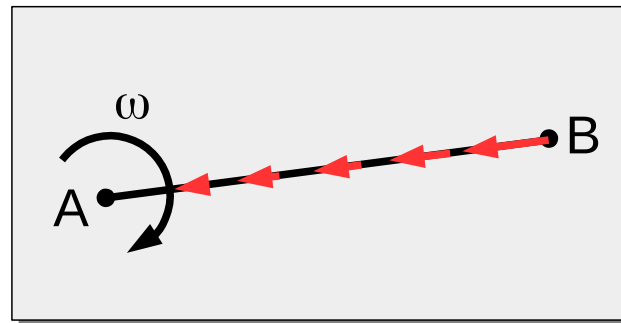
## Example



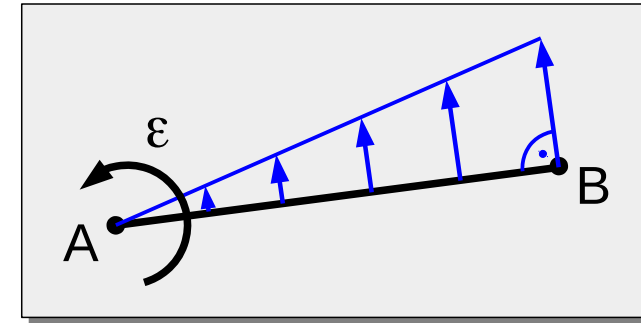
=



+



+



$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}_{BA}^n + \vec{a}_{BA}^t$$

absolute acceleration of point B

absolute acceleration of point A

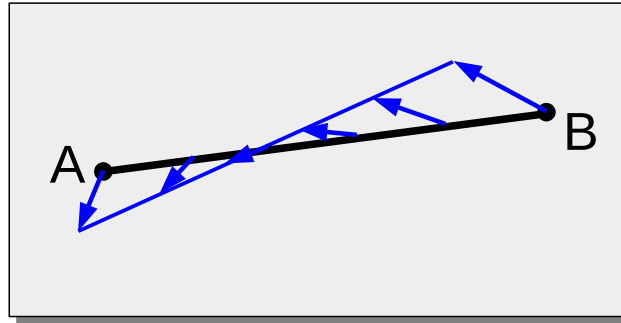
Angular acceleration of point B in rotation around point A.

Centripetal acceleration (normal)

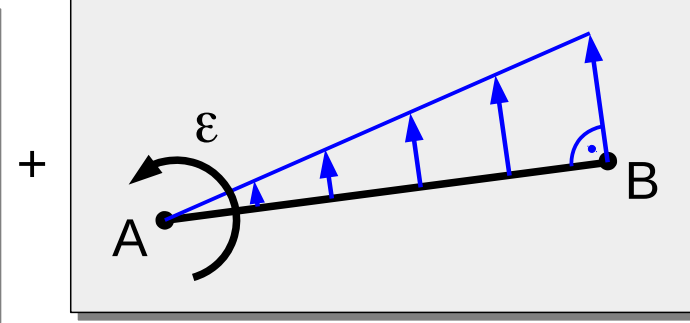
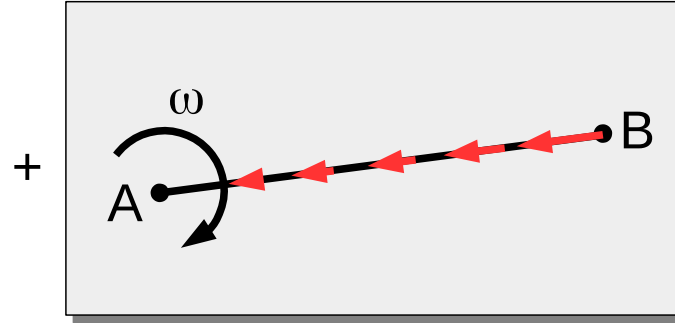
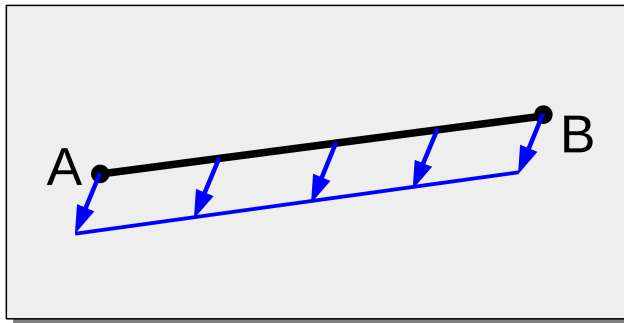
Rotary acceleration (tangential)

# Acceleration decomposition method

## Example



=



$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}_{BA}^n + \vec{a}_{BA}^t$$

