



Faculty of Automotive and Construction Machinery Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Theory of Machines and Automatic Control Winter 2018/2019

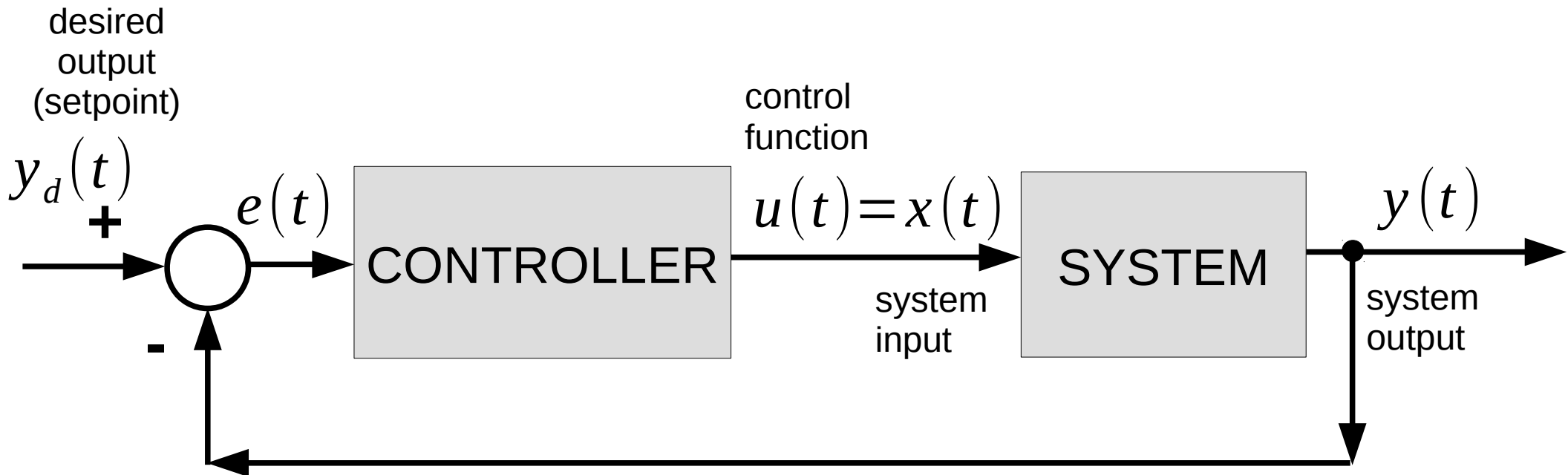
Lecturer: Sebastian Korczak, PhD, Eng.

Lecture 12

PID controller. Stability.

Materials license: only for educational purposes of Warsaw University of Technology students.

Closed loop control



PID transfer functions

Controller	Transfer function
Proportional (P)	k_P
Integral (I)	$\frac{1}{T_i s}$
Ideal derivative (D)	$T_d s$
Real derivative (D)	$\frac{T_d s}{T s + 1}$

PID transfer functions

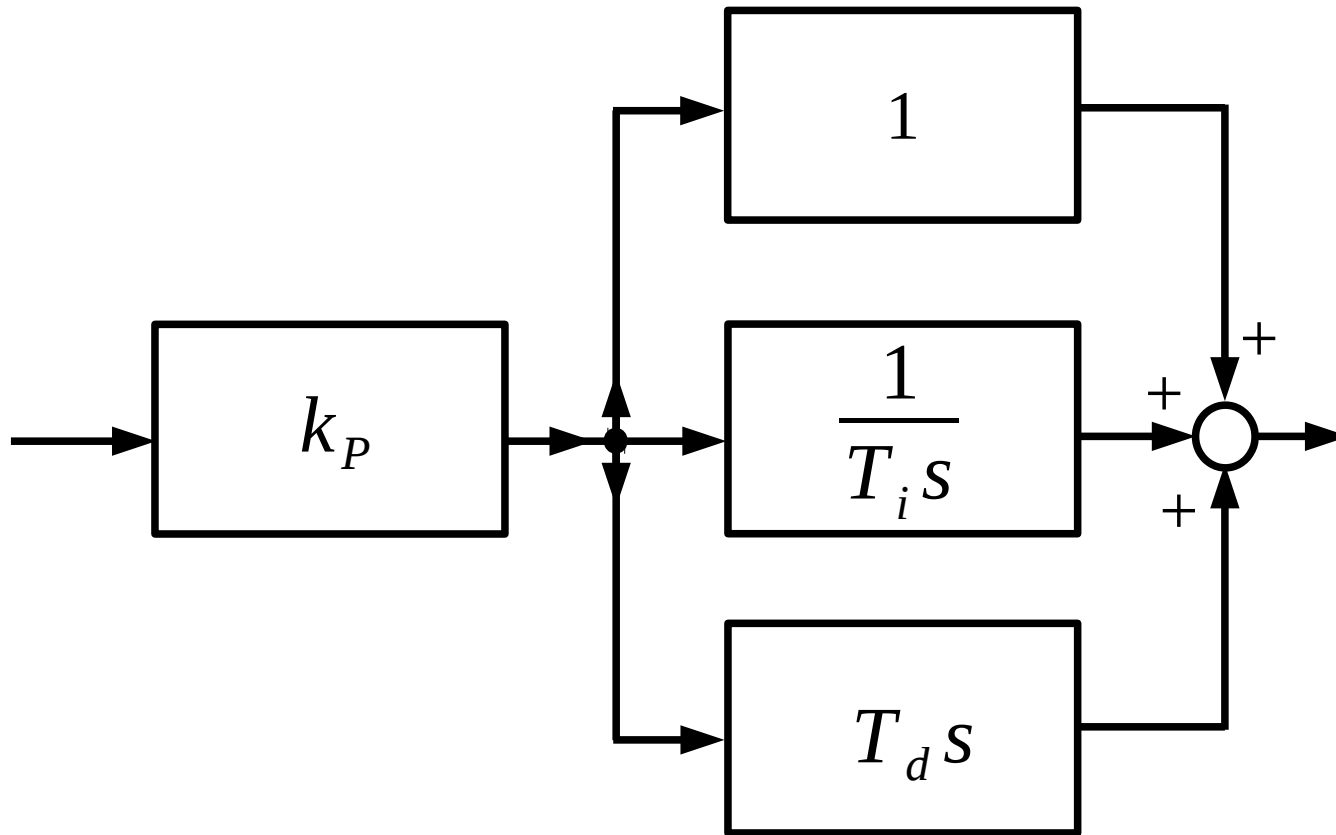
Controller	Transfer function
Proportional-integral-derivative (PID) <u>in standard form</u> with ideal derivative	$k_P \left(1 + \frac{1}{T_i s} + T_d s \right)$
Proportional-integral-derivative (PID) <u>in parallel form</u> with ideal derivative	$k_P + k_i \frac{1}{s} + k_d s$

PID transfer functions

Controller	Transfer function
Proportional-integral-derivative (PID) <u>in standard form</u> with real derivative	$k_P \left(1 + \frac{1}{T_i s} + \frac{T_d s}{Ts + 1} \right)$
Proportional-integral-derivative (PID) <u>in parallel form</u> with real derivative	$k_P + k_i \frac{1}{s} + k_d \frac{s}{Ts + 1}$

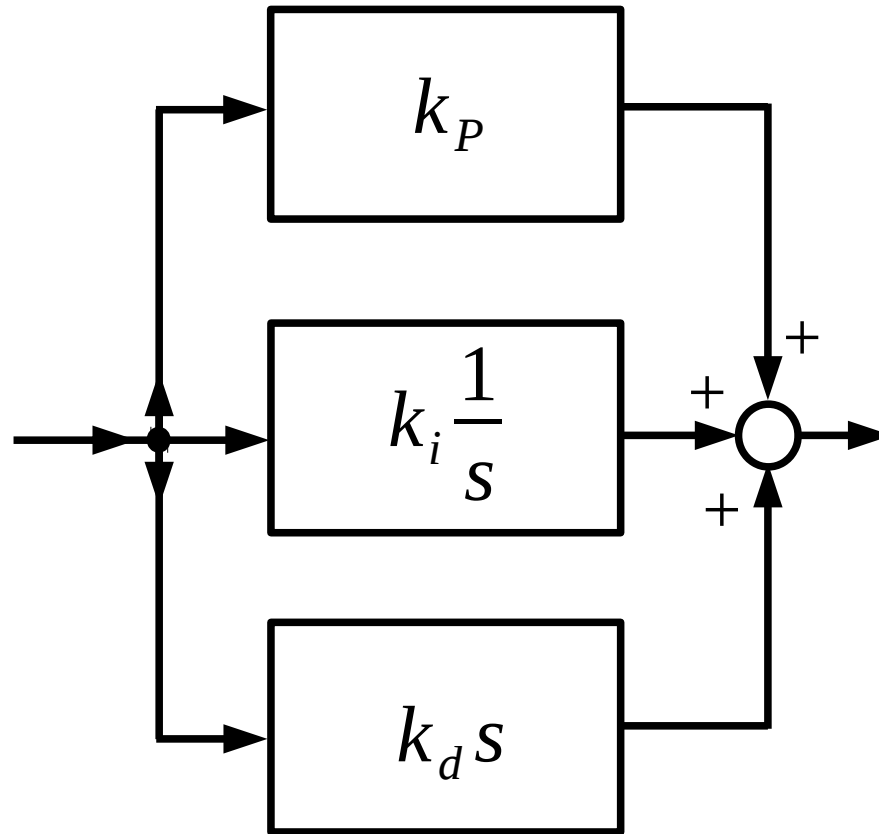
PID CONTROLLER

standard form with ideal derivative

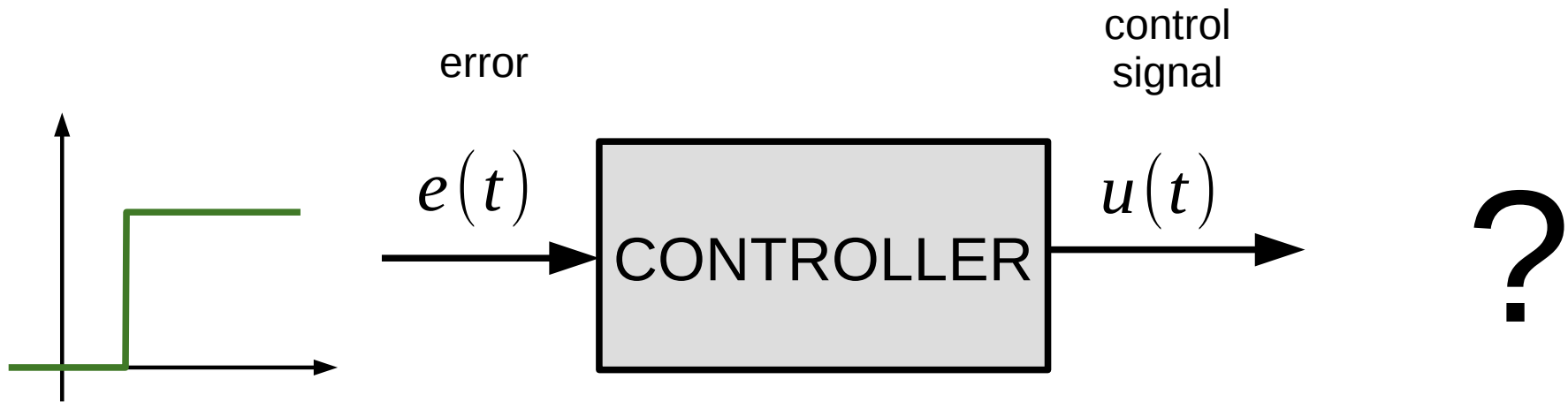


PID CONTROLLER

parallel form with ideal derivative

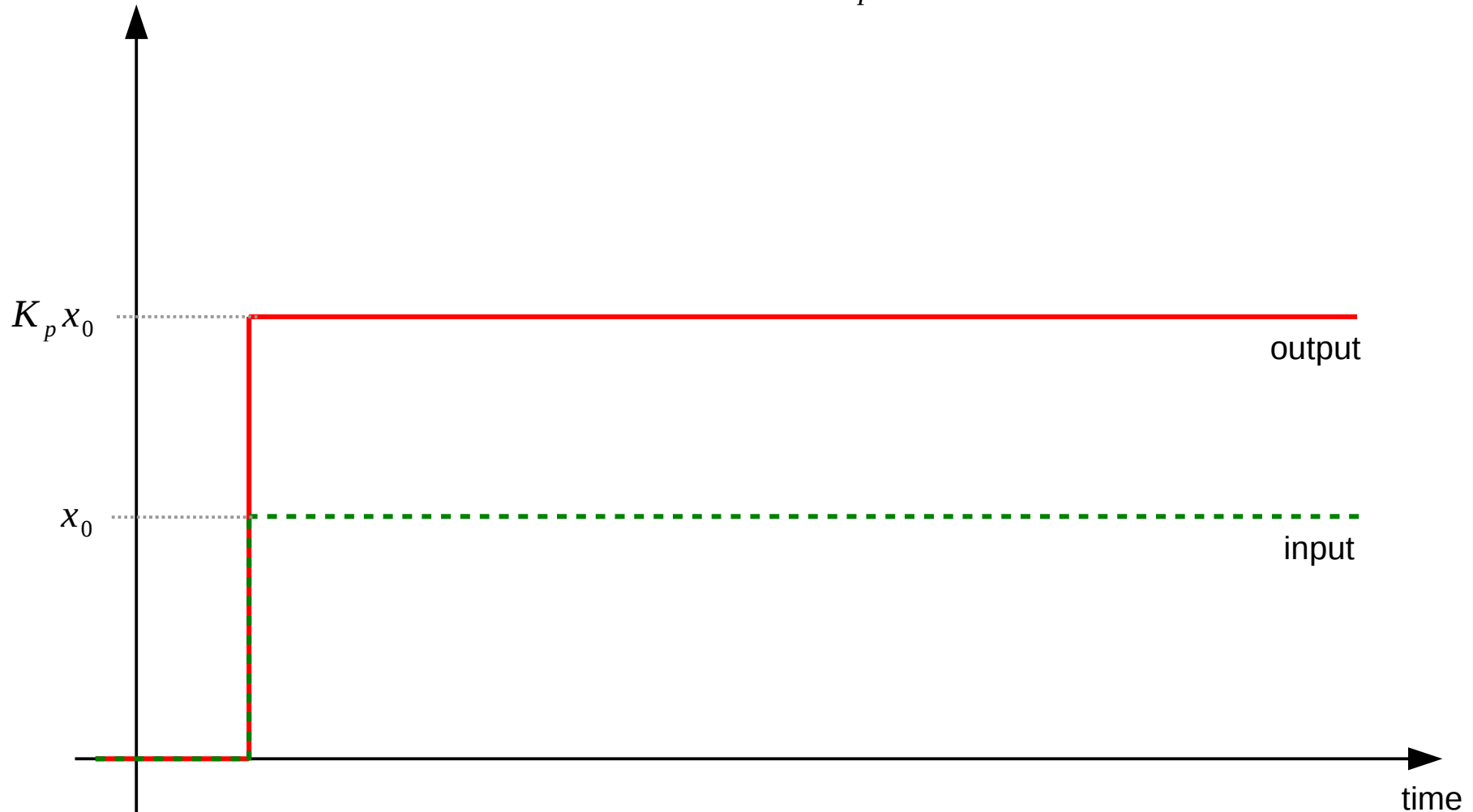


PID CONTROLLER step responses



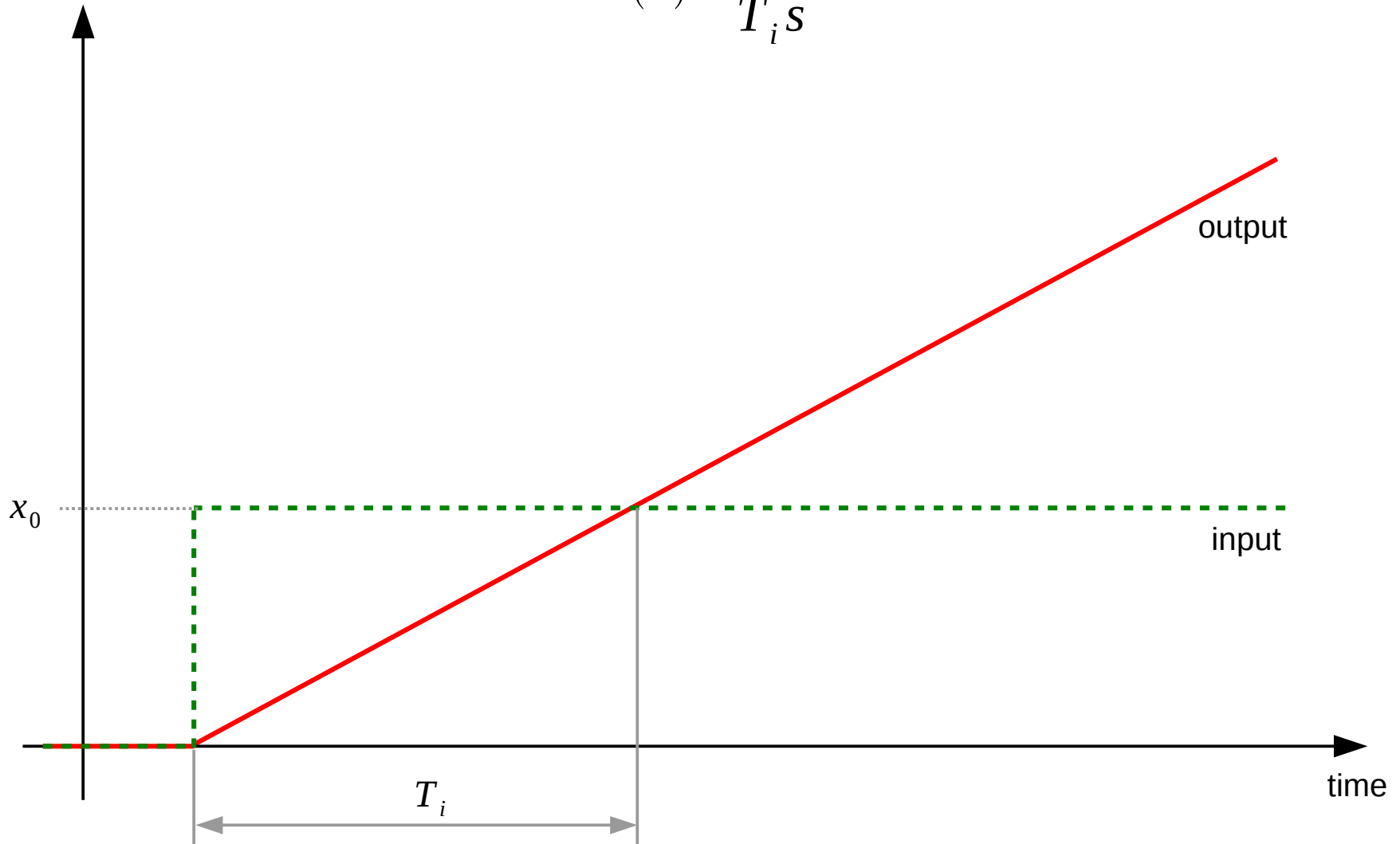
P - CONTROLLER

$$G(s) = K_p$$



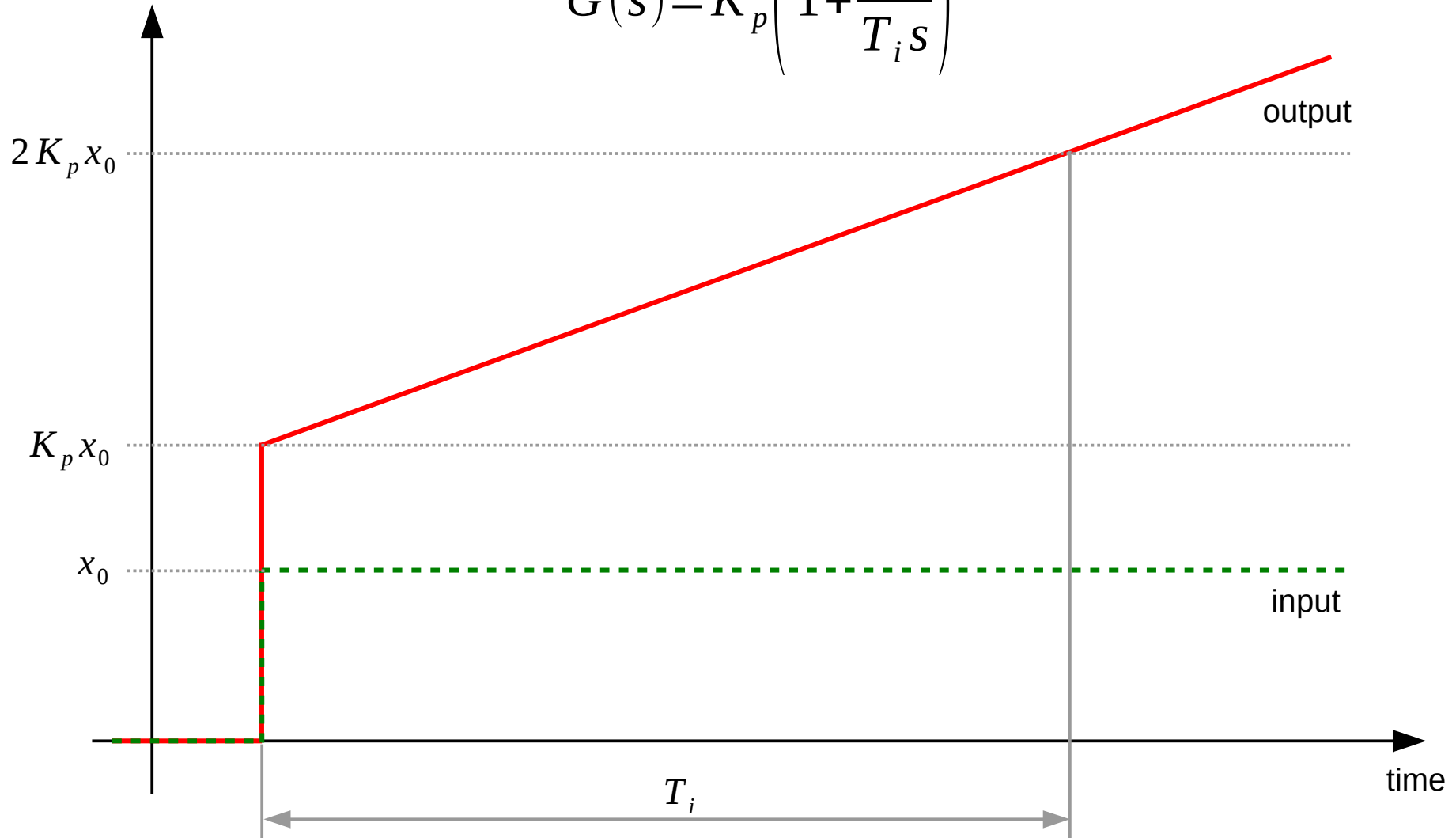
I - CONTROLLER

$$G(s) = \frac{1}{T_i s}$$



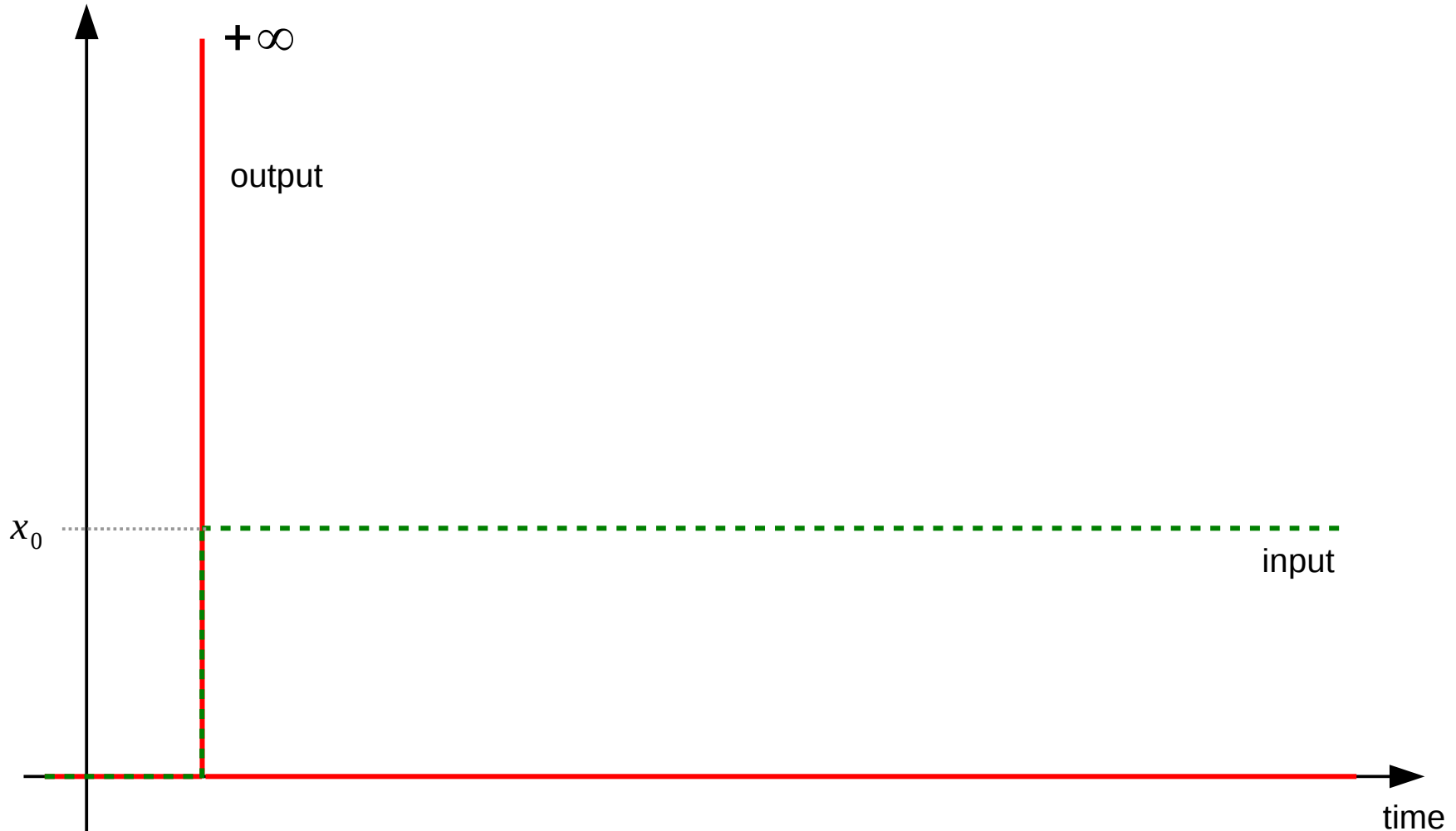
PI - CONTROLLER

$$G(s) = K_p \left(1 + \frac{1}{T_i s} \right)$$



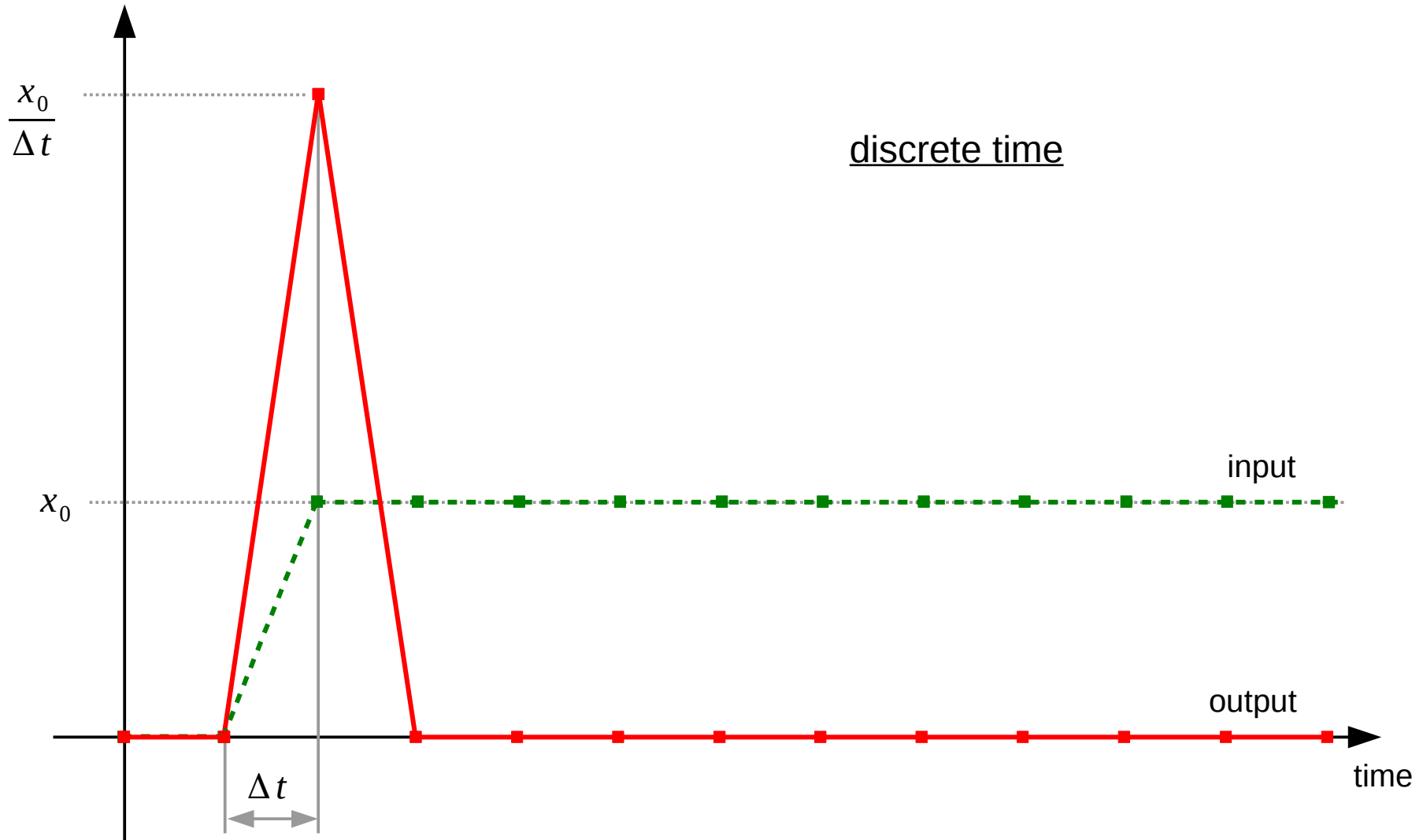
D - CONTROLLER

$$G(s) = T_d s$$



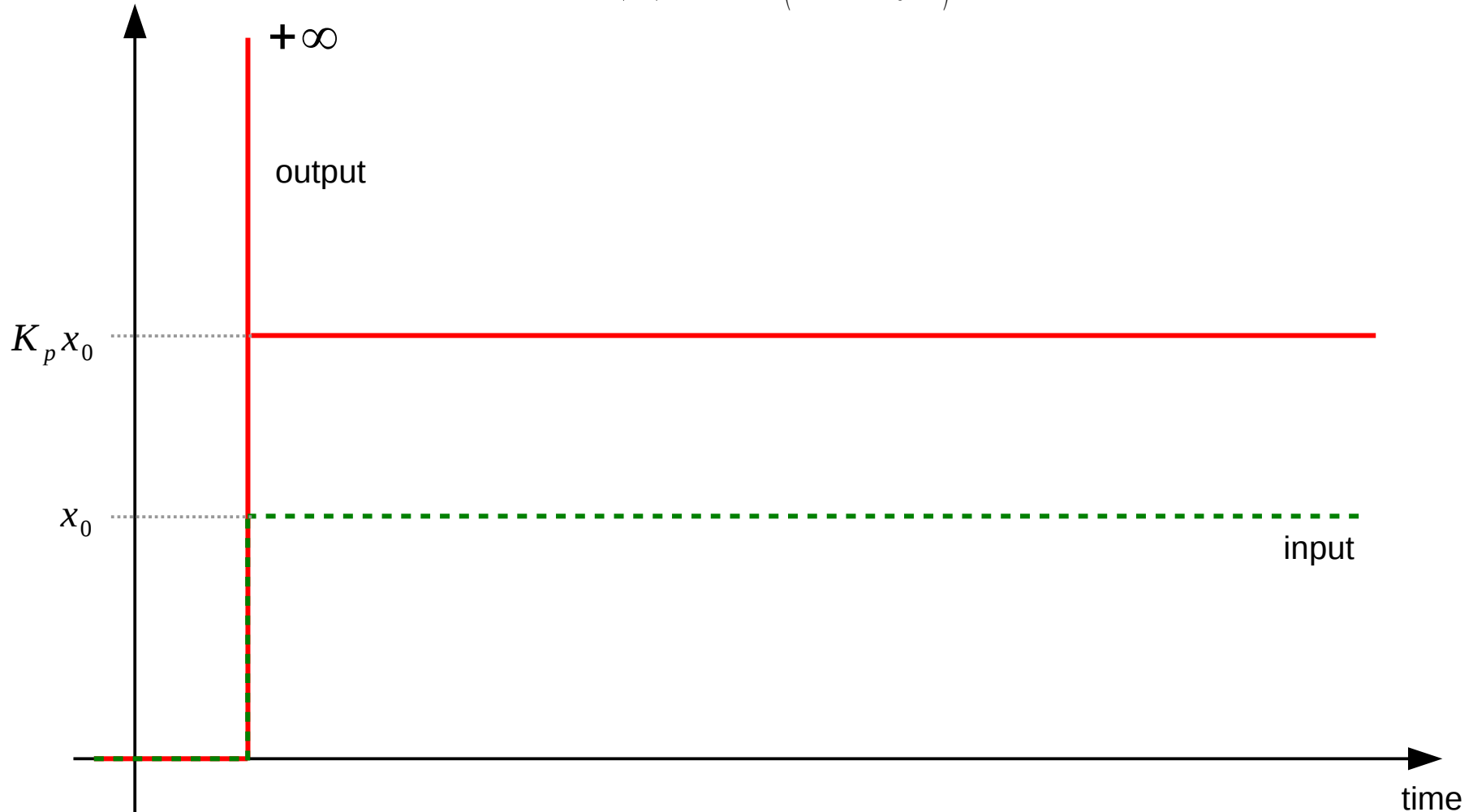
D - CONTROLLER

$$G(s) = T_d s$$



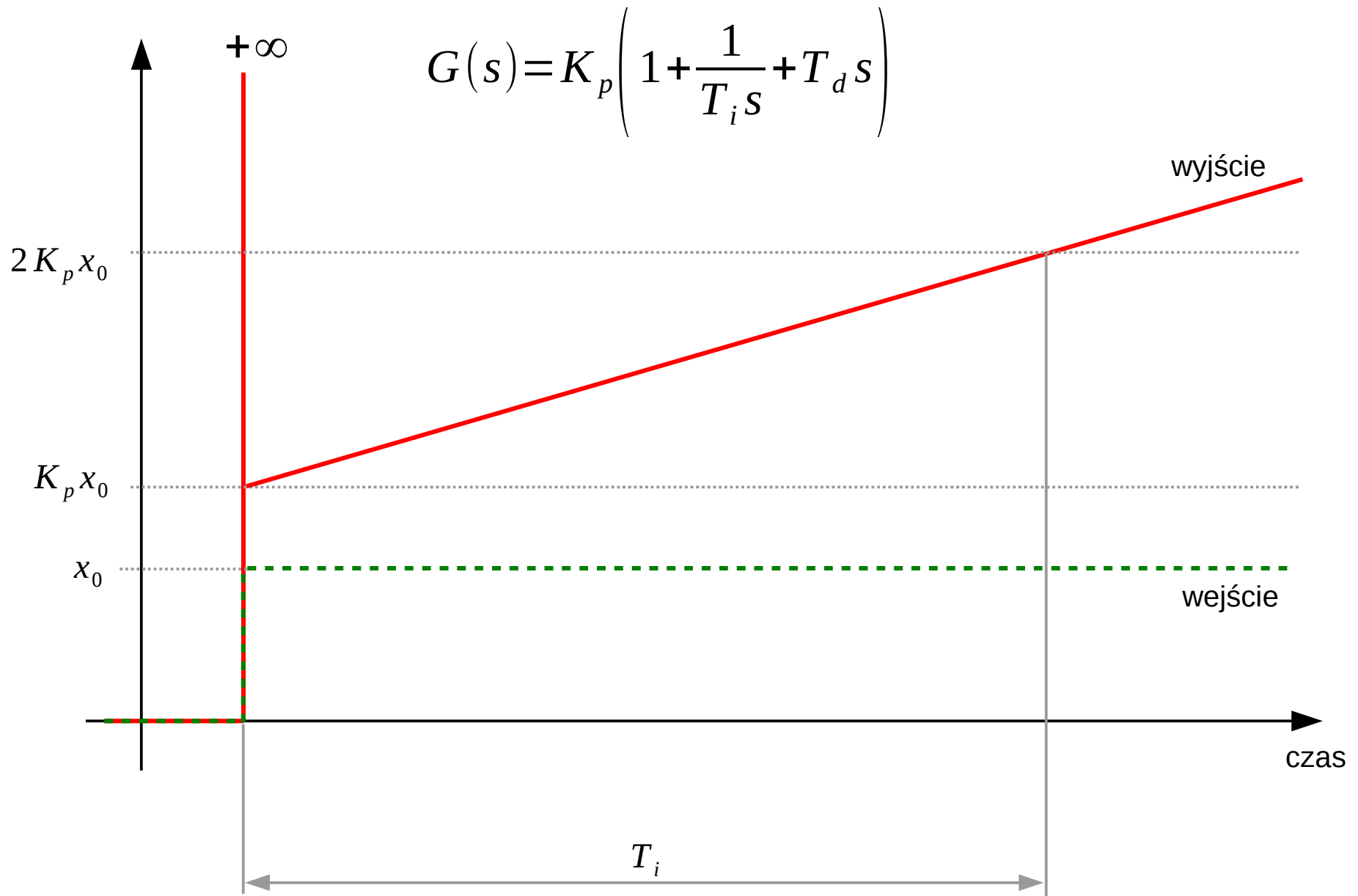
PD - CONTROLLER

$$G(s) = K_P(1 + T_d s)$$



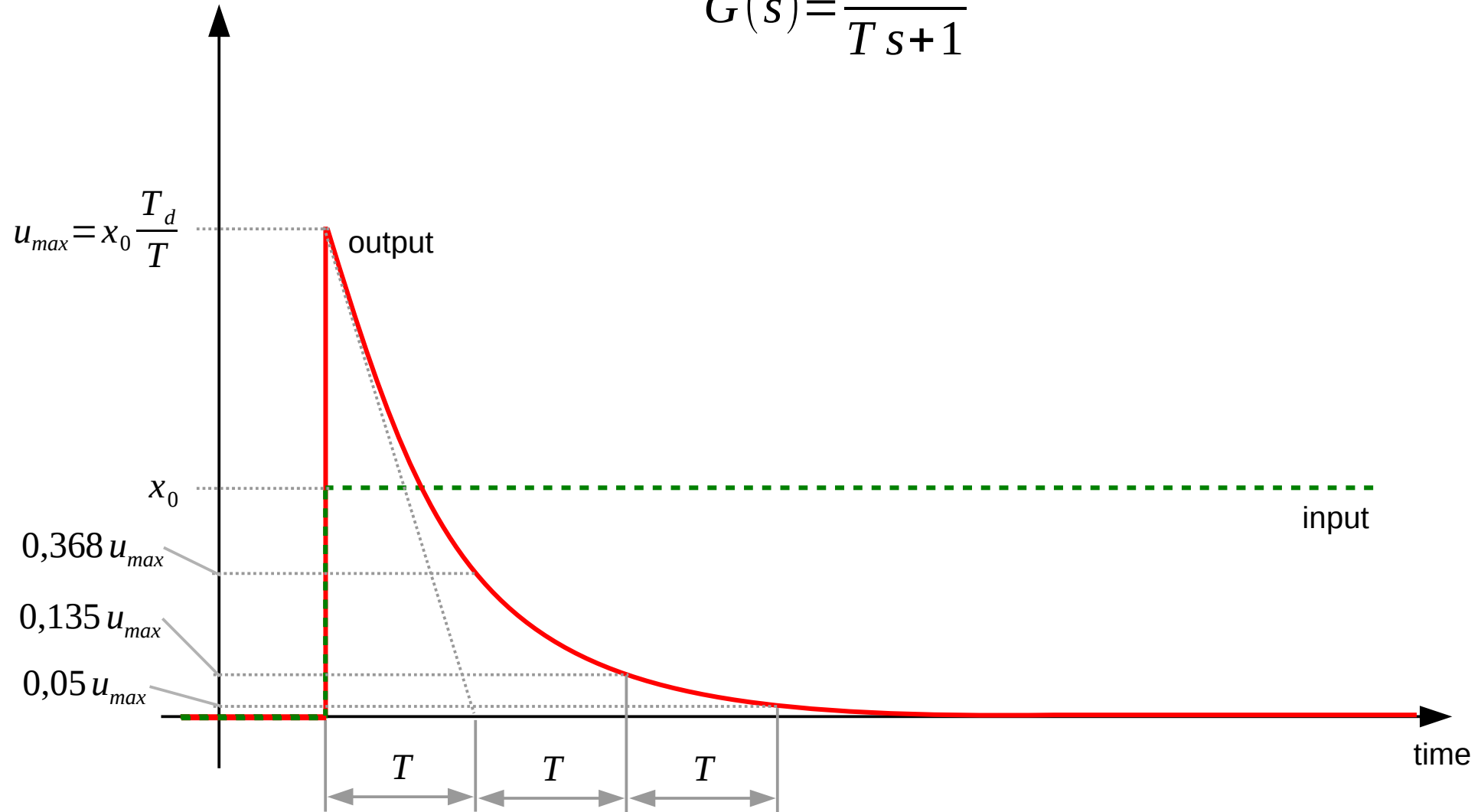
PID – CONTROLLER

standard form, ideal derivative



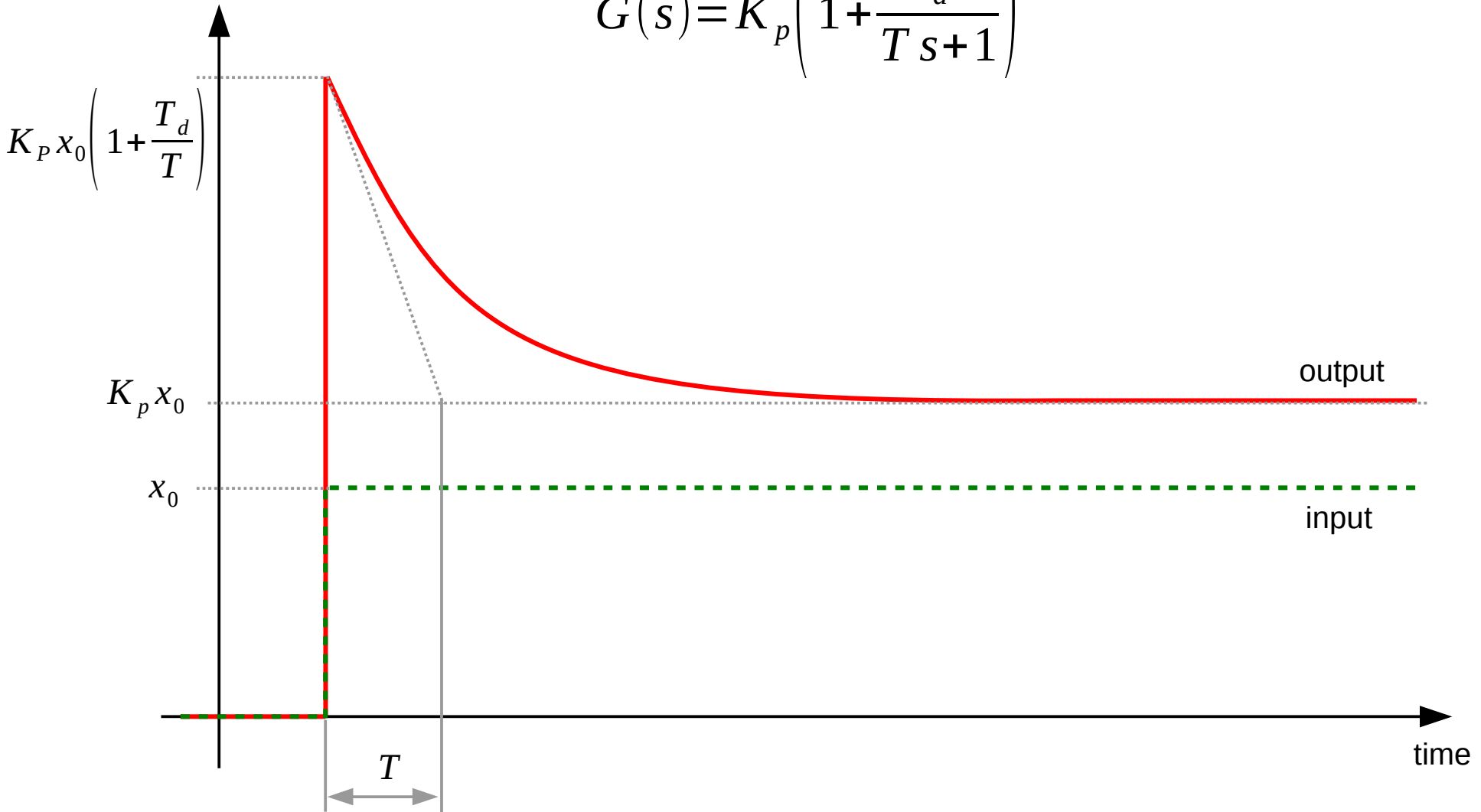
D - CONTROLLER

$$G(s) = \frac{T_d s}{T s + 1}$$



PD - CONTROLLER

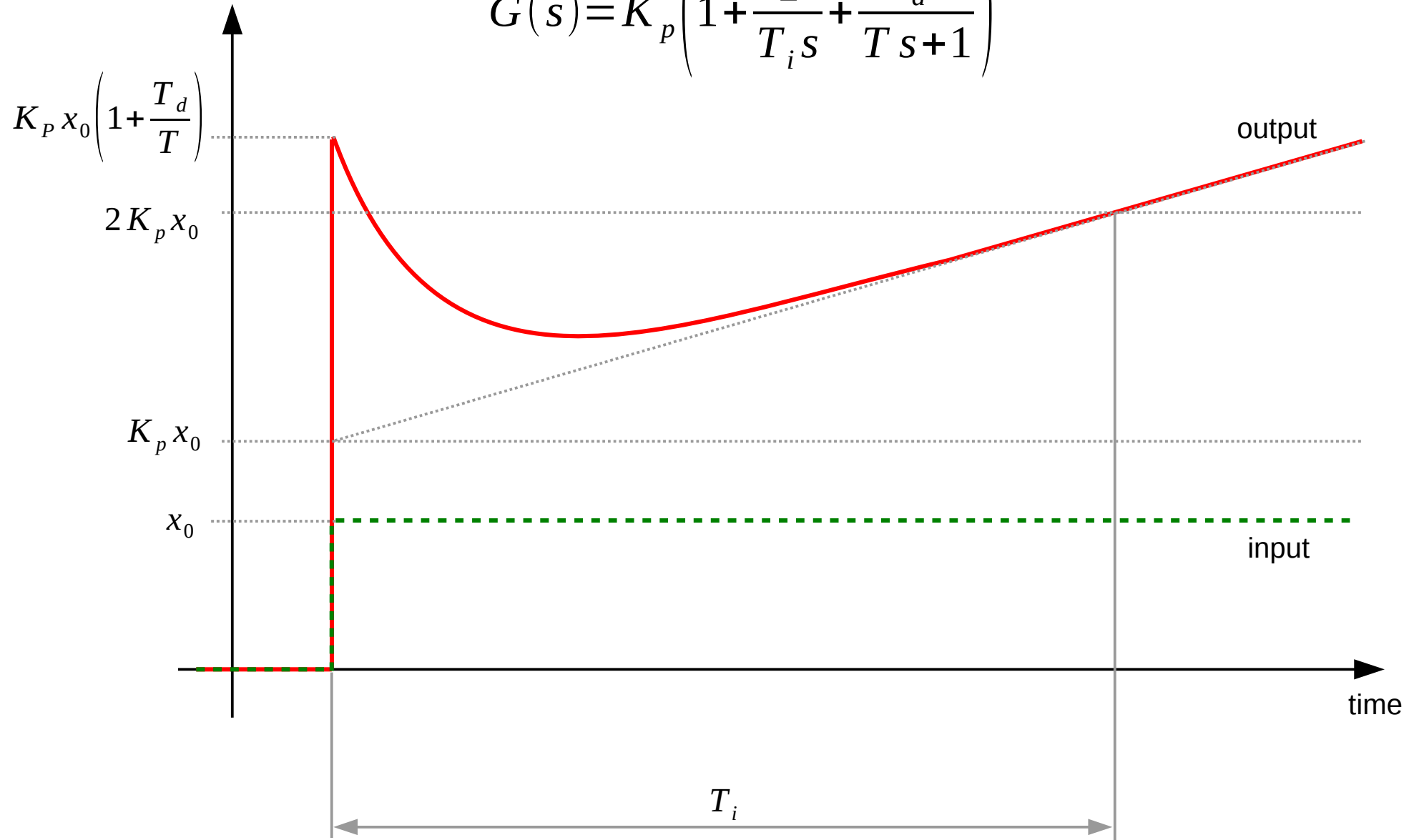
$$G(s) = K_p \left(1 + \frac{T_d s}{T s + 1} \right)$$