Centripetal acceleration  
(normal)

Rotary acceleration  
(tangential)

$$\vec{a}_{BA} = \vec{\varepsilon} \times \vec{AB}$$

$$\vec{a}_{BA} = \vec{\omega} \times (\vec{\omega} \times \vec{AB}) = -\omega^2 \vec{AB}$$



## Acceleration scheme (diagram)

*Acceleration scheme of a rigid body – geometry created by the ends of its acceleration vectors moved to the common starting point (acceleration scheme's pole).*

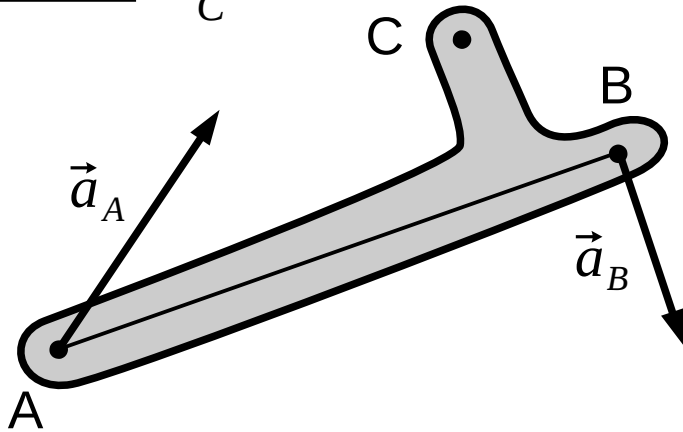
*Acceleration scheme is similar to the corresponding rigid body: it is scaled and rotated by  $(180^\circ - \psi)$  angle in the direction of body's angular velocity if  $\text{sgn}\omega = \text{sgn}\varepsilon$  (or opposite direction if  $\text{sgn}\omega \neq \text{sgn}\varepsilon$ ).*

# Acceleration scheme method

## Example

Given:  $\bar{a}_A$  and  $\bar{a}_B$  + geometry

Searched:  $\bar{a}_C$

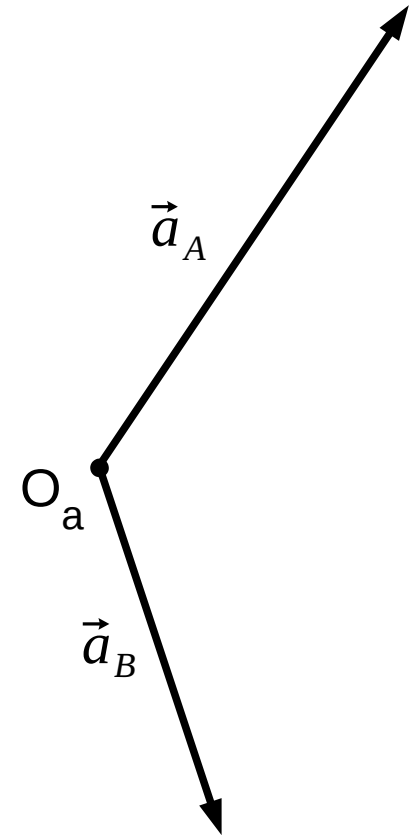
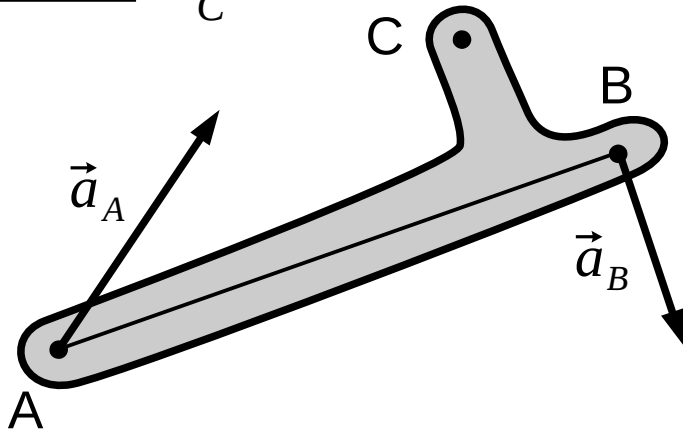