PID – CONTROLLER

standard form, real derivative

$$G(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{T s + 1} \right)$$



PID CONTROLLER

important notes

Proportional term – necessary part of the controller, creates a main part of control signal that bring output of the system closer to desired value; higher K_p coefficient gives lower errors; control signal is based on present error;

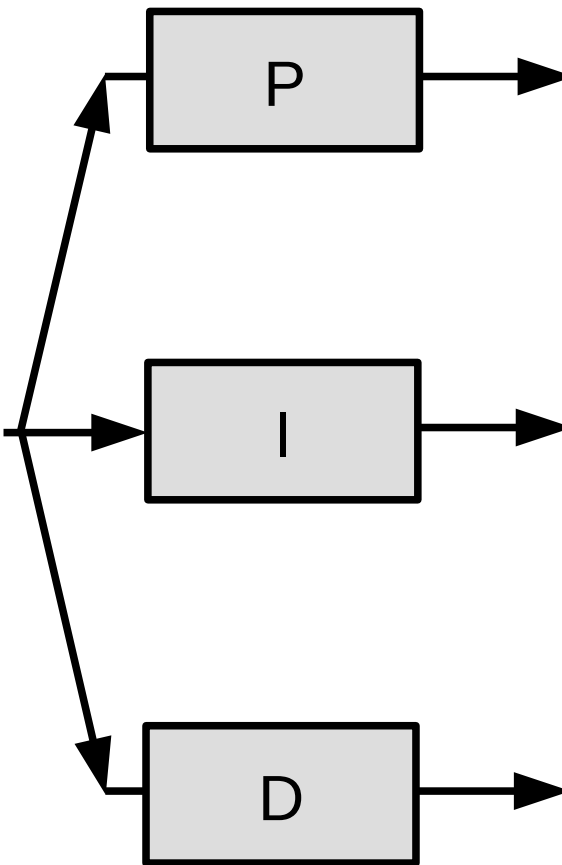
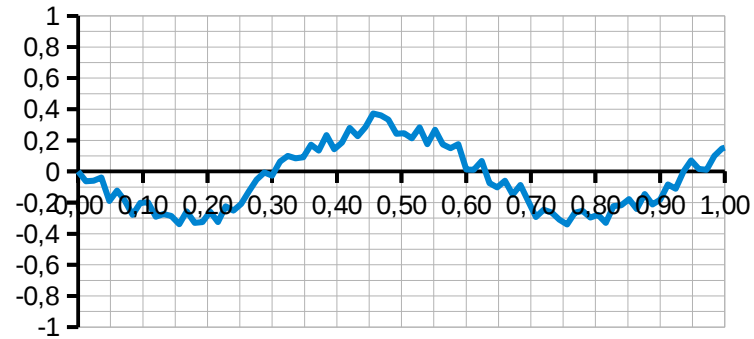
Integral term – this part of the controller accumulates error; for nonzero error control signal increases that helps to achieve zero error; control signal is based on past error values; “integral windup” problem;

Derivative term – this part of the controller reacts on error changes; for constant error control signal is zero; control signal is based on the trend of feature error; this term is very sensitive to noise;

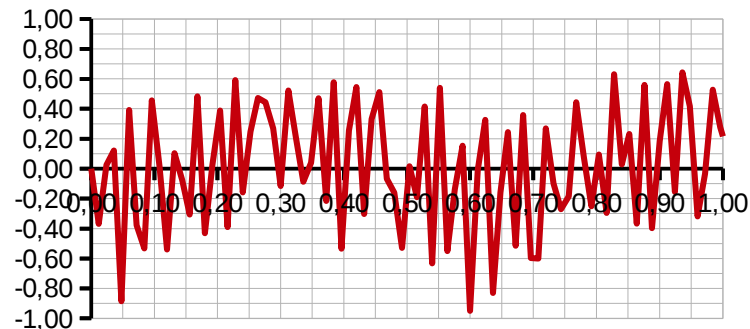
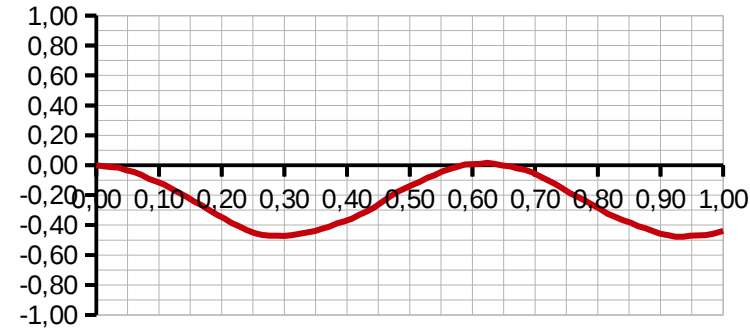
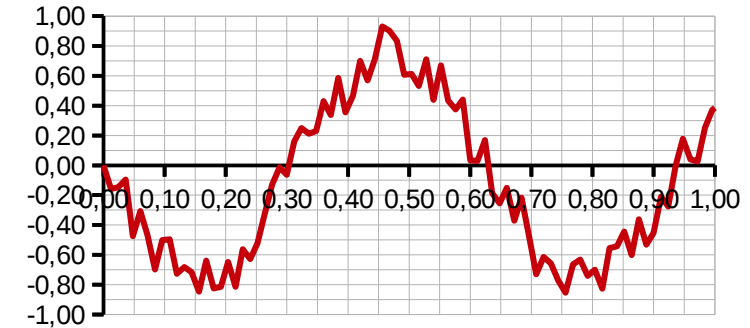
PID CONTROLLER