# Acceleration scheme method

## Example

Given:  $\vec{a}_A$  and  $\vec{a}_B$  + geometry

Searched:  $\vec{a}_C$



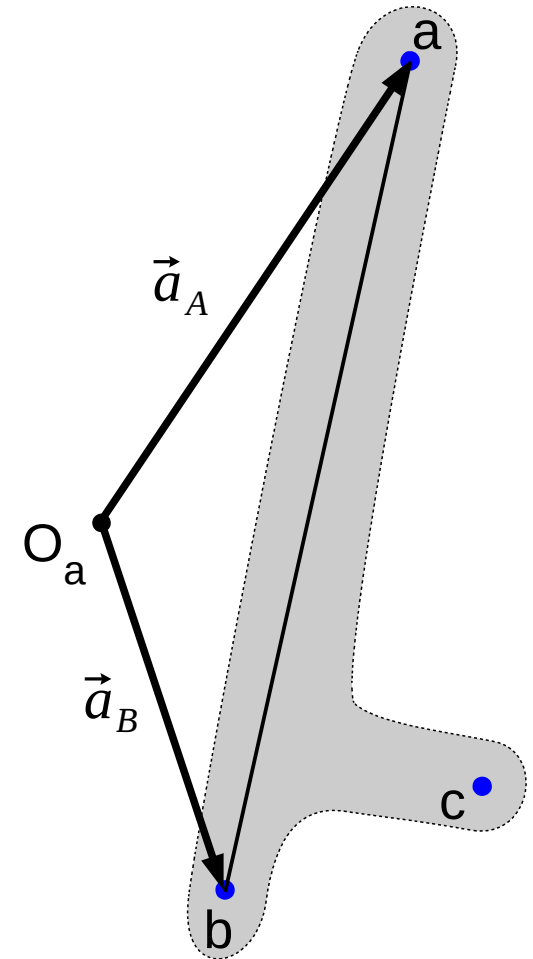
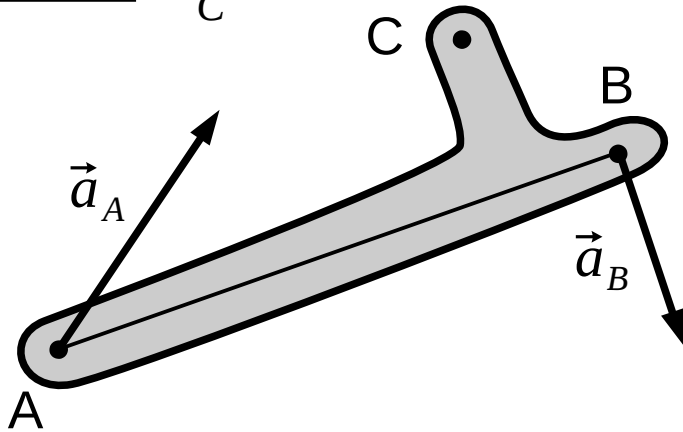
acceleration scale, e.g.: 1cm  $\rightarrow$  1m/s

# Acceleration scheme method

## Example

Given:  $\vec{a}_A$  and  $\vec{a}_B$  + geometry

Searched:  $\vec{a}_C$



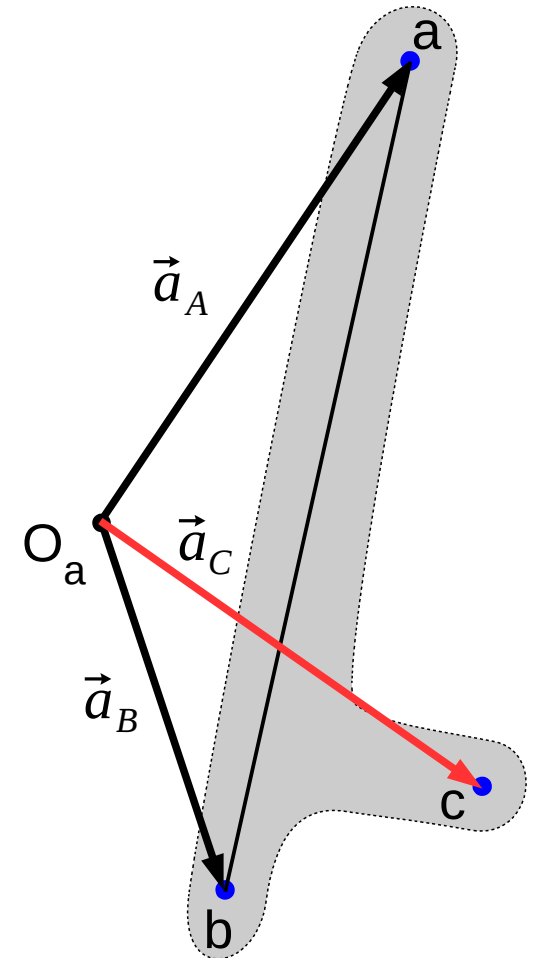
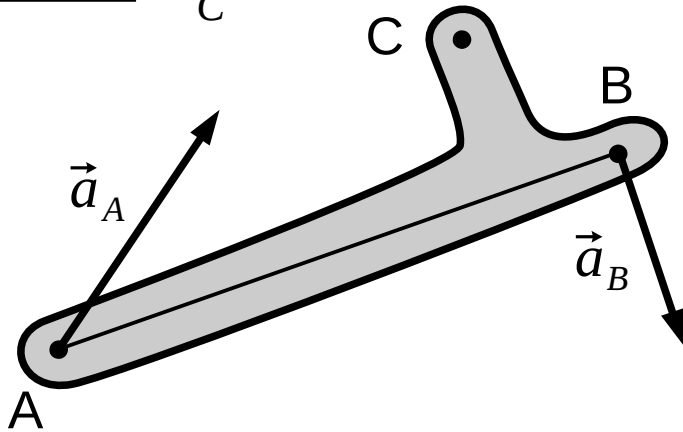
acceleration scale, e.g.: 1cm  $\rightarrow$  1m/s