Influence of errors onto control signal

Control error



Control signal



PID CONTROLLER

integral windup problem

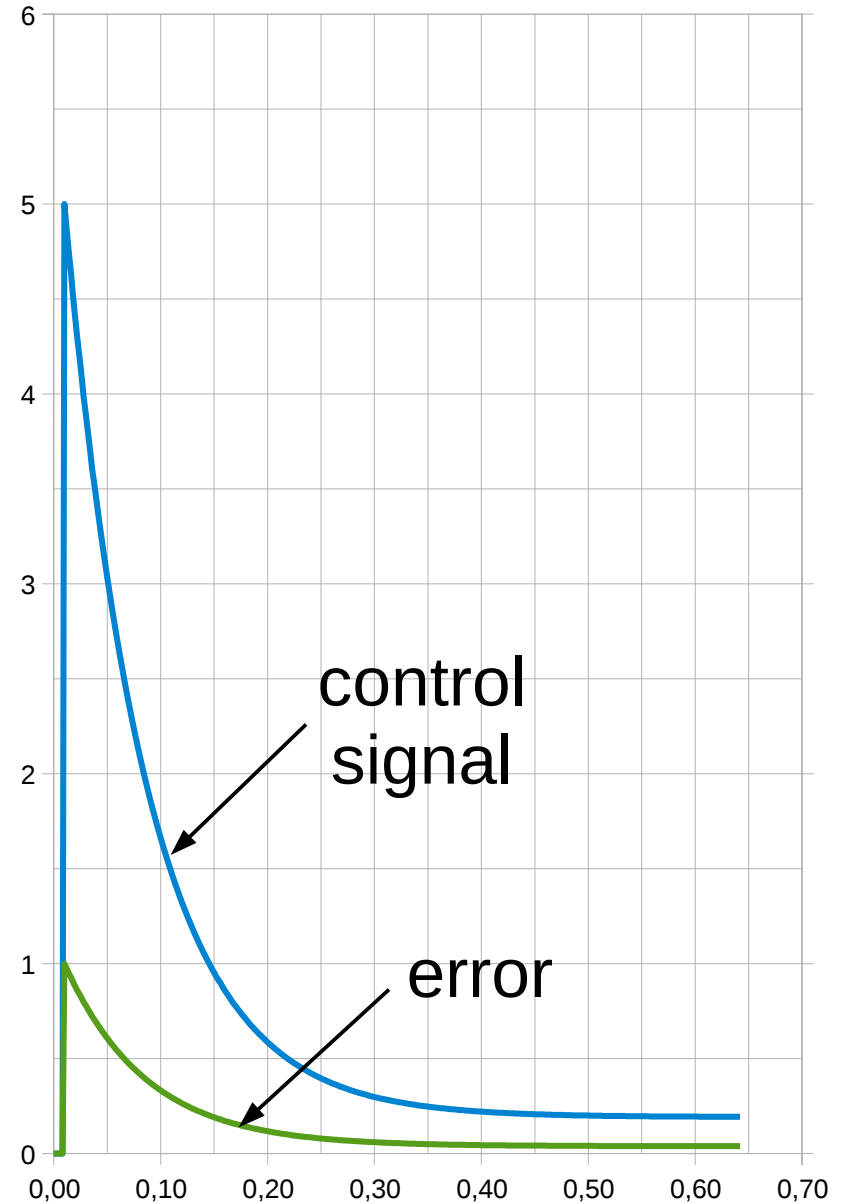
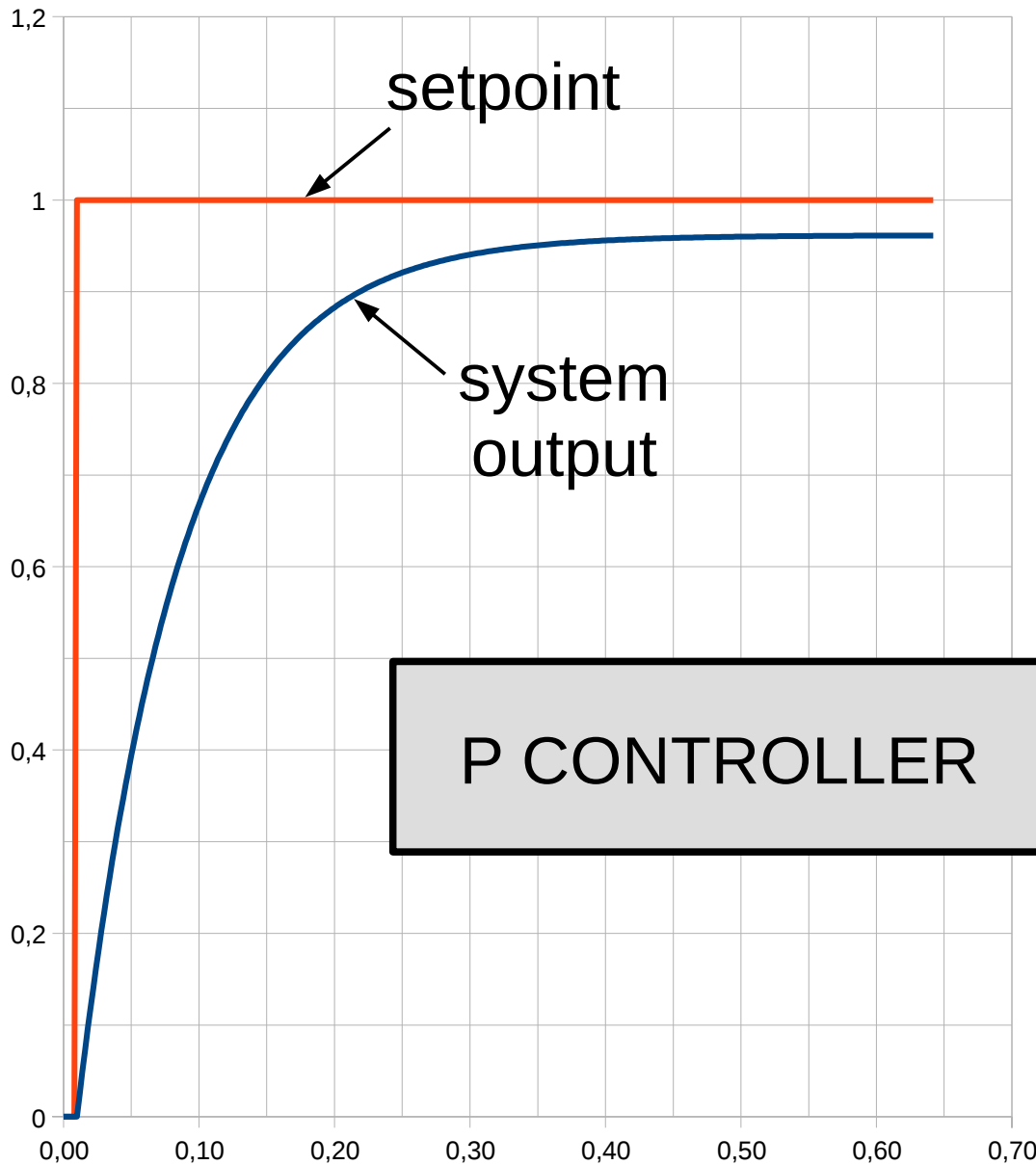
After a large change in a setpoint the integral term can produce very large control signal (higher than maximum possible) – system input is very high until accumulated error goes back close to zero.

Possible solution: disabling and zeroing integral term outside the small region around the setpoint.

Additional reading: google: “integral anti-windup for pi controllers”

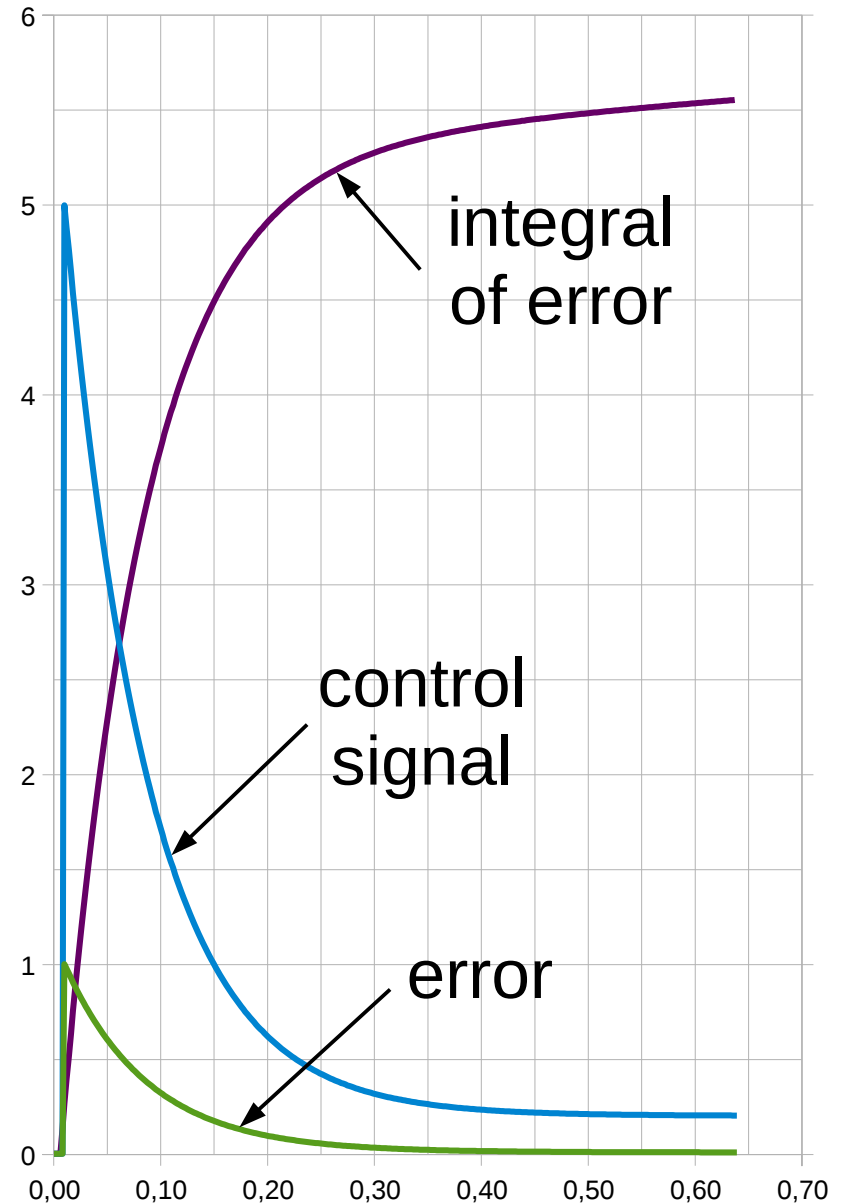
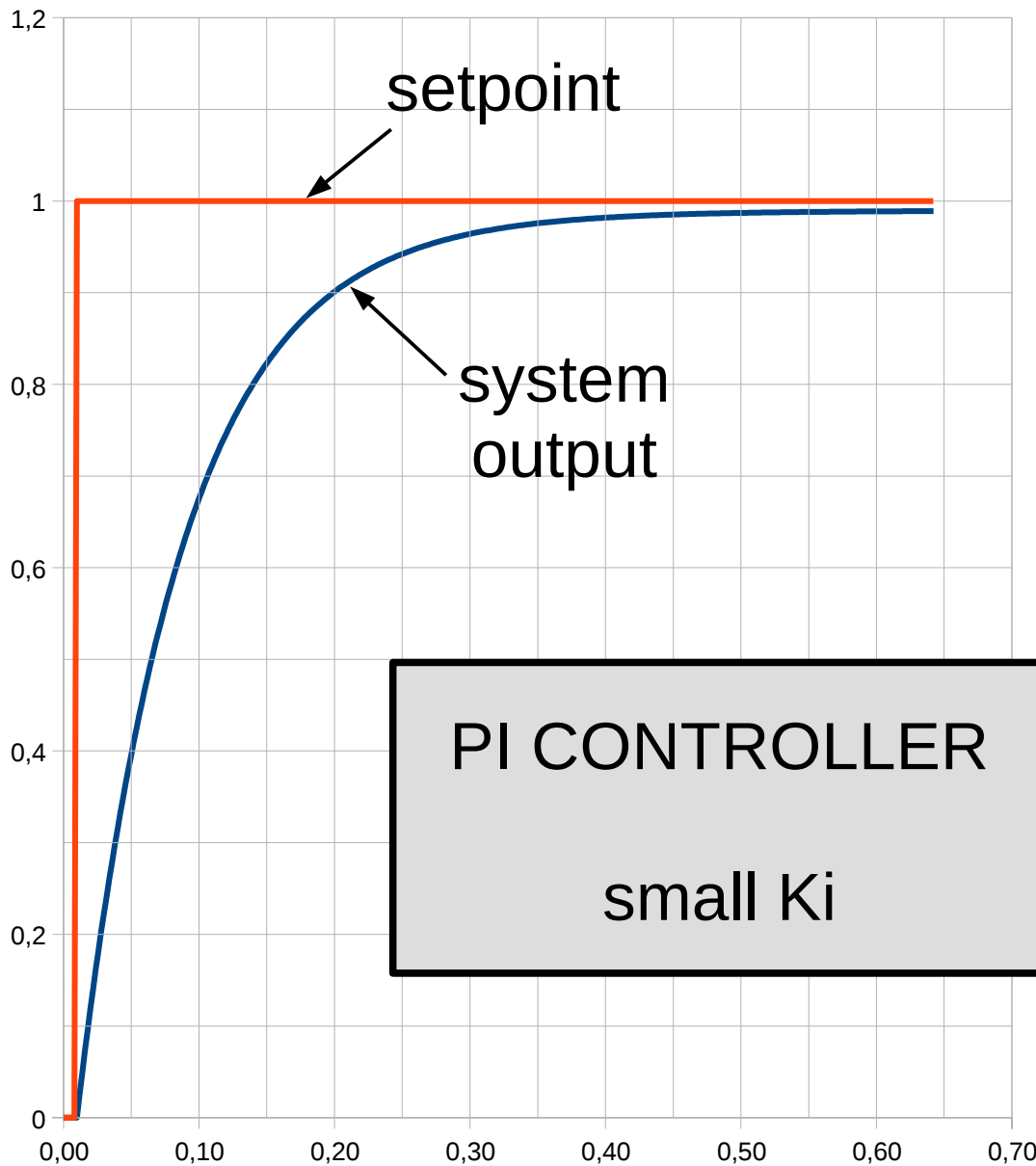
PID CONTROLLER

integral windup problem example (1st order inertial system + parallel PID)