# Acceleration scheme method

## Example

Given:  $\vec{a}_A$  and  $\vec{a}_B$  + geometry

Searched:  $\vec{a}_C$



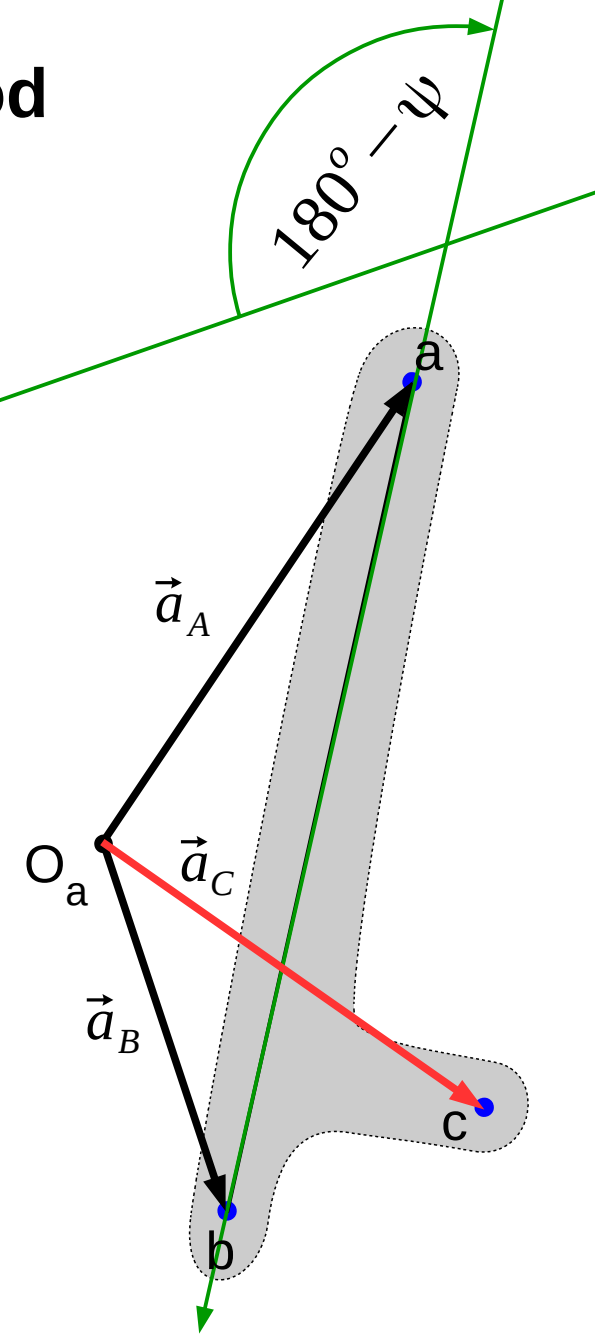
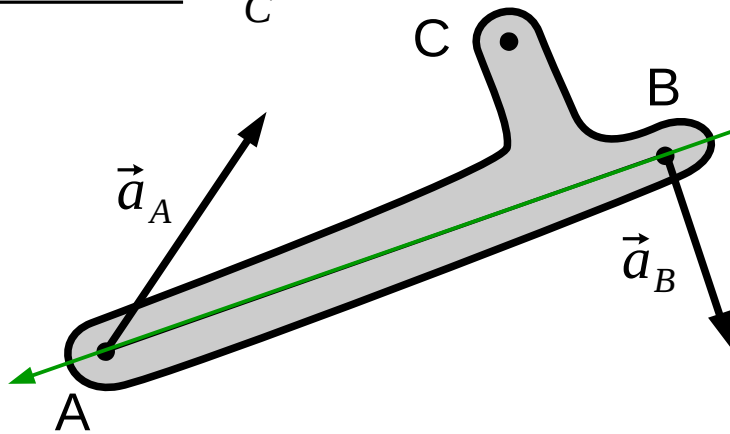
acceleration scale, e.g.: 1cm  $\rightarrow$  1m/s

# Acceleration scheme method

## Example

Given:  $\bar{a}_A$  and  $\bar{a}_B$  + geometry

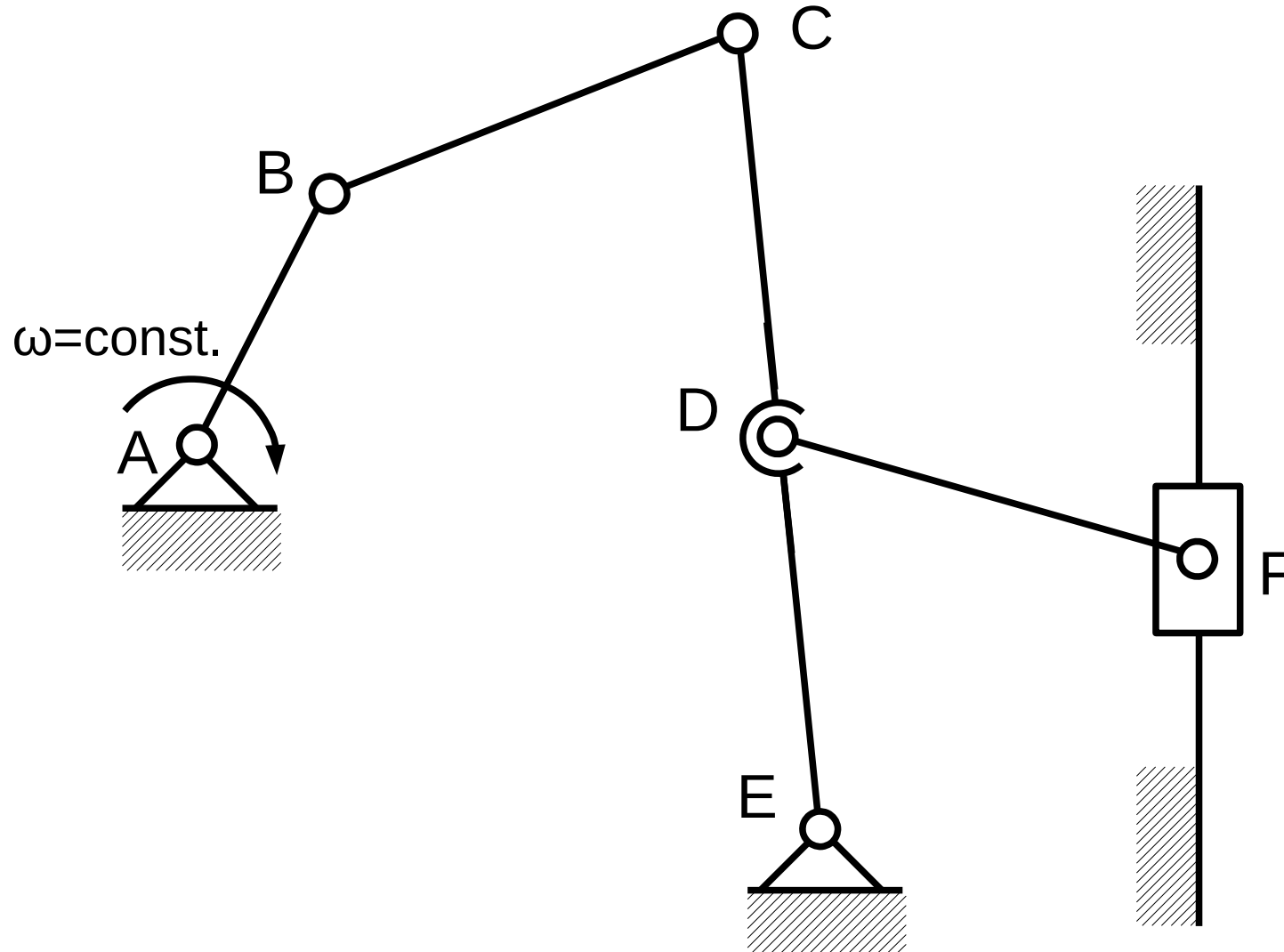
Searched:  $\bar{a}_C$



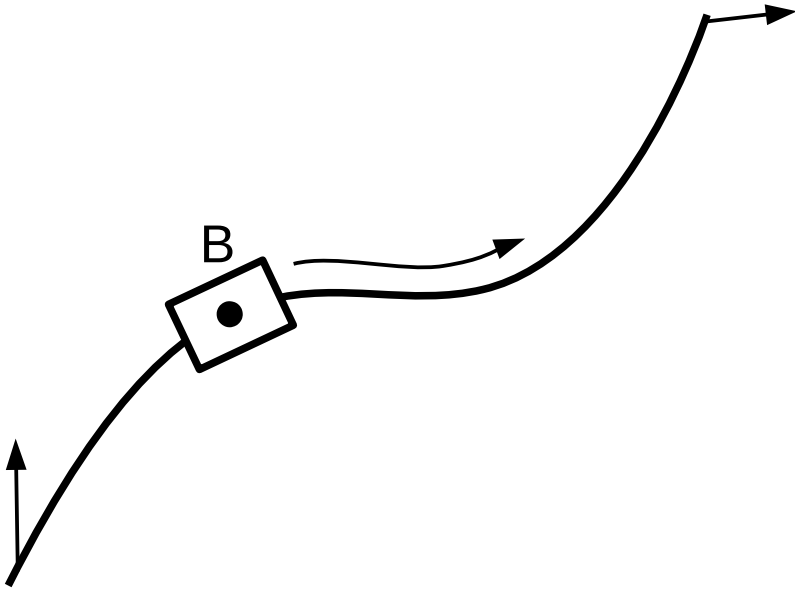
acceleration scale, e.g.: 1cm  $\rightarrow$  1m/s