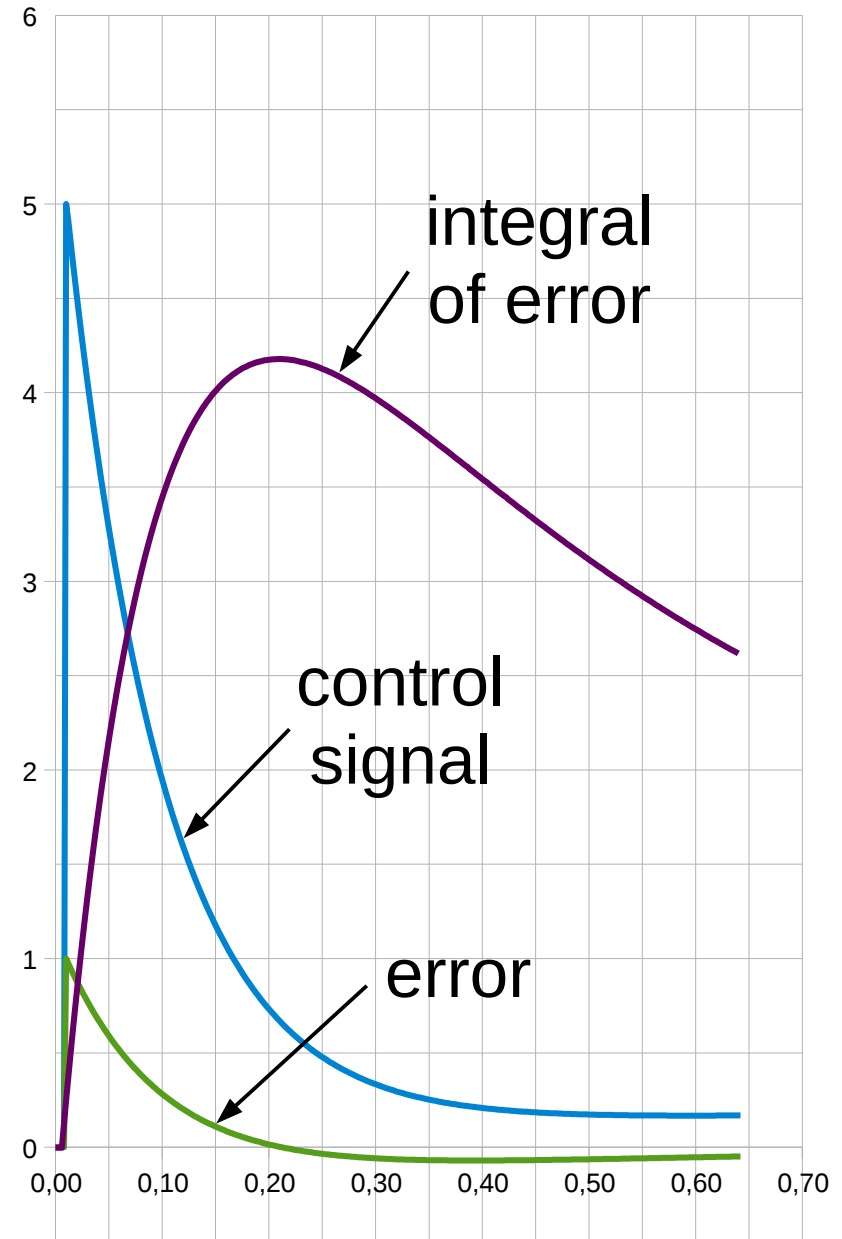
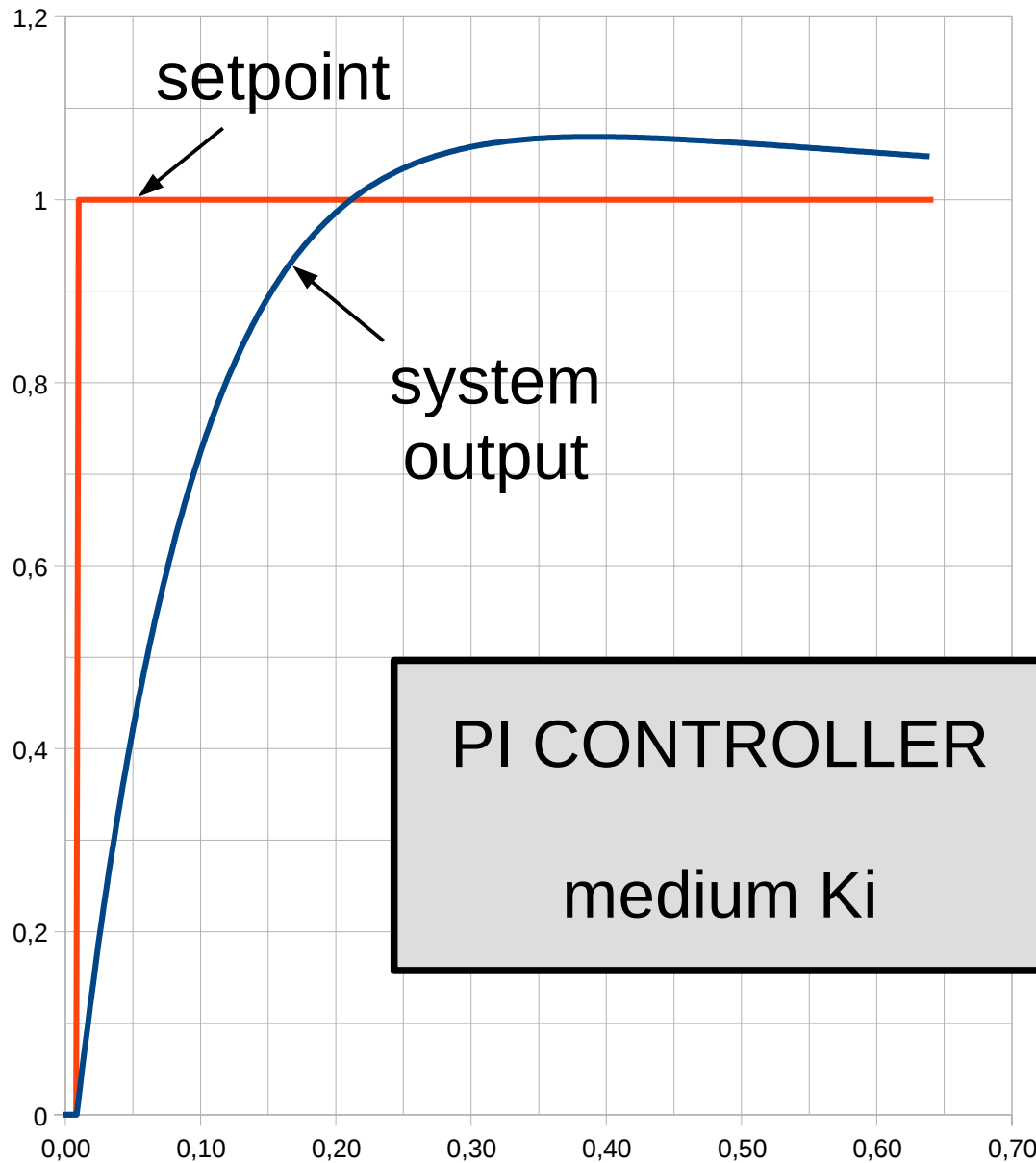
PID CONTROLLER

integral windup problem example (1st order inertial system + parallel PID)



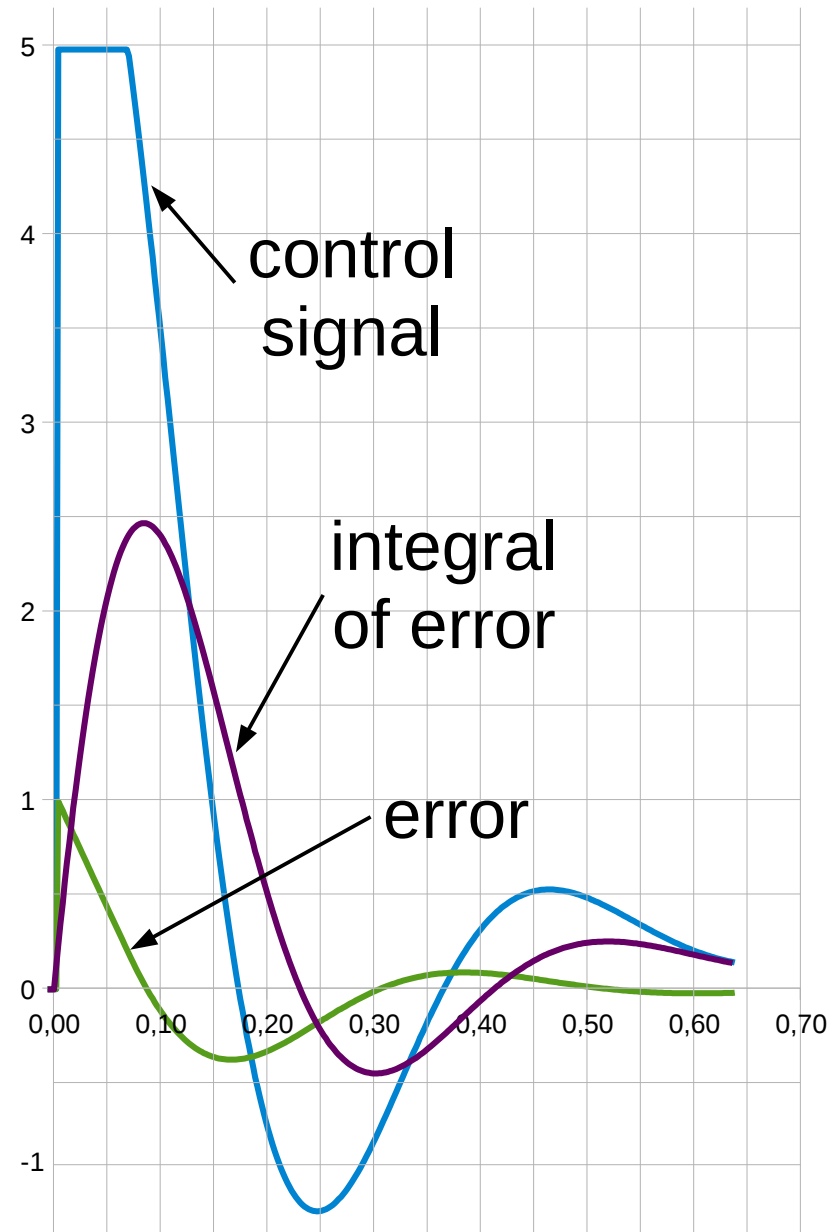
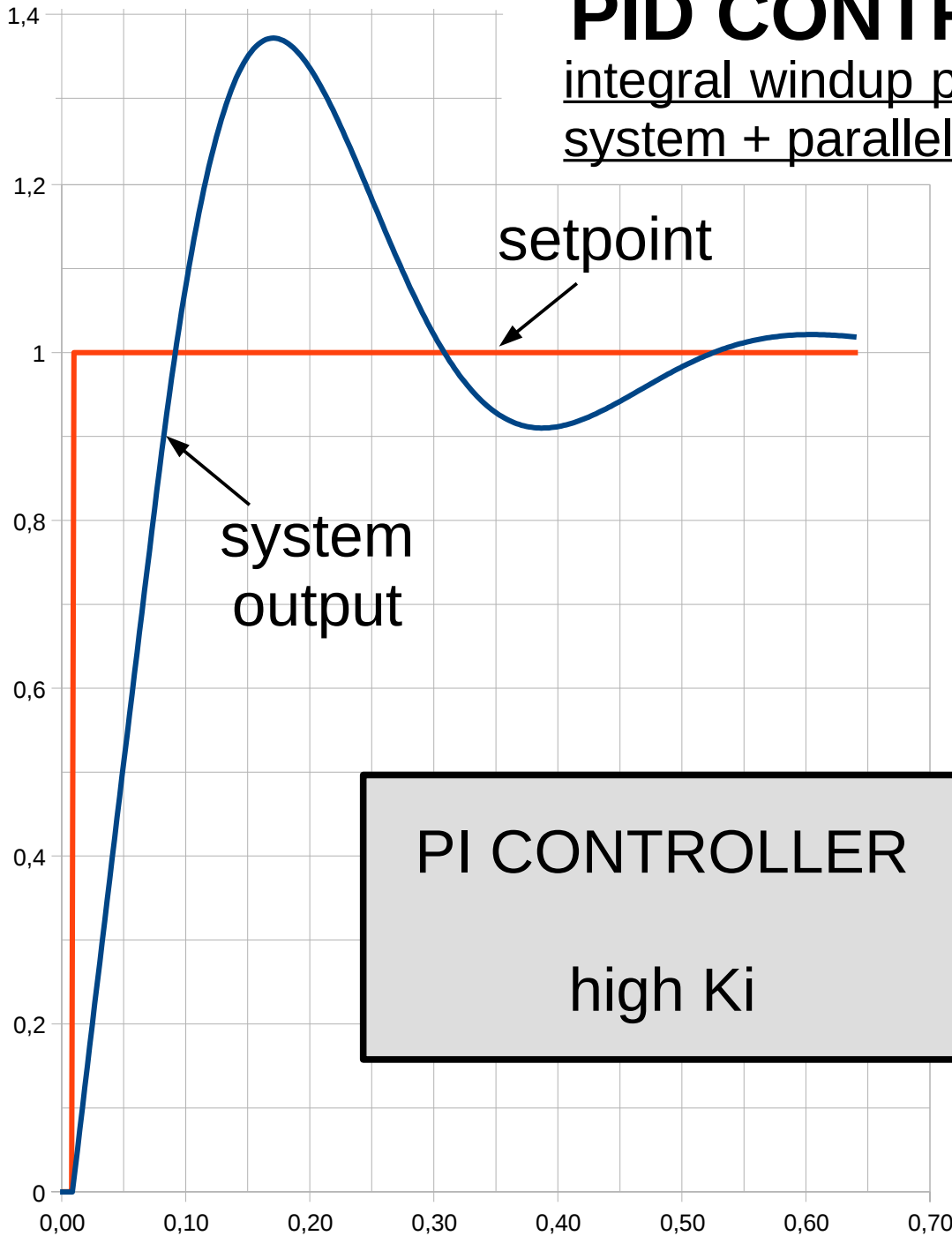
PID CONTROLLER

integral windup problem example (1st order inertial system + parallel PID)

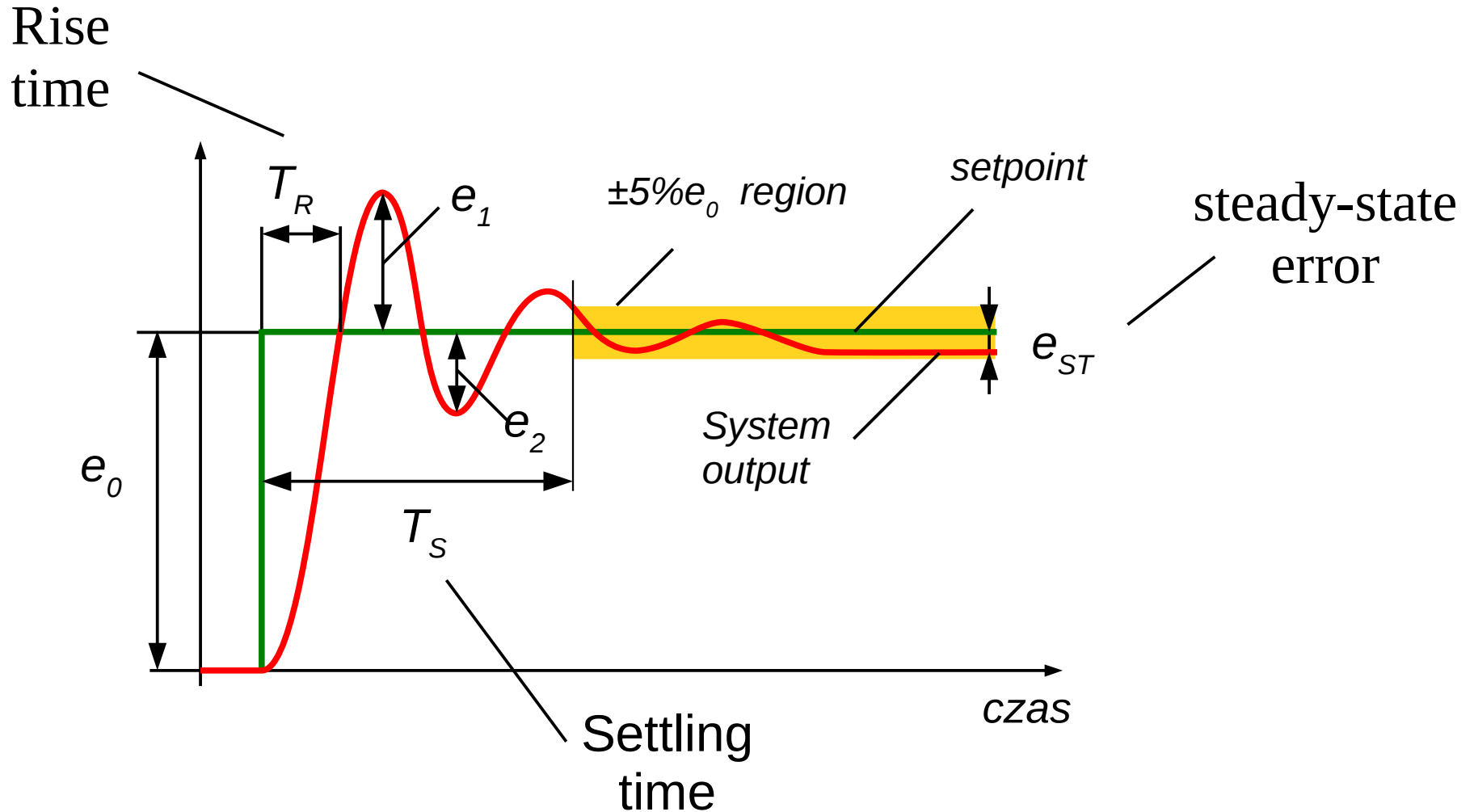


PID CONTROLLER

integral windup problem example (1st order inertial system + parallel PID)



Quality of the control process



Overshoot: $w = \frac{e_1}{e_0} 100\%$

Damping: $d = \frac{e_2}{e_1} 100\%$

PID CONTROLLER

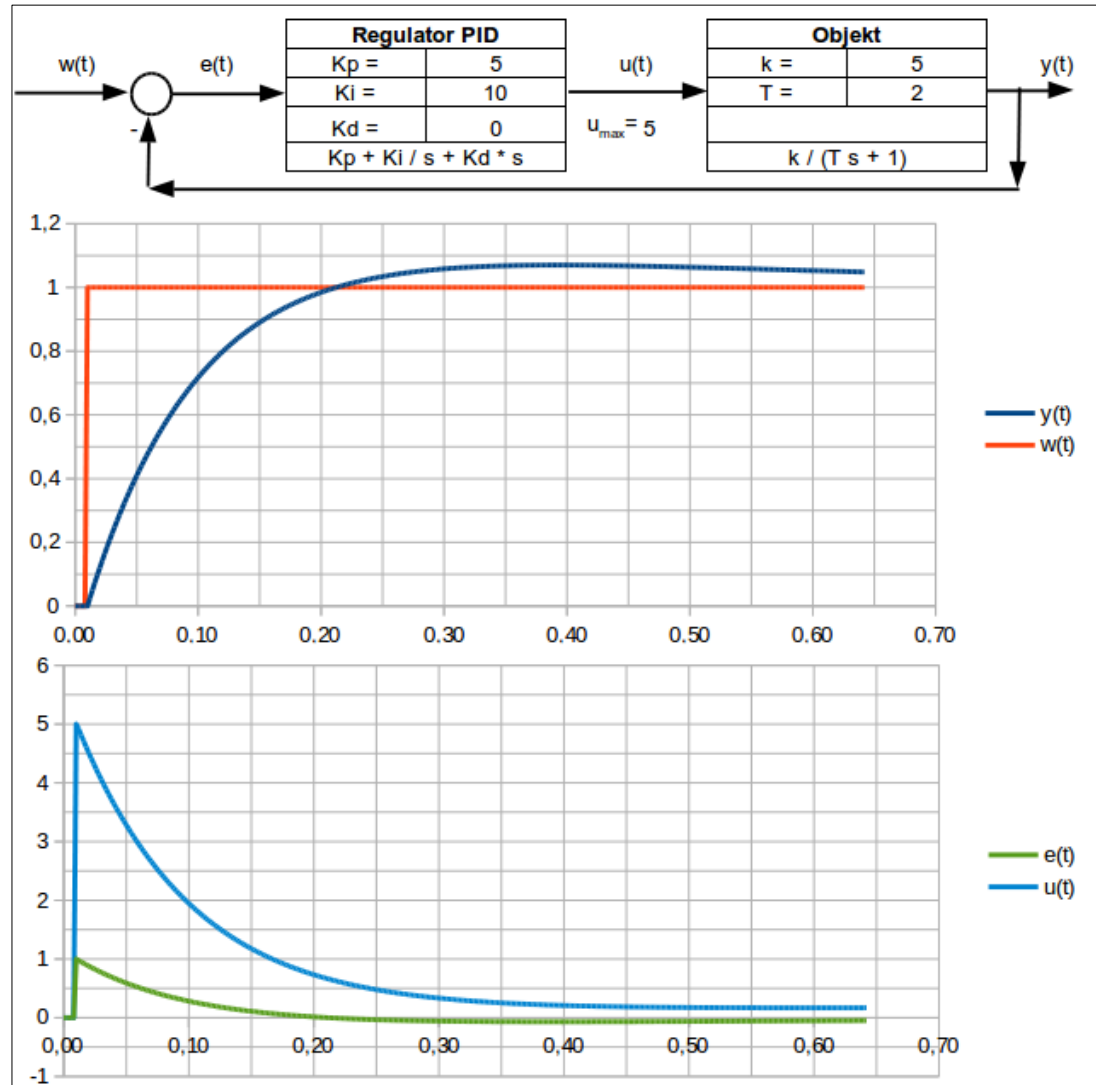
tuning methods

Analytical	With a simulation	Experimental
<p>1st step: calculation of the system's reduced transfer function</p> <p>2nd step: calculation of the system's step response</p> <p>3rd step: tuning of the K_p, K_i and K_d coefficients to obtain desired shape of step response</p>	<p>1st step: calculation of the system's reduced transfer function</p> <p>2nd step: numerical implementation of the system's reduced transfer function</p> <p>3rd step: tuning of the K_p, K_i and K_d coefficients to obtain desired shape of the system's simulated outputs</p>	<p>Manual tuning</p> <p>or</p> <p>methods:</p> <ul style="list-style-type: none">• Ziegler-Nichols<ul style="list-style-type: none">• Pessen• Cohen-Coon• Åström–Hägglund

PID CONTROLLER

interactive simulation and tuning

Download spreadsheet file from the website



PID CONTROLLER

Ziegler-Nichols tuning method (PID in standard form)

1. Disable integral and derivative terms of the controller. Set proportional gain to small value.
2. Observe a step response of the output of control loop. Go to point 3, if you observe stable and consistent oscillations. If not, increase proportional gain and repeat step 2.

PID CONTROLLER

Ziegler-Nichols tuning method (PID in standard form)

1. Disable integral and derivative terms of the controller. Set proportional gain to small value.
2. Observe a step response of the output of control loop. Go to point 3, if you observe stable and consistent oscillations. If not, increase proportional gain and repeat step 2.
3. For the ultimate gain K_u from step 2 and oscillation period T_u calculate parameters of the controller according to the table:

	k_p	T_i	T_d
Classic Ziegler-Nichols	$0.6 K_u$	$0.5 T_u$	$0.125 T_u$
Pessen	$0.7 K_u$	$0.4 T_u$	$0.15 T_u$
no overshoot	$0.2 K_u$	$0.5 T_u$	$0.333 T_u$

PID CONTROLLER

programming

```
dt = 0.1
p_error = 0.
sum = 0.
Kp = 2.
Ki = 0.5
Kd = 0.01
start:
```

```
    setpoint = ...
```

```
    measurement = ...
```

```
    error = setpoint - measurement
```

```
    sum = sum + error * dt
```

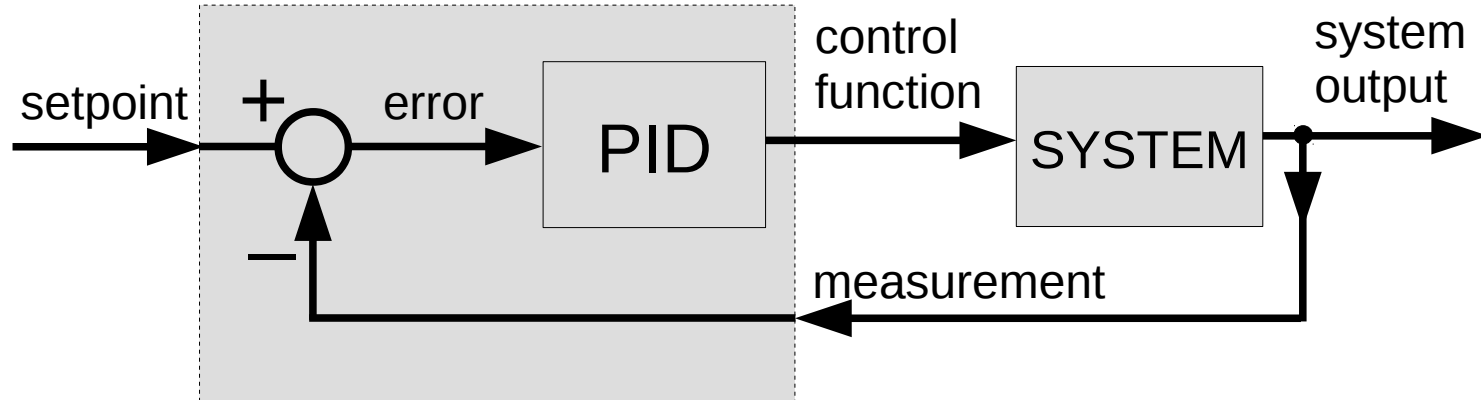
```
    derivative = (error - p_error) / dt
```

```
    output = Kp*error + Ki*sum + Kd*derivative
```

```
    p_error = error
```

```
    wait(dt)
```

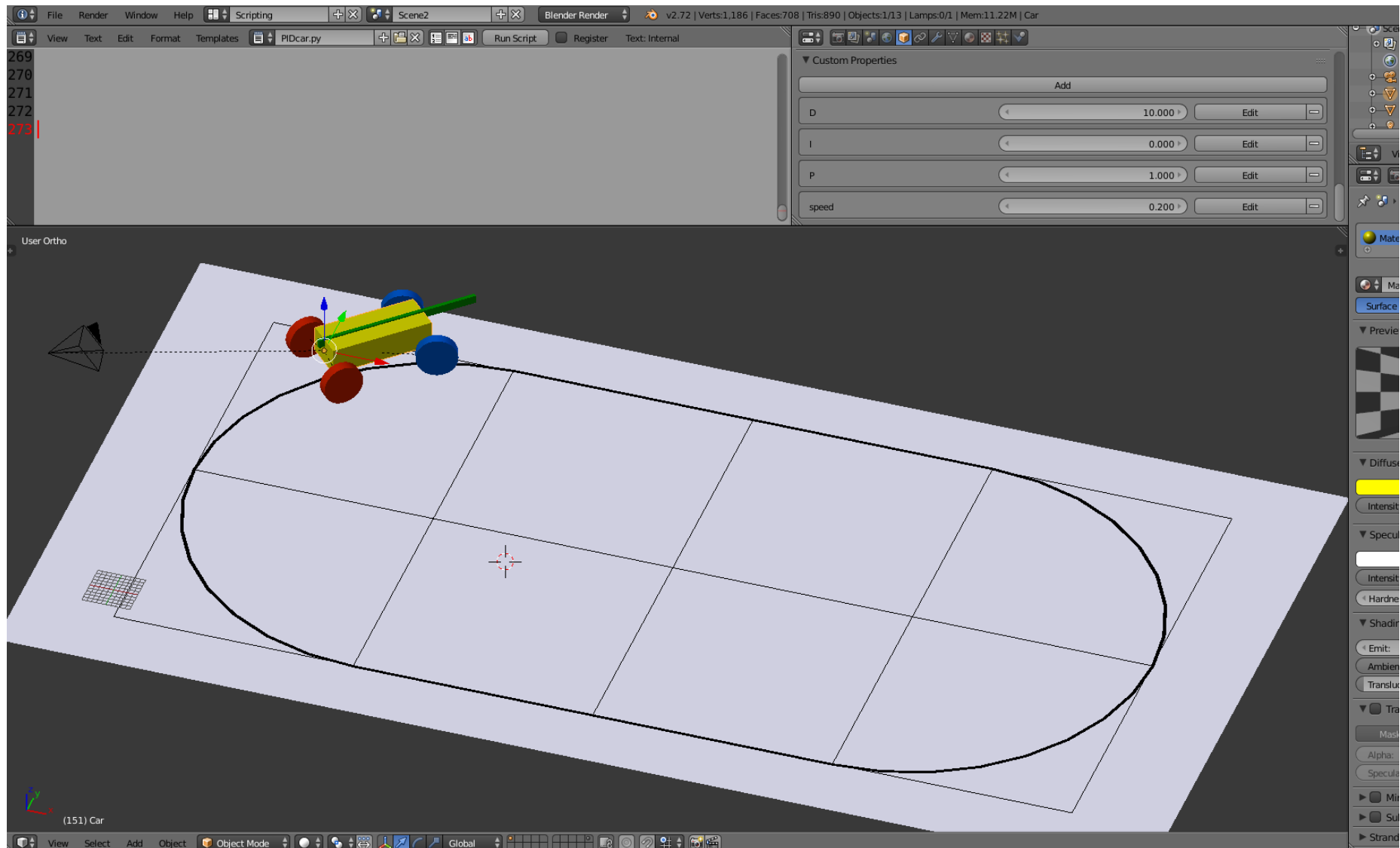
```
    goto start
```



PID CONTROLLER

interactive simulation

PID for a car position control – real-time simulation



Stability

Stability

In mathematics:

- stability theory
- numerical stability
- stability in
geometric theory

In engineering:

- BIBO stability
- stability in flight
dynamics
- ship stability

Stability

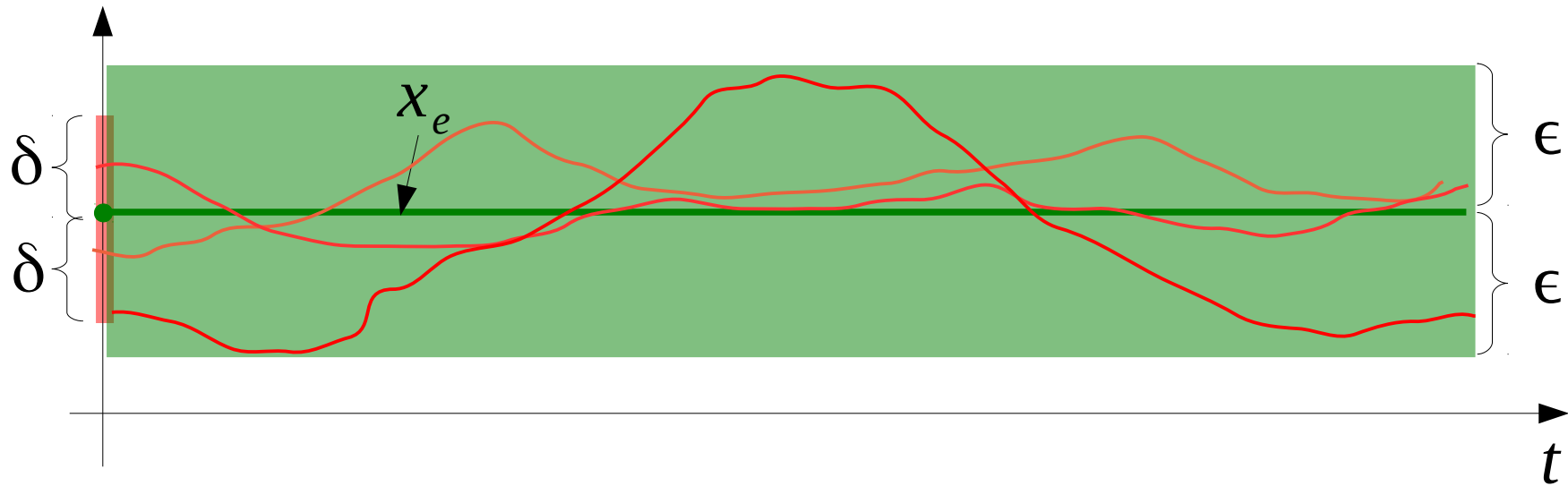
Stability theory (math) - study of the stability of differential equations' and dynamical systems' trajectories under small perturbations of initial conditions

- Lyapunov stability
- asymptotic stability
- orbital stability
- structural stability

Lyapunov stability

$$\dot{x}(t) = f(x(t))$$

$$f(x_e) = 0, \quad x_e - \text{equilibrium}$$

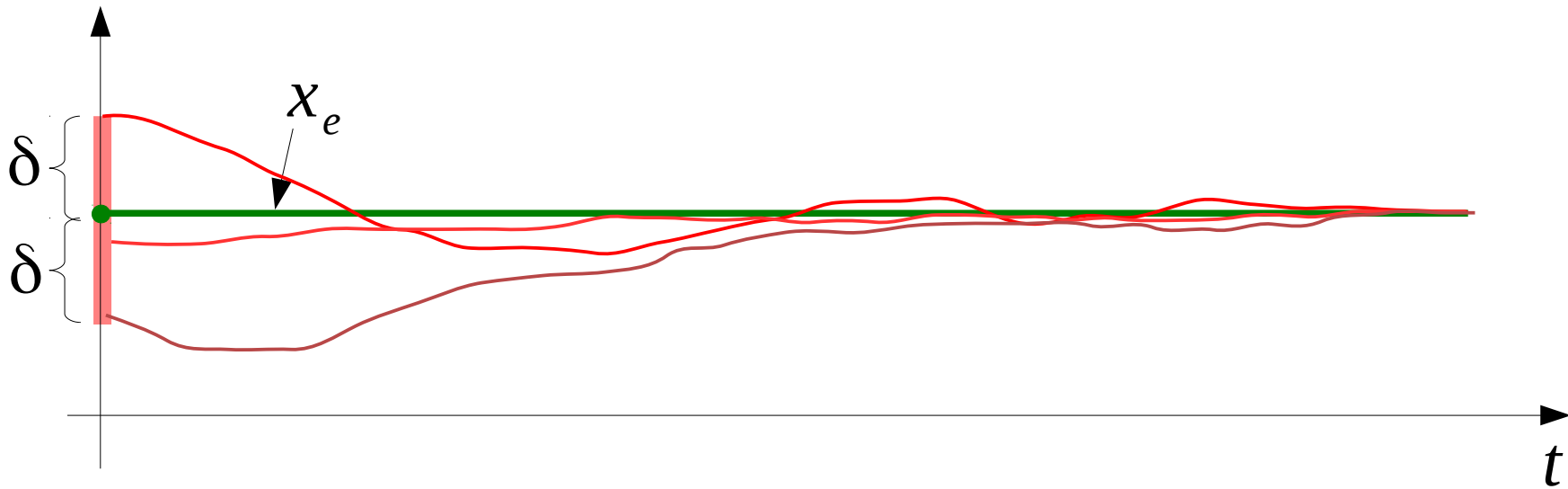


$$\forall_{t \geq 0} \quad \forall_{\epsilon > 0} \quad \exists_{\delta > 0} \quad \text{if } \|x(0) - x_e\| < \delta, \text{ then } \|x(t) - x_e\| < \epsilon$$

Asymptotic stability

$$\dot{x}(t) = f(x(t))$$

$$f(x_e) = 0, \quad x_e - \text{equilibrium}$$



$$\forall t \geq 0 \quad \forall \epsilon > 0 \quad \exists \delta > 0 \quad \text{if } \|x(0) - x_e\| < \delta, \text{ then } \lim_{t \rightarrow \infty} \|x(t) - x_e\| = 0$$

BIBO stability

Bounded Input, Bounded Output stability (in signal processing and control theory)

a LTI SISO system is called BIBO stable if its output will stay bounded for any bounded input.

$x(t)$ - input

$y(t)$ - output

$$\exists_{0 < A < \infty} \exists_{0 < B < \infty} \forall_{t \geq 0} \text{ if } |x(t)| \leq A, \text{ then } |y(t)| \leq B$$

STABILITY CRITERIA

General stability criterion

Hurwitz criterion

Nyquist stability criterion

General stability criterion

General stability criterion

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{s - p_1}$$

p_1 - pole of transfer function

$$x(t) = \delta(t)$$

$$y(t) = L^{-1}\{x(s)G(s)\} = L^{-1}\left\{1 \frac{1}{s - p_1}\right\} = e^{p_1 t}$$