# Accelerations

## Example

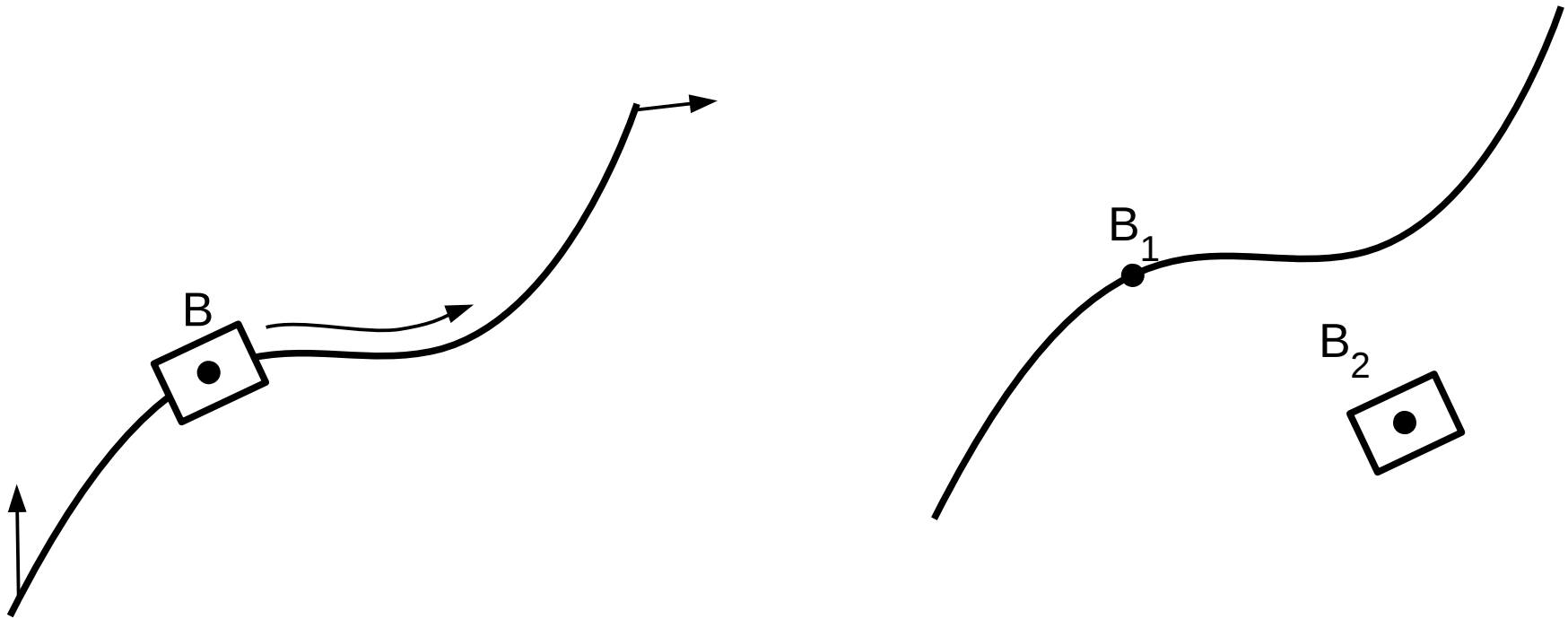


# Accelerations in relative motion

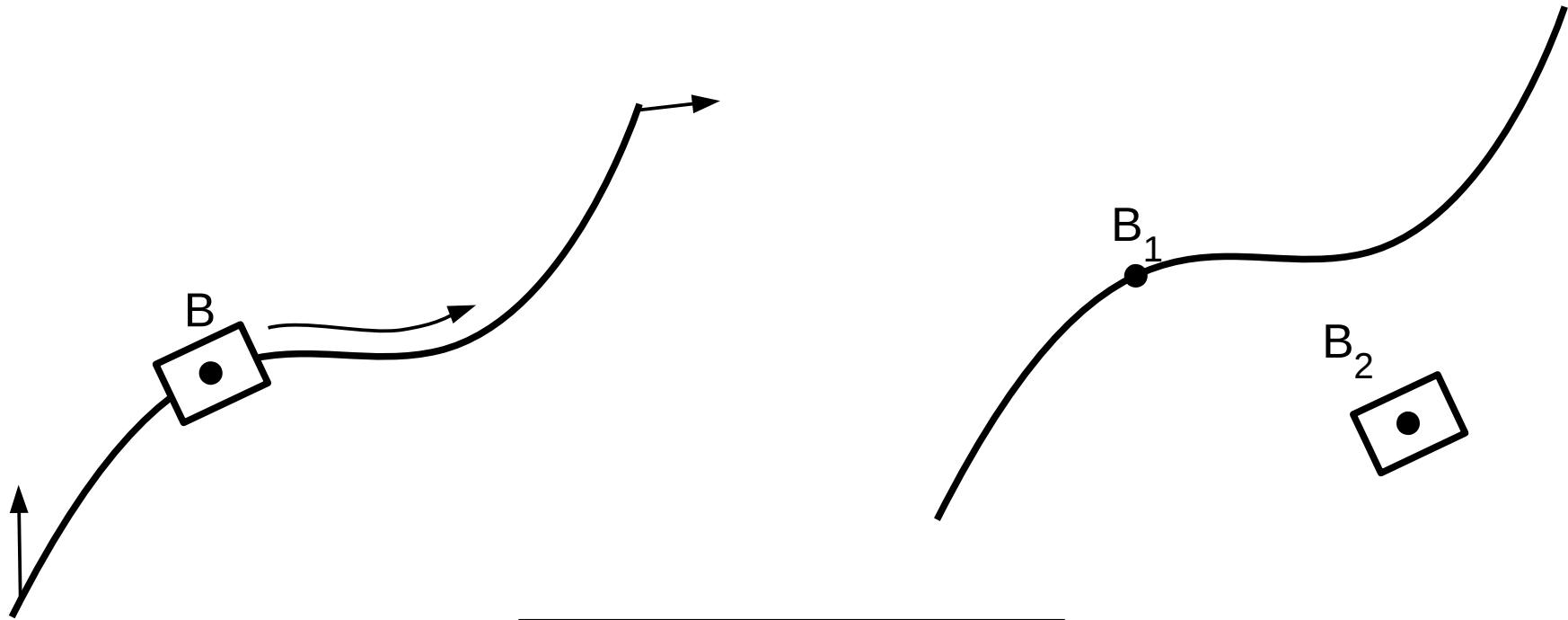




# Accelerations in relative motion



# Accelerations in relative motion



$$\vec{a}_{B2} = \vec{a}_{B1}^u + \vec{a}_{B2B1}^w + \vec{a}^c$$

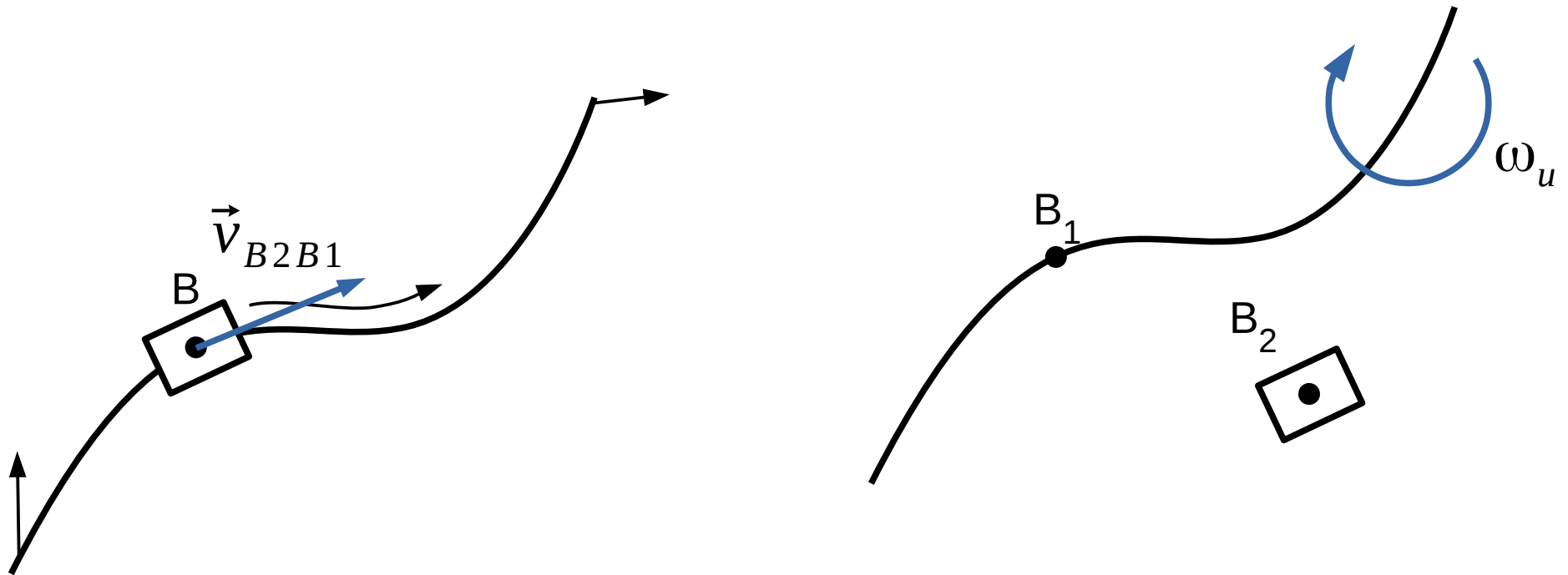
absolute acceleration  
of point B2

Transportation acceleration  
(absolute acceleration of  
point B1)

Relative  
acceleration

Coriolis  
acceleration

# Accelerations in relative motion



$$\vec{a}_{B2} = \vec{a}_{B1}^u + \vec{a}_{B2B1}^w + \vec{a}^c$$

absolute acceleration  