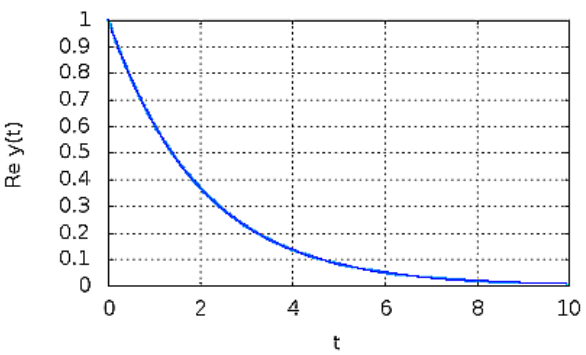
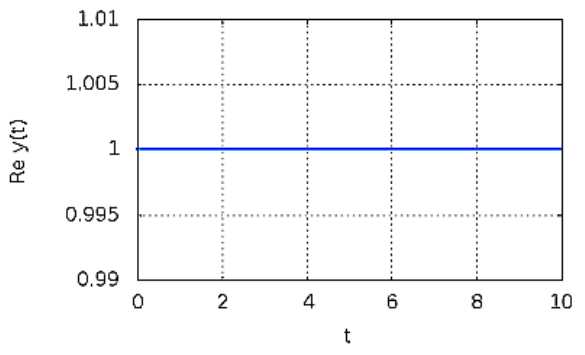
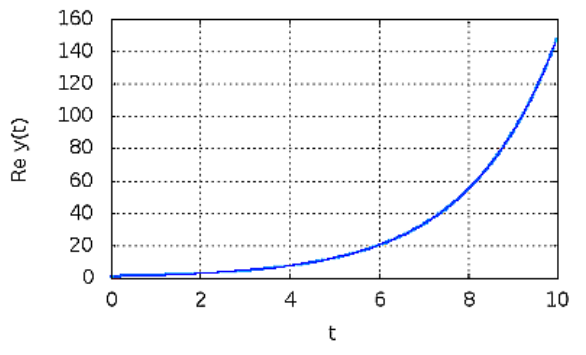
$$y(t) = e^{(a_1 + jb_1)t} = e^{a_1 t} e^{jb_1 t} = e^{a_1 t} (\cos b_1 t + j \sin b_1 t)$$

$$\text{Re } y(t) = e^{a_1 t} \cos b_1 t$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{s - p_1}$$

$$\Re y(t) = e^{a_1 t} \cos b_1 t$$

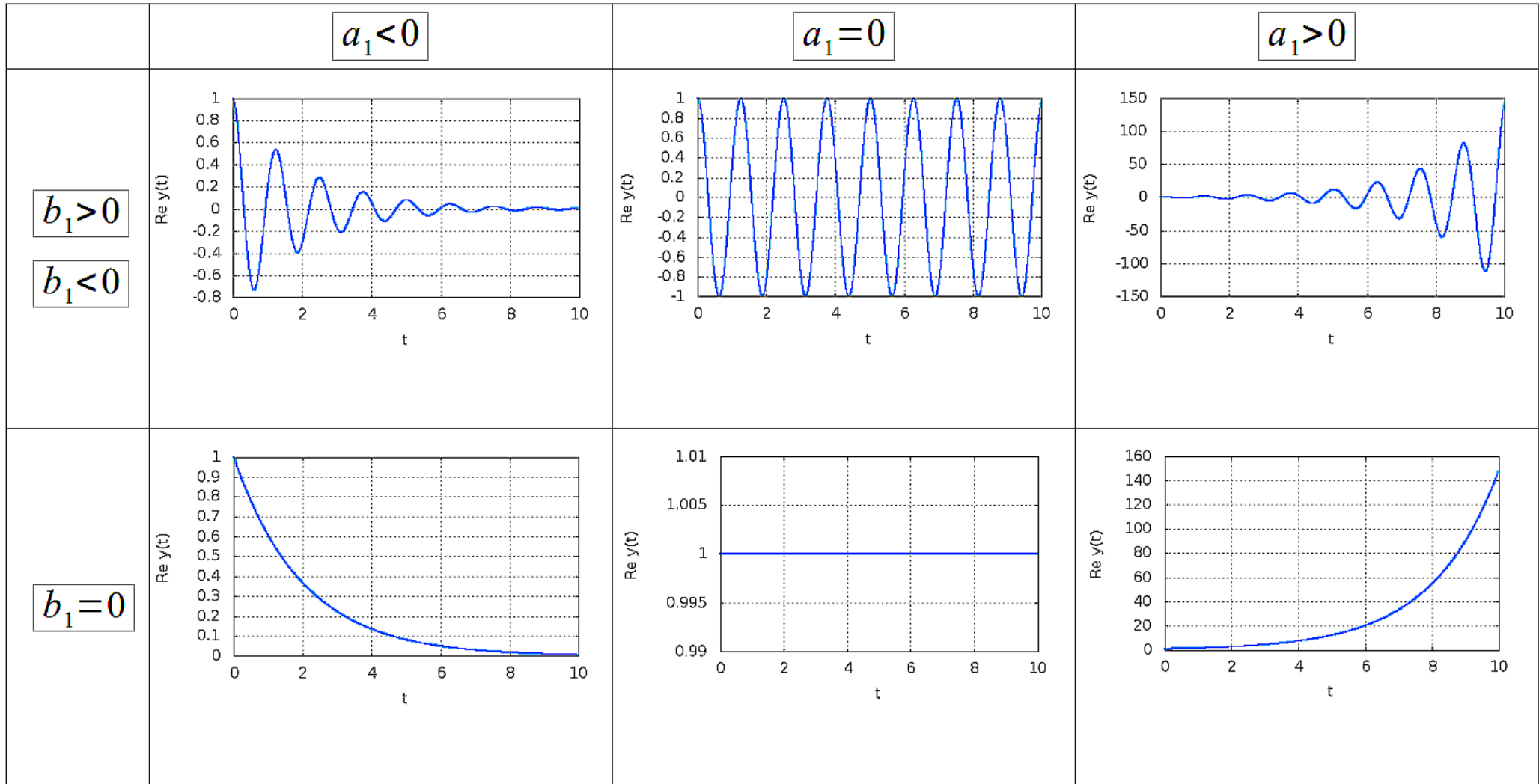
$$\Re(p_1) = a_1, \quad \Im(p_1) = b_1$$

	$a_1 < 0$	$a_1 = 0$	$a_1 > 0$
$b_1 > 0$ $b_1 < 0$			
$b_1 = 0$			

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{s - p_1}$$

$$\Re y(t) = e^{a_1 t} \cos b_1 t$$

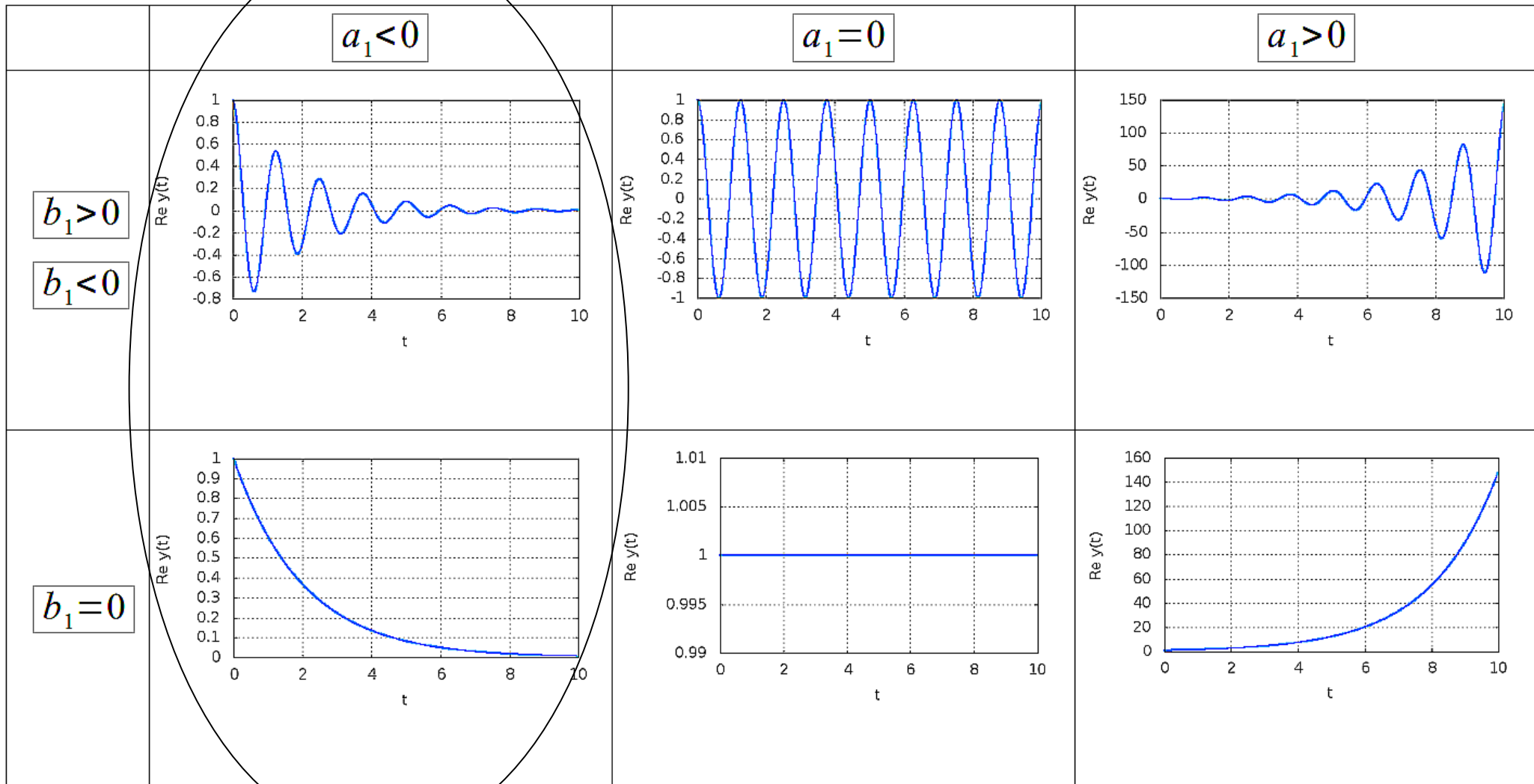
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$$\Re(p_1) = a_1, \quad \Im(p_1) = b_1$$



asymptotically stable $\Re(p_1) < 0$

General stability criterion

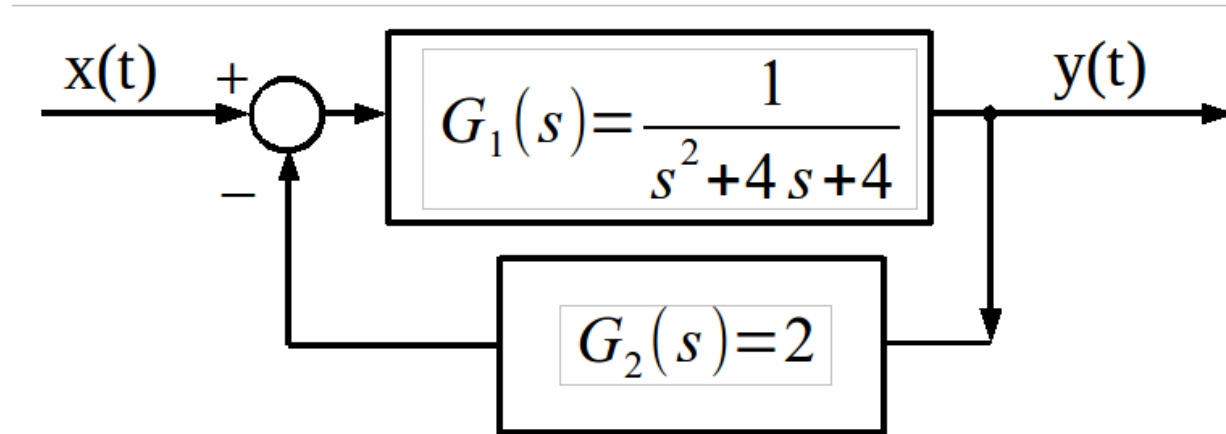
LTI SISO system is stable, if real parts of all transfer function's poles are less than zero.

$$G(s) = \frac{(s - z_1)(s - z_2) \dots (s - z_m)}{(s - p_1)(s - p_2) \dots (s - p_n)}$$

$G(s)$ is stable if: $\text{Re } p_1 < 0 \wedge \text{Re } p_2 < 0 \wedge \dots \wedge \text{Re } p_n < 0$

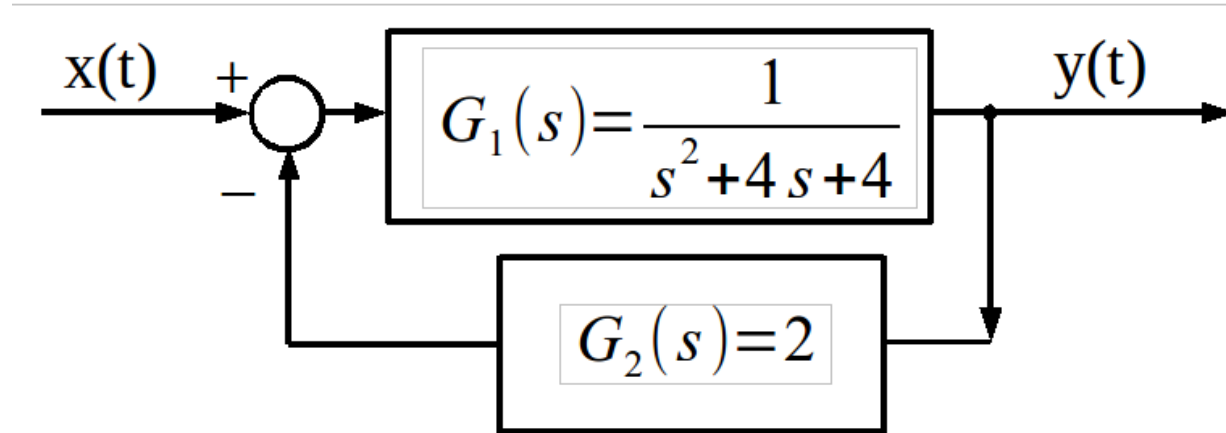
Example 1

Check stability of the presented system using the general stability criterion.



Example 1

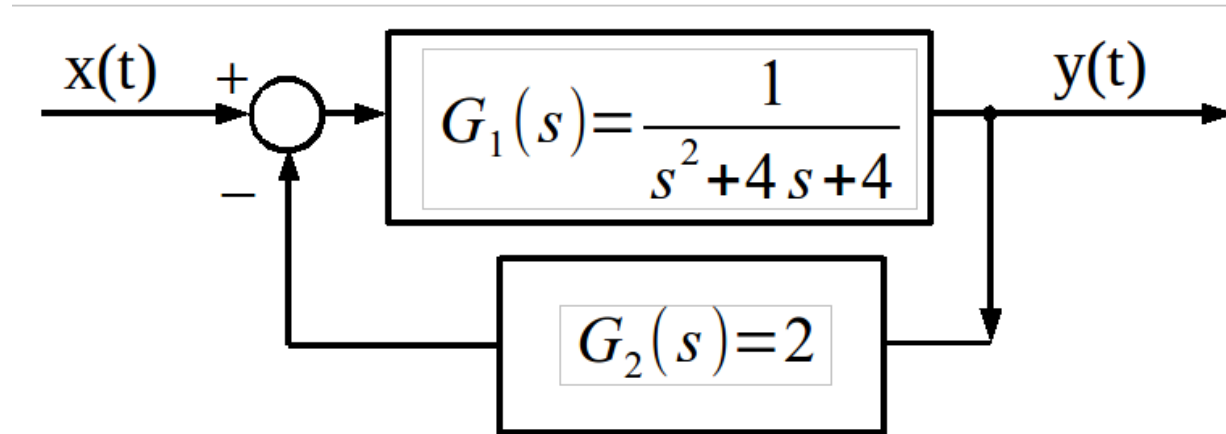
Check stability of the presented system using general stability criterion



$$G(s) = \frac{Y(s)}{X(s)} = \frac{G_1}{1 + G_1 G_2} = \frac{1}{s^2 + 4s + 6} = \frac{1}{(s - s_1)(s - s_2)}$$

Example 1

Check stability of the presented system using general stability criterion

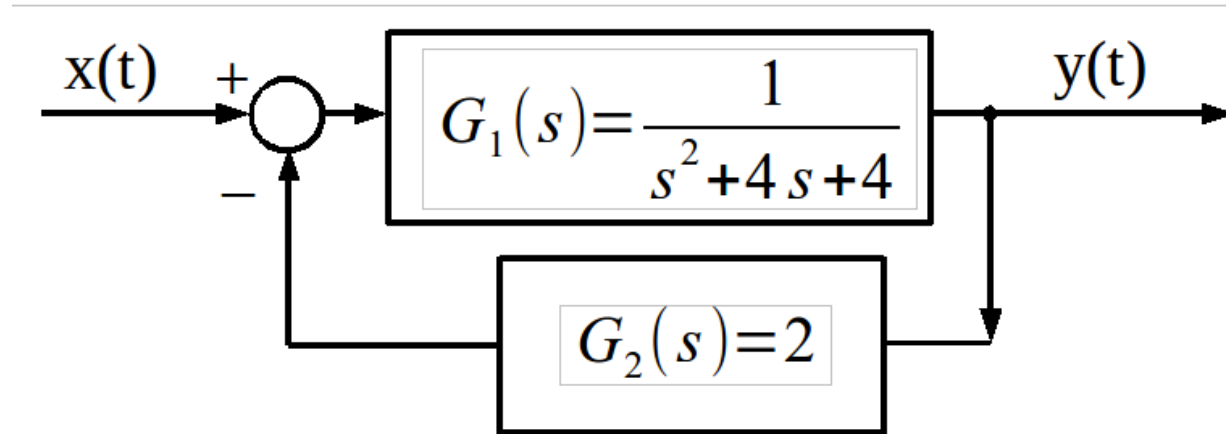


$$G(s) = \frac{Y(s)}{X(s)} = \frac{G_1}{1 + G_1 G_2} = \frac{1}{s^2 + 4s + 6} = \frac{1}{(s - s_1)(s - s_2)}$$

$$s_1 = -2 - 2\sqrt{2}j, \quad s_2 = -2 + \sqrt{2}j$$

Example 1

Check stability of the presented system using general stability criterion



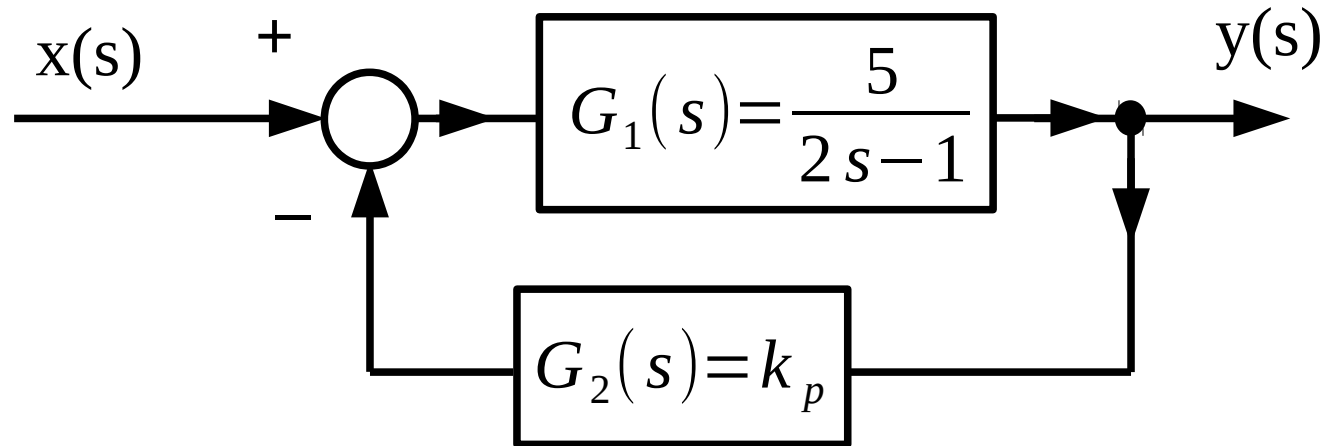
$$G(s) = \frac{Y(s)}{X(s)} = \frac{G_1}{1 + G_1 G_2} = \frac{1}{s^2 + 4s + 6} = \frac{1}{(s - s_1)(s - s_2)}$$

$$s_1 = -2 - 2\sqrt{2}j, \quad s_2 = -2 + \sqrt{2}j$$

$\Re(s_1) < 0 \wedge \Re(s_2) < 0 \Rightarrow$ system is stable from general stability criterion

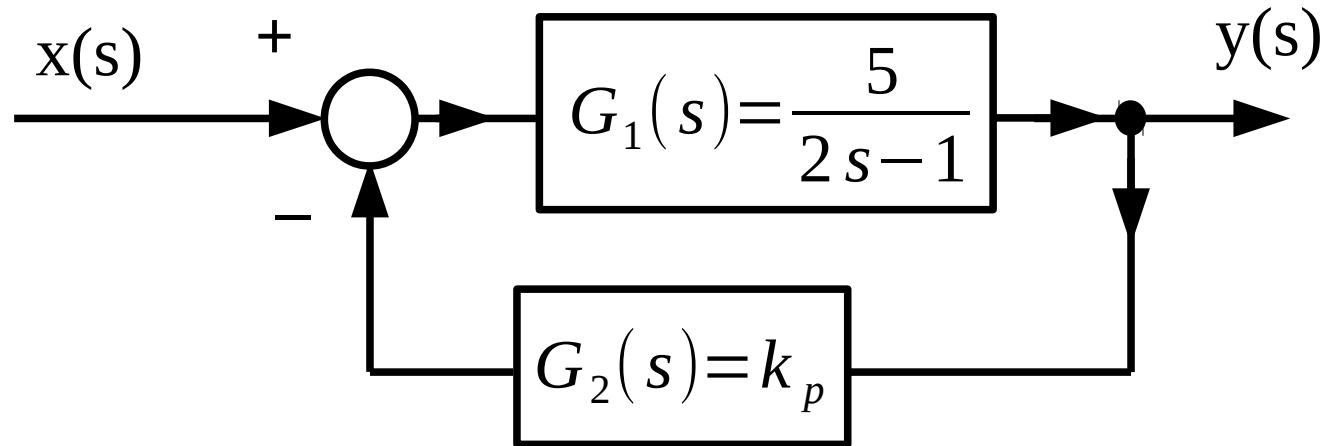
Example 2

Choose values of K_p to obtain system stability using general stability criterion



Example 2

Choose values of K_p to obtain system stability
using general stability criterion

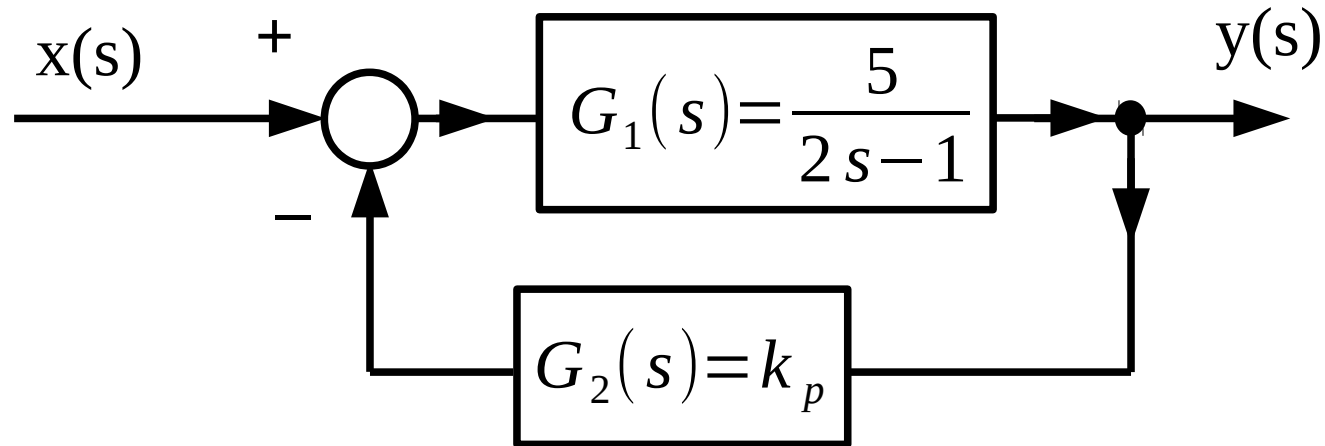


$$G(s) = \frac{Y(s)}{X(s)} = \frac{G_1}{1 + G_1 G_2} = \frac{5}{2} \frac{1}{s - \left(\frac{1}{2} - \frac{5}{2} k_p \right)}$$

$$p_1 = \left(\frac{1}{2} - \frac{5}{2} k_p \right)$$

Example 2

Choose values of K_p to obtain system stability
using general stability criterion



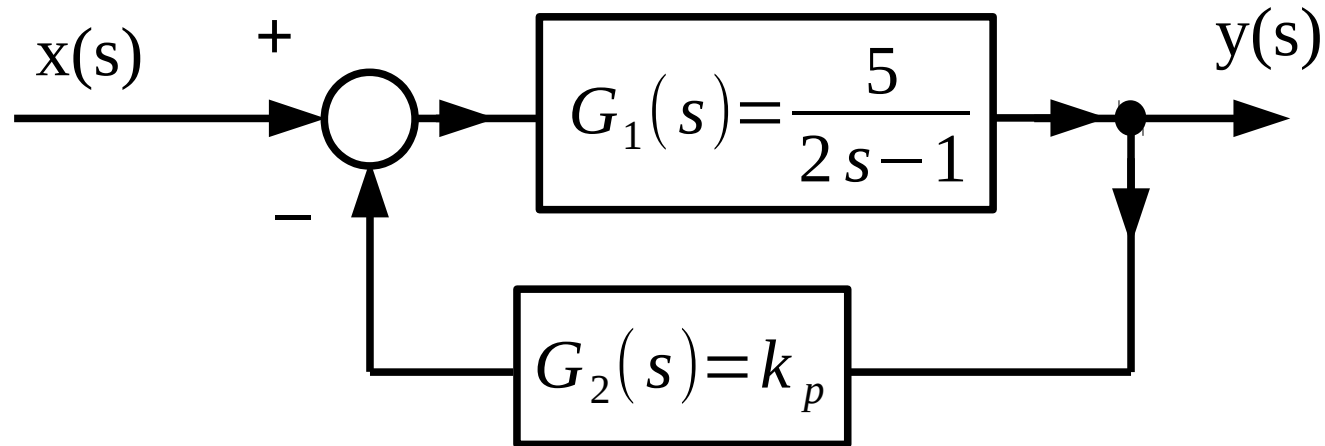
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$$p_1 = \left(\frac{1}{2} - \frac{5}{2} k_p \right)$$

System is stable, if $\Re(p_1) < 0$

Example 2

Choose values of K_p to obtain system stability
using general stability criterion



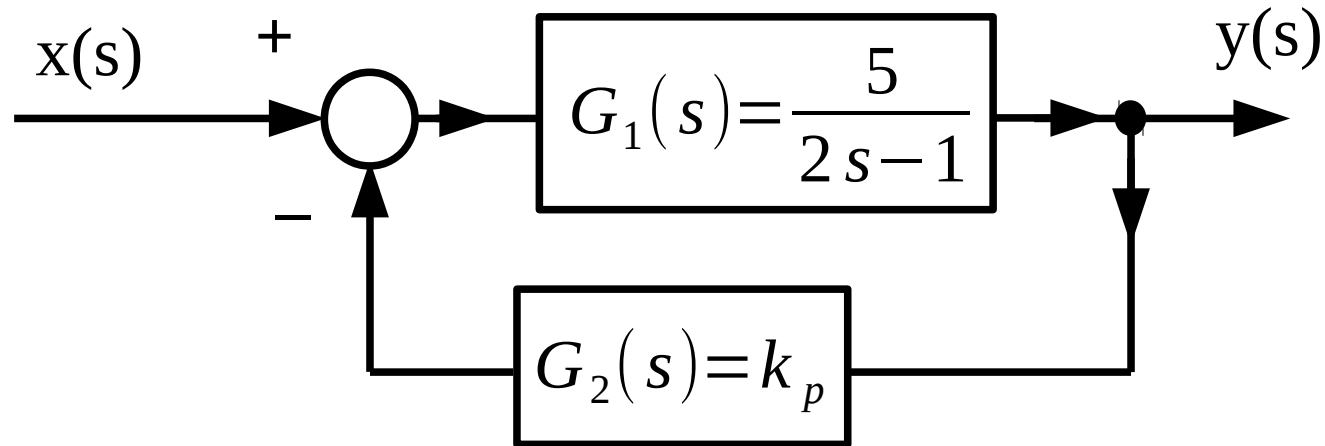
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$$p_1 = \left(\frac{1}{2} - \frac{5}{2} k_p \right)$$

System is stable, if $\Re(p_1) < 0 \Rightarrow k_p > \frac{1}{5}$

Example 2

Choose values of K_p to obtain system stability
using general stability criterion

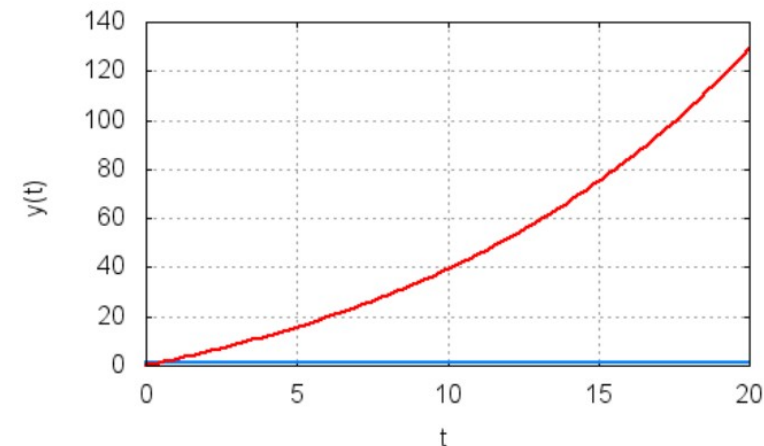


$$G(s) = \frac{Y(s)}{X(s)} = \frac{G_1}{1 + G_1 G_2} = \frac{5}{2} \frac{1}{s - \left(\frac{1}{2} - \frac{5}{2} k_p \right)}$$

$$p_1 = \left(\frac{1}{2} - \frac{5}{2} k_p \right)$$

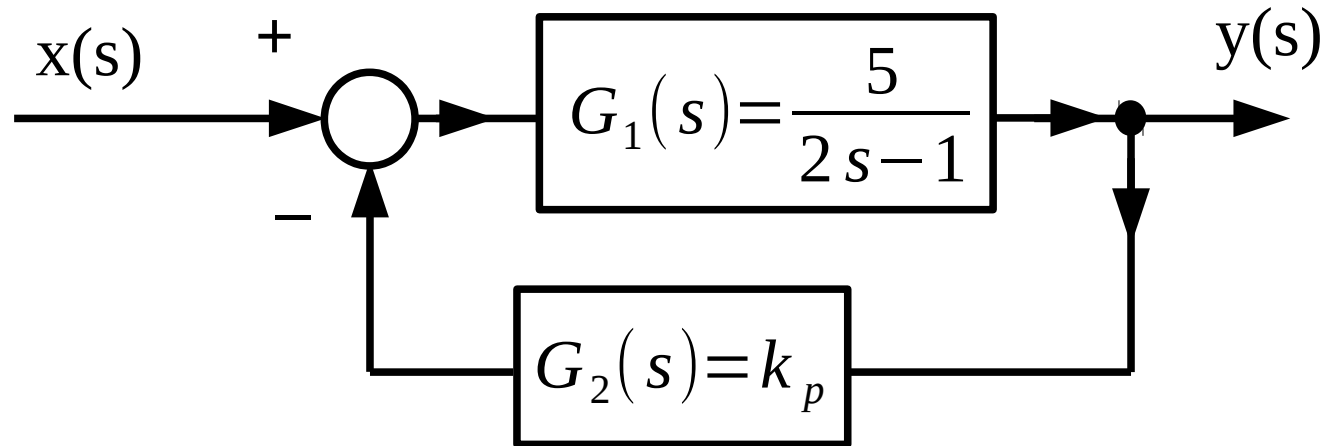
System is stable, if $\Re(p_1) < 0 \Rightarrow k_p > \frac{1}{5}$

$k_p = \frac{1}{6}$ (unstable)



Example 2

Choose values of K_p to obtain system stability
using general stability criterion

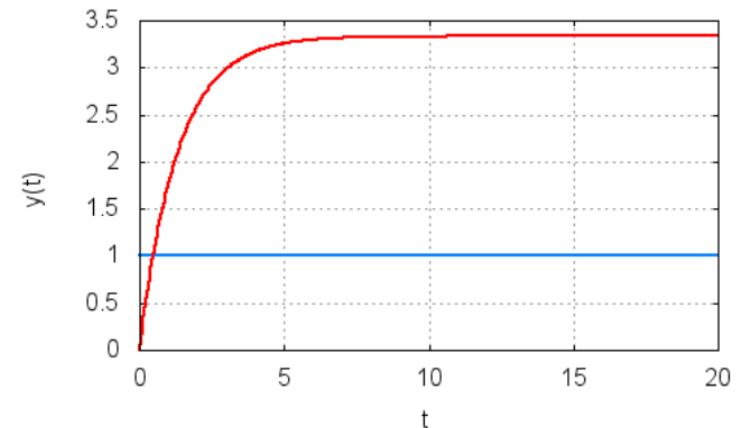


$$G(s) = \frac{Y(s)}{X(s)} = \frac{G_1}{1 + G_1 G_2} = \frac{5}{2} \frac{1}{s - \left(\frac{1}{2} - \frac{5}{2} k_p \right)}$$

$$p_1 = \left(\frac{1}{2} - \frac{5}{2} k_p \right)$$

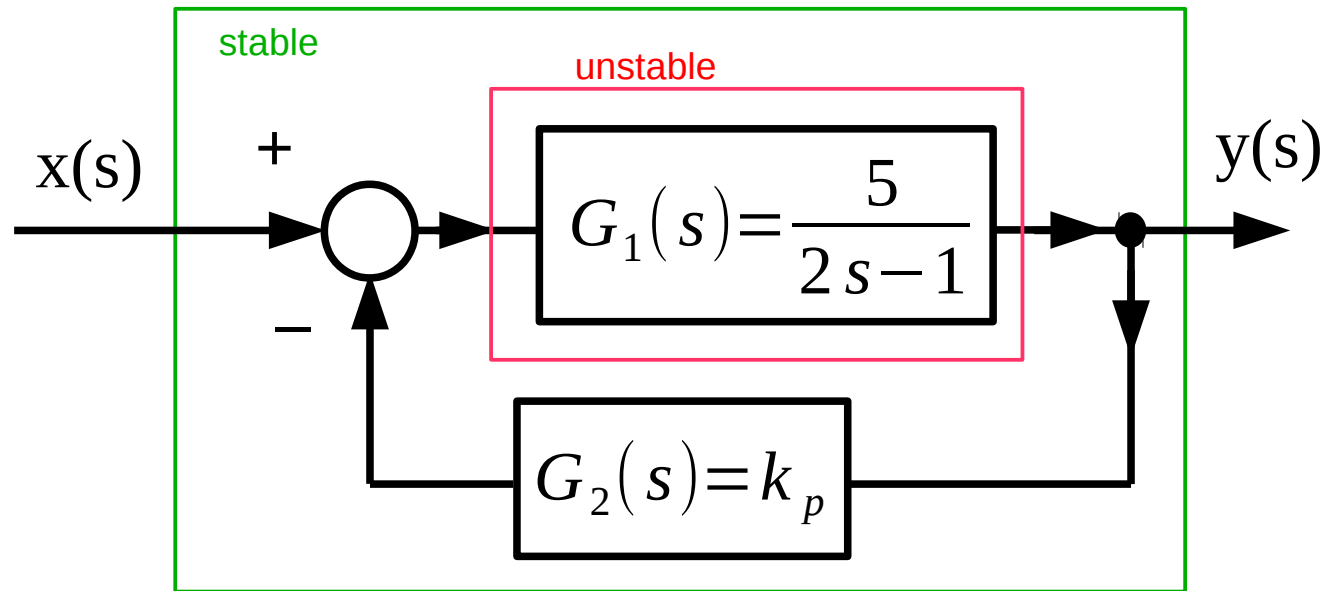
System is stable, if $\Re(p_1) < 0 \Rightarrow k_p > \frac{1}{5}$

$k_p = \frac{1}{2}$ (stable)



Example 2

Choose values of K_p to obtain system stability
using general stability criterion

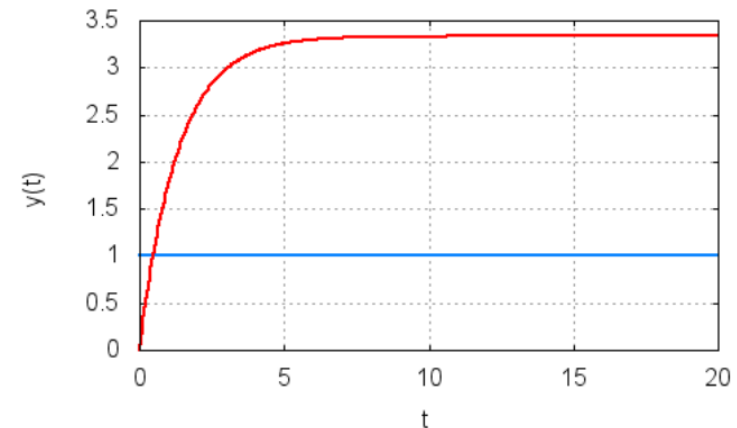


$$G(s) = \frac{Y(s)}{X(s)} = \frac{G_1}{1 + G_1 G_2} = \frac{5}{2} \frac{1}{s - \left(\frac{1}{2} - \frac{5}{2} k_p \right)}$$

$$p_1 = \left(\frac{1}{2} - \frac{5}{2} k_p \right)$$

System is stable, if $\Re(p_1) < 0 \Rightarrow k_p > \frac{1}{5}$

$k_p = \frac{1}{2}$ (stable)



List of important exam topics

- Mechanism mobility calculation
 - Procedure analytical method
 - Equation of machine motion
 - Nonuniformity and flyweel
 - Transfer function of LTI SISO
- Step response & Bode Plot of a given system
 - Block diagram algebra
 - PID controller & tuning
 - General stability criterion
 - Hurwitz criterion
 - particular Nyquist criterion