of point B2

Transportation acceleration  
(absolute acceleration of  
point B1)

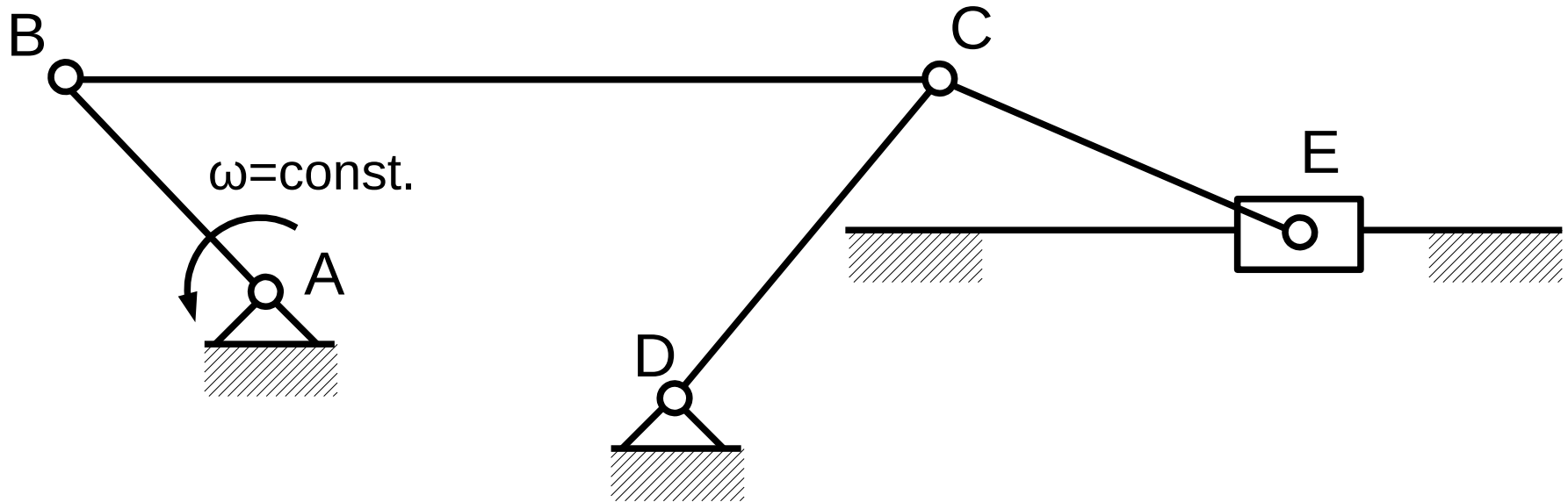
Relative  
acceleration

Coriolis  
acceleration

$$\vec{a}^c = 2 \vec{\omega}_u \times \vec{v}_{B2B1}$$

# Accelerations

## Example



# Accelerations in relative motion

## Example

